Lattice QED$_3$ with staggered and Wilson fermions: finite density and topology

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based on

arXiv:2509.20558 (PRD 2026),
with E. O. Rosanowski, A. Crippa, L. Funcke, P. V. Itaborai, K. Jansen
&
with S. Bharadwaj, E. O. Rosanowski, A. Di Tucci, L. Funcke, K. Jansen and D. Luo,
arXiv: 2504.21828, arXiv: 2603.05616

QIS-HENP @ C3NT Wuhan, China, June 16, 2026

Brief outline

- Why study QED_3: Two Perspectives
- Hamiltonian lattice formulation of QED_3
- Part I: Chern number topology with Wilson fermions
- Part II: Density-induced level crossing with Staggered fermions on Quantum Hardware
- Summary & outlook

WHY QUANTUM-SIMULATE QED_3 AT FINITE DENSITY ?

Particle physics perspective

- An important step toward simulating QCD on a QC
- QED_3 retains key features of QCD, like confinement
- Hamiltonian formulation is free of the numerical sign problem

[E. O. Rosanowski, A. Crippa, L. Funcke, P. V. Itaborai, K. Jansen, **S.S.,** PRD (2026)]

[J. Bender et.al., PRR (2023)],

[G. Gyawali et.al., arXiv:2410.06557],

[M. Meth, et.al., Nat. Phys. (2025)],

[A. Crippa, S. Romiti, et. al, Comm. Phys. (2025)],

[J. Cobos, et. al., arXiv:2507.08088],

[P. Majcen, et.al., arXiv:2602.04948] ...

Condensed matter perspective

- QED_3 has given insights into the study of quantum spin liquids

[F. F. Assaad et.al., Nature (2010)],

[Yin-Chen He et. al. PRX (2017)],

[M. Hermele et. al., PRB (2024)]

- Important for realising topological phases with non-trivial Chern numbers

[S. Sen., PRD 2020],

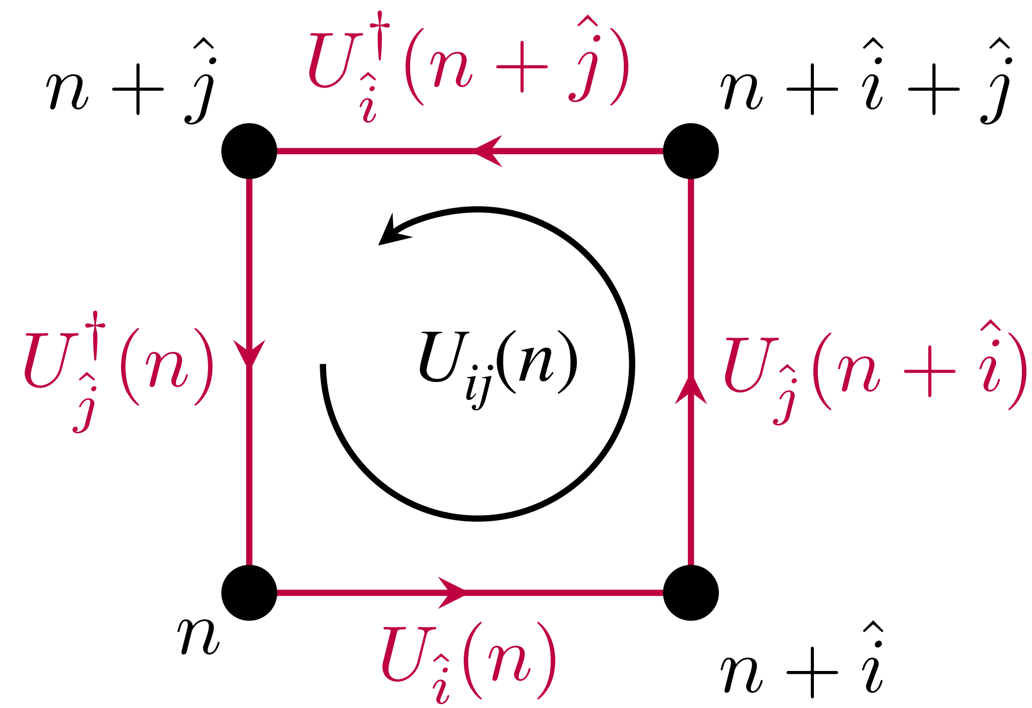
[S. Bharadwaj, E. O. Rosanowski, **S.S.**, A. Di Tucci, L. Funcke, K. Jansen and D. Luo, arXiv: 2504.21828, arXiv:2603.05616]

Hamiltonian lattice formulation of QED$_3$

$$\hat{H} = \underbrace{\hat{H}_m}_{\text{mass}} + \underbrace{\hat{H}_{\text{kin}}}_{\text{hopping}} + \underbrace{\hat{H}_\mu}_{\text{density}} + \underbrace{\hat{H}_E}_{\text{E-fields}} + \underbrace{\hat{H}_B}_{\text{plaquettes}}$$

Gauge fields

$\cos B \propto$ Re $\Big\{$

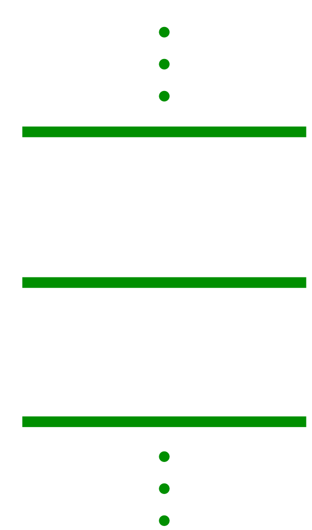


$\Big\}$

$$E\,|\,k\rangle = k\,|\,k\rangle$$

$$[E_i(n), U_j(n')] = \delta_{ij}\delta_{nn'}U_i(n)$$

$$[E_i(n), U_j^\dagger(n')] = -\,\delta_{ij}\delta_{nn'}U_i^\dagger(n)$$



HAMILTONIAN LATTICE FORMULATION OF QED_3

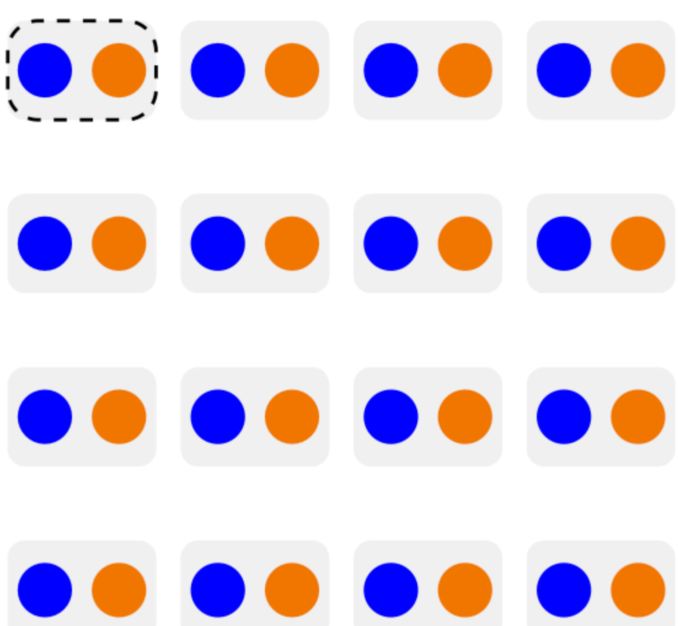
$$\hat{H} = \underbrace{\hat{H}_m}_{\text{mass}} + \underbrace{\hat{H}_{\text{kin}}}_{\text{hopping}} + \underbrace{\hat{H}_\mu}_{\text{density}} + \underbrace{\hat{H}_E}_{\text{E-fields}} + \underbrace{\hat{H}_B}_{\text{plaquettes}}$$

Fermions

Wilson discretisation

[K. G. Wilson (1974)]

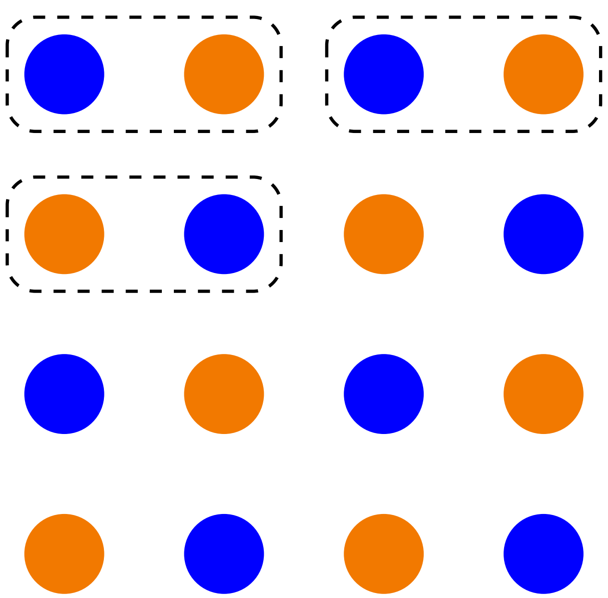
- No doublers
- No chiral symmetry
- Expensive (more degrees of freedom per lattice site)



Staggered discretisation

[J. Kogut, L. Susskind (1974)]

- Some doublers
- Some chiral symmetry
- Cheaper to simulate (less degrees of freedom per lattice site)



Part I

WHAT DOES IT TAKE TO STUDY CHERN NUMBER TOPOLOGY ?

Wilson Fermion Discretisation:

S. Bharadwaj, E. O. Rosanowski, **S.S.**, A. Di Tucci, L. Funcke, K. Jansen and D. Luo, arXiv: 2504.21828, arXiv:2603.05616

Part I : Chern Topology via Wilson fermions

The old connection from field theory

- **Callan & Harvey (1985):** established the (2+1)D bulk Chern–Simons current and the (1+1)D edge-mode anomaly as the two faces of one massive fermion, locked together by anomaly inflow
- **Kaplan (1992):** Saw this as a way to sidestep Nielsen-Ninomiya theorem as a way to simulate: (i) doubler-free, (ii) chiral symmetry on the lattice via domain wall fermions
- **Goltermann, Jansen & Kaplan (1993):** Computed the induced-CS current on the lattice with Wilson fermion in 2+1 D and mass coupling to domain wall - showed transitions across values of Wilson mass

Renewing connections to condensed matter physics

- **Sen (2020):** (i) tuned the induced CS current via lattice anisotropy, accessing new topological phases
- (ii) showed the analogy between Chern-insulator physics and the lattice domain wall CS-level changes - attributed difference to discrete time direction

Part I : Chern Topology via Wilson fermions

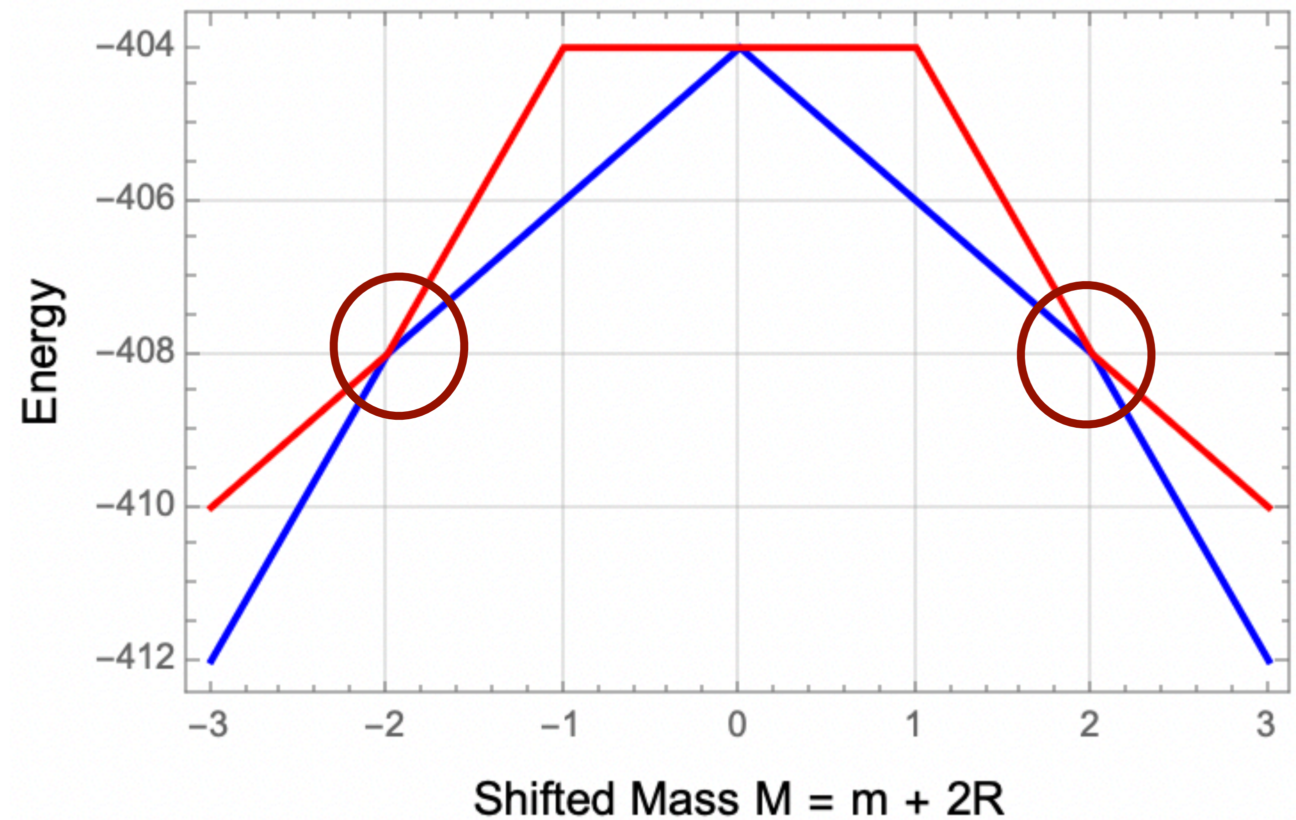
This work

- Start with the continuous-time Hamiltonian 2+1 D Wilson + dynamical $\mathbb{Z}_2$ gauge fields on a spatial lattice
- **Directly access the gauge-invariant GS** - compute the **Chern number** two ways (analytic momentum-space + many-body ED) to characterise the **topological phases**
- Based on arguments of time-reversal-invariance due to doublers - **exclude Staggered fermions** to see this effect.
- Study the **phase digram** with multiple flavours and **finite density**

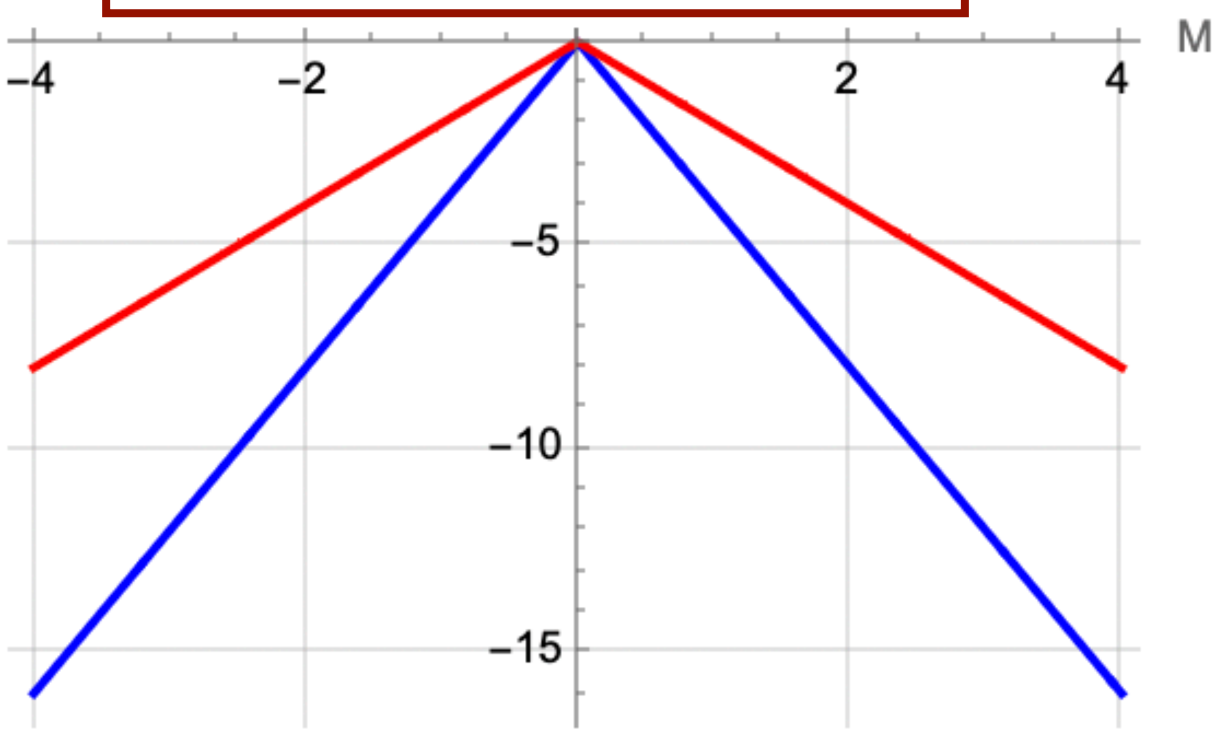
Fermion Discretization	**Time-reversal broken?**
Continuum $N_f = 1$ Dirac	✓ for any m
Continuum $N_f = 2$ Dirac	✗ iff $m_1 = -m_2$
Lattice $N_f = 1$ Staggered	✗ for any m
Lattice $N_f = 1$ Wilson	✓ for any (m, R)
Lattice $N_f = 2$ Wilson	✓ unless $(m_1, R_1) = -(m_2, R_2)$

Level crossings across discretisations

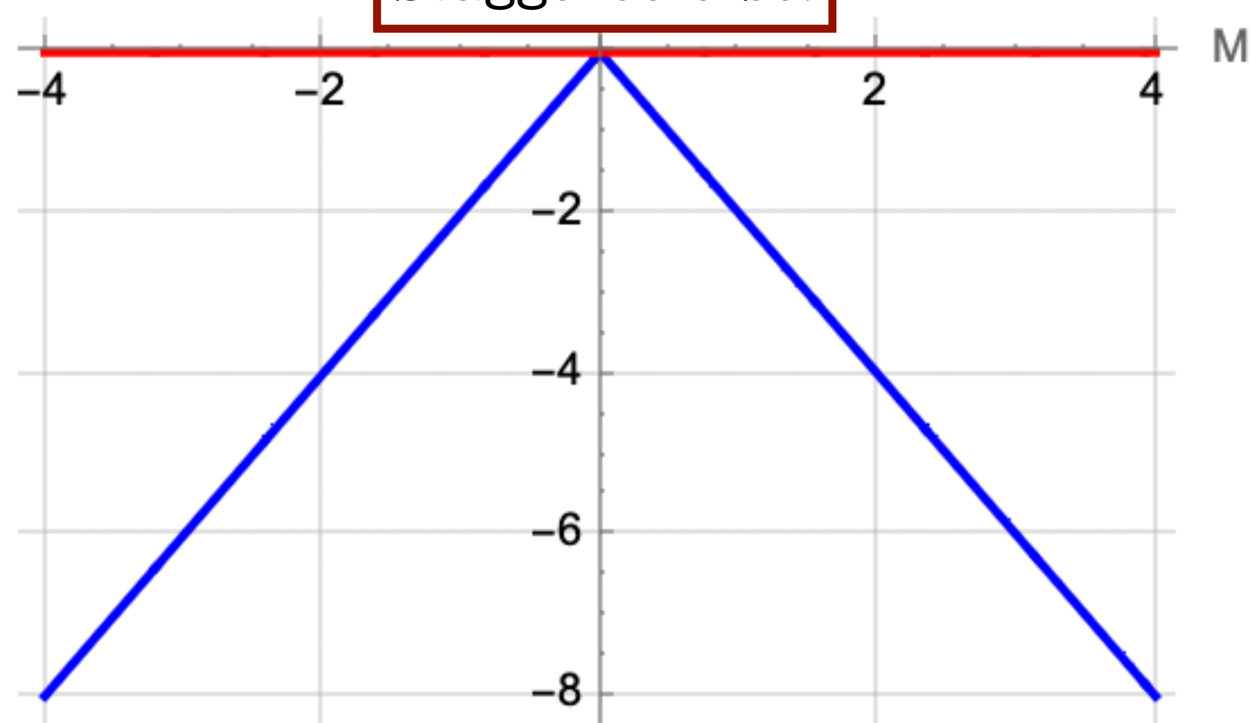
GS – ES_1 vs Wilson mass



Naive disc. (Wilson with R=0)



Staggered disc.

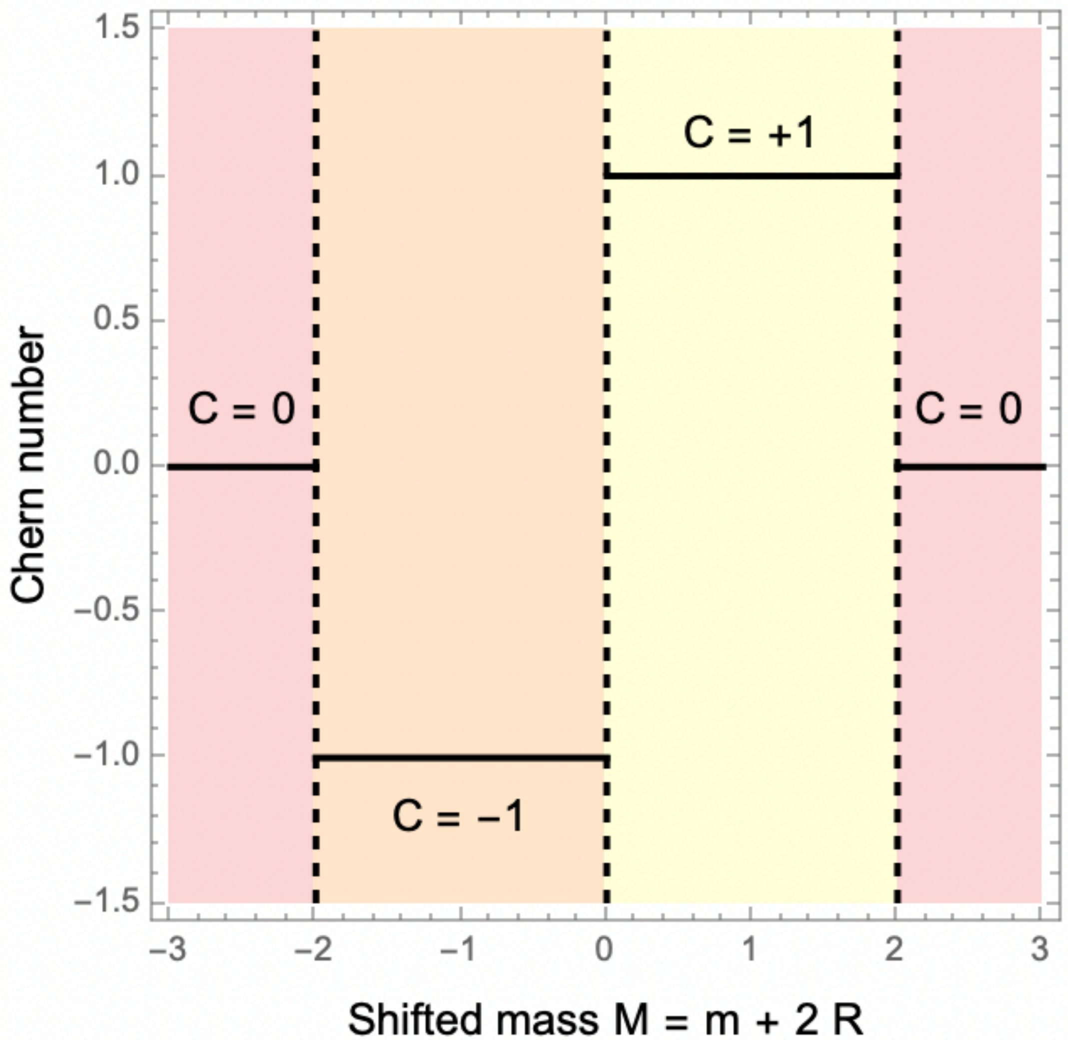


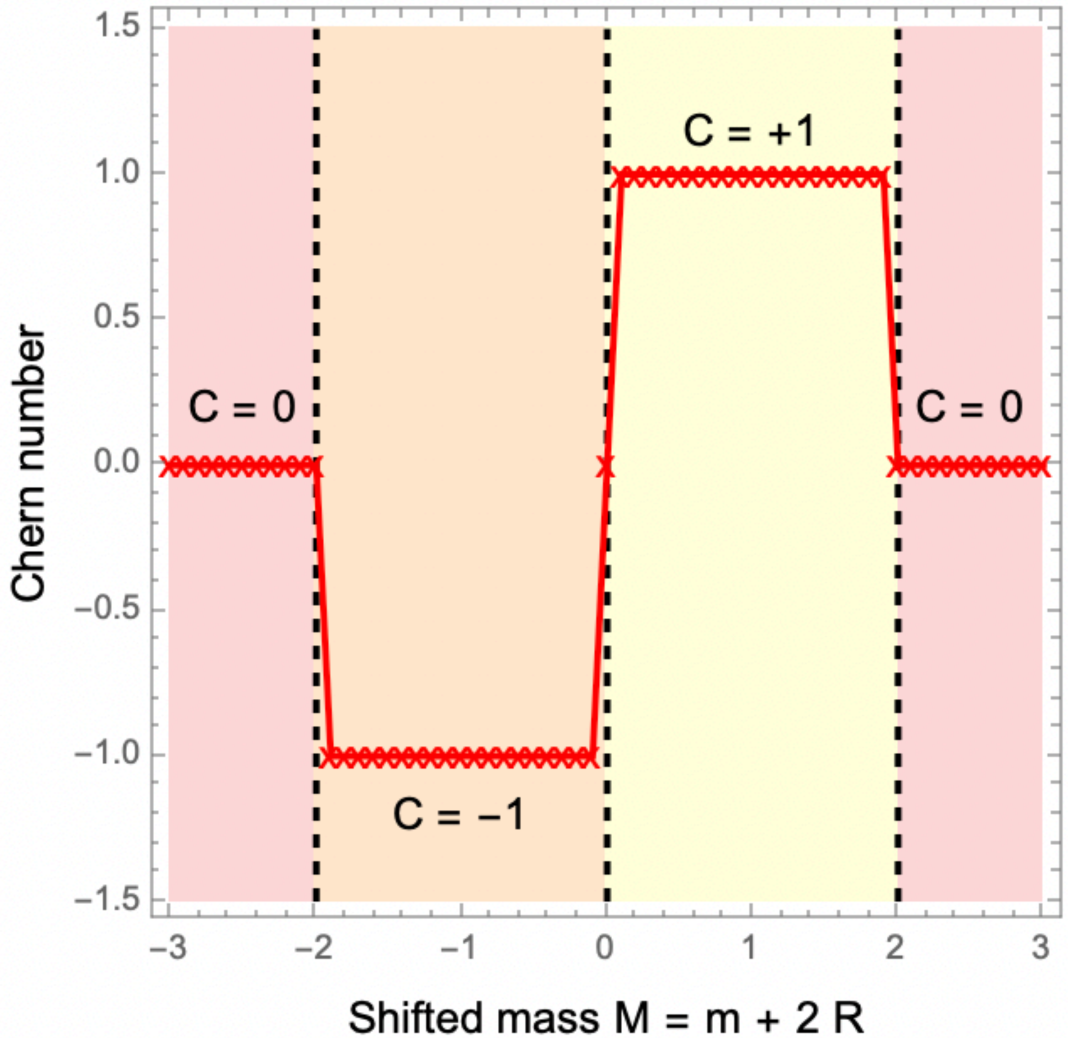
Only display trivial insulator phases across M=+/-2

Level crossings —> rich phase diagram

- Topological phases of $N_f = 1$ QED_3 Wilson fermions

- Consistent picture with S. Sen (PRD 2020) & L. Mazza et.al.,(NJP 2012)





Analytic Chern number computation

ED results many-body Chern number with dynamical $\mathbb{Z}_2$ on a 2×2 lattice

MULTIPLE FLAVOURS AND FINITE DENSITY

- Topological phases of $N_f = 2$ QED_3 Wilson fermions at finite density

- Consider two decompositions of the massive flavours: Single and Triplet

$$H_f = \frac{1}{2}\sum_r \left[\psi_a(r)^\dagger\,\gamma^0\left(i\gamma^k + R\right)U_k(r)\,\psi_a(r+\hat{k}) + \text{h.c.}\right]$$

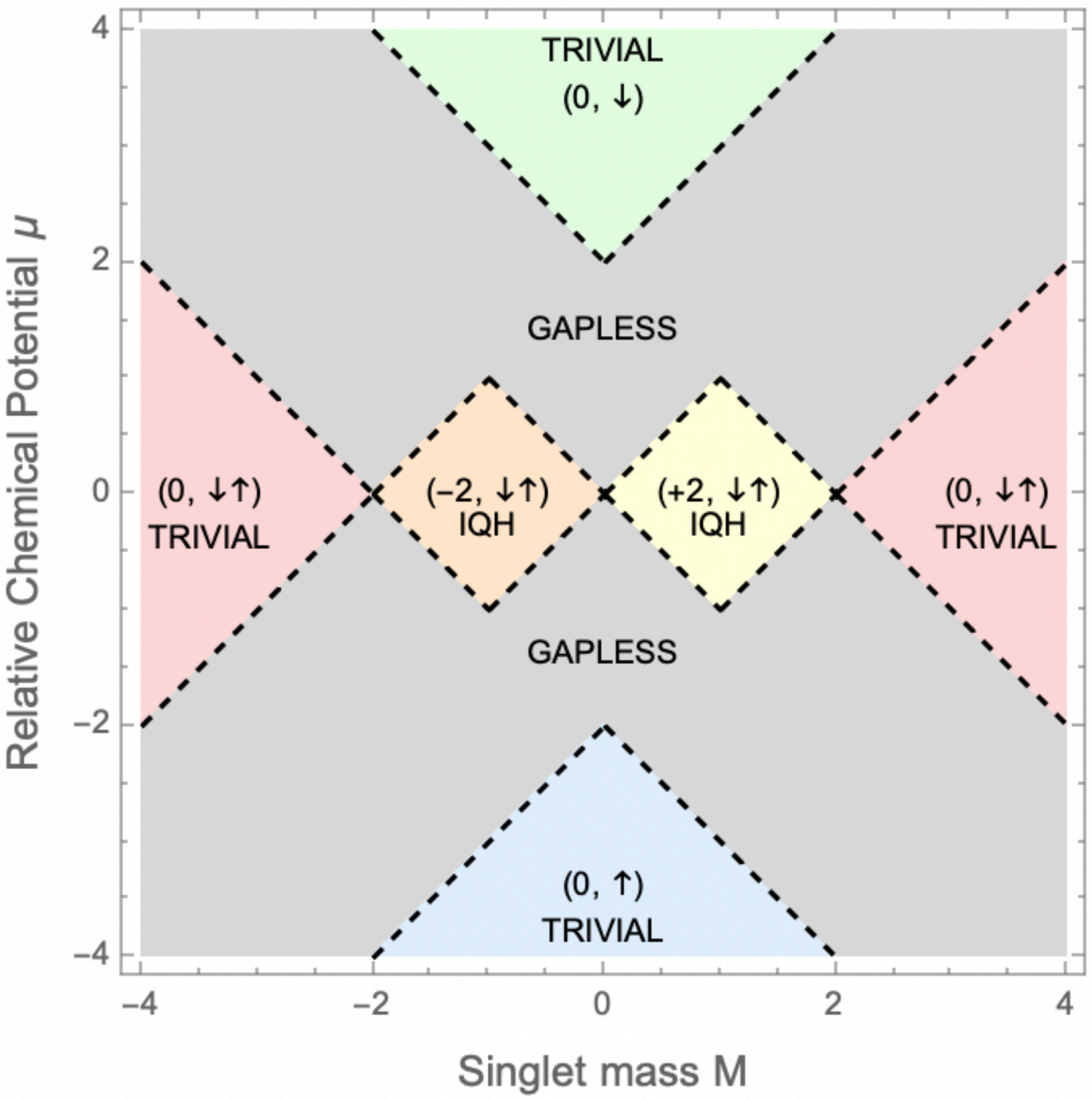
$$+ \sum_r M_a\,\psi_a(r)^\dagger\gamma^0\psi_a(r) + \mu\left(\psi_1^\dagger\psi_1 - \psi_2^\dagger\psi_2\right),$$

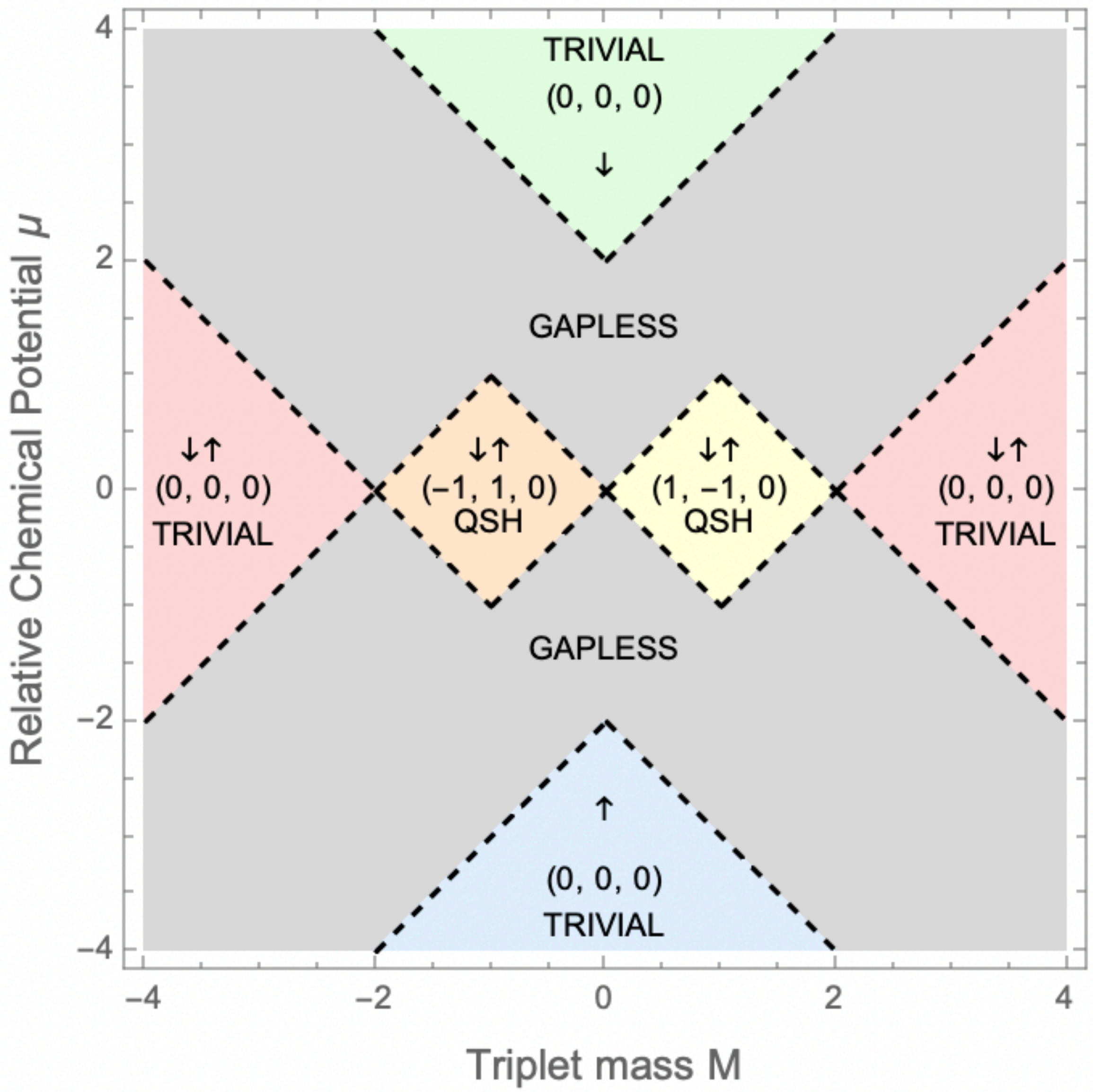
- For two flavours: $M_1 = M_2 = M$ (singlet) & $M_1 = -M_2 = M$ (triplet)

- Chemical potential couples to $\Delta N = N_1 - N_2$

- Analytic computation of the Chern-number using momentum space Hamiltonian

PROPOSED PHASE DIAGRAM AT FINITE DENSITY





Integer Quantum Hall (IQH) phases for singlet case with $M_1 = M_2 = M$

Quantum Spin Hall (QSH) phases for triplet case with $M_1 = -M_2 = M$

Part II

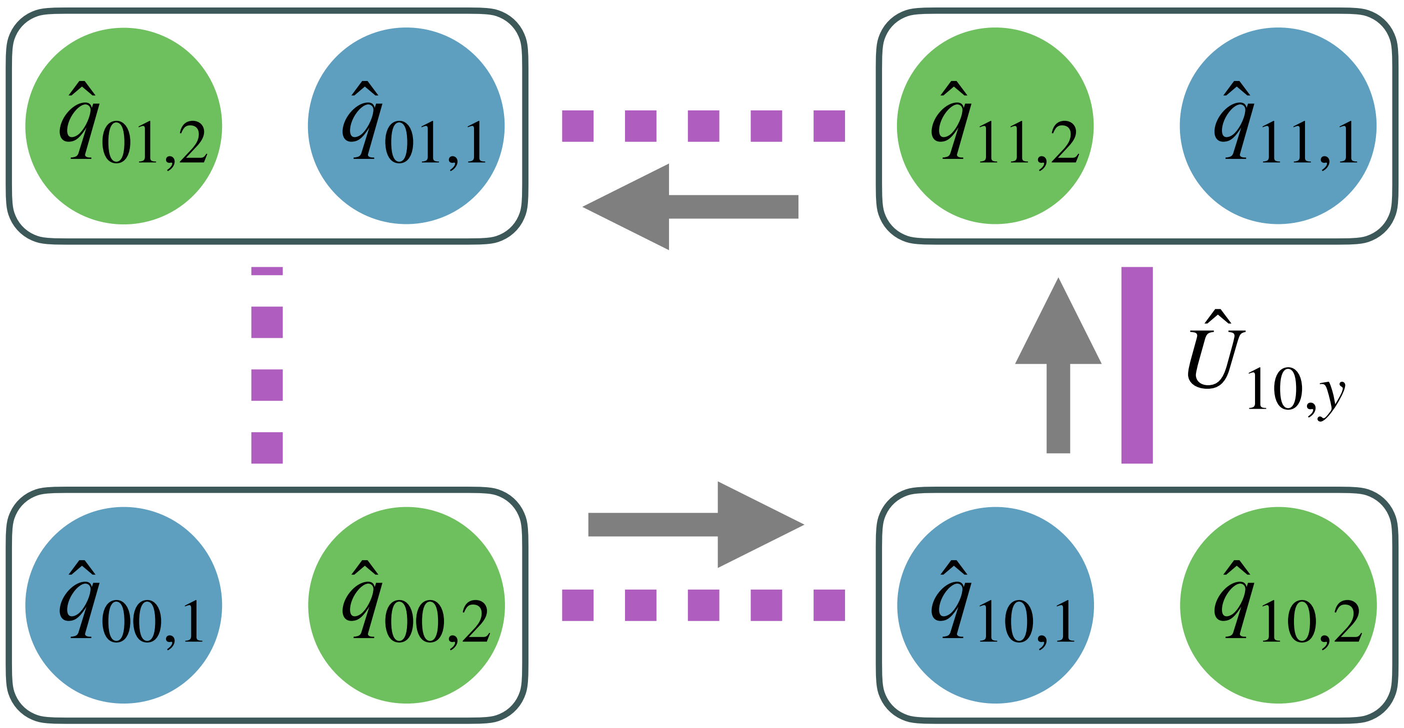
WHAT DOES IT TAKE TO QUANTUM-SIMULATE QED_3 AT FINITE DENSITY?

Currently? - Staggered Fermions:

[E. O. Rosanowski, A. Crippa, L. Funcke, P. V. Itaborai, K. Jansen, **S.S.,** PRD (2026)]

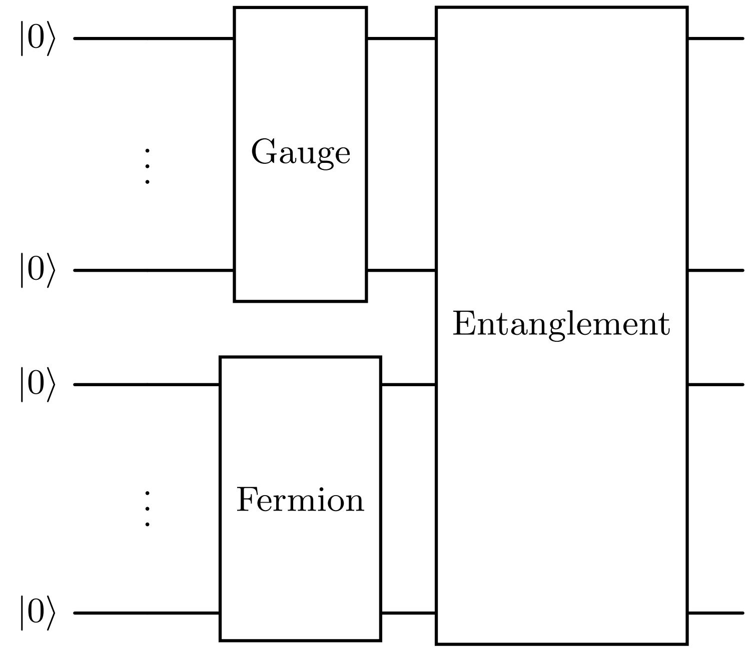
QED$_3$ on a Quantum Computer

Lattice setup: $N_f = 2$ staggered fermions on 2×2 lattice with OBC and one dynamical gauge link - Gauss law explicitly encoded



- Jordan-Wigner transformation maps fermions to qubits
  [P. Jordan, E. Wigner (1928)]
- Gauge fields get truncated : $U(1) \rightarrow \mathbb{Z}_{2l+1}$ with $l = 1$
- Gray encoding gives physical states: $|-1\rangle \rightarrow |00\rangle$, $|0\rangle \rightarrow |10\rangle$,
  [F. Gray (1953)],[O. Di Matteo, et.al., PRA (2021)]
  $|1\rangle \rightarrow |11\rangle$

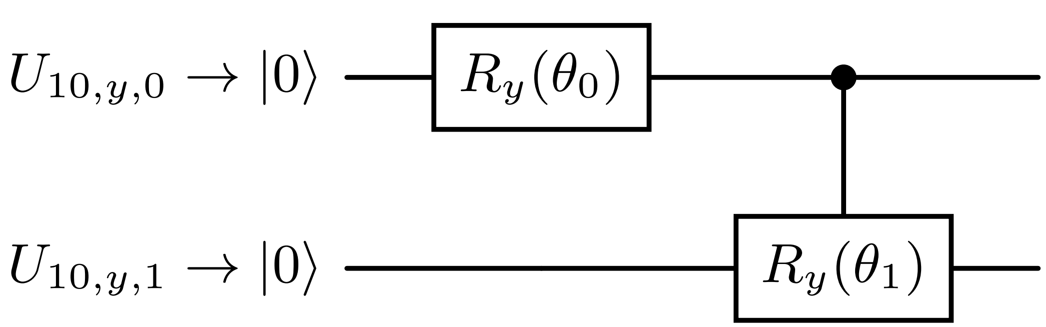
The quantum circuit ansatz



Gauge field encoding

[O. Di Matteo, et.al., PRA (2021)]

[A. Crippa, S. Romiti, et. al, Comm. Phys. (2025)]

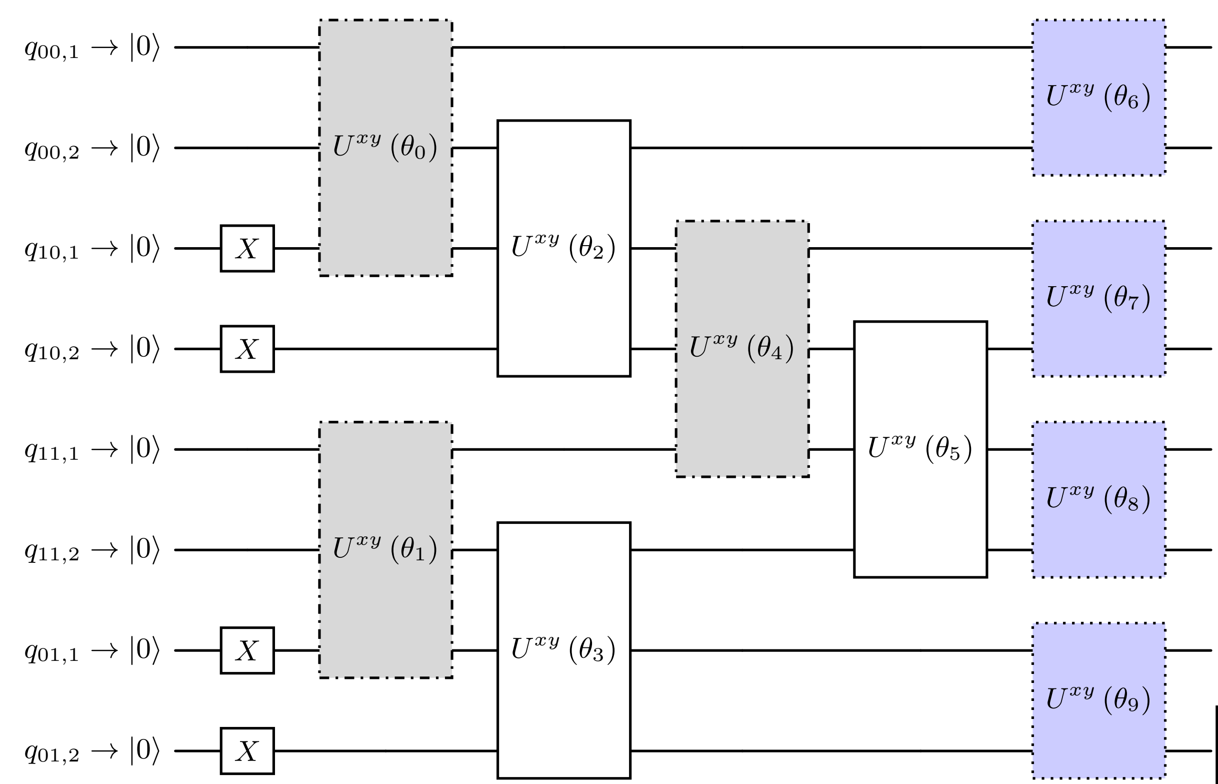


$\lceil \log_2 N \rceil$ qubits to represent N E-field eigenstates

* ! susceptible to hardware noise !

Fermion encoding

[S. Schuster et.al., PRD (2024)]



Variational Quantum Eigensolver

[A. Peruzzo et.al., Nat. Comm. (2014)]

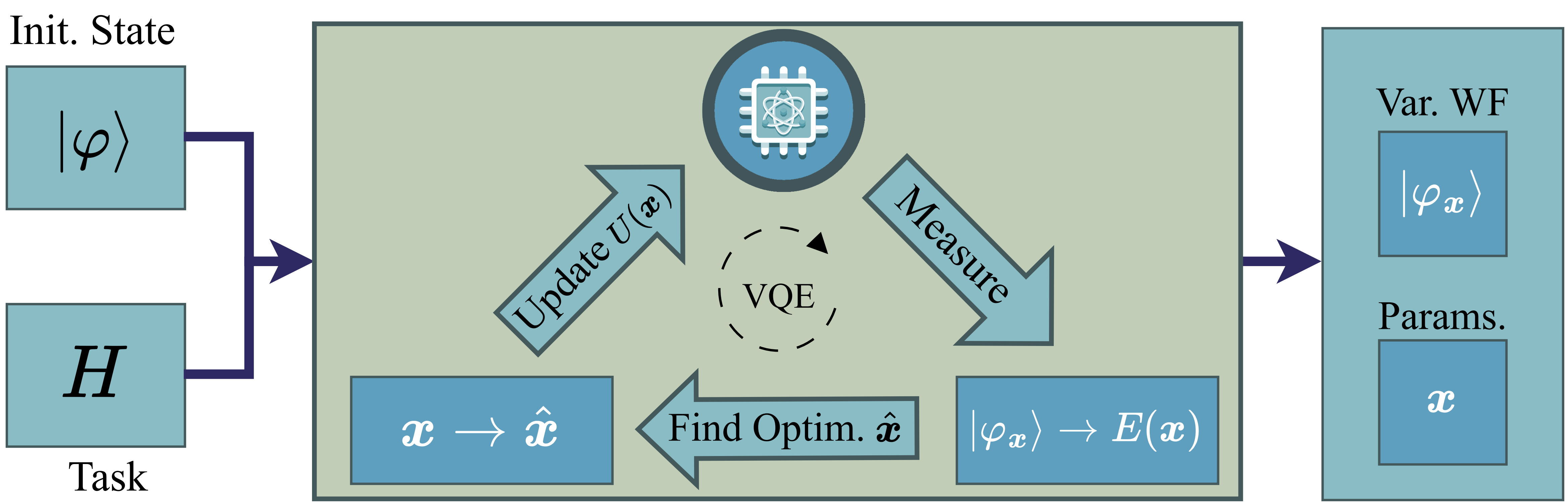


Image credit : adapted from Kim A. Nicoli et.al., NeurIPS 2023

Goal

Find the ground state of the Hamiltonian

How ?

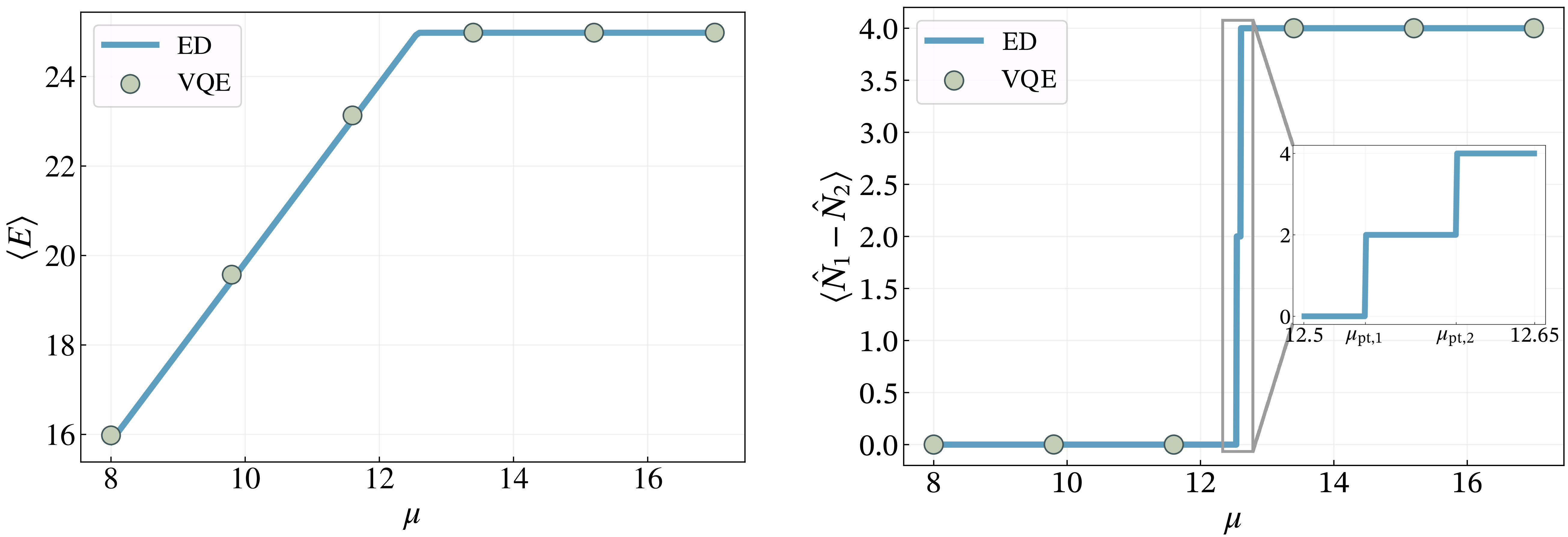
By finding a parameterized wave function $|\varphi_x\rangle$ that minimizes

$$E_x = \langle \varphi_x | H | \varphi_x \rangle$$

RESULTS I: CLASSICAL SIMULATIONS

[E. O. Rosanowski, A. Crippa, L. Funcke, P. Itaborai, K. Jansen, **S.S.**, arXiv: 2509.20558]

- Setup: 2×2 lattice with open boundary conditions, $N_f = 2$, $g = 5$
- Observables: ground-state energy and number operator difference



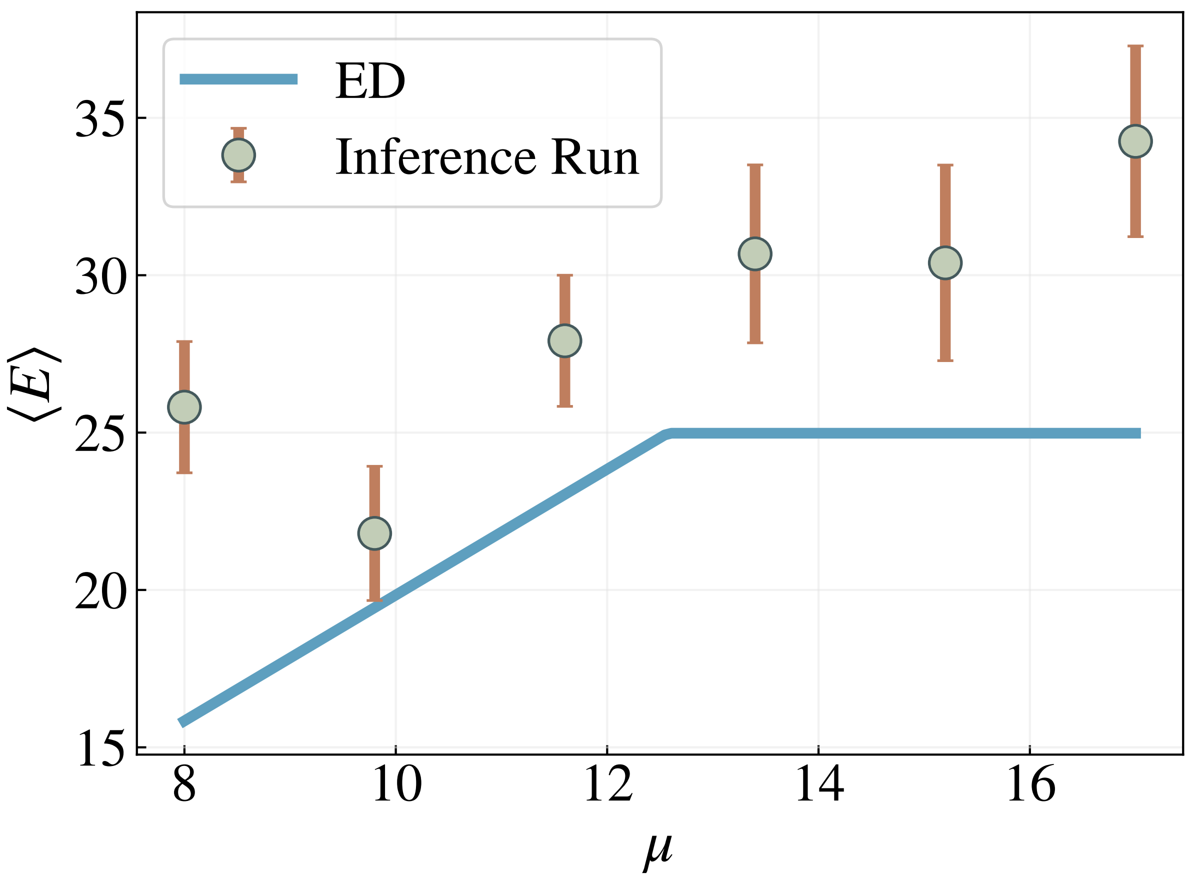
Noise-free classical simulation showing the lowest-energy result out of 10 runs

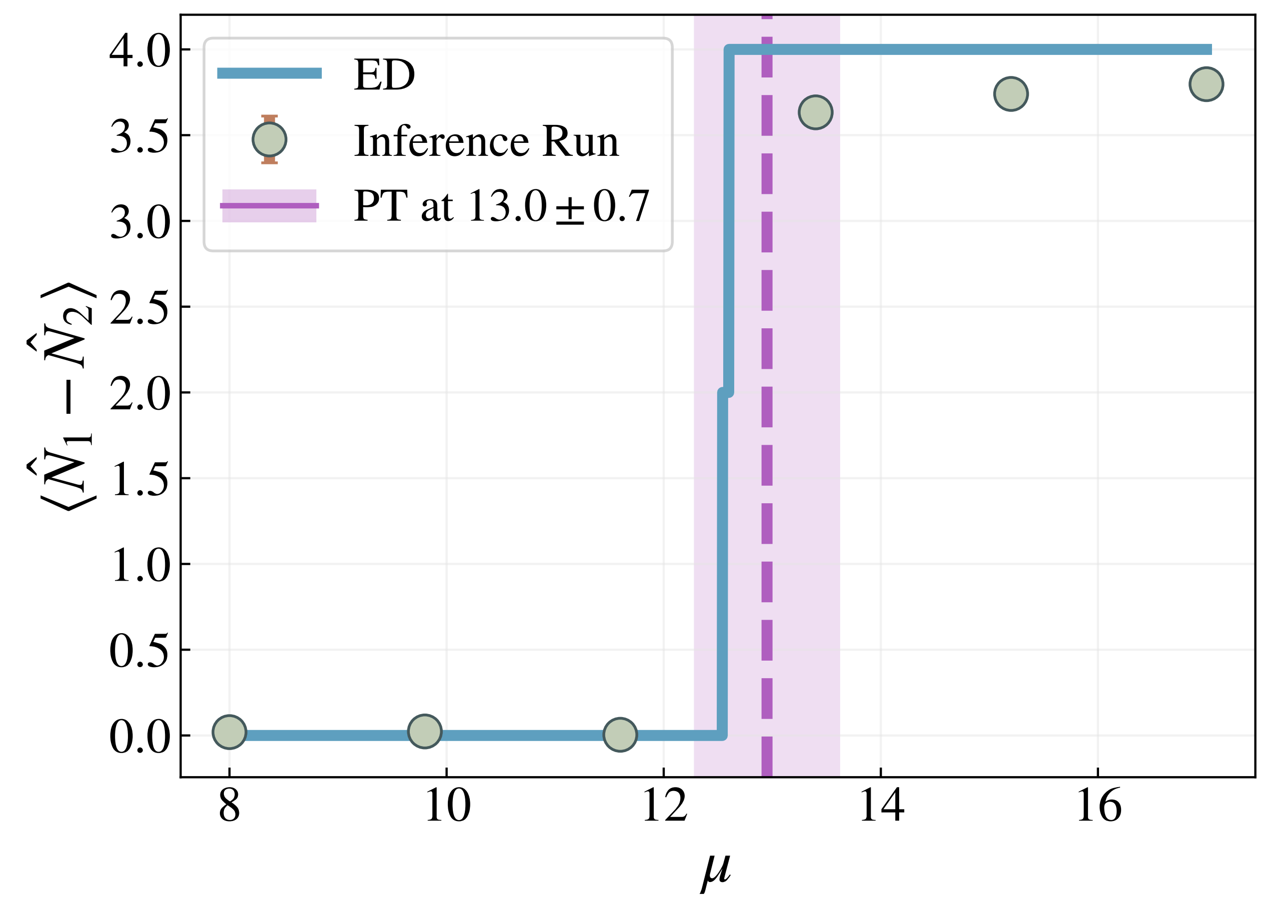
Results II: Inference results

[E. O. Rosanowski, A. Crippa, L. Funcke, P. Itaborai, K. Jansen, **S.S.**, arXiv: 2509.20558]

Measurements on ibm_marrakesh with $n_{\text{shots}} = 8{,}192$ (ZNE applied)

[K. Temme, et.al., PRL (2017)]





Noise sensitivity of the Hamiltonian operator $\gg$ Number operator

$O(10^3)$ $O(1)$

State sensitivity

[P. J. Coles et.al., (2019)]

Channel sensitivity

[G. Benenti et.al., JPB (2010)], [A. Hashim et.al., npj (2023)]

Summary & outlook

- Path toward QED_3 with Wilson fermions on a quantum computer
  - Showed that staggered disc does not support non-trivial Chern number
  - ED on 2X2 with gauge fields shows non-trivial Chern number
  - Mapped new phase diagram across μ and M
- Successful implementation of QED_3 on a quantum computer with Staggered fermions
  - Density induced level-crossings for multiple flavours of staggered
  - Inference results on IBM-hardware

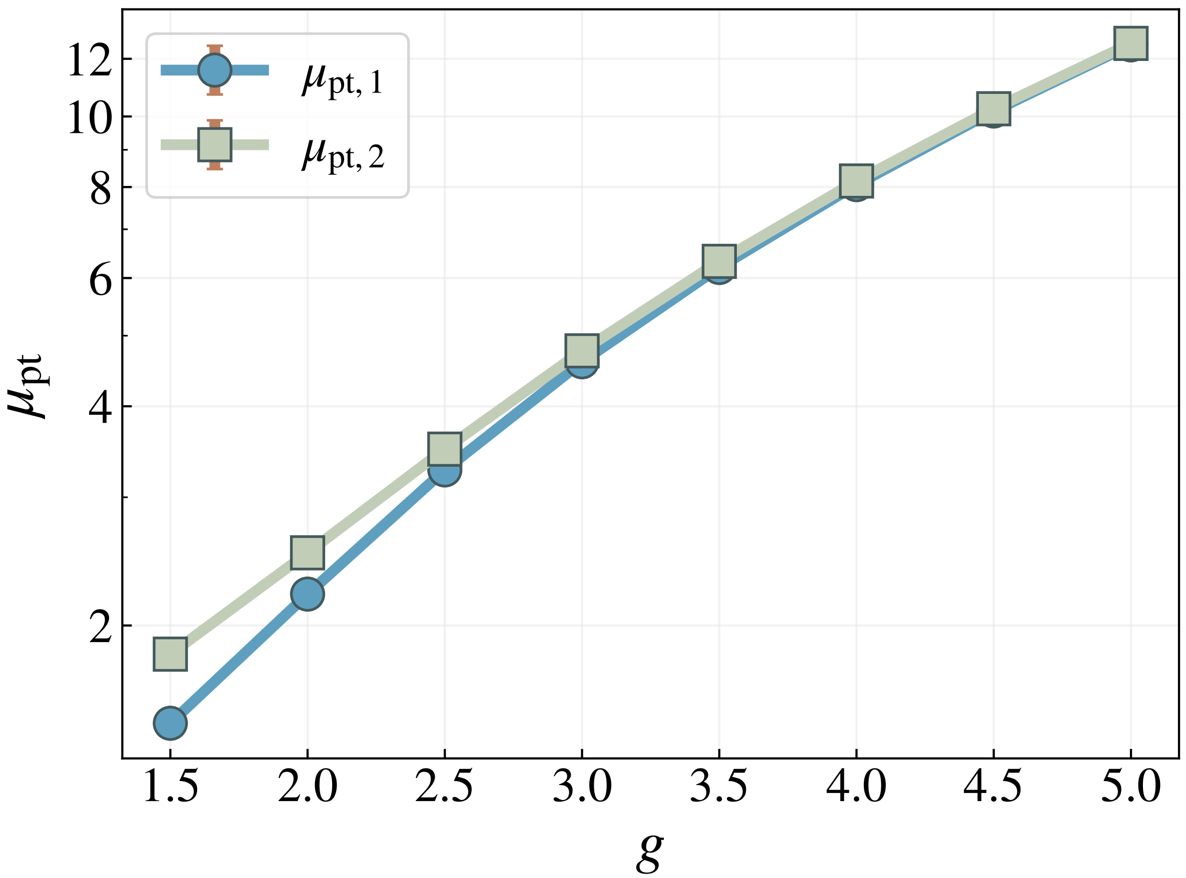
Thank you !

BACKUP SLIDES

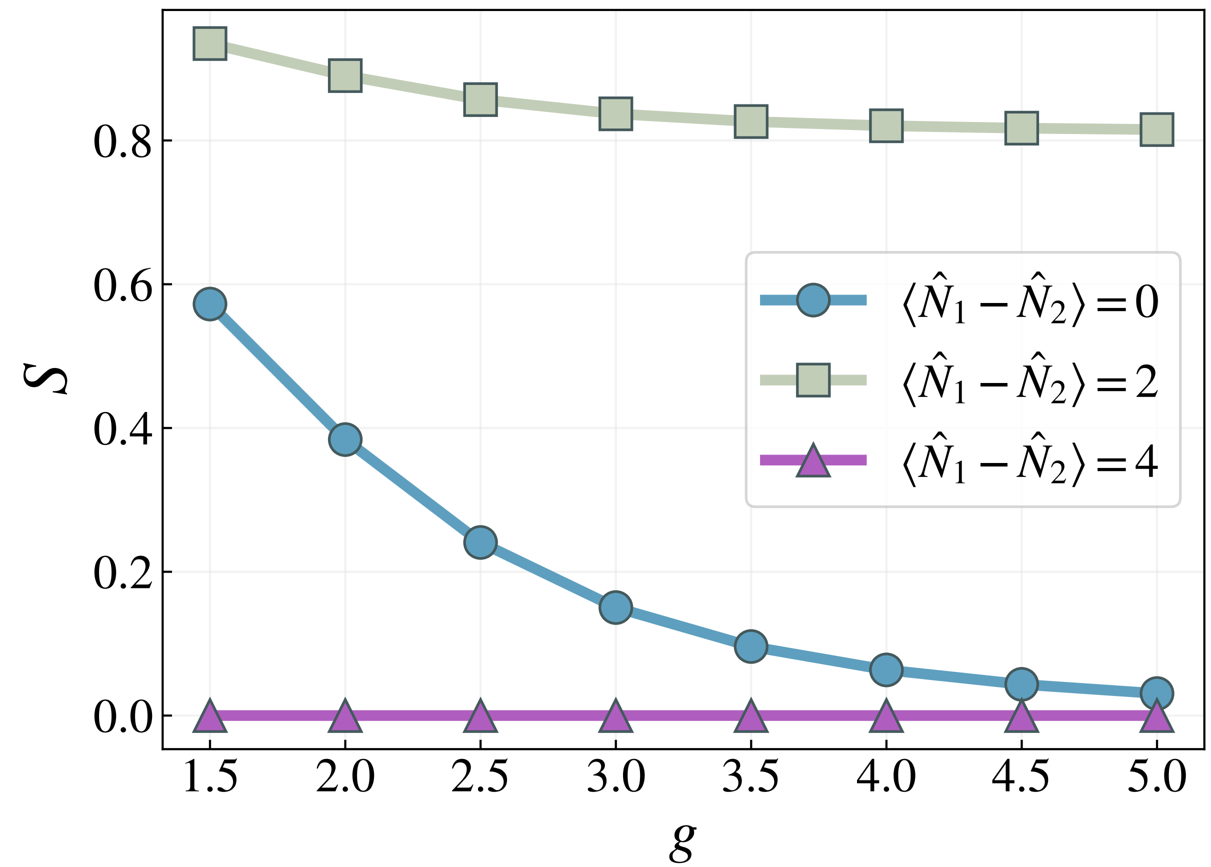
Additional analysis

Intermediate region : $\langle \hat{N}_1 - \hat{N}_2 \rangle = 2$

Transitions

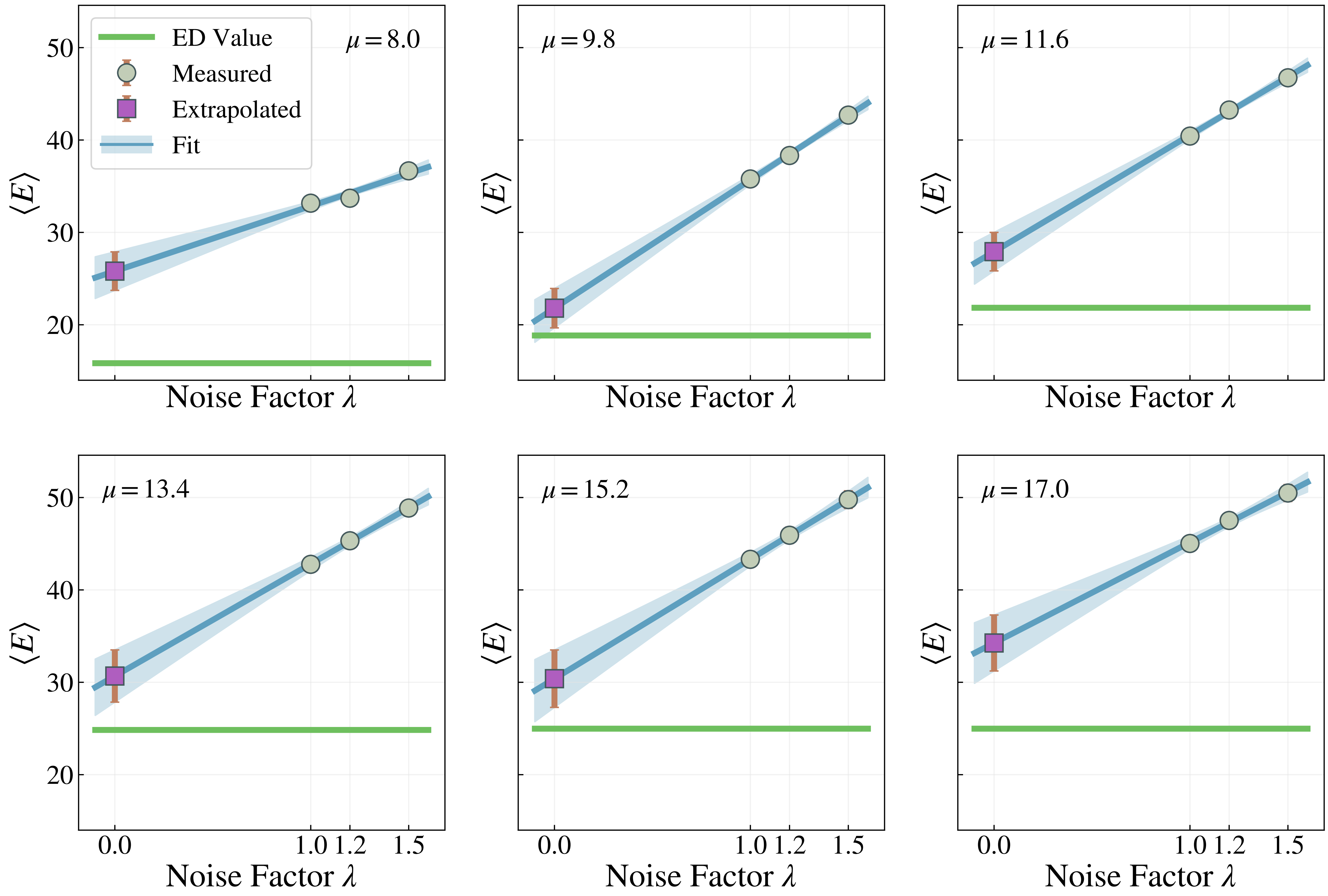


Entanglement



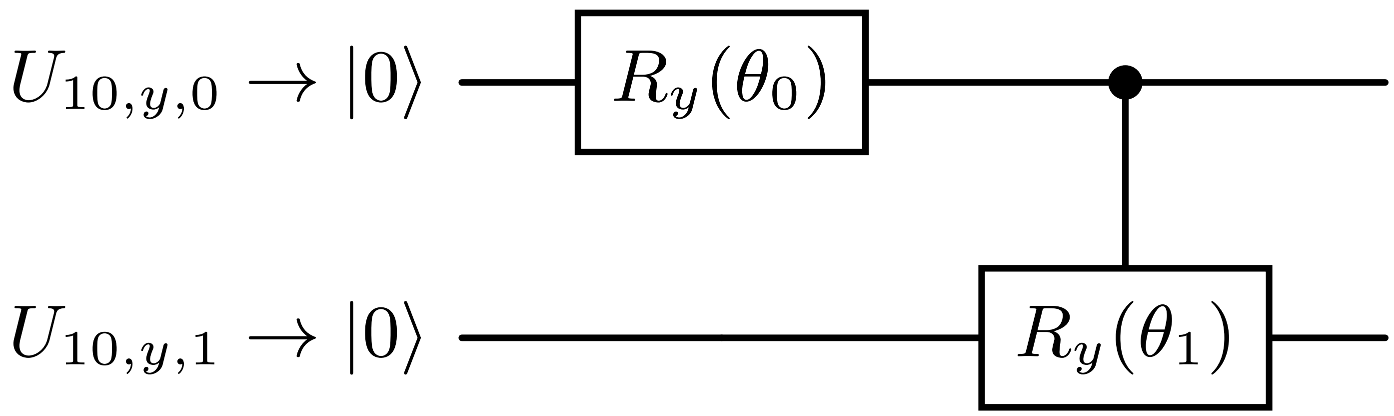
[E. Rosanowski, A. Crippa, L. Funcke, P. Itaborai, K. Jansen, S. Singh, arXiv: 2509.20558]

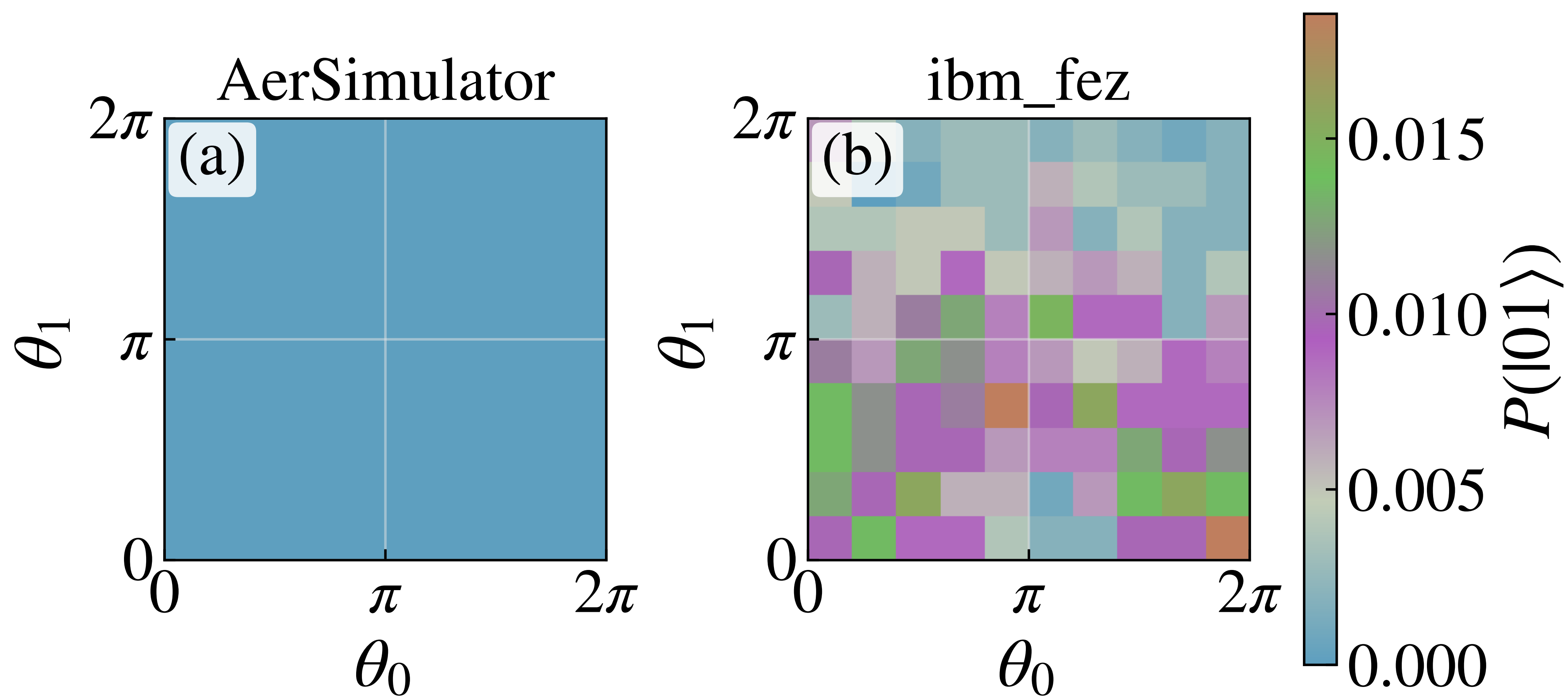
Zero Noise Extrapolation



[E. Rosanowski, A. Crippa, L. Funcke, P. Itaborai, K. Jansen, S. Singh, arXiv: 2509.20558]

LEAKAGE OUT OF GAUGE INVARIANT SECTOR





RESOURCE AND TRUNCATION

System	N_f	Depth	#qubits	#parameters
2×2 OBC	2	69	10	24
2×2 PBC	2	644	18	88
2×2 OBC	1	26	6	11
2×2 PBC	1	601	14	67
2×4 OBC	1	211	14	42

TABLE I. Ansatz circuit properties for different system sizes at $n_{\text{flavors}} = 2$. Here, *Depth* refers to the layers of multi-qubit gates. The compilation of the circuits is performed for the device **ibm_marrakesh** that is introduced in Section IV C.