Discrete Non-Abelian lattice gauge theories: a quantum simulation playground.

Matteo Wauters

QIS-HENP,

Wuhan, June 2026

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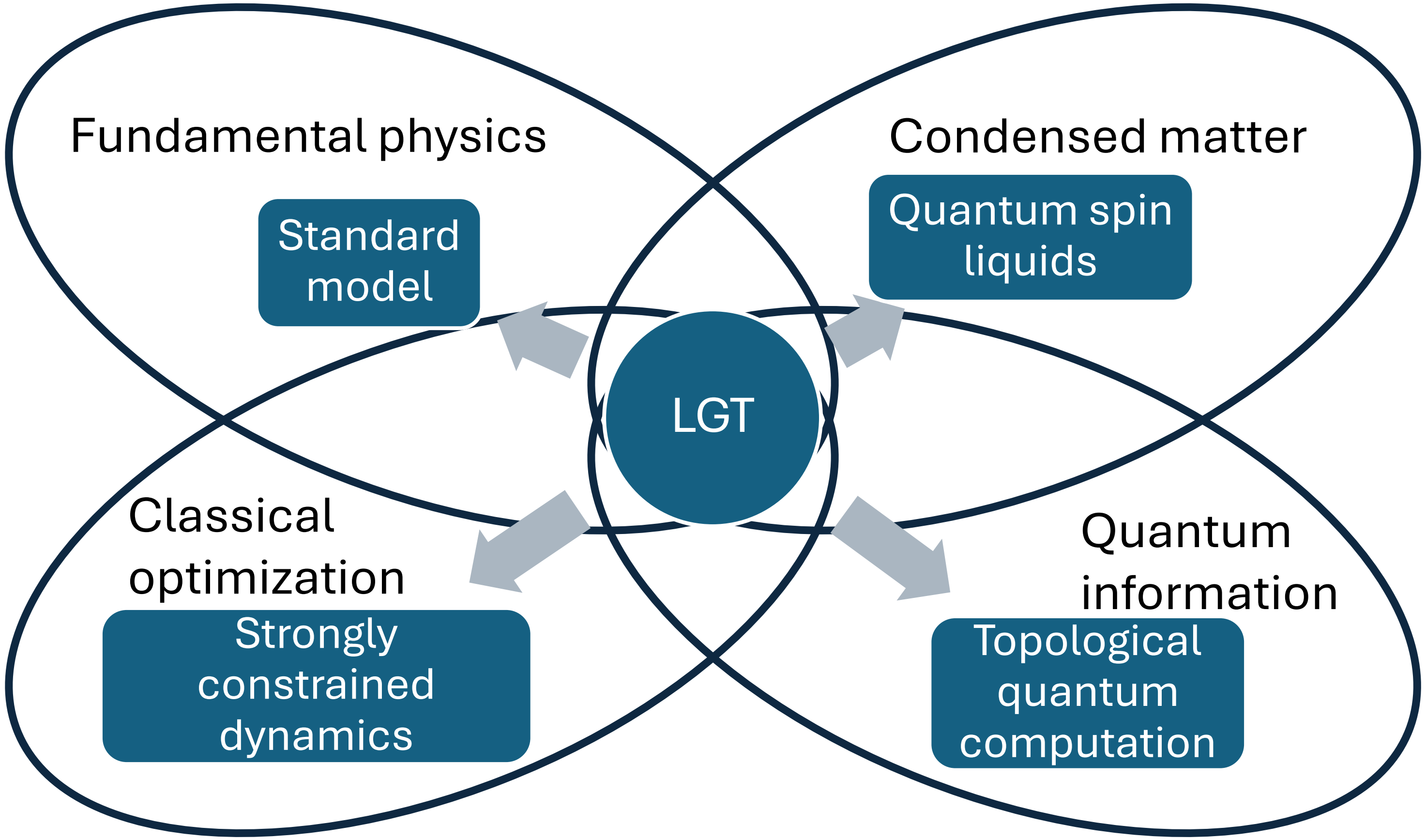
Lattice gauge theories: a many-body perspective



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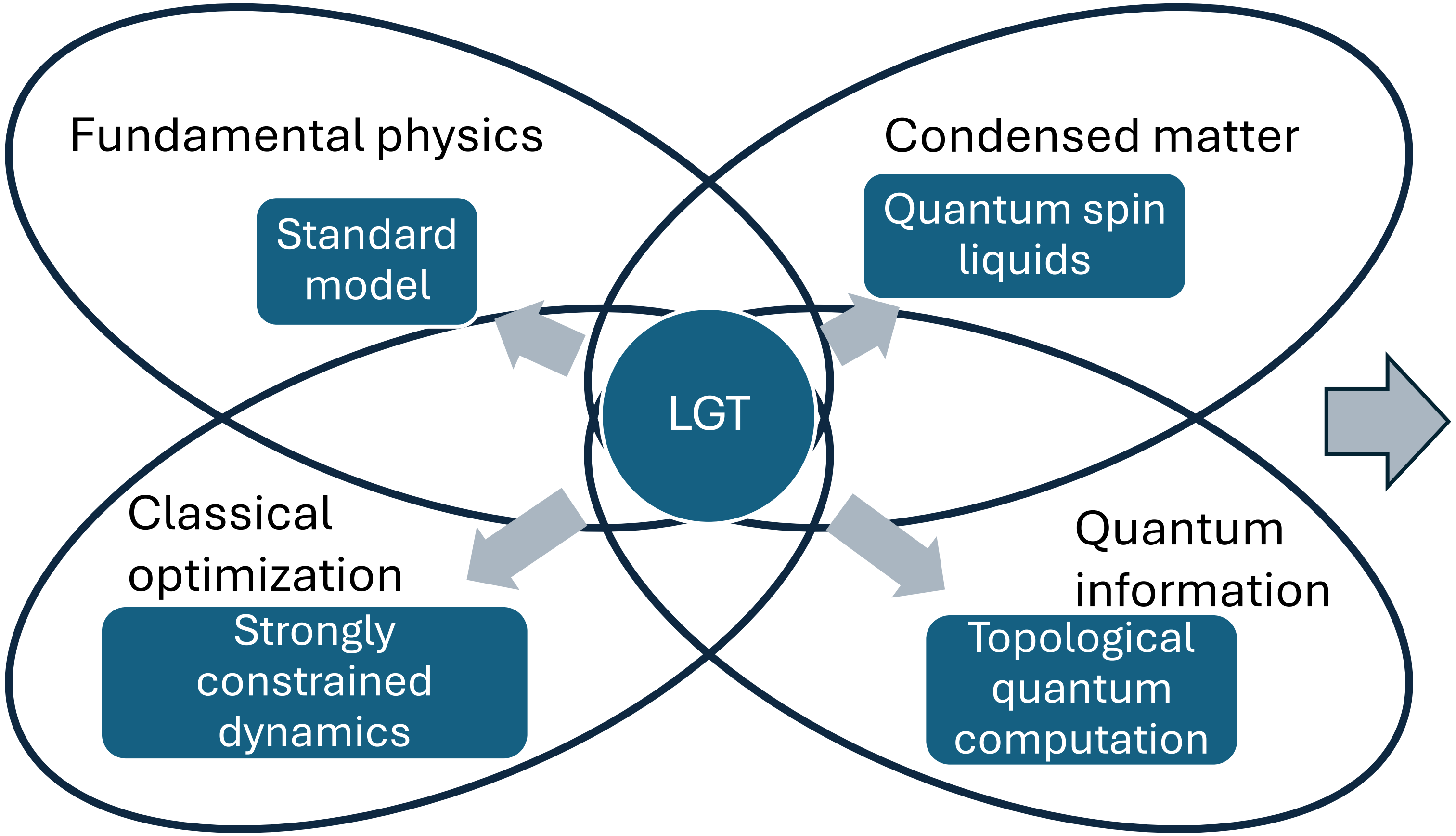
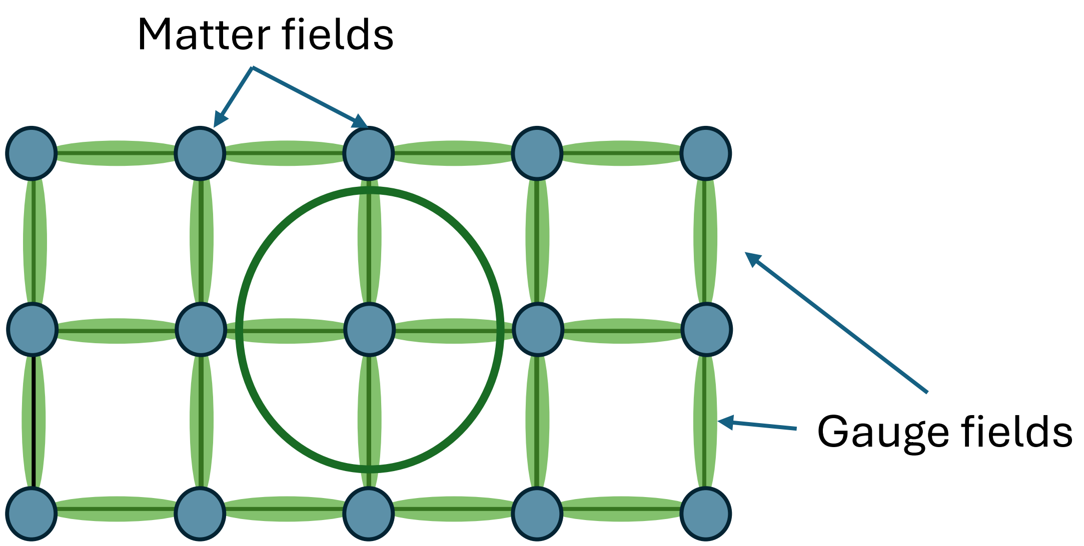


Quantum simulations of many-body physics

1. Go beyond classical methods
2. Hardware and software benchmark
3. Synthetic quantum systems

Quantum simulations of lattice gauge theories



Local symmetry

$U(1)$: $\boldsymbol{\nabla} \cdot \boldsymbol{E} = \rho$

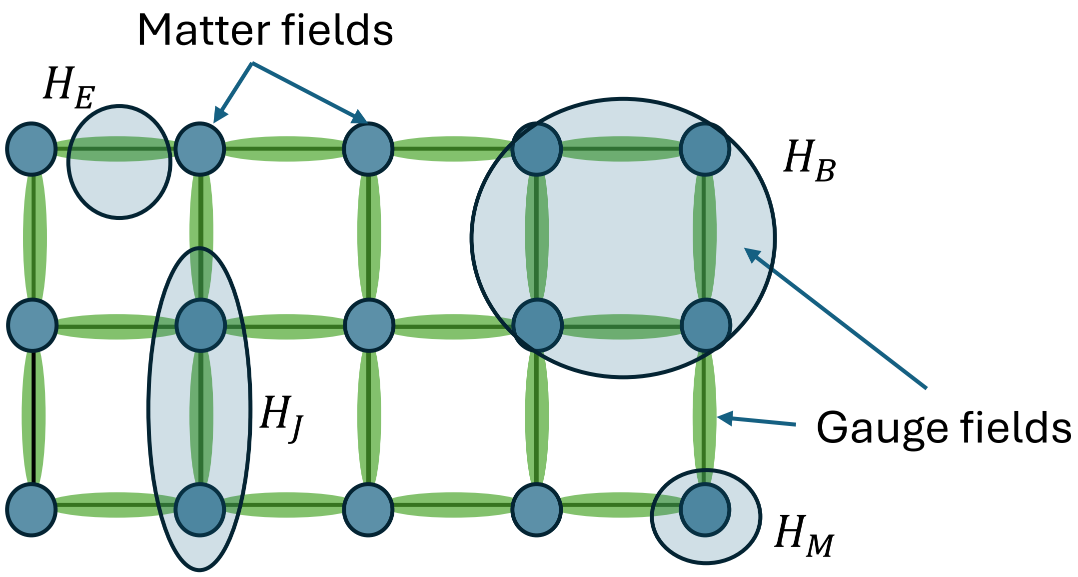
Lattice $\Rightarrow \sum_k \left(E_{\mathbf{r}+\mathbf{e}_k} - E_{\mathbf{r}-\mathbf{e}_k}\right) - \psi_{\mathbf{r}}^\dagger \psi_{\mathbf{r}} = 0$

Gauge group G

$\Theta_{\mathbf{r}}(g) H \Theta_{\mathbf{r}}^\dagger(g) = H \;\; \forall \mathbf{r} \in \mathbb{Z}^d, \forall g \in G$

Extensive number of symmetries

Quantum simulations of lattice gauge theories



$$H_J = \psi_n^\dagger U_{n,m} \psi_m + \mathrm{H.c.}$$

$$H_E = \sum_{j \in \mathrm{irreps}} \alpha_j \Pi_j$$

$$H_B = \mathrm{Tr}\, U_1 U_2 U_3^\dagger U_4^\dagger + \mathrm{H.c.}$$

$$H_M = M(-1)^n \psi_n^\dagger \psi_n$$

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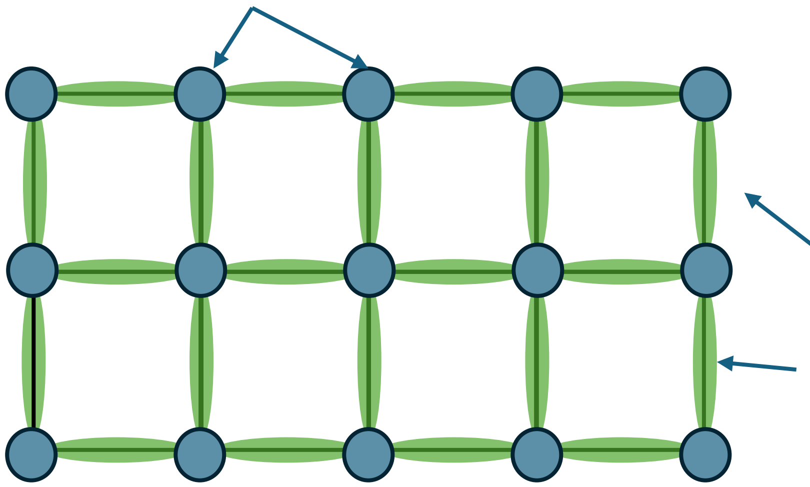

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Quantum simulations of LGTs are cool…

- Strongly constraint dynamics (ETH violations)
- Rich phenomenology
  - Ground state (topological order, confinement, anyons)
  - Real-time dynamics (scattering, string breaking, anyon braiding)

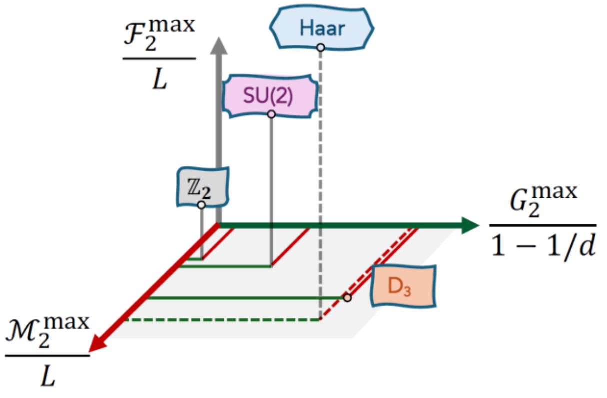
… but challenging

- K-body interactions
- Large local Hilbert space
- Preservation of gauge symmetries.

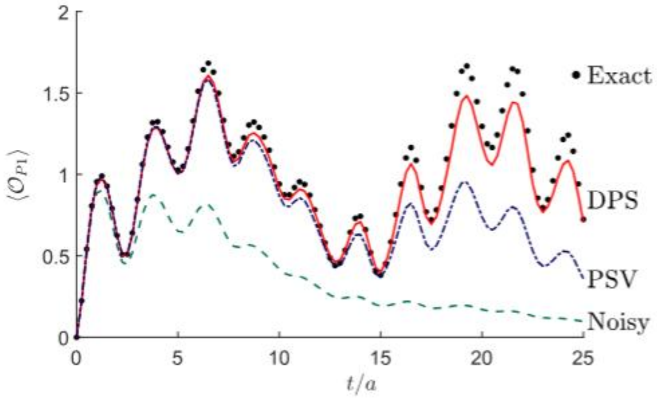
Discrete non-abelian LGTs?

- Exact encoding in discrete d.o.f. (qudits)
- Simpler (?) than Lie groups but still rich physics
- Non-Abelian extension of $\mathbb{Z}_N$-symmetric models
- Quantum complexity with non-Abelian symmetries

Quantum resources

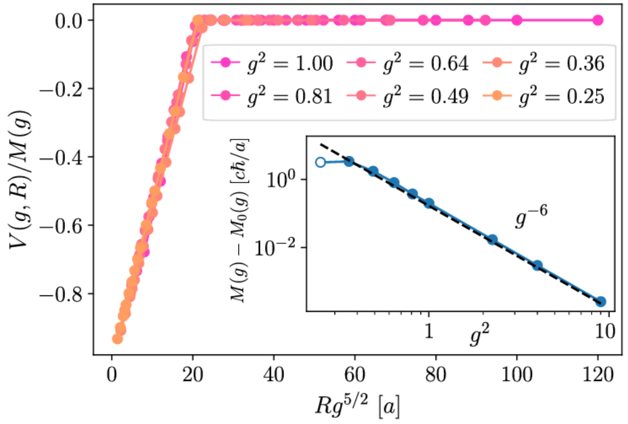


Error mitigation through monitored dynamics



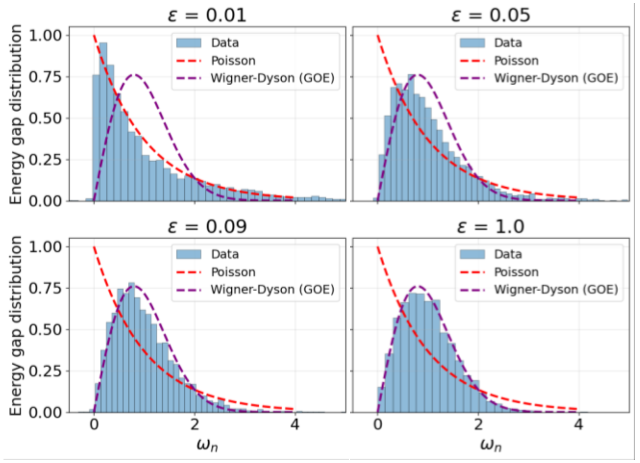
E Ballini, J Mildenberger, **MM Wauters**, P Hauke
Quantum (2025)

Confinement and color screening



PP Popov, et al arXiv:2512.00152

Eigenstate thermalization



M Vaia, at al, in preparation

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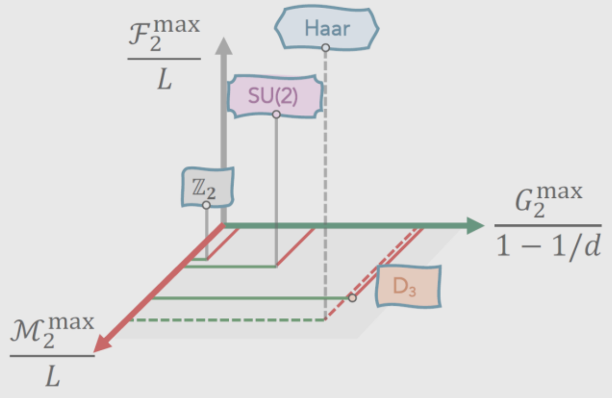

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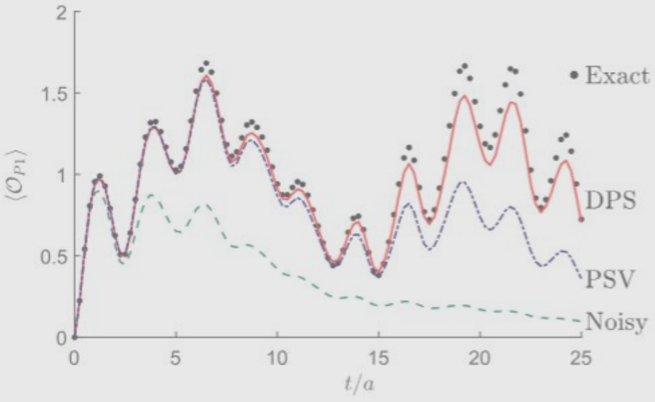

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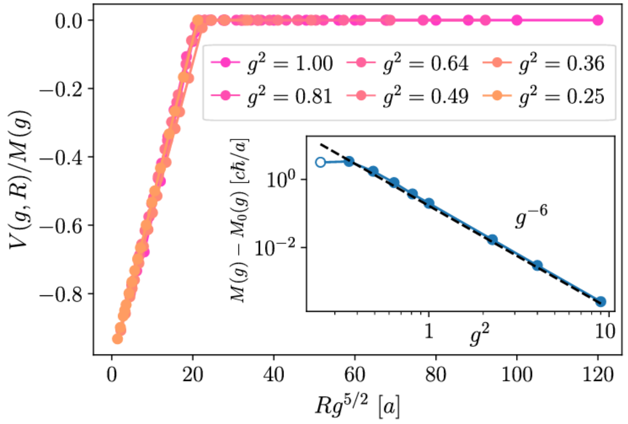

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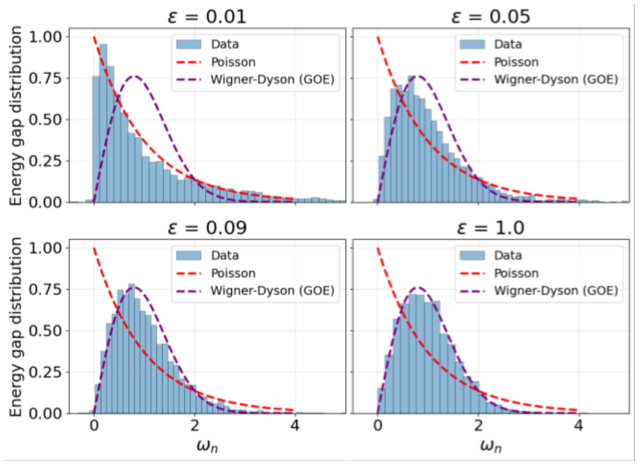

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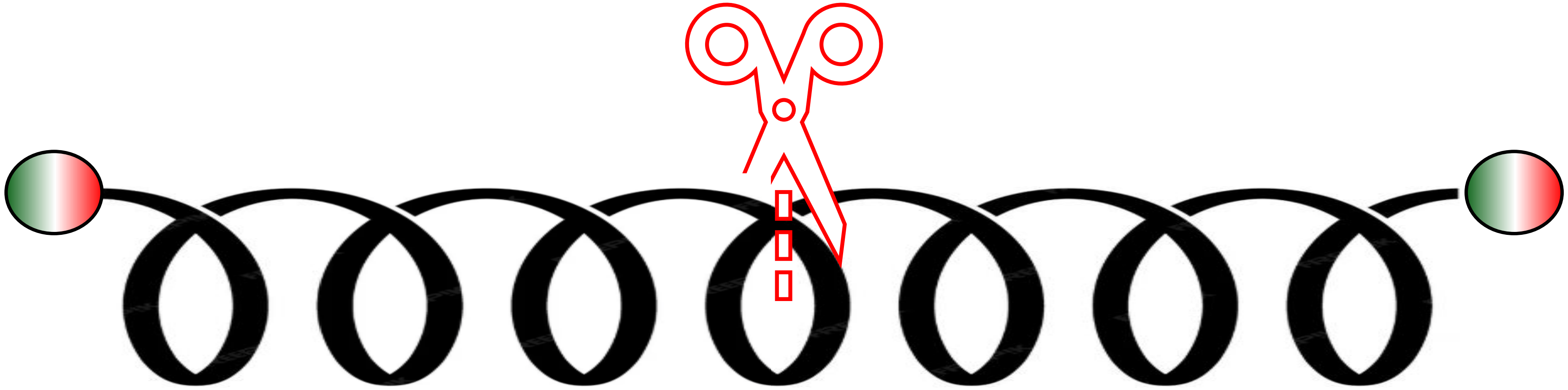


Confinement and color screening

String breaking without charges

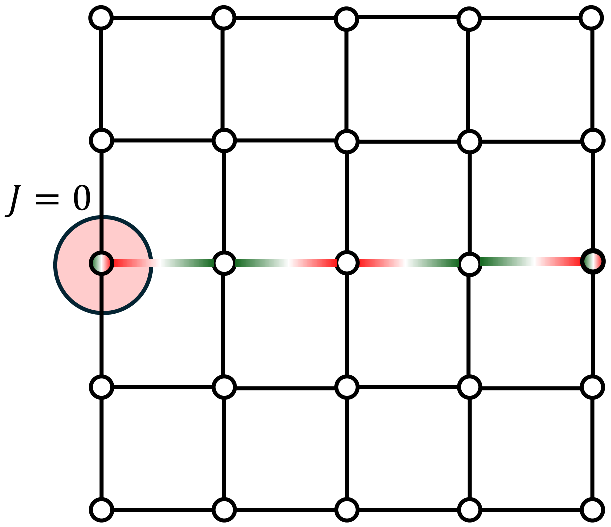
arXiv: 2512.00152

Confinement and string formation

There are no isolated particles in Nature with non-vanishing color charge i.e. all asymptotic particle states are color singlets. The Confinement Problem in Lattice Gauge Theory J. Greensite, 2003

SU(2)

$$H_E = g^2 \sum_l S_l^2$$



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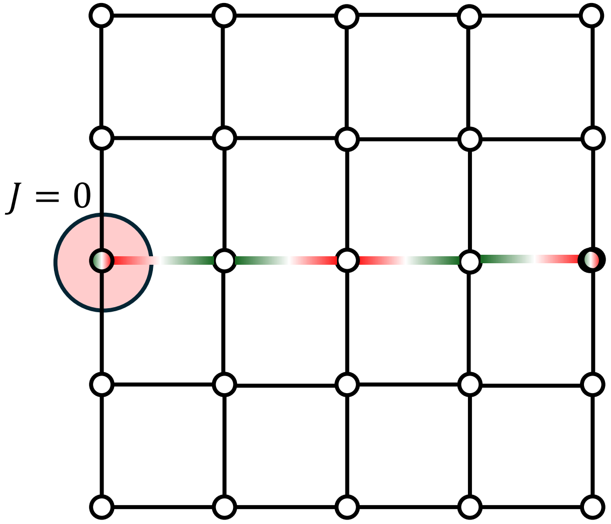
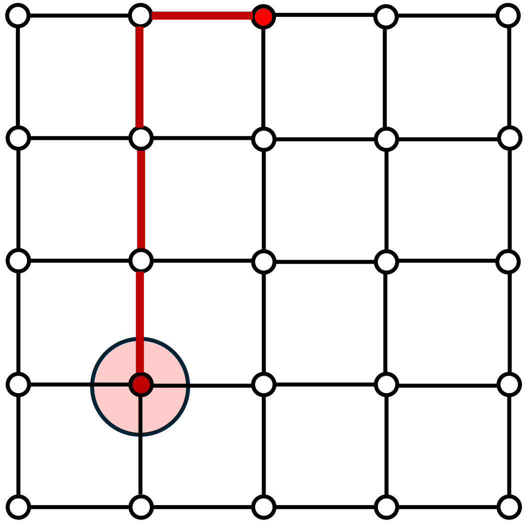


$\mathbb{Z}_2$

$$H_E = -g^2 \sum_l \sigma_l^x$$



$$e^{i\pi n_v} \prod_{i \in v} \sigma_i^x = 1$$

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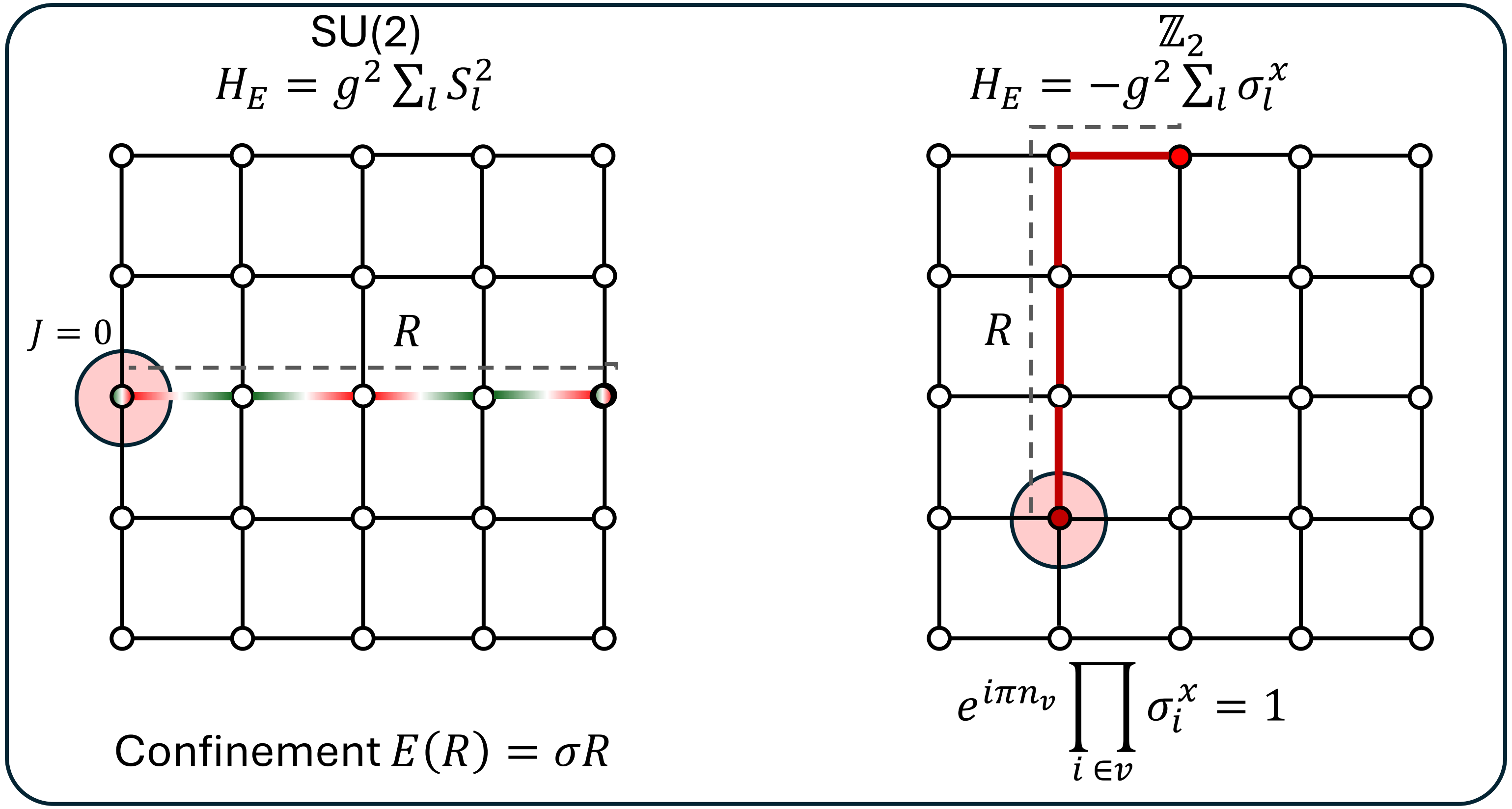

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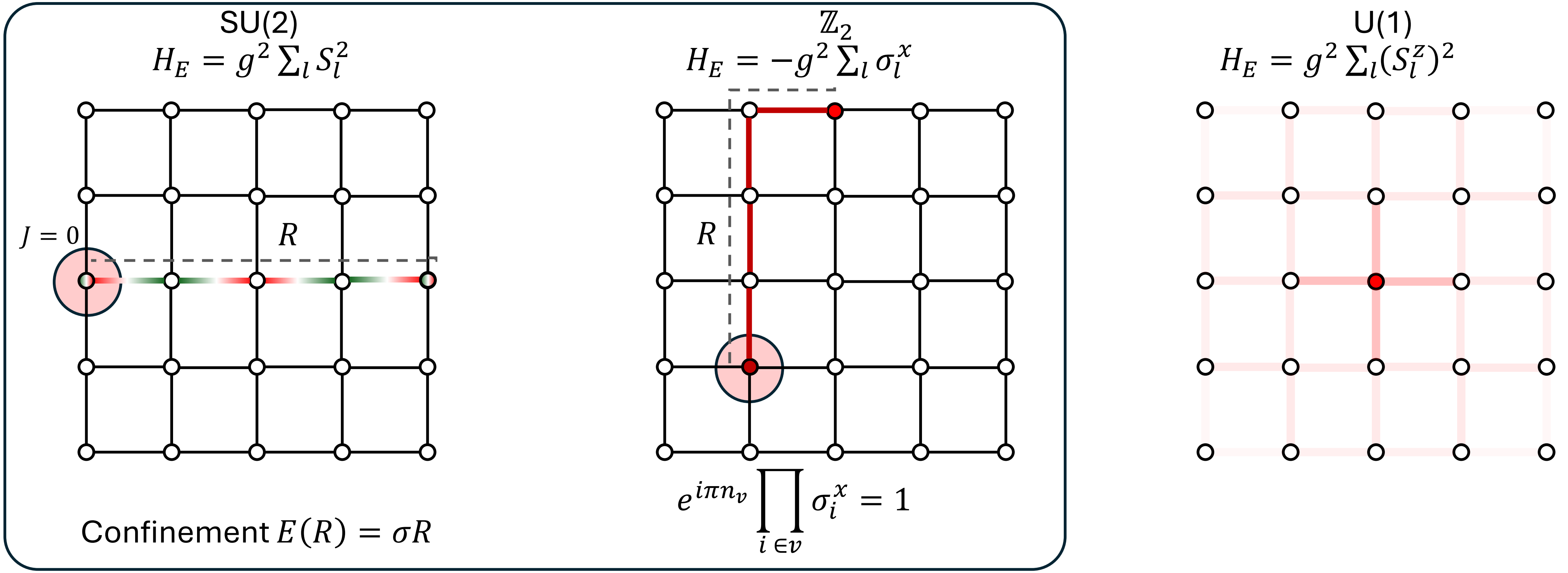
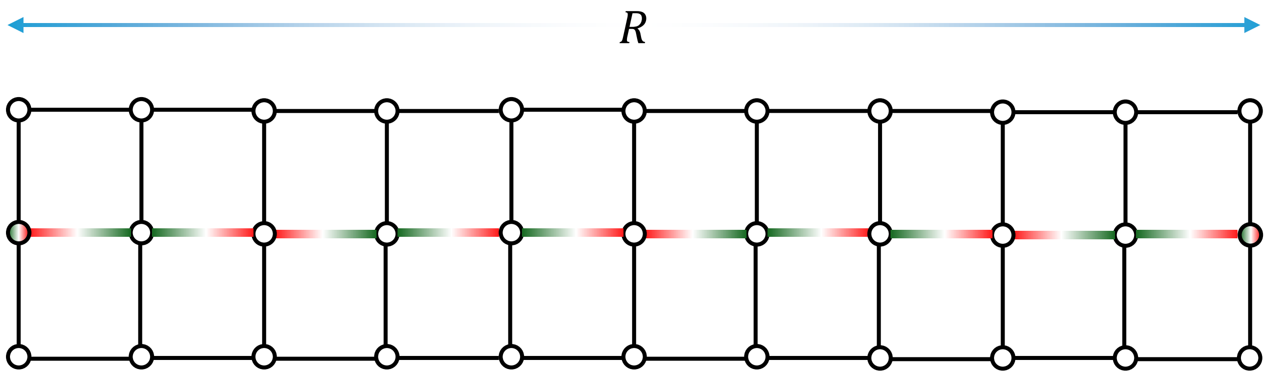


String breaking by pair nucleation



Competition in the Hamiltonian!

$$H_M = m \sum_n (-1)^n \psi_n^\dagger \psi_n$$

$$H_E = g^2 \sum_l E_l^2$$

When $(R-2)g^2E^2 > 2m$, the GS changes

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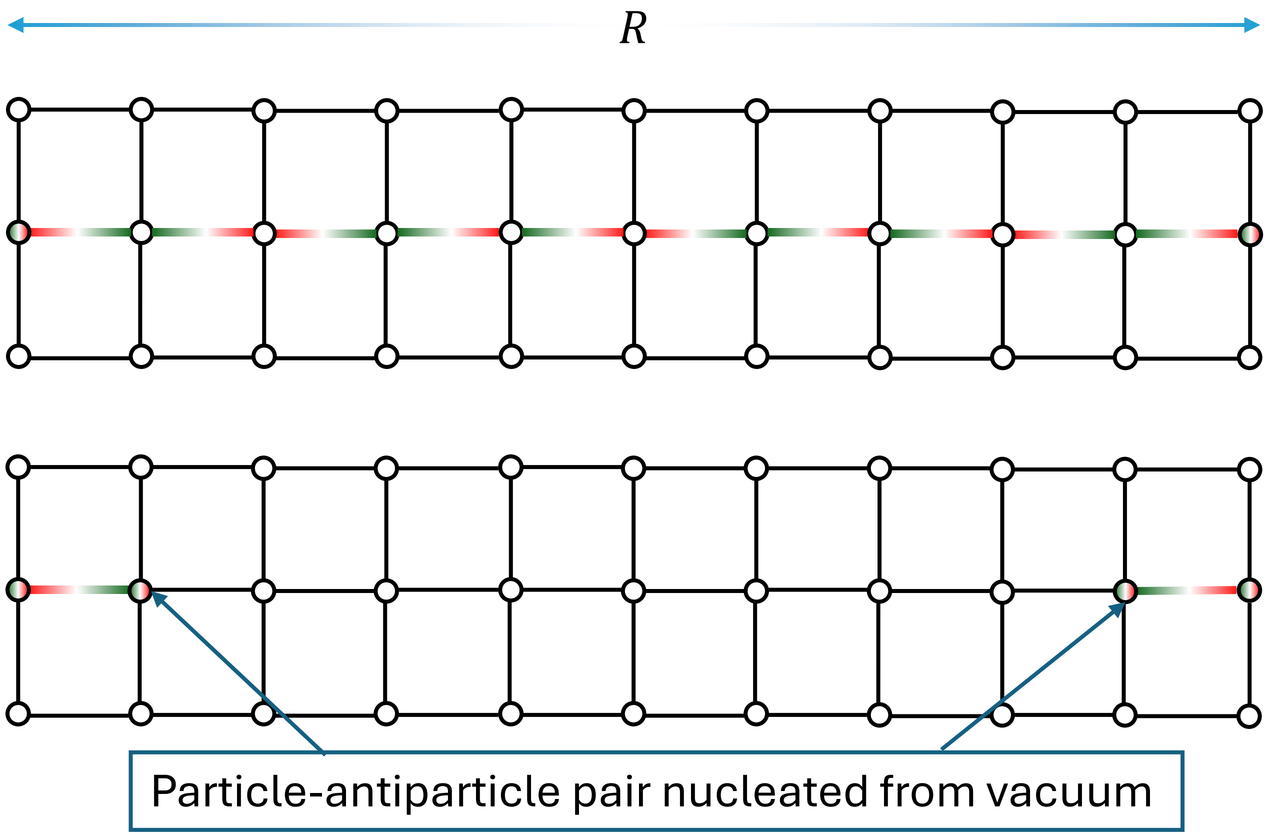

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String breaking by pair nucleation

Visualizing dynamics of charges and strings in (2 + 1)D lattice gauge theories

T. A. Cochran, B. Jobst, E. Rosenberg, Y. D. Lensky, G. Gyawali, N. Eassa, M. Will, A. Szasz, D. Abanin, R. Acharya, L. Aghababaie Beni, T. I. Andersen, M. Ansmann, F. Arute, K. Arya, A. Asfaw, J. Atalaya, R. Babbush, B. Ballard, J. C. Bardin, A. Bengtsson, A. Bilmes, A. Bourassa, J. Bovaird, … P. Roushan + Show authors

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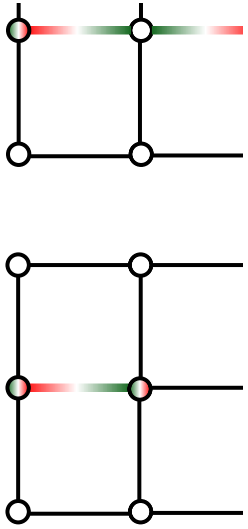

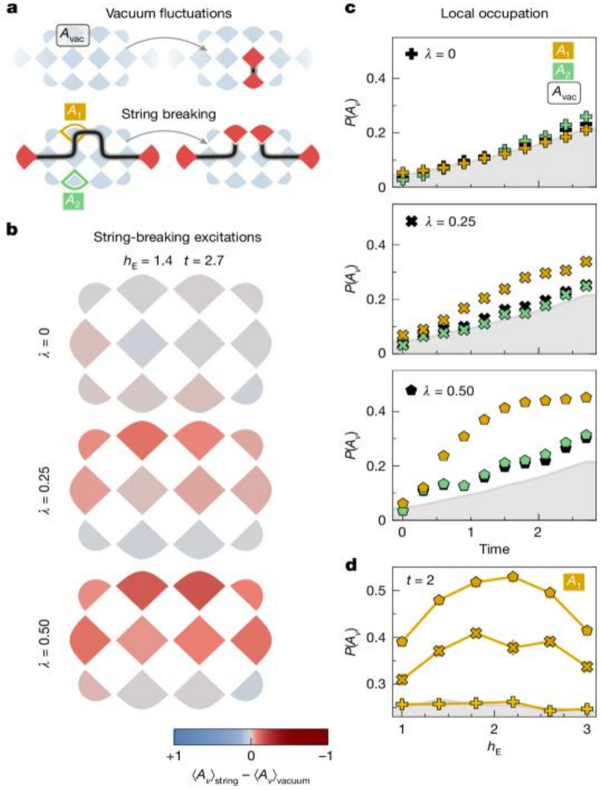

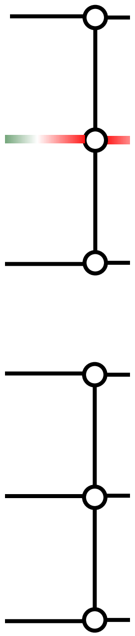

Observation of string breaking on a (2 + 1)D Rydberg quantum simulator

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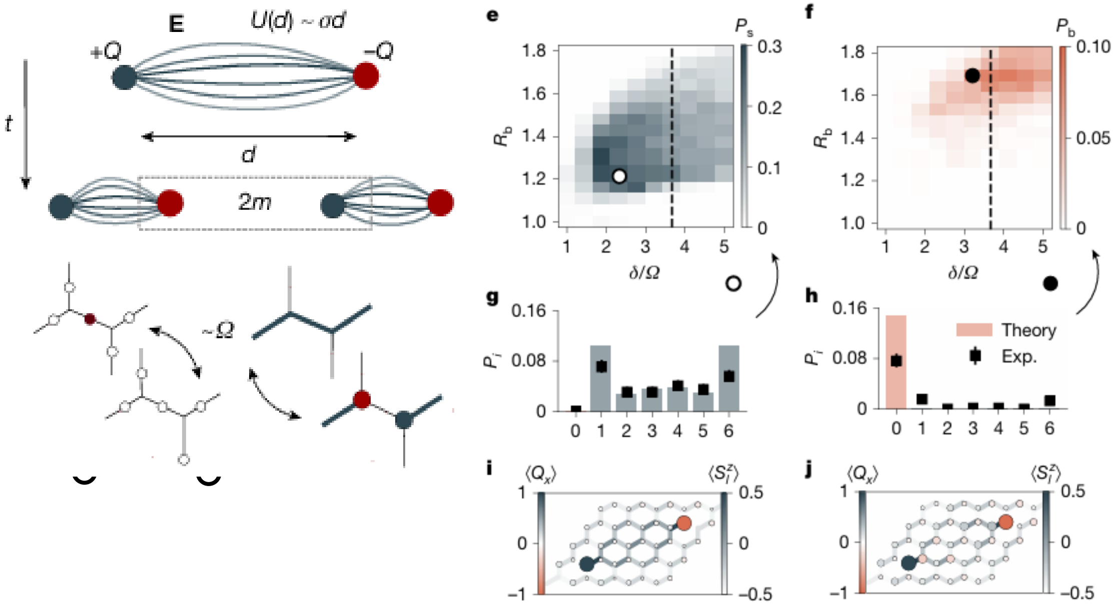

niltonian!

ψ_n

anges

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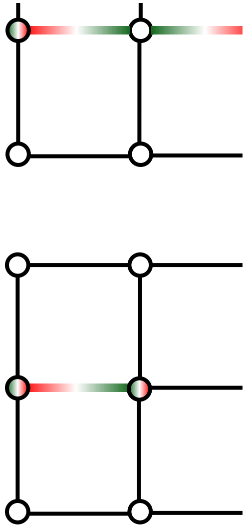

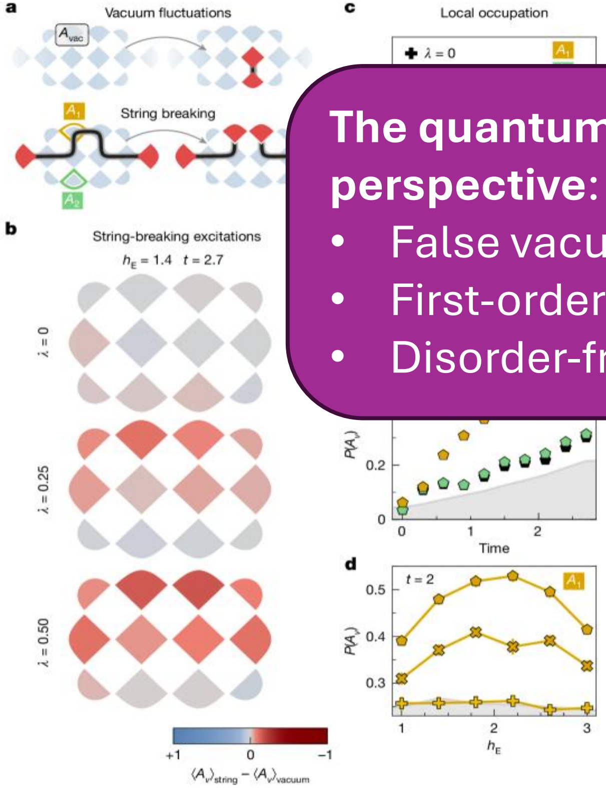

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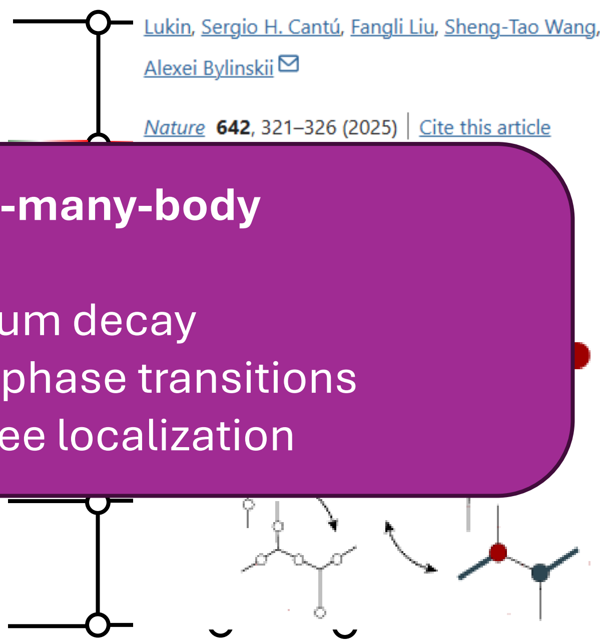
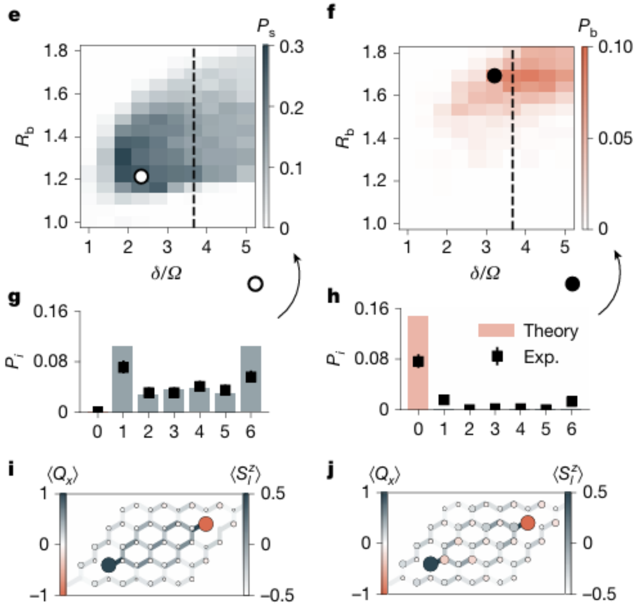




anges

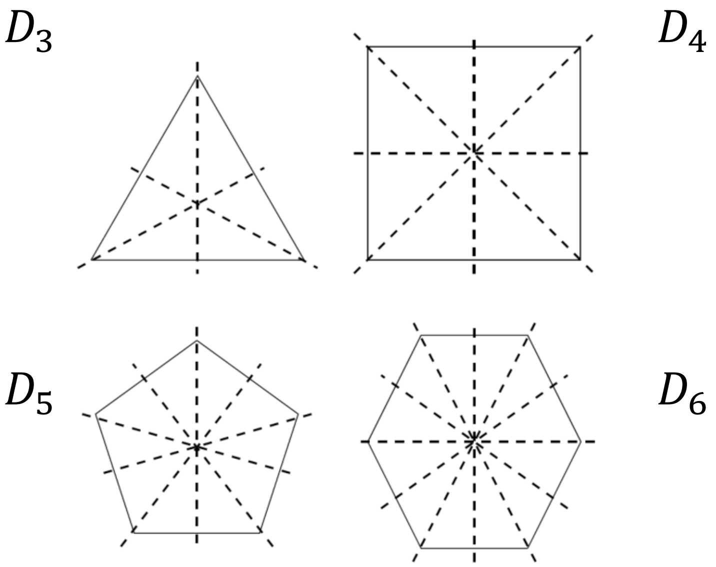
The quantum-many-body perspective:

- False vacuum decay
- First-order phase transitions
- Disorder-free localization

Dihedral groups D_N - the boring slide

D_N: symmetry group of a regular polygon with N sides

$$s = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \qquad r = \begin{bmatrix} \exp\left(\frac{2\pi i}{N}\right) & 0 \\ 0 & \exp\left(-\frac{2\pi i}{N}\right) \end{bmatrix}$$



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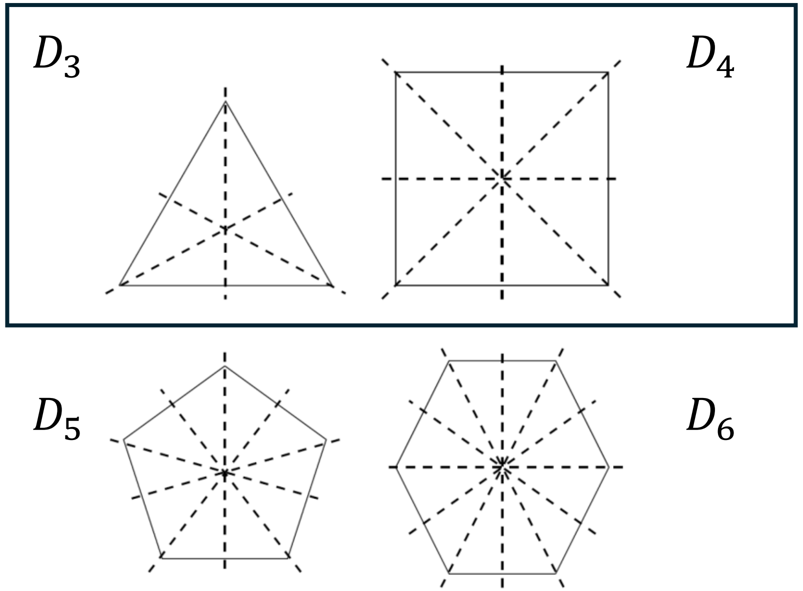

D_3	D_4	$\dim(j)$
e	e	1
τ	τ	2
p	p	1
	p_1	1
	p_2	1

Fundamental irrep → τ

# of colors → 2

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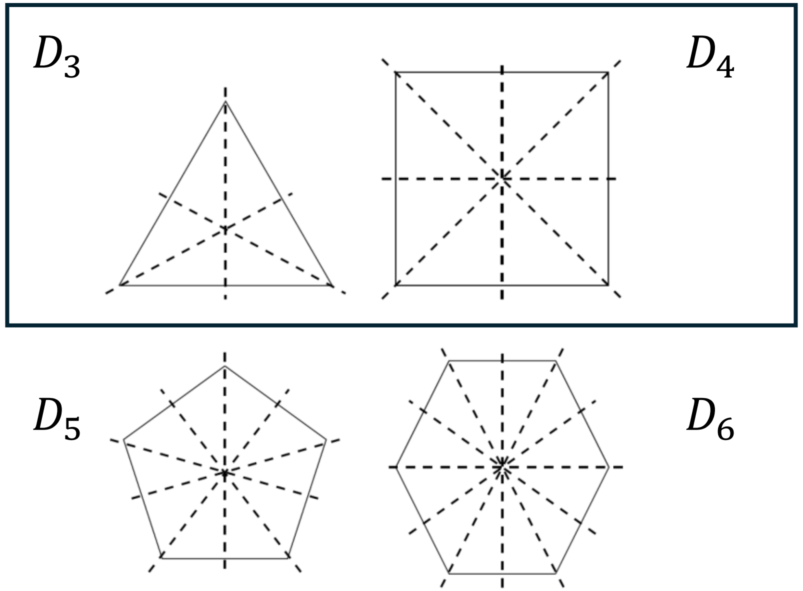
Two conjugate basis:

- Group elements $|g\rangle \Rightarrow$ magnetic fluxes, particle hopping, group connection
- Irreps $|jmn\rangle \Rightarrow$ Electric energy, gauge transformations

Gauge invariance = vertices transform trivially (group singlets)



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Fusion rules and gauge invariant states in D_3

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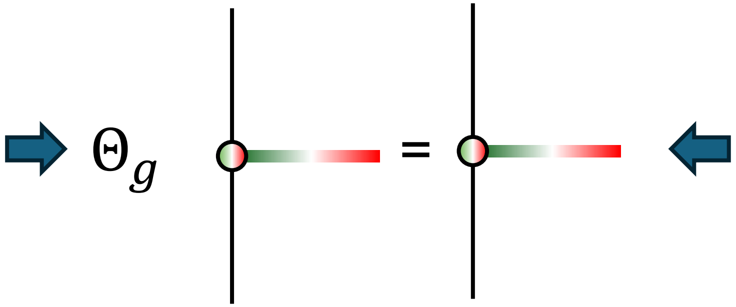

Fusion rules say how we can combine irreps together.
E.g. SU(2): $\frac{1}{2} \otimes \frac{1}{2} = 0 \oplus 1$

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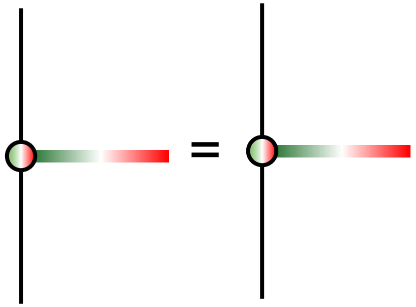

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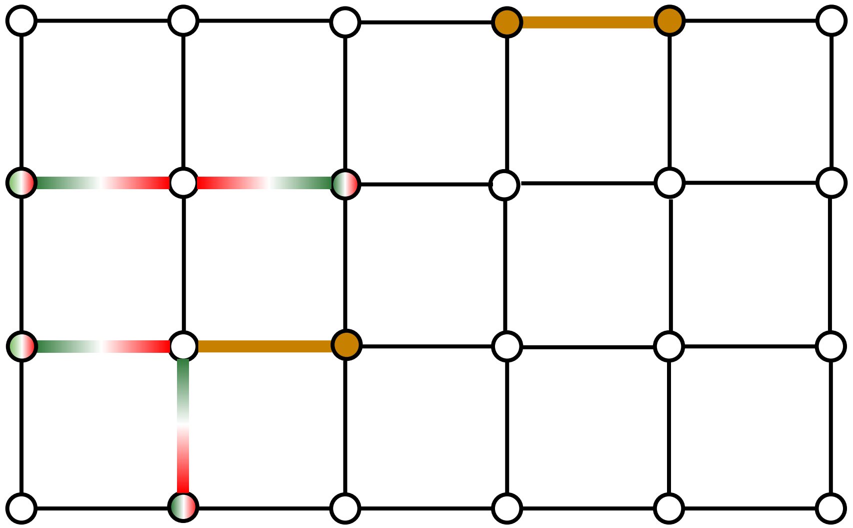
Charge transformation →

no particles	**0** irrep
single fermion	τ irrep
2 fermions	p irrep



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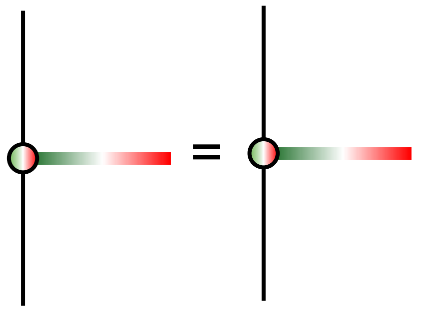

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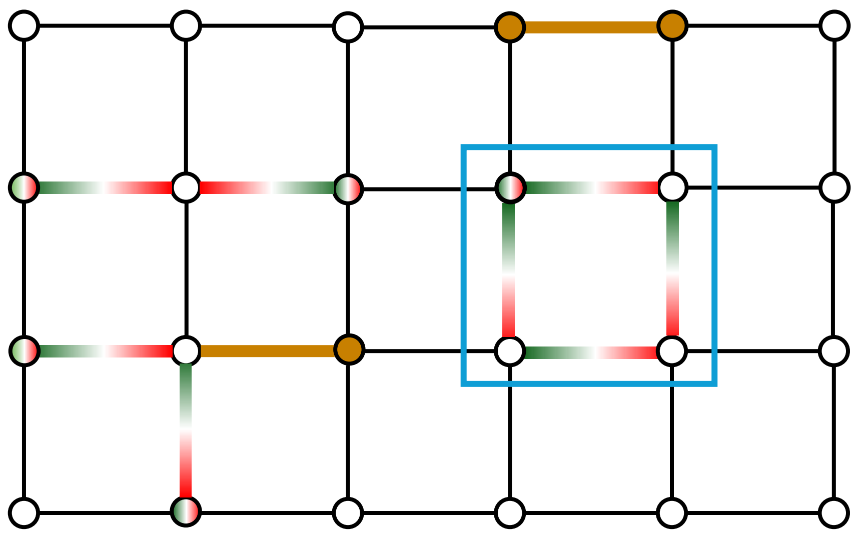
D_3: $\tau \otimes \tau = 0 \oplus \tau \oplus p$,

but

D_4: $\tau \notin \tau \otimes \tau$

➡ Depends on the center group



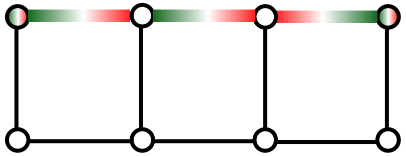
Ladder and GSs of D3 and D4

The simple $\tau \in \tau \otimes \tau$ or $\tau \notin \tau \otimes \tau$ has a huge impact on the physics of the LGT

We take a ladder with static charges on the upper corners, pure gauge Hamiltonian

$$H = H_E + H_B = g^2 \sum_l E_l^2 + \frac{2}{g^2} \sum_p \mathrm{Re}(\mathrm{Tr}(U_{p,1} U_{p,2} U_{p,3}^\dagger U_{p,4}^\dagger))$$

Let's consider the ground state at strong coupling $g^2 > 1$



$E_{gs} \simeq 3g^2 \Delta_\tau$

D_3

$\tau \in \tau \otimes \tau$

D_4

$\tau \notin \tau \otimes \tau$

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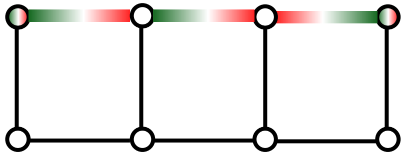

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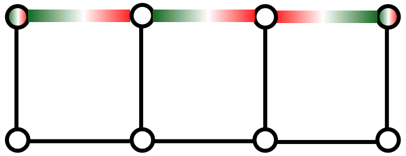

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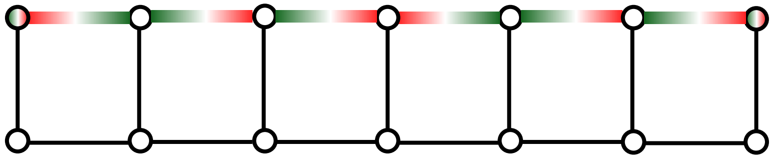
D_3

$\tau \in \tau \otimes \tau$

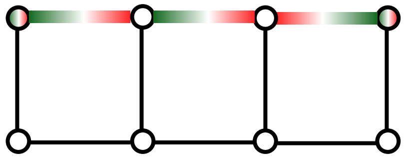


$E_{gs} \simeq 6g^2\Delta_\tau$

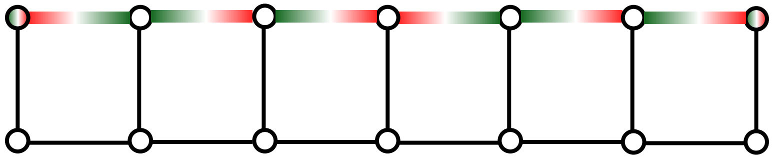
D_4

$\tau \notin \tau \otimes \tau$

$E_{gs} \simeq 3g^2\Delta_\tau$



$E_{gs} \simeq 6g^2\Delta_\tau$



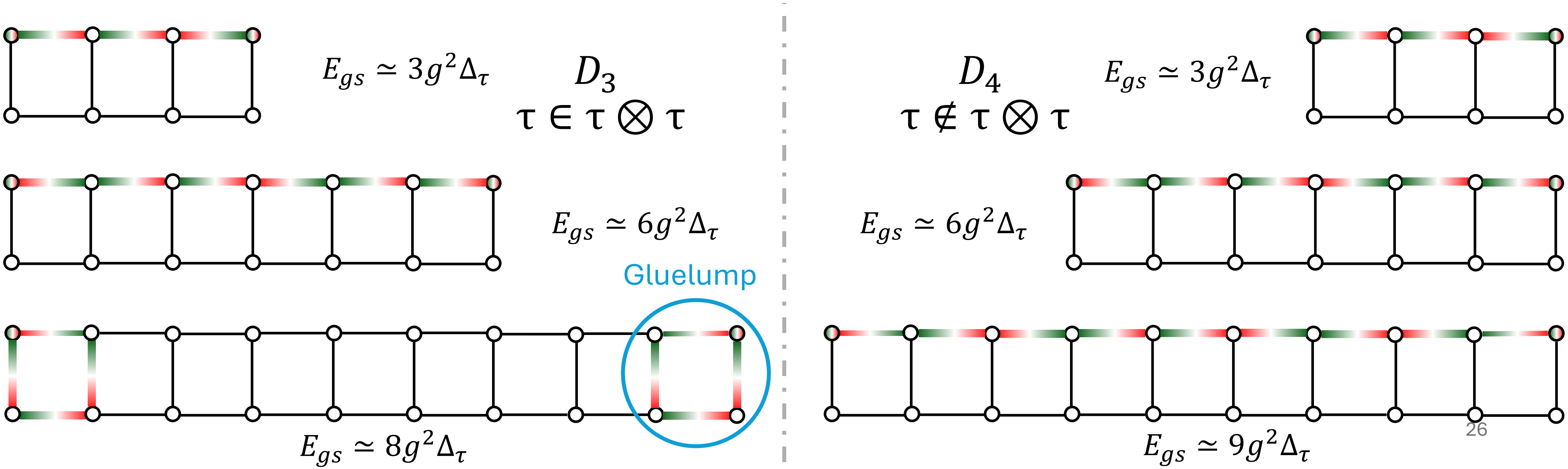
Ladder and GSs of D3 and D4

The simple $\tau \in \tau \otimes \tau$ or $\tau \notin \tau \otimes \tau$ has a huge impact on the physics of the LGT

We take a ladder with static charges on the upper corners, pure gauge Hamiltonian

$$H = H_E + H_B = g^2 \sum_l E_l^2 + \frac{2}{g^2} \sum_p \mathrm{Re}(\mathrm{Tr}(U_{p,1} U_{p,2} U_{p,3}^\dagger U_{p,4}^\dagger))$$

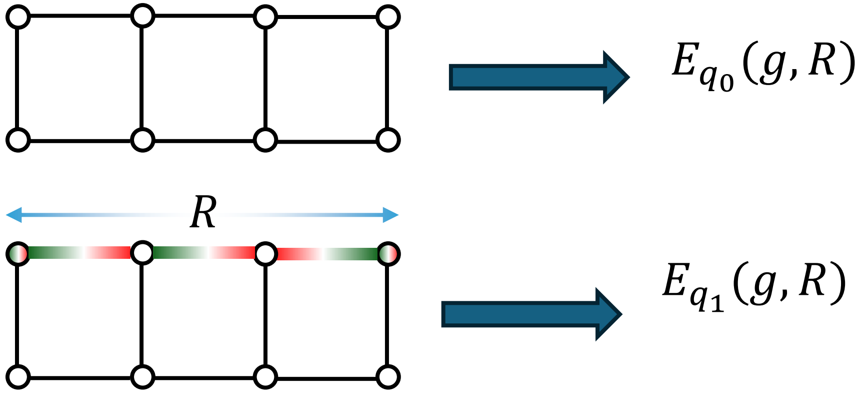
Let's consider the ground state at strong coupling $g^2 > 1$



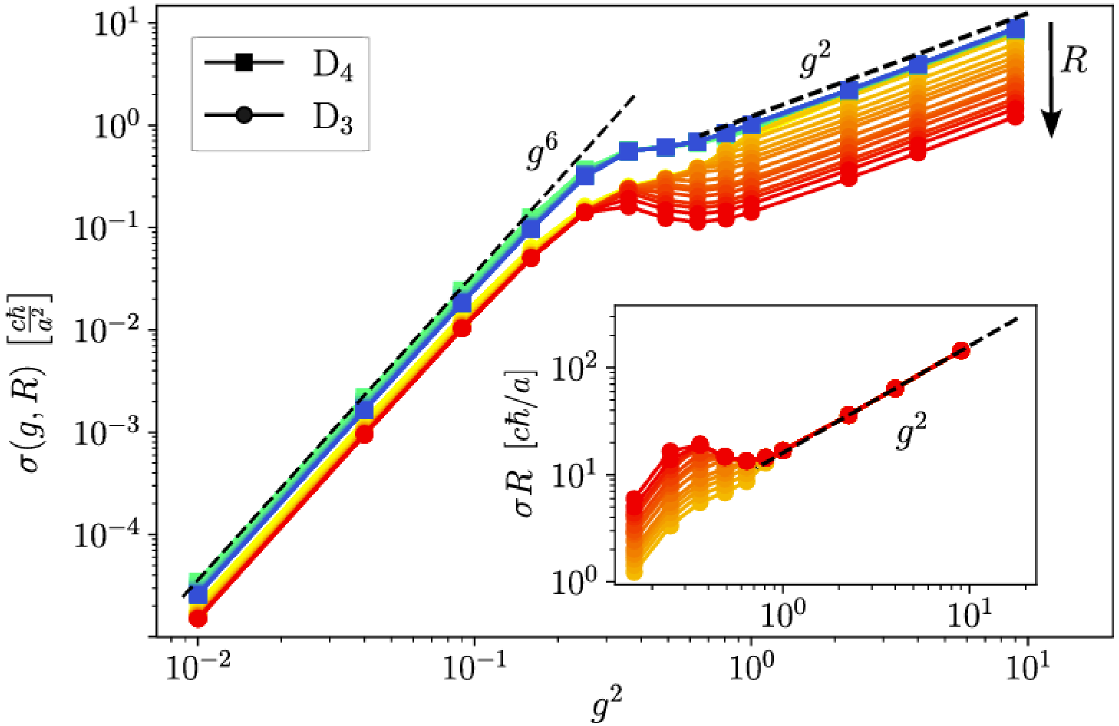
String tension

$$H = H_E + H_B = g^2 \sum_l E_l^2 + \frac{1}{g^2} \sum_p \mathrm{Re}(\mathrm{Tr}(U_{p,1}U_{p,2}U_{p,3}^\dagger U_{p,4}^\dagger))$$

DMRG study



String tension: $\sigma(g,R) = \frac{E_{q=1}(g,R) - E_{q=0}(g,R)}{R}$



The weak confinement $\sigma \propto g^6$ at low g^2 is due to the ladder geometry.

In D_4, σ is independent of R because of the field string connecting the charges

In D_3, σ decreasing with growing R indicates the string breaking!

Gluelumps mass and potential

$$H = H_E + H_B = ag^2 \sum_l E_l^2 + \frac{1}{ag^2} \sum_p \mathrm{Re}(\mathrm{Tr}(U_{p,1} U_{p,2} U_{p,3}^\dagger U_{p,4}^\dagger))$$

Renormalized contribution to the mass of the two gluelumps:

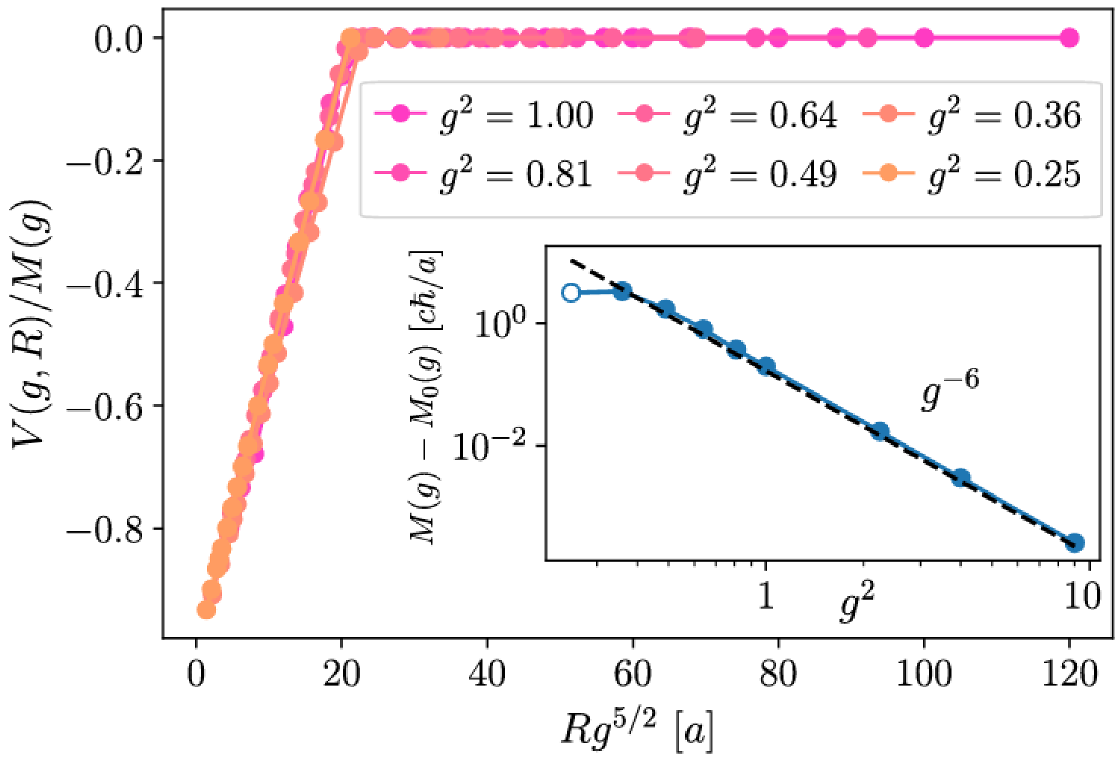
$$[E_{q=1}(g, R \to \infty) - E_{q=0}(g, R \to \infty)] = 2M(g)$$

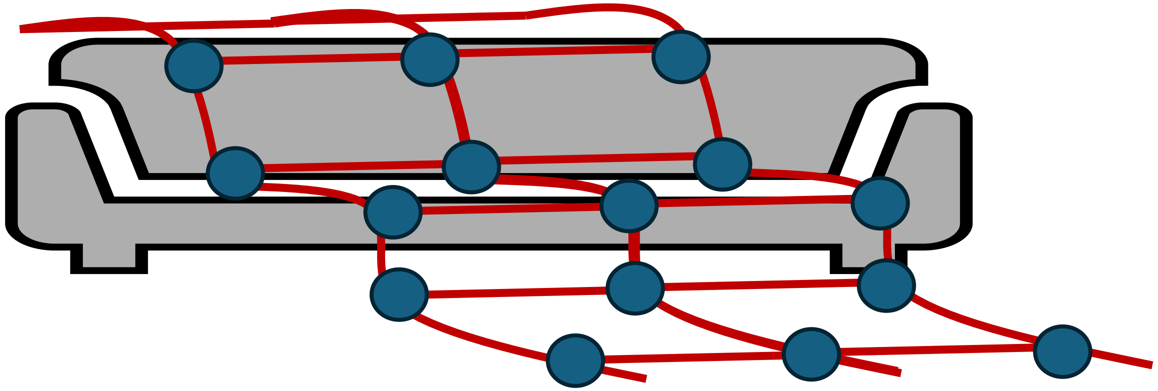
$$M(g) \simeq M_0(g) = 4g^2, \text{for } g^2 > 1$$

Scaling analysis of the interaction potential between gluelumps

$$V(g,R) = E_{q=1}(g,R) - E_{q=0}(g,R) - 2M(g)$$

Scaling with lattice spacing a indicates **a finite potential in the continuum limit**! ⇒ measurable scattering amplitudes.



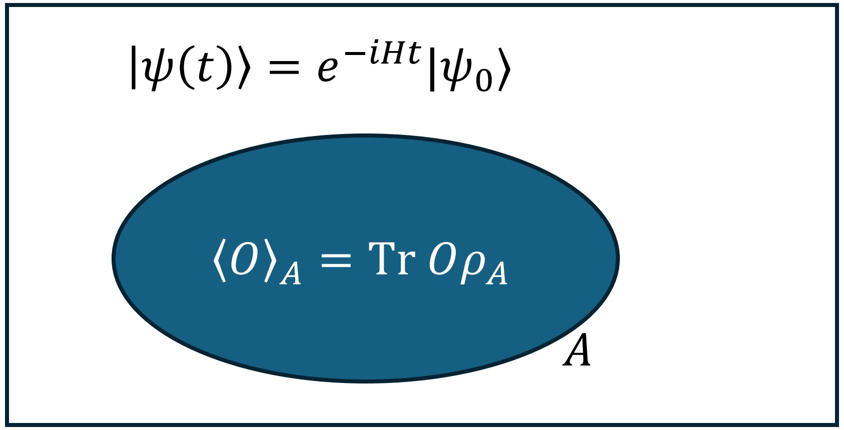


Effective thermalization

How do non-Abelian LGTs relax?

M.Sc. Thesis of Marco Vaia

Thermalization in lattice gauge theories



Eigenstate Thermalization hypothesis (ETH):

$$\langle O(t \to \infty)\rangle_A \simeq \frac{1}{Z}\sum_n e^{-\beta_{\text{eff}}E_n}\langle n|O|n\rangle$$

$$H|n\rangle = E_n|n\rangle$$

β_{eff} depends on $\langle\psi_0|H|\psi_0\rangle$

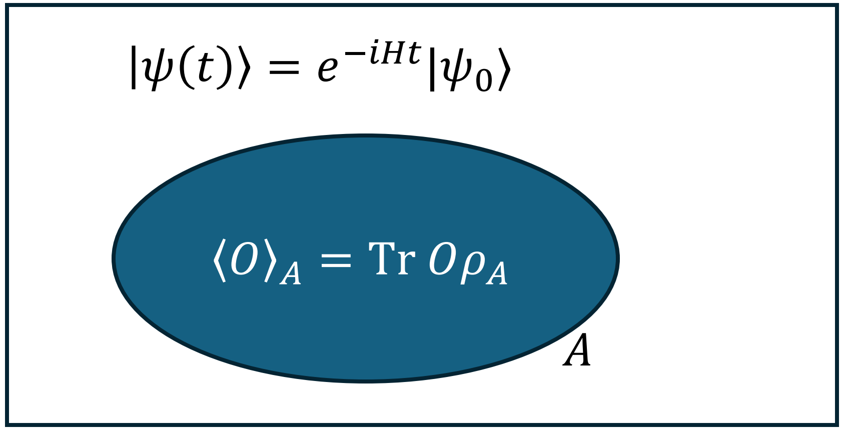
✓ Non-integrable (chaotic)

✕ Integrable and MBL

What to look for?

- Relaxation of local observables
- **Energy gap distribution**
- Spectral form factor
- **Entanglement spectrum**

Thermalization in lattice gauge theories



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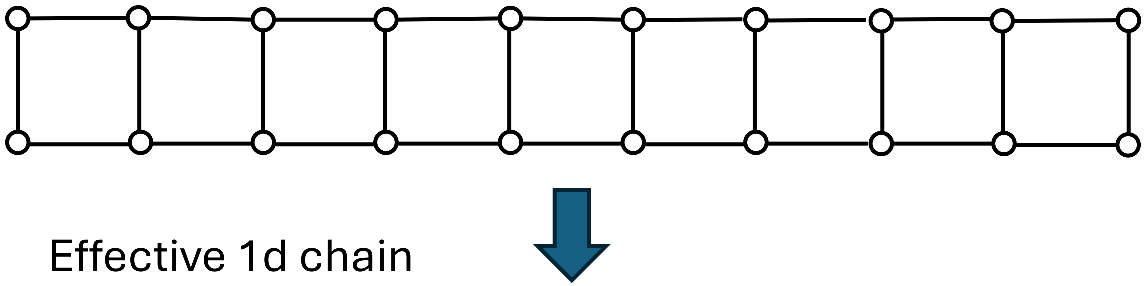
The quantum-many-body perspective:

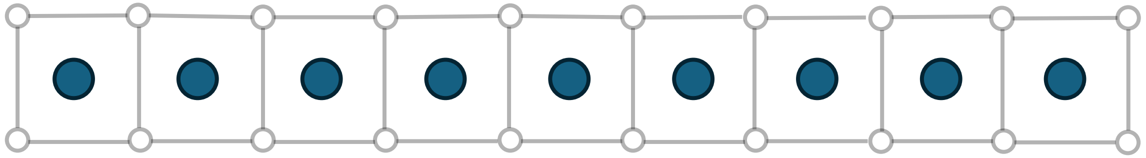
- Thermalization with extensive number of non-Abelian conserved charges
- ETH violation -> scar states

Thermalization in lattice gauge theories

Pure gauge LGT ladder



Effective 1d chain



Dual lattice

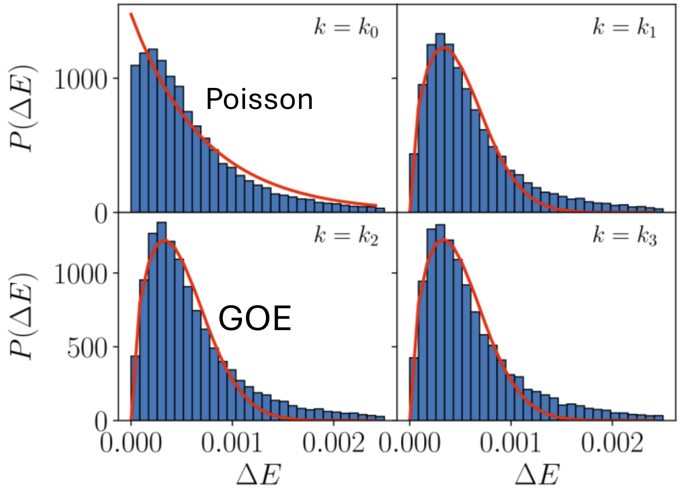
$\mathbb{Z}_N$

$$H_{\text{eff}} = -g^2 \sum_j \left[Z_j^\dagger Z_{j+1} + 2Z_j + \text{H.c.}\right] - \frac{1}{g^2} \sum_j \left[X_j + X_j^\dagger\right]$$

Clock model with longitudinal and transverse field

SU(2) (truncation to $J = 1/2$)

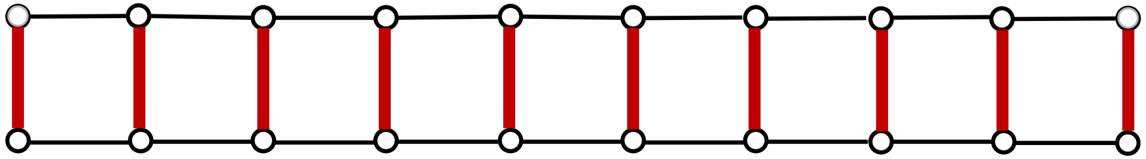
$$H_{\text{eff}} = -\frac{3g^2}{2} \sum_j \left[Z_j Z_{j+1} + 2Z_j\right] - 1/g^2 \sum_j \left(1 - 3Z_{j-1}\right) X_j \left(1 - 3Z_{j+1}\right)$$



Yao PRD 2021

Thermalization in lattice gauge theories

Pure gauge LGT ladder

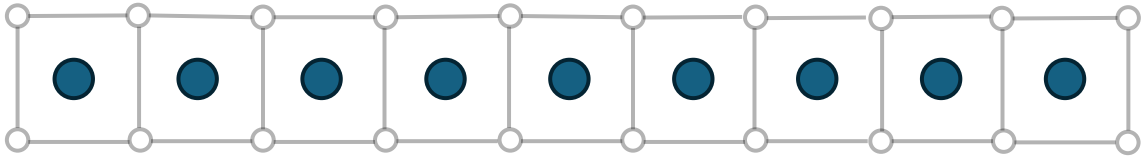


Gauge-fixed D_3 ladder

$$H = -\frac{1}{2g^2}\sum_r \mathrm{Tr}\left[U_r^\dagger U_{r+1}\right] + \mathrm{H.c.}$$

$$+g^2 \sum_r \sum_J \alpha_J \left[P_r^J + 2\prod_{r'<r} P_{r'}^J\right]$$

Effective 1d chain



Dual lattice

$\mathbb{Z}_N$

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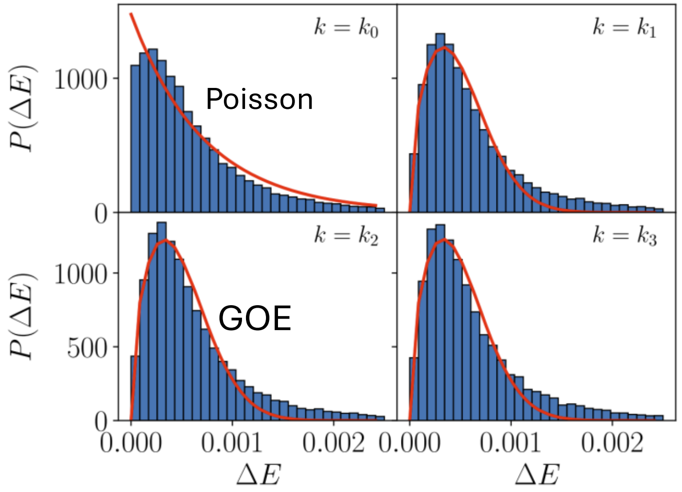

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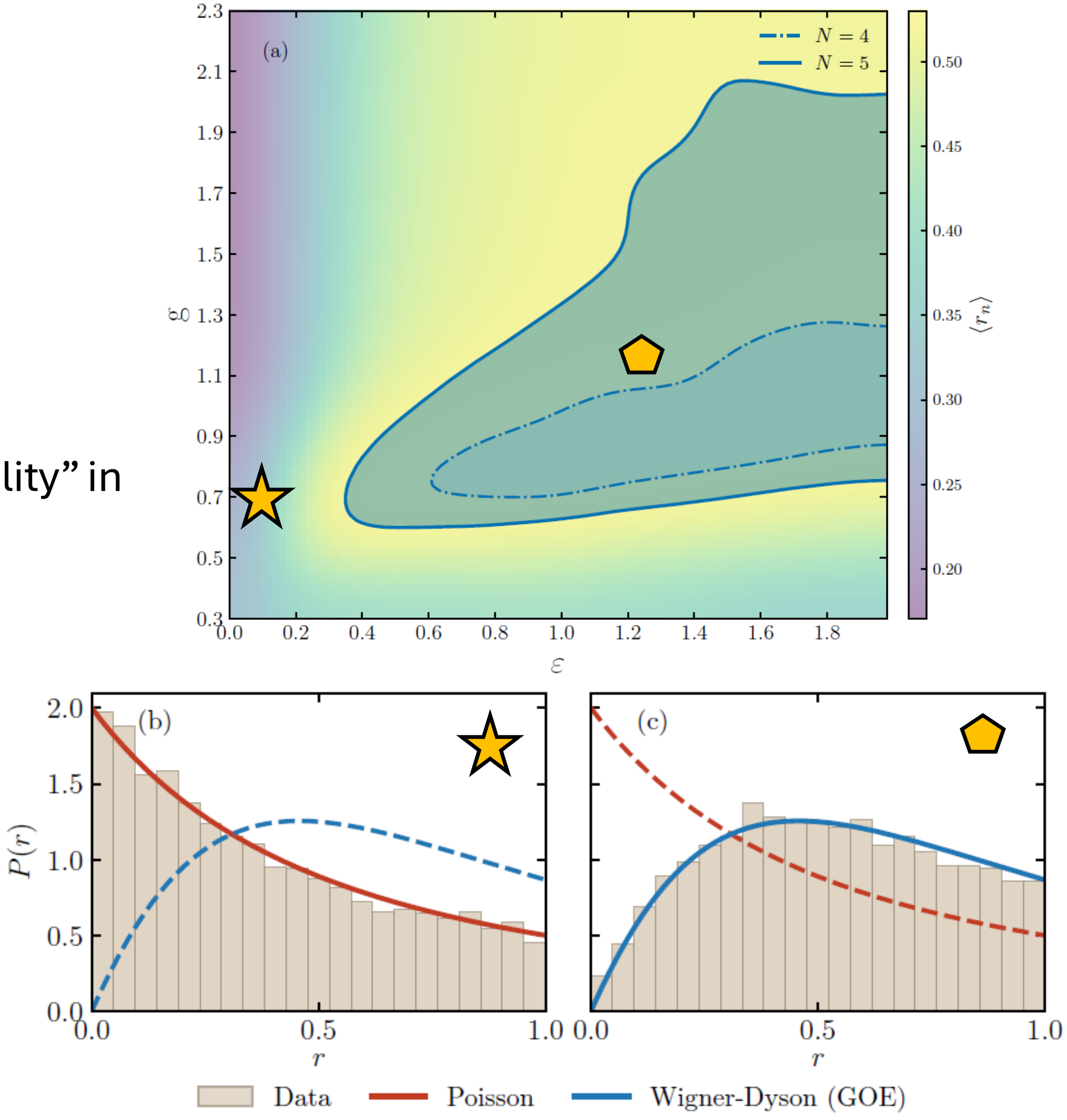
D_3 with a twist

$$H = -\frac{1}{2g^2}\sum_r \mathrm{Tr}\left[U_r^\dagger C U_{r+1} C^\dagger\right] + \mathrm{H.c.}$$
$$+g^2 \sum_r \sum_J \alpha_J \left[P_r^J + 2\prod_{r'<r} P_{r'}^J\right]$$

$C(\varepsilon) = \frac{e^{-\frac{i\pi}{4}}}{\sqrt{2}}\left[\mathbb{I} + \frac{i\varepsilon}{\sqrt{3}}(\sigma^x + \sigma^y - \sigma^z)\right] \Rightarrow$ introduces "chirality" in the interaction and breaks gauge invariance

Minimum gap ratio

$$r_n = \min\left(\frac{\mathrm{E}_{n+1} - \mathrm{E}_n}{\mathrm{E}_n - \mathrm{E}_{n-1}}, \frac{\mathrm{E}_n - \mathrm{E}_{n-1}}{\mathrm{E}_{n+1} - \mathrm{E}_n}\right)$$



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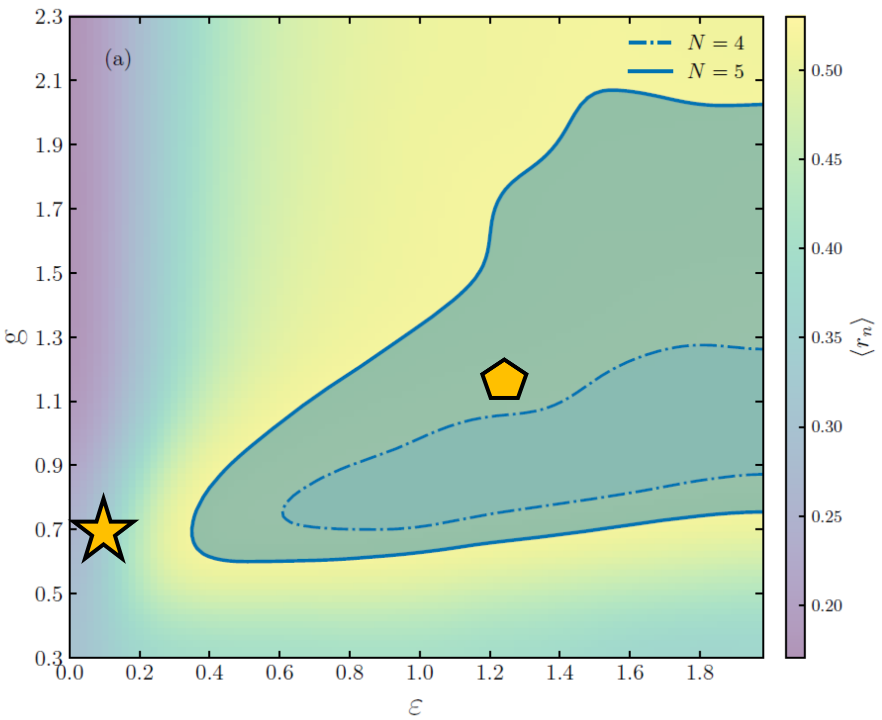
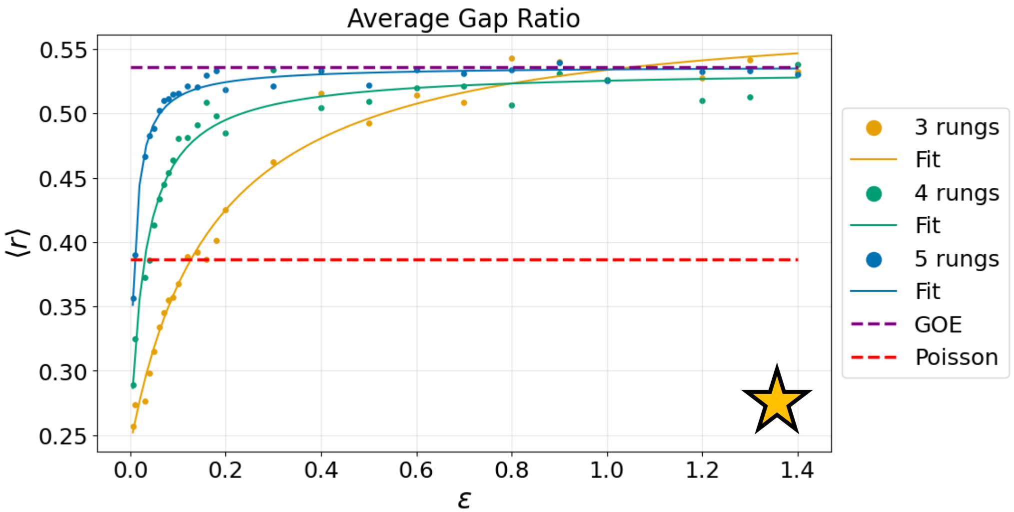
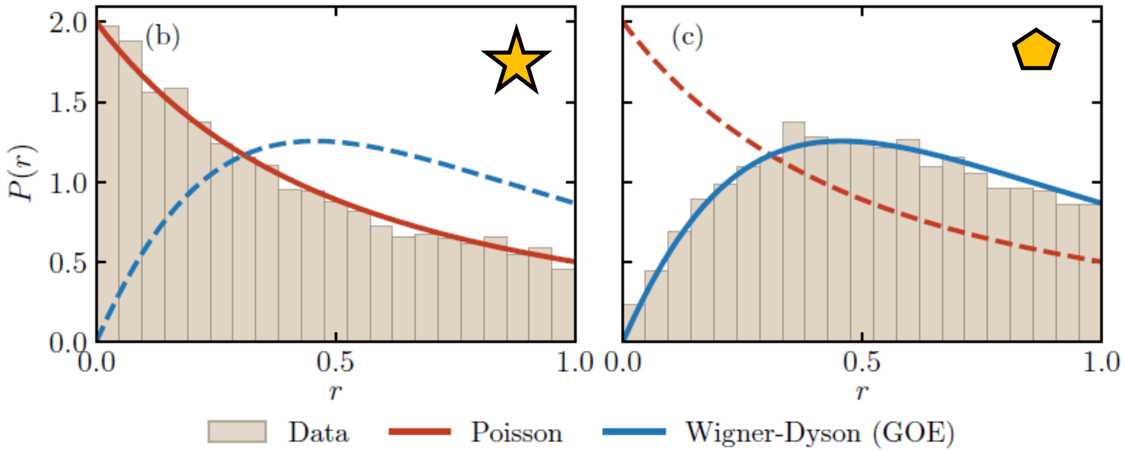






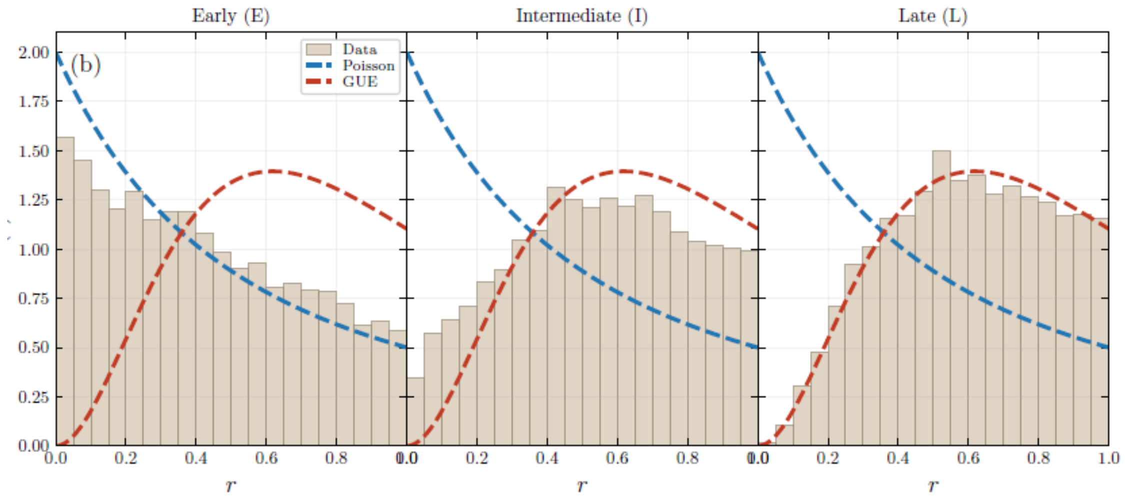
D_3 with a twist

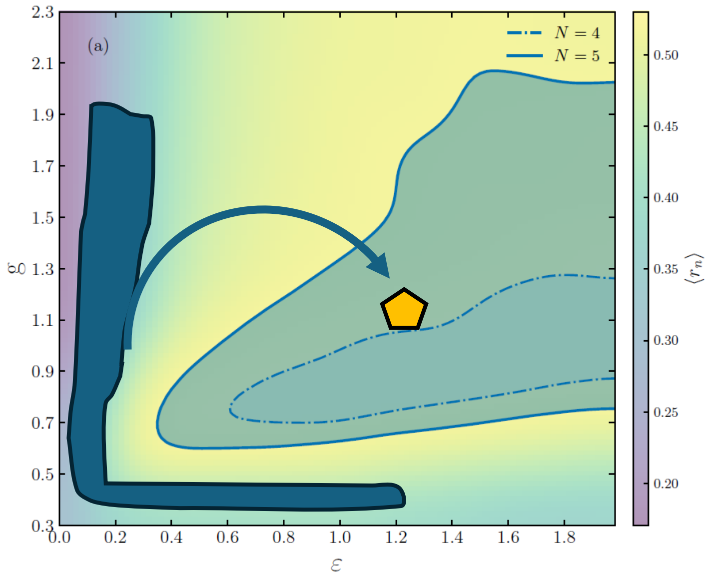
Thermalization in the dynamics: entanglement spectrum

$$|\psi(t)\rangle \Rightarrow \rho_{\mathrm{A}}(t) = \mathrm{Tr}_{\bar{\mathrm{A}}}|\psi(t)\rangle\langle\psi(t)|$$

$$\rho_{A(t)} = e^{-H_{\mathrm{ent}}(t)}$$

Level spacing statistics on $H_{\mathrm{ent}}(t)$





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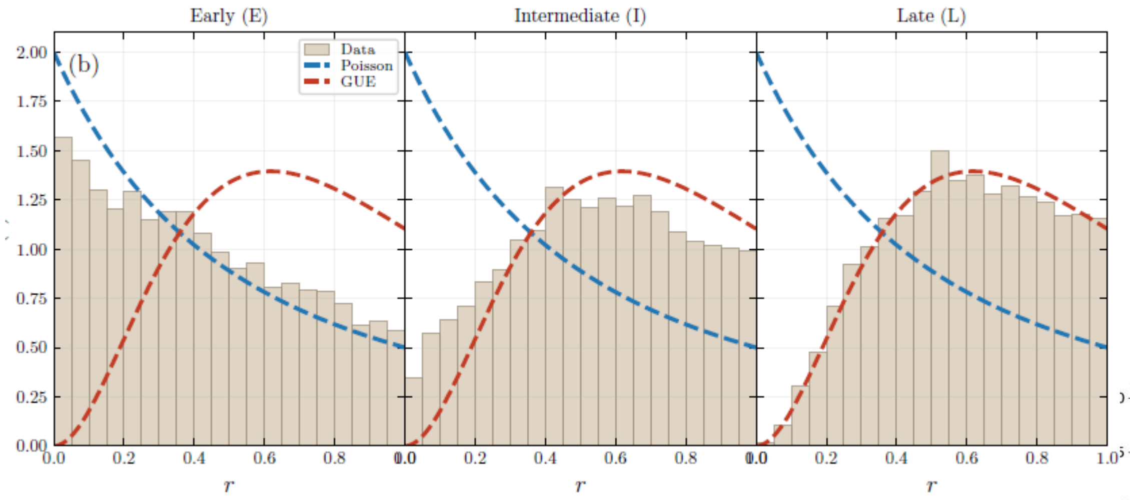
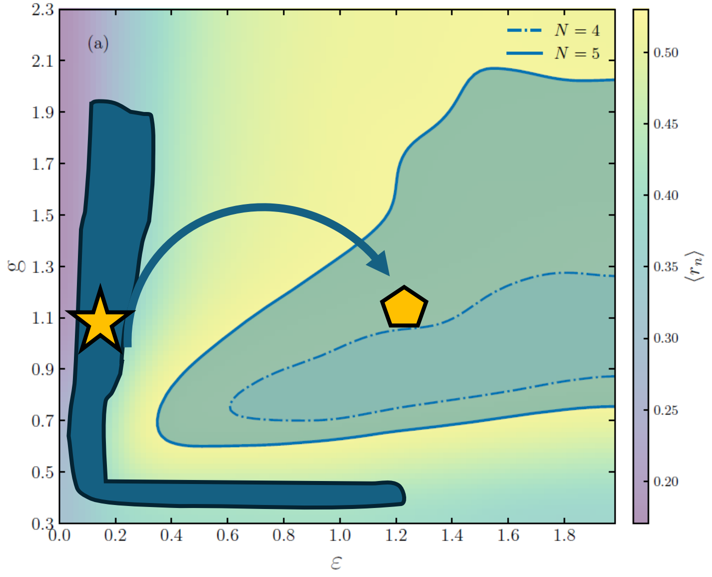
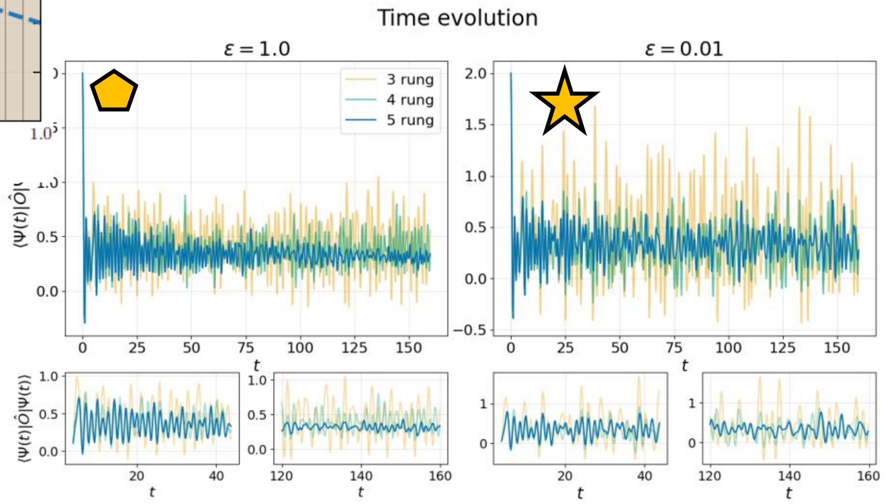




Dynamics

$|\psi_0\rangle = |GS\rangle$ of $H(g \ll 1)$

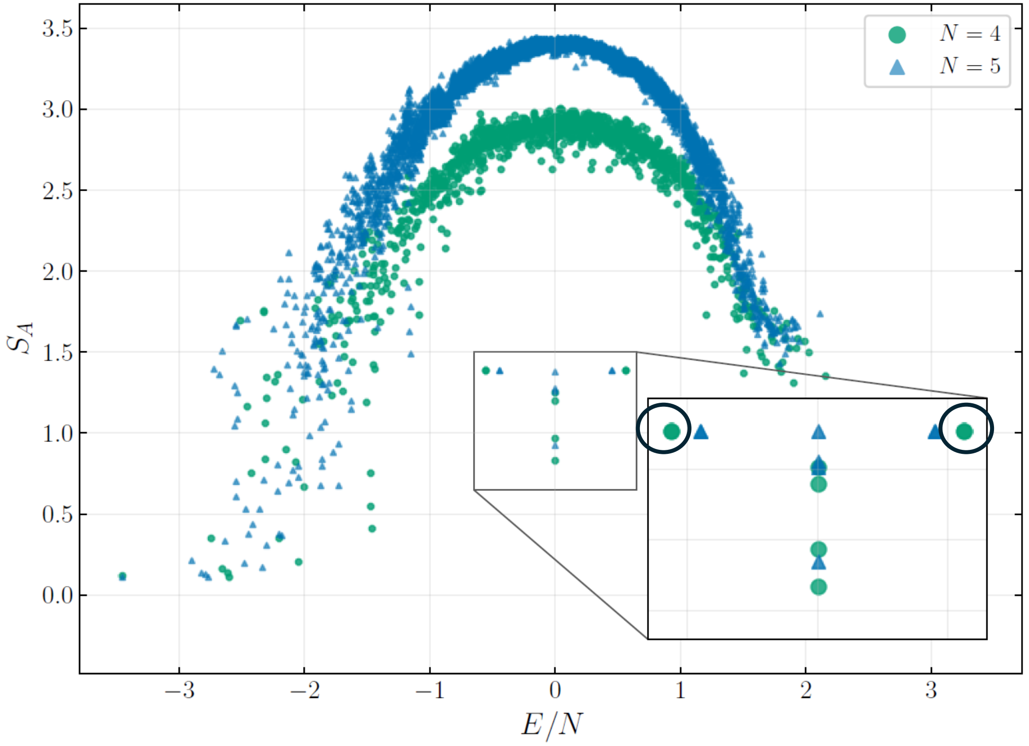
Local observable $O = 2\mathrm{Re}\left[\mathrm{Tr}\left(U_r^\dagger U_{r+1}\right)\right]$

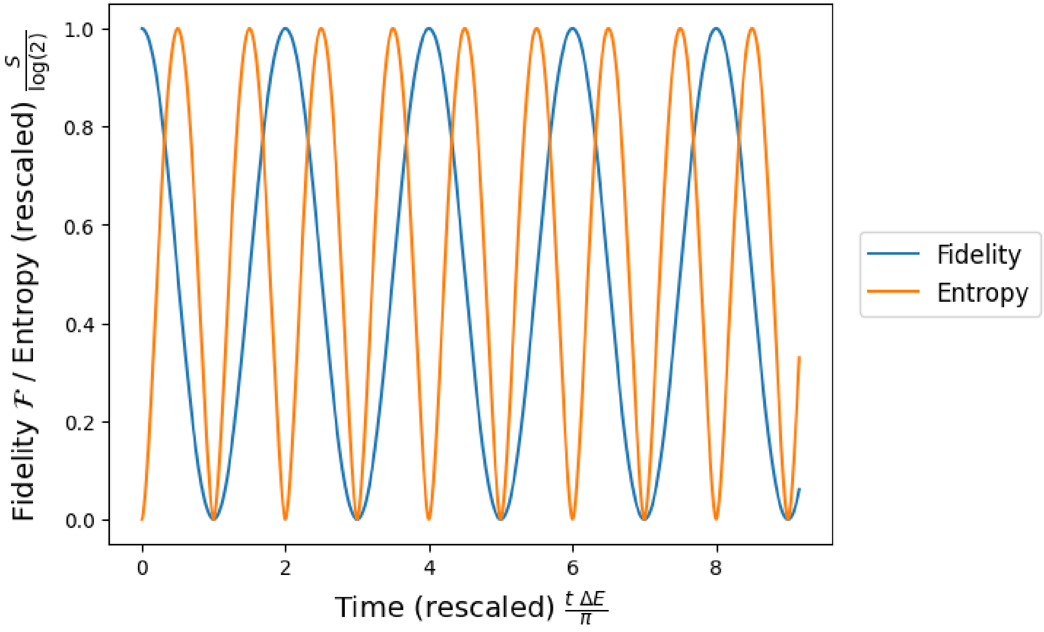


Better size scaling needed

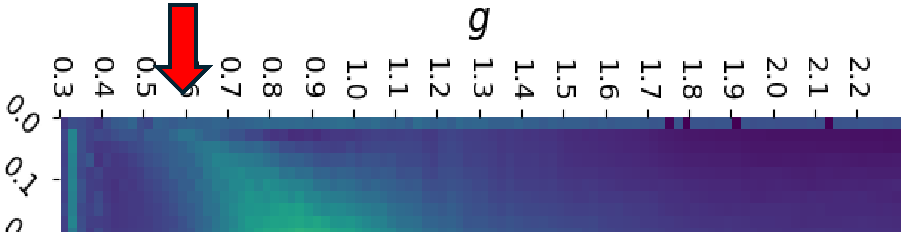
D_3 with a twist

ETH violation ⇒ **scar states**





From non-integrability to quantum resources?



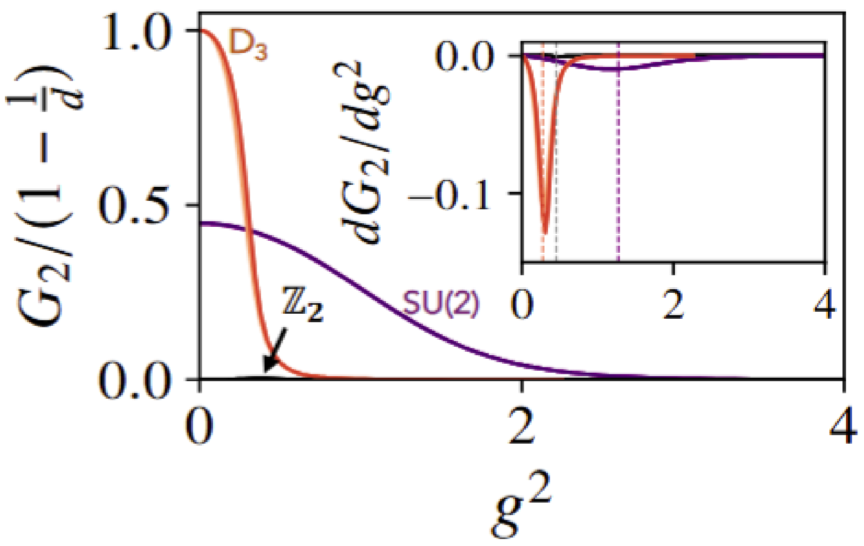
Quantum resources

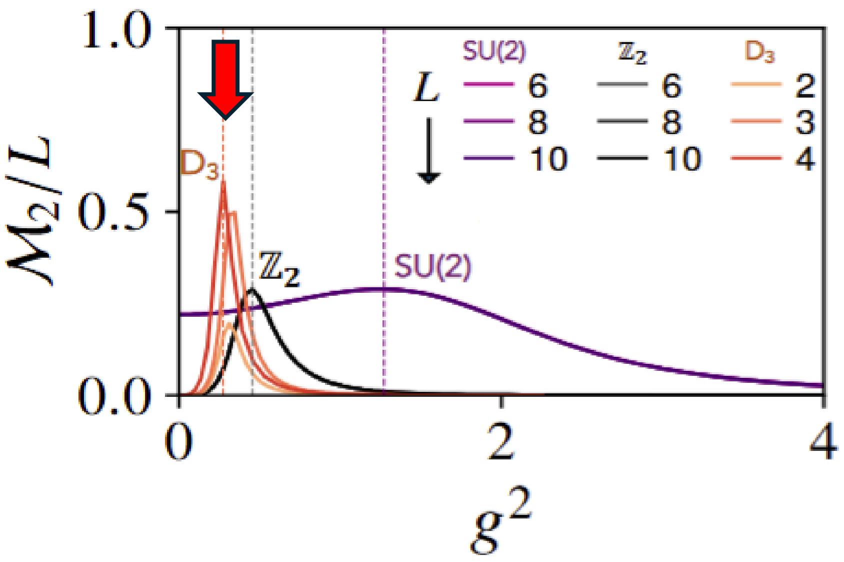
Generalized geometric measure (multipartite entanglement):

$$G_2(|\psi\rangle) = 1 - \max_{|\pi\rangle \in \mathcal{S}_2} |\langle \pi | \psi \rangle|^2$$

Stabilizer Reny Entropy (quantum magic)

$$\mathcal{M}_k(|\psi\rangle) = \frac{1}{1-k} \log \left[\sum_{P \in \mathcal{P}_L} \frac{|\langle \psi | P | \psi \rangle|^{2k}}{d^L} \right]$$





$\mathcal{S}_2$: set of 2-separale states

$\mathcal{P}_L$: generalized Pauli group on L d.o.f. with d local dimension

arXiv:2510.07385

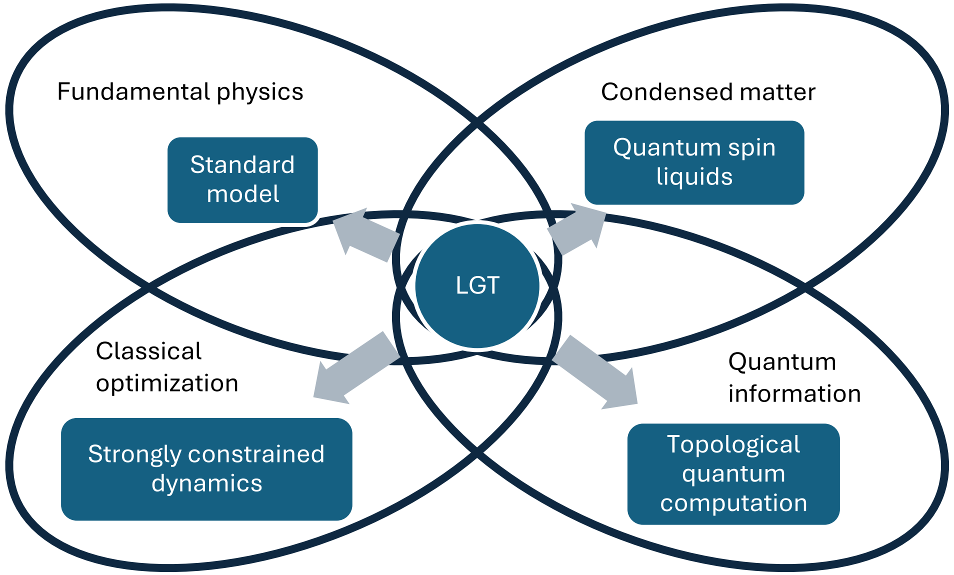
Conclusions

Lattice gauge theories are cool many-body models.

- Bridge between quantum-many body, high-energy physics, quantum information, classical optimization
- Rich physics
- Ideal benchmarks for quantum computers and simulators

Outlook:

- Confinement and screening in the dynamics
- String breaking
- Experiments



Acknowledgements

Marco Vaia

Thermalization

Edoardo Ballini
UniTn->UIBK

Philipp Hauke

Error mitigation

Julius Mildenberger
Uni. Leiden

Confinement

Michele
Burrello UNiPI

Pietro Silvi
UniPD

Pavel Popov
ICFO->?

Alberto Bottarelli
UniTn->free labour

Quantum
resources

Gopal Chandra Santra
Forschungszentrum Jülich

Funded by the European Union. Views and opinions expressed are however those of the author(s) only and do not necessarily reflect those of the European Union or the European Commission. Neither the European Union nor the granting authority can be held responsible for them.

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Cont

Thanks for the attention*!

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*If you didn't pay attention, I hope you have slept well, at least

Role of the centre

Why are Gluelump allowed in D_3 and not in D_4?

It depends on the centre of the group.

The center Z of a group G is an Abelian subgroup that commutes with every other element of G, i.e.,

$$Z = \{h \in G : \forall g \in G, hg = gh\}$$

D_3 centre is trivial, i.e., it is the identity element, while

D_4 centre is $\mathbb{Z}_2$: $Z = \left\{ \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \right\}$.

$$s = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \qquad r = \begin{bmatrix} \exp\left(\frac{2\pi i}{\mathrm{N}}\right) & 0 \\ 0 & \exp\left(-\frac{2\pi i}{\mathrm{N}}\right) \end{bmatrix}$$

We have shown that the presence of the non-trivial Z_2 centre gives vanishing Clebsh-Gordan coefficients for $\tau \otimes \tau \to \tau$.

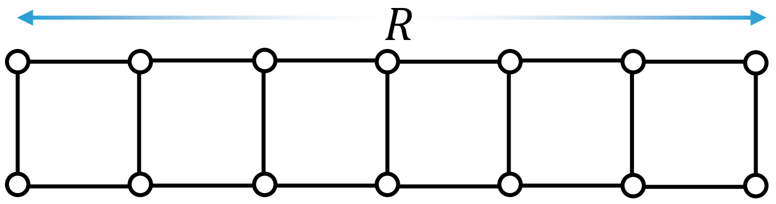
↓

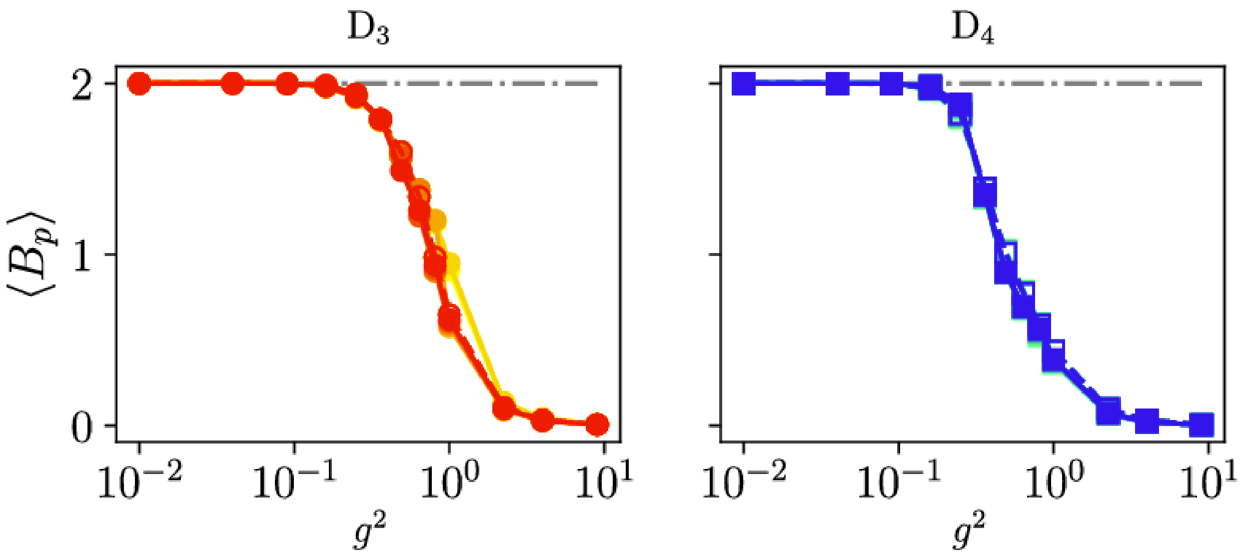
This happens for all even N

Phase diagram

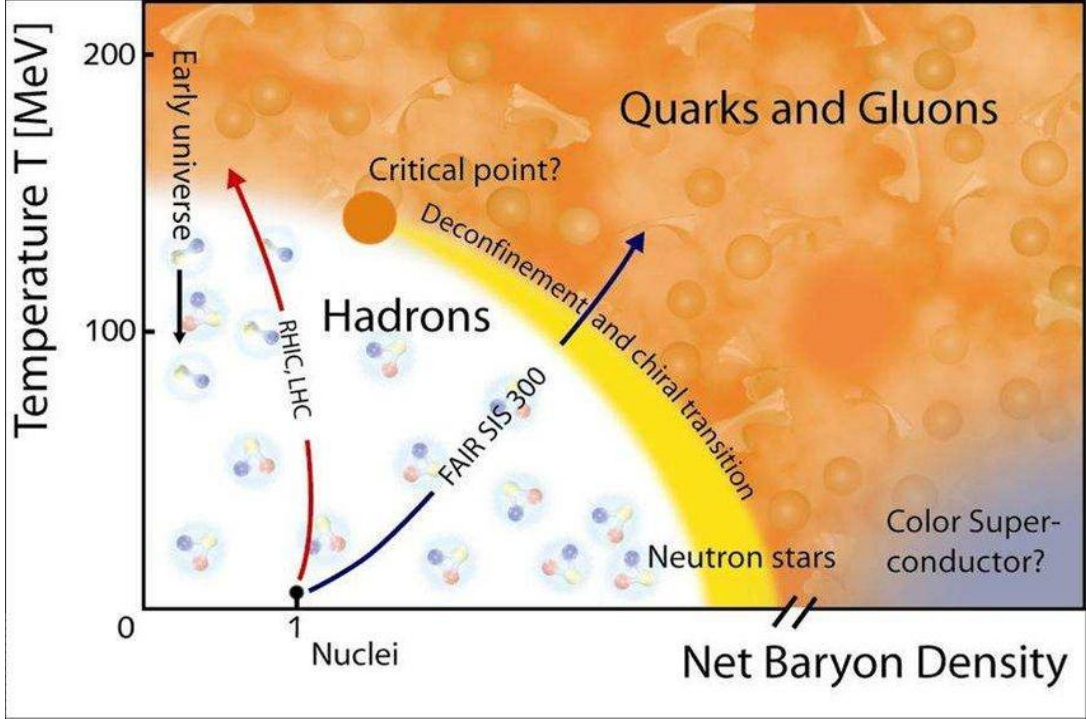
$$H = H_E + H_B = g^2 \sum_l E_l^2 + \frac{1}{g^2} \sum_p \mathrm{Re}(\mathrm{Tr}(U_{p,1} U_{p,2} U_{p,3}^\dagger U_{p,4}^\dagger))$$

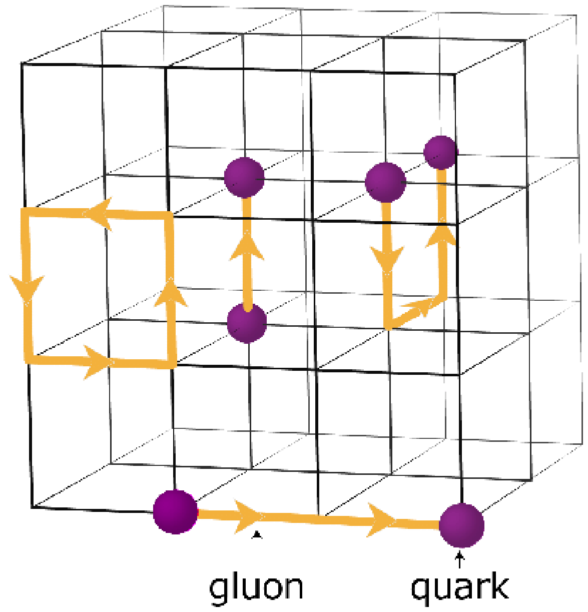
Magnetic energy density for different system sizes R, with the presence and without presence of static charges on the corners





Lattice QCD





Credit: Lattice QCD GPU Inverters on ROCm Platform

- Gauge theories are the backbone of the standard model
- QCD Lagrangian on a discrete spacetime grid, Wick rotation to Euclidean time;
- Observables are calculated using the Path Integral formalism;
- Monte Carlo methods for probability distribution of gauge configurations.

Local symmetries in lattice gauge theories

Gauss's Law for the electric field $\boldsymbol{\nabla} \cdot \boldsymbol{E} = -\rho$

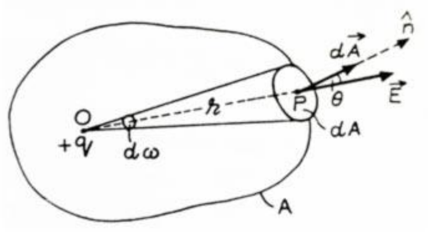
$$d\Phi_E = \vec{\mathbf{E}} \bullet d\vec{\mathbf{A}} = E\, dA \cos\theta\,,$$

$$d\Phi_E = \frac{q}{4\pi\varepsilon_0} \frac{dA \cos\theta}{r^2}$$

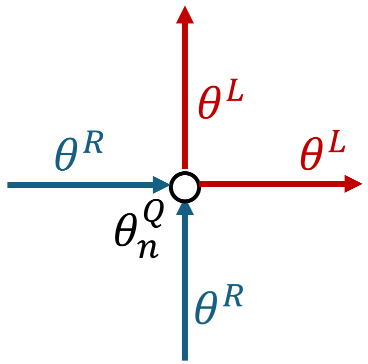
$$d\Phi_E = \frac{q}{4\pi\varepsilon_0} d\omega$$

$$\Phi_E = \oint \vec{\mathbf{E}} \bullet d\vec{\mathbf{A}} = \frac{q}{4\pi\varepsilon_0} \oint d\omega$$

$$\boxed{\Phi_E = \frac{q}{\varepsilon_0}}\,.$$



Gauss's Law in general LGT



Local symmetry: $\Theta_{g,n} = \prod_i \theta_{g,i}^R \prod_o \theta_{g,o}^L \;\; \theta_{g,n}^Q$

$$\theta_g^R |h\rangle = |hg^{-1}\rangle, \qquad \theta_g^L |h\rangle = |gh\rangle$$

$$[H, \Theta_{g,n}] = 0 \; \forall g, n$$

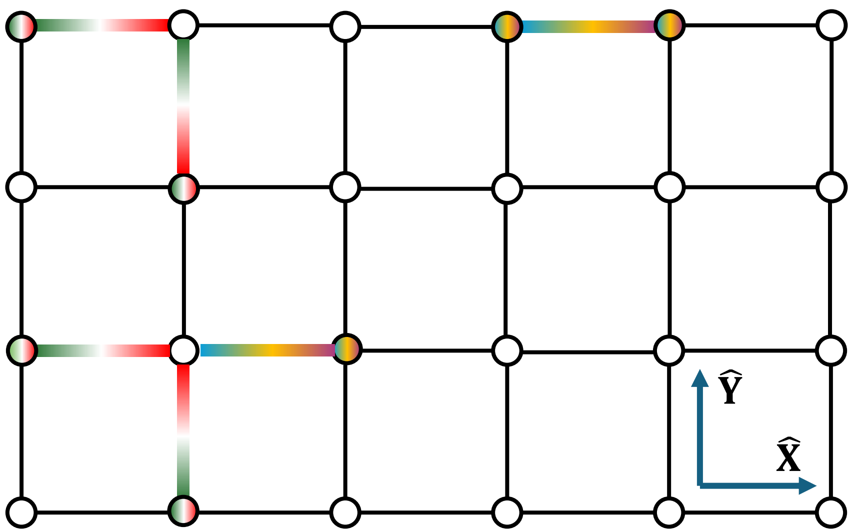
$$\Theta_{g,n} |\psi\rangle_{\text{phys}} = \alpha_{g,n} |\psi\rangle_{\text{phys}}$$

The set of phases $\alpha_{g,n}$ defines the gauge sector

Fusion rules

Electric fields on the links live in the Hilbert space of the irreducible representations (irreps) of the group.

SU(2):
- no particles → **0** irrep
- single fermion → **½-spin**
- 2 fermions → **1-spin**
- ⋮



The fusion rules tell us how to construct gauge invariant states

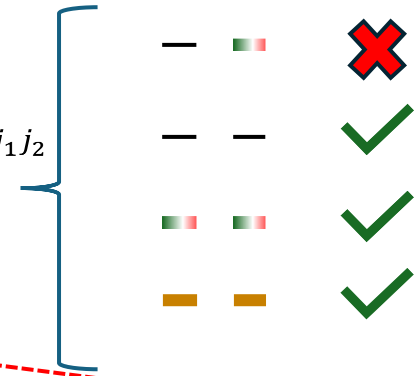
1/2×1/2	1, +1	1, 0	0, 0	1, −1
+1/2 +1/2	1			
+1/2 −1/2		1/2	1/2	
−1/2 +1/2		1/2	−1/2	
−1/2 −1/2				1

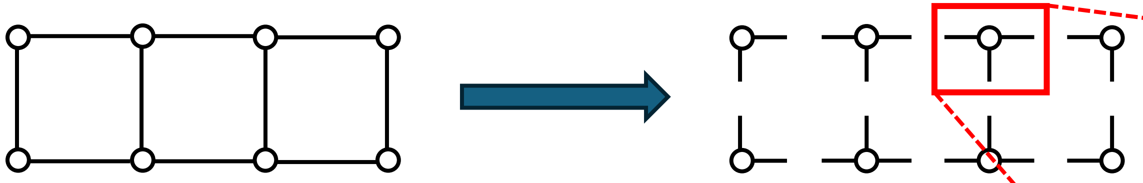
$$1/2 \otimes 1/2 = 0 \oplus 1$$

DMRG simulations

We wrote the Hamiltonian in the rishon formalism

$U_{mn} = |j; m, n\rangle \quad U_{mn} = |j_1; m\rangle |j_2; n\rangle \delta_{j_1 j_2}$





Gauge invariant basis

$$|1\rangle = |0,0,0\rangle \quad |2\rangle = \frac{|r,0,r\rangle + |g,0,g\rangle}{\sqrt{2}}$$

$$|3\rangle = \frac{|0,r,r\rangle + |0,g,g\rangle}{\sqrt{2}} \quad |4\rangle = \frac{|r,g,0\rangle + |g,r,0\rangle}{\sqrt{2}}$$

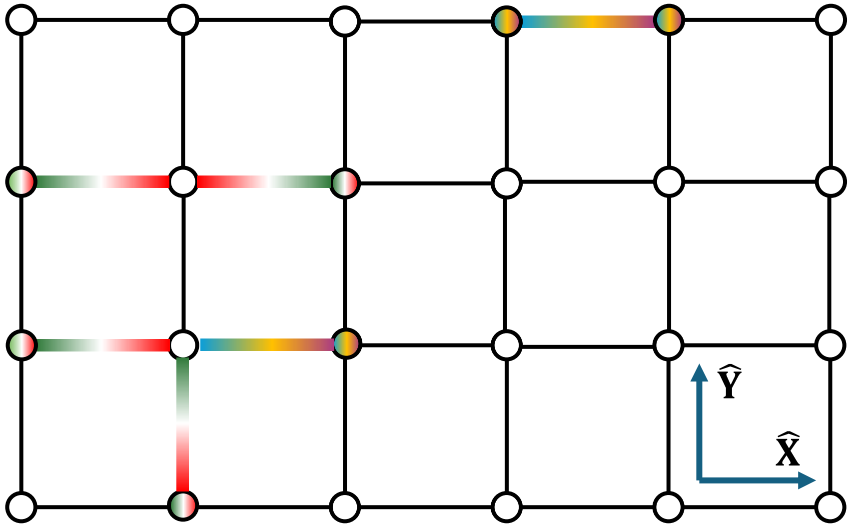
$$|5\rangle = \frac{|r,r,g\rangle + |g,g,r\rangle}{\sqrt{2}} \quad |6\rangle = \frac{|r,p,r\rangle - |g,p,g\rangle}{\sqrt{2}}$$

$$|7\rangle = \frac{|p,r,r\rangle - |p,g,g\rangle}{\sqrt{2}} \quad |8\rangle = \frac{|r,g,p\rangle - |g,r,p\rangle}{\sqrt{2}}$$

$$|9\rangle = |p,p,0\rangle \quad |10\rangle = |p,0,p\rangle \quad |11\rangle = |0,p,p\rangle \ .$$

Fusion rules and gauge invariant states in D_3

D_3: → no particles — **0** irrep
→ single fermion — τ irrep
→ 2 fermions — p irrep



$$SU(2):\ 1/2 \otimes 1/2 = 0 \oplus 1$$

One important difference with SU(2)

$$D_3:\ \tau \otimes \tau = 0 \oplus \boxed{\tau} \oplus p,$$

but

$$D_4:\ \tau \notin \tau \otimes \tau$$