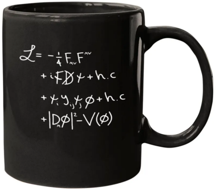
Simulating particle production from classical fields using quantum computation



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June 17

C3NT workshop: Quantum Information Science in High Energy Nuclear Physics, June 14-19, 2026

Based on Iván Cuntín, Wenyang Qian and BW, work in progress.

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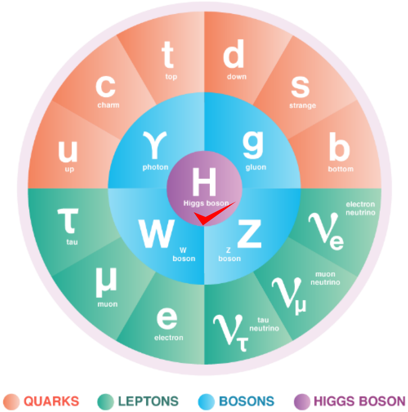
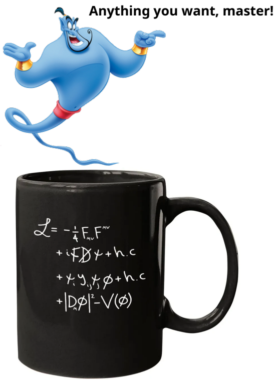
Classical Fields vs Particles

The standard model of particle physics

- **The experimental completion of the SM at the LHC**





- **In principle**, everying in SM can be calculated from

$$\mathcal{L} = \mathcal{L}_{QCD} + \mathcal{L}_{EW}$$

.

Real-time dynamics and thermalization

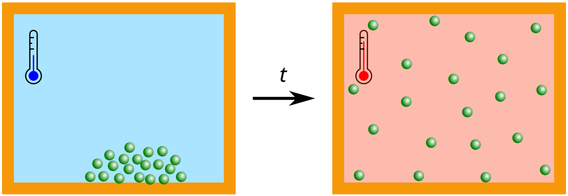
- **In practice**, there are many open issues, especially in QCD:

  1. Hadronic/nuclear wave functions
  2. PDFs from QCD first principles
  3. Hadronization

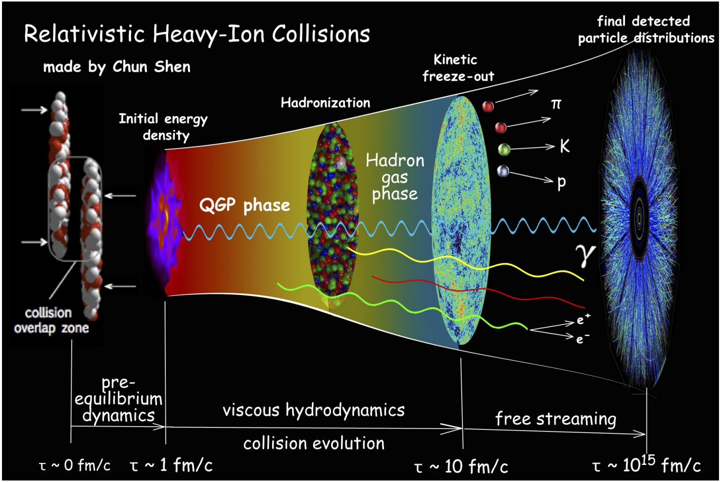
  ...

Quantum computing could provide a way out (?)

- This talk focuses on another one: **thermalization**



Real-time dynamics in heavy-ion collisions



Least understood stage: from to COLD nuclei to HOT QGP ($\tau < 1$ fm/c)

Complete description from QCD first principles?

- **One needs:**

1. Solve the hadronic or nuclear wave functions from H: $|\psi\rangle$
2. Evaluate the evoluiton operator:

$$U(t, t_0) = e^{-iH(t-t_0)}$$

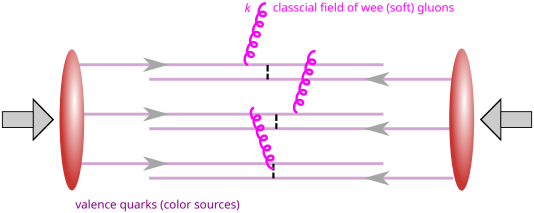
3. Calculate the observable: $\hat{O} = T^{\mu\nu}$, a hadron, a jet, $\cdots$

$$O(t) \equiv \langle\psi|U^\dagger(t, t_0)\hat{O}U(t, t_0)|\psi\rangle$$

We are already stuck at step 1: $|\psi\rangle$ is not completely known.

Let us start with Color-Glass Condensate initial state

- **Let us take the partonic picture (to replace step 1):**



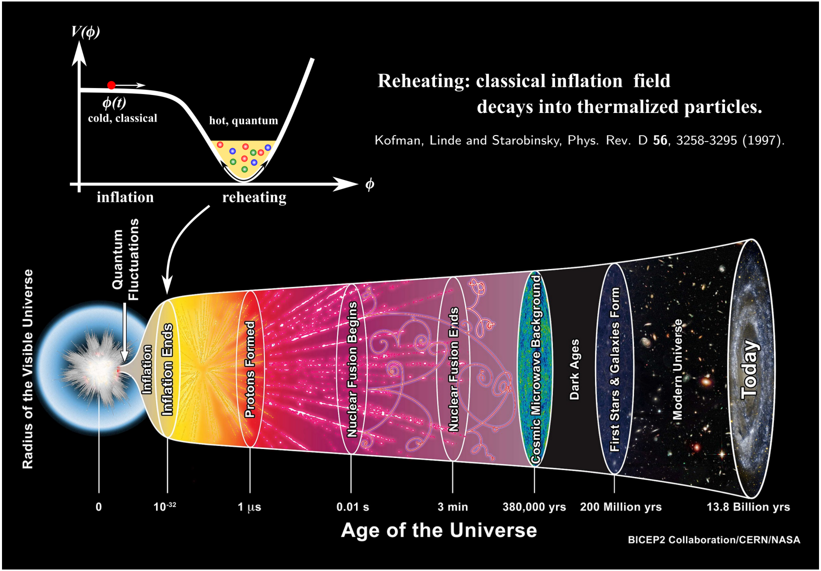
Bjorken, Lect. Notes Phys. **56**, 93 (1976).

1. The valence quarks pass through each other.
2. Produce a dense field of soft gluons $\Rightarrow$ classical field description.
3. In CGC: $k \sim Q_s > \Lambda_{QCD}$

A comprehensive review: Kovchegov and Levin, Camb. Monogr. Part. Phys. Nucl. Phys. Cosmol. **33**, 1 (2012). .

How do classical fields generate thermalized particles?

Another example: reheating in early universe



The main topic of this talk

- **Real-time dynamics in QFT starting from a coherent field**

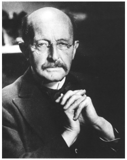
- **The QFT:** ϕ^4 theory as a warm-up

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 - \frac{\lambda}{4!}\phi^4.$$

- **Why quantum computing?**

1. In classical field approximation, T depends on UV cutoff

   Thermal equilibrium distribution: $f_p = \frac{T}{\underbrace{\omega_p - \mu}_{O(\hbar^0)}}$

   The ultraviolet catastrophe was remedied by Planck ☞

   T. Epelbaum, F. Gelis, N. Tanji and BW, Phys. Rev. D **90**, no.12, 125032 (2014) [arXiv:1409.0701 [hep-ph]].

2. Classical field + vacuum quantum fluctuations is nonrenormalizable.

   T. Epelbaum, F. Gelis and BW, Phys. Rev. D **90**, no.6, 065029 (2014) [arXiv:1402.0115 [hep-ph]].

Scalar field theory on the lattice

The spatial lattice

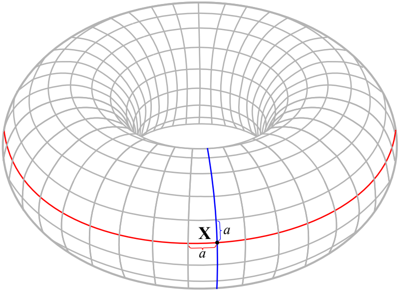
- Take t **to be continuous and x to take discrete values**

- The QFT lives on **a lattice in d spatial dimensions:**

$$\mathbb{V} \equiv \left\{ \mathbf{x} = a\mathbf{n} \;\middle|\; n_j \in \{0, 1, \ldots, N-1\}, j = 1, \ldots, d \right\},$$

where a is the lattice spacing, $L = aN$, and $\mathbf{n} = (n_1, n_2, \ldots, n_d)$ denotes a d-dimensional integer vector.

- Impose **periodic boundary conditions $\Rightarrow$ d-dimensional torus**



- **The corresponding momentum-space lattice:**

$$\mathbb{P} \equiv \left\{ \mathbf{p} = \Delta p\, \mathbf{n} \,\middle|\, n_j \in \left\{ -\left\lfloor \tfrac{N-1}{2} \right\rfloor, \ldots, \left\lfloor \tfrac{N}{2} \right\rfloor \right\},\ j = 1, \ldots, d \right\},$$

where the momentum-space lattice spacing $\Delta p \equiv 2\pi/L$, and the *floor function* is defined as $\lfloor x \rfloor$ = the largest integer n such that $n \leq x$, yielding

$$\left\lfloor \tfrac{N-1}{2} \right\rfloor, \ldots, \left\lfloor \tfrac{N}{2} \right\rfloor \Big\} = \begin{cases} \{-(N-1)/2, \cdots, (N-1)/2\} & \text{if } N \text{ is odd} \\ \{-N/2+1, \cdots, N/2\} & \text{if } N \text{ is even} \end{cases}.$$

- **Momentum-space quantities are periodic with period $2\pi/a$:**

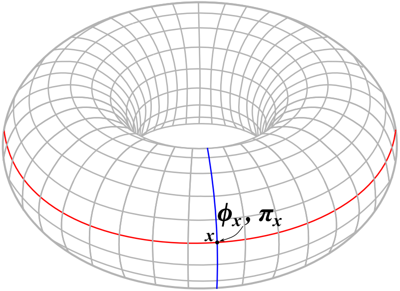
$$O_{\mathbf{p}} = O_{\mathbf{p}+\frac{2\pi}{a}\hat{e}_j}, \qquad j = 1, \ldots, d$$

where $\hat{e}_j$ is the unit vector along the j-th spatial axis.

- **The Kronecker delta in momentum space:**

$$\delta_{\mathbf{p},\mathbf{q}} = 1 \quad \text{if } \mathbf{p} - \mathbf{q} = \tfrac{2\pi}{a}\mathbf{n} \text{ for some } \mathbf{n} \in \mathbb{Z}^d, \quad \text{and 0 otherwise.}$$

The scalar field on the lattice



- **The Hamiltonian:** $H = a^d \sum_{\mathbf{x}} \left[\frac{1}{2}\hat{\pi}_{\mathbf{x}}^2 + \frac{1}{2} m^2 \hat{\phi}_{\mathbf{x}}^2 + \frac{1}{2}(\nabla \hat{\phi}_{\mathbf{x}})^2 + V(\hat{\phi}_{\mathbf{x}}) \right]$

with $\sum_{\mathbf{x}} (\nabla_j \hat{\phi}_{\mathbf{x}})^2 = -\sum_{\mathbf{x}} \hat{\phi}_{\mathbf{x}} \left[\hat{\phi}_{\mathbf{x}+a\hat{e}_j} - 2\hat{\phi}_{\mathbf{x}} + \hat{\phi}_{\mathbf{x}-a\hat{e}_j} \right] / a^2$.

- **Canonical commutation relations:**

$$[\hat{\phi}_{\mathbf{x}}, \hat{\phi}_{\mathbf{y}}] = [\hat{\pi}_{\mathbf{x}}, \hat{\pi}_{\mathbf{y}}] = 0, \qquad [\hat{\phi}_{\mathbf{x}}, \hat{\pi}_{\mathbf{y}}] = i a^{-d} \delta_{\mathbf{x},\mathbf{y}}.$$

- **Define $\hat{a}_{\mathbf{p}}$ and $\hat{a}^{\dagger}_{\mathbf{p}}$**

$$H = \sum_{\mathbf{p}} \omega_{\mathbf{p}} \left[\hat{a}^{\dagger}_{\mathbf{p}} \hat{a}_{\mathbf{p}} + \frac{1}{2} \right] + a^d \sum_{\mathbf{x}} V(\hat{\phi}_{\mathbf{x}})$$

with

$$\omega_p = \sqrt{m^2 + |\mathbf{p}_a|^2} \qquad \text{with } p^j_a \equiv \frac{2}{a} \sin\left(\frac{ap^j}{2}\right).$$

$$[\hat{a}_{\mathbf{p}}, \hat{a}_{\mathbf{p}'}] = 0 = [\hat{a}_{\mathbf{p}}, \hat{a}_{\mathbf{p}'}], \qquad [\hat{a}_{\mathbf{p}}, \hat{a}^{\dagger}_{\mathbf{p}'}] = \delta^{(d)}_{\mathbf{p},\mathbf{p}'}$$

and

$$\hat{\phi}_{\mathbf{x}} = \frac{1}{L^{\frac{d}{2}}} \sum_{\mathbf{p}} \frac{e^{i\mathbf{p}\cdot\mathbf{x}}}{\sqrt{2\omega_{\mathbf{p}}}} (\hat{a}_{\mathbf{p}} + \hat{a}^{\dagger}_{-\mathbf{p}}), \qquad \hat{\pi}_{\mathbf{x}} = -\frac{i}{L^{\frac{d}{2}}} \sum_{\mathbf{p}} e^{i\mathbf{p}\cdot\mathbf{x}} \sqrt{\frac{\omega_{\mathbf{p}}}{2}} (\hat{a}_{\mathbf{p}} - \hat{a}^{\dagger}_{-\mathbf{p}}).$$

- $\hat{a}^{\dagger}_{\mathbf{p}} \hat{a}_{\mathbf{p}}$: the number density operator in phase space

Jordan-Lee-Preskill Basis

- **Representation using eigenstates of $\hat{\phi}_{\mathbf{x}}$ and $\hat{\pi}_{\mathbf{x}}$** (take $a = 1$)

$$\hat{\phi}_{\mathbf{x}}|\phi_{\mathbf{x}}\rangle = \phi_{\mathbf{x}}|\phi_{\mathbf{x}}\rangle, \qquad \hat{\pi}_{\mathbf{x}}|\pi_{\mathbf{x}}\rangle = \pi_{\mathbf{x}}|\pi_{\mathbf{x}}\rangle$$

with

$$\langle\phi_{\mathbf{x}}|\phi'_{\mathbf{x}}\rangle = \delta(\phi_{\mathbf{x}} - \phi'_{\mathbf{x}}) \text{ and } \int d\phi_{\mathbf{x}}|\phi_{\mathbf{x}}\rangle\langle\phi_{\mathbf{x}}| = \mathbb{1}.$$

(One may use qumodes, see, e.g, Jack Y. Araz on Sep 9 and Simon Jonathan Williams on Sep 12.)

- **$\phi_{\mathbf{x}}$ only takes discrete values:**

$$\phi_{\mathbf{x}} \in \left\{ \Delta\phi\left(-\frac{N_\phi - 1}{2} + j \right) \middle| j = 0, 1, \cdots, N_\phi - 1 \right\}$$
$$= \underbrace{\{-\hat{\phi}_{\max}, -\hat{\phi}_{\max} + \Delta\phi, \cdots, \hat{\phi}_{\max} - \Delta\phi, \hat{\phi}_{\max}\}}_{N_\phi} \text{ with } \Delta\phi = 2\hat{\phi}_{\max}/(N_\phi - 1)$$

Jordan, Lee and Preskill, Science **336**, 1130-1133 (2012); Quant. Inf. Comput. **14**, 1014-1080 (2014).

where $\langle\phi_{\mathbf{x}}|\phi'_{\mathbf{x}}\rangle = \Delta\phi^{-1}\delta_{\phi_{\mathbf{x}},\phi'_{\mathbf{x}}}$ and $|\phi_{\mathbf{x}}\rangle \equiv |j\rangle/\sqrt{\Delta\phi}$.

• Representation of ϕ_{x} by qubits

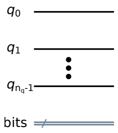
$$\hat{\phi}_{\mathrm{x}} = -\frac{\Delta\phi}{2}\sum_{i=0}^{n_q-1} 2^i Z_i$$

where $n_q = \log_2\left(\frac{2\hat{\phi}_{\max}}{\Delta\phi} + 1\right) = \log_2 N_\phi$, Z_i is the Z gate acting on the i^{th} qubit. The eigenstate of ϕ is mapped onto the computation basis:

$$\hat{\phi}_{\mathrm{x}}|j\rangle \equiv \hat{\phi}_{\mathrm{x}}|q_{n-1}q_{n-2}\cdots q_0\rangle = \Big(\underbrace{-\hat{\phi}_{\max} + \Delta\phi\sum_{i=0}^{n_q-1} 2^i q_i}_{j}\Big)|q_{n-1}q_{n-2}\cdots q_0\rangle = \phi_{\mathrm{x}}|j\rangle$$

that is,

$$\sqrt{\Delta\phi}|\phi_{\mathrm{x}}\rangle \equiv |j\rangle = |q_{n_q-1}\cdots q_0\rangle =$$



in `Qiskit` convention.

- **Eigenstates of $\hat{\pi}_{\mathbf{x}}$**:

$$|\pi_{\mathbf{x}}\rangle = \int d\phi_{\mathbf{x}} e^{i\pi_{\mathbf{x}}\phi_{\mathbf{x}}} |\phi_{\mathbf{x}}\rangle$$

$$\Rightarrow |\pi_{\mathbf{x}}\rangle = \Delta\phi \sum_{\phi_{\mathbf{x}}} e^{i\pi_{\mathbf{x}}\phi_{\mathbf{x}}} |\phi_{\mathbf{x}}\rangle = \sqrt{\Delta\phi} \sum_{j=0}^{N_\phi - 1} e^{i\pi_{\mathbf{x}}\Delta\phi(-\frac{N_\phi - 1}{2} + j)} |j\rangle$$

with

$$\pi_{\mathbf{x}} \Delta\phi N_\phi = 2\pi \left(-\frac{N_\phi - 1}{2} + k \right) \qquad \text{with } k = 0, 1, \cdots, N_\phi - 1.$$

- **The normalized conjugate momentum eigenstate**:

$$|k\rangle_\pi \equiv \frac{1}{\sqrt{N_\phi \Delta\phi}} |\pi_{\mathbf{x}}\rangle = \frac{1}{\sqrt{N_\phi}} \sum_{j=0}^{N_\phi - 1} e^{i\frac{2\pi}{N_\phi}(-\frac{N_\phi - 1}{2} + k)(-\frac{N_\phi - 1}{2} + j)} |j\rangle \equiv F_{jk} |j\rangle.$$

- **Relating $\hat{\pi}_{\mathbf{x}}$ to $\hat{\phi}_{\mathbf{x}}$:**

$$\begin{aligned}\langle j|\hat{\pi}_{\mathbf{x}}|j'\rangle &= \sum_{k,k'} \langle j|k\rangle_{\pi\pi}\langle k|\hat{\pi}_{\mathbf{x}}|k'\rangle_{\pi\pi}\langle k'|j'\rangle \\ &= \frac{2\pi}{N_\phi \Delta\phi}\sum_k \left(-\frac{N_\phi - 1}{2} + k\right) F_{jk}F^\dagger_{kj'} \\ \Leftrightarrow \hat{\pi}_{\mathbf{x}} &= \frac{2\pi}{N_\phi (\Delta\phi)^2}\hat{F}\hat{\phi}_{\mathbf{x}}\hat{F}^\dagger \equiv \mu \hat{F}\hat{\phi}_{\mathbf{x}}\hat{F}^\dagger\end{aligned}$$

where $\hat{F}$ is given by

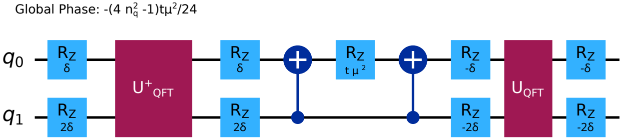
$$\hat{F} \equiv \frac{1}{\sqrt{N_\phi}}\sum_{j,k=0}^{N_\phi - 1} e^{i\frac{2\pi}{N_\phi}\left(-\frac{N_\phi - 1}{2}+j\right)\left(-\frac{N_\phi - 1}{2}+k\right)}|j\rangle\langle k|.$$

- **Implementing $\hat{F}$ using QFT:**

$$\hat{F} = e^{-\frac{i}{2}\delta(N_\phi - 1)}\prod_{j=0}^{n-1} R_z^j(-2^j\delta)U_{QFT}\prod_{k=0}^{n-1} R_z^k(-2^k\delta)$$

with $\delta \equiv \pi(N_\phi - 1)/N_\phi$ and $U_{QFT}|k\rangle = \frac{1}{\sqrt{N_\phi}}\sum_{j=0}^{N_\phi - 1} e^{i\frac{2\pi}{N_\phi}kj}|j\rangle$.

- **Example of a circuit for $e^{\frac{i}{2}\hat{\pi}_x^2 t}$ with $n_q = 2$:**



- **Scalar field theory is tractale:**

TABLE I. CNOT gates count of the Trotter steps required for the implementation of the ϕ^4 evolution operator. All-to-all qubit connectivity is assumed.

Operator	$e^{-i\Phi\theta}$	$e^{-i\Phi^2\theta}$	$e^{-i\Pi^2\theta}$	$e^{-i\Phi_j\Phi_k\theta}$	$e^{-i\Phi^4\theta}$
Number of CNOTs	0	$n_q^2 - n_q$	$3n_q^2 - 3n_q$	$2n_q^2$	$\frac{1}{4}n_q^4 - \frac{3}{2}n_q^3 + \frac{15}{4}n_q^2 - \frac{5}{2}n_q$

Taken from A. C. Y. Li, A. Macridin, S. Mrenna and P. Spentzouris, Phys. Rev. A **107**, no.3, 032603 (2023).

Harmonic oscillator basis

- **The hamiltonian:**

$$
\begin{aligned}
H &= \sum_{\mathbf{p}} \omega_{\mathbf{p}} \left[\hat{a}^\dagger_{\mathbf{p}} \hat{a}_{\mathbf{p}} + \frac{1}{2} \right] + a^d \sum_{\mathbf{x}} \frac{\lambda}{4!} \hat{\phi}^4_{\mathbf{x}} \\
&= \sum_{\mathbf{p}} \omega_{\mathbf{p}} \left[\hat{a}^\dagger_{\mathbf{p}} \hat{a}_{\mathbf{p}} + \frac{1}{2} \right] + a^d \sum_{\mathbf{x}} \frac{\lambda}{4!} \prod_{j=1}^{4} \frac{1}{L^{\frac{d}{2}}} \sum_{\mathbf{p}_j} \frac{e^{i\mathbf{p}_j \cdot \mathbf{x}}}{\sqrt{2\omega_{\mathbf{p}_j}}} (\hat{a}_{\mathbf{p}_j} + \hat{a}^\dagger_{-\mathbf{p}_j}) \\
&= \sum_{\mathbf{p}} \omega_{\mathbf{p}} \left[\hat{a}^\dagger_{\mathbf{p}} \hat{a}_{\mathbf{p}} + \frac{1}{2} \right] + \frac{\lambda}{4! L^d} \prod_{j=1}^{4} \sum_{\mathbf{p}_j} \frac{1}{\sqrt{2\omega_{\mathbf{p}_j}}} (\hat{a}_{\mathbf{p}_j} + \hat{a}^\dagger_{-\mathbf{p}_j}) \delta_{\mathbf{p}_1+\mathbf{p}_2+\mathbf{p}_3+\mathbf{p}_4,0}
\end{aligned}
$$

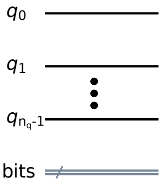
- **The eigenstates of $\hat{a}^\dagger_{\mathbf{p}} \hat{a}_{\mathbf{p}}$:** $\hat{a}^\dagger_{\mathbf{p}} \hat{a}_{\mathbf{p}} |n\rangle = n|n\rangle$ with $n = 0, 1, \cdots$

$$
\hat{a}_{\mathbf{p}} = \sum_{n=0}^{\infty} \sqrt{n+1} |n\rangle\langle n+1|, \qquad \hat{a}^\dagger_{\mathbf{p}} = \sum_{n=0}^{\infty} \sqrt{n+1} |n+1\rangle\langle n|
$$

- **Let us truncate** with $n = 0, 1, \cdots, n_\varphi$

- **Using binary encoding:** number of qubits $n_q = \log_2(n_\phi + 1)$ for each $\mathbf{p}$

$$|n\rangle = \left| \sum_{i=0}^{n_q} 2^i q_i \right\rangle =$$



L. Veis, J. Višňák, H. Nishizawa, H. Nakai and J. Pittner, Int. J. Quant. Chem. **116**, no.18, 1328-1336 (2016).

N. Klco and M. J. Savage, Phys. Rev. A **99**, no.5, 052335 (2019) [arXiv:1808.10378 [quant-ph]].

- **The complete basis for the truncated Hilbert space:**

$$|\mathbb{n}\rangle = \bigotimes_{\mathbf{p}} |n_{\mathbf{p}}\rangle = \bigotimes_{\mathbf{p}} \bigotimes_{i} |q_i\rangle$$

As no real quantum devices are available, evaluate these matrices on PC below.

Decay of classical fields into particles

Setup the problem

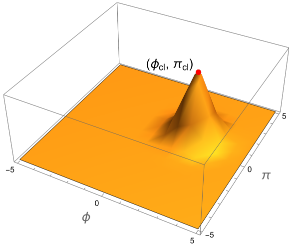
- **The initial state:** coherent state (the most classical quantum state)

$$|\alpha_{\mathbf{p}}\rangle = e^{\alpha_{\mathbf{p}}\hat{a}^\dagger_{\mathbf{p}} - \alpha^*_{\mathbf{p}}\hat{a}_{\mathbf{p}}}|0\rangle = e^{-|\alpha_{\mathbf{p}}|^2/2}\sum_{n=0}^{\infty}\frac{\alpha^n_{\mathbf{p}}}{\sqrt{n!}}|n\rangle$$

where $\alpha_{\mathbf{p}}$ is a complex function of $\mathbf{p}$ and $\hat{a}_{\mathbf{p}}|\alpha\rangle = \alpha_{\mathbf{p}}|\alpha\rangle$, such that

$$\phi_{cl} \equiv \langle\alpha|\hat{\phi}_{\mathbf{x}}|\alpha\rangle = \frac{1}{L^{\frac{d}{2}}}\sum_{\mathbf{p}}\frac{1}{\sqrt{2\omega_{\mathbf{p}}}}\left(\alpha_{\mathbf{p}}e^{i\mathbf{p}\cdot\mathbf{x}} + \alpha^*_{\mathbf{p}}e^{-i\mathbf{p}\cdot\mathbf{x}}\right),$$

$$\pi_{cl} \equiv \langle\alpha|\hat{\pi}_{\mathbf{x}}|\alpha\rangle = -\frac{i}{L^{\frac{d}{2}}}\sum_{\mathbf{p}}\sqrt{\frac{\omega_{\mathbf{p}}}{2}}\left(\alpha_{\mathbf{p}}e^{i\mathbf{p}\cdot\mathbf{x}} - \alpha^*_{\mathbf{p}}e^{-i\mathbf{p}\cdot\mathbf{x}}\right).$$



R. J. Glauber, Phys. Rev. **131**, 2766-2788 (1963).

- **The system:** spatially homogeneous like early universe

$$|\psi(t)\rangle = e^{iHt}|\alpha\rangle$$

with

$$|\alpha\rangle = |0\rangle \otimes \cdots \otimes |0\rangle \otimes \underbrace{|\alpha\rangle}_{\mathbf{p}=\mathbf{0}} \otimes |0\rangle \cdots \otimes |0\rangle.$$

- **The observables**

1. The phase-space distribution

$$f_{\mathbf{p}}(t) \equiv \frac{1}{L^d}\langle\psi(t)|a_{\mathbf{p}}^{\dagger}a_{\mathbf{p}}|\psi(t)\rangle = \langle\psi(t)|\hat{a}_{\mathbf{p}}^{\dagger}\hat{a}_{\mathbf{p}}|\psi(t)\rangle$$

Note that we only study the weak-coupling cases for now.

2. The classical fields

$$\phi_{cl}(t) \equiv \langle\psi(t)|\hat{\phi}_{\mathbf{x}}|\psi(t)\rangle, \qquad \pi_{cl}(t) \equiv \langle\psi(t)|\hat{\pi}_{\mathbf{x}}|\psi(t)\rangle$$

In classical field theory

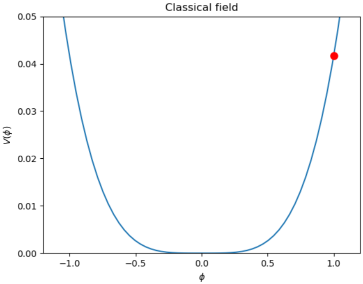
- **The equation of motion:**

$$(\partial^2 + m^2)\phi(t) + \frac{\lambda}{6}\phi^3 = 0$$

with the initial conditions

$$\phi(0) = \langle\alpha|\hat{\phi}_{\mathbf{x}}|\alpha\rangle, \qquad \pi(0) = \langle\alpha|\hat{\pi}_{\mathbf{x}}|\alpha\rangle.$$

- **Oscillate all the time:** $\phi(0) = 1, \pi(0) = 0$ for $m = 1, \lambda = 1$.



Thermal field theory on the lattice

- **In thermal equilibrium,** the expectation value of any observable $\hat{O}$

$$\langle \hat{O} \rangle_\beta \equiv Z_\beta^{-1} \mathrm{Tr}[e^{-\beta \hat{H}} \hat{O}],$$

where $\beta = 1/T$ with T the temperature and the partition function

$$Z_\beta \equiv \mathrm{Tr}[e^{-\beta \hat{H}}].$$

Here, the trace can be carried out by summing the expectation values over the complete orthonormal states either in coordinate space or in momentum space.

Qian and BW, JHEP **07**, 166 (2024); Cuntín, Qian and BW, PoS **ICHEP2024**, 630 (2025).

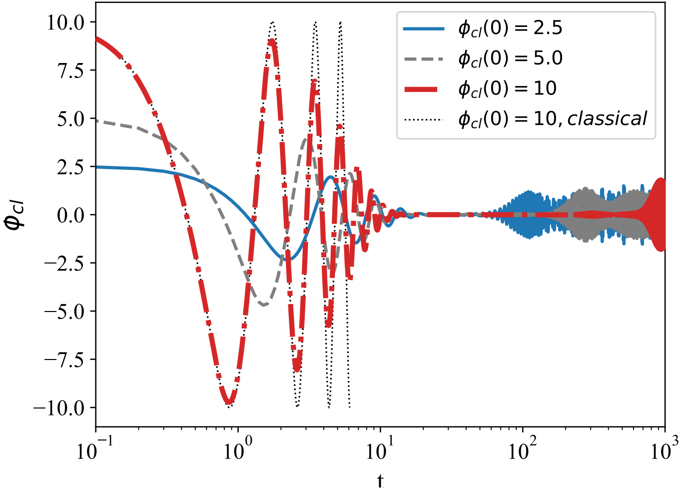
- **Quantum Poincaré recurrence:** the state returns arbitrarily close to its initial configuration after sufficiently long times in a finite, close system with a discrete energy spectrum.

P. Bocchieri and A. Loinger, Physical Review 107, 337 (1957).

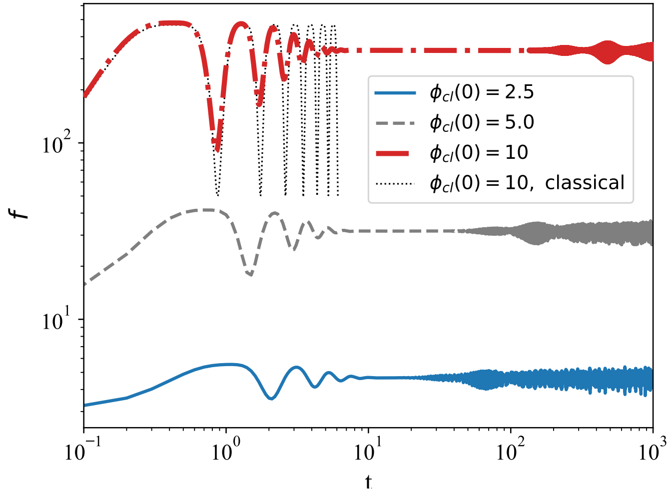
At finite N, the system does not thermalize in a strict sense.

One-mode system (quantum mechanics)

- **The expectation of ϕ:** $H = \frac{1}{2}\hat{\pi}_{\mathbf{x}}^2 + \frac{1}{2}m^2\hat{\phi}_{\mathbf{x}}^2 + V(\hat{\phi}_{\mathbf{x}})$ with $m = 1, \lambda = 1$.

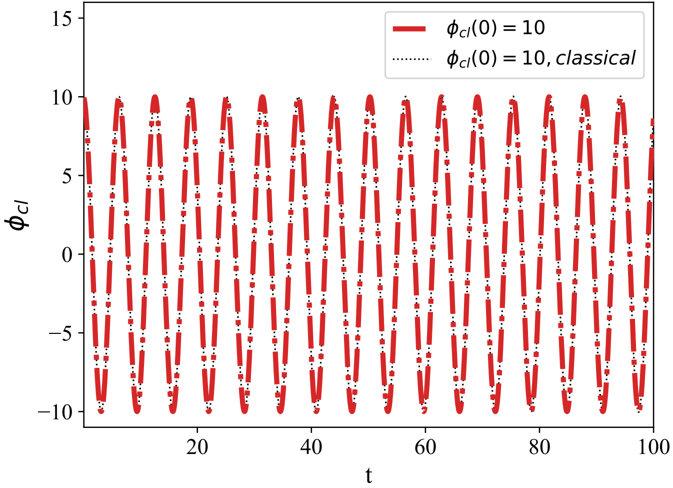


- **The particle number $n = f$:** with $m = 1, \lambda = 1$.



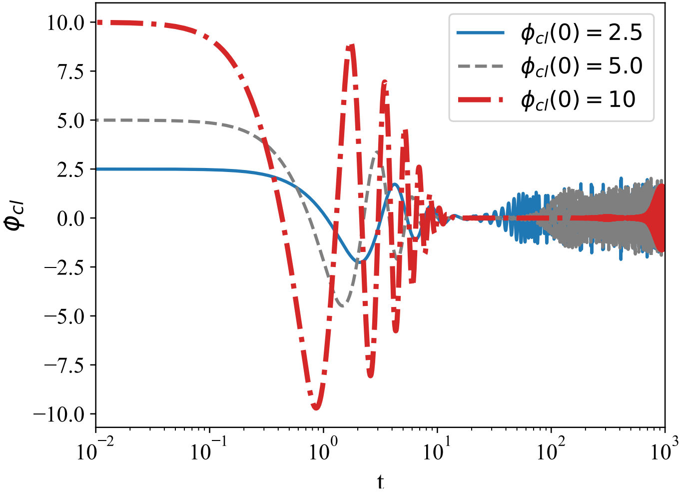
Other observables like pressure have similar behavior.

- **It is consequence of interactions:** with $m = 1, \lambda = 0$.

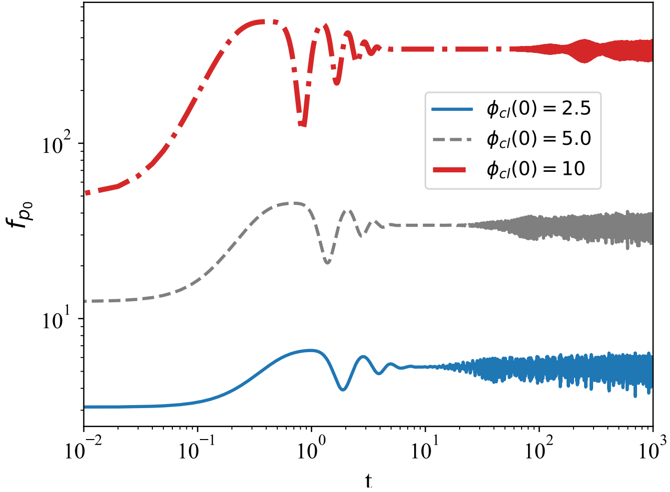


Lattice QFT N=2 in $(1+1)D$

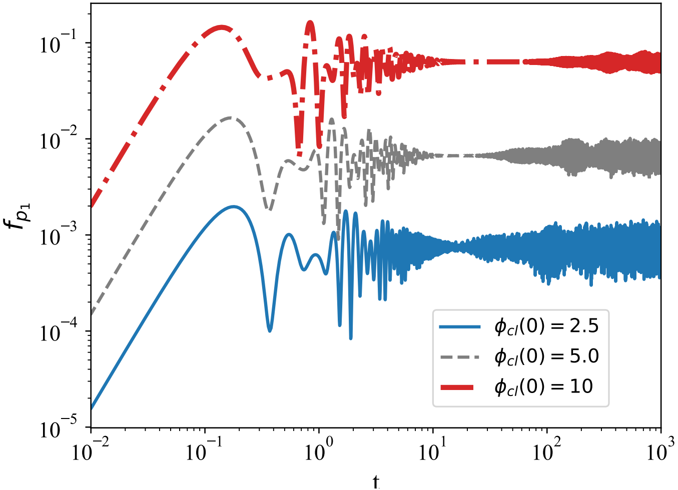
- **The expectation of ϕ:** with $m = 1, \lambda = 1, a = 0.25$.



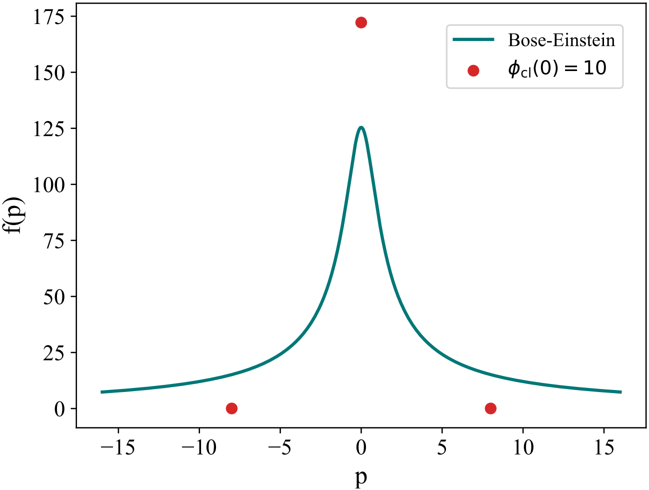
- **The particle number f at $p_0 = 0$:** with $m = 1, \lambda = 1, a = 0.25$.



- **The particle number f at $p_1 = 2/a$:** with $m = 1, \lambda = 1, a = 0.25$.

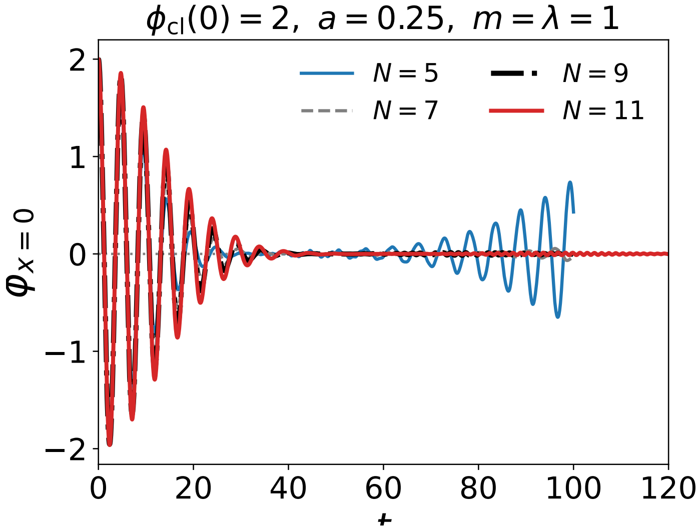


• Reach a equilibrate stage before signal to Poincaré recurrence, not BE



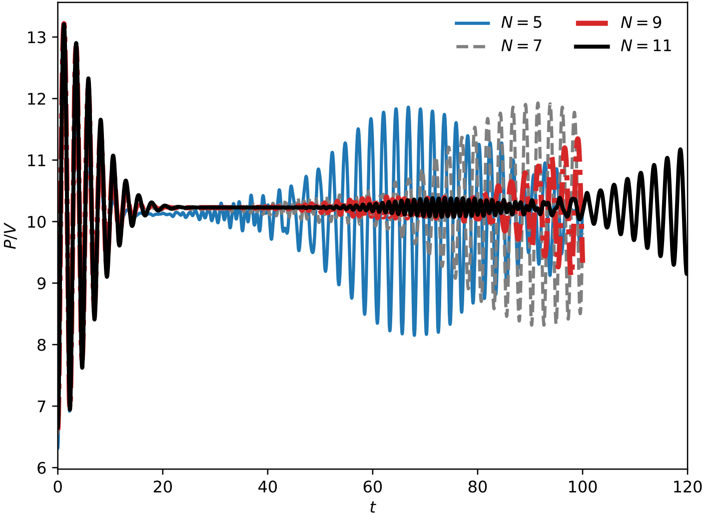
Lattice QFT at larger N with Tensor Networks

- **The expectation of ϕ:** with $m = 1, \lambda = 1, a = 0.25$.



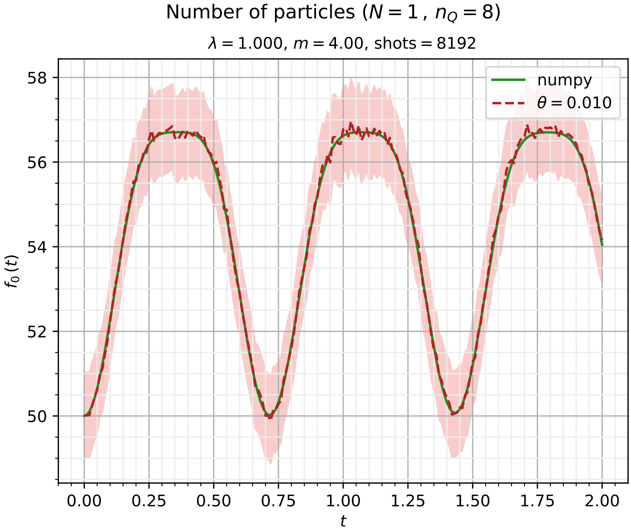
Lattice QFT at larger N with Tensor Networks

- **The pressure:** with $m = 1, \lambda = 1, a = 0.25$.



Quantum Simulations on Quantum Simulators

- **f for $\phi_{cl}(0) = 5$:** with $m = 1, \lambda = 1$.



Summary and Outlook

- We observe equilibration of particle production from highly occupied classical fields (coherent states).
- The occupation-number distribution $f_{\mathbf{p}}$ and the pressure P exhibit equilibration, but their late-time values do not coincide with a Bose–Einstein distribution.
- These results may provide new insight into the emergence of hydrodynamic behavior in small collision systems containing only a few hundred particles.
- Many important questions remain open:
  1. How does the equilibration behavior evolve in the large-volume (thermodynamic) limit, $N \to \infty$?
  2. Can few-body quantum systems thermalize, for example through the eigenstate thermalization hypothesis (ETH)?
  3. How and when does Bose–Einstein thermalization emerge in lattice quantum field theory?

Back up slides

Eigenstate Thermalization

Eigenstate Thermalization Hypothesis (ETH) is a proposed mechanism explaining why an isolated quantum many-body system:

$$A(t) \equiv \langle A \rangle = |c_\alpha|^2 A_{\alpha\alpha} + \sum_{\alpha \neq \beta} c_\beta c_\alpha^* e^{-i(E_\beta - E_\alpha)t} A_{\alpha\beta}$$

$$\lim_{t\to\infty} \frac{1}{t} \int_0^t A(t) \to |c_\alpha|^2 A_{\alpha\alpha}$$

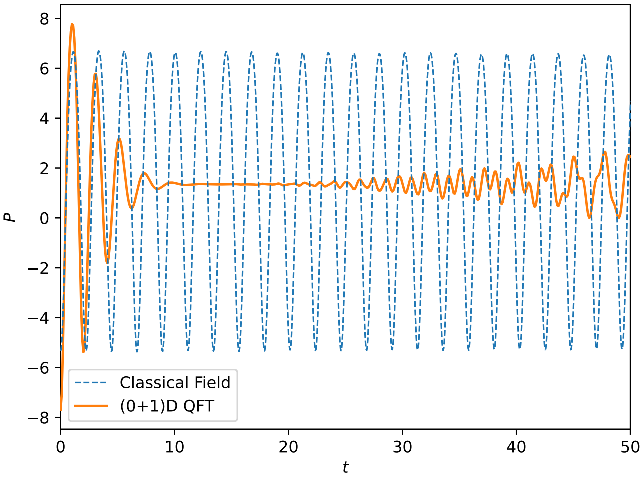
If $A_{\alpha\alpha}$ is a smooth function of E and $A_{\alpha\beta}$ is small for $\alpha \neq \beta$,

$$|c_\alpha|^2 A_{\alpha\alpha} \approx a(E)|c_\alpha|^2 = a(E),$$

where $a(E)$ is a function of the energy E only and therefore it is "thermal".

M. Srednicki, Phys. Rev. E **50**, 888 [arXiv:cond-mat/9403051 [cond-mat]].

• The pressure:

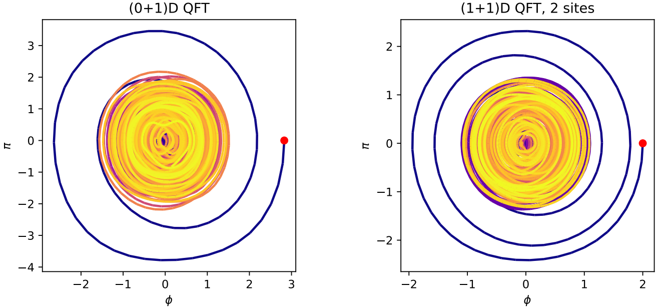


In classical field theory, $P \equiv \frac{1}{2}\pi^2 - \frac{m}{2}\phi^2 - \frac{\lambda}{4!}\phi^4$.

In the (1+1)D QFT

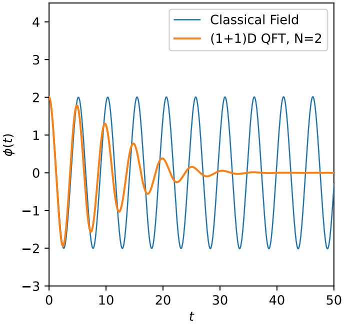
- **Phase-space diagram for $\alpha = 2 = \alpha^*$:**

$$\phi(0) = 2\sqrt{2/N} \text{ and } \pi(0) = 0$$



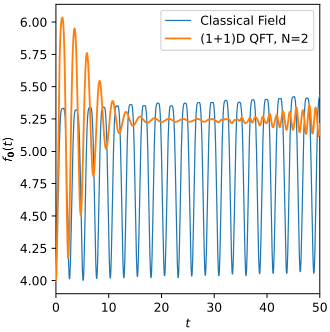
- **The scalar field:**

$$\phi(t) = \langle\psi(t)|\hat{\phi}|\psi(t)\rangle$$



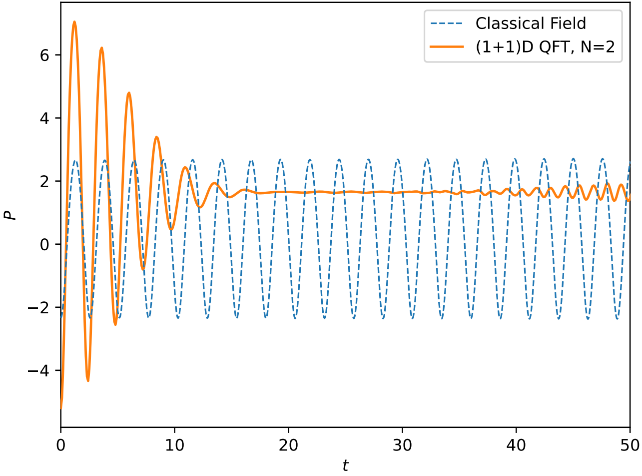
- **Phase-space distribution:**

$$f_{\mathbf{p}}(t) = \langle\psi(t)|\hat{a}^\dagger_{\mathbf{p}}\hat{a}_{\mathbf{p}}|\psi(t)\rangle$$



Here, in classical field theory $f_0(t) \equiv (\phi(t)^2 + \pi(t)^2)$.

- **The pressure:**



In classical field theory, $P \equiv \frac{1}{2}\pi^2 - \frac{m}{2}\phi^2 - \frac{\lambda}{4!}\phi^4$.

• Compare with time average: