QUANTUM COMPUTING AND SIMULATION CENTER

ICSC Centro Nazionale di Ricerca in HPC, Big Data and Quantum Computing

TENSOR NETWORK ALGORITHMS FOR HIGH-DIMENSIONAL HEP SIMULATIONS

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University of Padova

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QUANTUM FLAGSHIP

UNIVERSITÀ DI PADOVA

TENSOR NETWORKS STATES

$$\psi_{\alpha_1,\alpha_2,\ldots\alpha_N} \qquad \mathcal{O}(d^N)$$

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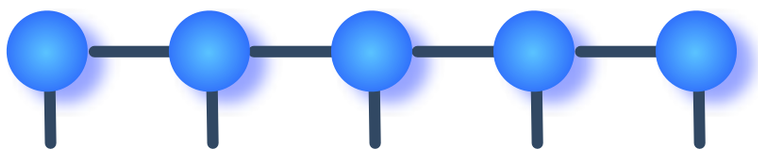

$$A^{\beta_1}_{\alpha_1} A^{\beta_1\beta_2}_{\alpha_2} \ldots A^{\beta_{N-1}}_{\alpha_N} \qquad \mathcal{O}(Ndm^2)$$

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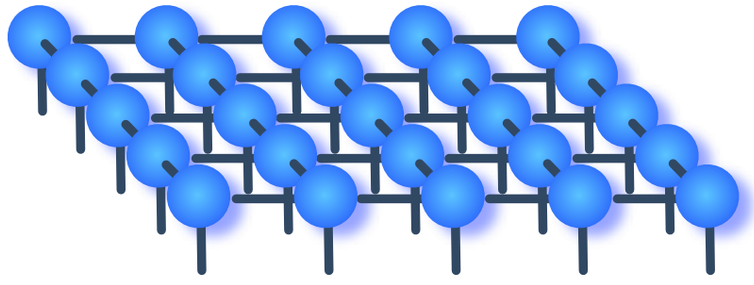
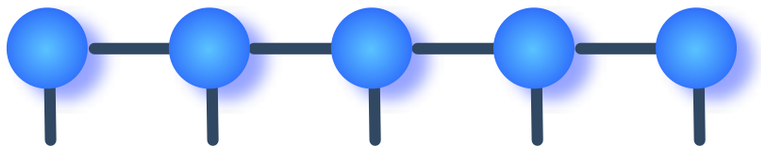


PEPS



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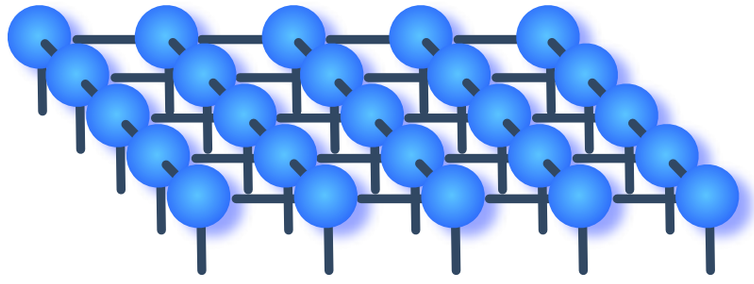

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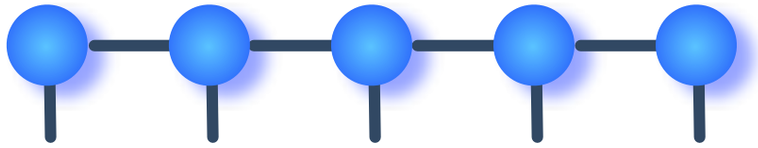

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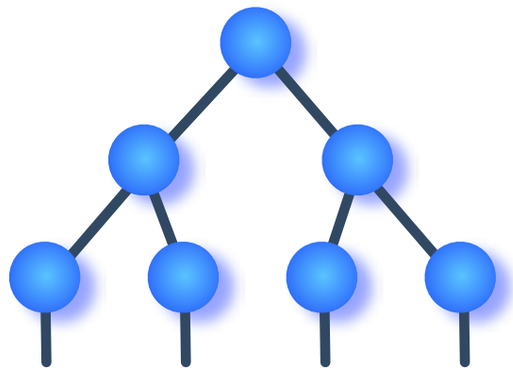

Tree Tensor Network

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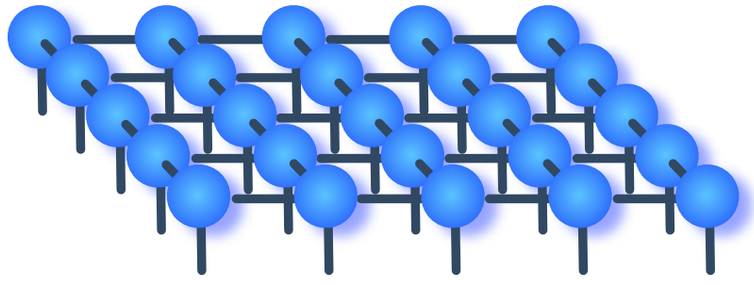

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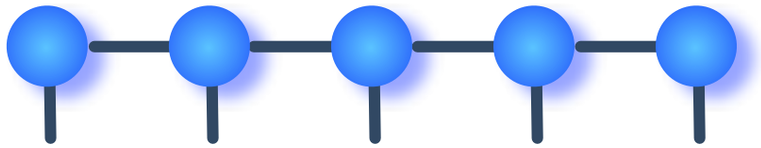

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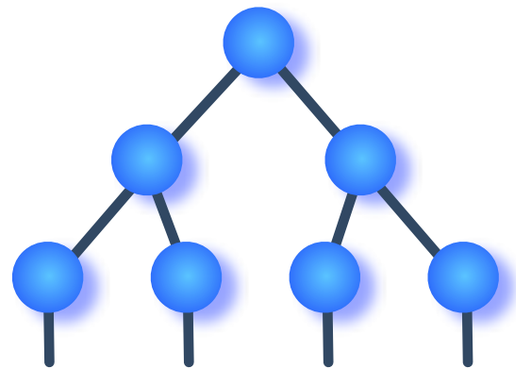
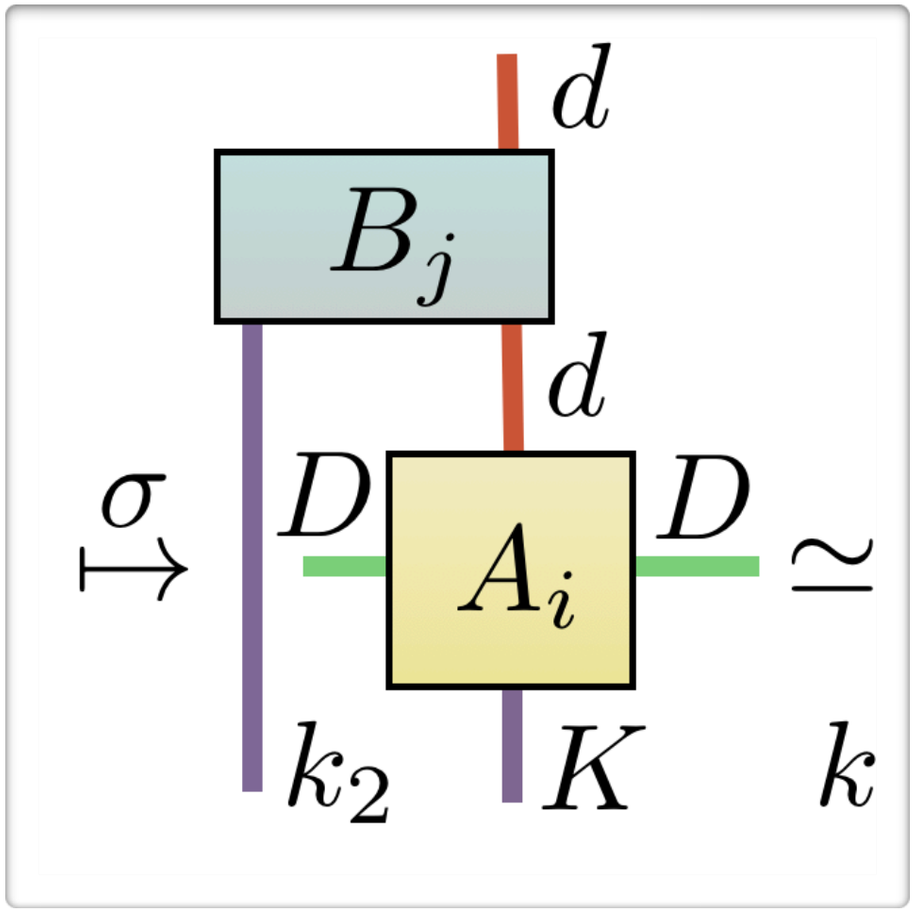


Tree Tensor Network

Tensor networks states are a compressed description of the system tunable between mean field and exact

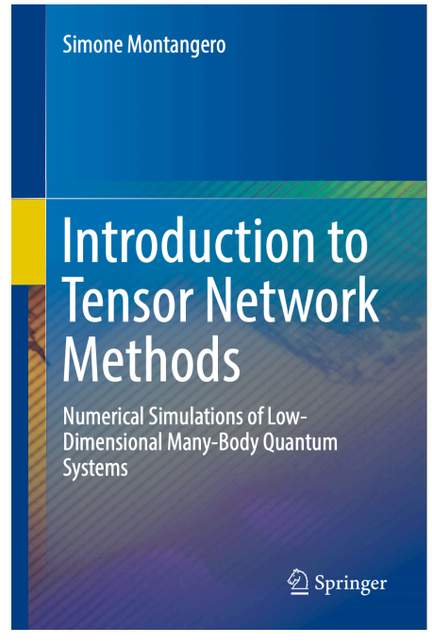
TENSOR NETWORK ALGORITHMS



- *State of the art in 1D (poly effort)*
- *No sign problem*
- *Extended to open quantum systems*
- *Machine Learning & Big Data*
- *Compression of LLM*
- *Extended to lattice gauge theories*
- *Thousands of low-entangled qubits*

U. Schollwock, RMP (2005)

I. Glasser, et al. PRX (2018) *A. Cichocki, ECM (2013)*



ENTANGLEMENT OF PURE MANY-BODY QUANTUM SYSTEMS

For pure states:

$$\mathcal{S} = -\mathrm{Tr}\,\rho \log \rho$$

Von Neumann Entropy

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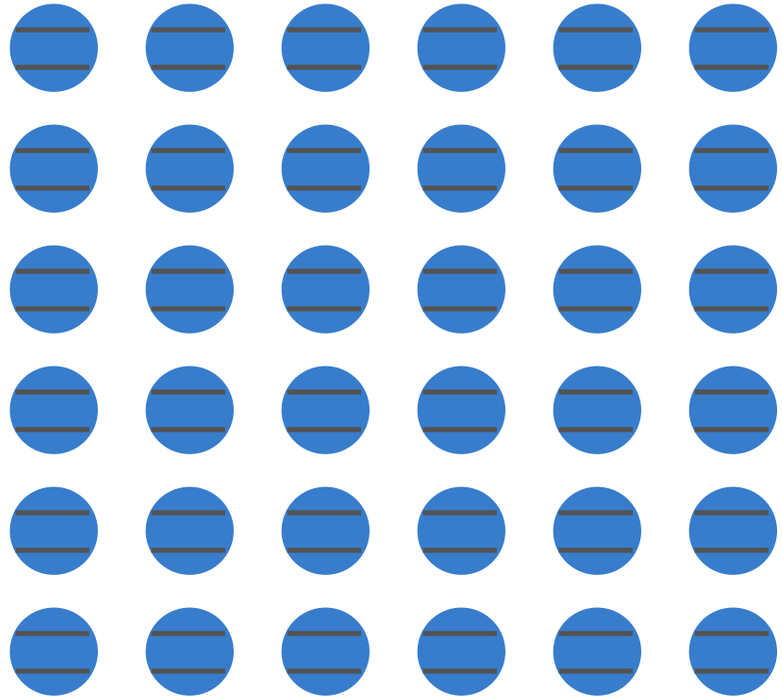

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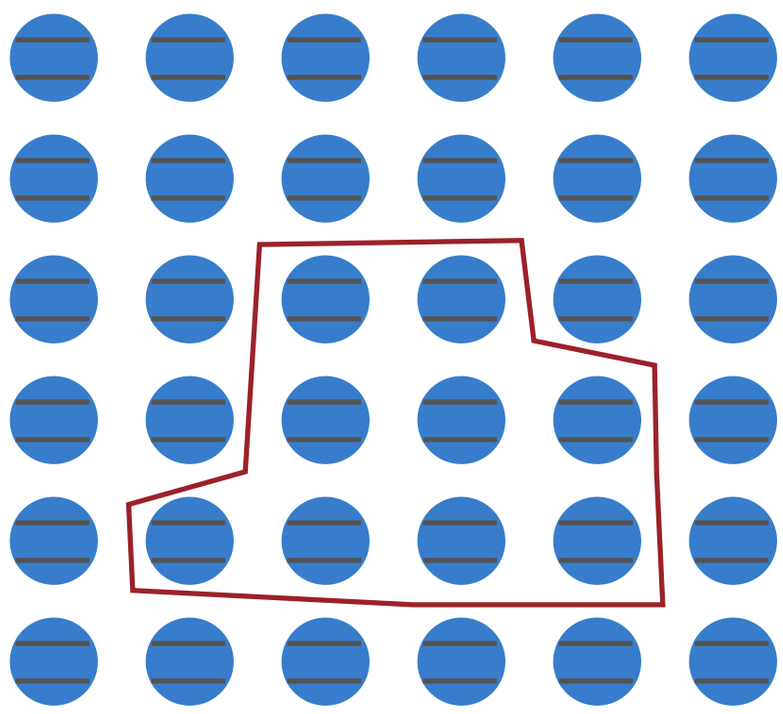

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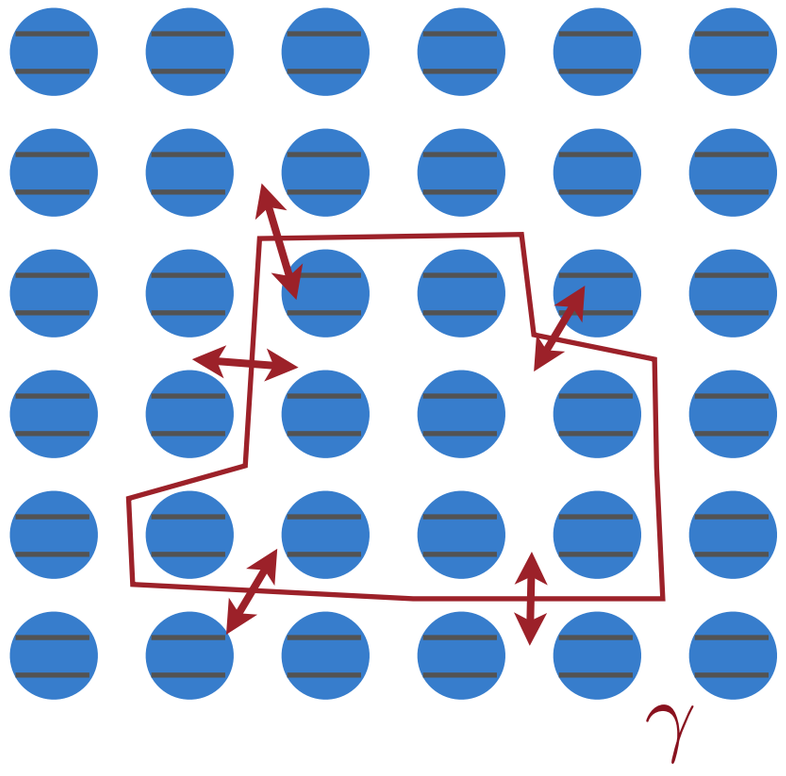

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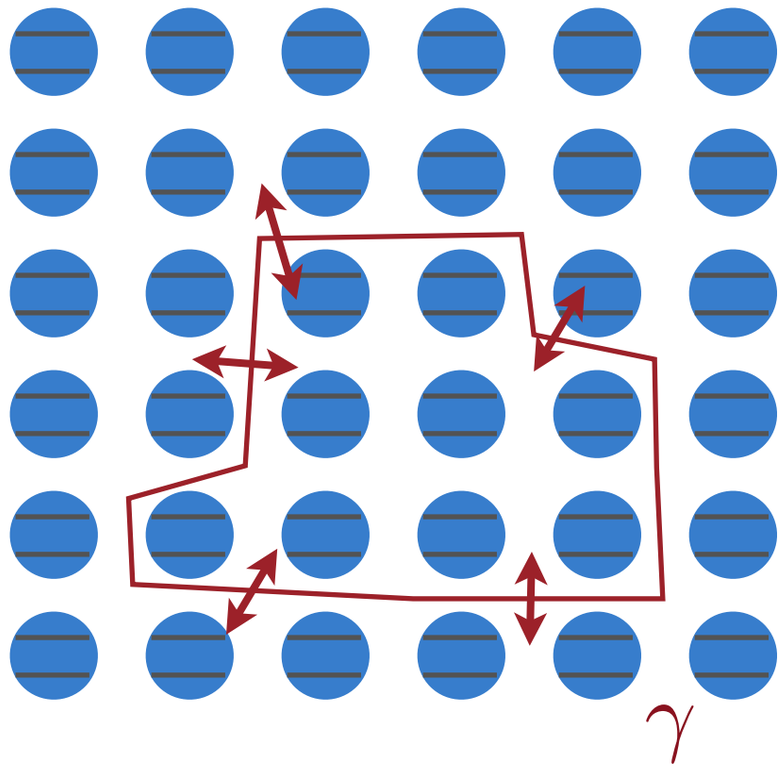

Area law

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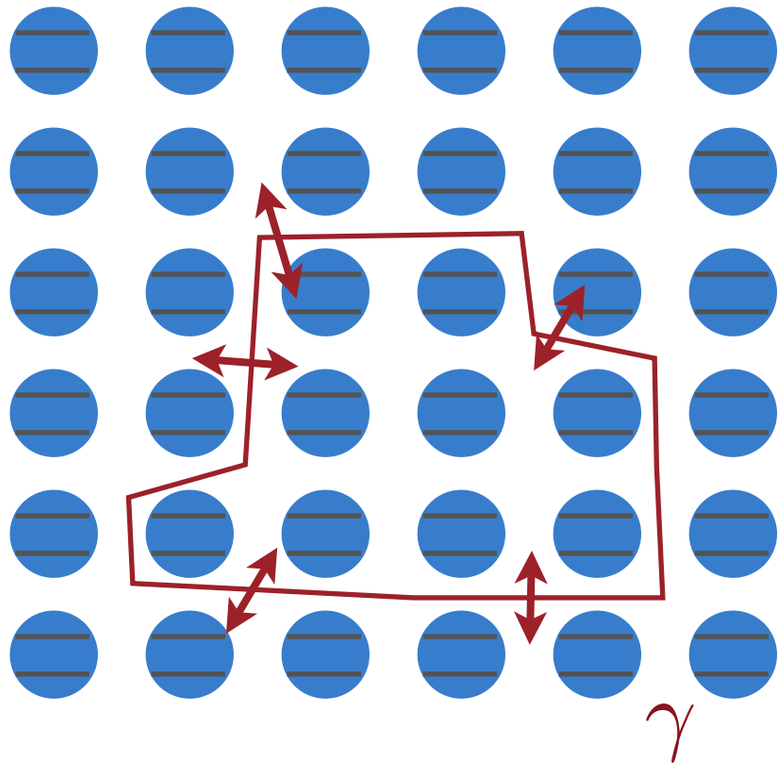

Area law

$$\mathcal{S} \propto \gamma$$

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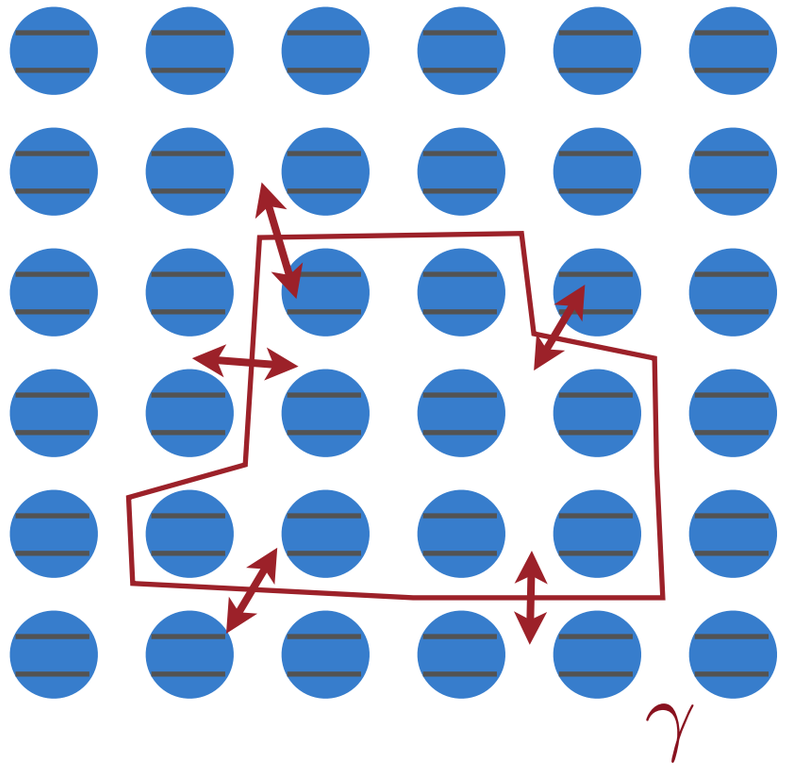

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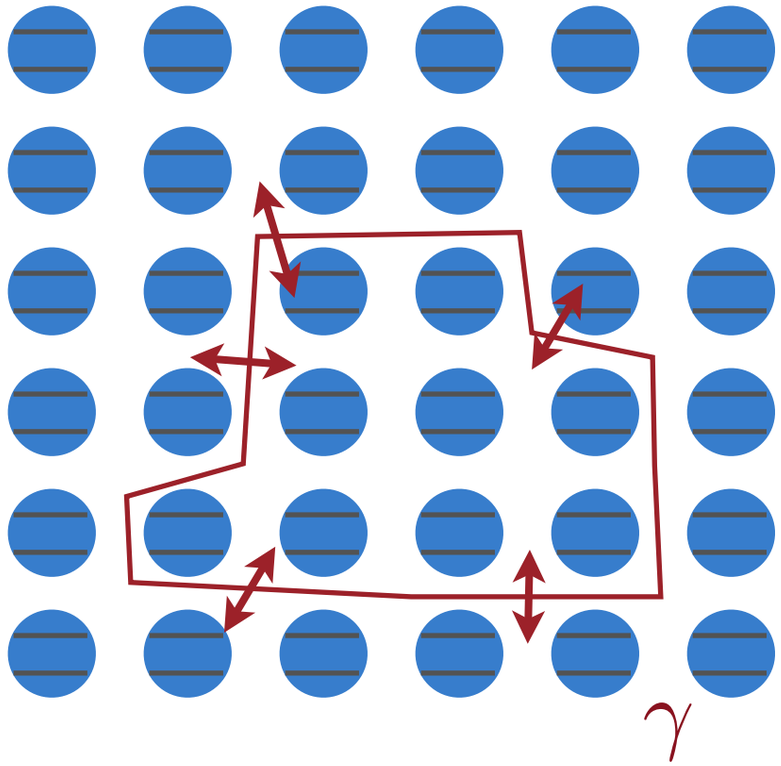

Area law

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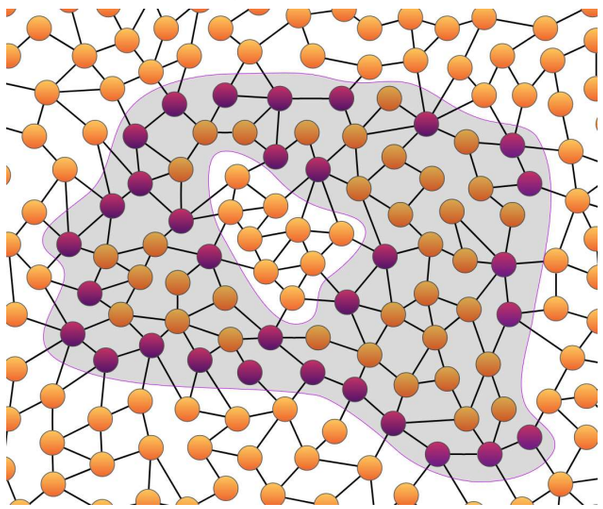
1D critical systems:

$$\mathcal{S} = \frac{c}{3} \log N$$

AREA LAWS AND TENSOR NETWORKS

$$\gamma \propto L$$



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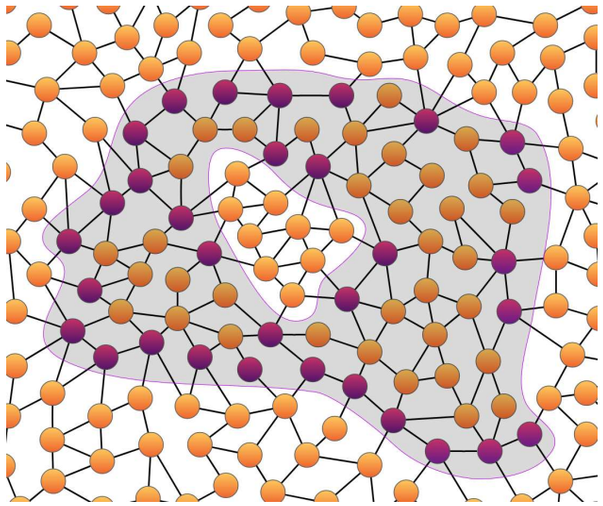

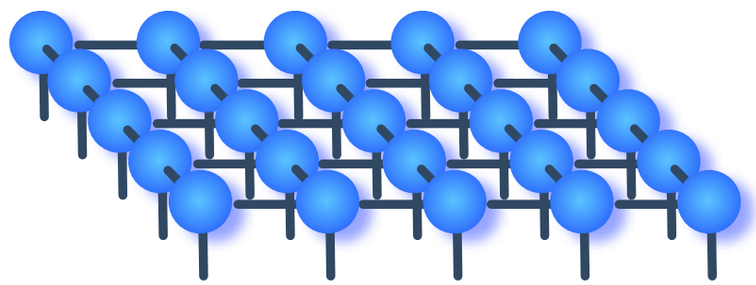

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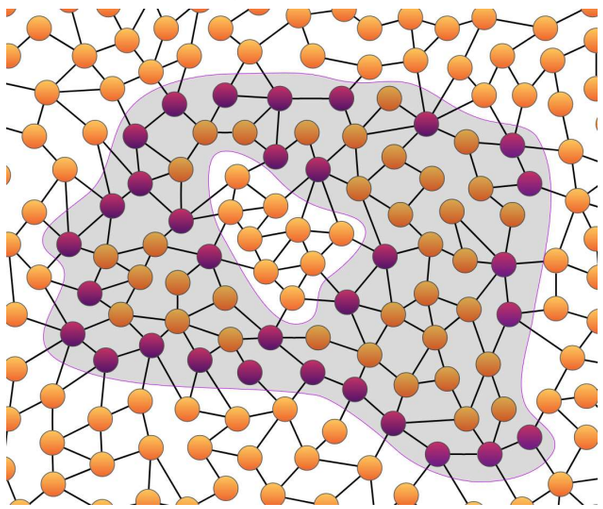

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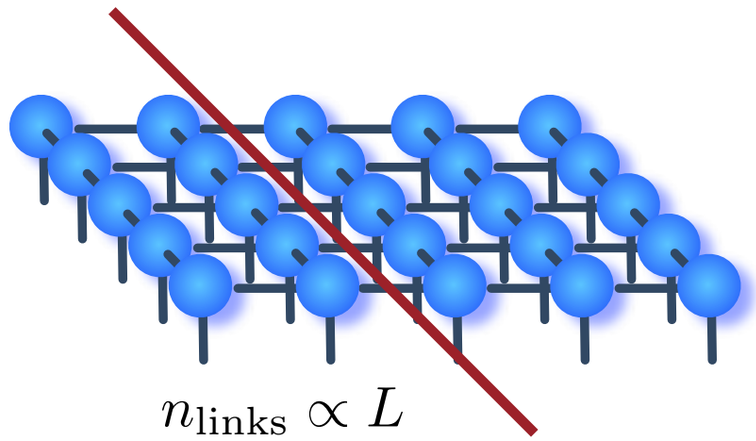

$n_{\text{links}} \propto L$

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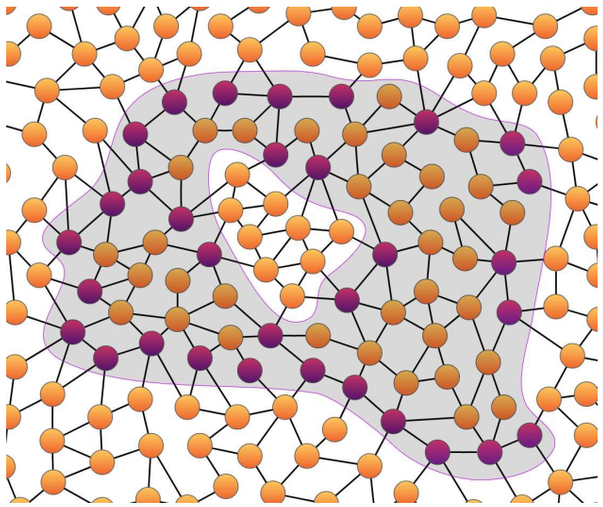

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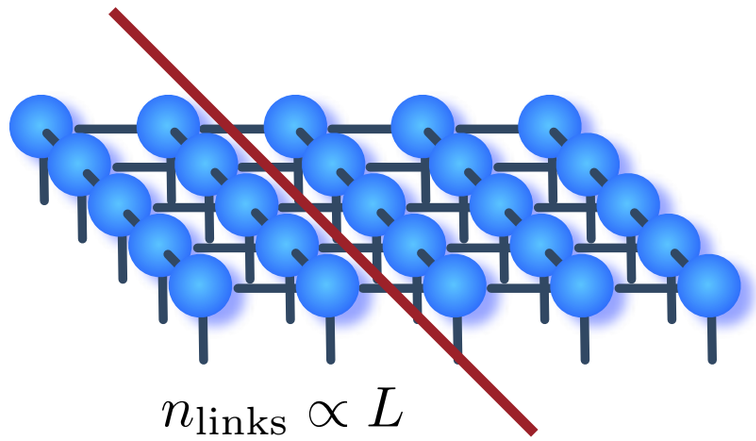

$n_{\text{links}} \propto L$

$$\mathcal{S}_{\max} \propto n_{\text{links}} \times \log m$$

AREA LAWS AND TENSOR NETWORKS

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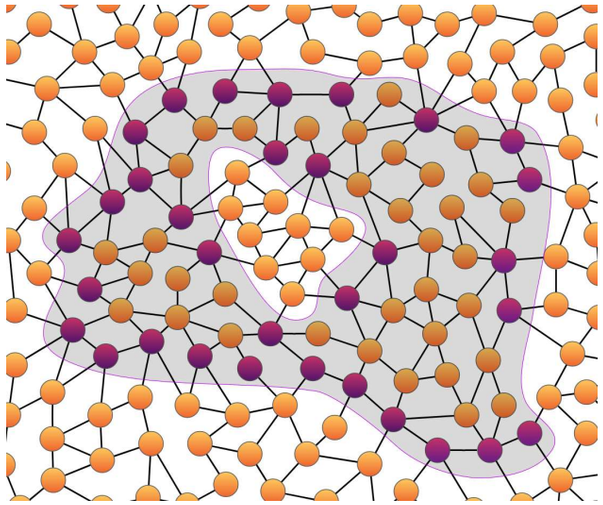

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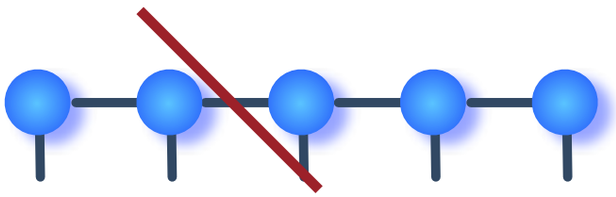

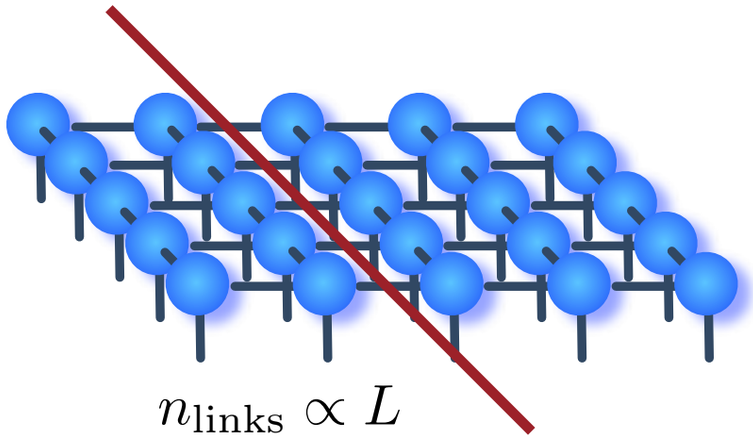

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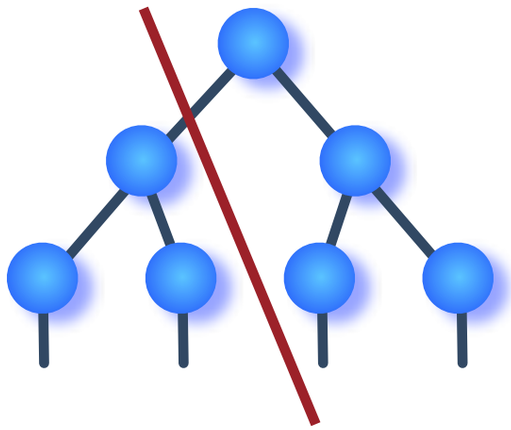

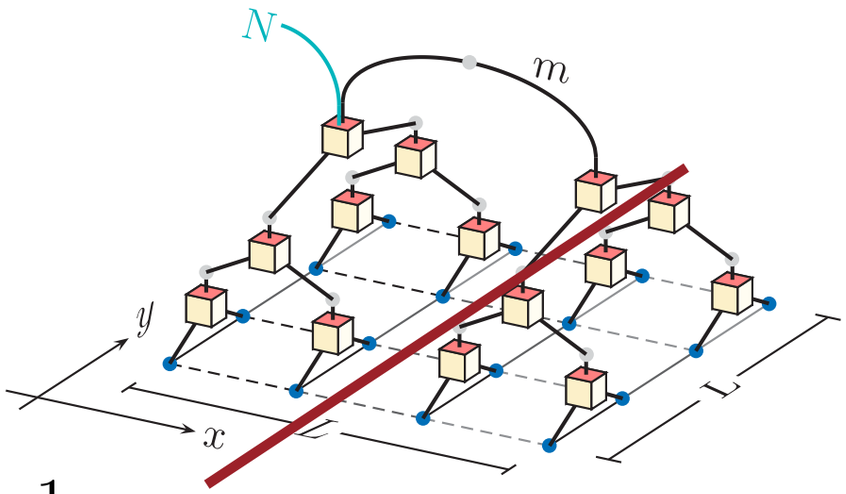

$n_{\text{links}} = 1$

AREA LAWS AND TENSOR NETWORKS

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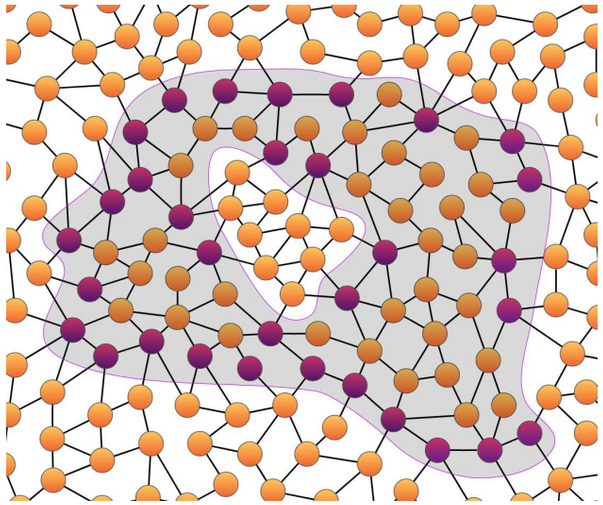

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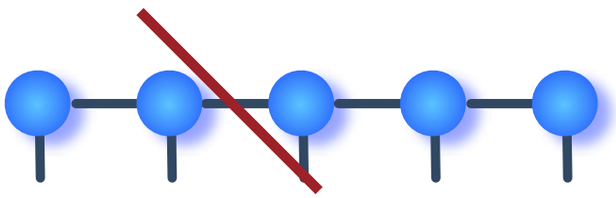

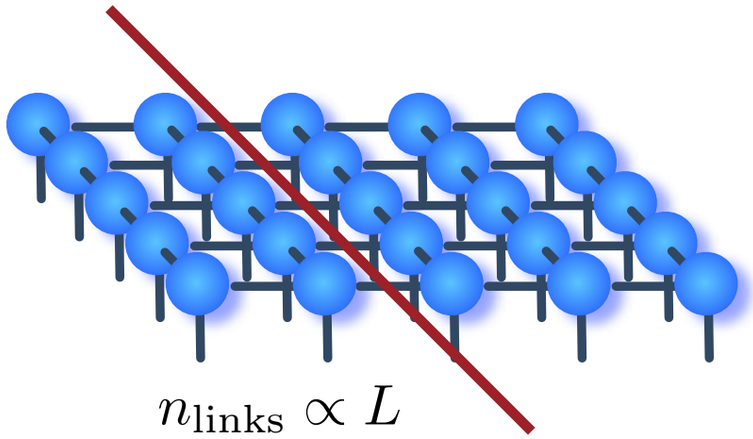

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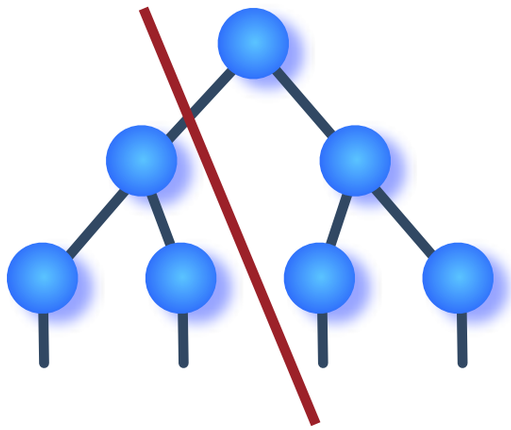
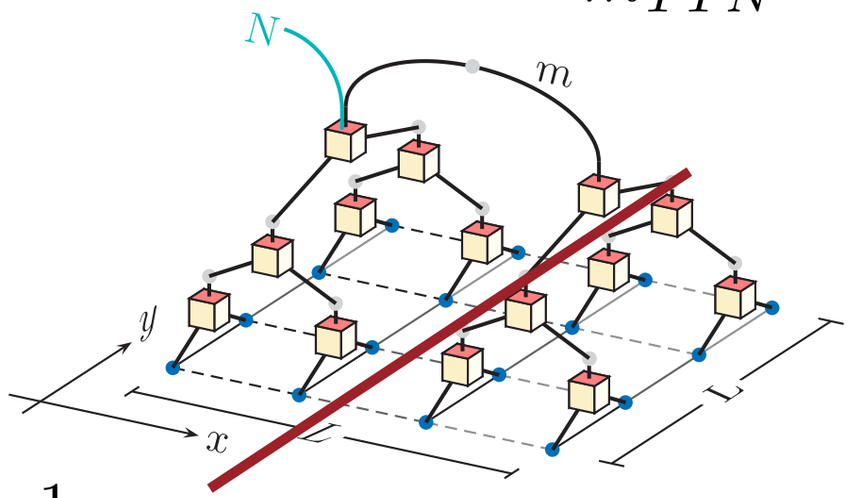


$n_{\text{links}} = 1$

$m_{TTN} \sim e^{cL}$



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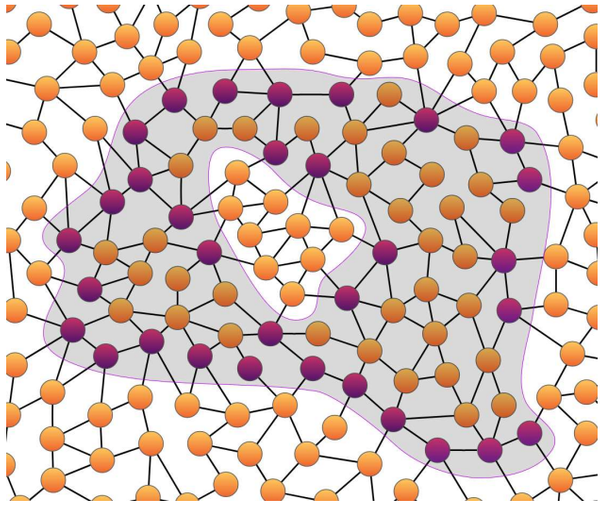

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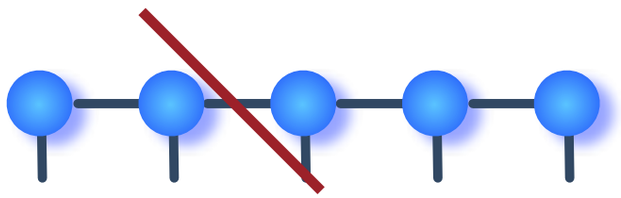

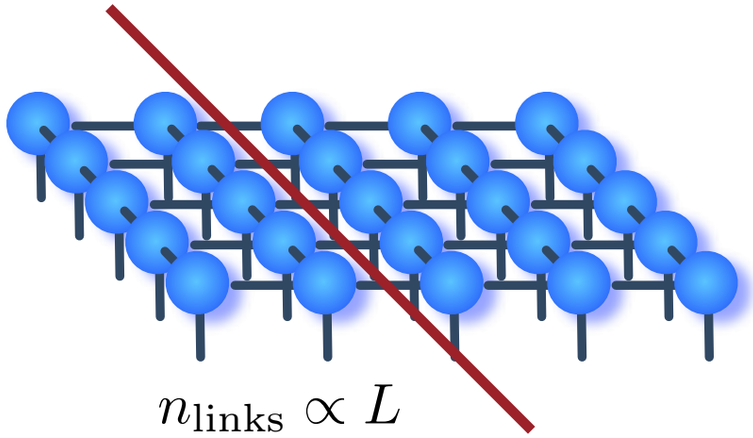

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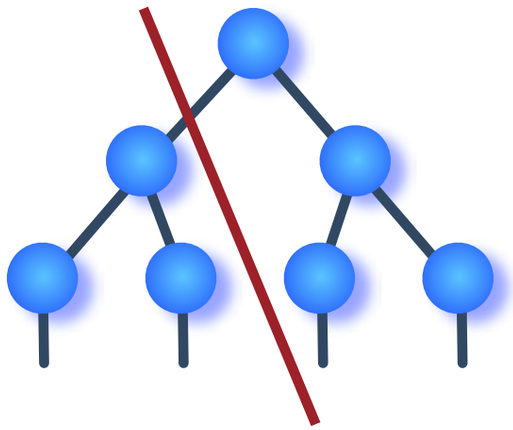

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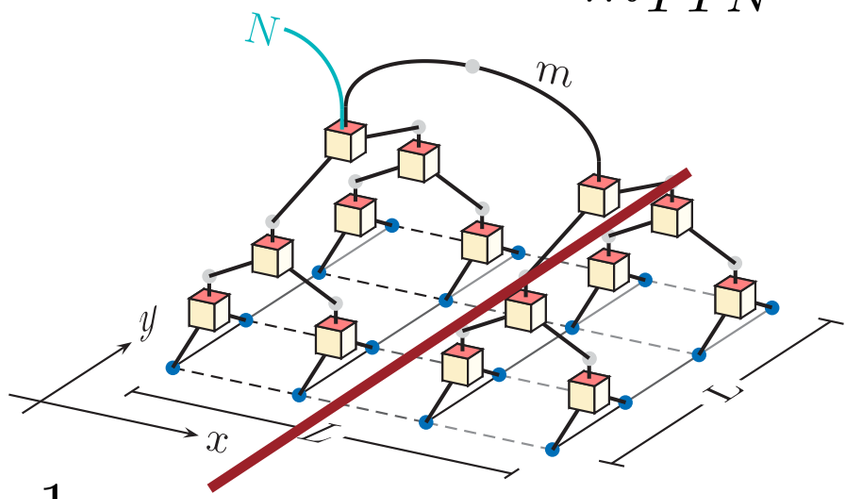

Tensor Network	Complexity	Area law in 2D	Typical Bond dimensions	Exact contractable
MPS / DMRG	$\mathcal{O}\{\chi^3\}$	No (Only in 1D)	> 10.000	Yes ($\mathcal{O}\{\chi^3\}$)
TTN	$\mathcal{O}\{\chi^4\}$	No (Only in 1D)	$\approx 1.000 - 2.000$	Yes ($\mathcal{O}\{\chi^4\}$)
PEPS	$\mathcal{O}\{\chi^{10}\}$	Yes	~ 10	No ($\mathcal{O}\{\chi^L\}$)
MERA	$\mathcal{O}\{\chi^8\}$ (1D), $\mathcal{O}\{\chi^{16}\}$ (2D)	Yes	~ 10	Yes ($\mathcal{O}\{\chi^8\}$)

AREA LAWS AND TENSOR NETWORKS

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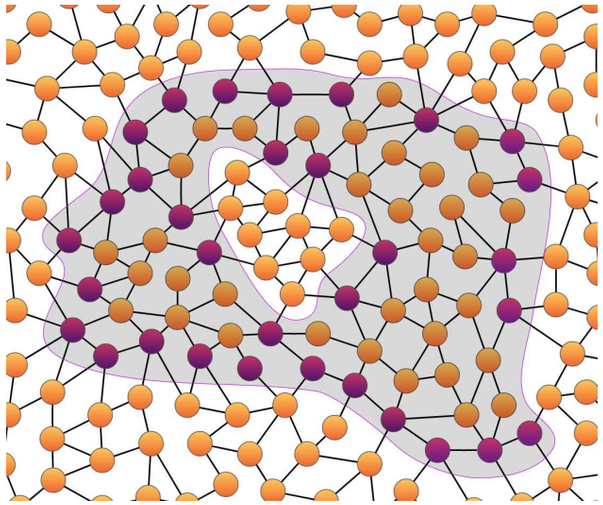

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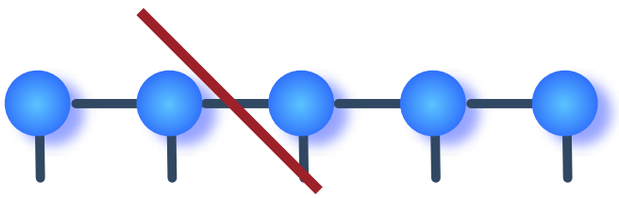

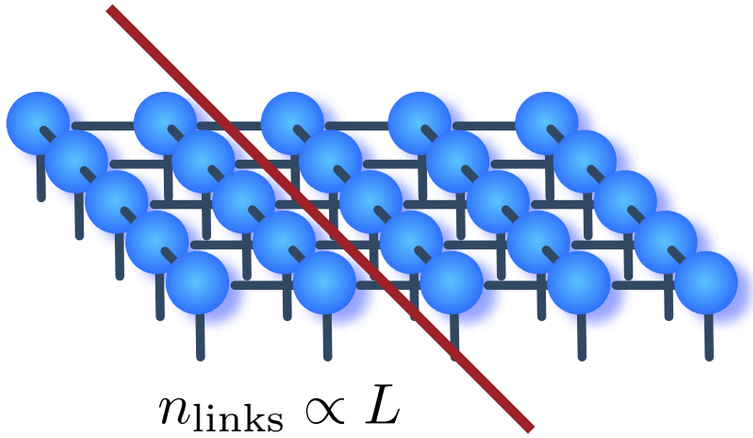

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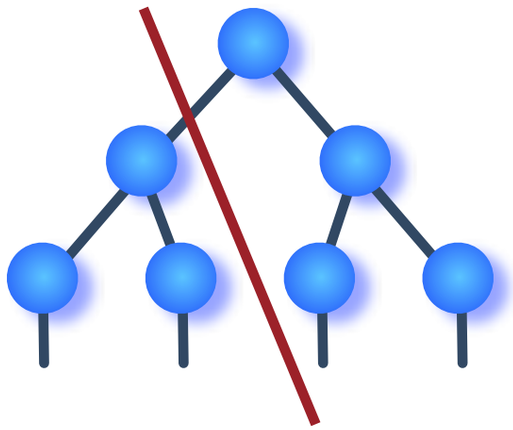

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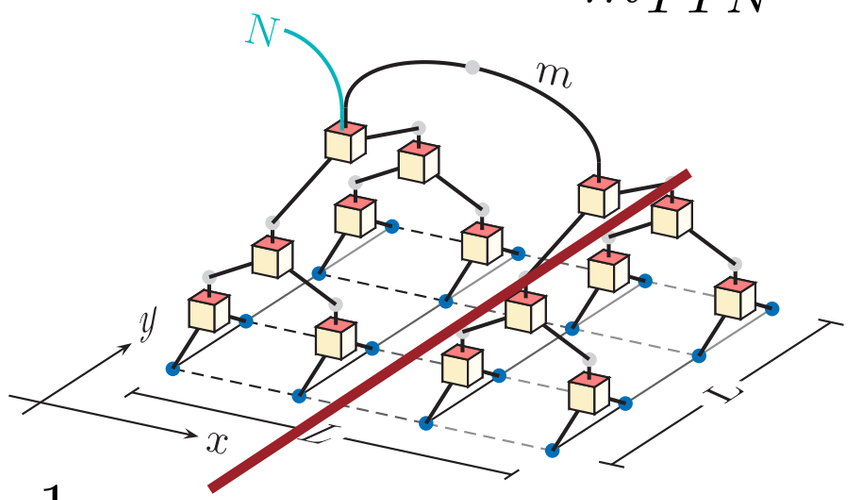

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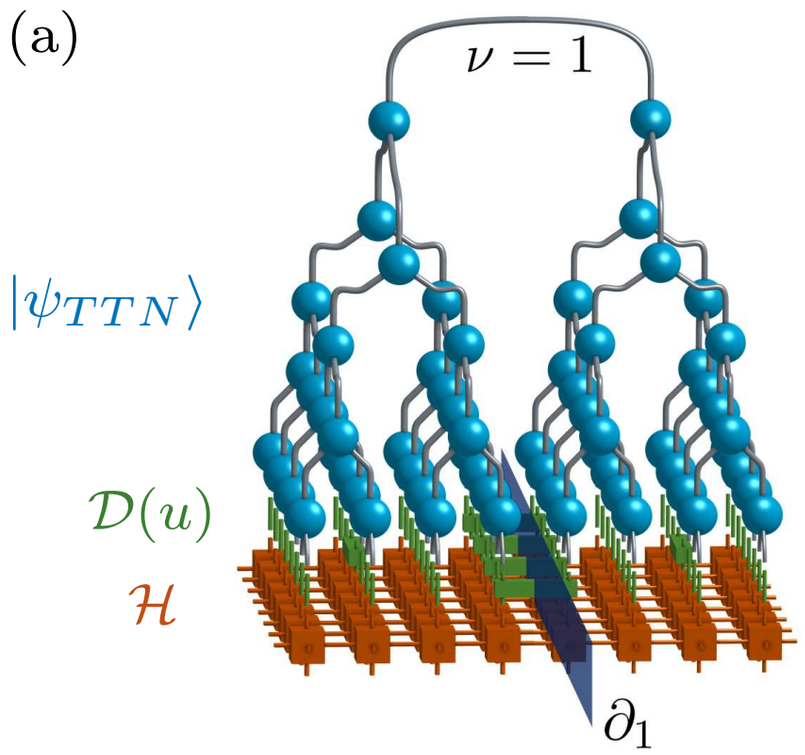
TTN	$\mathcal{O}\{\chi^4\}$	No (Only in 1D)	$\approx 1.000 - 2.000$	Yes ($\mathcal{O}\{\chi^4\}$)
PEPS	$\mathcal{O}\{\chi^{10}\}$	Yes	~ 10	No ($\mathcal{O}\{\chi^L\}$)
MERA	$\mathcal{O}\{\chi^8\}$ (1D), $\mathcal{O}\{\chi^{16}\}$ (2D)	Yes	~ 10	Yes ($\mathcal{O}\{\chi^8\}$)
aTTN	$\mathcal{O}\{\chi^4\}$	Yes*	≈ 500	Yes ($\mathcal{O}\{\chi^4\}$)

AUGMENTED TREE TENSOR NETWORKS



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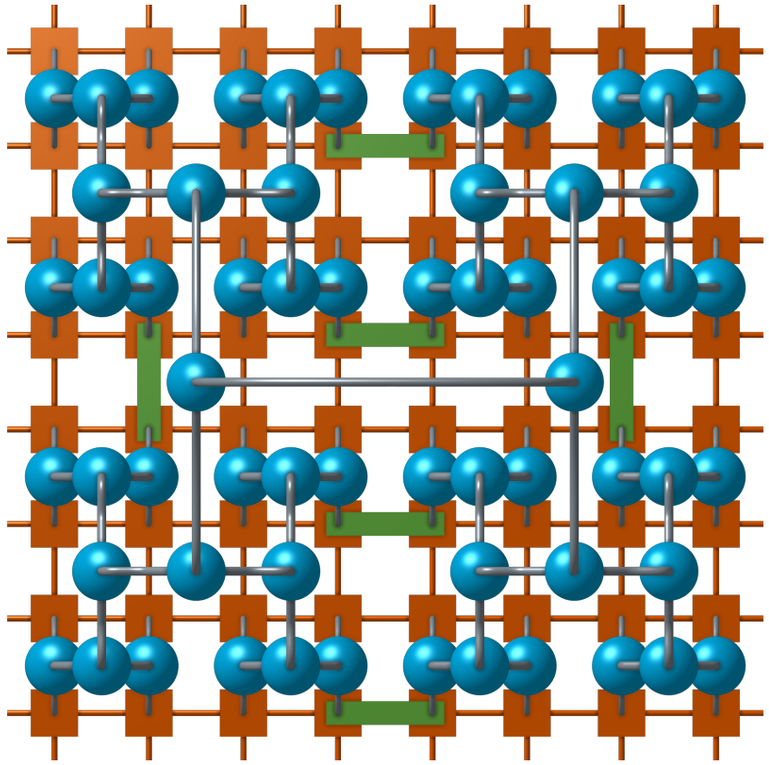

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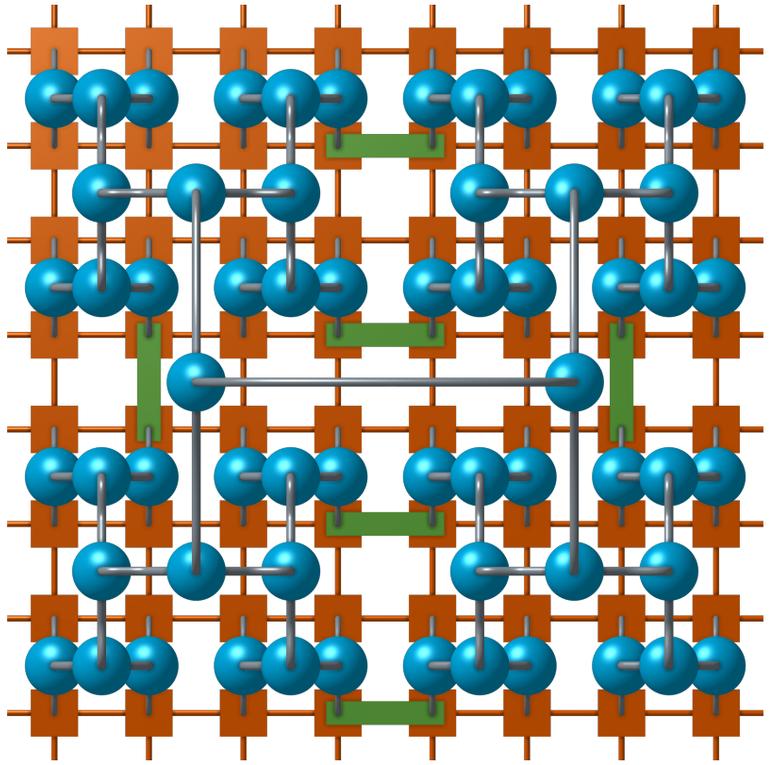

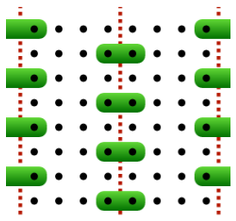

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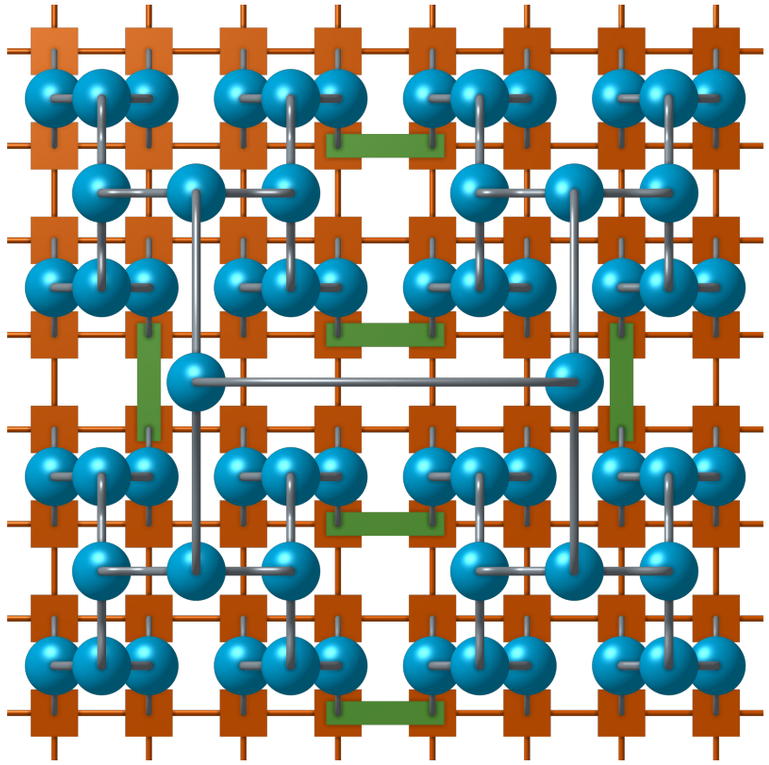

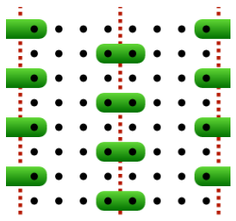

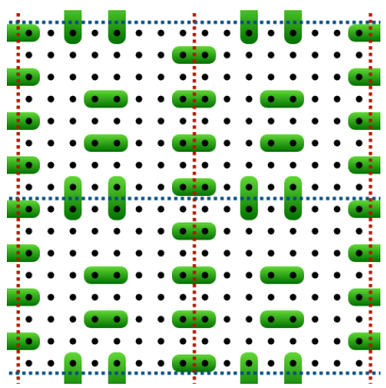

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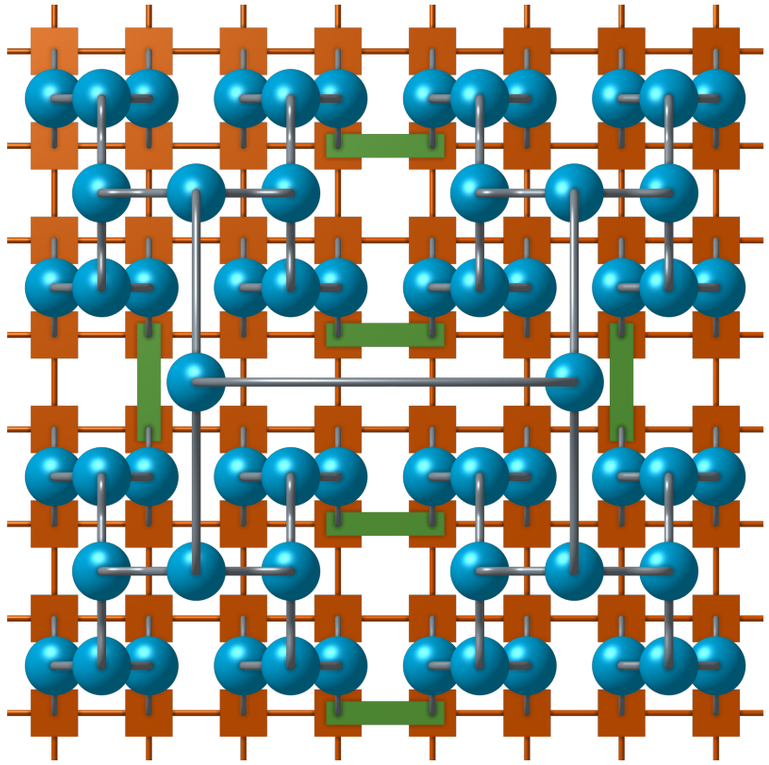

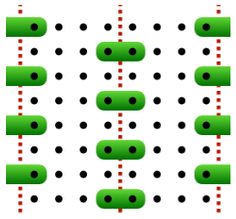

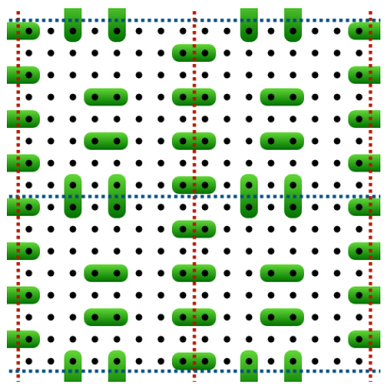

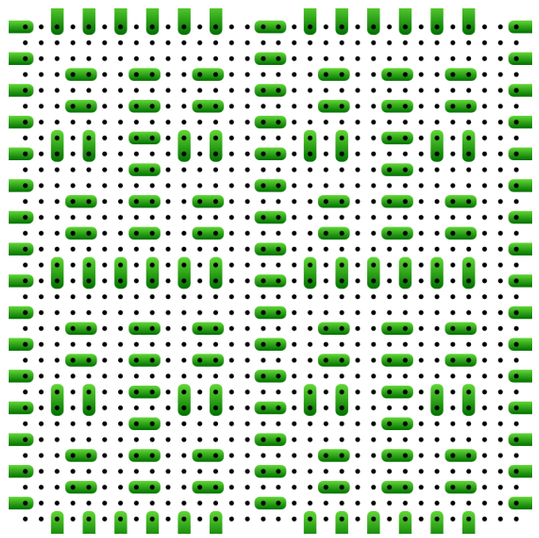

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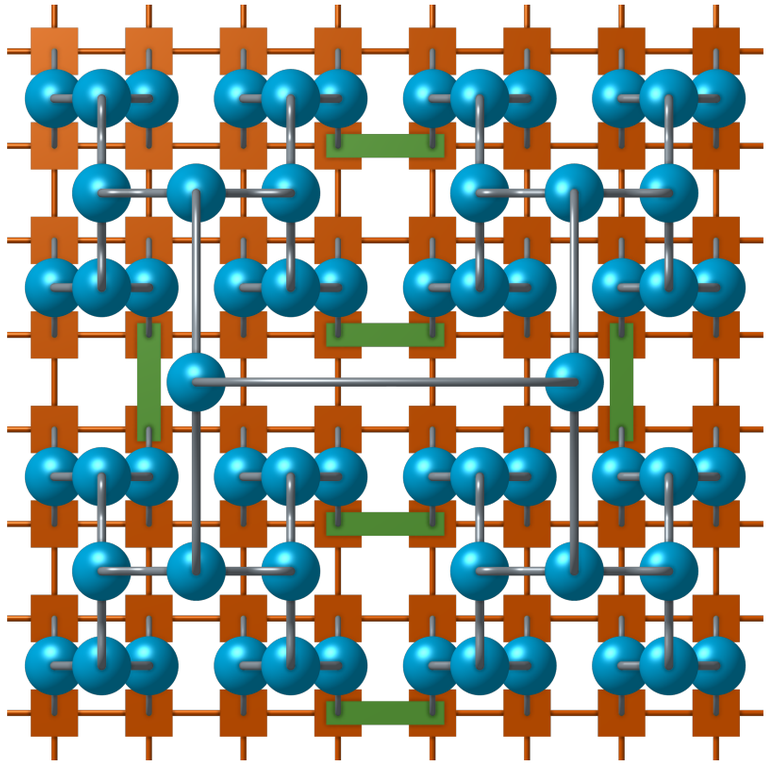

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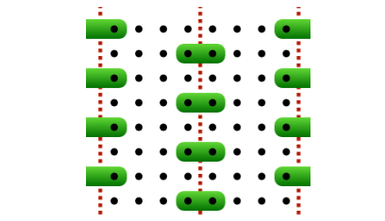

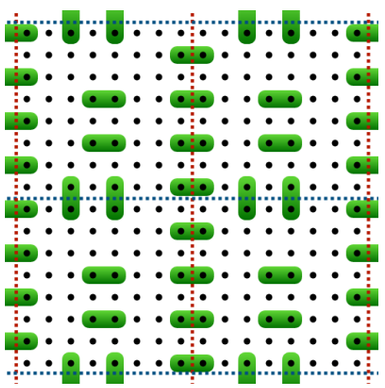

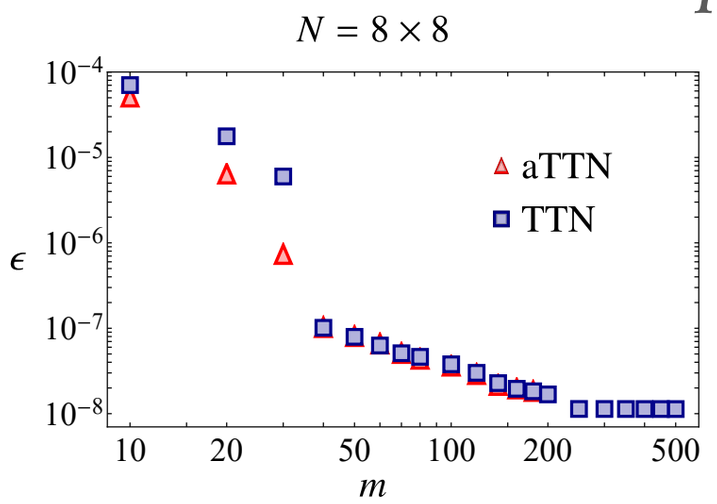

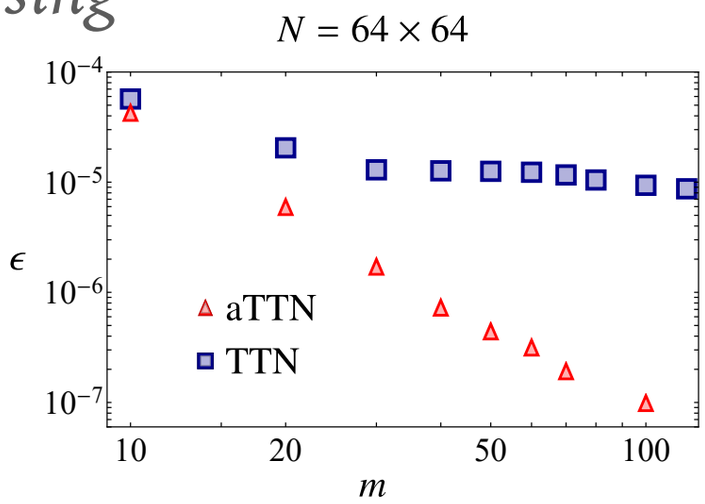

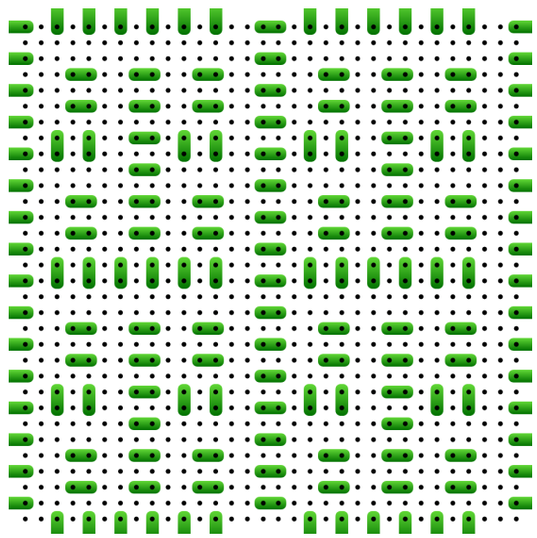

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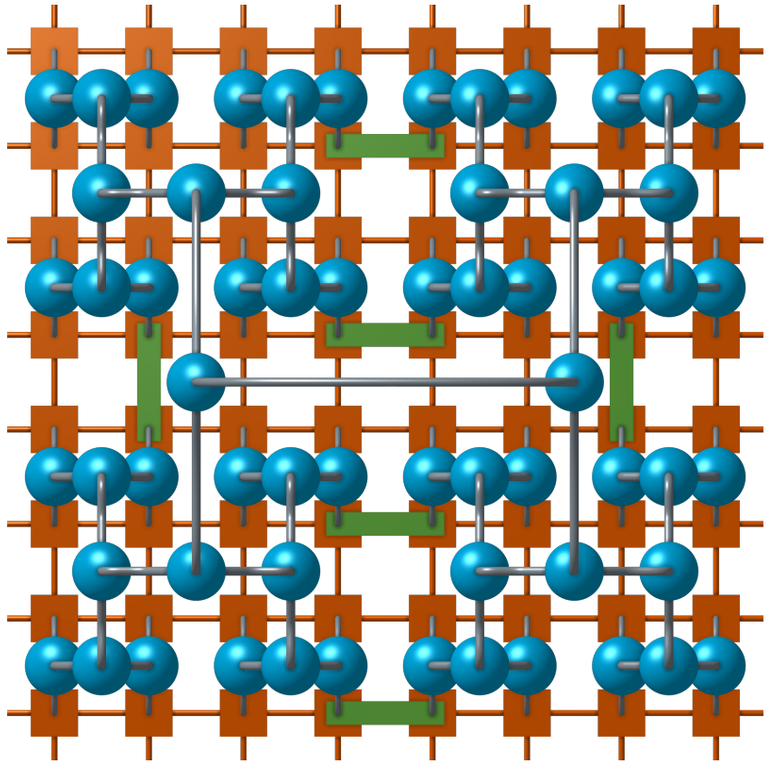


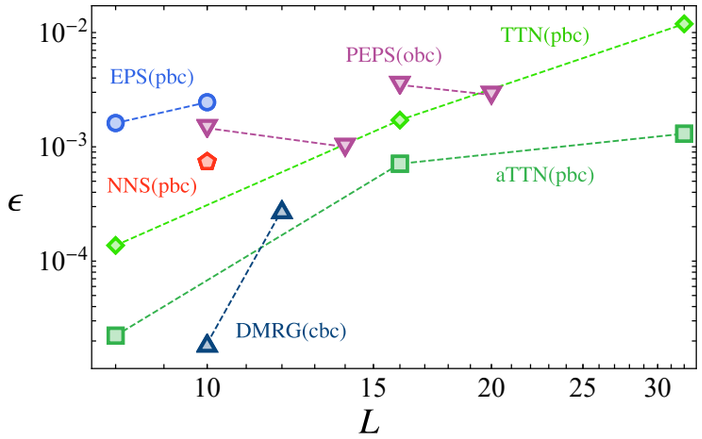
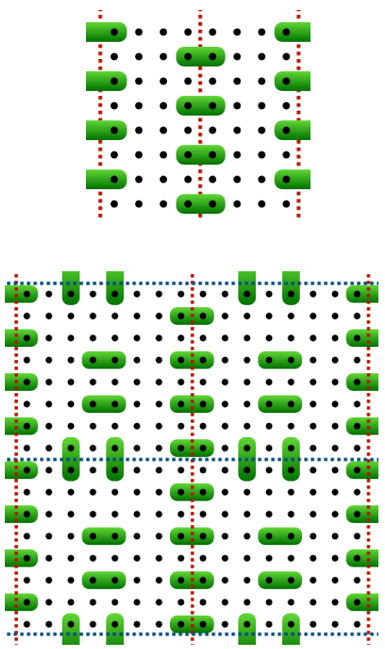
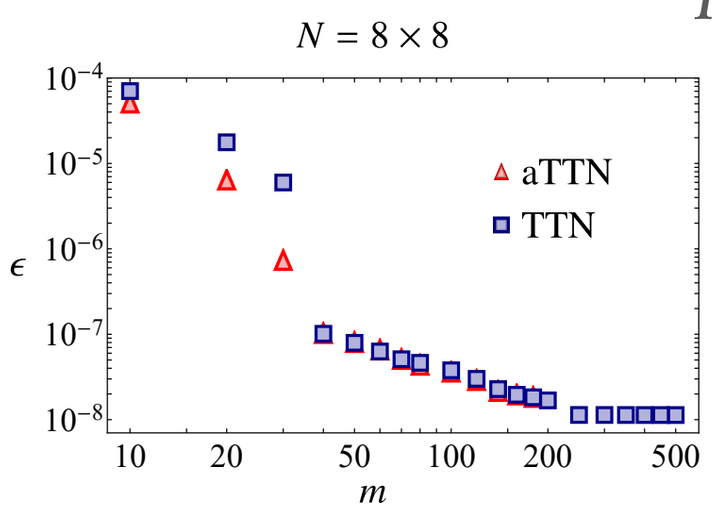
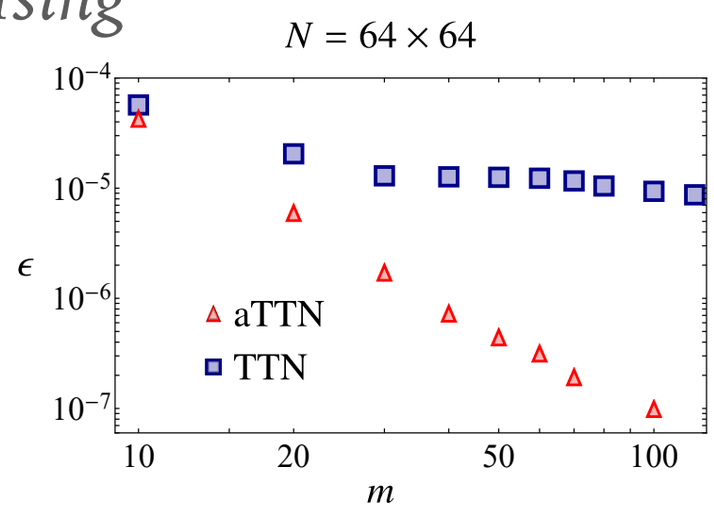


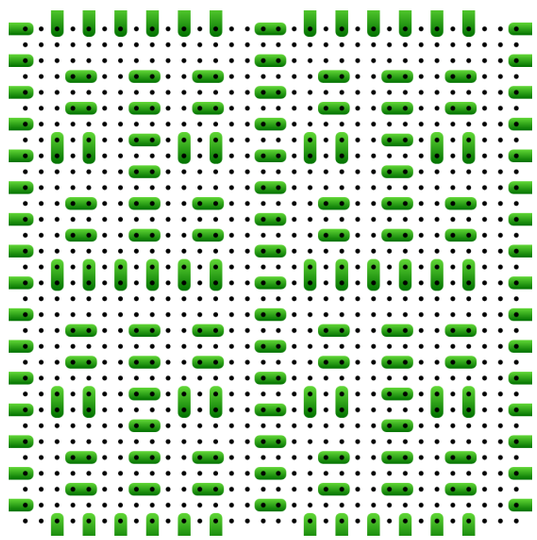
FIG. 2: Relative error ϵ of the 2D Heisenberg ground-state energy as a funciotn of the system linear size L compared with the best available estimates obtained by MC [21] for the TTN, aTTN, NNS [59], EPS [60], PEPS [61], 2D-DMRG [57]. Depending on the method open (obc), cylindrical (cbc) or periodic (pbc) boundary conditions have been chosen. For each datapoint, we compare the Monte Carlo result with the same boundary conditions.

Ising









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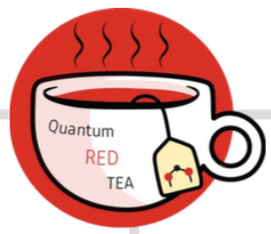
Overview Quantum TEA



Quantum **T**ensor network **E**mulator **A**pplications

Can I just have tensor networks for quantum information?

How to emulate a digital quantum circuits?

+

www.quantumtea.it
baltig.infn.it/quantum_tea

Courtesy of Daniel Jaschke, INFN Padova

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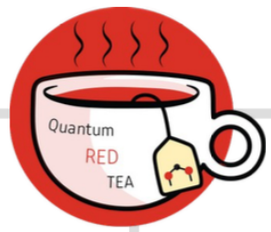

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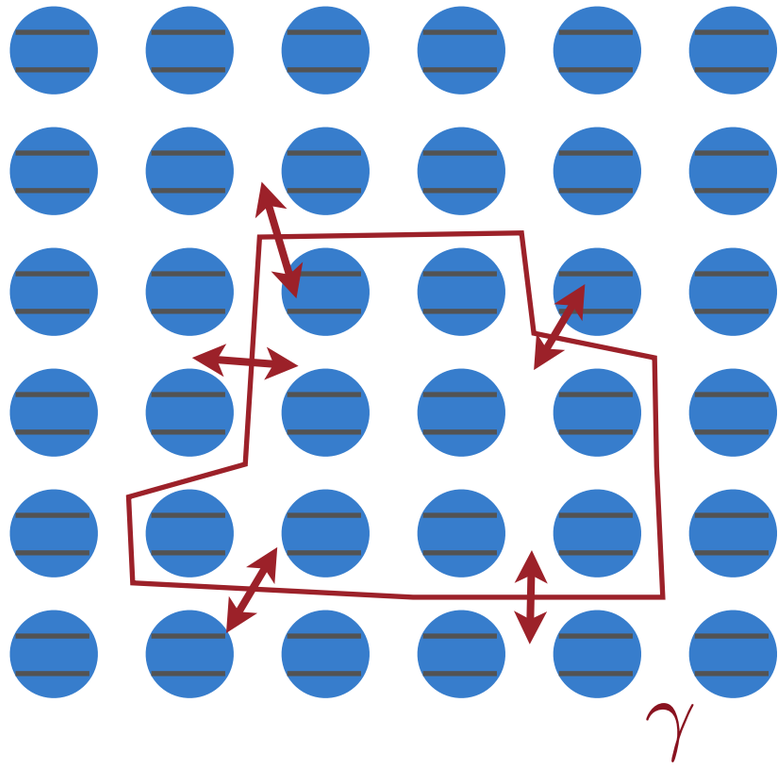
ENTANGLEMENT OF MIXED MANY-BODY QUANTUM SYSTEMS

For pure states:

$$\mathcal{S} = -\mathrm{Tr}\,\rho \log \rho$$

Von Neumann Entropy



Area law

$$\mathcal{S} \propto \gamma$$

$$\mathcal{S} \propto N^{(D-1)}$$

1D critical systems:

$$\mathcal{S} = \frac{c}{3} \log N$$

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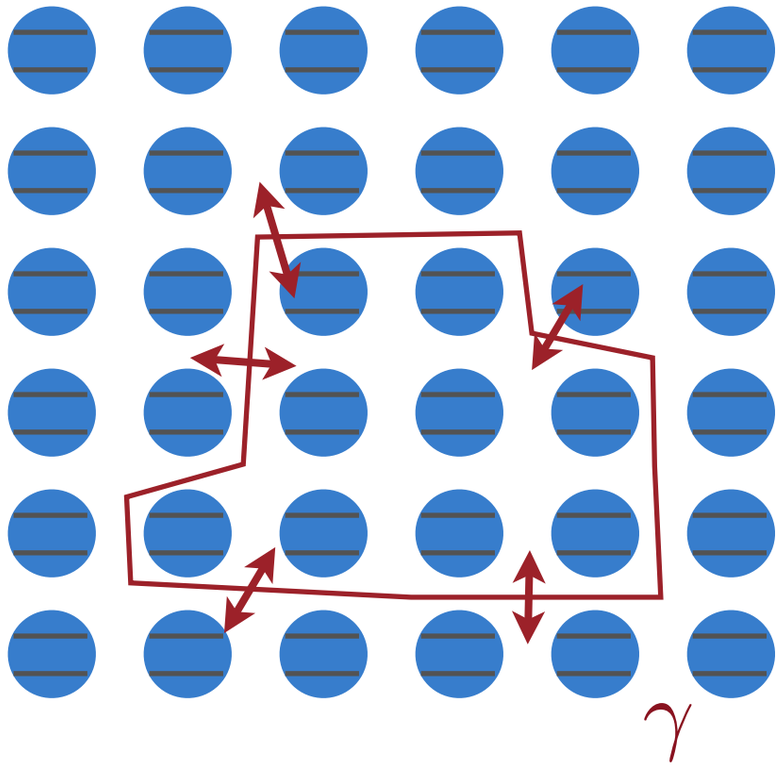

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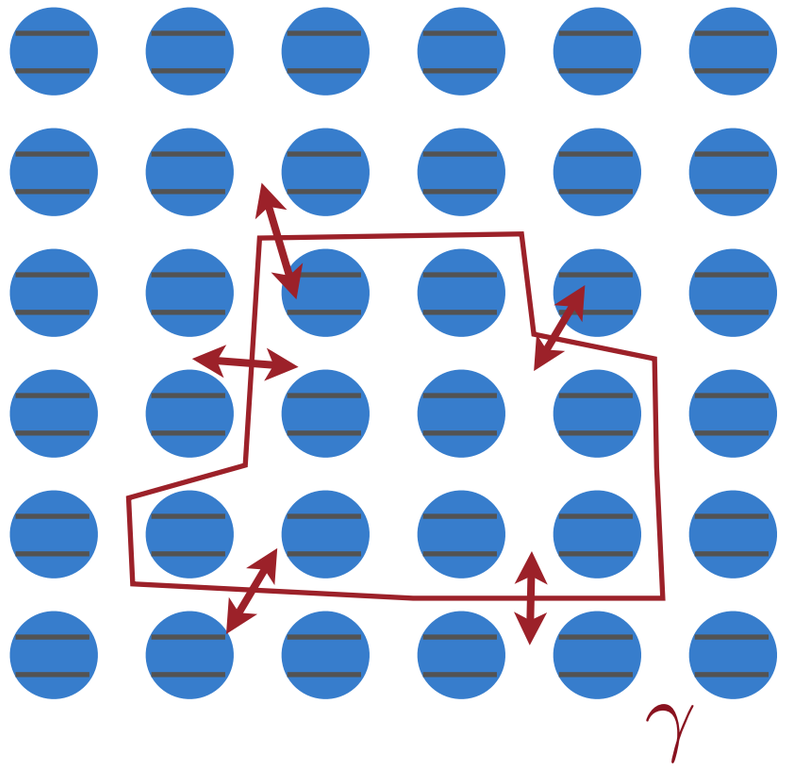

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Entanglement of formation

C.H. Bennet et al. PRA 1996

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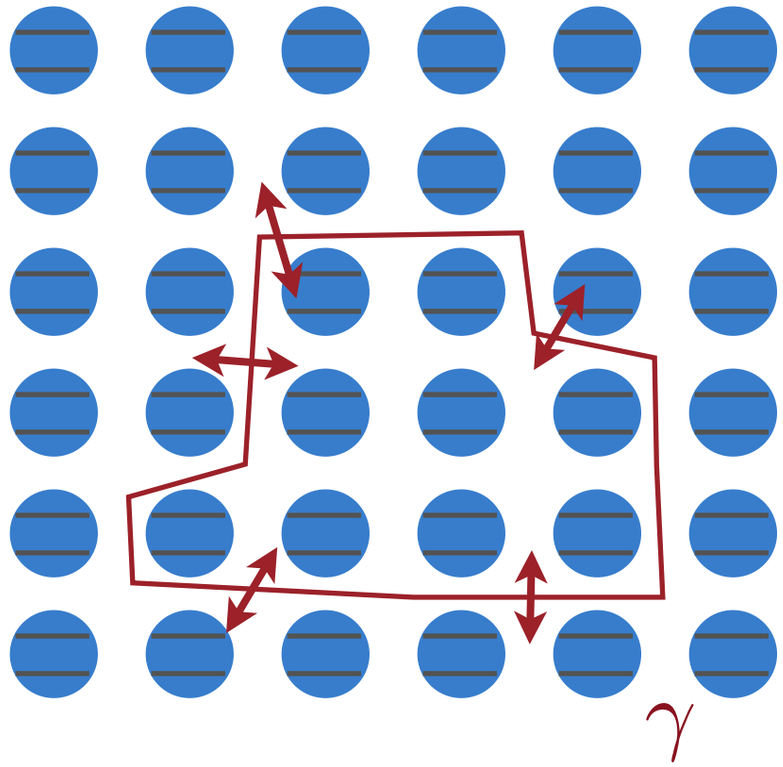

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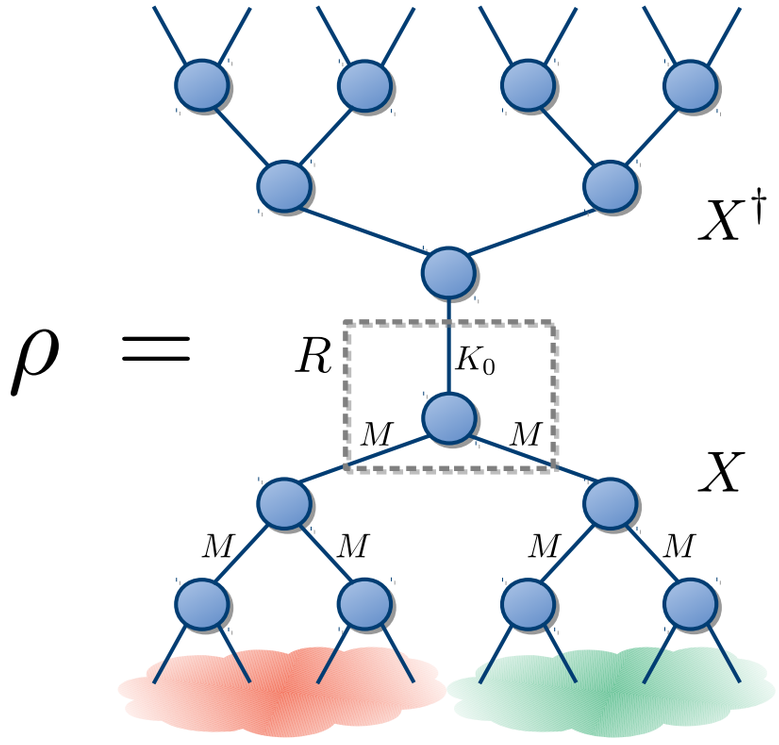
$$E_F(\rho, T) \overset{?}{\propto} \log N^{c/3}$$

TREE TENSOR OPERATORS



L. Arceci, P. Silvi, and S. Montangero PRL (2022)

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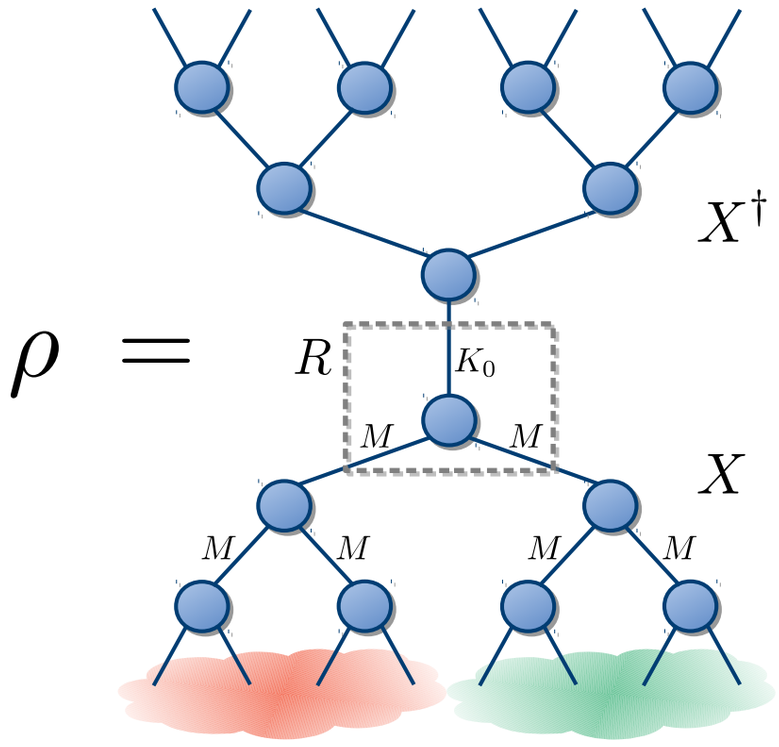

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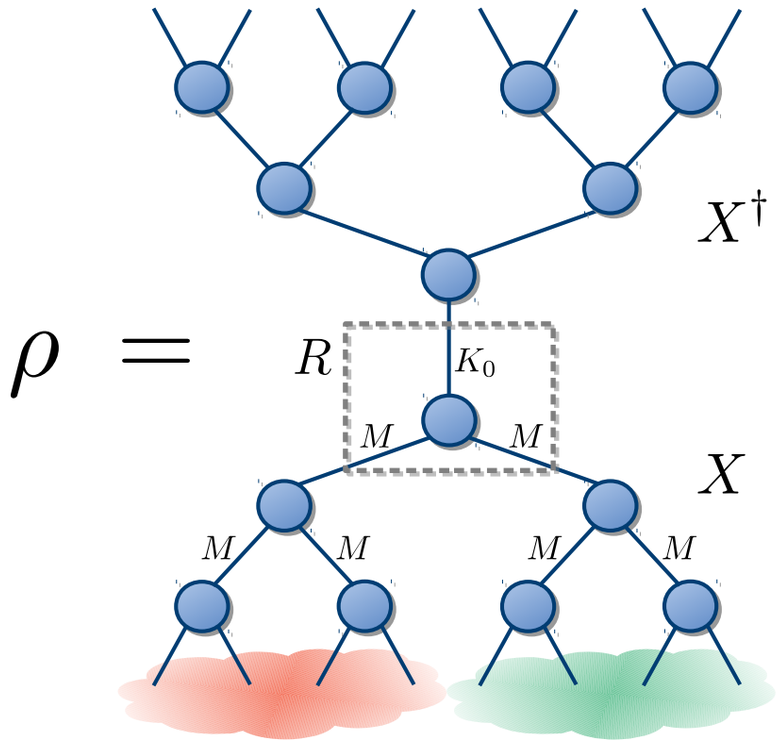

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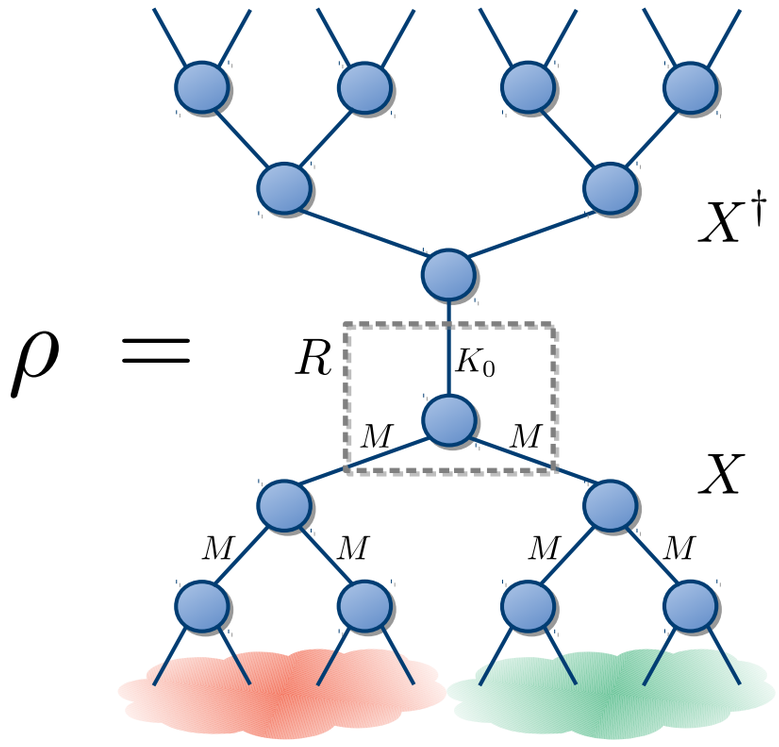

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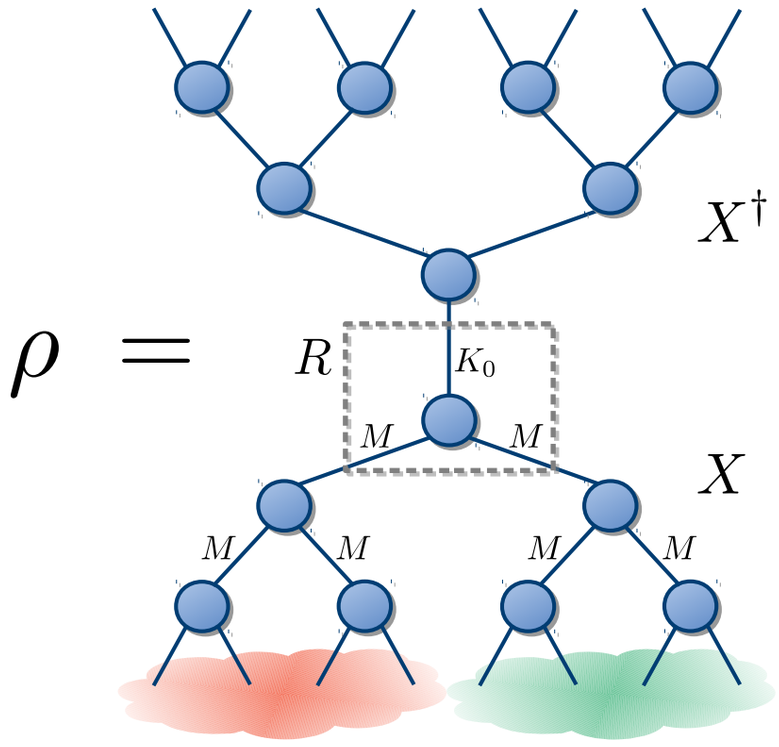

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(Mixture of K_0 Boltzmann factors)

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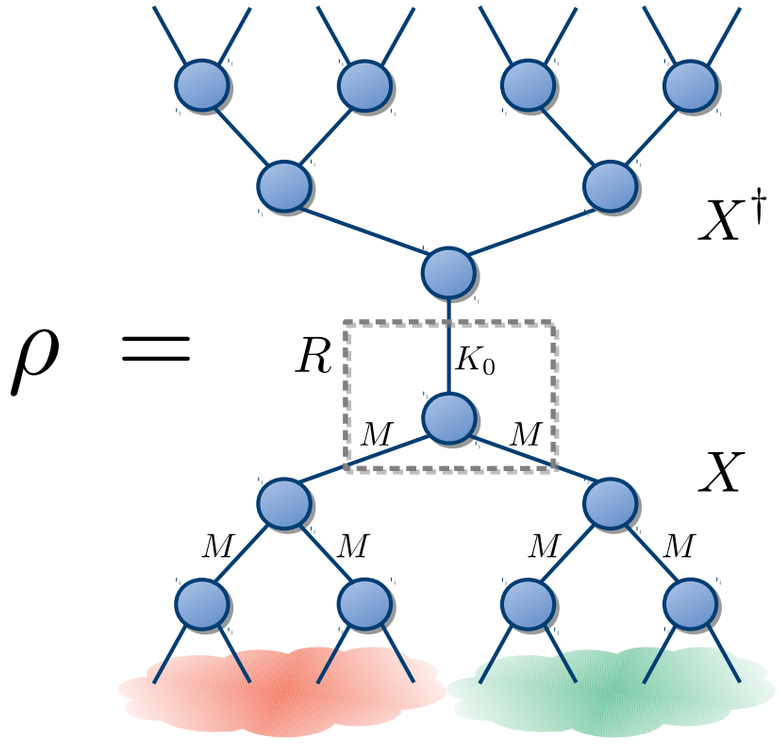

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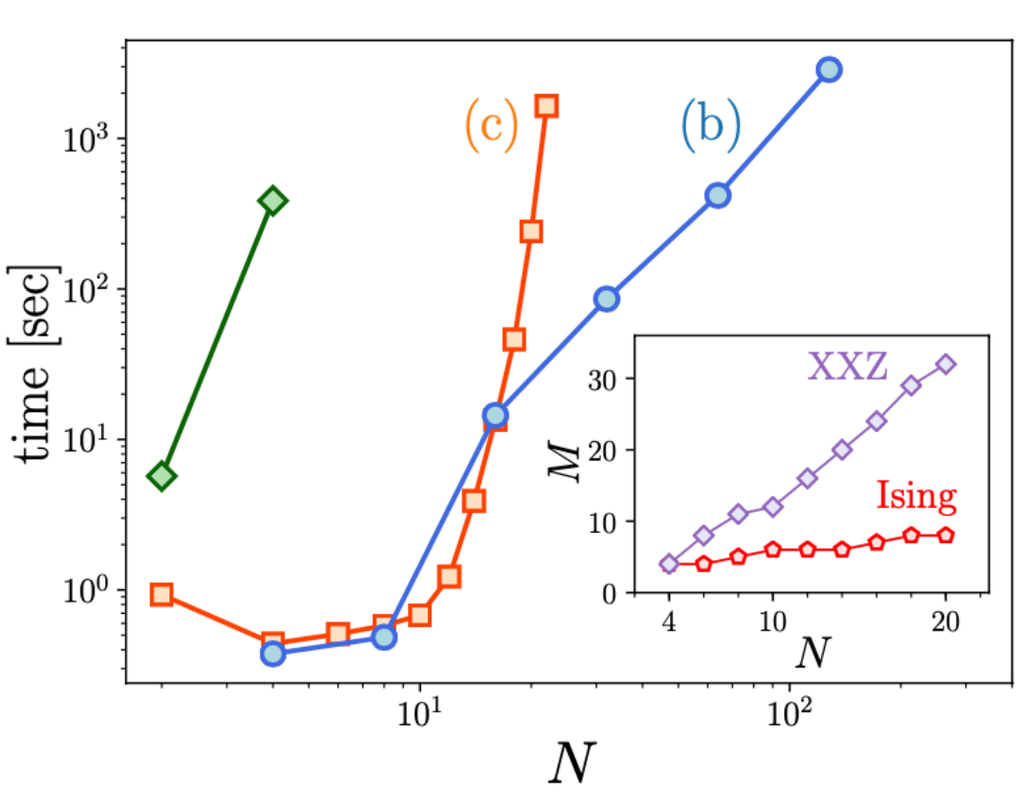

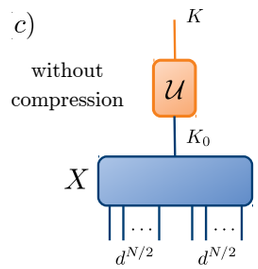

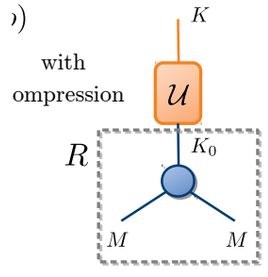

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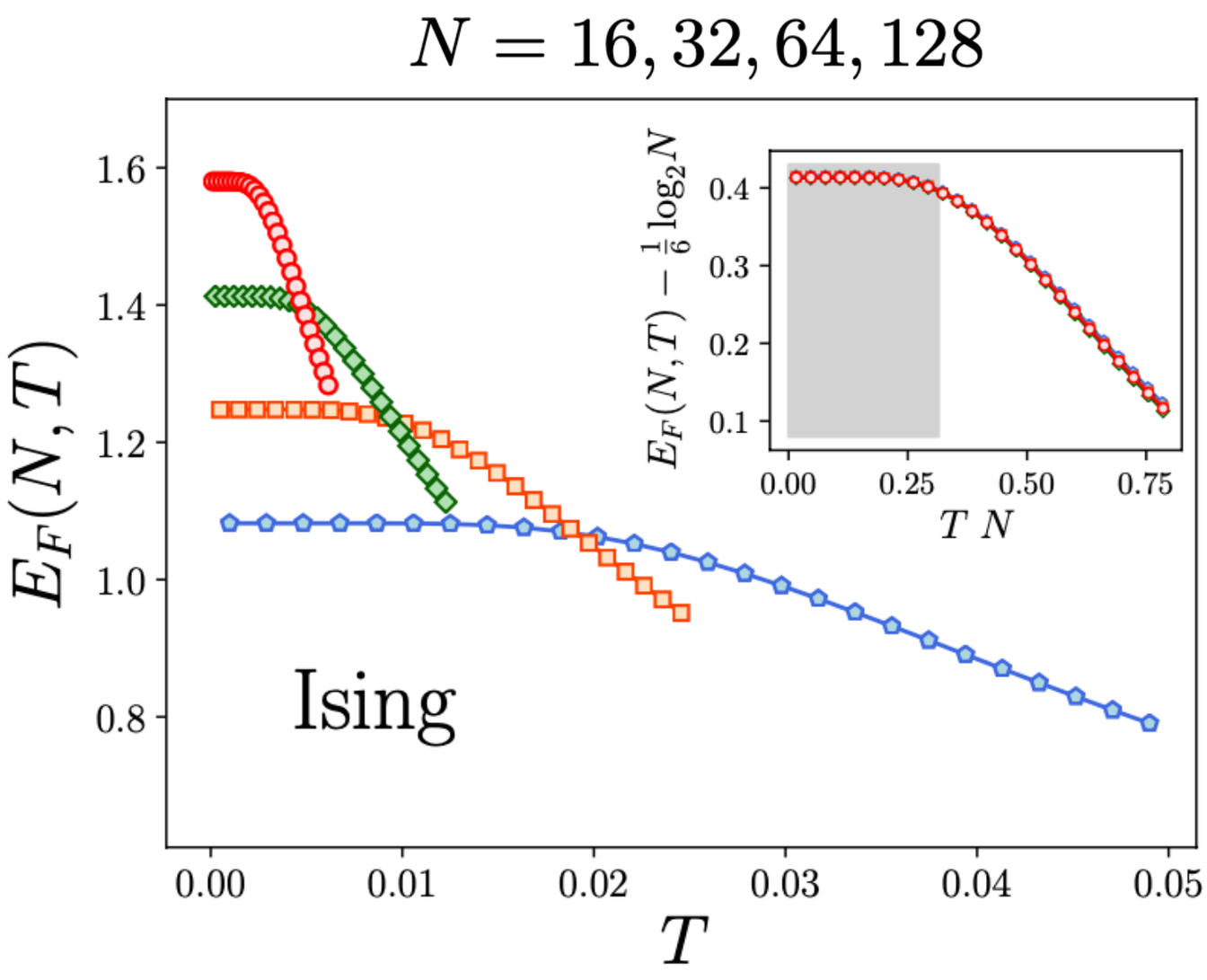
CONFORMAL SCALING OF ENTANGLEMENT OF FORMATION

L. Arceci, P. Silvi, and S. Montangero PRL (2022)

CONFORMAL SCALING OF ENTANGLEMENT OF FORMATION

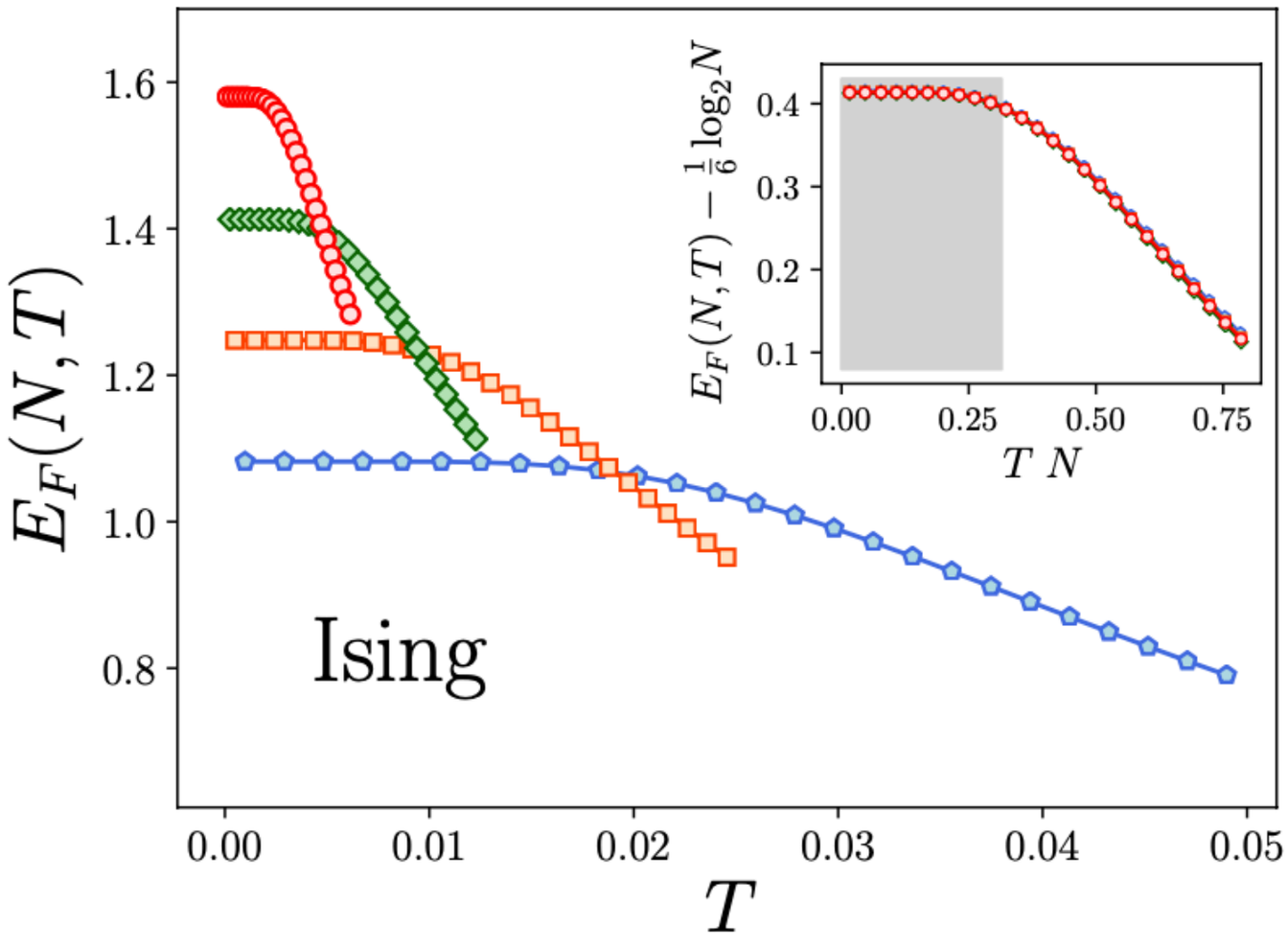


L. Arceci, P. Silvi, and S. Montangero PRL (2022)

CONFORMAL SCALING OF ENTANGLEMENT OF FORMATION

$$E_F = \log(N^{c/3} f(TN^z))$$

$N = 16, 32, 64, 128$



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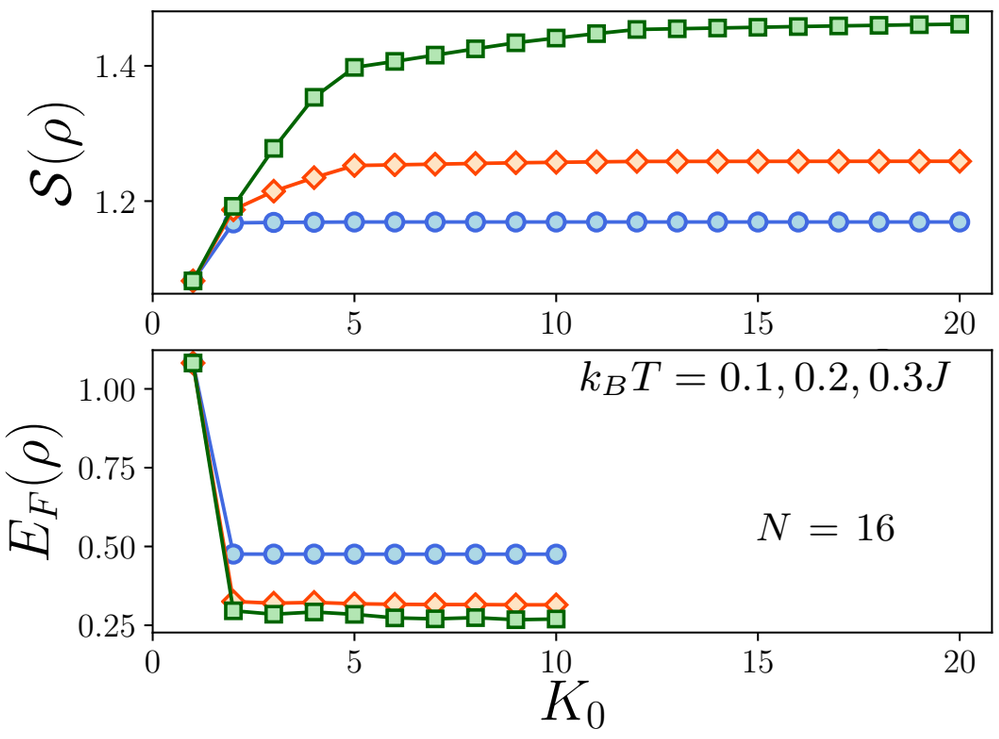

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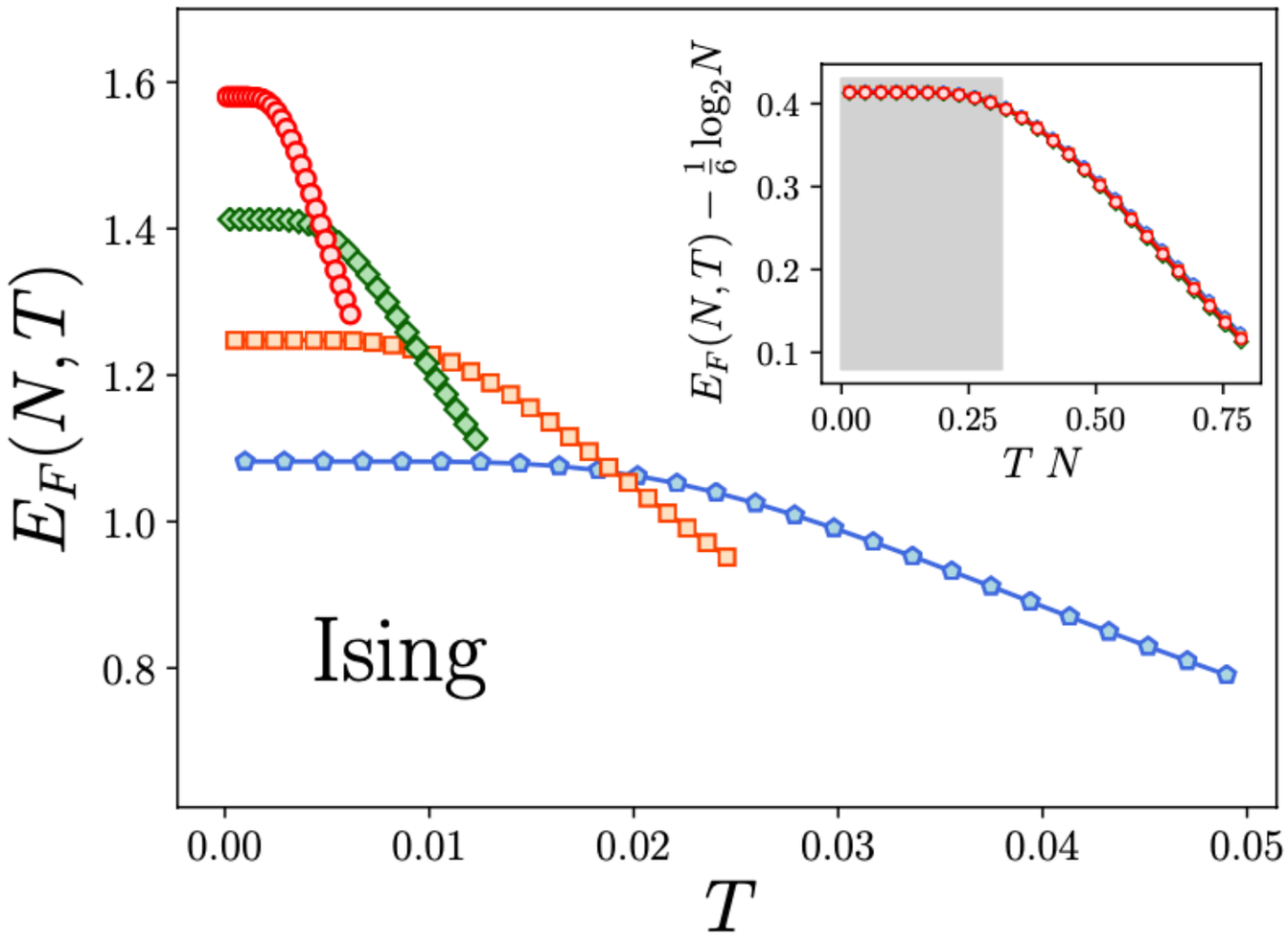

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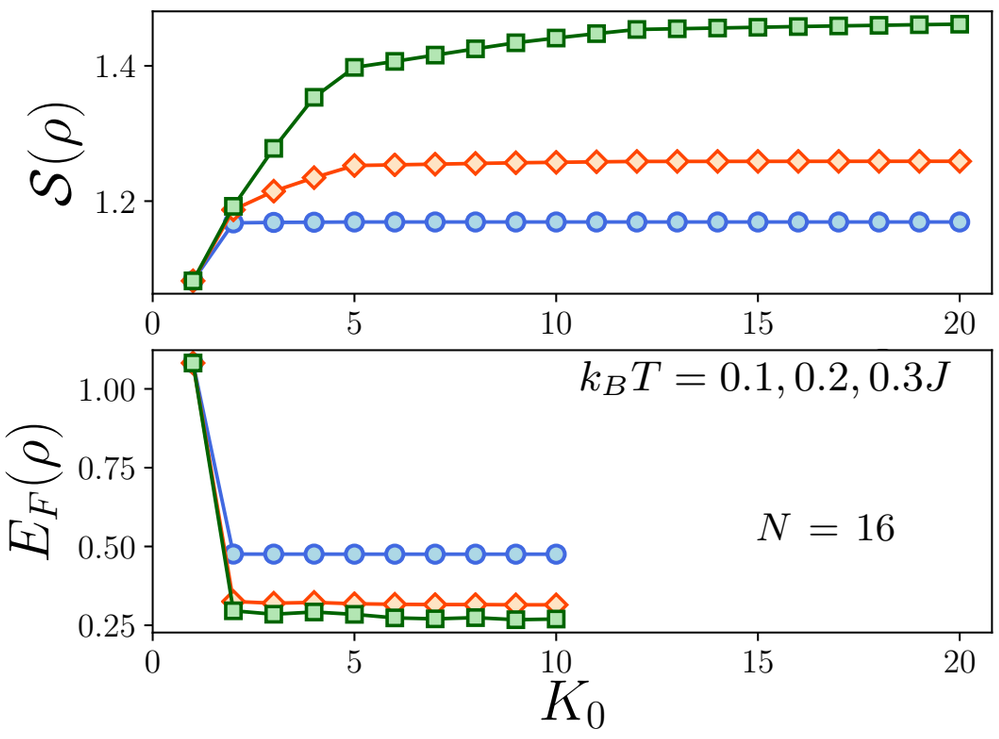

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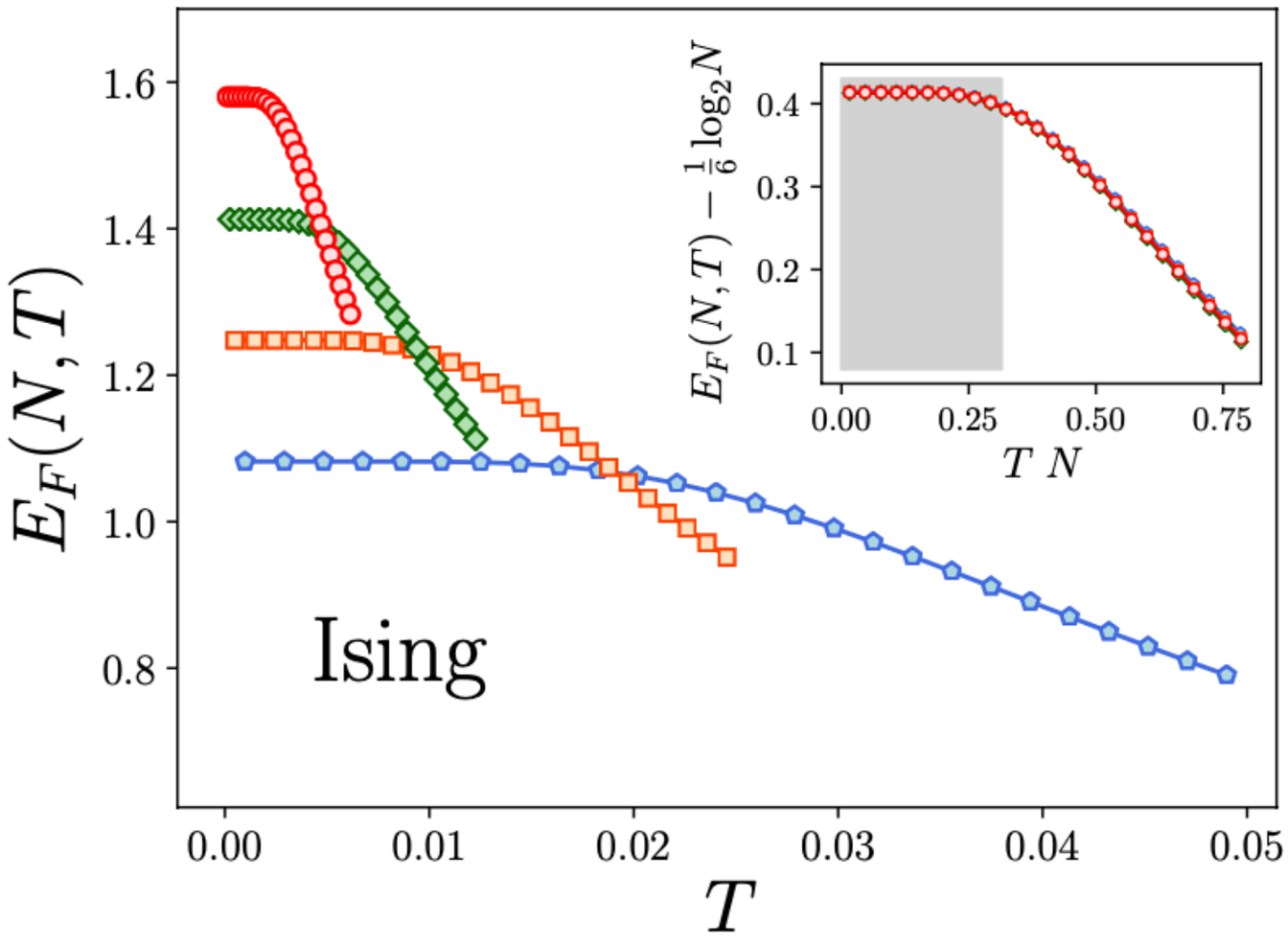

Numerical complexity depends on entropy, not entanglement!

L. Arceci, P. Silvi, and S. Montangero PRL (2022)

ADIABATIC QUANTUM COMPUTING

- *Preparation of the system in an "easy" state*

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- *Preparation of the system in an "easy" state* $\downarrow\downarrow\downarrow \ldots \downarrow\downarrow\downarrow$
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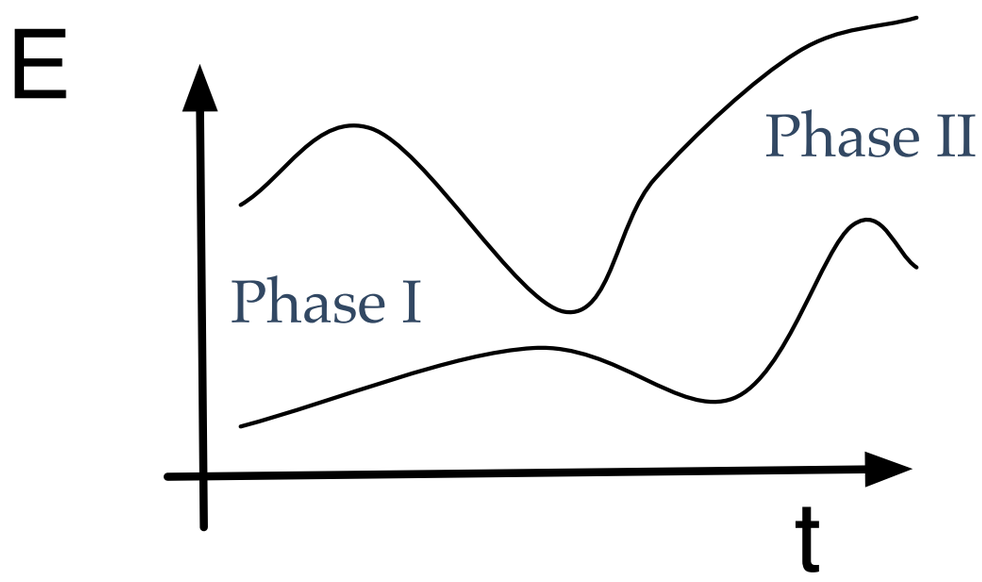

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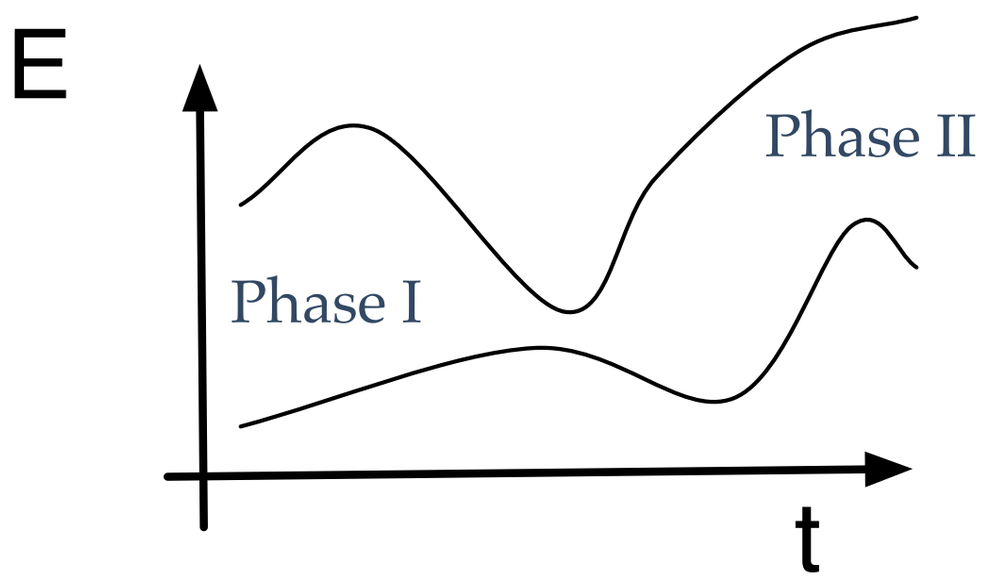

Adiabatic strategy

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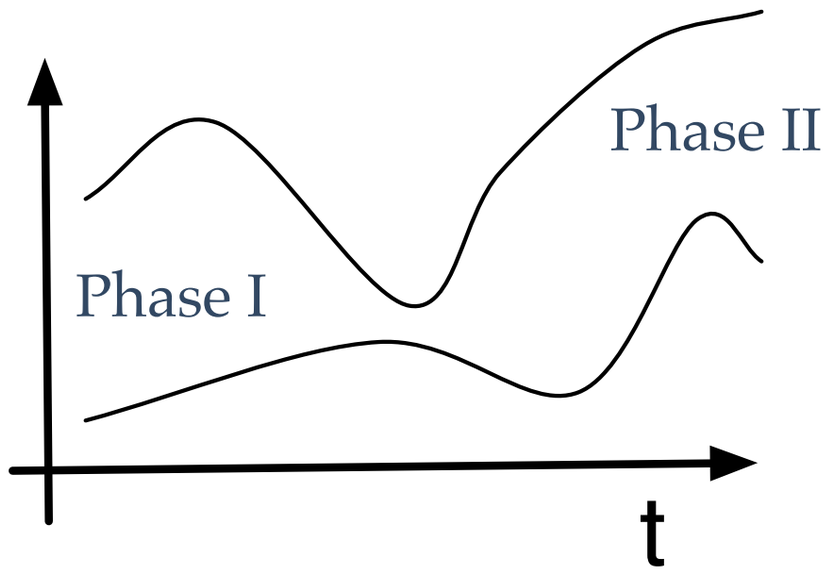
Slow



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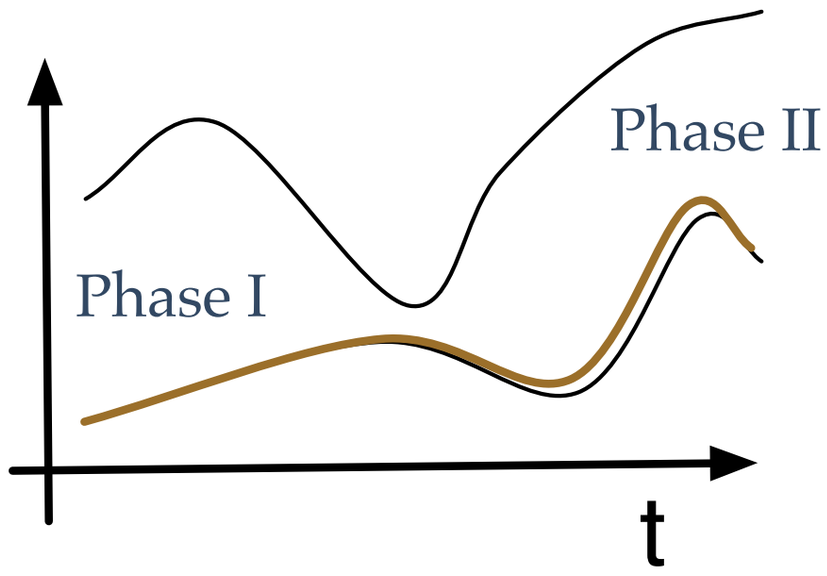

Adiabatic strategy

OPTIMAL CROSSING OF QPT

RedCRAB

Optimal control

P. Doria et al PRL (2010), T. Caneva ***et al. PRA (2014)***

OPTIMAL CROSSING OF QPT

System

RedCRAB

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$$i\frac{\partial}{\partial t}|\psi(t)\rangle = (H_0 + f(t)H_1)\,|\psi(t)\rangle$$

P. Doria et al PRL (2010), T. Caneva ***et al.*** ***PRA (2014)***

OPTIMAL CROSSING OF QPT

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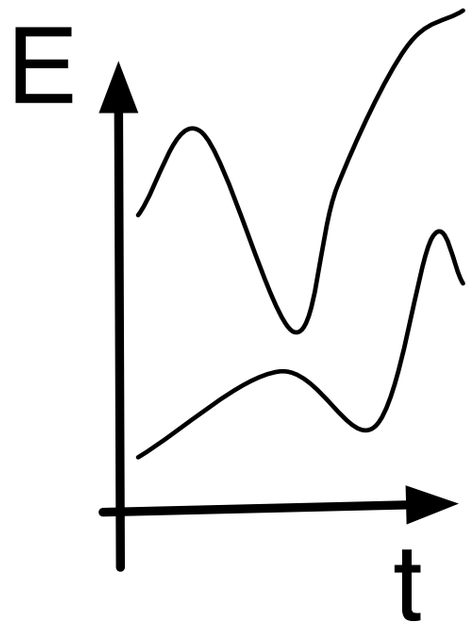

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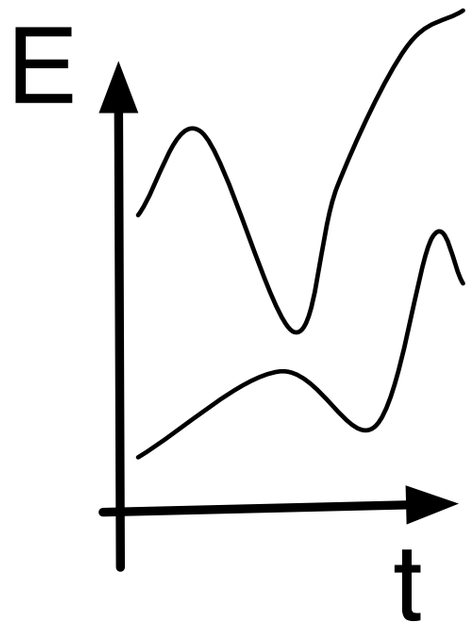

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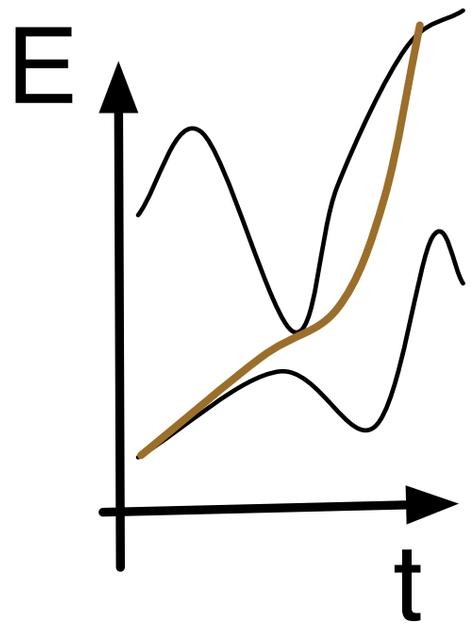

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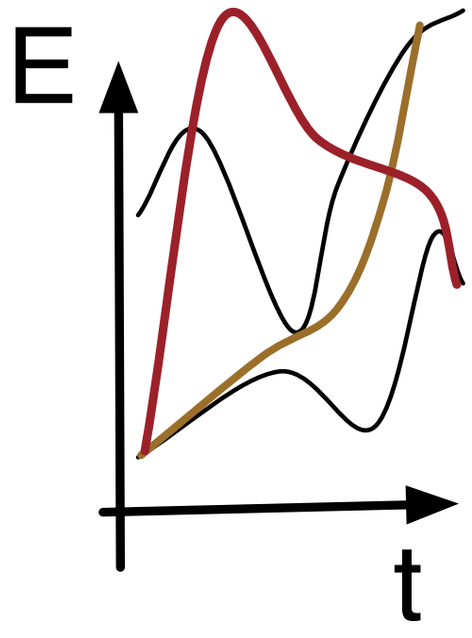

Optimal control

P. Doria et al PRL (2010), T. Caneva ***et al.*** ***PRA (2014)***

OPTIMAL CROSSING OF QPT

Optimal control of complex atomic quantum systems

S. van Frank[1], M. Bonneau[1], J. Schmiedmayer[1],
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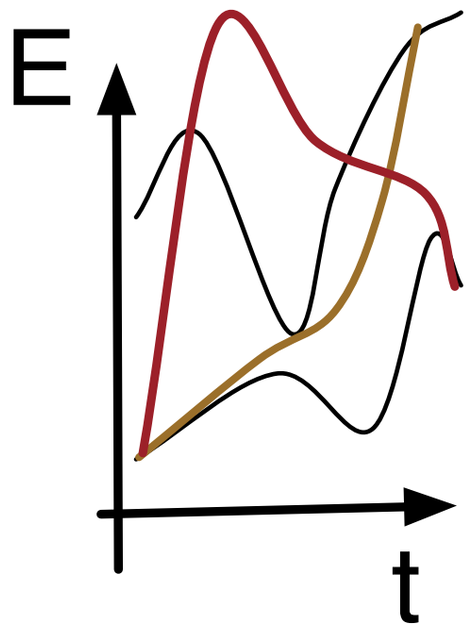

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Optimal control

Fast closed-loop optimal control of ultracold atoms in an optical lattice

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[2]*QSTAR Center for Quantum Science and Technology, Largo Enrico Fermi 2, I-50125 Arcetri, Italy*

T. Calarco[3] and S. Montangero[3]
[3]*Institut für Quanteninformationsverarbeitung, Universität Ulm, D-89069 Ulm, Germany*
(Dated: July 30, 2019)

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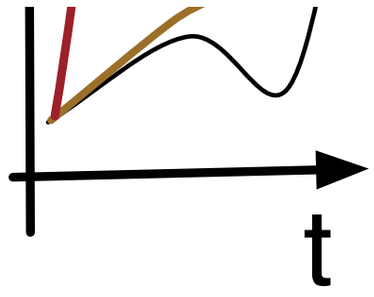

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Generation and manipulation of Schrödinger cat states in Rydberg atom arrays

A. Omran,[1,*] H. Levine,[1,*] A. Keesling,[1] G. Semeghini,[1] T. T. Wang,[1,2] S. Ebadi,[1] H. Bernien,[3] A. S. Zibrov,[1] H. Pichler,[1,4] S. Choi,[5] J. Cui,[6] M. Rossignolo,[7] P. Rembold,[6] S. Montangero,[8] T. Calarco,[6,9] M. Endres,[10] M. Greiner,[1] V. Vuletić,[11] and M. D. Lukin[1,†]

HIGHER DIMENSIONS

CONFINEMENT IN 2D ISING MODEL

$$H = -J \sum_{\langle i,j \rangle} \sigma_i^z \sigma_j^z - g \sum_i \sigma_i^x$$

$$|\downarrow \ldots \downarrow\rangle$$

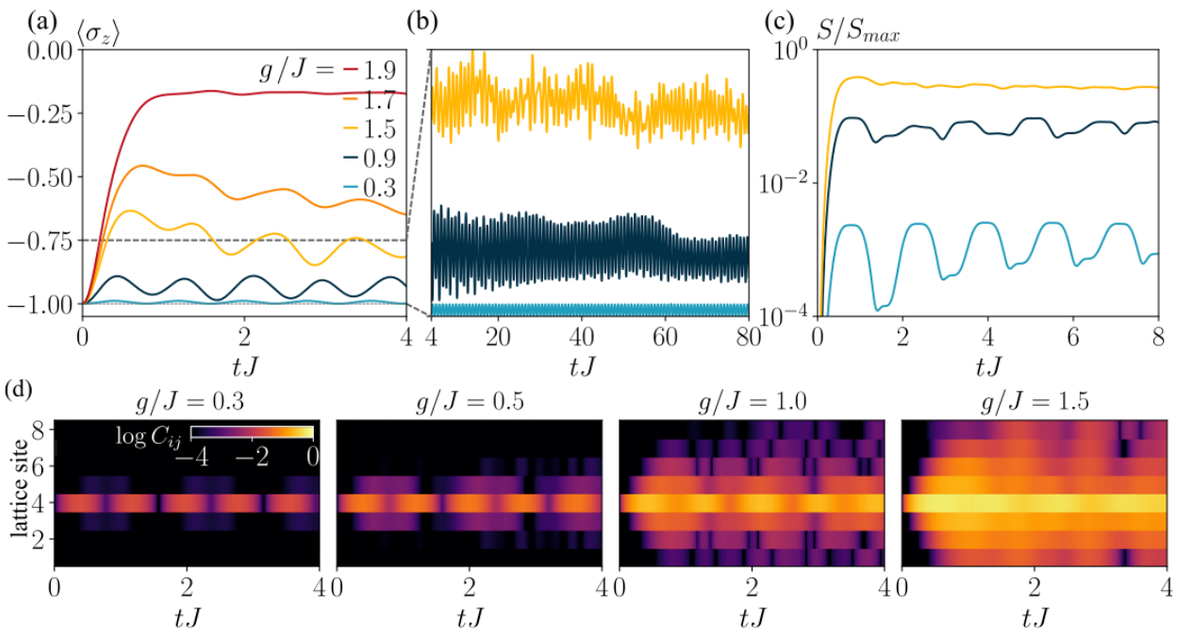

FIG. 1. Confinement in the 2D quantum Ising model. (a) Short time dependence of magnetization $\langle \sigma^z(t) \rangle$ for a set of g/J. (b) $\langle \sigma^z(t) \rangle$ for longer times and small g/J. (c) Time dependence of entanglement entropy S for a bipartition of two neighbouring spins and the rest of the system. $S_{\max}$ is the maximal entanglement entropy of the bipartition (in this case $S_{\max} = 2 \ln 2$). (d) Time dependence of a horizontal cut of the connected correlation function C_{ij} where $i = 4$ for a set of g/J.

L. Pavešić et al. arxiv: 2406.11979, PRB (2025)

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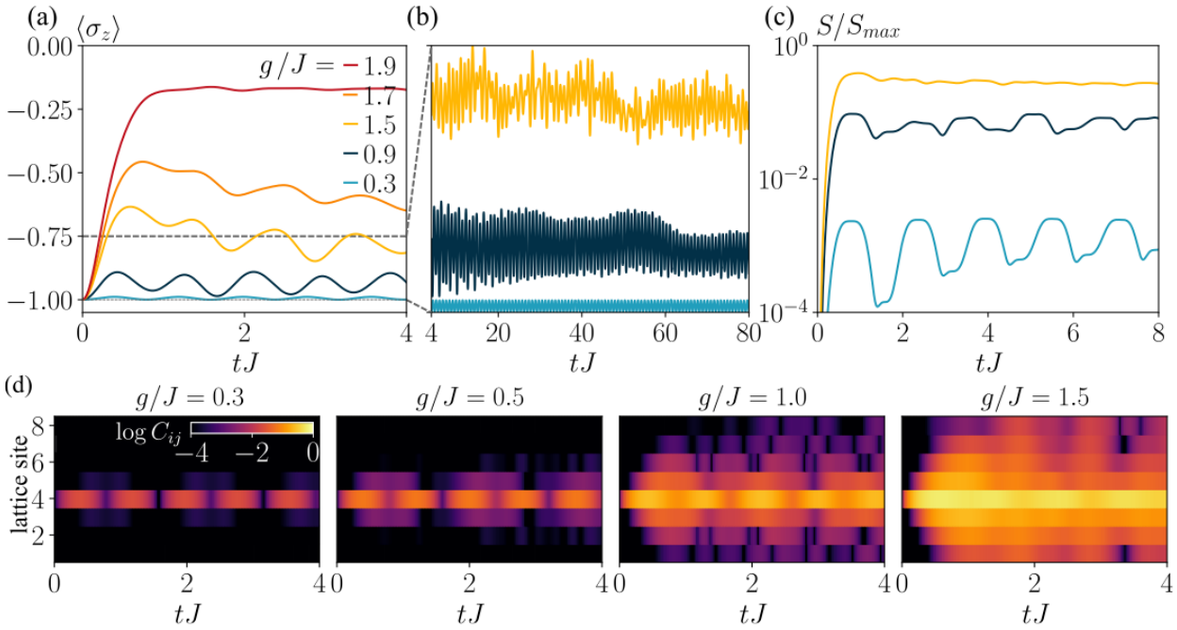

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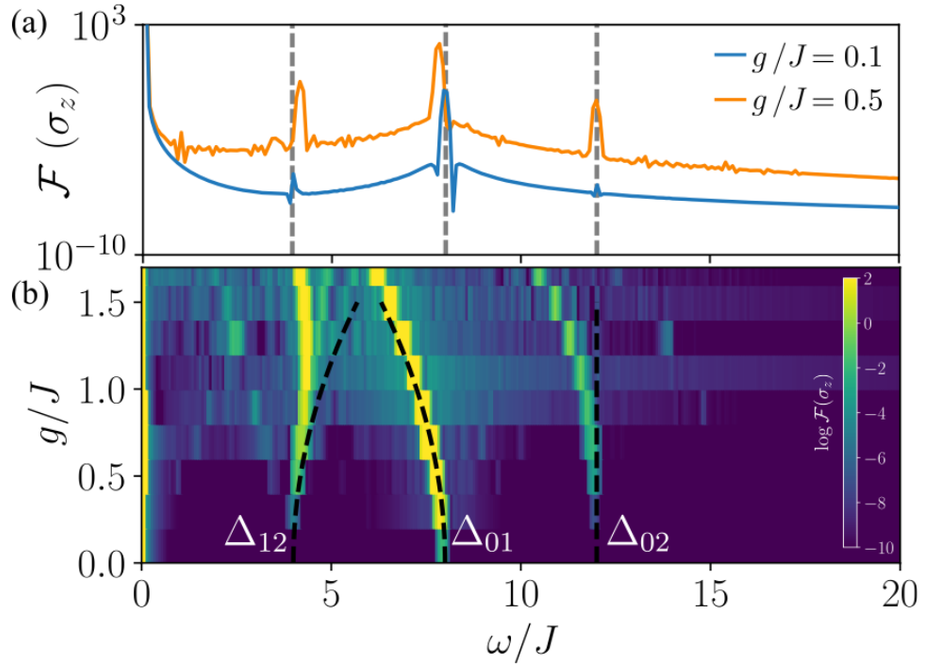

FIG. 2. Spectrum of σ^z, given by the Fourier transform $\mathcal{F}(\sigma^z)$. (a) Spectrum of magnetization for two cases of g/J. Dashed vertical lines correspond to $\omega/J = 4, 8$, and 12. (b) Heatmap of the spectrum for a range of g/J. Black dashed lines correspond to the transition energies obtained by perturbation theory.

$$E_0 = -\frac{g^2}{8J} N^2 + \mathcal{O}(g^4),$$

$$E_1 = 8J - \frac{g^2}{8J}\left(N^2 + 6\right) + \mathcal{O}(g^4),$$

$$E_2 = 12J - \frac{g^2}{8J} N^2 + \mathcal{O}(g^4).$$

INTERFACE PHYSICS IN 2D ISING MODEL

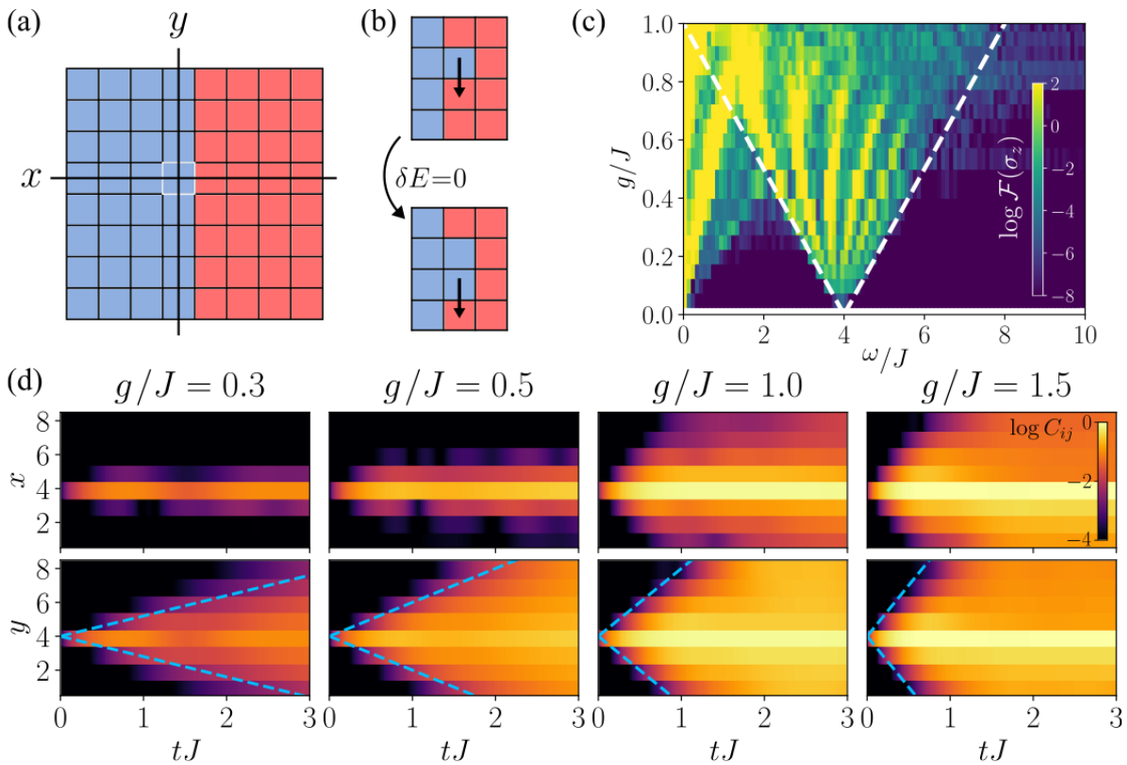

FIG. 3. Spread of correlations near an interface. (a) Sketch of the initial state, with blue representing ↑ and red ↓ spins. Black lines indicate the cuts shown in (d). (b) Sketch of the resonant process along the interface. (c) Heatmap of the spectral density of $\langle \sigma^z \rangle (t)$ for a spin at the interface for a range of g/J. White dashed lines correspond to the transition energies of a freely propagating edge mode. (d) Horizontal (x) and vertical (y) cuts of the connected correlations function C_{ij} with respect to the spin at $(4, 4)$ (white square in (a)). Blue dashed lines are $\pm 4gt$, showing that the interface mode carries the correlations.

L. Pavešić et al. arxiv: 2406.11979, PRB (2025)

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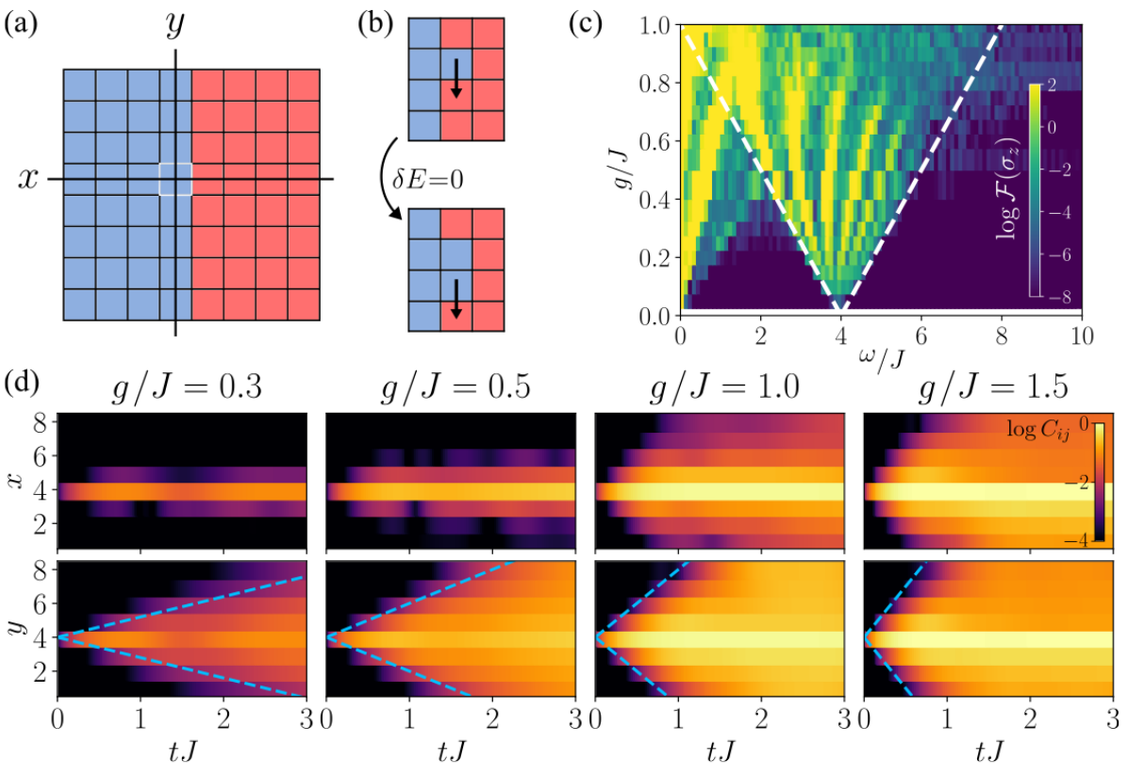

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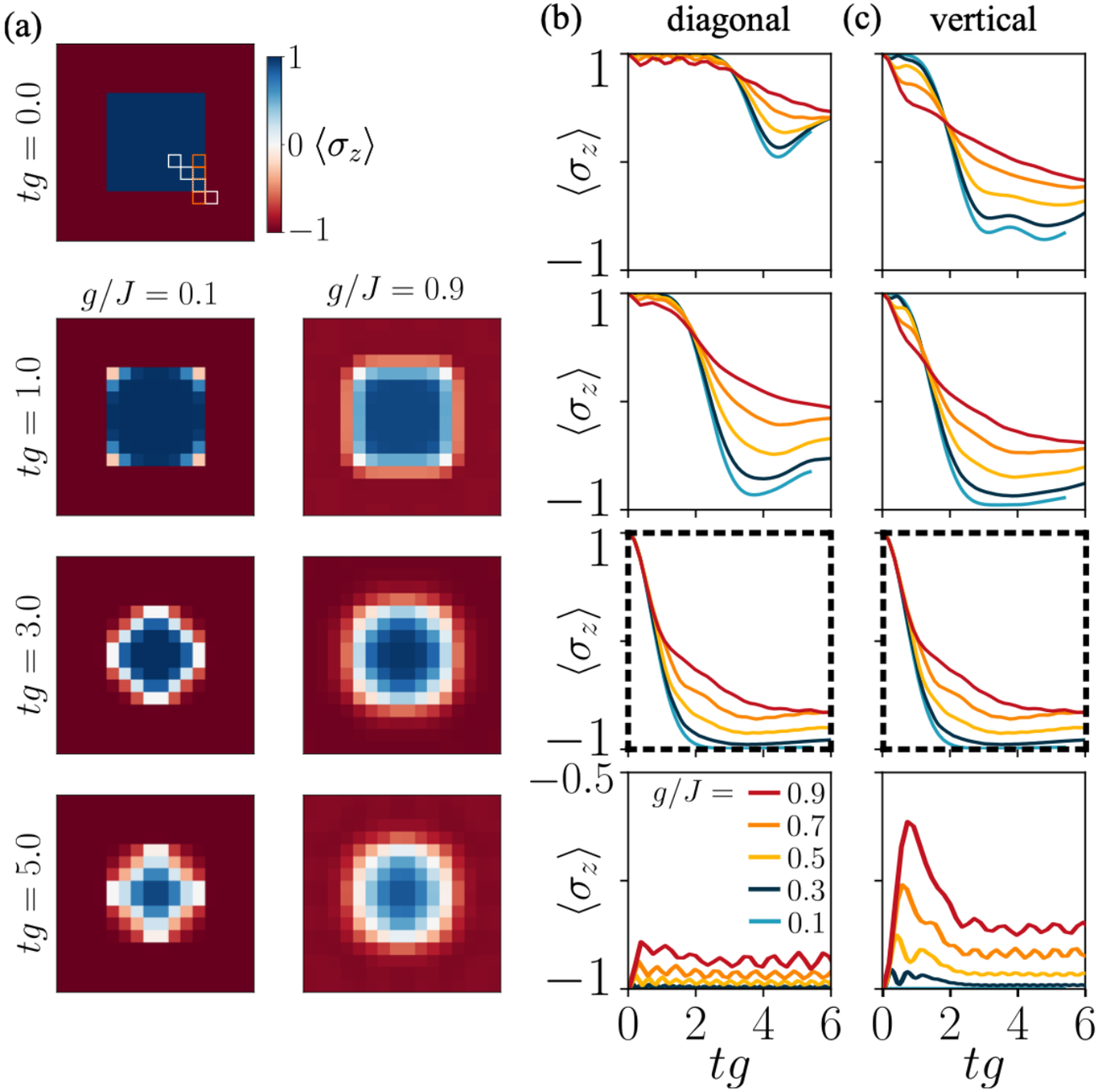

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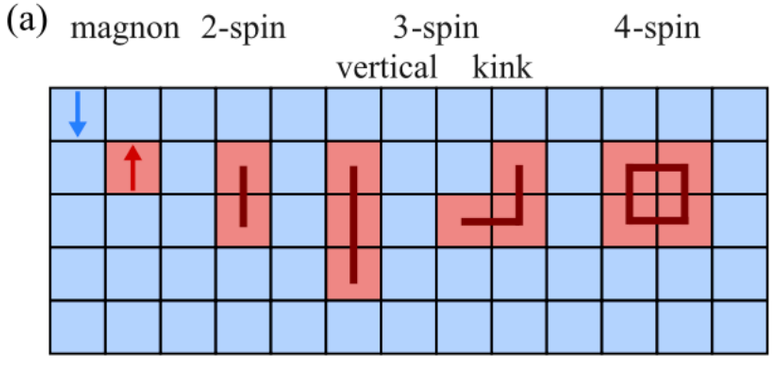
STATE PREPARATION FOR SCATTERING SIMULATIONS



$$H = -J \sum_{\langle ij \rangle} Z_i Z_j - g \sum_i X_i - h \sum_i Z_i$$

L. Pavešić, M. F. Di Liberto, SM . arXiv:2509.02702

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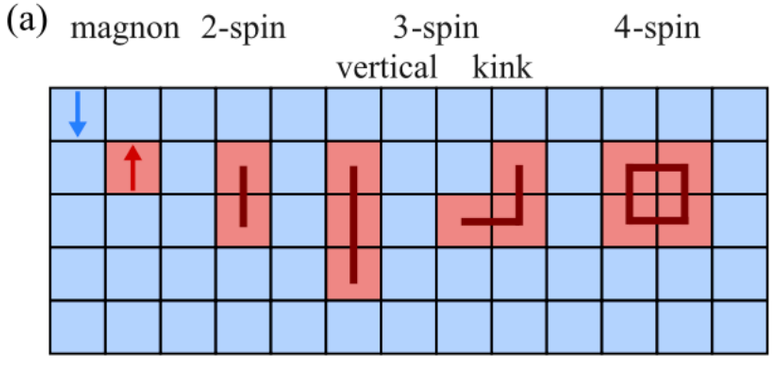

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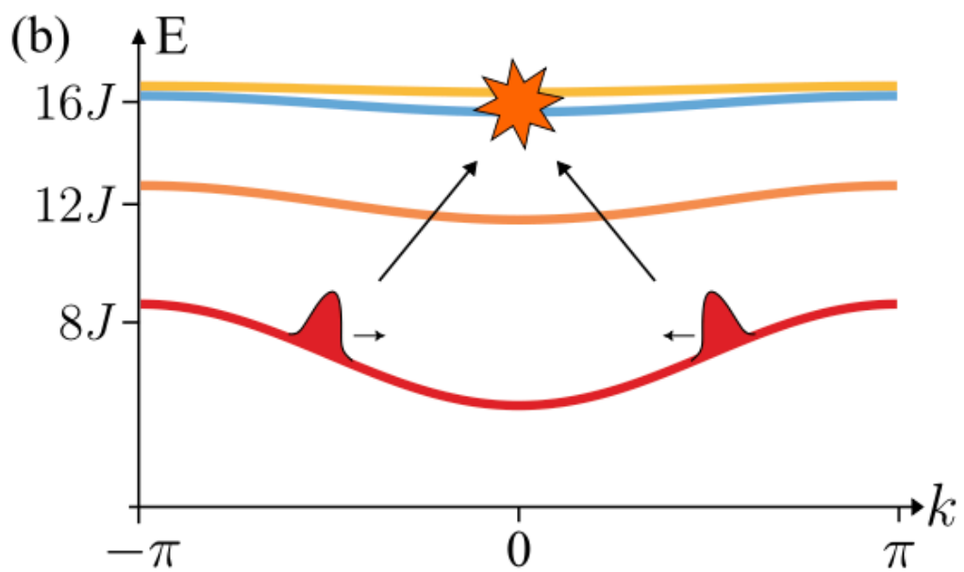

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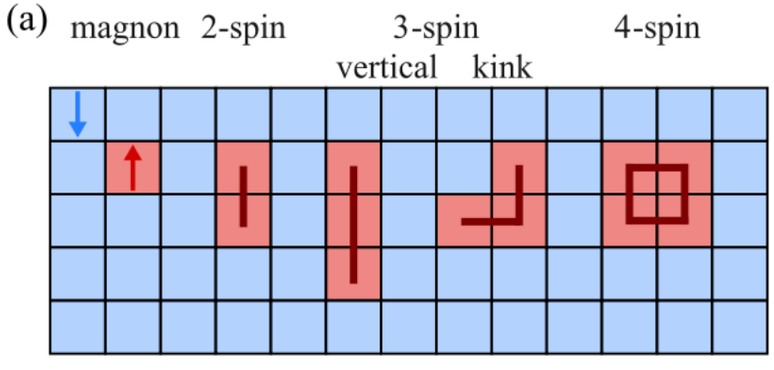
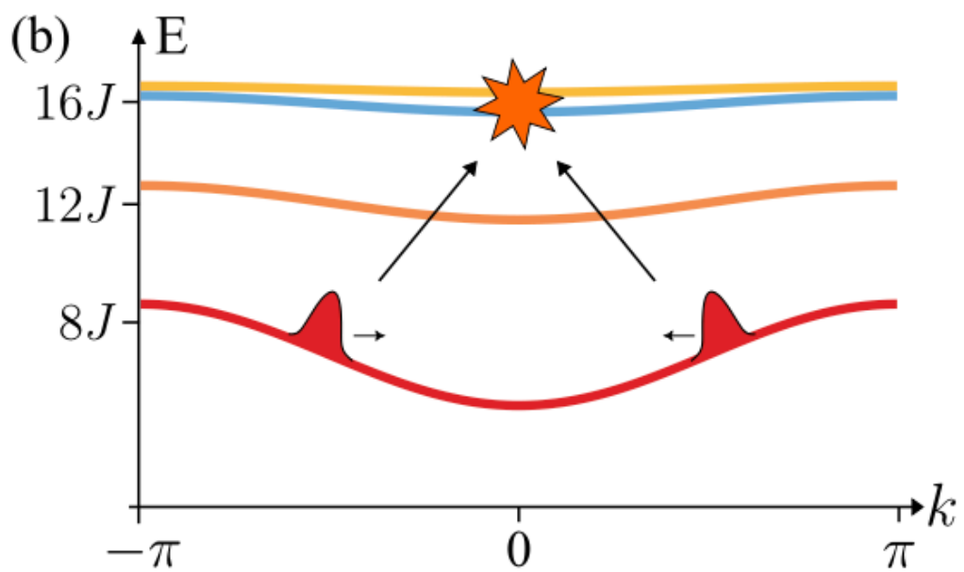


$$g = 0,$$

$$|\mathbf{r}_0, \mathbf{k}\rangle = \frac{1}{N} \sum_{\mathbf{r}} f(\mathbf{r}, \mathbf{r}_0) e^{i\mathbf{k}\cdot\mathbf{r}} X_{\mathbf{r}} |0\rangle$$

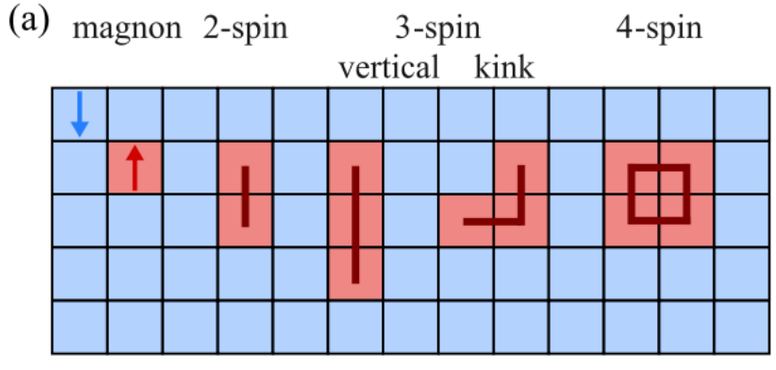
$$f(\mathbf{r}, \mathbf{r}_0) = \frac{1}{\sigma\sqrt{\pi}} e^{\frac{-(\mathbf{r}-\mathbf{r}_0)^2}{2\sigma^2}}$$

$$H = -J \sum_{\langle ij \rangle} Z_i Z_j - g \sum_i X_i - h \sum_i Z_i$$



L. Pavešić, M. F. Di Liberto, SM . arXiv:2509.02702

STATE PREPARATION FOR SCATTERING SIMULATIONS



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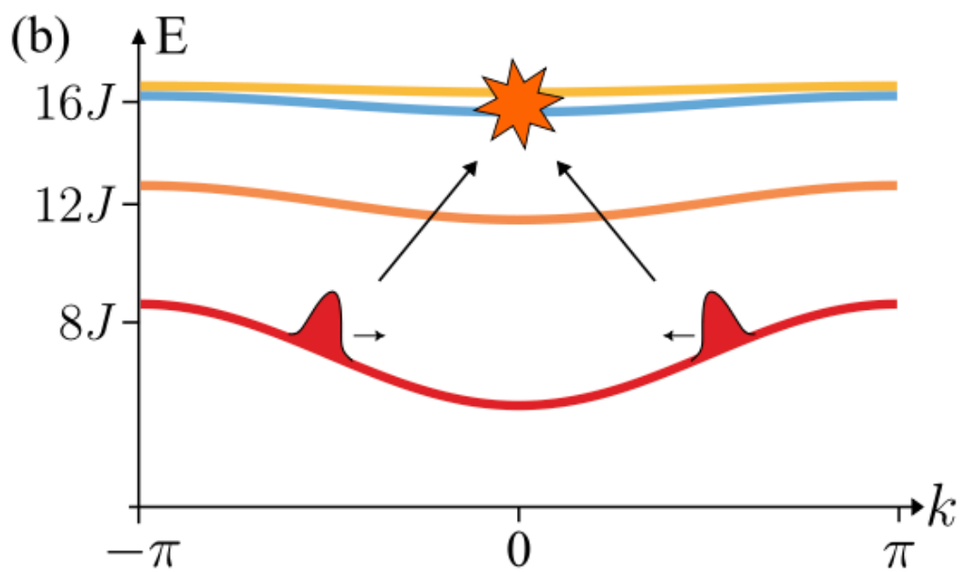
$$g(t) = g^* \sin^2\left(\frac{\pi}{2}\sin^2\left(\frac{\pi}{2}\frac{t}{\tau}\right)\right)$$



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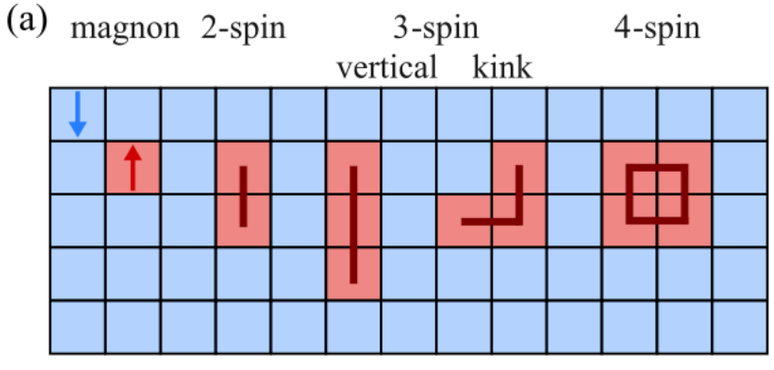

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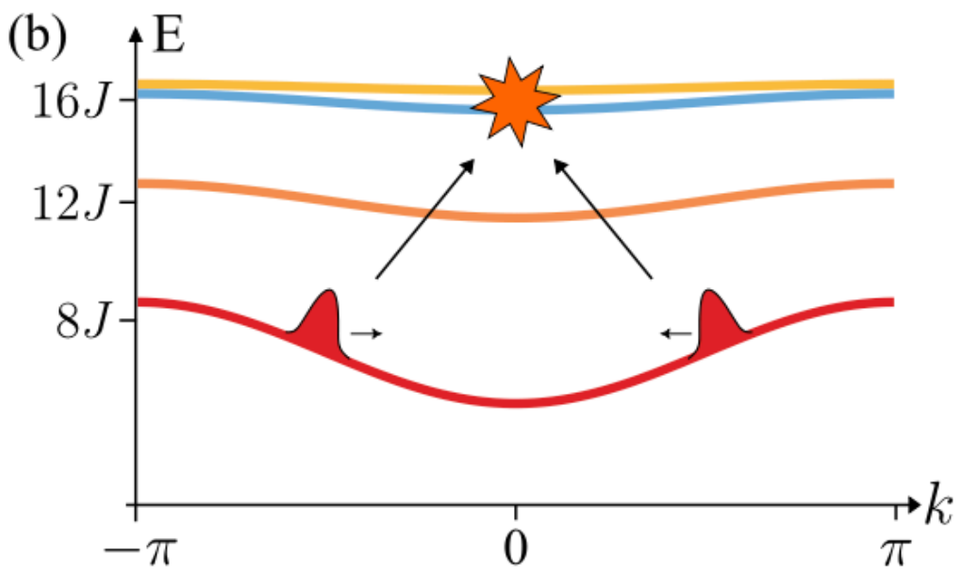
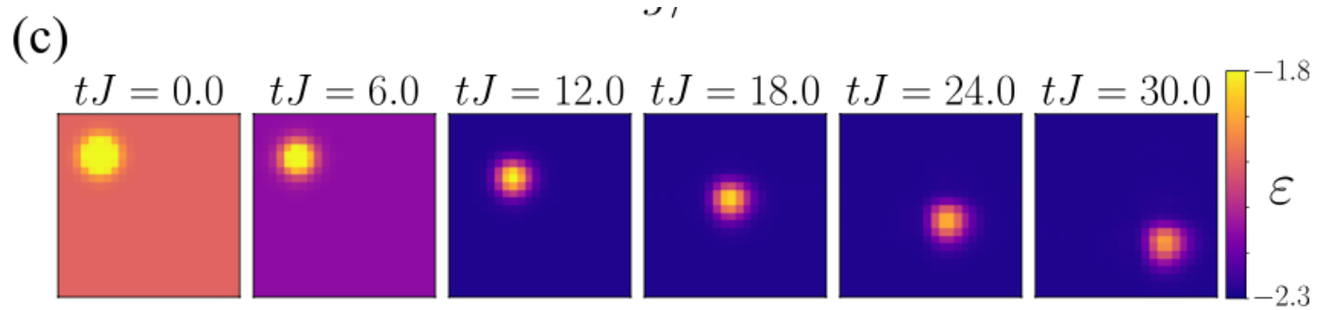
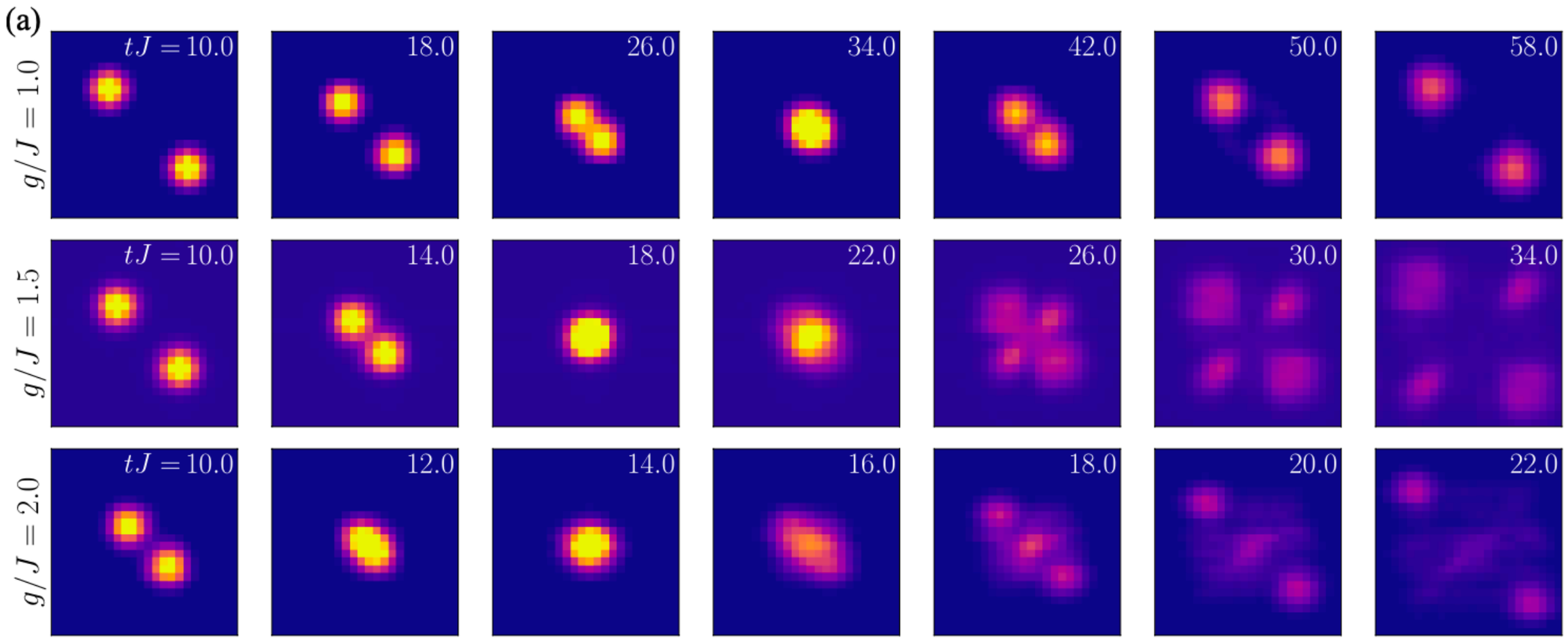




$\tau J = 10$, $\mathbf{k} = (\frac{\pi}{2}, \frac{\pi}{2})$ and $g/J = 1.5$.

L. Pavešić, M. F. Di Liberto, SM . arXiv:2509.02702

2D+1 SCATTERING IN ISING MODEL VIA TTN



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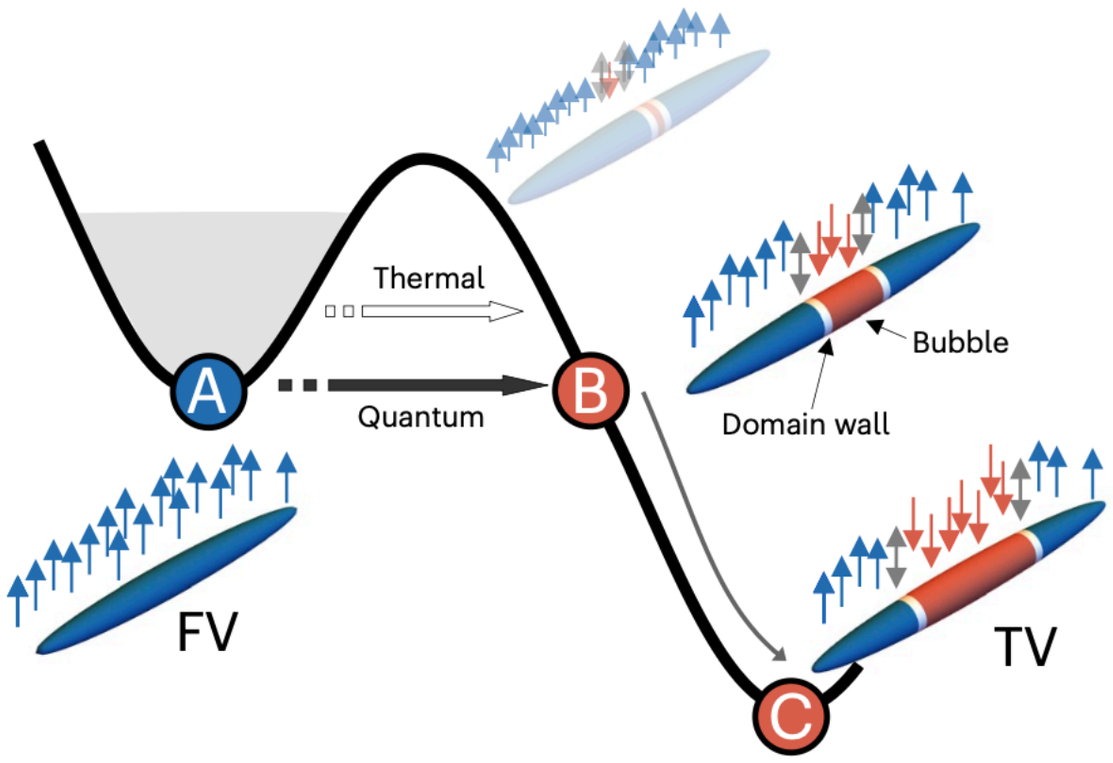
L. Pavešić, M. F. Di Liberto, SM . arXiv:2509.02702 Nat. Comm. (2026)

FALSE VACUUM DECAY



C. Callan; S. Coleman "Fate of the false vacuum. II. First quantum corrections". *Physical Review D*. **D16** (6): 1762–68 (1977)..

Turner, M. S.; Wilczek, F. "Is our vacuum metastable?" (PDF). Nature. 298 (5875): 633–634 (1982).

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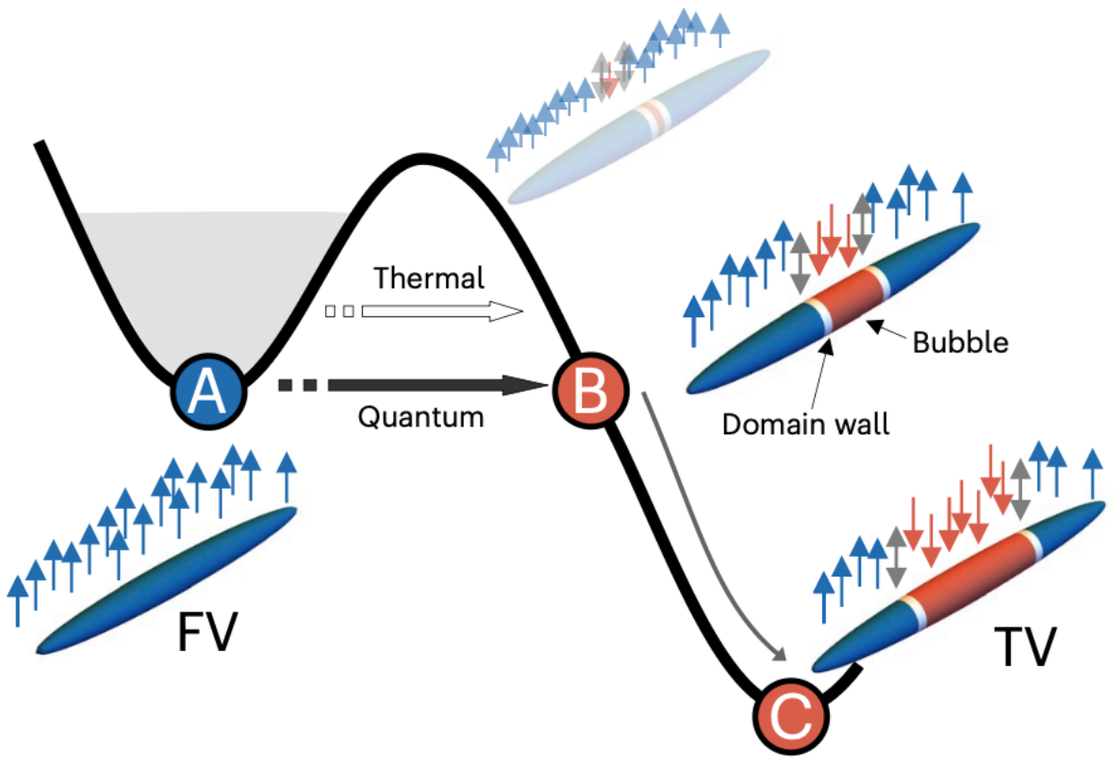

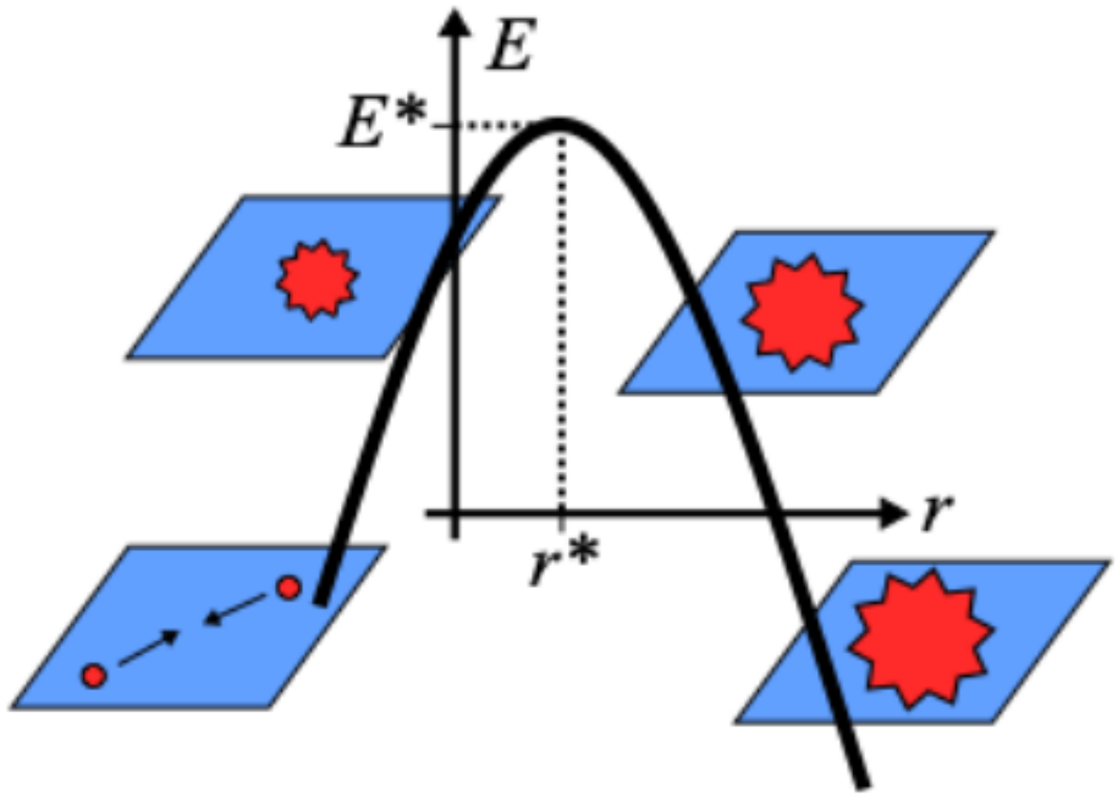

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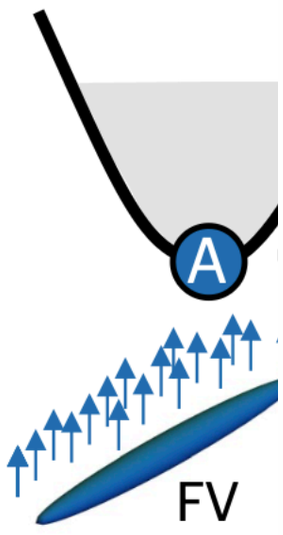

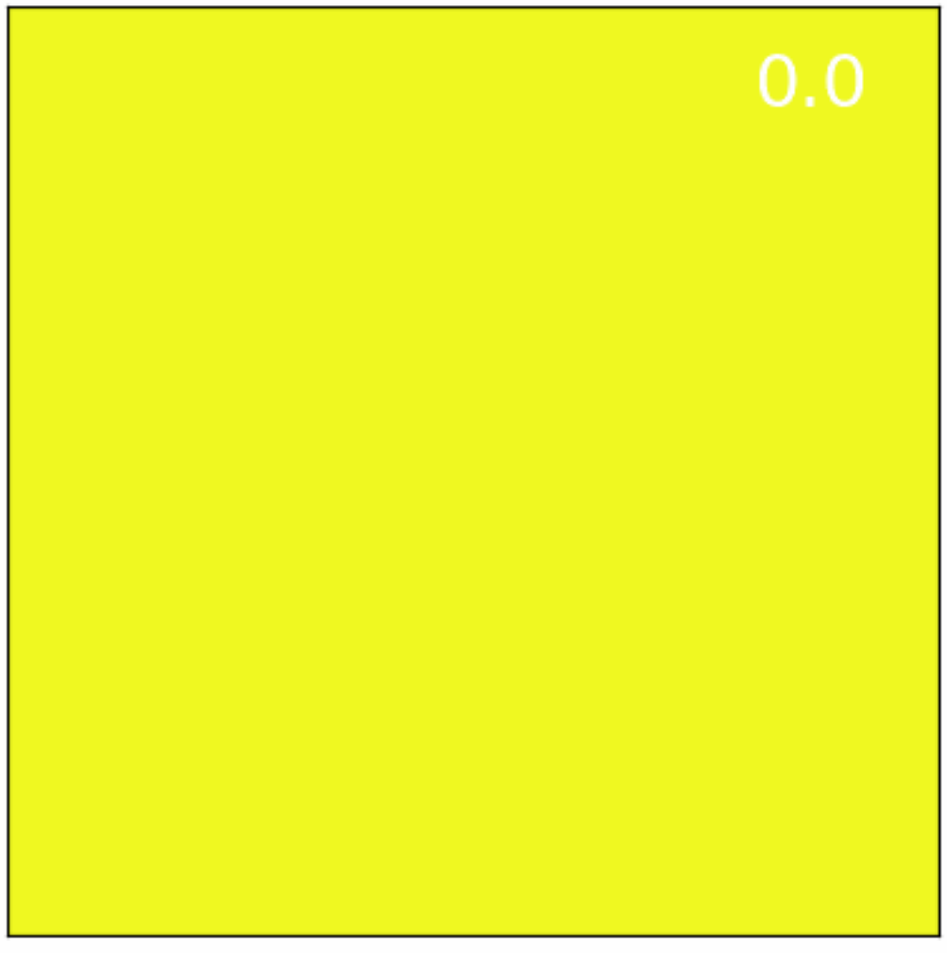

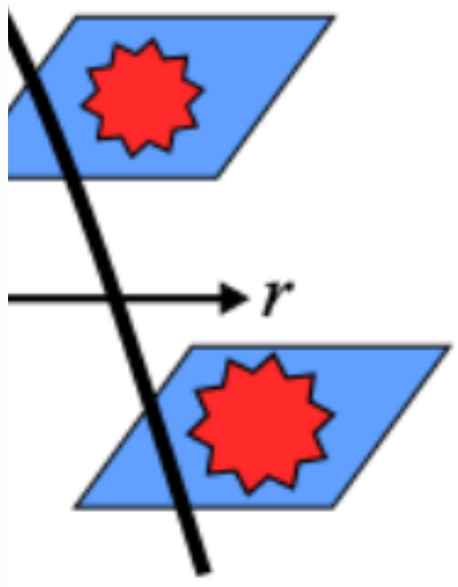

C. Callan; S. Coleman "F .

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FALSE VACUUM DECAY IN THE 2D+1 ISING MODEL

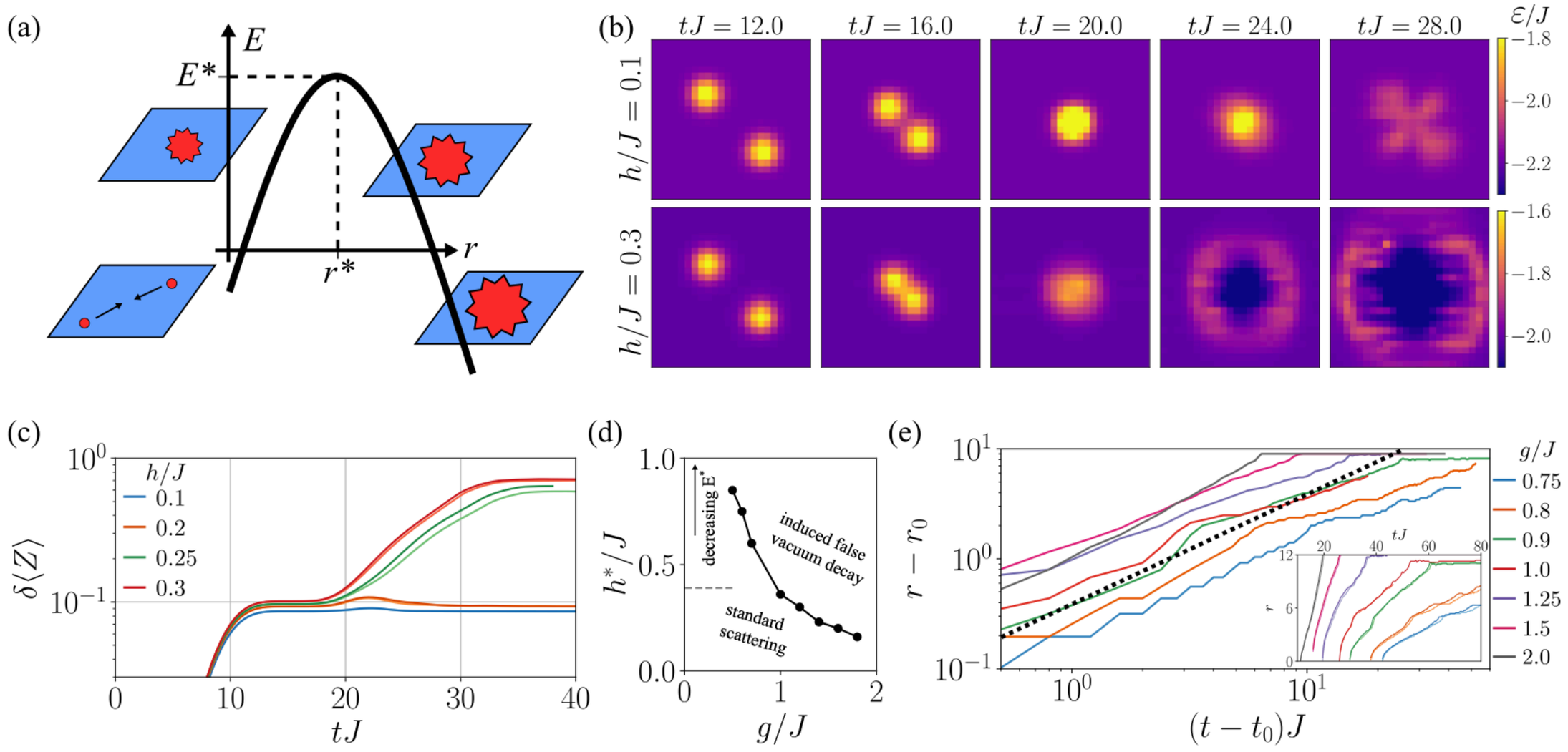
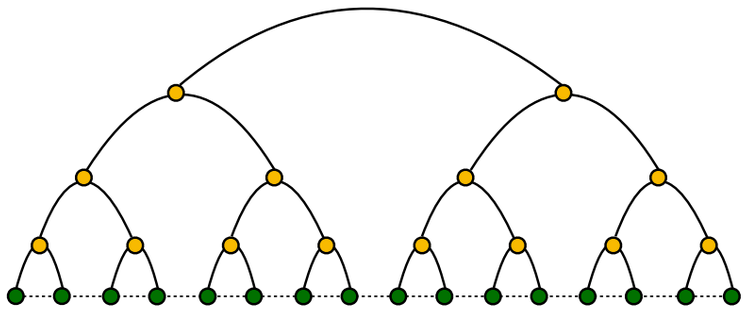
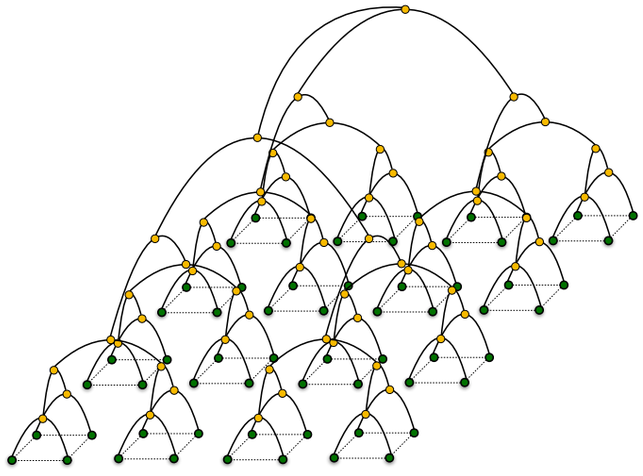


FIG. 3. Scattering in the false vacuum. (a) A sketch of the energy landscape of a true vacuum bubble with radius r. (b) The evolution of the energy density during scattering with $h/J = 0.1$ and $h/J = 0.3$, both with $g/J = 1.5$ and $\mathbf{k} = \left(\frac{\pi}{2}, \frac{\pi}{2}\right)$. (c) The change in magnetization per site with varying h for $g/J = 1.5$ and $\mathbf{k} = \left(\frac{\pi}{2}, \frac{\pi}{2}\right)$. (d) The dependence of the threshold value h^* on g/J. The gray dashed line corresponds to the threshold h^* expected for $g \to 0$. See text for details. (e) The evolution of the radius of the true vacuum bubble in log-log scale for varying g, with some longitudinal field larger than the threshold value. We subtract the time t_0 when the radius reaches $r_0 = 3$ on the horizontal, and r_0 on the vertical axis. The black dashed line corresponds to linear growth and acts as a guide to the eye. (Inset) The evolution of the radius in lin-lin scale. In (c) and (e), the curves with different shades of the same color correspond to simulations with different bond dimension, darker is bigger. Here we use 150 and 200.

L. Pavešić, M. F. Di Liberto, SM . arXiv:2509.02702 Nat. Comm. (2026)

3D TREE TENSOR NETWORK





T. Felser, P. Silvi, M. Collura,
S. Montangero
PRX (2020)

G. Magnifico, T. Felser, P. Silvi, and S. Montangero
Nat. Comm. (2021)

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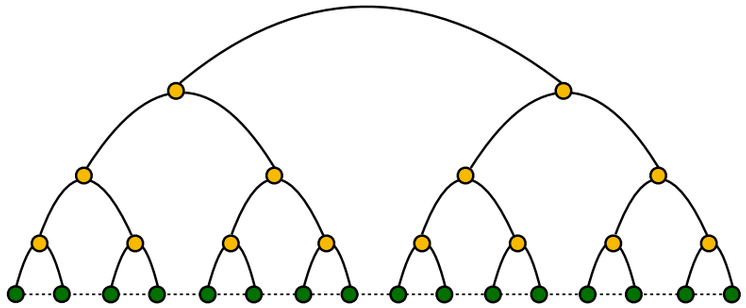

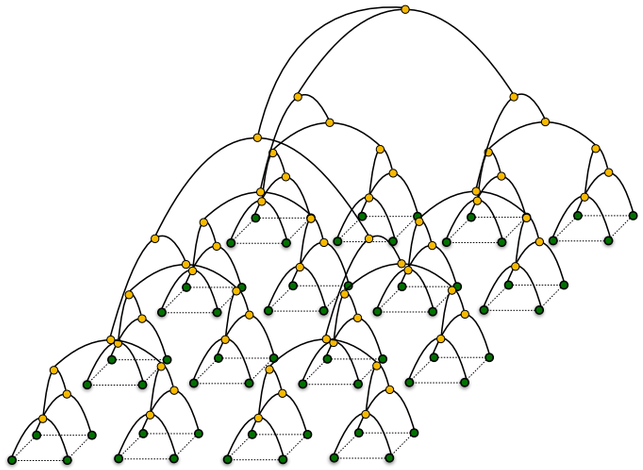

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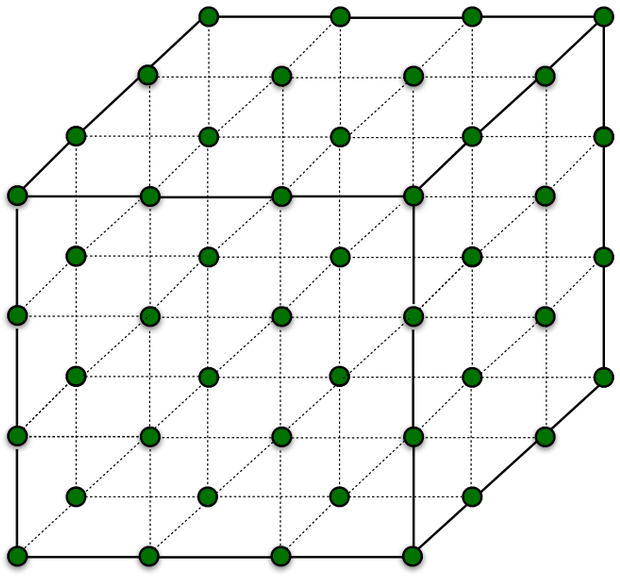

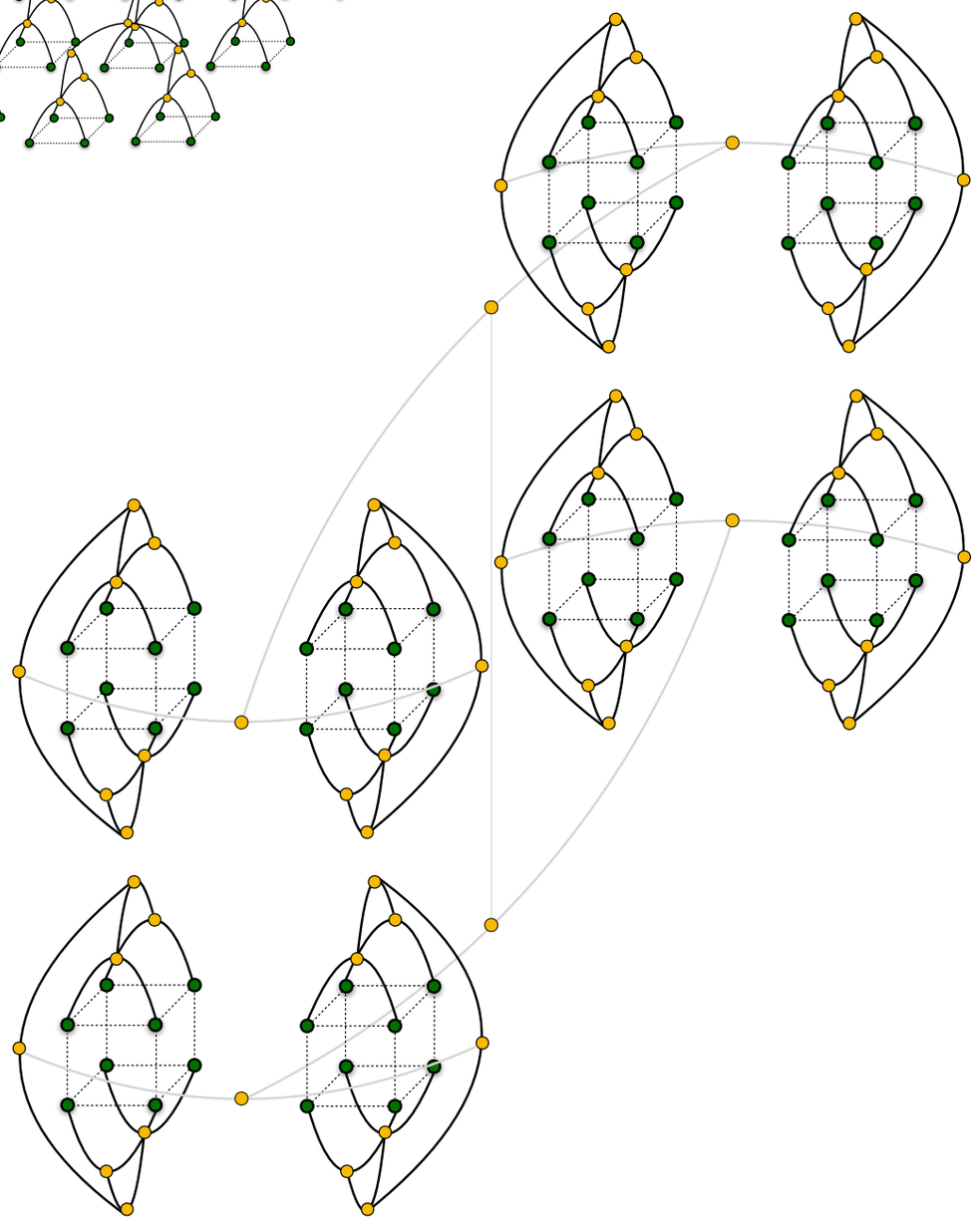

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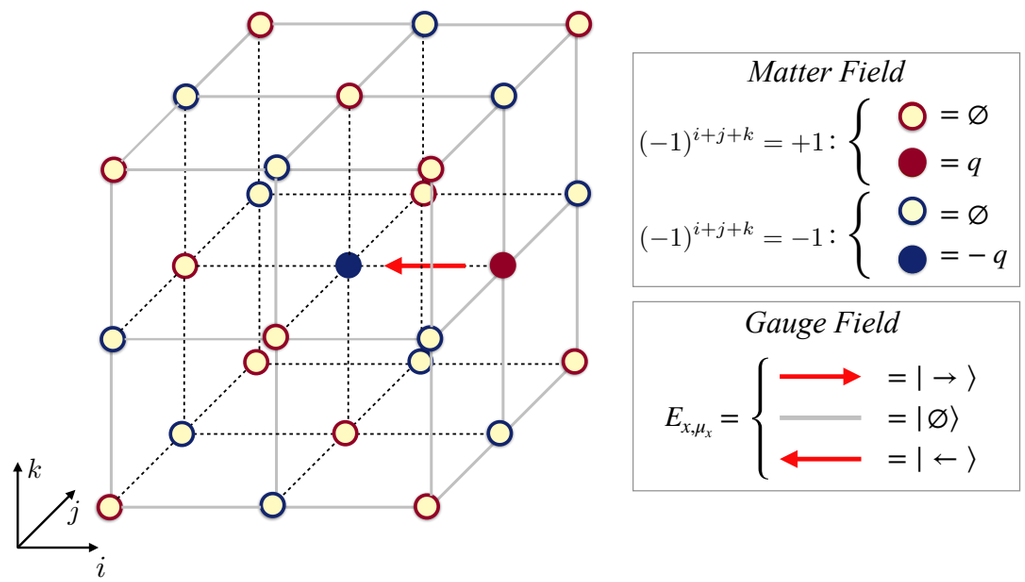
Nat. Comm. (2021)

3D QUANTUM-LINK FORMULATION OF QED

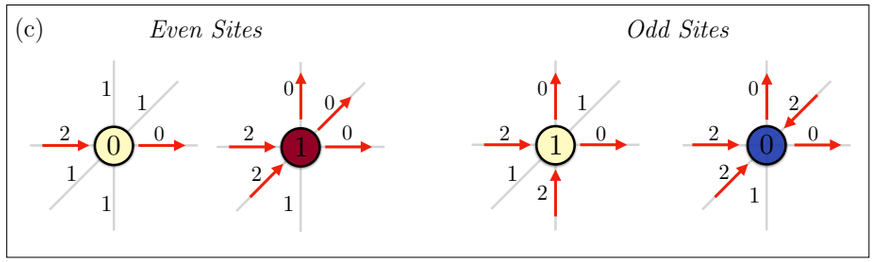
$$\hat{H} = -t \sum_{x,\mu} \left(\hat{\psi}_x^\dagger \, \hat{U}_{x,\mu} \, \hat{\psi}_{x+\mu} + \text{H.c.} \right)$$

$$+ m \sum_x (-1)^x \hat{\psi}_x^\dagger \hat{\psi}_x + \frac{g_e^2}{2} \sum_{x,\mu} \hat{E}_{x,\mu}^2$$

$$- \frac{g_m^2}{2} \sum_x \left(\square_{\mu_x,\mu_y} + \square_{\mu_x,\mu_z} + \square_{\mu_y,\mu_z} + \text{H.c.} \right)$$



$$\hat{G}_x = \hat{\psi}_x^\dagger \hat{\psi}_x - \frac{1-(-1)^x}{2} - \sum_\mu \hat{E}_{x,\mu}$$



$$H_{pen} = \nu \sum_{x,\mu} \left(1 - \delta_{2,\hat{L}_{x,\mu}} \right)$$

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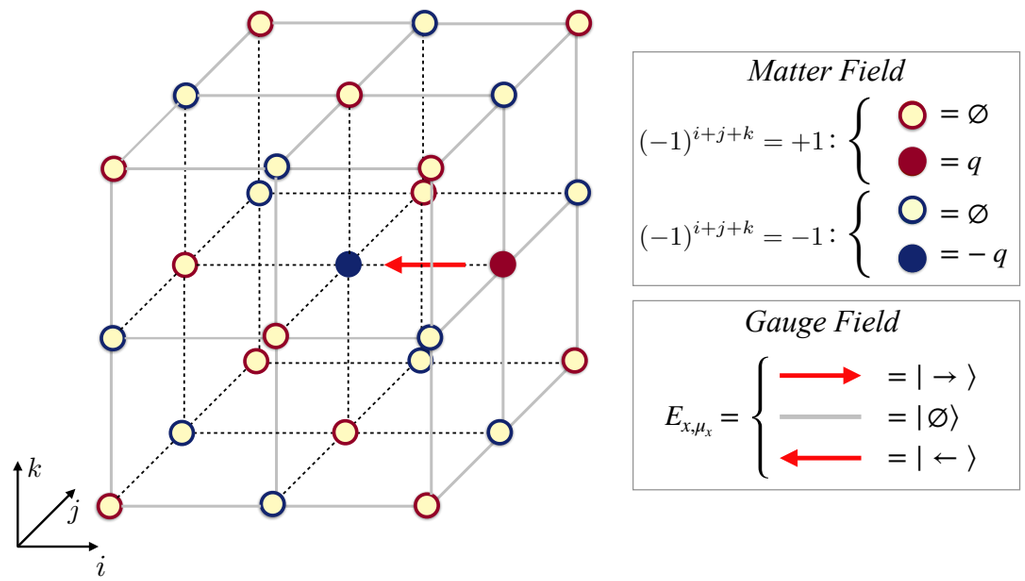

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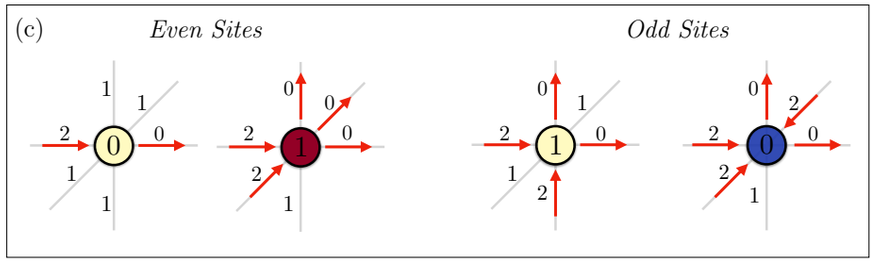
Local dimension 267, up to 12288 Hamiltonian operators



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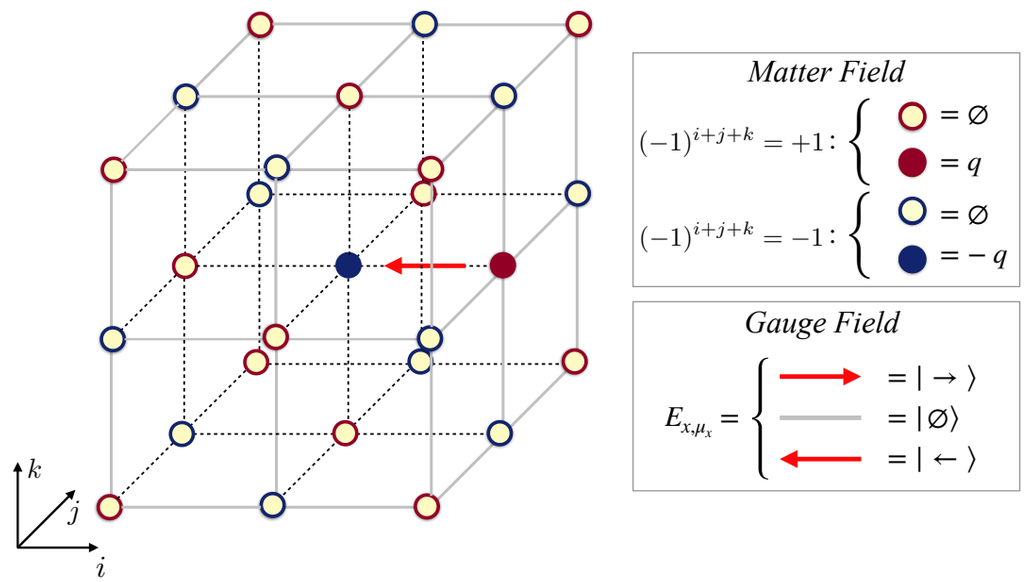

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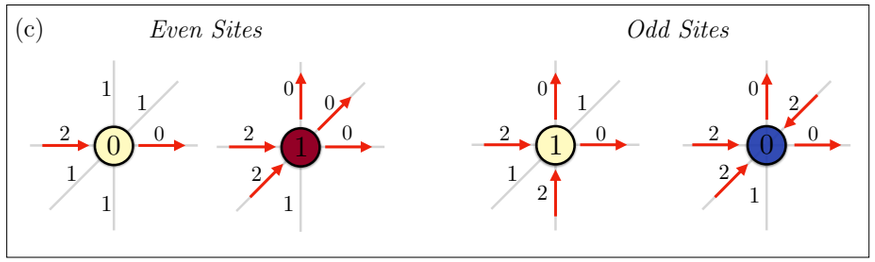
Local dimension 267, up to 12288 Hamiltonian operators

Up to 5 weeks x 64 cores of computational time



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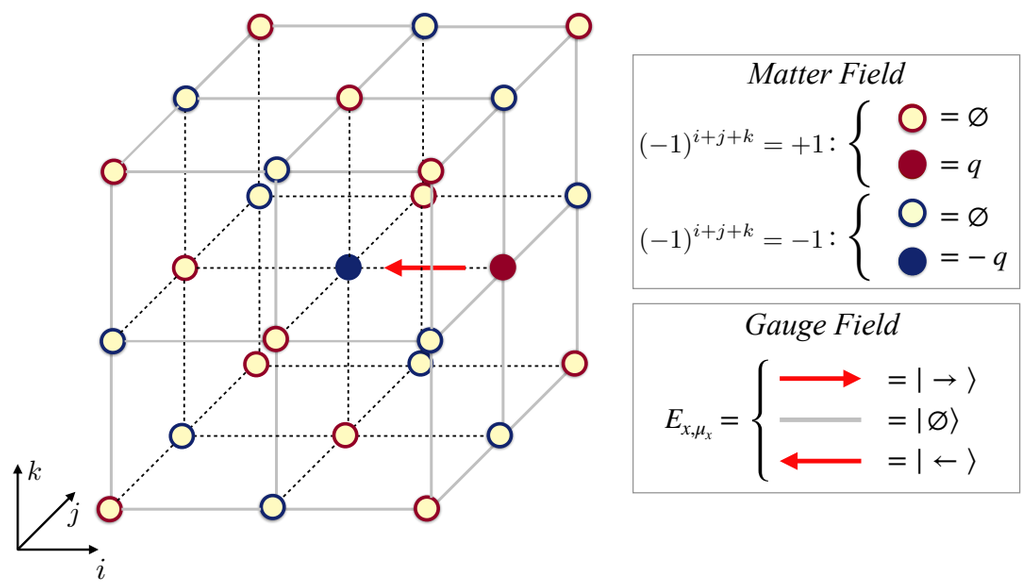

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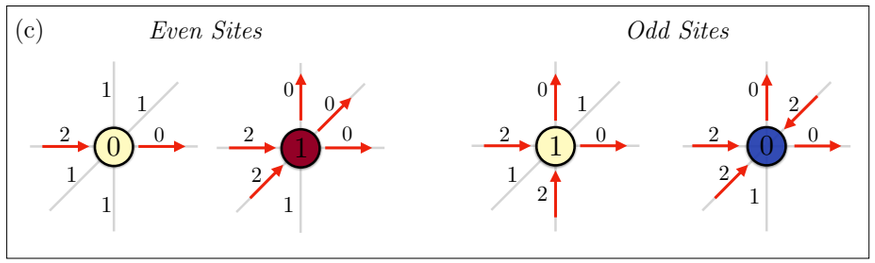

Local dimension 267, up to 12288 Hamiltonian operators

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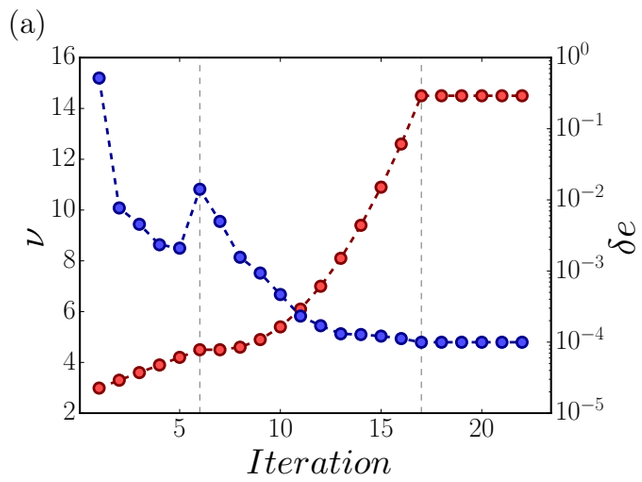
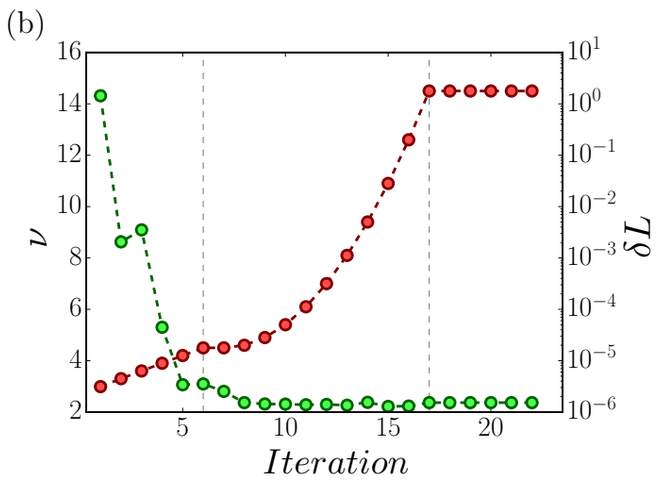
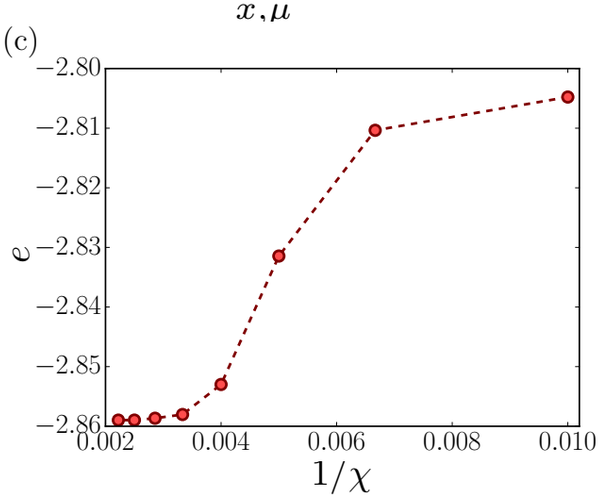
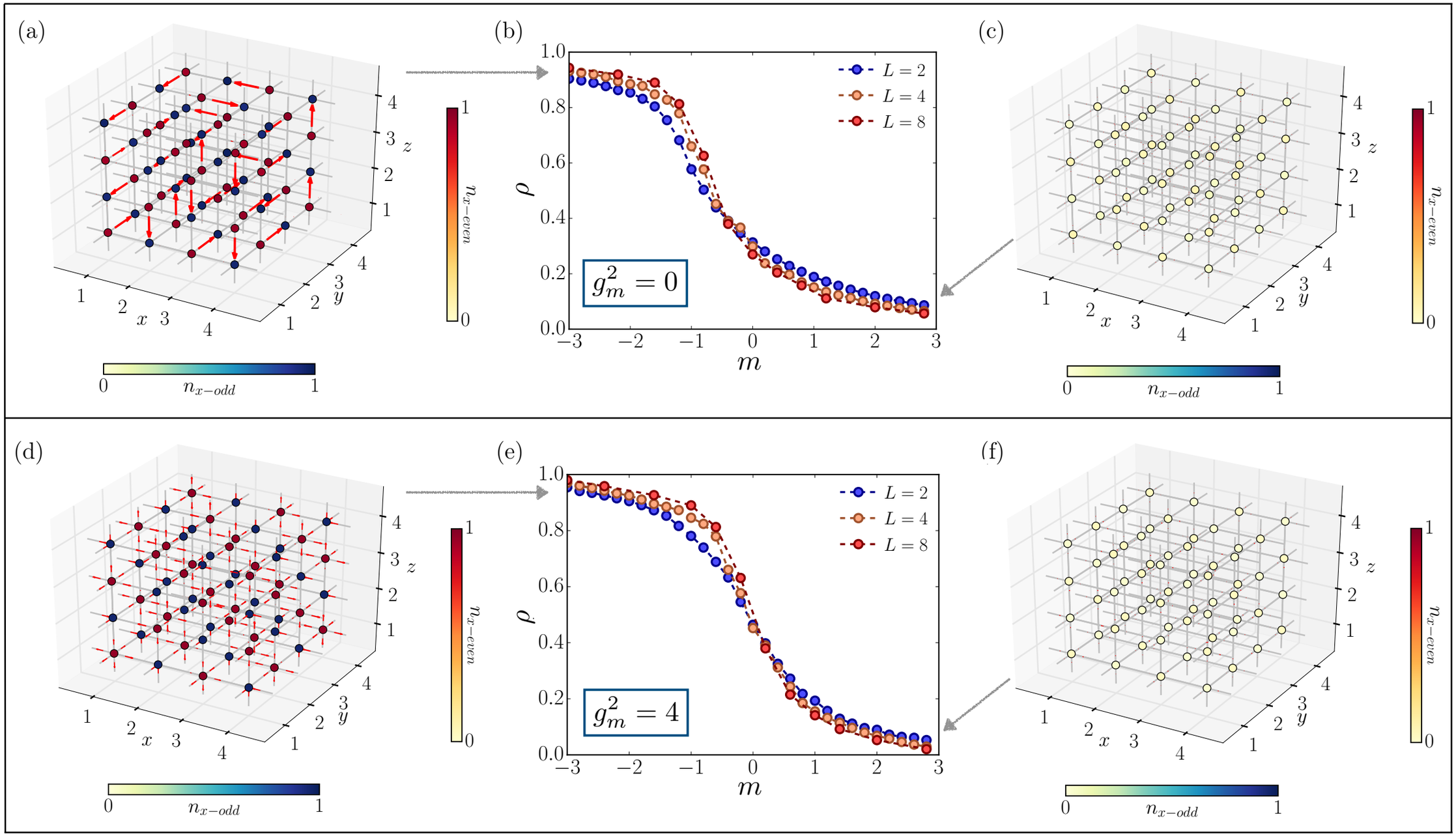






QUANTUM PHASES



$$m_c \approx +0.22$$

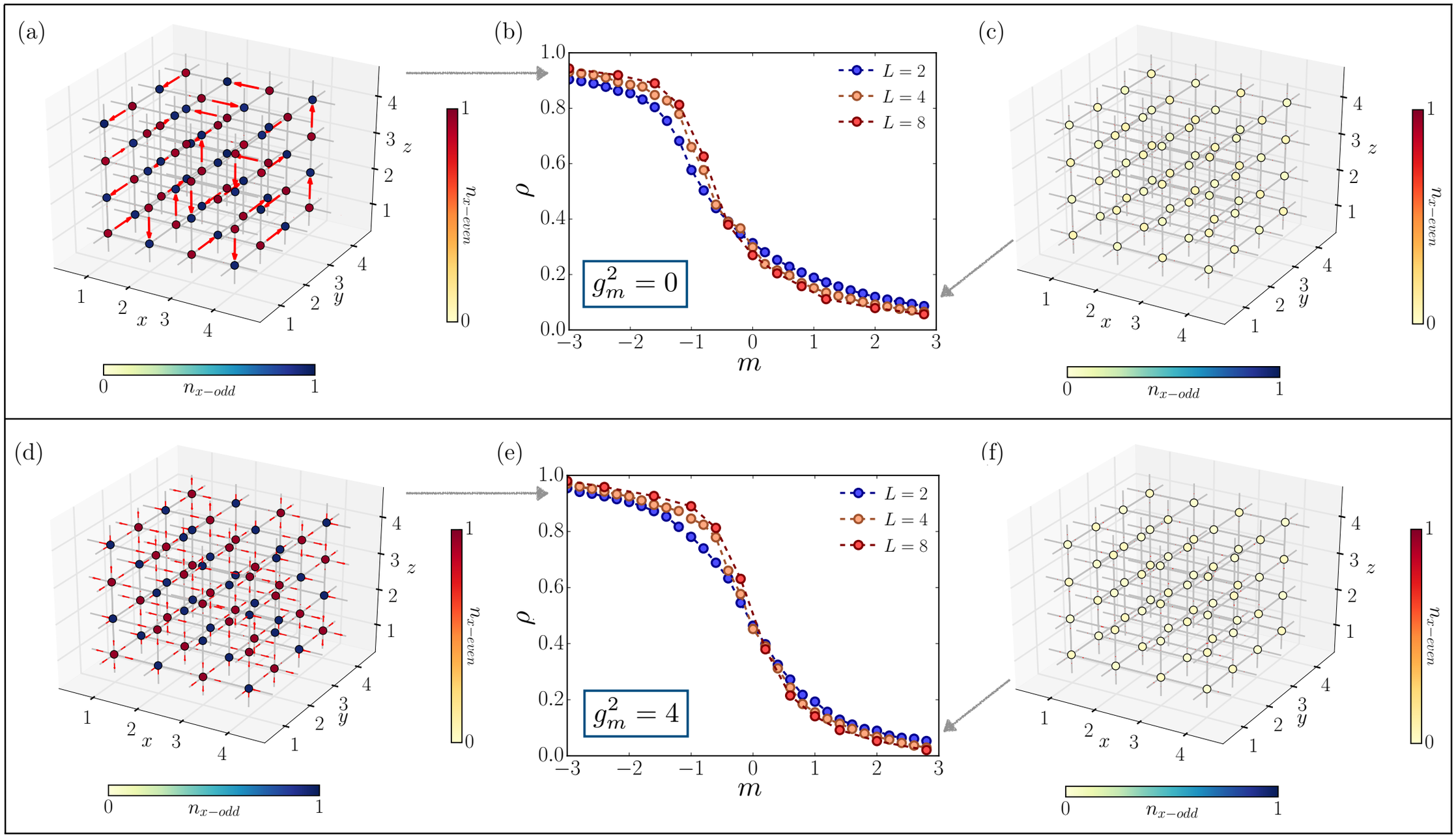
$$g_m^2 = 8/g_e^2$$

QUANTUM PHASES

Hilbert space of

~8 ^ 4 qubits!

4Kb QRAM



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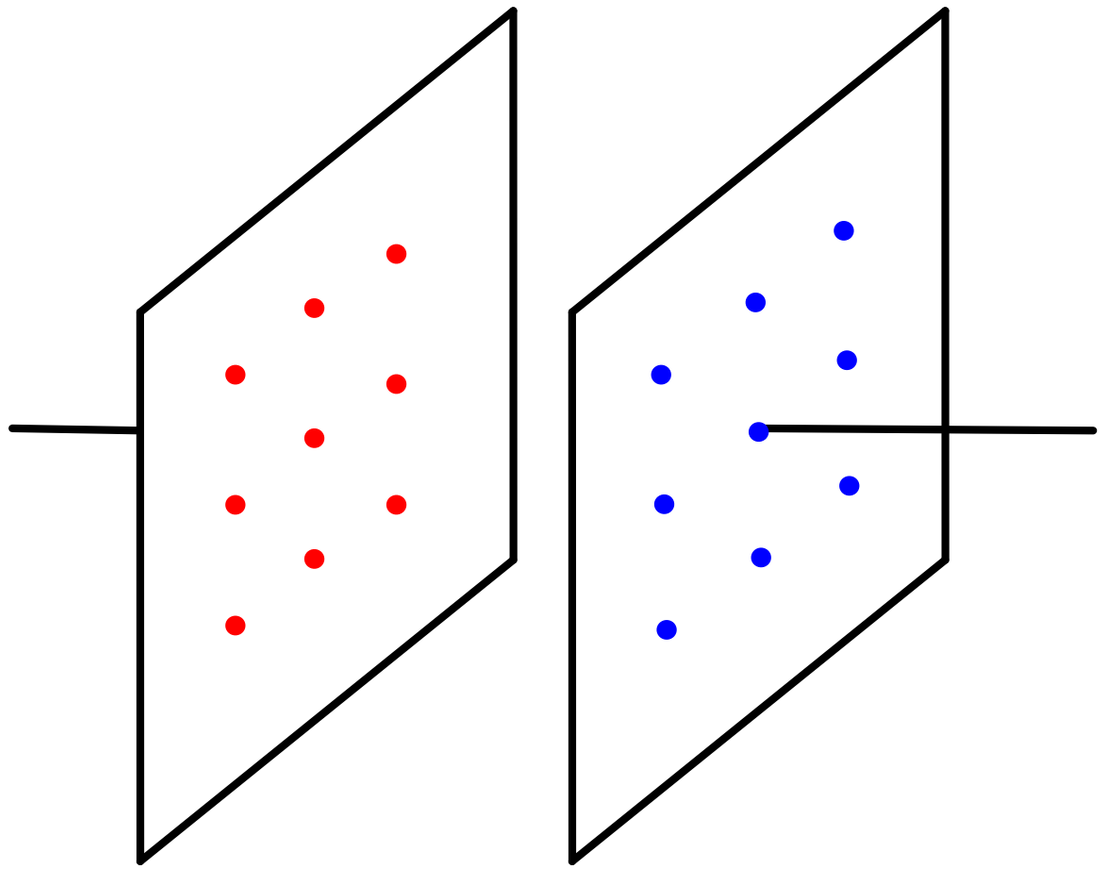
SCREENING



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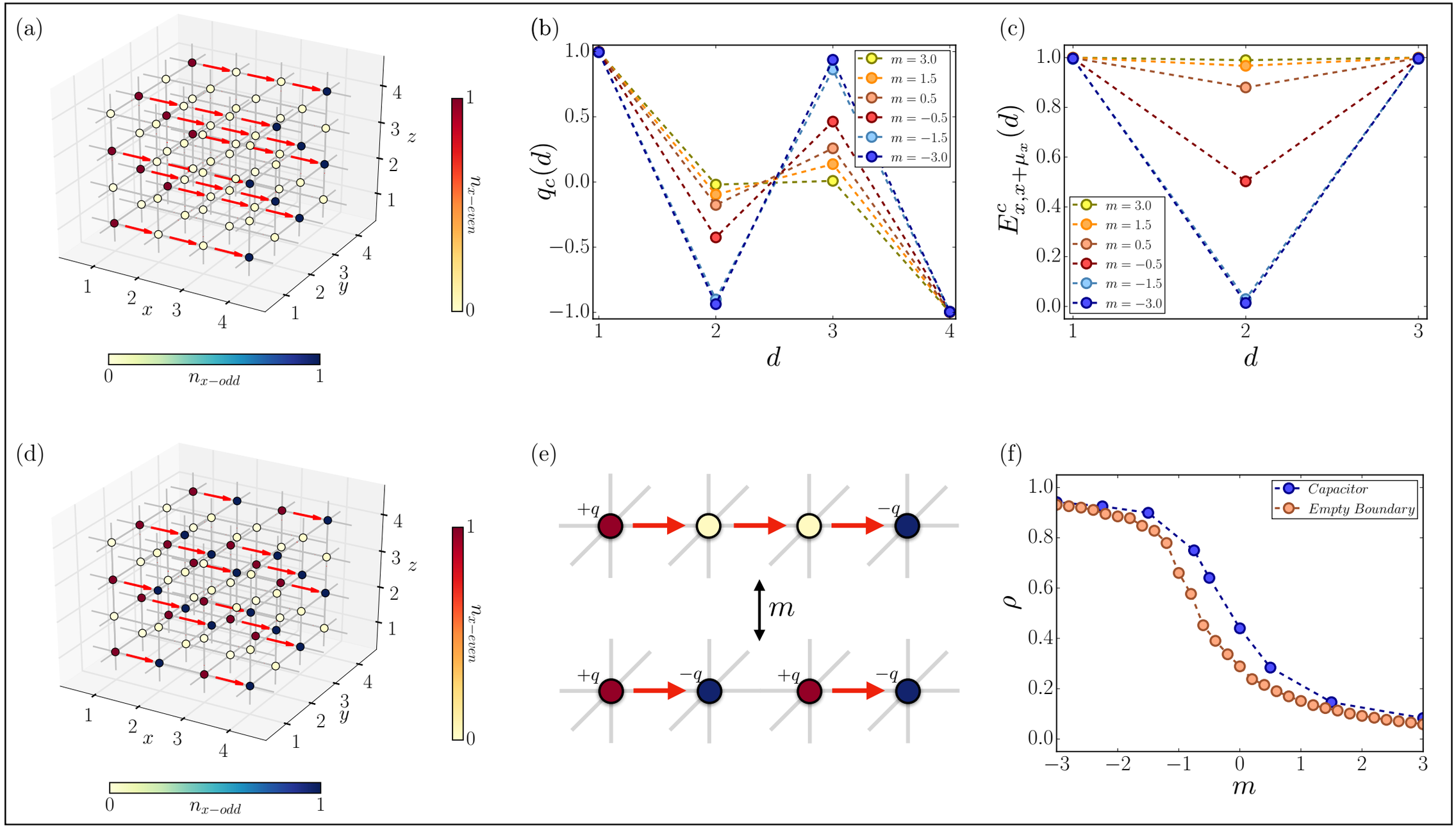
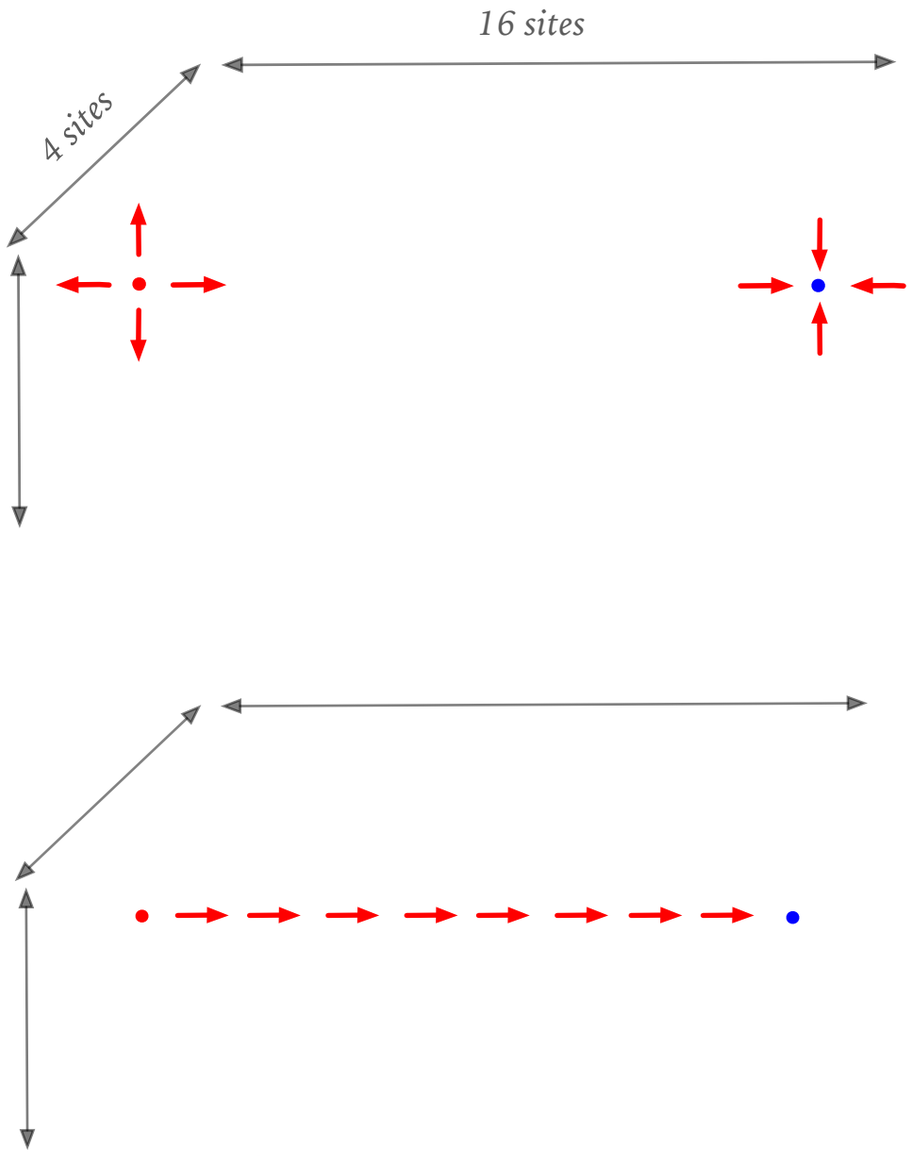
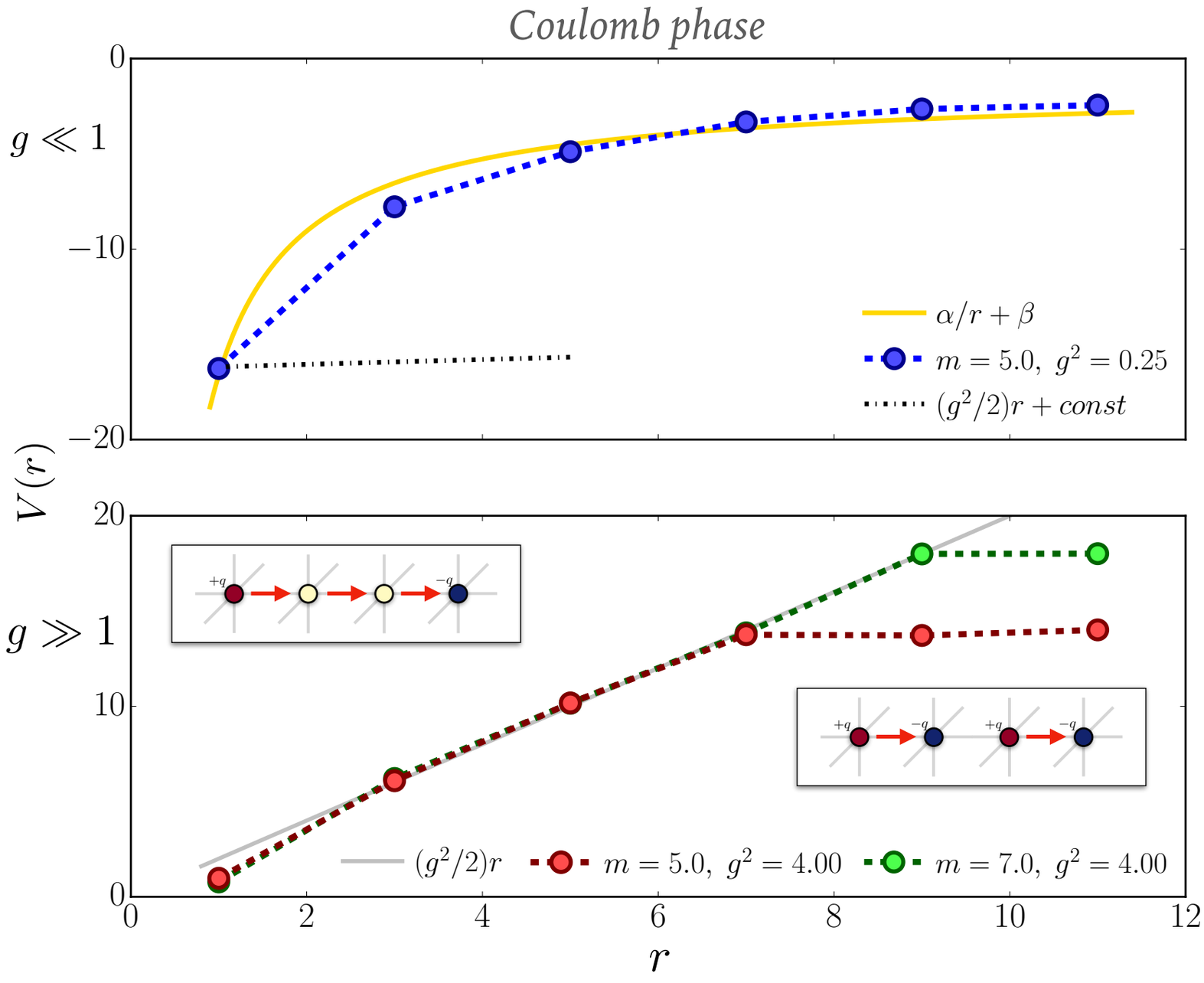

(a)
n_{x-even}
n_{x-odd}
(b)
$q_c(d)$
d
$m = 3.0$
$m = 1.5$
$m = 0.5$
$m = -0.5$
$m = -1.5$
$m = -3.0$
(c)
$E^c_{x,x+\mu_x}(d)$
d
(d)
n_{x-even}
n_{x-odd}
(e)
$+q$
$-q$
m
(f)
ρ
m
Capacitor
Empty Boundary

CONFINEMENT

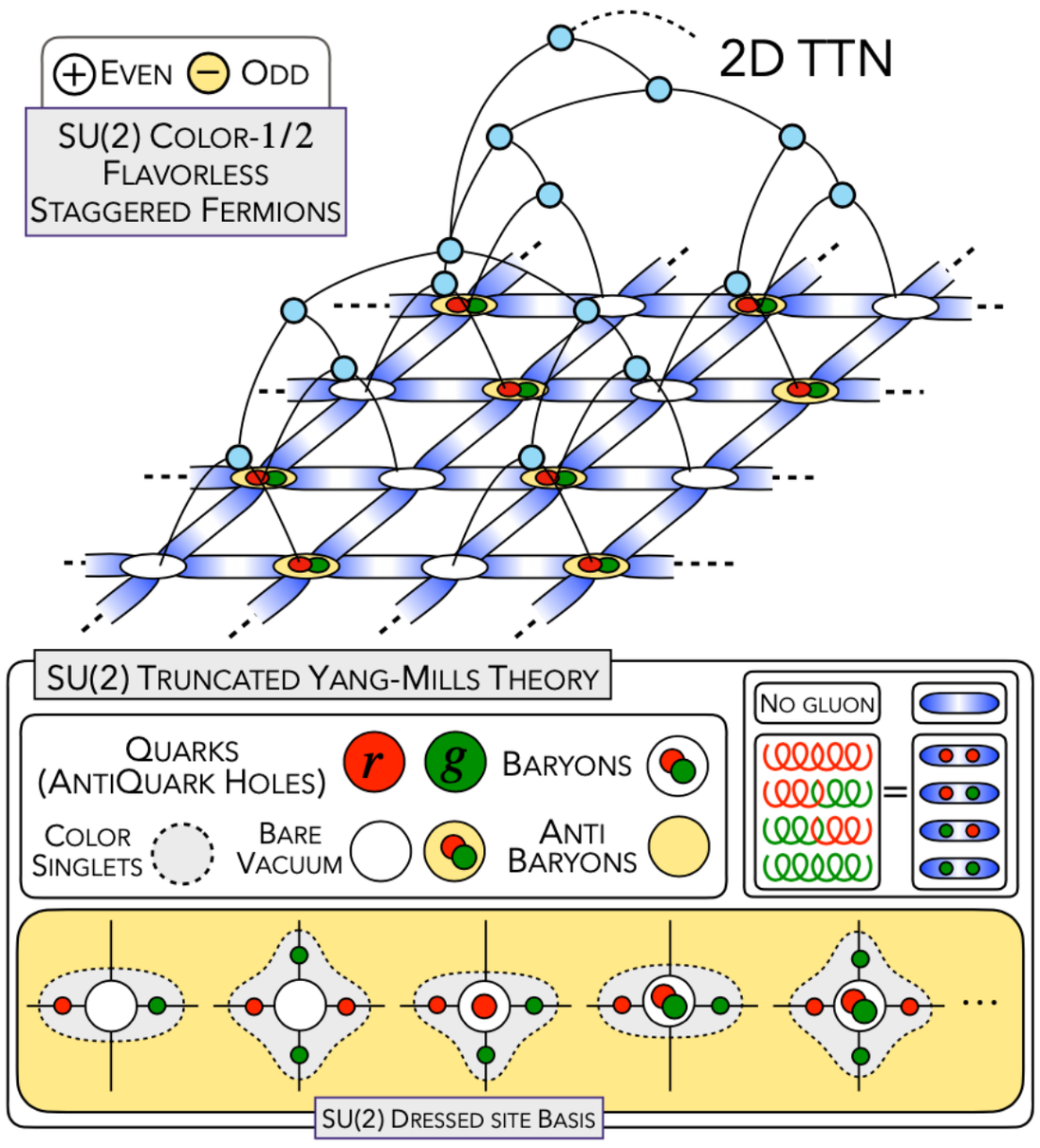




G. Magnifico, T. Felser, P. Silvi, and S. Montangero
Nat. Comm. (2021)

$$g_e^2 = g^2/a,\ g_m^2 = 8/(g^2 a)$$

2+1D NON-ABELIAN LGT

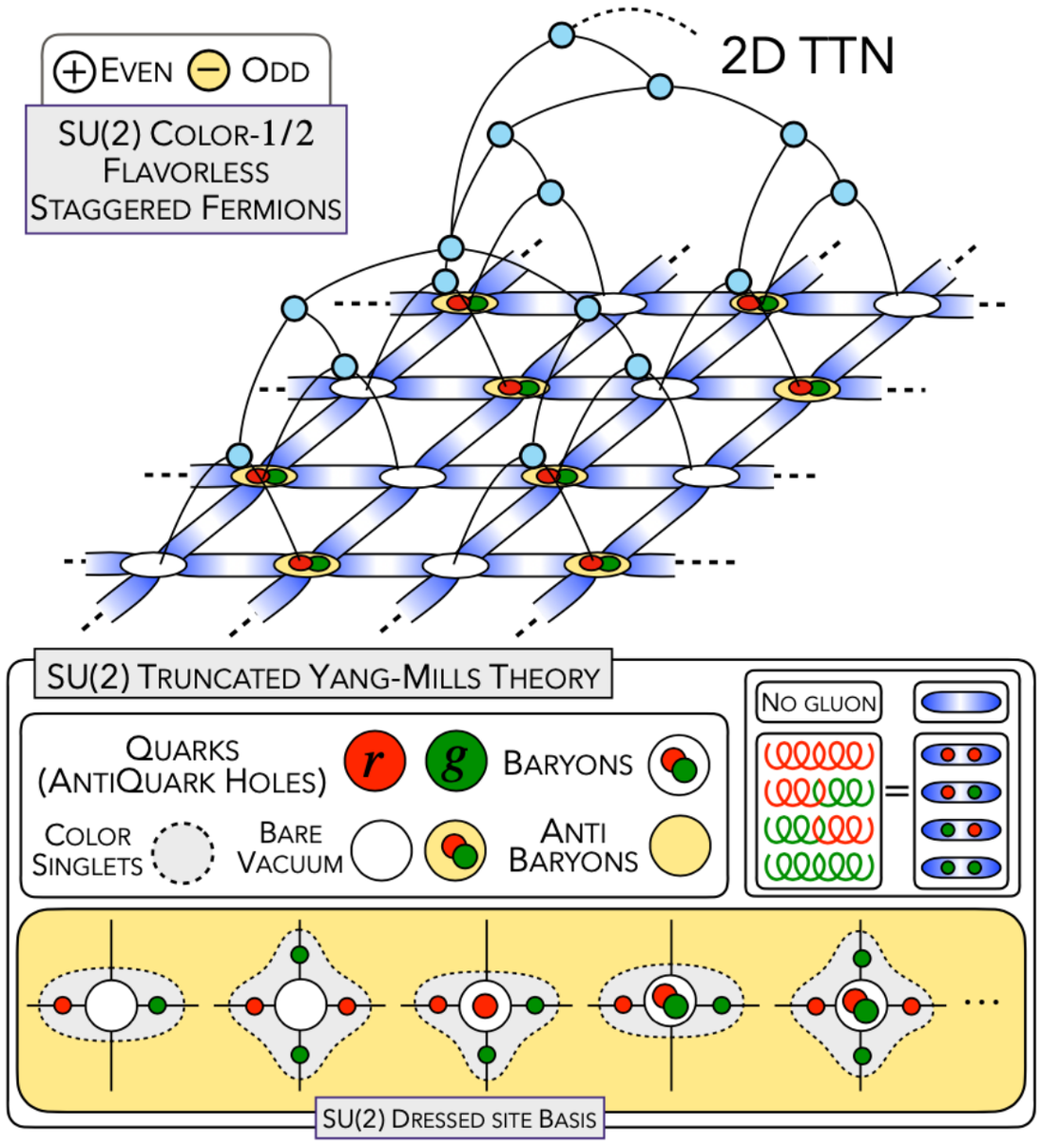


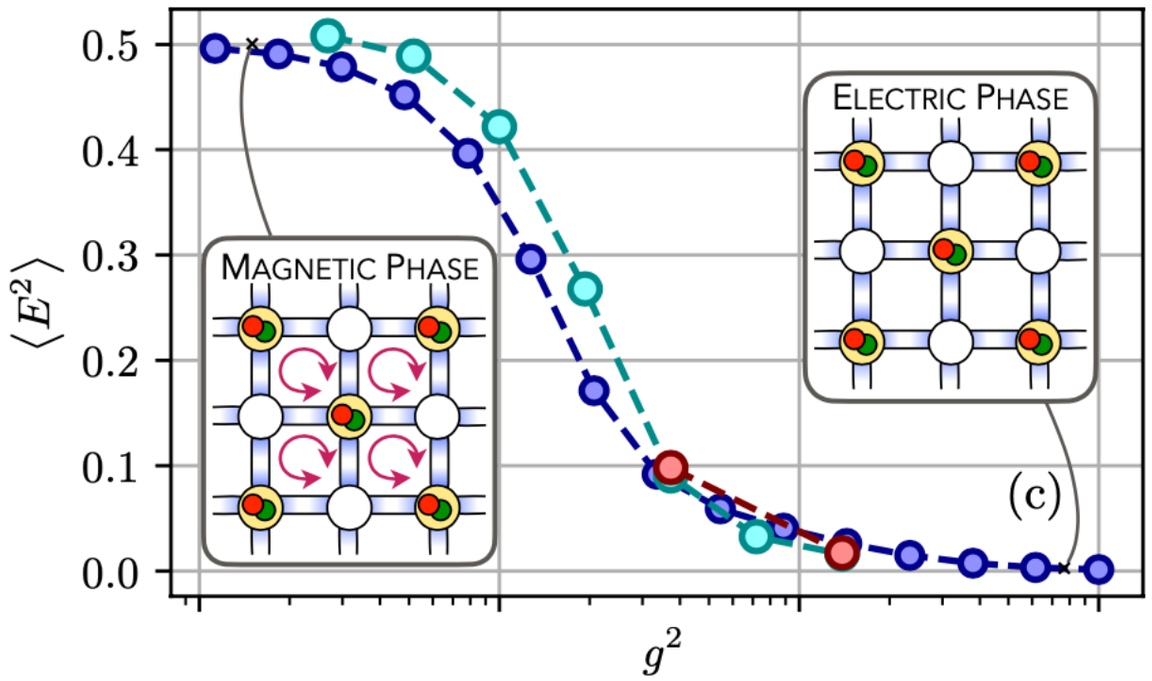
$\langle E^2 \rangle$

(2+1)D SU(2) Yang-Mills Lattice Gauge Theory at finite density via tensor networks

G. Cataldi, G. Magnifico, P. Silvi, SM *arXiv:2307.09396*

2+1D NON-ABELIAN LGT





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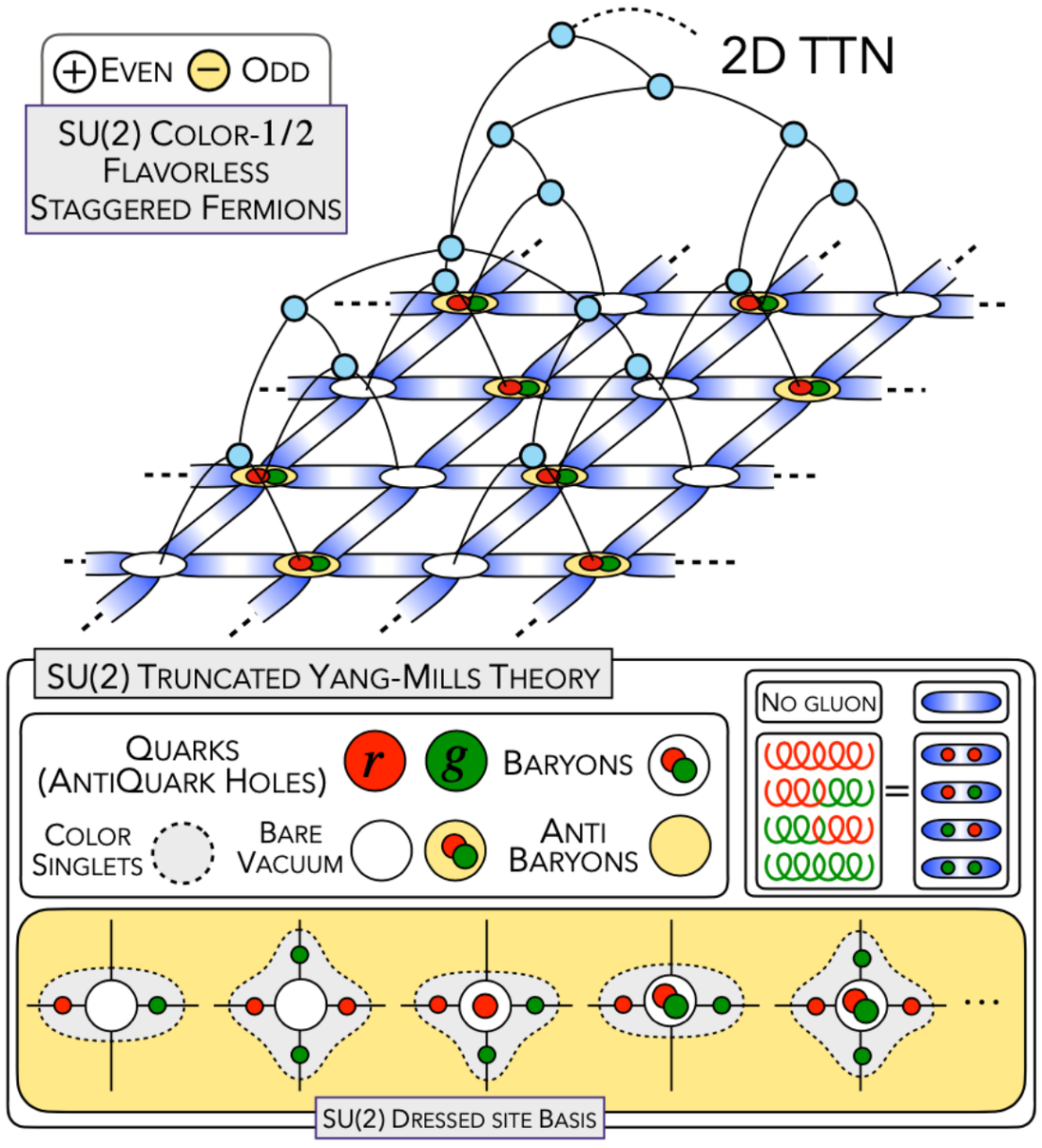

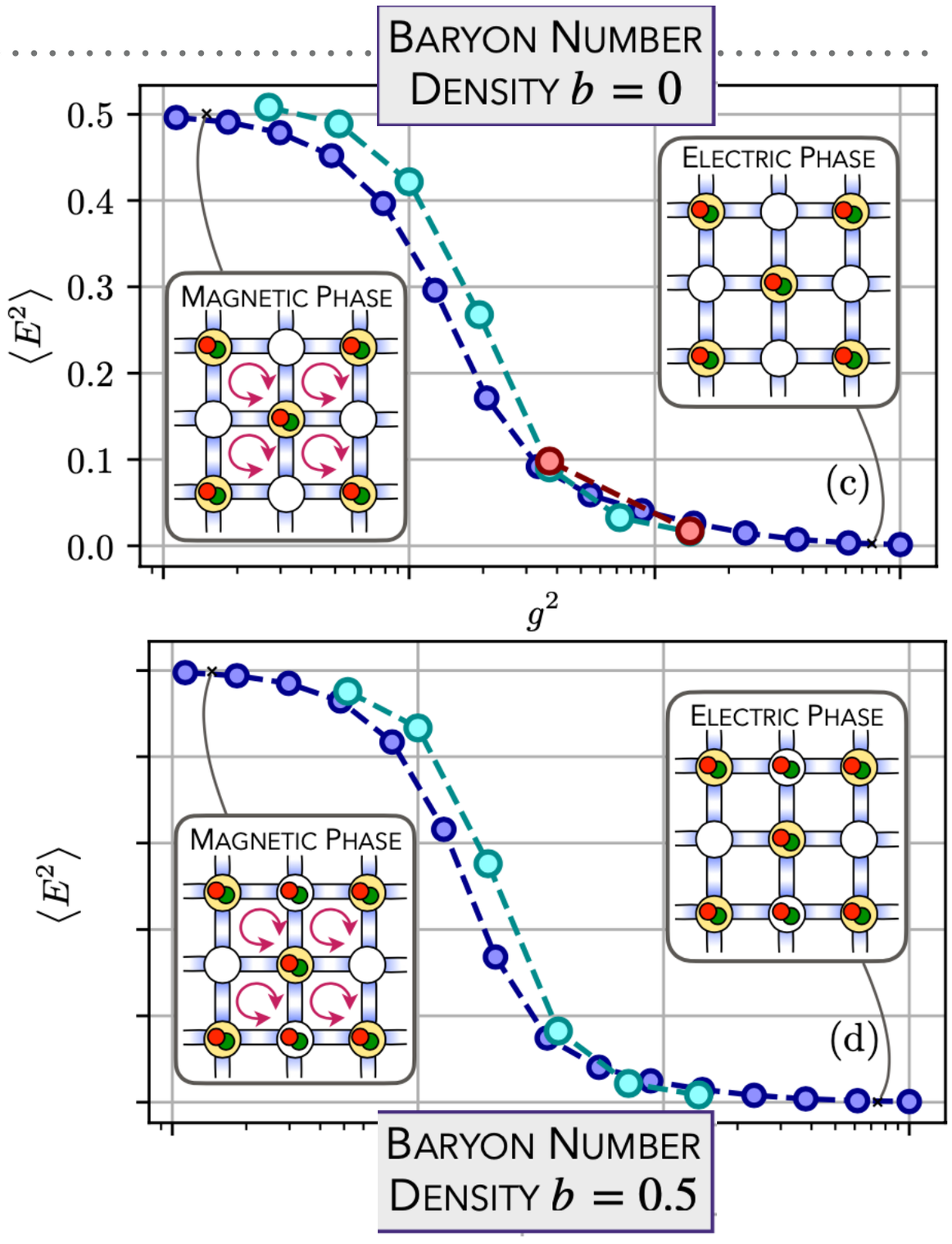

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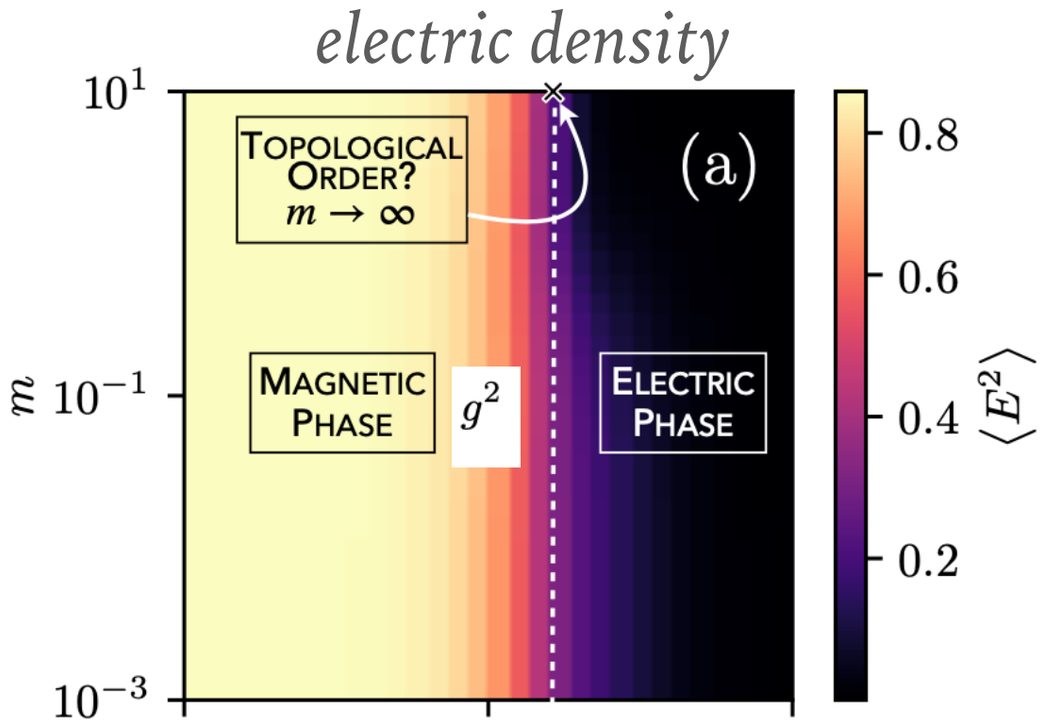
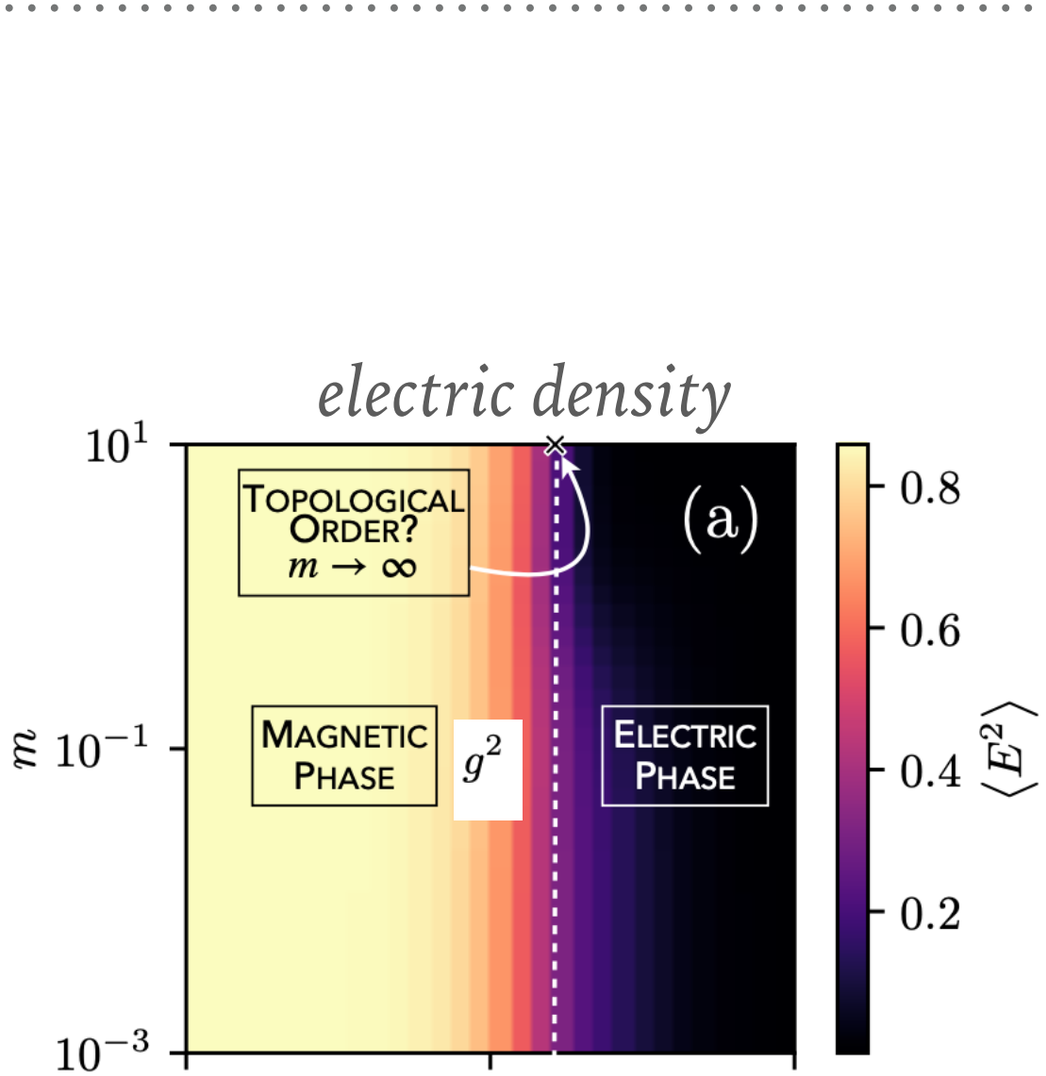
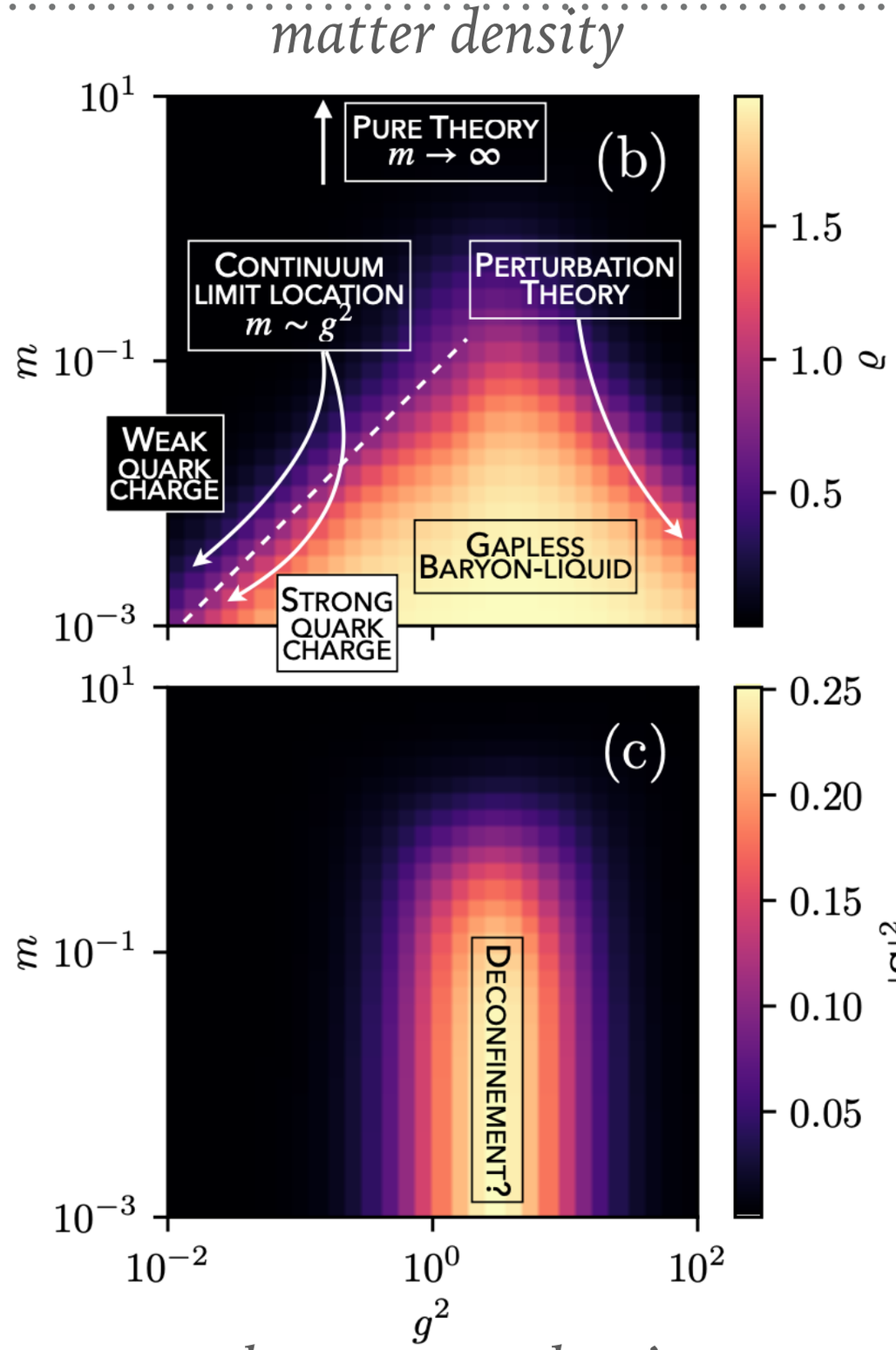
2D+1 SU(2) FINITE-SIZE PHASE DIAGRAM



2D+1 SU(2) FINITE-SIZE PHASE DIAGRAM





ROADMAP FOR QUANTUM(-INSPIRED) LGT SIMULATION

ROADMAP FOR QUANTUM(-INSPIRED) LGT SIMULATION

3. arXiv:2407.03058 [pdf, other]

hep-lat cond-mat.str-el hep-th physics.comp-ph quant-ph

Tensor Networks for Lattice Gauge Theories beyond one dimension: a Roadmap

Authors: Giuseppe Magnifico, Giovanni Cataldi, Marco Rigobello, Peter Majcen, Daniel Jaschke, Pietro Silvi, Simone Montangero

ROADMAP FOR QUANTUM(-INSPIRED) LGT SIMULATION

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Tensor Networks for Lattice Gauge The Roadmap

Authors: Giuseppe Magnifico, Giovanni Cataldi, M
Pietro Silvi, Simone Montangero

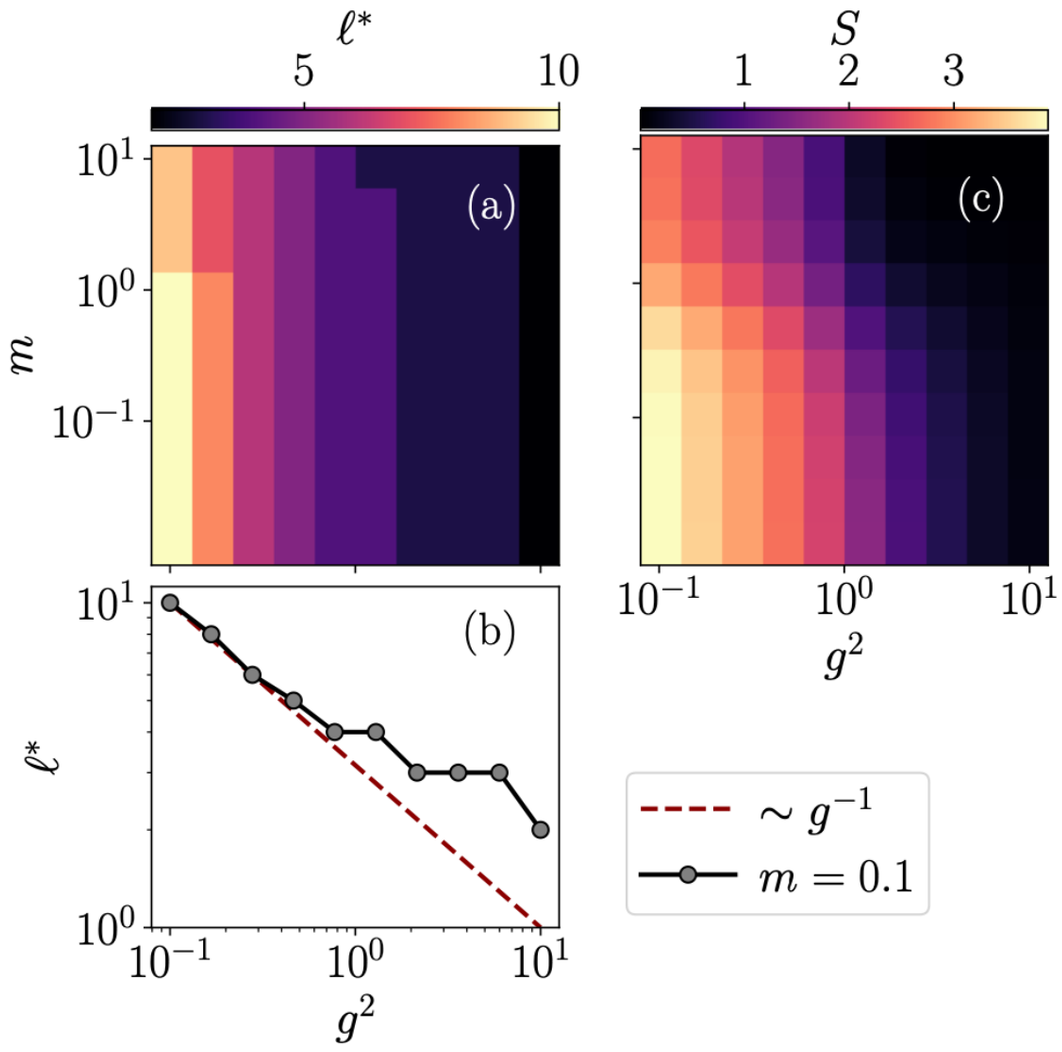

FIG. 4. Exact diagonalization of a QED plaquette for a grid of masses and couplings, $m \in [10^{-2}, 10^1]$ and $g^2 \in [10^{-1}, 10^1]$. (a,b) Minimal gauge truncation ℓ^* required to reach a precision $\epsilon_{\text{trunc}} = 10^{-5}$ in the magnetic energy $\langle \text{Re}\, \hat{U}_\square \rangle$. (c) Corresponding entanglement entropy S associated with a symmetric bipartition of the plaquette.

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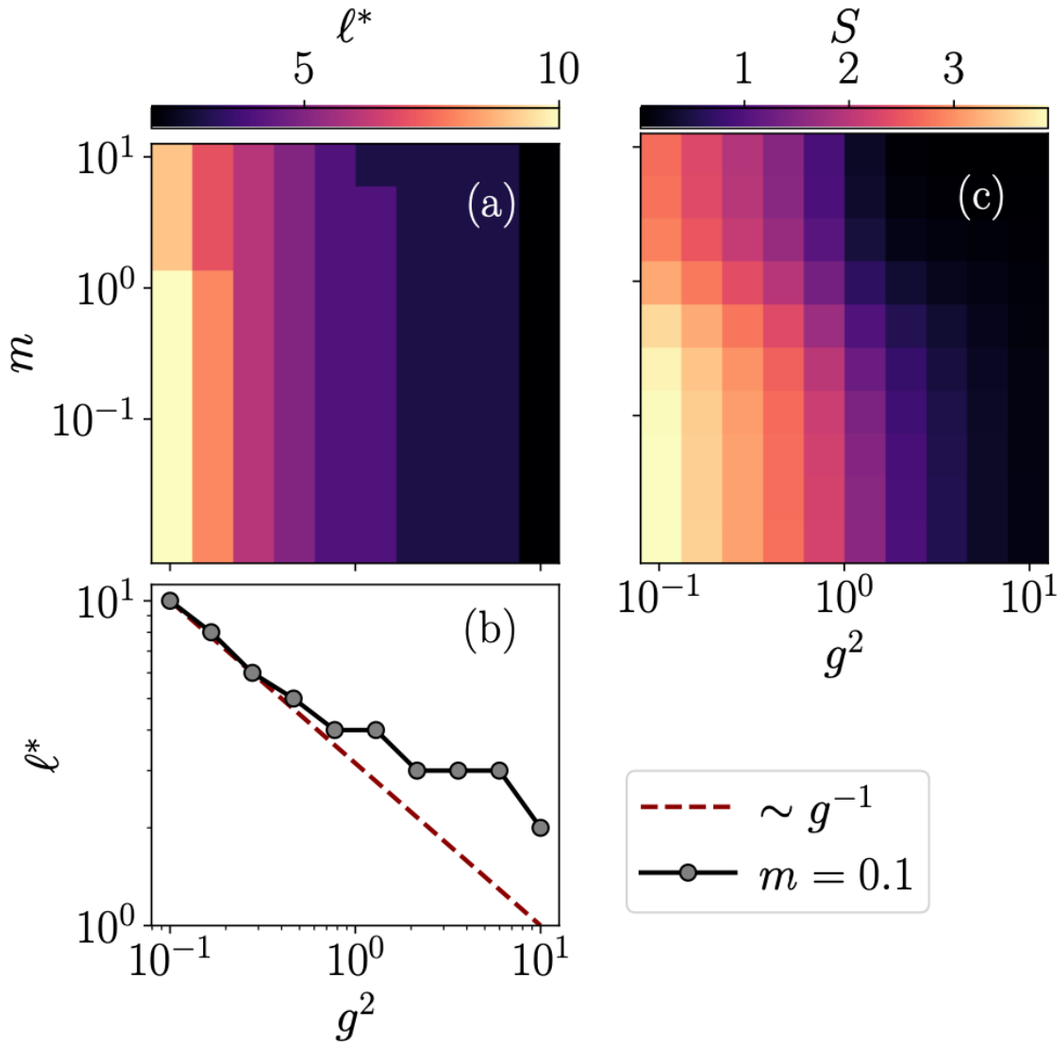
Tensor Networks for Lattice Gauge The Roadmap

Authors: Giuseppe Magnifico, Giovanni Cataldi, M
Pietro Silvi, Simone Montangero

ℓ	d					
	$(2+1)$-dimensions			$(3+1)$-dimensions		
	U(1)	SU(2)	SU(3)	U(1)	SU(2)	SU(3)
1	35	30	164	267	178	3096
2	165	168	752	3437	3670	52476
3	455	600	3738	18487	35280	813438
4	969	1650	19878	64953	214958	17490134
5	1771	3822	43698	177155	967466	69232482
6	2925	7840	82128	408421	3509062	228461186
7	4495	14688	212496	835311	10828494	1245755754
8	6545	25650	333538	1561841	29473038	2782999996

TABLE I. Dressed site Hilbert space dimension d for increasing number ℓ of allowed electric energy density levels in some 2- and 3-dimensional paradigmatic LGTs with dynamical matter and gauge groups U(1), SU(2), and SU(3).



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ROADMAP FOR QUANTUM(-INSPIRED) LGT SIMULATION

3. arXiv:2407.030[illegible] [pdf, other]

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Tensor Netv
Roadmap

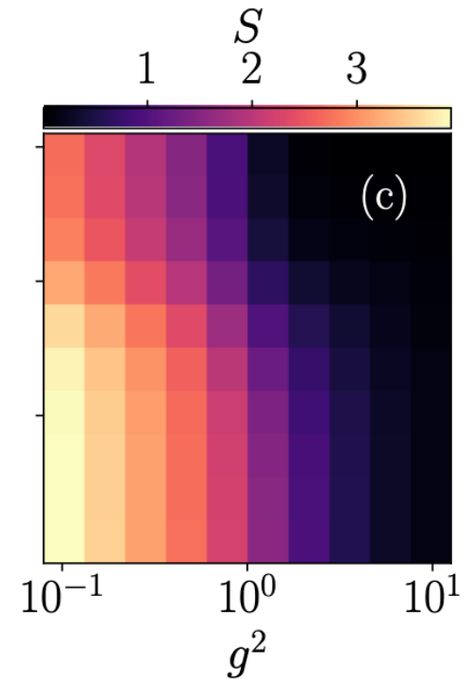
Authors: Giuse
Pietro Silvi, Sim

ℓ	(2 + 1)-dimensions		
	U(1)	SU(2)	SU(3
1	35	30	16
2	165	168	75
3	455	600	373
4	969	1650	1987
5	1771	3822	4369
6	2925	7840	8212
7	4495	14688	21249
8	6545	25650	33353

TABLE I. Dressed site creasing number ℓ of a in some 2- and 3-dime namical matter and gauge groups U(1), SU(2), and SU(3).

System size	χ	Factor	Estimated walltime
64×64	450	T_{base}	4.16 days
64×64	900	$16 \cdot T_{\text{base}}$	66.6 days
256×256	450	$28 \cdot T_{\text{base}}$	116.5 days
256×256	900	$448 \cdot T_{\text{base}}$	5.1 years
$16 \times 16 \times 16$	450	$4 \cdot T_{\text{base}}$	16.6 days
$16 \times 16 \times 16$	900	$64 \cdot T_{\text{base}}$	266 days
$64 \times 64 \times 64$	450	$1984 \cdot T_{\text{base}}$	23 years
$64 \times 64 \times 64$	900	$31744 \cdot T_{\text{base}}$	362 years

TABLE II. *Estimated simulation time.* We derive the baseline from a single-tensor optimization of a 64×64 quantum Ising simulation with $\mathbb{Z}_2$ symmetry taking 7192s on a A100 GPU. Further, we assume that single-tensor update, one tensor and one GPU per MPI thread, and 50 sweeps for the baseline. To extrapolate to larger systems, we assume a scaling with $\mathcal{O}(\chi^4 N^{D-1})$ as well as seven (thirty-one) tensors per MPI thread for 256×256 ($64 \times 64 \times 64$) systems. The empirical scalings are approximately a factor of 2.3 for doubling the system size and 13 for doubling the bond dimension, which we obtain from smaller simulations with $\chi = 225$ and for 32×32 qubits. The times are valid for any $d < \chi$.



a QED plaquette for a grid
$^{-2}, 10^1]$ and $g^2 \in [10^{-1}, 10^1]$.
ℓ^* required to reach a preci-
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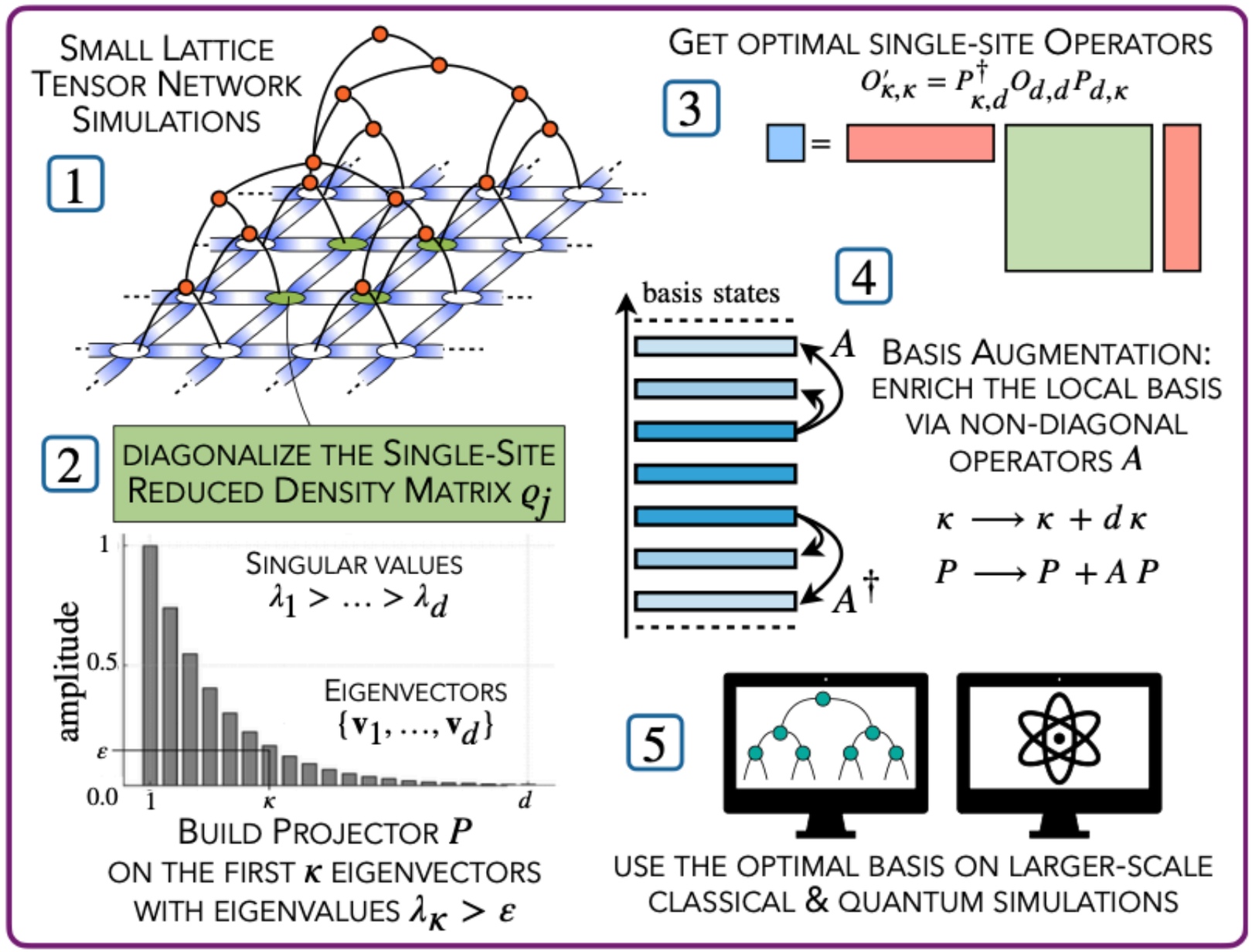
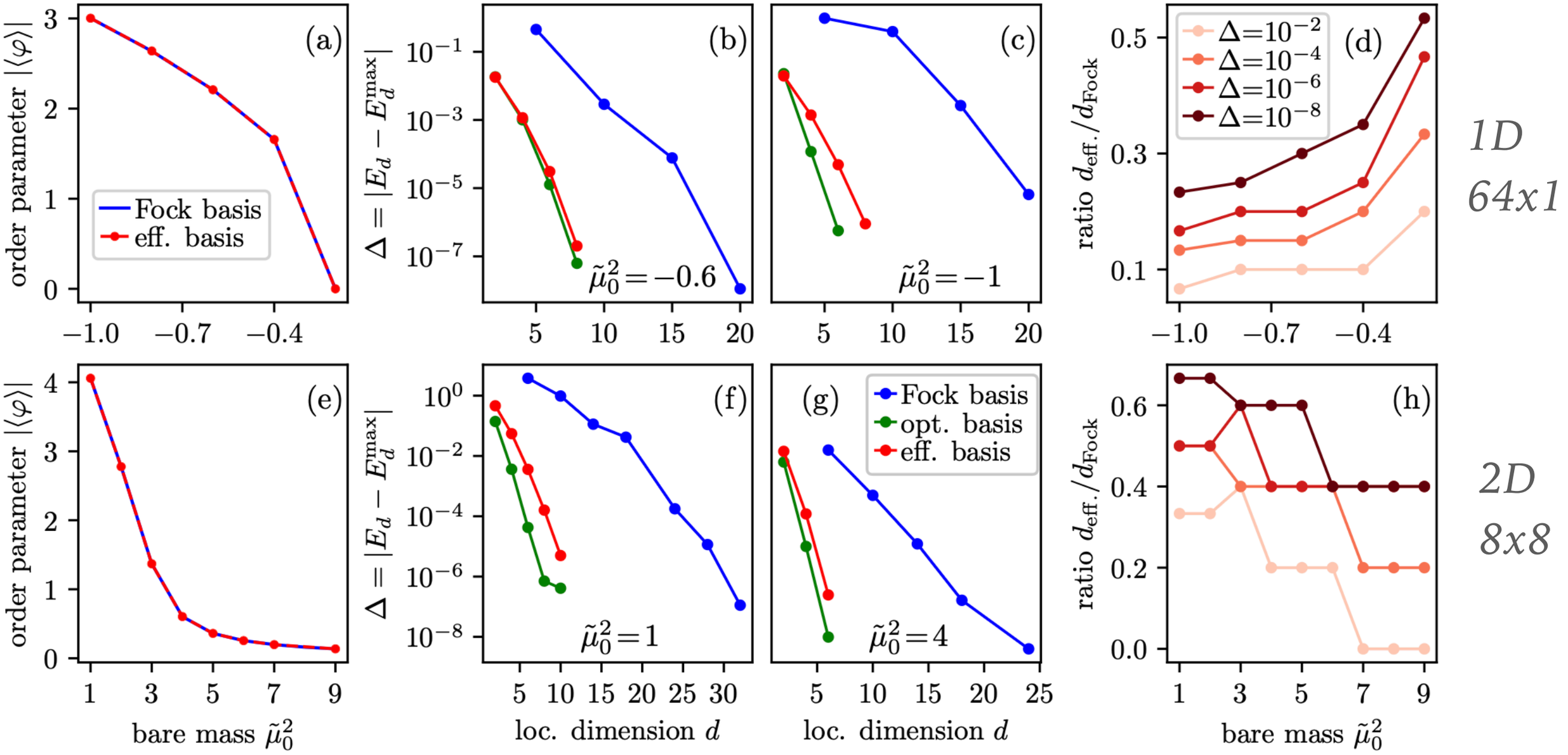
OPTIMAL TRUNCATED BASIS FOR QUANTUM SIMULATIONS



P. Majcen et al. arXiv: 2509.17975

PHI4 MODEL



1D 64x1

2D 8x8

$$\hat{\mathrm{H}} = \sum_{\mathbf{n},k} \left[\frac{\hat{\pi}_{\mathbf{n}}^2}{2a} + \frac{\left(\hat{\varphi}_{\mathbf{n}+\boldsymbol{\mu}_k} - \hat{\varphi}_{\mathbf{n}}\right)^2}{2a} + \frac{a\mu_0^2}{2}\hat{\varphi}_{\mathbf{n}}^2 + \frac{a\lambda}{4!}\hat{\varphi}_{\mathbf{n}}^4 \right]$$

TAKE HOME MESSAGES

- Tensor network algorithms can be used to benchmark, verify, support and guide quantum simulations/computations
- High-dimensional tensor network simulations are becoming increasingly efficient and able to investigate interesting physics
- Entanglement of mixed many-body states can be quantified
- Scalability to full HPC will be necessary to produce quantitative results
- Interesting developments also in other directions (classical optimisers/annealers)
- Tensor network machine learning is competitive with DNN

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Galileo Galilei

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Thank you for your attention!

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Ilaria Siloi Carmelo Mordini
Daniel Jaschke Giuseppe Calajò
Giuseppe Magnifico Simone Notarnicola
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Lorenzo Maffi Luka Pavešić
Marco Rigobello Giovanni Cataldi
Marco Ballarin Mattia Morgavi
Peter Majcen Alice Pagano
Nora Reinić Marco Tesoro
Asmita Datta Rocco Barač

Positions available!

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QUANTERA

INFN Istituto Nazionale di Fisica Nucleare

DFG Deutsche Forschungsgemeinschaft

QRyd Demo

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T-NiSQ
Tensor Networks in Simulation of Quantum matter

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