

Further Study on the Spin and kHz QPOs of Millisecond Pulsars in LMXBs

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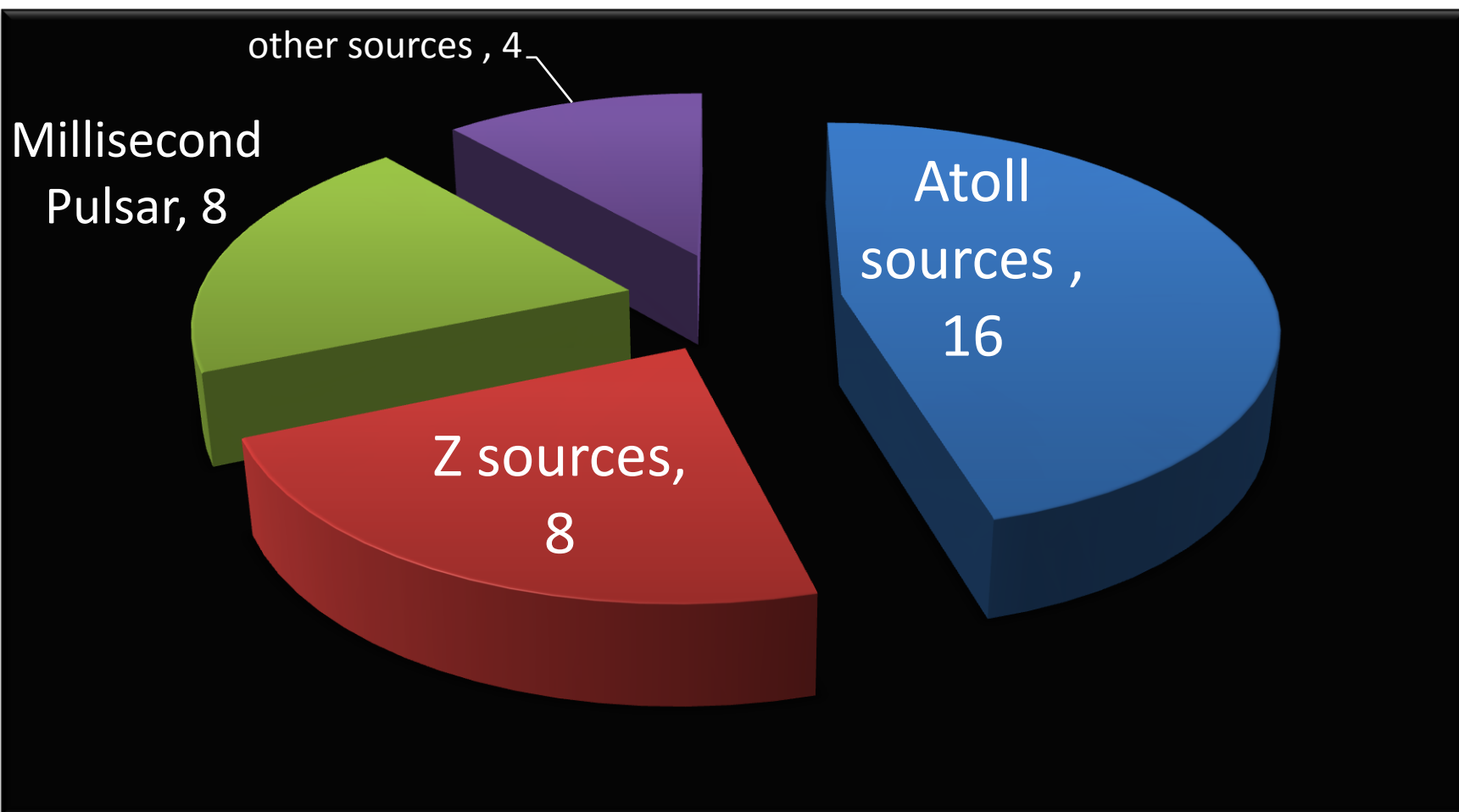
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Outline

- kHz QPOs in LMXBs (Updated)
- Correlations Between the twin QPOs
- Some models
- Correlations Between Spin Frequencies and kHz QPOs of Millisecond Pulsars in LMXBs
- Further Studies
- Preliminary Conclusion

kHz QPOs by RXTE 1996-2008



Correlations Between the twin QPOs

$$\nu_1 = a \left(\frac{\nu_2}{1000 \text{ Hz}} \right)^b \text{ Hz} \quad a = 724 \pm 3, b = 1.9 \pm 0.1$$

Psaltis et al 1998

$$a = 722 \pm 0.7, b = 1.85 \pm 0.01$$

Yin et al. 2005

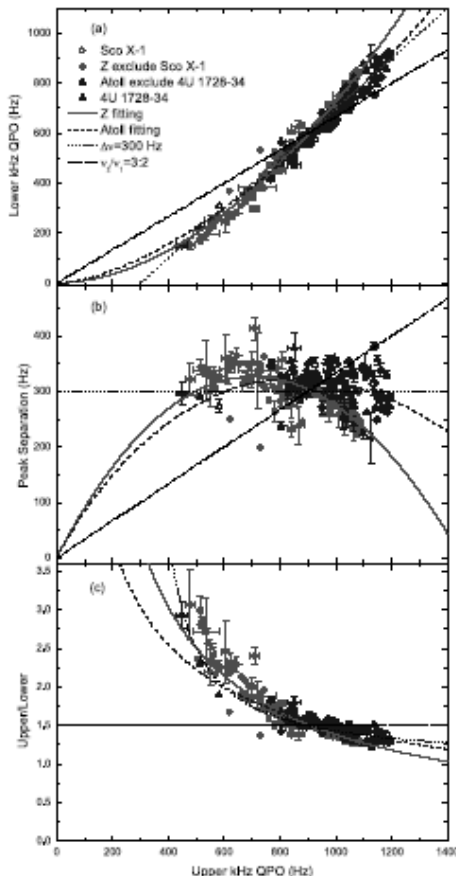


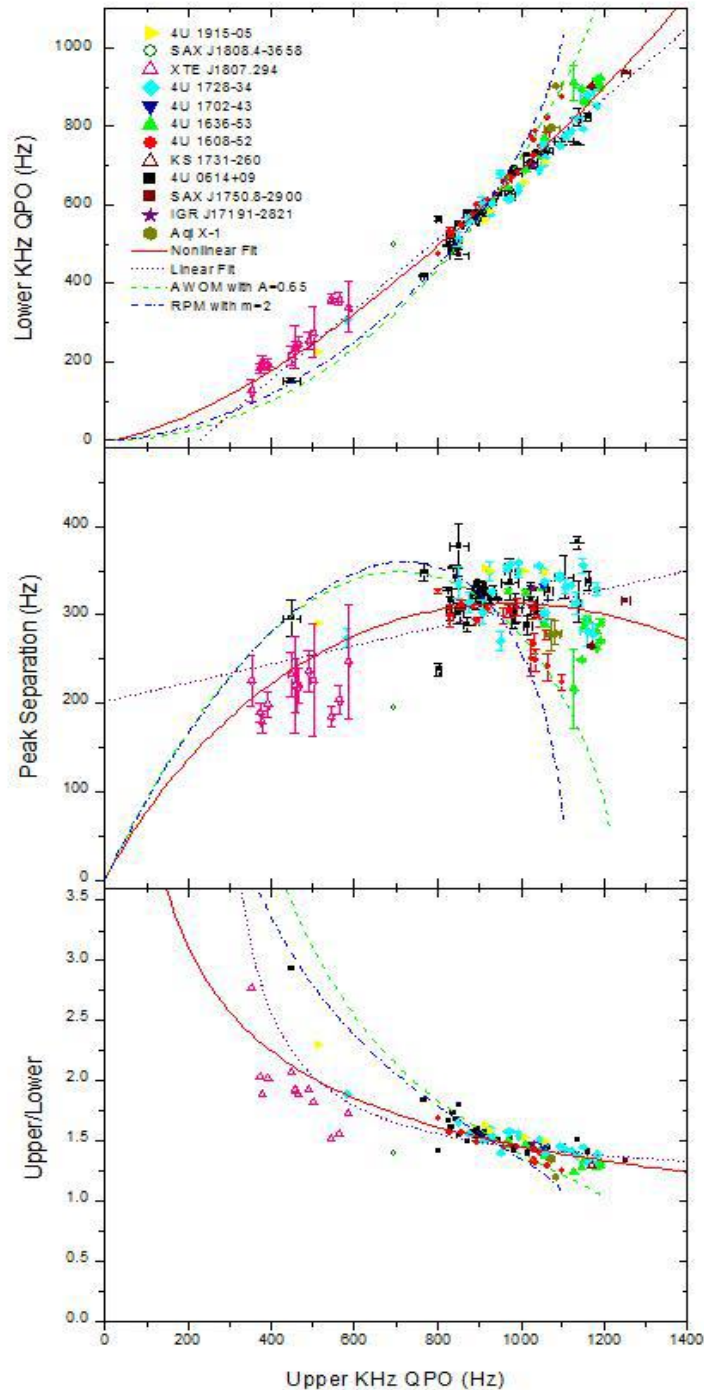
Table 2. The twin kHz QPO correlation $\nu_1 = a(\nu_2/1000 \text{ Hz})^b$.

Sources ^a	a (Hz)	b
All Z samples	724.99 ± 2.52	1.86 ± 0.03
Z ($\nu_2 < 840 \text{ Hz}$)	812.09 ± 2.59	2.20 ± 0.10
Z ($\nu_2 > 840 \text{ Hz}$)	722.72 ± 1.45	1.79 ± 0.03
Sco X-1	721.95 ± 0.69	1.85 ± 0.01
All Atoll samples	683.48 ± 3.01	1.61 ± 0.04
4U 1728-34	667.86 ± 5.59	1.51 ± 0.07
All Z and Atoll samples	699.13 ± 2.23	1.68 ± 0.02

Zhang, C.M., et al., 2006

Figure 1. Plots of (a) ν_1 versus ν_2 , (b) $\Delta\nu$ versus ν_2 and (c) ν_2/ν_1 versus ν_2 . The Z [Atoll] fitting line represents the fitted correlation between the pair kHz QPO frequencies for the Z [Atoll] sources as $\nu_1 = (724.99 \pm 2.52 \text{ Hz}) (\nu_2/1000 \text{ Hz})^{1.86 \pm 0.03}$ [$\nu_1 = (683.48 \pm 3.01 \text{ Hz}) (\nu_2/1000 \text{ Hz})^{1.61 \pm 0.04}$].

Correlations between the twin QPOs with spin frequencies



$$\nu_1 = a \left(\frac{\nu_2}{1000 \text{ Hz}} \right)^b \text{ Hz}$$

$$a = 687.2 \pm 3.47, b = 1.47 \pm 0.03$$

$$\nu_2 = A \nu_1 + B$$

$$A = 1.069 \pm 0.02, B = 257.07 \pm 13.88$$

Follow Belloni 2005

Some models

- Beat model
- Non-linear resonance model
- Relativistic precession model
- Alfven wave oscillation model
- Other models

Beat model

Miller et al. 1998

$$v_2 - v_1 = v_{spin}$$

Lamb & Miller 2001

$$v_2 - v_1 < v_{spin}$$

Lamb & Miller 2003

$$v_2 - v_1 = \frac{1}{2} v_{spin}$$

$$\Delta v = v_2 - v_1 \sim \text{const.}$$

Non-linear resonance model

- Kluzniak & Abramowicz 2001
- Abramowicz & Kluzniak 2001
- In given radius, orbit frequency and epicyclic frequency form an integral ratio or commensurability.

$$\nu_2 / \nu_1 = 3/2$$

Black hole QPOs: 3:2

Relativistic precession model

Stella & Vietri 1999

Precession Model for KHz QPO

$$\nu_2 = \nu_{kepler}$$

$$\nu_1 = \nu_{precession} = \nu_2 \left(1 - \sqrt{1 - \frac{3R_s}{r}} \right)$$

$\Delta \nu = \nu_2 - \nu_1$ is not const.

Alfven wave oscillation model

$$v_A(r) = \frac{v_A(r)}{2\pi r} = \frac{1}{2\pi r} \frac{B(r)}{\sqrt{4\pi\rho}} = \frac{B_s (R/r)^3}{2\pi r} \sqrt{\frac{Sv_{ff}}{4\pi\dot{M}}} \propto \sqrt{S} \quad (\text{Shapiro \& Teukolsky 1983})$$

Dipole Magnetic Field

$$\rho = \dot{M} / (Sv_{ff})$$

$$v_{ff} = \sqrt{2GM/r} = c\sqrt{R_s/r}$$

$R_s \sim$ Schwarzschild radius

$$S_r = 4\pi r^2$$

$$S_p = 4\pi R^2 (1 - \cos\theta_c), \quad \sin^2\theta_c = R/r \equiv X$$

$$R \ll r, \quad S_p \cong \frac{2\pi R^2}{r}$$

$$S_r = 4\pi R^2 X^{-2}$$

$$S_p = 4\pi R^2 (1 - \sqrt{1-X})$$

$$v_2 = \sqrt{\frac{GM}{4\pi^2 r^3}} = v_A(S_r) = 1850(\text{Hz}) AX^{3/2}$$

$$v_1 = v_A(S_p) = v_2 \sqrt{S_p/S_r} = v_2 X \sqrt{1 - \sqrt{1-X}}$$

$$\frac{v_1}{v_2} = X \sqrt{1 - \sqrt{1-X}}$$

$$\Delta v = v_2 - v_1 = v_2 (1 - X \sqrt{1 - \sqrt{1-X}})$$

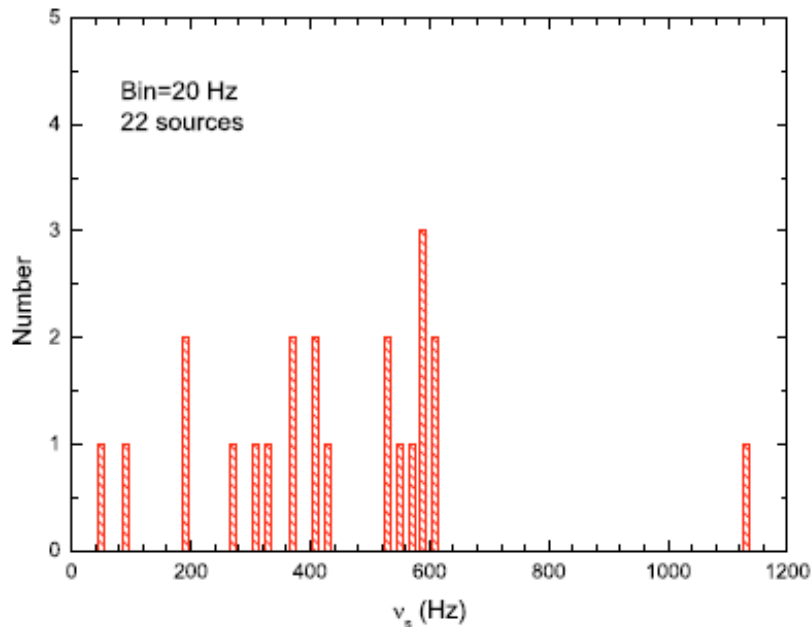
$$A = \frac{m^{1/2}}{R_6^{3/2}}; \quad X = \frac{R}{r}$$

m NS mass in solar mass

R_6 NS radius in 10^6 cm

Correlations Between Spin Frequencies and kHz QPOs

Spin frequencies detected in 24 LMXBs from 45Hz (Vireal & Strohmayer 2004) to 1122Hz (Kaaret 2007)

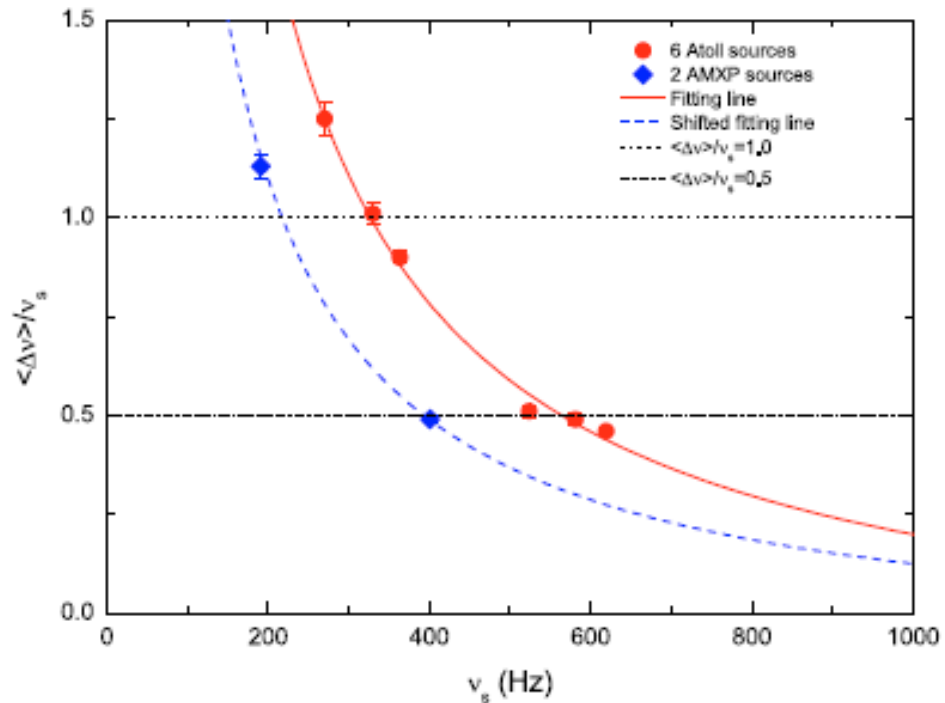
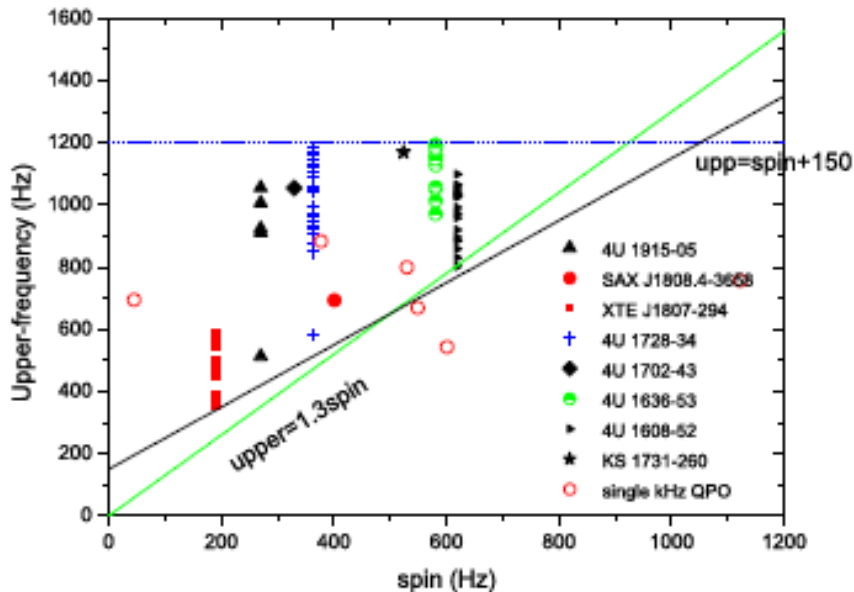


Distribution of the spin frequencies of the 22 neutron stars in LMXBs

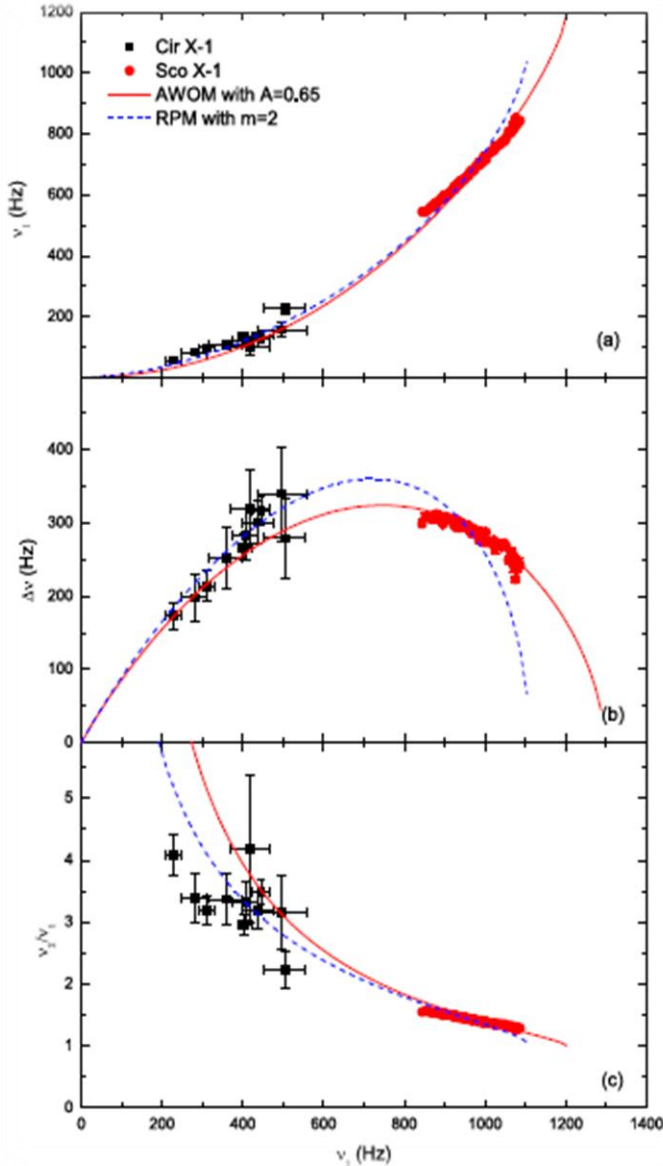
$$\langle \Delta \nu \rangle \sim \nu_s$$

Yin et al. 2007

1. $\nu_{2\min} > 1.3\nu_s$ Only in twin kHz QPO, not single
2. $\langle \Delta \nu \rangle = -(0.19 \pm 0.05)\nu_s + (389.40 \pm 21.67) \text{ Hz}$ in 6 atoll sources
3. Shift a factor of 1.5, good fit to AMXP



Further studies

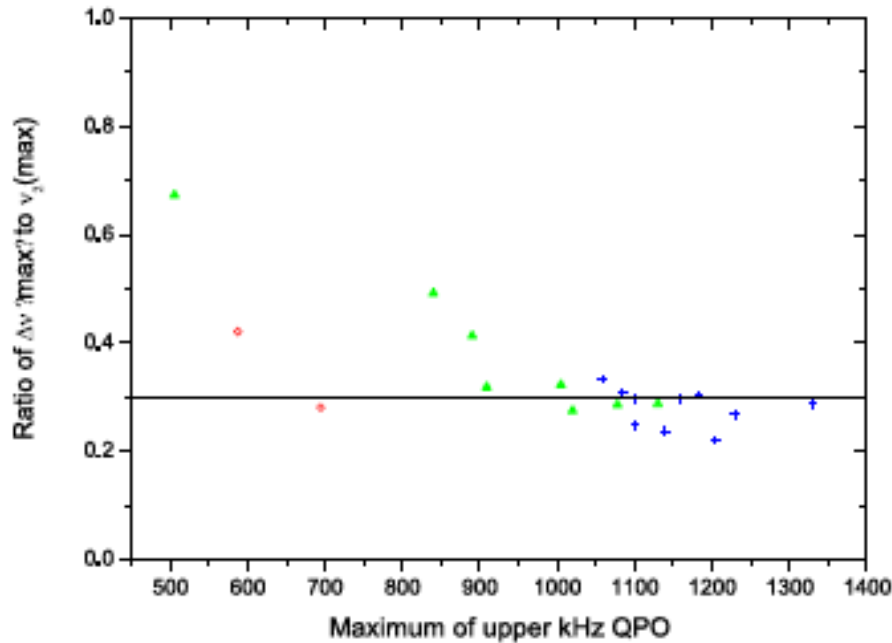


Cir X-1 and Sco X-1 are two extreme Z sources.

The (b) represents an obvious tendency of increasing firstly (Cir X-1) and then decreasing (Sco X-1) with the frequency. That is consistent with AWOM ($A=0.65$) and RPM ($m=2$).

(C) acts a good fit of detected data and can reject 3:2 relationship

0.29 ratio



The ratio between the maximum peak separation and maximum upper frequency versus the maximum upper frequency diagram.

$$\nu_2 = 1850AX^{3/2} (\text{Hz}) = 1295\left(\frac{A}{0.7}\right)X^{3/2} (\text{Hz})$$

$$\nu_1 = \nu_2 X^{4/3} \left[1 - \sqrt{1-X}\right]^{1/2}$$

$$\frac{\nu_2}{\nu_1} = f(X) / X^{5/4}, \quad f(X) = \left[1 + \sqrt{1-X}\right]^{1/2}$$

$$1 \leq f(X) \leq 1.4$$

$$\Delta\nu = \nu_2 - \nu_1 = \nu_2 \left[1 - X^{5/4} / (1 + \sqrt{1-X})\right]^{1/2}$$

$$= (1850A)F(X)$$

$$F(X) = X^{3/2} \left[1 - X^{5/4} / (1 + \sqrt{1-X})\right]^{1/2}$$

$$\nu_2(\text{max}) = 1850A \quad \text{and} \quad \Delta\nu(\text{max}) = 525A$$

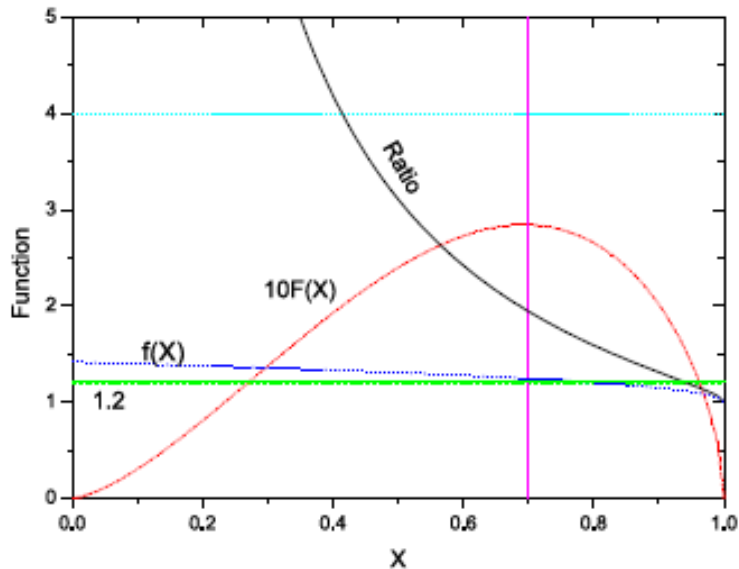
$$\frac{\Delta\nu(\text{max})}{\nu_2(\text{max})} = 0.29$$

$$\langle \nu_2 / \nu_1 \rangle = 1.5$$

Detected averaged twin kHz QPOs ratio Zhang et al. 2006

$$\frac{\nu_2}{\nu_1} = f(X) / X^{5/4}, f(X) = \left[1 + \sqrt{1 - X} \right]^{1/2}$$

$$\langle X \rangle = 0.8$$

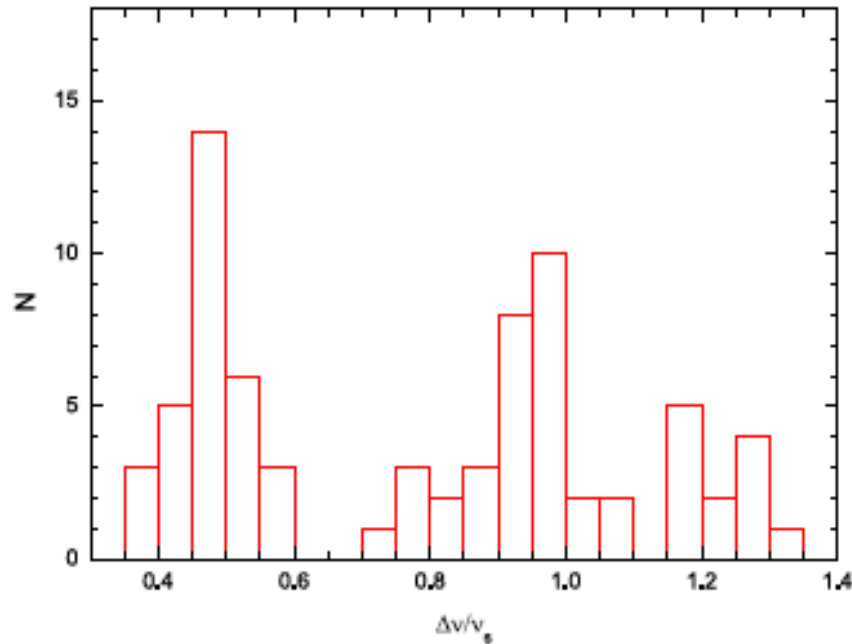


$$\langle A \rangle = 0.7 \quad \langle X \rangle = 0.8 \quad \langle f(X) \rangle = 1.2$$

$$\nu_1 = \frac{1000 \text{ Hz}}{f(X)(1.85 \text{ A})^{5/6}} \left(\frac{\nu_2}{1000} \right)^{11/6} \cong 670 (\text{Hz}) \left(\frac{\nu_2}{1000} \right)^{1.83}$$

The ratio of twin kHz QPOs asymptotically decreases with X; the separation function shows its maximum to occur at X=0.7.

Distribution of $\Delta\nu/\nu_s$



XTE J1807-294

4U 1702-43

4U 1728-34

4U1915-05

$$\nu_s < 400\text{Hz} \quad \langle \Delta\nu \rangle / \nu_s \sim 1$$

SAX J1808.4-3658

4U 1608-52

4U 1636-53

4U 1731-28

$$\nu_s > 400\text{Hz} \quad \langle \Delta\nu \rangle / \nu_s \sim 0.5$$

The ratio concentration around
0.5 and 1.0 is not clear

Anti-correlation: Yes or No?

If the anti-correlation is between $\langle \Delta \nu \rangle$ and ν_s real, we can use it to infer the averaged kHz QPO peak separations of sources or to roughly estimate the spin frequency (Z sources).

Name	Spin frequency (Hz)	Estimate average separation (Hz)
EXO 0748-676	45	380
XB 1254-690	95	370
XTE J1739-285	1122	160

However, this anti-correlation is still a conjecture since it is based on data from only six sources.

But if this result were confirmed, it means that the spin frequency would play a role in the mechanism that produces the kHz QPOs

Table 1. Ten sources with the twin kHz QPOs and spin frequencies. dn average ? nu2/nu1 average ?

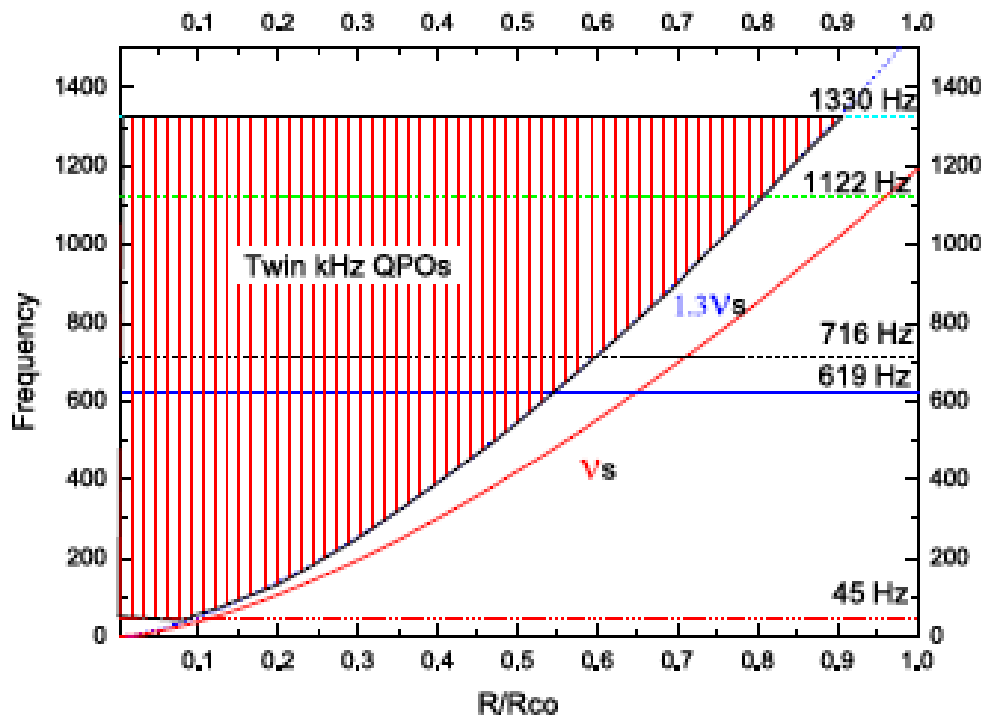
Sources ⁽¹⁾	ν_2 ⁽²⁾ (Hz)	$\Delta\nu$ ⁽³⁾ (Hz)	ν_2/ν_1 ⁽⁴⁾ (Hz)	ν_s ⁽⁵⁾ (Hz)	$\Delta\nu/\nu_s$	References ^(*)
Millisecond pulsars						
XTE J1807-294	353-587 (587)	179-247	1.51-2.78	191	0.94-1.29	[1]
SAX J1808.4-3658	694 (725)	195	1.39	401	0.49	[2]
Atoll sources						
4U 1608-52	802-1099 (1099)	224-327	1.26-1.69	619	0.36-0.53	[3]
4U 1636-53	971-1192 (1230)	217-329	1.24-1.51	581	0.37-0.57	[4]
4U 1702-43	1055 1085 (1085)	333	1.46	330	1.01	[5]
4U 1728-34	582-1183 (1183)	271-359	1.31-1.89	363	0.75-0.99	[6]
KS 1731-260	1169 (1205)	266	1.29	524	0.51	[7]
4U 1915-05	514-1055 (1265)	290-353	1.49-2.3	270	1.07-1.31	[8]
IGR J17191-2821	1030 (1030)	330	1.47	294	1.12	[9]
SAX J1750.8-2900	1253 (1253)	317	1.34	601	0.53	[10]
4U 0614+09	449-1162 (1162???)	238-382	1.38-2.93	415	0.57-0.92	[11]
Aql X-1	1074-1083 (1083)	278-280	1.35-1.35	550	0.51-0.51	[12]

$$\nu_2 \geq 1.3\nu_s \quad X_{\min} = (1.3\nu_s / 1850A)^{2/3}$$

We obtain a constrain condition for the ratio of twin kHz QPOs

$$\frac{\nu_2}{\nu_1} \leq \left[1 + \sqrt{1 - X_{\min}} \right]^{1/2} / X_{\min}^{5/4}$$

The domain which can produce the twin kHz QPOs



$$\nu_s = 1850A \left(\frac{R}{R_{co}} \right)^{3/2}$$

45 Hz	Lowest spin frequency
619 Hz	Highest burst frequency
716 Hz	Highest radio spin frequency
1122 Hz	Highest spin frequency
1330 Hz	Highest kHz QPO frequency

Conclusion

- Z, Atoll, AMXP, are systemically different or not?
- 0.29 ratio which is to obtain an average value to measure the separation over spin frequency
- Estimate the separation or the spin frequency
- Some correlations of twin kHz QPOs

THANKS FOR YOUR
“PATIENCE”.
IT IS URGENT TO HAVE
“SUPPER”!

