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A quantum hydrodynamical description for scrambling and many-body chaos

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- The background of the slide is a photograph of a large, modern brick building with a prominent clock tower. The building has large glass windows and is surrounded by green trees. The sky is clear and blue.
- ▶ Basic introduction of quantum chaos
 - ▶ Quantum hydrodynamics and SK EFT
 - ▶ Hydrodynamics theory for chaotic systems
 - ▶ Correlation functions and pole skipping
 - ▶ Summary and discussion



Some facts about quantum chaos

Chaotic phenomena are ubiquitous in nature. While much has been learned about chaos at a classical level, its characterizations and manifestations at a quantum level are far less understood, especially in many-body systems.

One key question: how to characterize chaos at quantum level without clear definition of moving orbits in the phase space?

One possible way is to study so-called **out-of-time ordered** correlation functions (OTOC) !

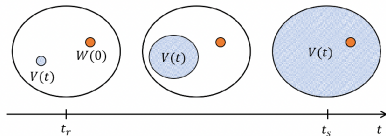
If V and W are two generic few-body operators, one can define the following quantity:

$$\begin{aligned} C(t) &= -\langle [V(t), W(0)]^2 \rangle = C_1(t) - C_2(t) \\ C_1(t) &= \langle V(t)W(0)W(0)V(t) \rangle + \langle W(0)V(t)V(t)W(0) \rangle \\ C_2(t) &= \langle V(t)W(0)V(t)W(0) \rangle + \langle W(0)V(t)W(0)V(t) \rangle \end{aligned} \quad (1)$$

$C_1(t) \implies$ time ordered correlation $C_2(t) \implies$ out-of-time ordered correlation

Typical behavior of OTOC

Physically one can interpret the typical behavior of OTOC as due to scrambling.



- $t \lesssim t_r$: $C(t) \sim \frac{1}{\mathcal{N}}$
- $t_r \ll t \ll t_s$: $C(t) \sim \frac{1}{\mathcal{N}} e^{\lambda t}$
- $t \gg t_s$: $C(t) = \text{const.}$

t_r : relaxation time t_s : scrambling time $\mathcal{N}$: total number of degrees of the system

The exponent λ is the so-called Lyapunov exponent and has been shown to be bounded in a generic system JHEP 08 (2016) 106 :

$$\lambda \leq \lambda_{\max} = \frac{2\pi k_B}{\beta_0 \hbar} \quad (2)$$

The system which saturates this upper bound will be referred to maximal chaos.

Further comments on OTOC

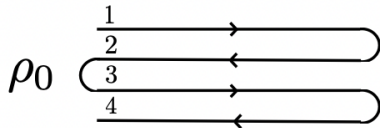


- When separate two operators also spatially, the exponential growth can be generalized to

$$C(t, \vec{x}) \equiv -\langle [V(t, \vec{x}), W(0)]^2 \rangle \sim \frac{1}{\mathcal{N}} e^{\lambda \left(t - \frac{|\vec{x}|}{v_B}\right)}, \quad (3)$$

v_B is often referred to as the butterfly velocity, characterizing the scrambling of an operator in space.

- Typical calculation of OTOC can be achieved by using following contour (4-point case):



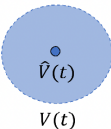
In reality, performing calculations using this kind of contour is quite complicated.

Possible hydrodynamical description of quantum chaos

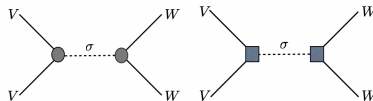


In this work, two fundamental proposals have been emphasized:

- To leading order in the limit $\mathcal{N} \rightarrow \infty$, a coarse-grained description is allowed in which the growth of the operator can be understood as building up a “cloud” of some effective field σ .
- The chaotic behavior of OTOCs can be understood from exchanging and propagation of σ . The only difference between time ordered and out-of time-ordered configurations is in the effective vertex.

$$V(t) = f(\hat{V}(t), \sigma) =$$


The diagram shows a light blue shaded circular region with a dashed blue border. Inside the circle is a small blue dot labeled $\hat{V}(t)$. Below the circle is the label $V(t)$.



There have been various hints for such an effective description (For example, in PRL 117(2016)111601, the Schwarzian is considered as the EFT for a hydrodynamic mode) .



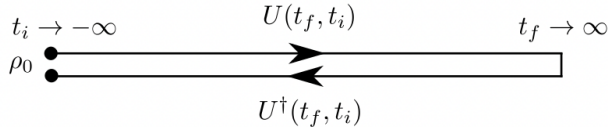
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- The background of the slide is a photograph of a large, modern university building with a prominent clock tower. The building is constructed of red brick and has large glass windows. The clock tower is tall and features two clock faces. The sky is clear and blue, and there are green trees in the foreground.
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Basic ideas of EFT construction

Consider a quantum many-body system in a “liquid” phase for which **the only gapless degrees of freedom are those associated with conserved quantities**. The effective theory for these low energy degrees of freedom is hydrodynamics.

This kind of hydrodynamics action is formulated as an effective theory for a general statistical system in some state ρ_0 defined on the closed time path (CTP), which can be shown in the following:



The effective action defined on such CTP can naturally contain dissipation effects.

With appropriate external sources introduced, the generating functional defined on CTP can be directly written as

$$Z[\phi_{1i}, \phi_{2i}] \equiv e^{W[\phi_{1i}, \phi_{2i}]} = \text{Tr} \left[\rho_0 \mathcal{P} \exp \left(i \int dt (\mathcal{O}_{1i}(t) \phi_{1i}(t) - \mathcal{O}_{2i}(t) \phi_{2i}(t)) \right) \right] \quad (4)$$

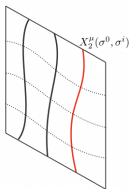
Typically, generating functional $W[\phi_{1i}, \phi_{2i}]$ can not be written as a local functional of external sources. **The non-locality can be interpreted as coming from integrating out certain gapless modes**, which are identified with the desired hydrodynamic modes. Thus, in order to obtain a local action we need to un-integrate such gapless modes. Due to this, the effective action I_{EFT} can be defined as

$$e^{W[\phi_1, \phi_2]} = \int D\varphi_1 D\varphi_2 e^{iI_{\text{EFT}}[\phi_1, \phi_2; \varphi_1, \varphi_2]} \quad (5)$$

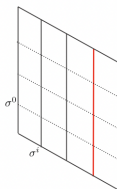
$\varphi_{1,2}$ can be identified as the desired hydrodynamical modes associated with conserved currents or energy-momentum tensor.

When discussing fluid with energy-momentum tensor, one interesting picture can be given as follows:

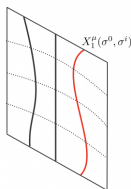
Physical spacetime₂



Fluid spacetime



Physical spacetime₁



$$\sigma^A = (\sigma^0, \sigma^i):$$

- $\sigma^i \Rightarrow$ labels fluid elements
- $\sigma^0 \Rightarrow$ “internal clock” carried by each fluid element

Based on this picture, the hydrodynamical degrees of freedom are given by $X_{1,2}^\mu(\sigma^A)$, which describe motions of fluid elements along two segments of the CTP contour.



Hydrodynamical variables on the CTP

Due to the picture discussed before, the proper time square of the motion is given by

$$-d\ell_s^2 = g_{s\mu\nu} \frac{\partial X_s^\mu}{\partial \sigma^0} \frac{\partial X_s^\nu}{\partial \sigma^0} (d\sigma^0)^2 \quad (6)$$

Thus, the fluid velocity can be given by

$$u_s^\mu(\sigma) = \frac{\delta X_s^\mu}{\delta \ell_s} = \frac{1}{b_s} \frac{\partial X_s^\mu}{\partial \sigma^0}, \quad b_s = \sqrt{-\frac{\partial X_s^\mu}{\partial \sigma^0} g_{s\mu\nu} \frac{\partial X_s^\nu}{\partial \sigma^0}}, \quad g_{s\mu\nu} u_s^\mu u_s^\nu = -1 \quad (7)$$

Also, when the fluid is charged, the corresponding chemical potential can be written as

$$\mu_s(\sigma) = u_s^\mu(\sigma) A_{s\mu}(X_s(\sigma)) + \frac{1}{b_s} \partial_0 \varphi_s(\sigma), \quad (8)$$

where $\varphi_s(\sigma)$ is the hydrodynamical degree of freedom corresponding to conserved currents.



Symmetry principles

Symmetry is the only guiding principle of EFT construction.

Symmetries from SK contour:

- A Z_2 reflection symmetry:

$$I_{\text{hydro}}^* [h_1, h_2, \beta] = -I_{\text{hydro}} [h_2, h_1, \beta]$$

- Normalization condition:

$$I_{\text{hydro}} [h_1 = h_2, \beta] = 0$$

- Well-defined path integral:

$$\text{Im } I_{\text{hydro}} \geq 0$$

Symmetries from fluid manifold:

- Reparameterizations of spatial manifolds of σ^A :

$$\sigma^i \rightarrow \sigma'^i (\sigma^i), \quad \sigma^0 \rightarrow \sigma^0$$

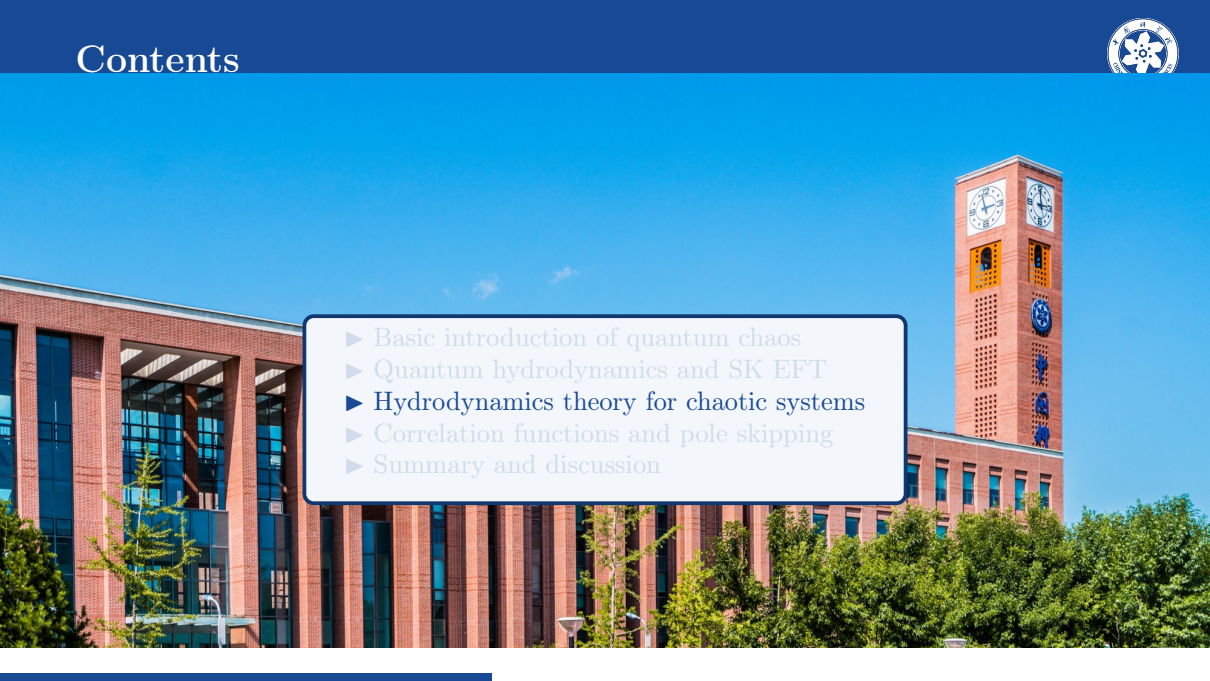
- Time-diffeomorphisms of σ^0 :

$$\sigma^0 \rightarrow \sigma'^0 = f(\sigma^0, \sigma^i), \quad \sigma^i \rightarrow \sigma^i$$

- Possible diagonal transformation for charged normal fluid.

Also, for thermal density matrix $\rho \sim e^{-\beta H}$, KMS conditions are also needed.



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System with only energy conservation

To simplify, this work consider a system with only energy conservation, in which case one set

$$X_{1,2}^i(\sigma^0, \sigma^i) = \sigma^i \implies \sigma^i = x^i \quad (9)$$

In this particular case, the remaining dynamical variables are then $X_{1,2}^0(\sigma^0, x^i)$ and inverse temperature $\beta(\sigma^0, x^i)$.

The equation of motion of $X_{1,2}^0$ equivalent to energy conservations. For notational simplicity, superscripts 0 will be dropped. In this case, the following external metric components can be written as

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = -e^2 (dt - w_i dx^i)^2 + (dx^i)^2, \quad g_{00} = -e^2, \quad g_{0i} = e^2 w_i \quad (10)$$

with induced metric in the fluid spacetime

$$ds^2 = h_{AB} d\sigma^A d\sigma^B = -E^2 (d\sigma - v_i dx^i)^2 + (dx^i)^2$$

$$E = e \partial_\sigma X, \quad v_i = \frac{1}{\partial_\sigma X} (w_i - \partial_i X) \quad (11)$$



Energy-momentum tensor and effective action

The stress tensor of the full system can be computed using the variation of I_{EFT} :

$$\mathcal{E} = -\hat{T}_0^0 = -\frac{1}{\sqrt{-g}} \left(e \frac{\delta I_{\text{EFT}}}{\delta e} - w_i \frac{\delta I_{\text{EFT}}}{\delta w_i} \right), \quad \mathcal{J}^i = -\hat{T}_0^i = \frac{1}{\sqrt{-g}} \frac{\delta I_{\text{EFT}}}{\delta w_i}, \quad (12)$$

with $\mathcal{E}$ energy density and $\mathcal{J}^i$ energy flux.

All the formalism discussed before is based on fluid manifold. In some cases, **writing the action directly in physical spacetime is helpful.**

Introducing $X = \frac{1}{2}(X_1 + X_2)$ and $X_a = X_1 - X_2$, where X can be interpreted as describing physical motions while X_a quantum statistical noises. To write the action in physical spacetime one identify $X = t$ and then invert $X(\sigma, x^i)$ to obtain $\sigma(t, x^i)$. Thus, in physical spacetime the most general Lagrangian can be written as

$$\mathcal{L}_{\text{EFT}} [\sigma, X_a] = -H \partial_t X_a - G_i \partial_i X_a + \frac{i}{2} M_1 (\partial_t X_a)^2 + \frac{i}{2} M_2 (\partial_i X_a)^2 + O(a^3). \quad (13)$$

Further comments on effective action $\mathcal{L}_{\text{EFT}}$



All the expansion coefficients in the $\mathcal{L}_{\text{EFT}}$ ($H, G_i, \dots$) are the functional of field $\sigma(t, x^i)$.

The dependence on σ of H, G_i is only through the local inverse temperature β

$$\beta = \frac{\beta_0}{\partial_t \sigma} \quad (14)$$

and its derivative, e.g.

$$H = H(\beta, \partial_t \beta, \partial_i^2 \beta, \dots) \quad (15)$$

The quantities H and G_i are respectively the dynamical part of the energy density and energy flux

$$\mathcal{E}_r = H + \dots, \quad \mathcal{J}_r^i = G_i + \dots \quad (16)$$

The equations of motions that follow from $\mathcal{L}_{\text{EFT}}$ then reduce to energy conservation

$$\partial_t H + \partial_i G_i = 0, \quad X_a = 0 \quad (17)$$



Near-equilibrium expansion

The equilibrium configuration is always the solution of energy conservation

$$\sigma = t, \quad \beta = \beta_0, \quad X_a = 0 \quad (18)$$

Expanding around such configuration one has

$$\sigma = t + \epsilon(t, x^i), \quad \beta = \beta_0 + \delta\beta, \quad \delta\beta = \beta_0(1 - \partial_t \epsilon), \quad X_a = -\epsilon_a(t, x^i) \quad (19)$$

Thus, at linear order in $\delta\beta$, one can write H and G_i as

$$H = f_1 \delta\beta = -\beta_0 f_1 \partial_t \epsilon, \quad G_i = h_1 \partial_i \delta\beta = -\beta_0 h_1 \partial_i \partial_t \epsilon \quad (20)$$

where f_1 and h_1 are differential operators. By doing so, the quadratic action can be written as

$$\mathcal{L}_{\text{hydro}} = \epsilon_a K \epsilon - \frac{i}{2} \epsilon_a M \epsilon_a, \quad K = \beta_0 (f_1 \partial_t + h_1 \partial_i^2) \partial_t, \quad M = M_1 \partial_t^2 + M_2 \partial_i^2 \quad (21)$$

and EOM becomes

$$(f_1 \partial_t + h_1 \partial_i^2) \partial_t \epsilon = 0, \quad \epsilon_a = 0 \quad (22)$$



Shift symmetry in 0+1-dimension

Key proposal in this work: imposing certain additional symmetry on the action gives rise to exponentially growing behavior of hydrodynamic variable $\sigma(t, x^i)$!

First consider a quantum mechanical system (0+1-dimension). Introducing an auxiliary variable $u(\sigma(t))$ through (the λ here is just Lyapunov exponent)

$$u = e^{-\lambda\sigma}, \quad \sigma = -\frac{1}{\lambda} \log u \quad (23)$$

The shift symmetry can thus be expressed as the invariance of $\mathcal{L}_{\text{EFT}}$ under

$$u \rightarrow u + a \quad (24)$$

for arbitrary constant a . At the linearized level (perturbation near equilibrium solution), this symmetry corresponds to the following solution

$$\sigma = t - \frac{a}{\lambda} e^{\lambda t} + \cdots, \quad (25)$$

which one wish to propose **as the origin of the chaotic behavior.**



Structure of $\mathcal{L}_{\text{EFT}}$ with shift symmetry

To characterize hydrodynamical theories with shift symmetry, note that for H to be invariant under such symmetry, H should depend on $u(\sigma)$ only through derivatives, i.e.

$$H = H(\partial_t u, \partial_t^2 u, \dots) \quad (26)$$

Accordingly, one can expand H in derivatives (primes denote t derivatives)

$$H = a_1 \frac{u''}{u'} + \frac{a_2}{2} \frac{u''^2}{u'^2} + a_3 \left(\frac{u''}{u'} \right)' + \dots \Rightarrow \quad (27)$$

$$H = \left(-\frac{a_1 \lambda \beta_0}{\beta} + \frac{\lambda^2 \beta_0^2 a_2}{2\beta^2} \right) + \left(-\frac{a_1}{\beta} + \lambda \beta_0 \frac{a_3 + a_2}{\beta^2} \right) \beta' + \frac{\frac{1}{2} a_2 + a_3}{\beta^2} \beta'^2 - \frac{a_3 \beta''}{\beta} \quad (28)$$

Setting $a_1 = 0$ and $a_3 = -a_2$, then H is related to Schwarzian (AdS₂/SYK!)

$$H = -a_2 \text{Sch}(u, t) = a_2 \left(\frac{1}{2} \frac{u''^2}{u'^2} - \left(\frac{u''}{u'} \right)' \right) = a_2 \left(\frac{\lambda^2 \beta_0^2}{2\beta^2} - \frac{\beta'^2}{2\beta^2} + \frac{\beta''}{\beta} \right) \quad (29)$$

General shift symmetry



More generally, the shift symmetry can be written as

$$u(t, x^i) \rightarrow u(t, x^i) + f(t, x^i) \quad (30)$$

for certain class of functions $f(t, x^i)$. Suppose f satisfies a general differential equation

$$\partial_t f = \kappa(\partial_i) f, \quad (31)$$

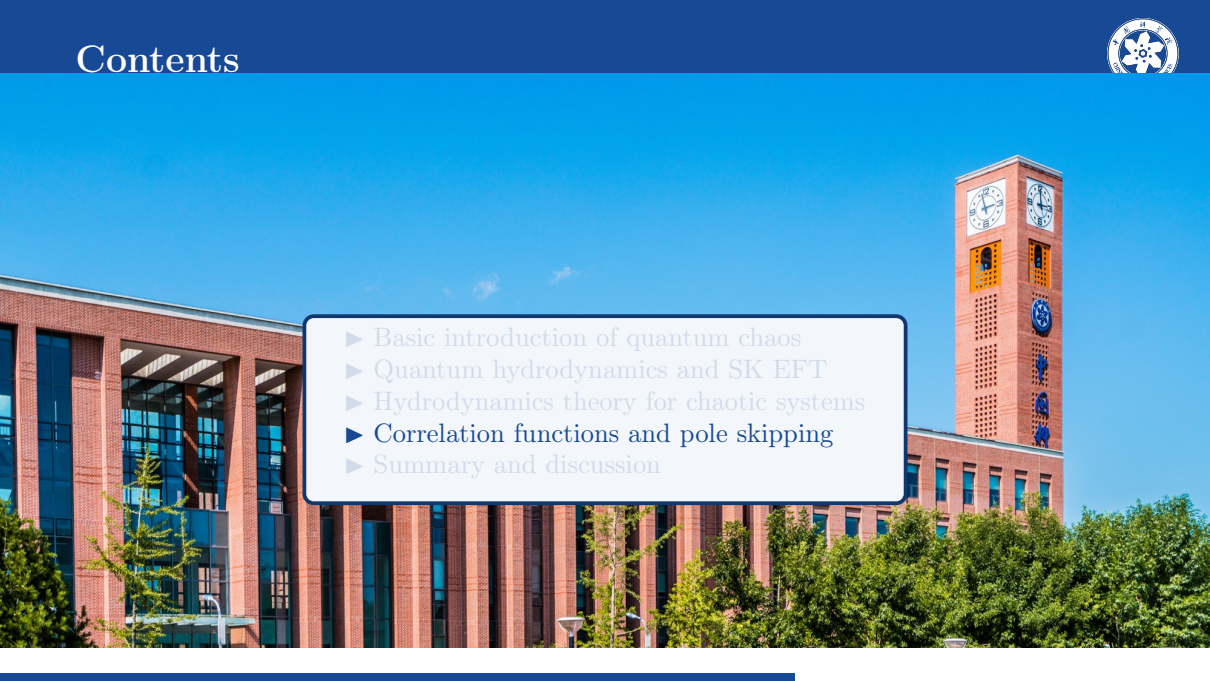
where differential operator $\kappa(\partial_i)$ built from spatial derivatives (constant solution is always allowed). At linearized level of perturbation around equilibrium state,

$$\sigma(t, x^i) = t - \frac{f}{\lambda} e^{\lambda t} \implies \partial_t \epsilon = -f e^{\lambda t} - \frac{1}{\lambda} \kappa(\partial_i) f e^{\lambda t} \equiv \tilde{\lambda}(\partial_i) \epsilon \quad (32)$$

is the solution of EOM, which exhibits the desired exponential growth.

In general, different chaotic systems can have different $\kappa(\partial_i)$ and thus $\lambda(\partial_i)$.



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Near-equilibrium two-point functions in chaos EFT



The near-equilibrium two-point functions of the hydrodynamical mode σ in the chaos EFT will clarify the implications of the various shift symmetries. The definitions are as follows

$$G_R(x) = i \langle \epsilon(x) \epsilon_a(0) \rangle \quad G_A(x) = i \langle \epsilon_a(x) \epsilon(0) \rangle = G_R(-x), \quad G_S(x) = \langle \epsilon(x) \epsilon(0) \rangle \quad (33)$$

In particular, at finite temperature, the fluctuation-dissipation relations ($\Longleftrightarrow$ KMS conditions) should be satisfied

$$G_S(x) = -\frac{i}{2} \coth \frac{i\beta_0 \partial_t}{2} (G_R(x) - G_A(x)) \quad (34)$$

The shortcut of calculation of the retarded Green's function is just inverting the differential operator $K = \beta_0 (\partial_t f_1 + h_1 \partial_i^2) \partial_t$ in the $\epsilon_a K \epsilon$ term in effective action, with appropriate boundary condition:

$$G_R(x) = - \int_C \frac{d^d k}{(2\pi)^d} \frac{e^{-i\omega t + i k_i x^i}}{K} \quad (35)$$



General structure of G_R in chaos EFT

Assuming the system has a general shift symmetry, then f_1 and h_1 in operator K can be written in a form

$$f_1 = \left(\partial_t - \tilde{\lambda}(\partial_i^2) \right) a(\partial_t, \partial_i^2), \quad h_1 = \left(\partial_t - \tilde{\lambda}(\partial_i^2) \right) b(\partial_t, \partial_i^2) \quad (36)$$

For convenience, one will denote

$$\mathcal{D}(\partial_t, \partial_i^2) \equiv -\frac{b(\partial_t, \partial_i^2)}{a(\partial_t, \partial_i^2)} = D_E + O(\partial_t, \partial_i^2) \quad (37)$$

The leading order term D_E can be interpreted as energy diffusion constant. Thus, in momentum space, the operator K can be written as

$$K = i\beta_0\omega a(\omega, k)(\omega - i\tilde{\lambda}(k))(\omega + i\mathcal{D}(\omega, k)k^2), \quad k^2 = k_i^2 \quad (38)$$

Pole structure: upper half ω -plane $\implies \omega = i\tilde{\lambda}$, lower half ω -plane $\implies \omega + i\mathcal{D}(\omega, k)k^2 = 0$.



Examples of different situations

- (0+1)-dimensional systems $\implies$ the exponential term comes from the pole at $\omega = i\lambda$:

$$K = i\beta_0\omega^2 a(\omega)(\omega - i\lambda) \implies G_R(t) = \theta(t) (ce^{\lambda t} + \dots), \quad c = -\frac{1}{\beta_0 a(i\lambda)\lambda^2} \quad (39)$$

- General $\tilde{\lambda}(k) \implies$ performing the ω integral at pole $\omega = i\tilde{\lambda}$:

$$G_R = -\theta(t) \frac{1}{\beta_0} \int \frac{d^{d-1}k}{(2\pi)^{d-1}} \frac{e^{\tilde{\lambda}(k)t + ik_i x^i}}{\tilde{\lambda}(\tilde{\lambda} + k^2 \mathcal{D}(i\tilde{\lambda}, k^2)) a(i\tilde{\lambda}, k^2)} \quad (40)$$

If $\tilde{\lambda} + k^2 \mathcal{D}(i\tilde{\lambda}, k^2) = 0$ has a solution at some $-k_C^2 < 0$, then define

$$\bar{\lambda} = \tilde{\lambda}(-k_C^2), \quad v_B = \bar{\lambda}/k_C \implies G_R = c\theta(t)e^{\bar{\lambda}\left(t - \frac{|\vec{x}|}{v_B}\right)} + \dots \quad (41)$$

Butterfly velocity v_B can be extracted directly from the retarded Green's function!



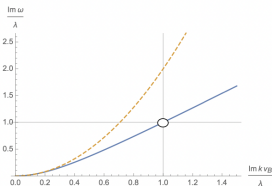
Phenomenon of pole skipping

Pole skipping is the generic characteristic phenomenon in quantum chaos system. $\implies$ "fingerprint" of the quantum scrambling in frequency space!

The physical definition of pole skipping:

$$G_R(\omega, k) = \frac{\mathcal{A}(\omega, k)}{\mathcal{B}(\omega, k)} \implies \mathcal{A}(i\lambda, ik_C) = 0 \quad \text{and} \quad \mathcal{B}(i\lambda, ik_C) = 0 \quad (42)$$

hold at some critical imaginary momentum $k = ik_C$.



At that particular momentum (imaginary), the pole is in fact skipped (the black circle on the blue line).

The importance of pole skipping is that one can directly calculate Lyapunov exponent λ and butterfly velocity v_B , without the need to calculate OTOCs. cf. JHEP 05 (2017) 125



Pole skipping from chaos EFT

The hydrodynamic origin of chaos predicts the phenomenon of pole-skipping in the response functions of the energy density and flux.

In chaos EFT, two-point functions of the energy density $\mathcal{E}$ and flux $\mathcal{J}^i$ can be expressed by two-point functions of σ field:

$$\begin{aligned} \mathcal{G}_R^{\mathcal{E}\mathcal{E}} &= \beta_0^2 f_1 h_1 \partial_t \partial_i^2 G_R, & \mathcal{G}_R^{\mathcal{E}\mathcal{J}^i} &= -\beta_0^2 f_1 h_1 \partial_t^2 \partial_i G_R \\ \mathcal{G}_R^{\mathcal{J}^i \mathcal{E}} &= -\beta_0^2 h_1 f_1 \partial_t^2 \partial_i G_R, & \mathcal{G}_R^{\mathcal{J}^i \mathcal{J}^j} &= -\beta_0^2 h_1^2 \partial_t^2 \partial_j \partial_i G_R \end{aligned} \quad (43)$$

By using the knowledge of $G_R(\omega, k)$, one find (**No exponential behavior any more!**)

$$\mathcal{G}_R^{\mathcal{E}\mathcal{E}}(\omega, k) = \beta_0 \frac{(\omega - i\tilde{\lambda}(k))k^2 b(\omega, k)}{\omega + i\mathcal{D}(\omega, k)k^2} \quad (44)$$

Key lessons from $\mathcal{G}_R^{\mathcal{E}\mathcal{E}}$: pole skipping $\iff \omega - i\tilde{\lambda}(k) = 0$, $\omega + i\mathcal{D}(\omega, k)k^2 = 0 \implies \lambda$ and v_B



General structure of OTOCs in large $\mathcal{N}$ limit

One can expand the generic few-body operator $V(t)$ in power series of $\epsilon(t) = \sigma(t) - t$ as

$$V(t) = \hat{V}(t) + L^{(1)}[\hat{V}\epsilon](t) + O(\epsilon^2) \quad \text{with} \quad L^{(1)}[\hat{V}\epsilon] = \sum_{n,m=0}^{\infty} c_{nm} \partial_t^n \hat{V} \partial_t^m \epsilon \quad (45)$$

Higher terms in ϵ will be neglected as they give subleading corrections in $1/\mathcal{N}$.

By using such expansion, **a general 4-point function can be factorized into 2-point functions of ϵ in the leading order of large $\mathcal{N}$ limit:**

$$G_{i_1 i_2 i_3 i_4} \sim \langle \mathcal{P} V_{i_1}(t_1) V_{i_2}(t_2) W_{i_3}(t_3) W_{i_4}(t_4) \rangle \Rightarrow 1 + \langle B_V^{i_1 i_2}(t_1, t_2) B_W^{i_3 i_4}(t_3, t_4) \rangle \quad (46)$$

With effective vertex for $\hat{V}\hat{V}$ coupling to ϵ B_V defined as (B_W is similar with $V \rightarrow W$)

$$B_V^{i_1 i_2} = \frac{1}{g_V} \left(L_{t_1}^{(1)} [g_V(t_{12}) \epsilon_{i_1}(t_1)] + L_{t_2}^{(1)} [g_V(t_{12}) \epsilon_{i_2}(t_2)] \right), \quad g_V(t_{12}) \equiv \langle \mathcal{P} \hat{V}_{i_1}(t_1) \hat{V}_{i_2}(t_2) \rangle \quad (47)$$



Shift symmetry for effective vertex

Require the effective vertex respect the shift symmetry, then it should be invariant under $\epsilon_i \rightarrow \epsilon_i + c e^{\lambda t}$ with c some constant. This implies that

$$L_{t_1}^{(1)} [g_V(t_{12}) e^{\lambda t_1}] + L_{t_2}^{(1)} [g_V(t_{12}) e^{\lambda t_2}] = 0 \quad (48)$$

After introducing

$$F_{\text{even}}(\lambda, t) = \sum_{n \text{ even}} f_n(\lambda) \partial_t^n g_V(t), \quad F_{\text{odd}}(\lambda, t) = \sum_{n \text{ odd}} f_n(\lambda) \partial_t^n g_V(t) \quad (49)$$

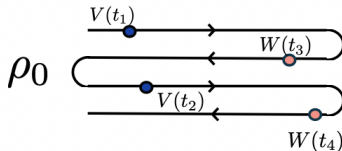
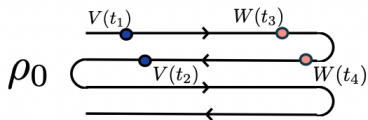
where $f_n = \sum_m c_{nm} \lambda^m$, then shift symmetry implies that

$$\frac{F_{\text{even}}(\lambda, t)}{F_{\text{odd}}(\lambda, t)} = -\tanh \frac{\lambda t}{2} \quad (50)$$

Key point: shift symmetry $\implies$ the distinct behaviors of TOCs and OTOCs.

TOCs and OTOCs

Considering the following case (Left: TOCs, Right: OTOCs):



Contribution of the exponential mode:

$$G_4 - 1 \sim B_W [\tilde{\epsilon}(t_3), \tilde{\epsilon}(t_4)]$$

$$\tilde{\epsilon}(t_3) = c_{12} e^{\lambda t_3}, \quad \tilde{\epsilon}(t_4) = c_{12} e^{\lambda t_4}$$

$$c_{12} = c_+ L_{t_1}^{(1)} [g_V e^{-\lambda t_1}] + c_- L_{t_2}^{(1)} [g_V e^{-\lambda t_2}] = 0$$

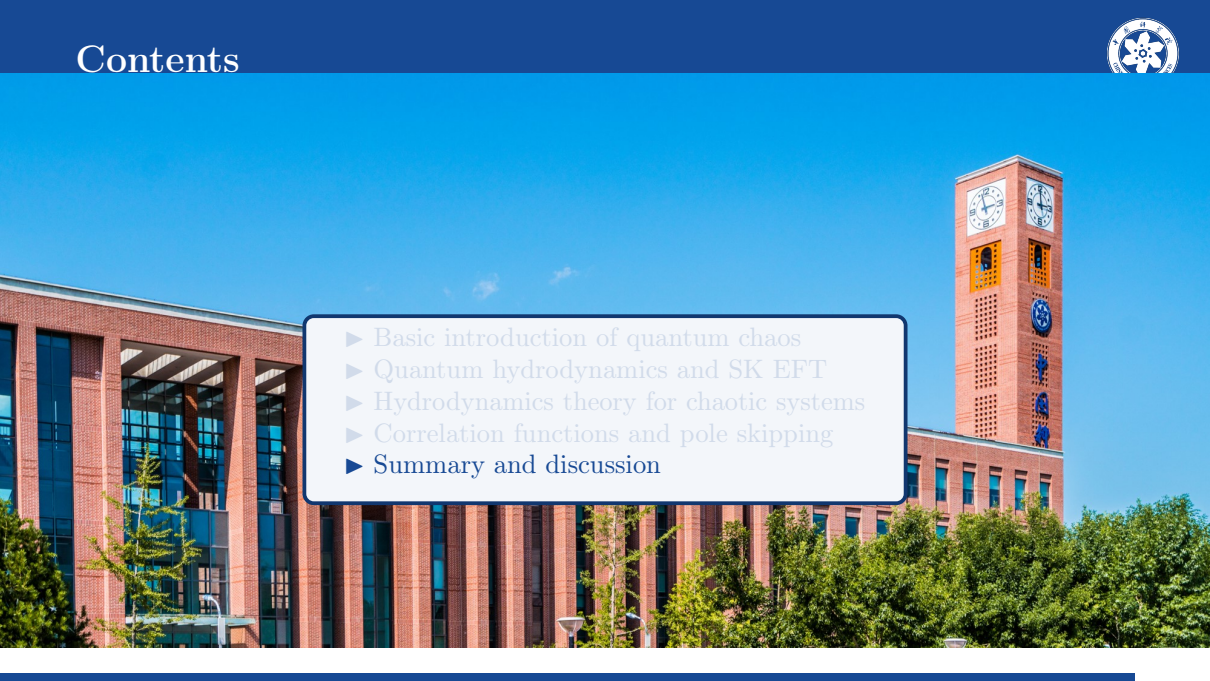
Contribution of the exponential mode:

$$H_4 - 1 \sim L_{t_2}^{(1)} L_{t_4}^{(1)} [g_V(t_{12}) g_W(t_{34}) e^{\lambda t_{42}}]$$

$$H_4 - 1 \sim e^{\lambda(t_3+t_4-t_1-t_2)/2} + \dots$$

Exponential growth in OTOCs!



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- The background of the slide is a photograph of a large, modern brick building with a prominent clock tower. The building has large glass windows and is surrounded by green trees. The sky is clear and blue.
- ▶ Basic introduction of quantum chaos
 - ▶ Quantum hydrodynamics and SK EFT
 - ▶ Hydrodynamics theory for chaotic systems
 - ▶ Correlation functions and pole skipping
 - ▶ Summary and discussion

- Quantum chaos can be effectively described by using hydrodynamic EFT and such EFT provides a unified formalism without involving any microscopic details.
- With a shift symmetry, the hydrodynamic theory has a mode which grows exponentially in time, and exhibits ballistic spreading with a butterfly velocity v_B .
- The shift symmetry prevents correlation functions of the energy density and flux, and TOCs of generic operators from having such exponential growth behavior.
- Correlation functions of energy-density and energy flux can be calculated by correlation functions of hydrodynamical mode.
- Pole skipping phenomenon in the energy density and flux two point functions can be used to extract Lyapunov exponent λ and butterfly velocity v_B directly without involving OTOCs.

- In this paper, only system with energy conservation is considered $\implies$ Can the discussion be generalized to systems with full energy-momentum conservation, with other conserved quantities, or with additional light modes?
- The precise scope of the applicability of such hydrodynamic description?
- Many systems considered in the previous studies are related to the maximally chaotic system. Are maximally chaotic systems distinguished from general chaotic systems?
- How can some possible stringy effects, which may lead the Lyapunov exponent to deviate from the maximal value, be incorporated in such hydrodynamical EFT formalism?
- Deeper understanding of shift symmetry is needed, especially its physical origin and nature. How can this shift symmetry be identified on the gravity side in holographic systems?