

Recent studies on generalized parton distributions and spin-1 structure functions

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November 6, 2025

Generalized Parton Distributions (GPDs)

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may skip

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BEPC, Tau-charm, KEKB

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HIAF, J-PARC, ...

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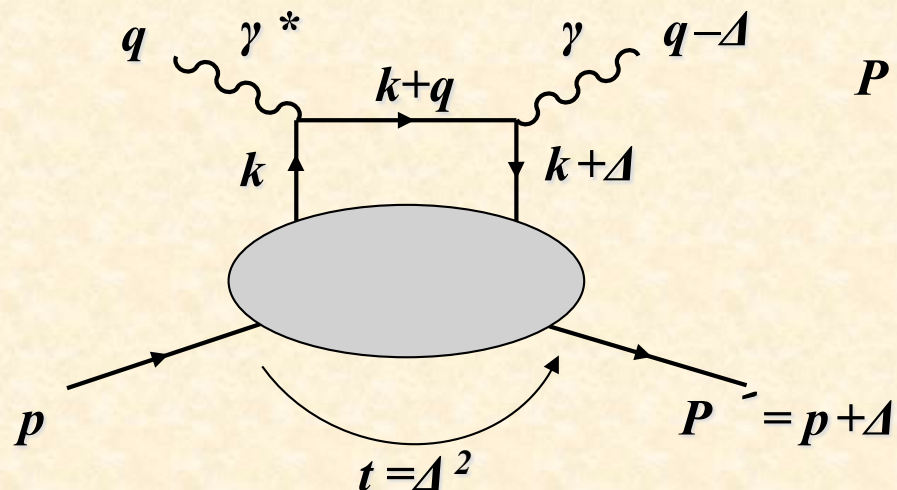
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may skip

Generalized Parton Distributions (GPDs)



$$P = \frac{p + p'}{2}, \quad \Delta = p' - p$$

Bjorken variable $x = \frac{Q^2}{2p \cdot q}$

Momentum transfer squared $t = \Delta^2$

Skewdness parameter $\xi = \frac{p^+ - p'^+}{p^+ + p'^+} = -\frac{\Delta^+}{2P^+}$

GPDs are defined as correlation of off-forward matrix:

$$\int \frac{dz^-}{4\pi} e^{ixP^+z^-} \langle p' | \bar{\psi}(-z/2) \gamma^+ \psi(z/2) | p \rangle \Big|_{z^+=0, \vec{z}_\perp=0} = \frac{1}{2P^+} \left[H(x, \xi, t) \bar{u}(p') \gamma^+ u(p) + E(x, \xi, t) \bar{u}(p') \frac{i\sigma^{+\alpha} \Delta_\alpha}{2M} u(p) \right]$$

$$\int \frac{dz^-}{4\pi} e^{ixP^+z^-} \langle p' | \bar{\psi}(-z/2) \gamma^+ \gamma_5 \psi(z/2) | p \rangle \Big|_{z^+=0, \vec{z}_\perp=0} = \frac{1}{2P^+} \left[\tilde{H}(x, \xi, t) \bar{u}(p') \gamma^+ \gamma_5 u(p) + \tilde{E}(x, \xi, t) \bar{u}(p') \frac{\gamma_5 \Delta^+}{2M} u(p) \right]$$

Forward limit: PDFs $H(x, \xi, t) \Big|_{\xi=t=0} = f(x), \quad \tilde{H}(x, \xi, t) \Big|_{\xi=t=0} = \Delta f(x),$

First moments: Form factors

Dirac and Pauli form factors F_1, F_2

$$\int_{-1}^1 dx H(x, \xi, t) = F_1(t), \quad \int_{-1}^1 dx E(x, \xi, t) = F_2(t)$$

Axial and Pseudoscalar form factors G_A, G_P

$$\int_{-1}^1 dx \tilde{H}(x, \xi, t) = g_A(t), \quad \int_{-1}^1 dx \tilde{E}(x, \xi, t) = g_P(t)$$

Second moments: Angular momenta

Sum rule: $J_q = \frac{1}{2} \int_{-1}^1 dx x [H_q(x, \xi, t=0) + E_q(x, \xi, t=0)], \quad J_q = \frac{1}{2} \Delta q + L_q$

$\Rightarrow$ probe L_q , key quantity to solve the spin puzzle!

Origin of hadron masses

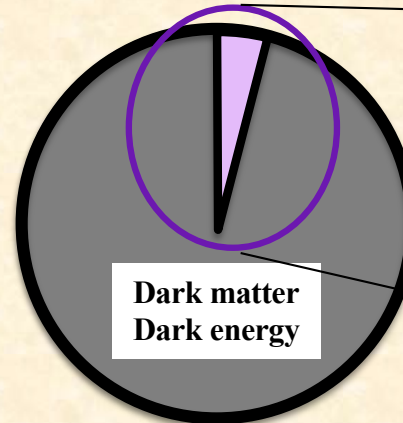
Mass and spin of the nucleon are two of fundamental quantities in physics.

Nucleon mass: $M = \left\langle p \left| \int d^3x T^{00}(x) \right| p \right\rangle$

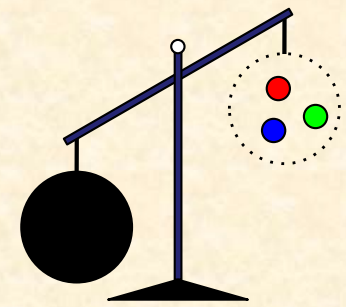
Energy-momentum tensor:

$$T^{\mu\nu}(x) = \frac{1}{2} \bar{q}(x) i \vec{D}^{(\mu} \gamma^{\nu)} q(x) + \frac{1}{4} g^{\mu\nu} F^2(x) - F^{\mu\alpha}(x) F^\nu{}_\alpha(x)$$

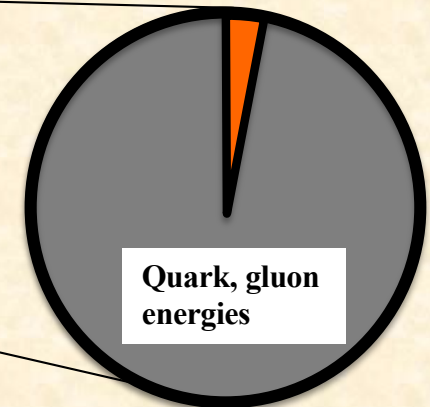
Ordinary matter
= Atoms $\simeq$ Nucleons



Dark matter



Quark mass

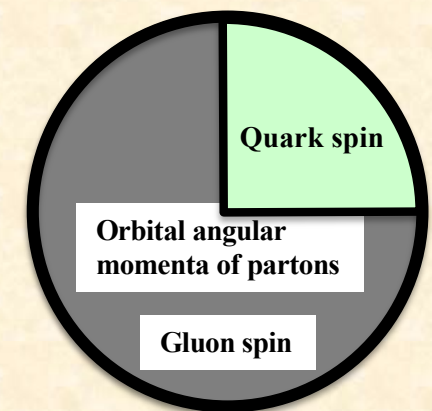


Origin of nucleon mass

Nucleon spin: $\frac{1}{2} = \left\langle p \left| J^3 \right| p \right\rangle$

3rd component of total angular momentum: $J^3 = \frac{1}{2} \epsilon^{3jk} \int d^3x M^{3jk}(x)$

Angular-momentum density: $M^{\alpha\mu\nu}(x) = T^{\alpha\nu}(x)x^\mu - T^{\alpha\mu}(x)x^\nu$

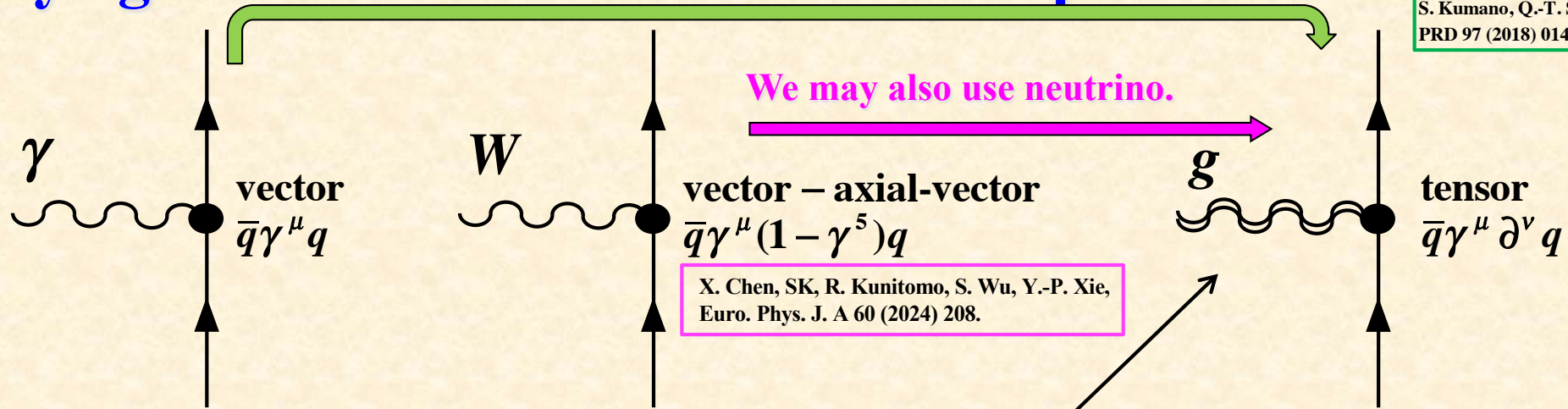


Origin of nucleon spin
("Dark spin")

Why “gravitational” interactions with quarks

We studied in 2017-2018.

S. Kumano, Q.-T. Song, O. Teryaev,
PRD 97 (2018) 014020.



X. Chen, SK, R. Kunitomo, S. Wu, Y.-P. Xie,
Euro. Phys. J. A 60 (2024) 208.

It is possible to probe gravitational sources
in the microscopic level without gravitons.

GPDs (Generalized Parton Distributions), GDAs (Generalized Distribution Amplitudes) = timelike GPDs

$$\int \frac{dz^-}{4\pi} e^{ixP^+z^-} \langle p' | \bar{q}(-z/2) \gamma^+ q(z/2) | p \rangle \Big|_{z^+=0, \vec{z}_\perp=0} = \frac{1}{2P^+} \left[H(x, \xi, t) \bar{u}(p') \gamma^+ u(p) + E(x, \xi, t) \bar{u}(p') \frac{i\sigma^{+\alpha} \Delta_\alpha}{2M} u(p) \right]$$

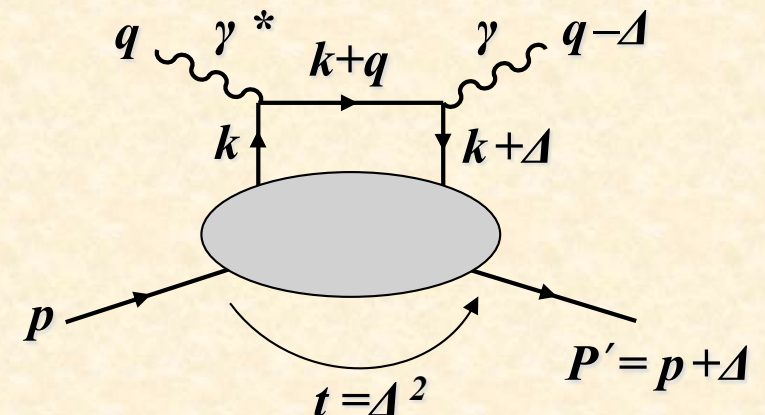
Non-local operator of GPDs/GDAs:

$$\begin{aligned} & (P^+)^n \int dx x^{n-1} \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \left[\bar{q}(-z/2) \gamma^+ q(z/2) \right] \Big|_{z^+=0, \vec{z}_\perp=0} \\ &= \left(i \frac{\partial}{\partial z^-} \right)^{n-1} \left[\bar{q}(-z/2) \gamma^+ q(z/2) \right] \Big|_{z=0} \\ &= \bar{q}(0) \gamma^+ \left(i \tilde{\partial}^+ \right)^{n-1} q(0) \end{aligned}$$

= energy-momentum tensor of a quark for $n = 2$
(electromagnetic for $n = 1$)

= source of gravity

Virtual Compton
or (timelike) two-photon process



Gravitational form factors and radii for pion

This is the first report on gravitational radii of hadrons from actual experimental measurements.

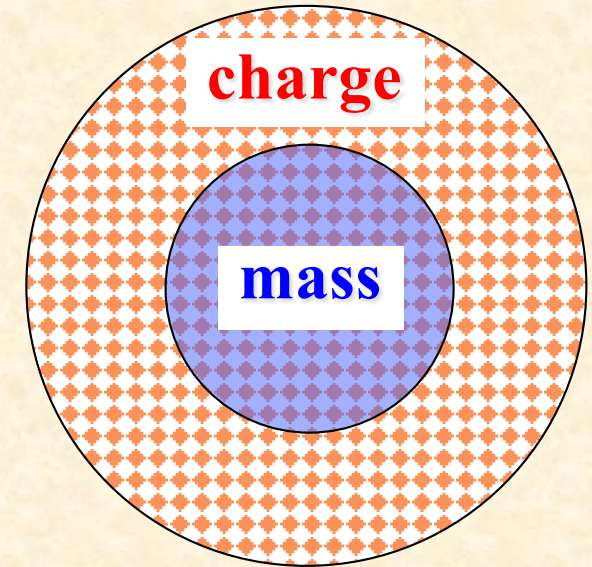
$$\sqrt{\langle r^2 \rangle_{\text{mass}}} = 0.32 \sim 0.39 \text{ fm}, \quad \sqrt{\langle r^2 \rangle_{\text{mech}}} = 0.82 \sim 0.88 \text{ fm}$$

$$\Leftrightarrow \sqrt{\langle r^2 \rangle_{\text{charge}}} = 0.672 \pm 0.008 \text{ fm}$$

SK, Q.-T. Song, O. Teryaev
PRD 97 (2018) 014020.

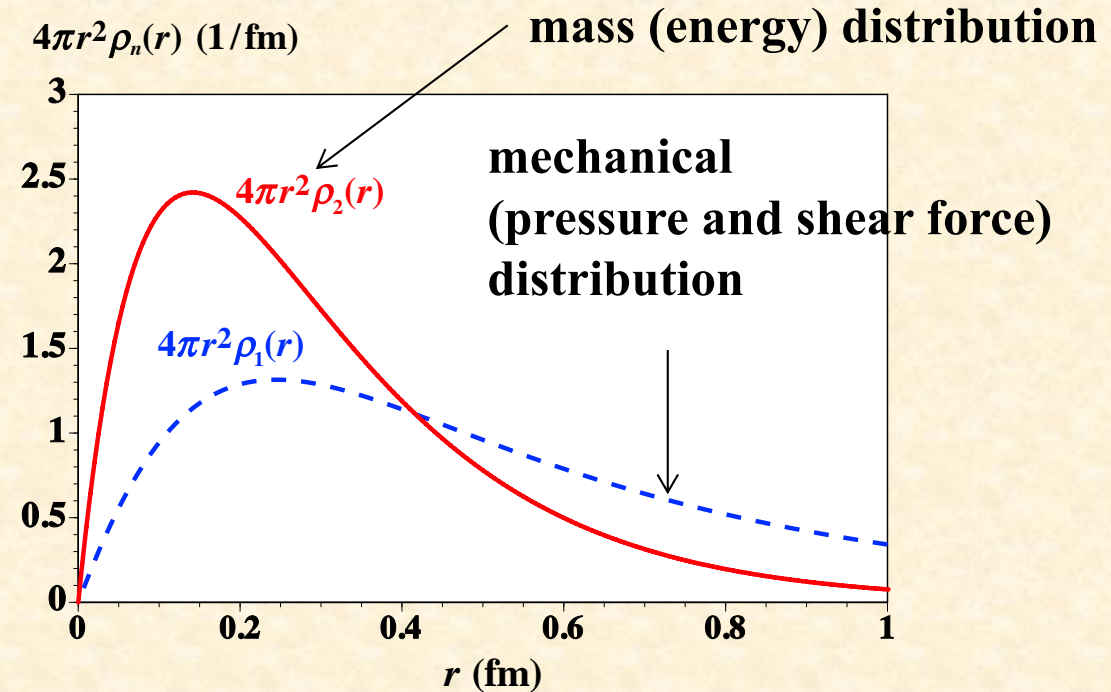
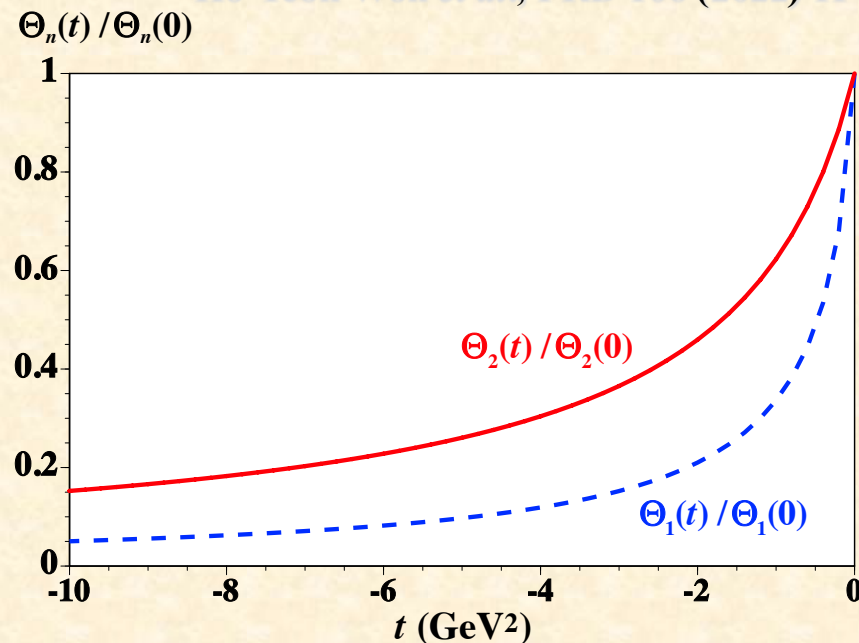
Related theoretical studies:

A. Freese and I. C. Cloet, PRC 100 (2019) 015201;
P. E. Shanahan and W. Detmold, PRD 99 (2019) 014511;
C. D. Roberts *et al.*, Prog. Part. Nucl. Phys. 120 (2021) 103883;
J.-L. Zhang *et al.* PLB 815 (2021) 136158;
June-Young Kim and Hyun-Chul Kim, PRD 104 (2021) 074019;
Ho-Yeon Won *et al.*, PRD 106 (2022) 114009.



Proton mass radius:

R. Wang, W. Kou, Y.-P. Xie, X. Chen,
PRD 103 (2021) L091501.



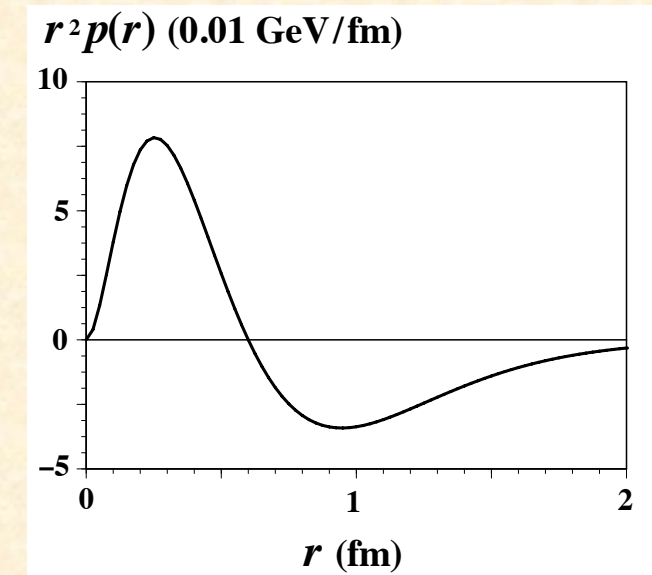
Nucleon pressure

$$\langle N(p') | T_q^{\mu\nu}(0) | N(p) \rangle = \bar{u}(p') \left[A \gamma^{(\mu} \bar{P}^{\nu)} + B \frac{\bar{P}^{(\mu} i \sigma^{\nu)\alpha} \Delta_\alpha}{2M} + D \frac{\Delta^\mu \Delta^\nu - g^{\mu\nu} \Delta^2}{M} + \bar{C} M g^{\mu\nu} \right] u(p)$$

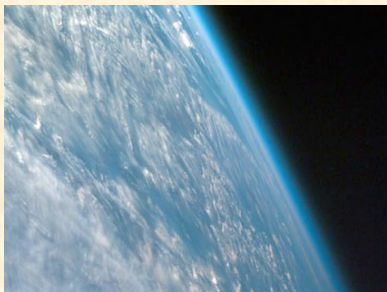
V. D. Burkert, L. Elouadrhiri, and F. X. Girod,
Nature **557** (2018) 396;

M. V. Polyakov and P. Schweitzer,
Int. J. Mod. Phys. A **33** (2018) 1830025;

C. Lorce, H. Moutarde, and A. P. Tranwinski,
Eur. Phys. J. C **79** (2019) 89.



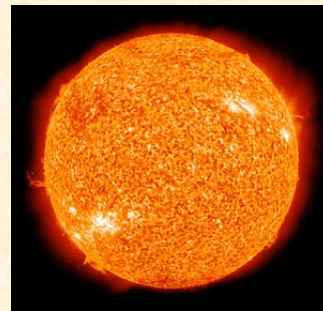
Highest pressure in nature 1 Pa (Pascal) = 1 N/m²



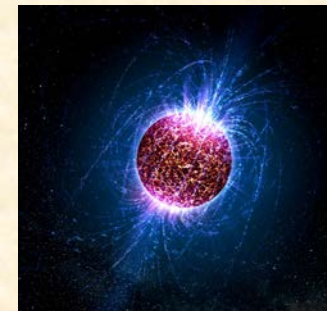
Earth atmosphere
10⁵ Pa = 1000 hPa



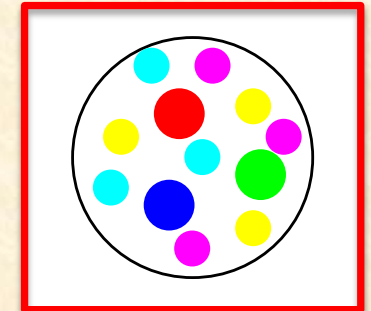
Center of earth
10¹¹ Pa = 100 GPa



Center of Sun
10¹⁶ Pa = 10 PPa



Neutron star
10³⁴ Pa



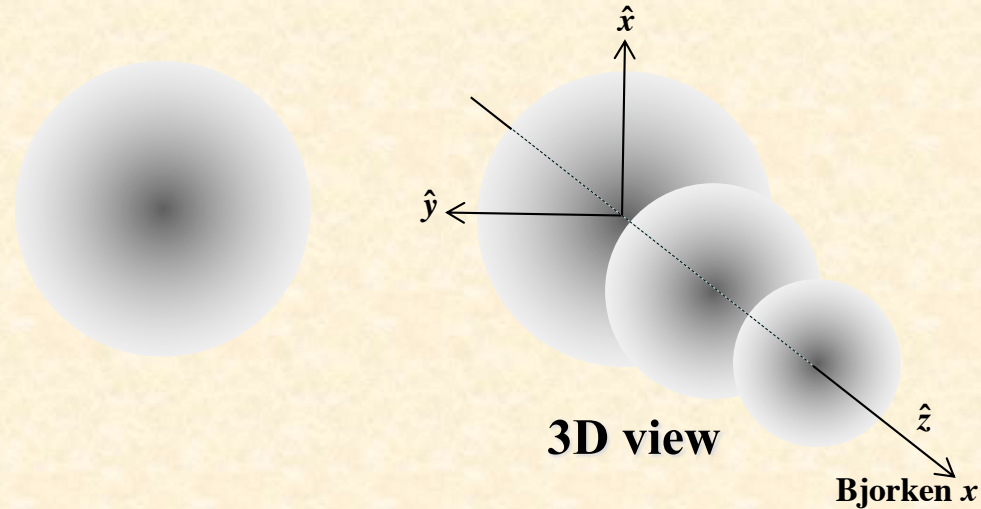
Hadron
10³⁵ Pa

Proton (hadrons) puzzle studies by hadron tomography

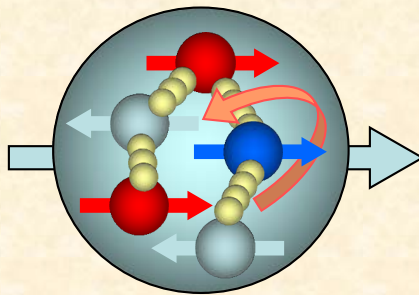
Hadron tomography



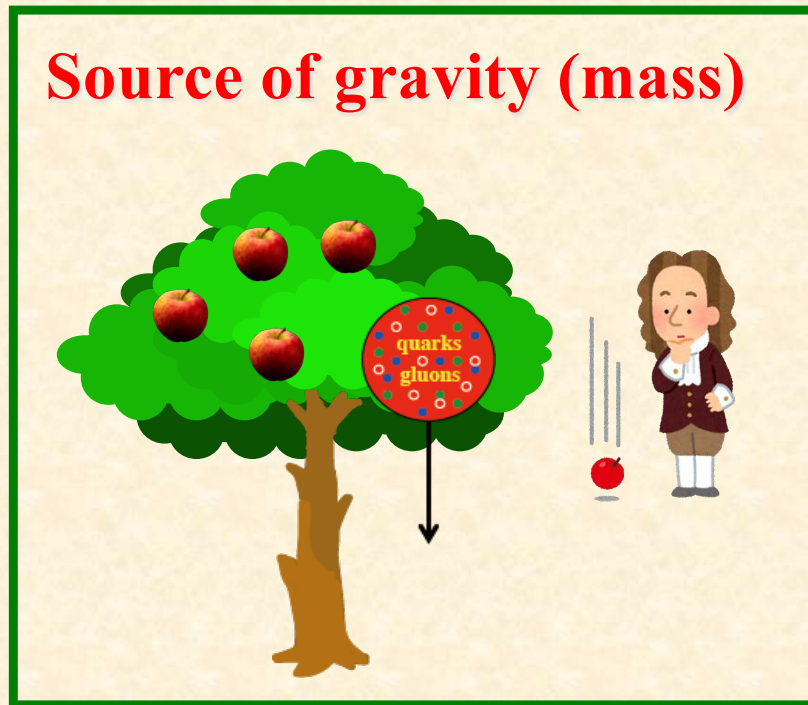
Proton radius puzzle



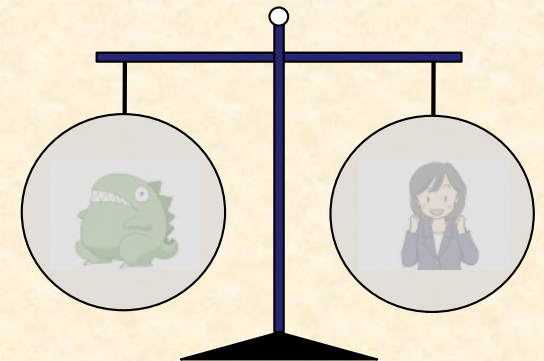
Origin of nucleon spin



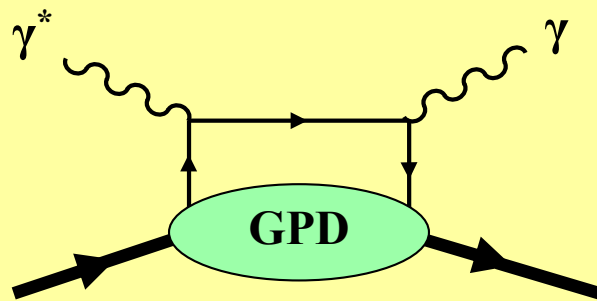
Source of gravity (mass)



Exotic hadrons



Charged-lepton scattering on spacelike GPDs

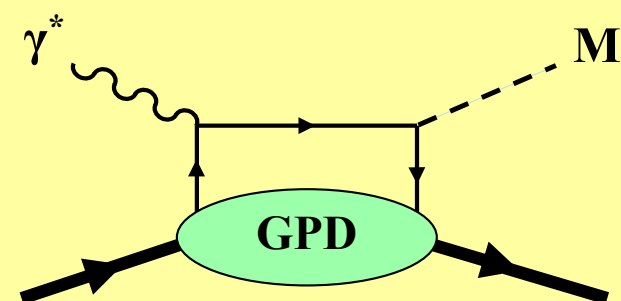
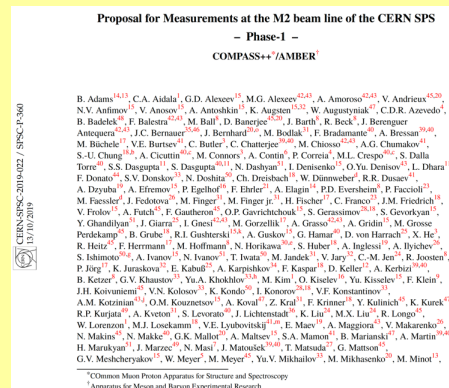


DVCS (Deeply Virtual Compton Scattering)

Jefferson Lab

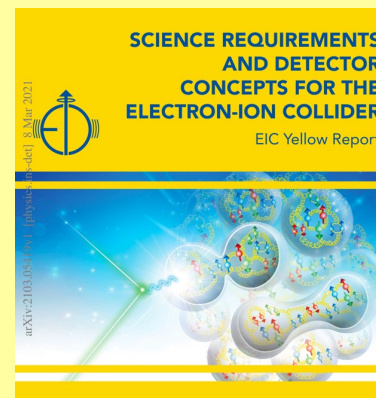


CERN-AMBER

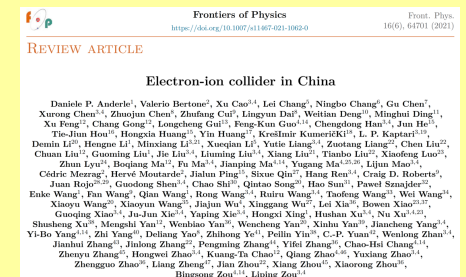


DVMP (Deeply Virtual Meson Production)

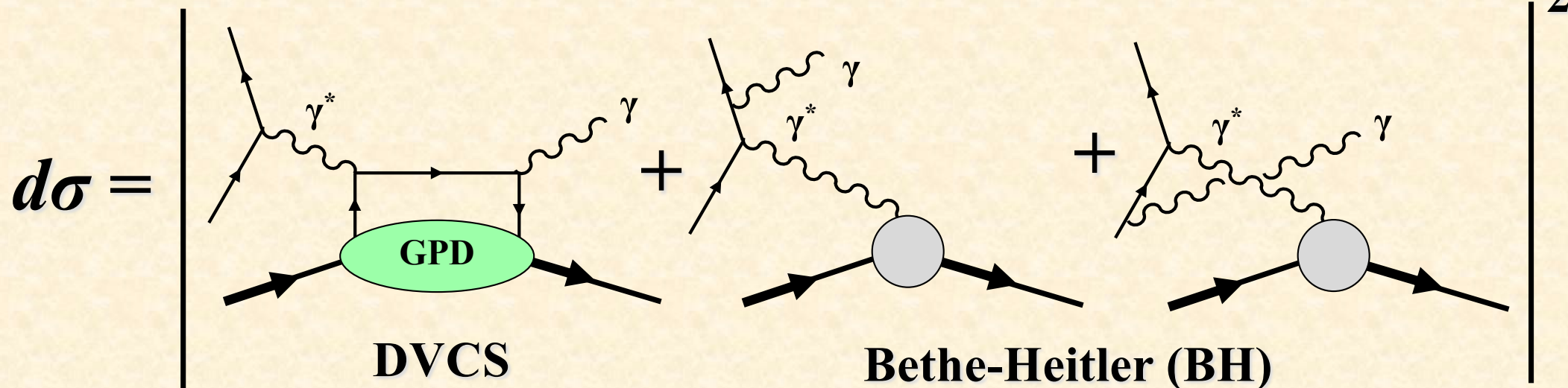
EIC-US



EicC



Deeply Virtual Compton Scattering (DVCS)



$$\frac{d\sigma(eN \rightarrow e'N'\gamma)}{dQ^2 dx dt d\phi} \propto |T_{DVCS} + T_{BH}|^2$$

e.g. Polarized beam: $d\sigma(e^\uparrow) - d\sigma(e^\downarrow) \propto T_{BH} * \text{Im}(T_{DVCS})$

$$\text{Re } \mathcal{H}_q = e_q^2 \mathcal{P} \int_0^1 dx \left[H^q(x, \xi, t) - H^q(-x, \xi, t) \right] \left(\frac{1}{\xi - x} + \frac{1}{\xi + x} \right)$$

$$\text{Im } \mathcal{H}_q = \pi e_q^2 \left[H^q(\xi, \xi, t) - H^q(-\xi, \xi, t) \right]$$

HERMES, JLab,
COMPASS/AMBER, EIC, EicC, ...

- Polarized beam, unpolarized target: $\text{Im}\{\mathcal{H}, \tilde{\mathcal{H}}, \mathcal{E}\}$
- Unpolarized beam, longitudinally-polarized target: $\text{Im}\{\mathcal{H}, \tilde{\mathcal{H}}\}$
- Polarized beam, longitudinally-polarized target: $\text{Re}\{\mathcal{H}, \tilde{\mathcal{H}}\}$
- Unpolarized beam, transversely-polarized target: $\text{Im}\{\mathcal{H}, \mathcal{E}\}$

Recent measurement at JLab

may skip

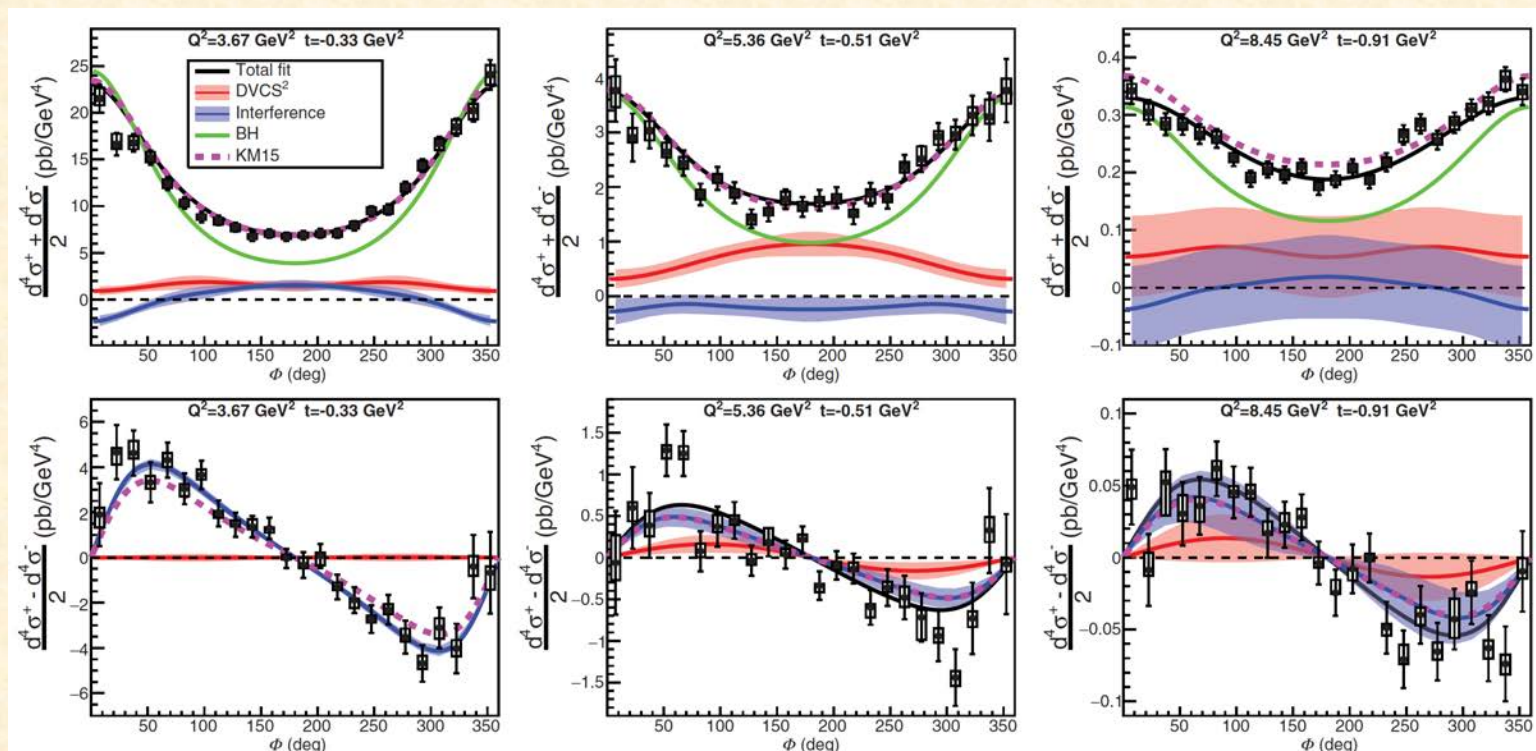
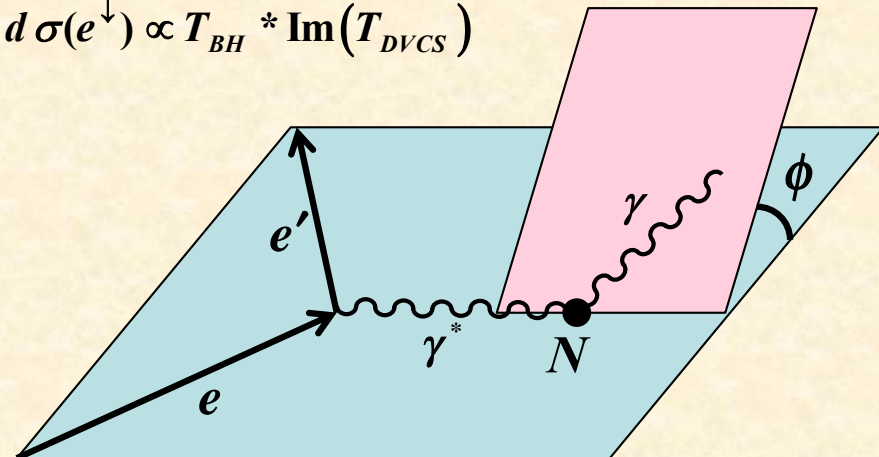
F. Georges et al., PRL 128 (2022) 252002.

$$\frac{d\sigma(eN \rightarrow e'N'\gamma)}{dQ^2 dx dt d\phi} \propto |T_{DVCS} + T_{BH}|^2, \text{ e.g. Polarized beam: } d\sigma(e^\uparrow) - d\sigma(e^\downarrow) \propto T_{BH} * \text{Im}(T_{DVCS})$$

$$\text{Re } \mathcal{H}_q = e_q^2 \mathcal{P} \int_0^1 dx \left[H^q(x, \xi, t) - H^q(-x, \xi, t) \right] \left(\frac{1}{\xi - x} + \frac{1}{\xi + x} \right)$$

$$\text{Im } \mathcal{H}_q = \pi e_q^2 \left[H^q(\xi, \xi, t) - H^q(-\xi, \xi, t) \right]$$

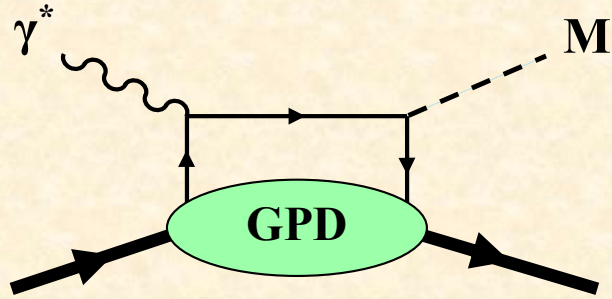
- Polarized beam, unpolarized target: $\text{Im}\{\mathcal{H}, \tilde{\mathcal{H}}, \mathcal{E}\}$



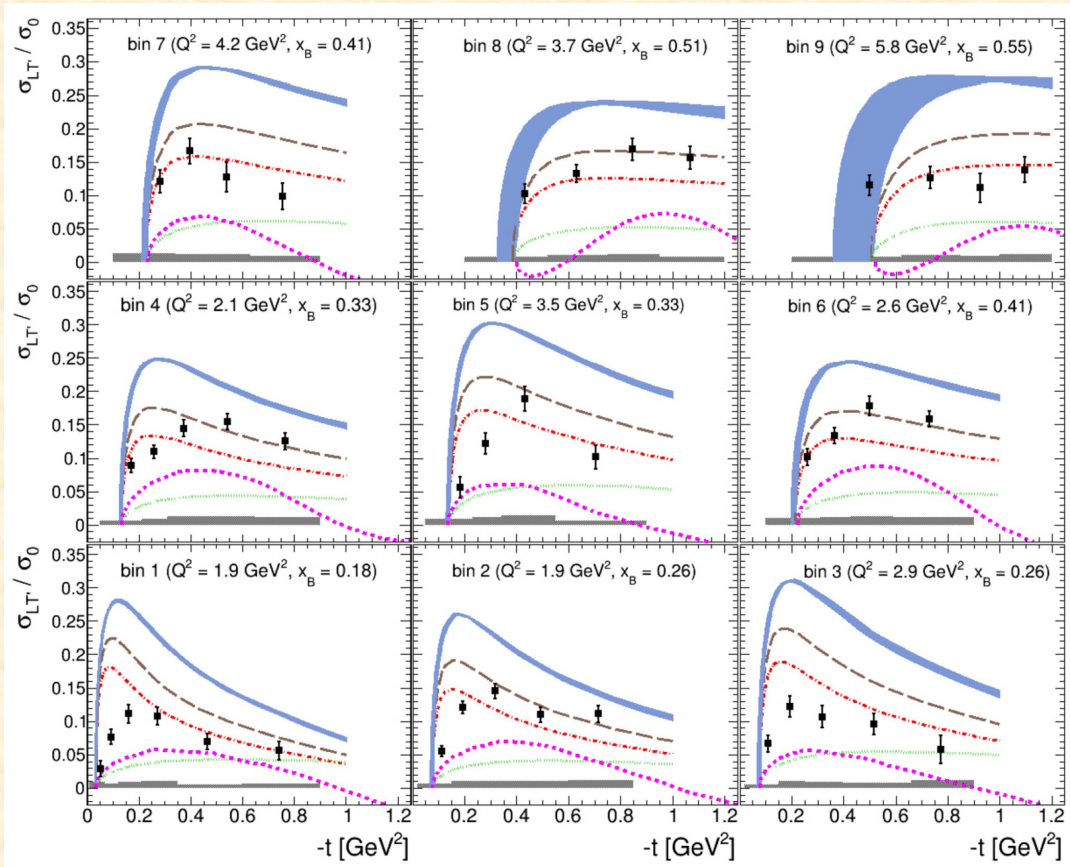
ϕ dependent cross sections

Deeply Virtual Meson Production (DVMP)

may skip



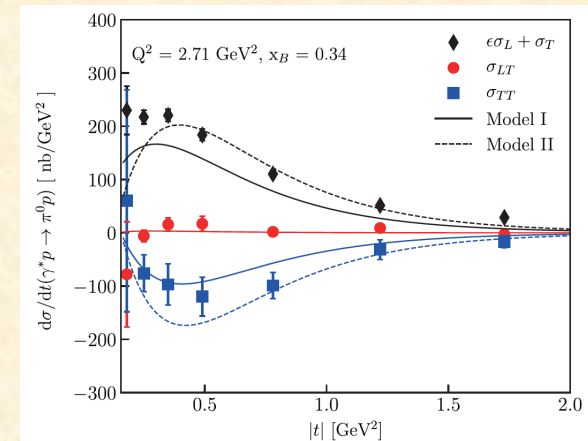
$$\frac{d\sigma(eN \rightarrow e'N'\gamma)}{dQ^2 dx dt d\phi} \propto \sigma_T + \varepsilon\sigma_L + \varepsilon\sigma_{TT} \cos(2\phi) + \sqrt{2\varepsilon(1+\varepsilon)}\sigma_{LT} \cos(\phi) + P_b \sqrt{2\varepsilon(1-\varepsilon)}\sigma_{LT'} \sin(\phi)$$



S. Diehl *et al.*, PLB 839 (2023) 137761.

	Meson	Flavor
$\mathcal{H}_T, \varepsilon_T$	π^+	$\Delta u - \Delta d$
	π^0	$2\Delta u + \Delta d$
	η	$2\Delta u - \Delta d + 2\Delta s$
$\mathcal{H}, \varepsilon$	ρ^+	$u - d$
	ρ^0	$2u + d$
	ω	$2u - d$
	ϕ	g

K. Joo, EIC Asia workshop (2024).



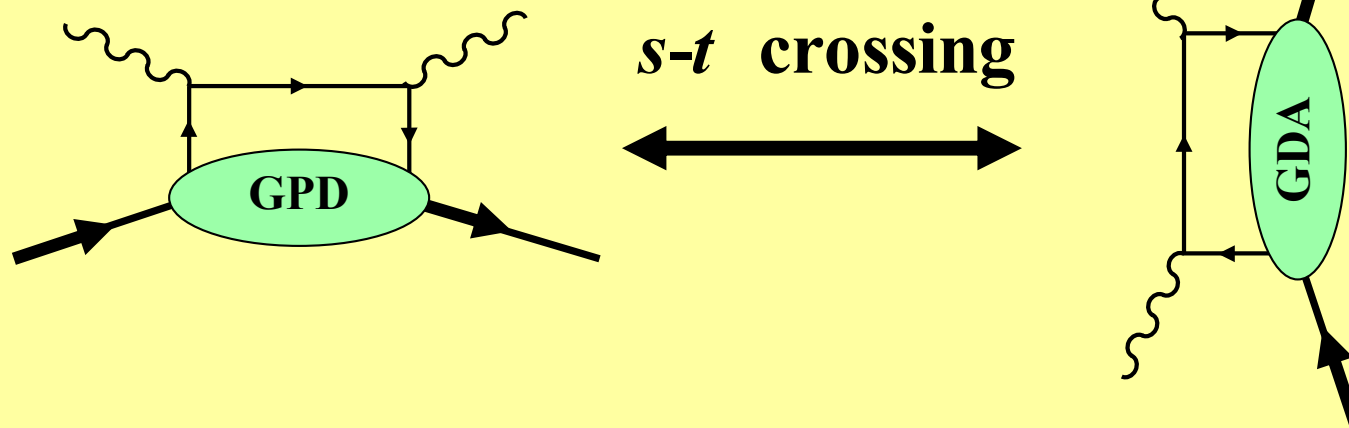
S. V. Goloskokov, Ya-Ping Xie, Xurong Chen, Chin. Phys. C 46 (2022) 123101.

e^+e^- facilities on timelike GPDs

Generalized Distribution Amplitudes
(GDAs = timelike GPDs)

Spacelike GPDs

GDA = Timelike GPDs



Extraction of GDAs and
gravitational form factors
from KEKB data.

SK, Q.-T. Song, O. Teryaev,
Phys. Rev. D 97 (2018) 014020.

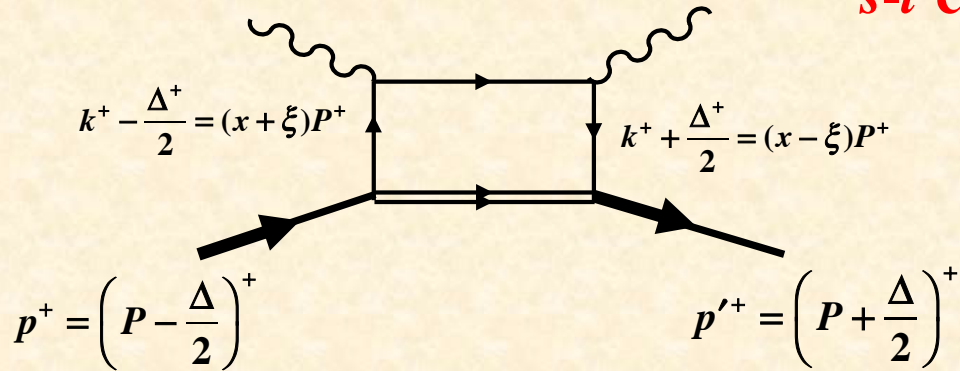
GPD $H_q^h(x, \xi, t)$ and GDA(= timelike GPD) $\Phi_q^{hh}(z, \zeta, W^2)$

$$\text{GPD: } H_q(x, \xi, t) = \int \frac{dy^-}{4\pi} e^{ixP^+y^-} \langle h(p') | \bar{\psi}(-y/2) \gamma^+ \psi(y/2) | h(p) \rangle \Big|_{y^+=0, \vec{y}_\perp=0}, \quad P^+ = \frac{(p+p')^+}{2}$$

$$\text{GDA: } \Phi_q(z, \zeta, s) = \int \frac{dy^-}{2\pi} e^{izP^+y^-} \langle h(p) \bar{h}(p') | \bar{\psi}(-y/2) \gamma^+ \psi(y/2) | 0 \rangle \Big|_{y^+=0, \vec{y}_\perp=0}$$

$$\text{DA: } \Phi_q^\pi(z, \zeta, s) = \int \frac{dy^-}{2\pi} e^{izP^+y^-} \langle \pi(p) | \bar{\psi}(-y/2) \gamma^+ \gamma_5 \psi(y/2) | 0 \rangle \Big|_{y^+=0, \vec{y}_\perp=0}$$

$H_q^h(x, \xi, t)$



$$P = \frac{p+p'}{2}, \quad \Delta = p' - p$$

Bjorken variable: $x = \frac{Q^2}{2p \cdot q}$

Momentum transfer squared: $t = \Delta^2$

Skewness parameter: $\xi = \frac{p^+ - p'^+}{p^+ + p'^+} = -\frac{\Delta^+}{2P^+}$

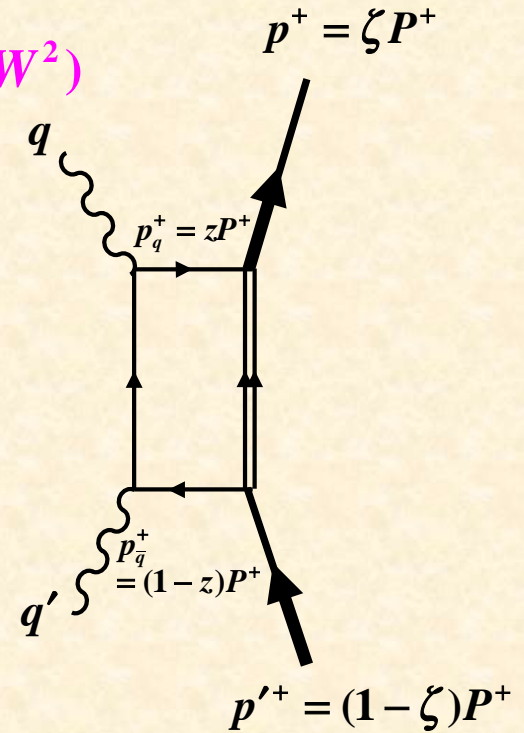
JLab / COMPASS

$\longleftrightarrow$
s-t crossing

$\Phi_q^{hh}(z, \zeta, W^2)$

$$\begin{aligned} z &\Leftrightarrow \frac{1-x/\xi}{2} \\ \zeta &\Leftrightarrow \frac{1-1/\xi}{2} \\ W^2 &\Leftrightarrow t \end{aligned}$$

KEKB



Bjorken variable for $\gamma\gamma^*$: $z = \frac{Q^2}{2q \cdot q'}$

Light-cone momentum ratio for a hadron in $h\bar{h}$: $\zeta = \frac{p^+}{p'^+} = \frac{1+\beta \cos \theta}{2}$

Invariant mass of $h\bar{h}$: $W^2 = (p+p')^2$

Cross section for $\gamma^* \gamma \rightarrow \pi^0 \pi^0$

$$\frac{d\sigma}{d(\cos\theta)} = \frac{1}{16\pi(s+Q^2)} \sqrt{1 - \frac{4m_\pi^2}{s}} \sum_{\lambda, \lambda'} |\mathcal{M}|^2$$

$$\mathcal{M} = \varepsilon_\mu^\lambda(q) \varepsilon_\nu^{\lambda'}(q') T^{\mu\nu} = e^2 A_{\lambda\lambda'}, \quad T^{\mu\nu} = i \int d^4\xi e^{-i\xi \cdot q} \langle \pi(p) \pi(p') | T J_{em}^\mu(\xi) J_{em}^\nu(0) | 0 \rangle$$

$$A_{\lambda\lambda'} = \frac{1}{e^2} \varepsilon_\mu^\lambda(q) \varepsilon_\nu^{\lambda'}(q') T^{\mu\nu} = -\varepsilon_\mu^\lambda(q) \varepsilon_\nu^{\lambda'}(q') g_T^{\mu\nu} \sum_q \frac{e_q^2}{2} \int_0^1 dz \frac{2z-1}{z(1-z)} \Phi_q^{\pi\pi}(z, \zeta, W^2)$$

GDA (timelike GPD): $\Phi_q^{\pi\pi}(z, \zeta, s) = \int \frac{dy^-}{2\pi} e^{izP^+y^-} \langle \pi(p) \pi(p') | \bar{\psi}(-y/2) \gamma^+ \psi(y/2) | 0 \rangle_{y^+=0, \vec{y}_\perp=0}$

$$\frac{d\sigma}{d(\cos\theta)} \simeq \frac{\pi\alpha^2}{4(s+Q^2)} \sqrt{1 - \frac{4m_\pi^2}{s}} |A_{++}|^2, \quad A_{++} = \sum_q \frac{e_q^2}{2} \int_0^1 dz \frac{2z-1}{z(1-z)} \Phi_q^{\pi\pi}(z, \zeta, W^2)$$

- **Continuum:** GDAs without intermediate-resonance contribution

$$\Phi_q^{\pi\pi}(z, \zeta, W^2) = N_\pi z^\alpha (1-z)^\alpha (2z-1) \zeta (1-\zeta) F_q^\pi(s)$$

$$F_q^\pi(s) = \frac{1}{[1 + (s - 4m_\pi^2) / \Lambda^2]^{n-1}}, \quad n = 2 \text{ according to constituent counting rule}$$

- **Resonances:** There exist resonance contributions to the cross section.

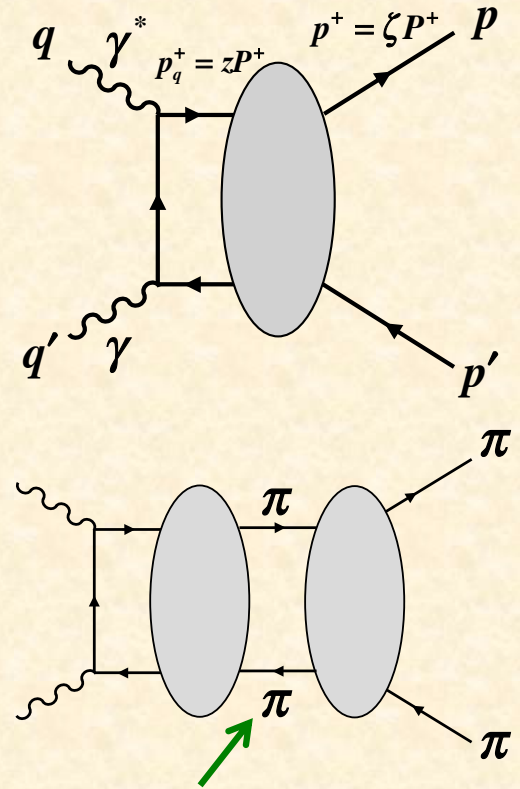
$$\sum_q \Phi_q^{\pi\pi}(z, \zeta, W^2) = 18 N_f z^\alpha (1-z)^\alpha (2z-1) [\tilde{B}_{10}(W) + \tilde{B}_{12}(W) P_2(\cos\theta)]$$

$$P_2(x) = \frac{1}{2}(3x^2 - 1)$$

$$\tilde{B}_{10}(W) = \text{resonance} [f_0(500), f_0(980)] + \text{continuum}$$

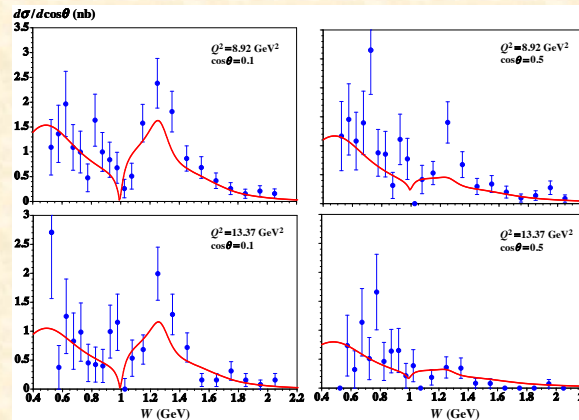
$$\tilde{B}_{12}(W) = \text{resonance} [f_2(1270)] + \text{continuum}$$

Belle measurements:
M. Masuda *et al.*,
PRD93 (2016) 032003.

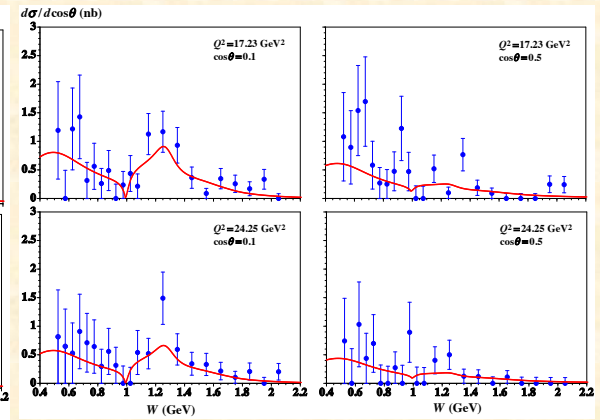


Including intermediate resonance contributions

$Q^2 = 8.92, 13.37 \text{ GeV}^2$



$Q^2 = 17.23, 24.25 \text{ GeV}^2$



Gravitational form factors and radii for pion

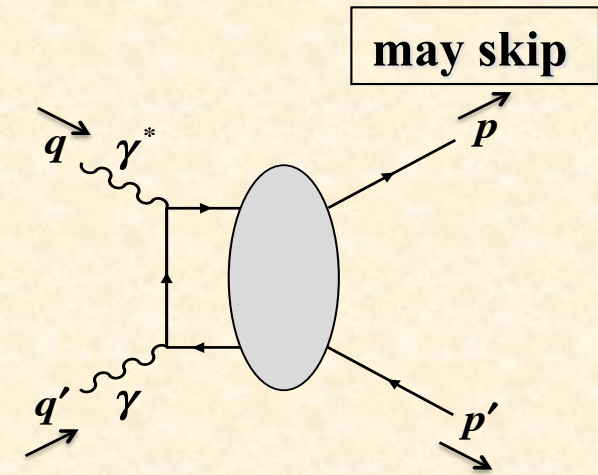
$$\int_0^1 dz (2z-1) \Phi_q^{\pi^0\pi^0}(z, \zeta, s) = \frac{2}{(P^+)^2} \langle \pi^0(p) \pi^0(p') | T_q^{++}(0) | 0 \rangle$$

$$\langle \pi^0(p) \pi^0(p') | T_q^{\mu\nu}(0) | 0 \rangle = \frac{1}{2} \left[(s g^{\mu\nu} - P^\mu P^\nu) \Theta_{1,q}(s) + \Delta^\mu \Delta^\nu \Theta_{2,q}(s) \right]$$

$$P = \frac{p+p'}{2}, \quad \Delta = p' - p$$

$T_q^{\mu\nu}$: energy-momentum tensor for quark

$\Theta_{1,q}, \Theta_{2,q}$: gravitational form factors for pion



See also Hyeon-Dong Son,
Hyun-Chul Kim, PRD90 (2014) 111901.

Analysis of $\gamma^* \gamma \rightarrow \pi^0 \pi^0$ cross section

- ⇒ Generalized distribution amplitudes $\Phi_q^{\pi^0\pi^0}(z, \zeta, s)$
- ⇒ Timelike gravitational form factors $\Theta_{1,q}(s), \Theta_{2,q}(s)$
- ⇒ Spacelike gravitational form factors $\Theta_{1,q}(t), \Theta_{2,q}(t)$
- ⇒ Gravitational radii of pion

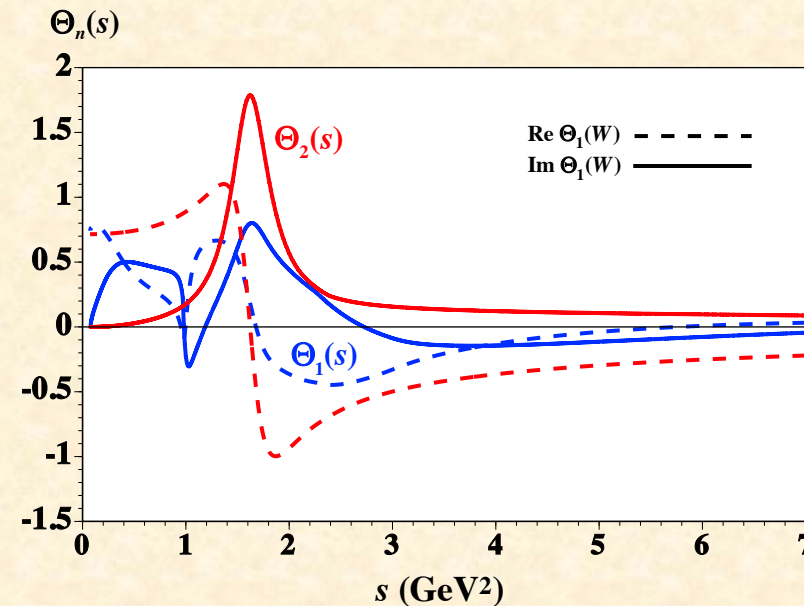
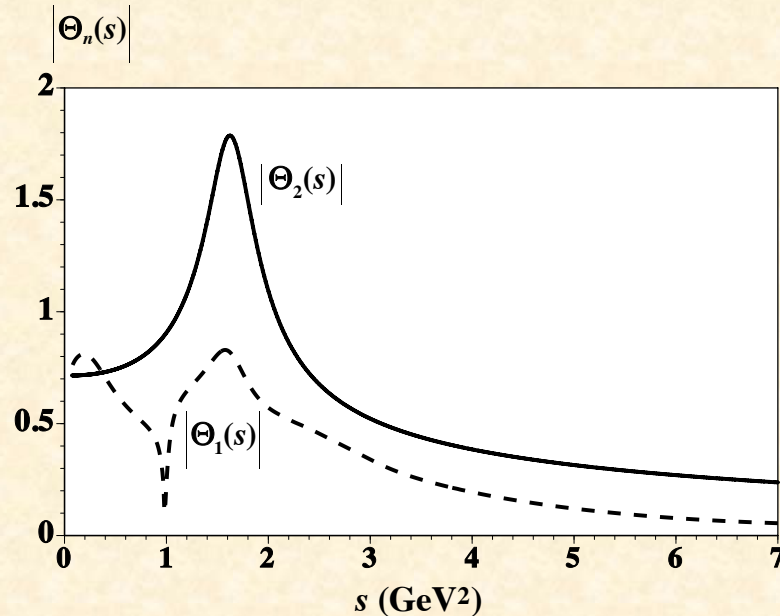
Gravitational form factors:

Original definition: H. Pagels, Phys. Rev. 144 (1966) 1250.

Operator relations: K. Tanaka, Phys. Rev. D 98 (2018) 034009;

Y. Hatta, A. Rajan, and K. Tanaka, JHEP 12 (2018) 008;

K. Tanaka, JHEP 01 (2019) 120.



Spacelike gravitational form factors and radii for pion

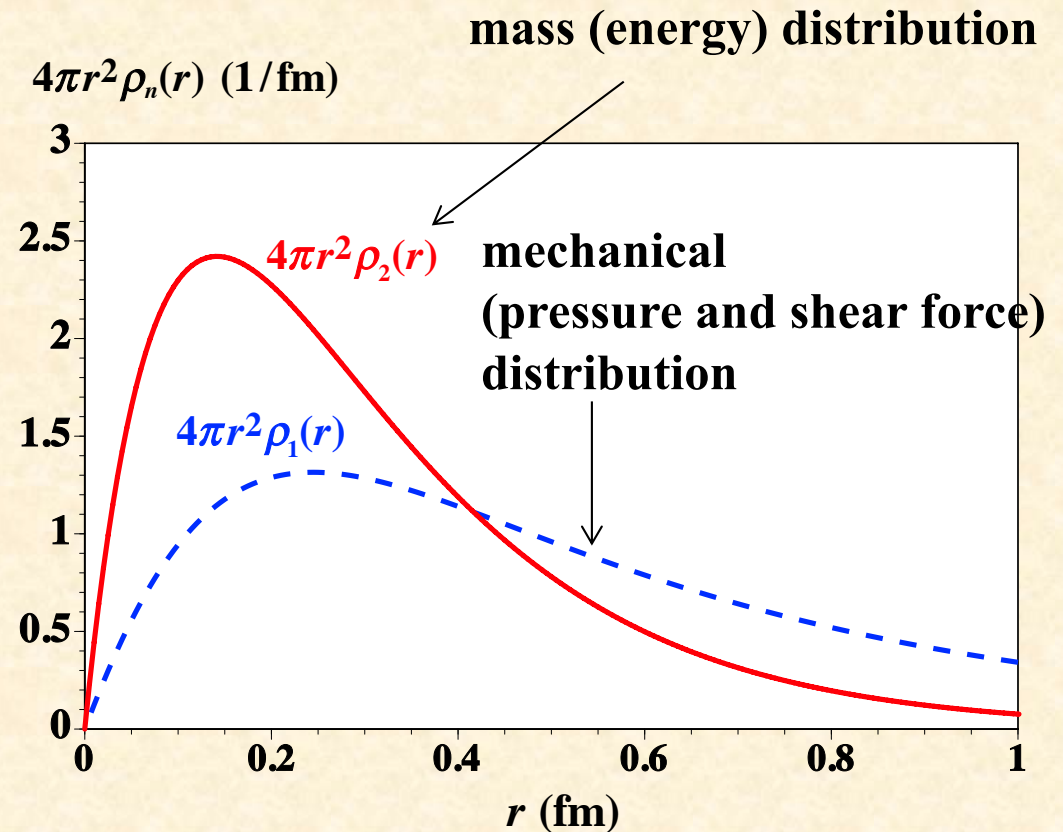
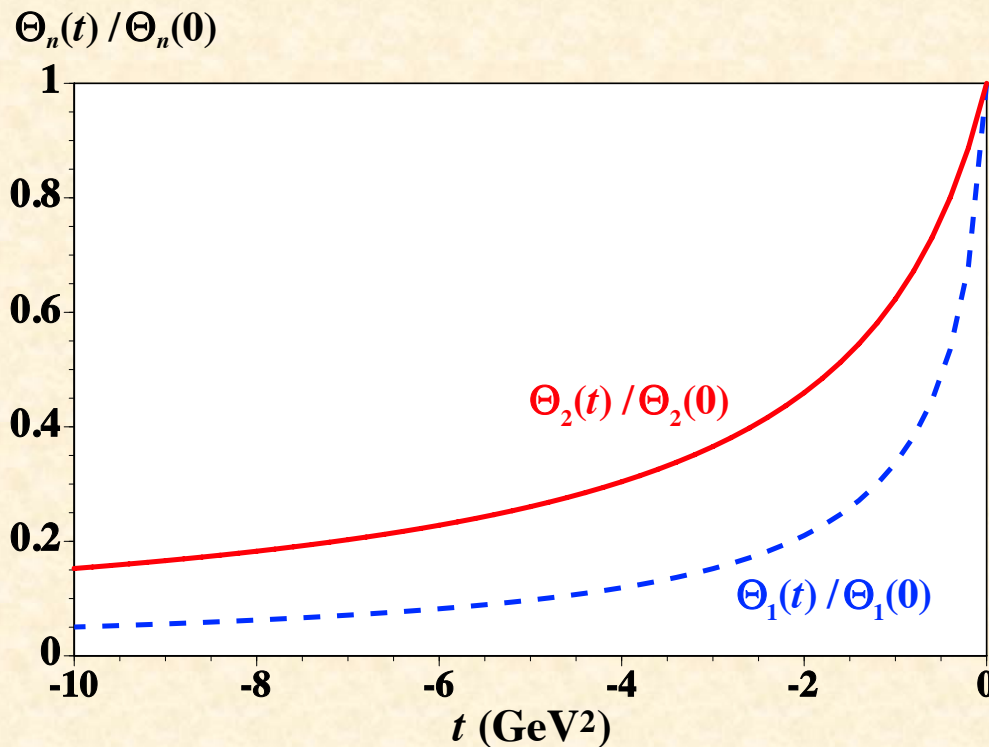
$$F(s) = \Theta_1(s), \Theta_1(s), \quad F(t) = \int_{4m_\pi^2}^{\infty} ds \frac{\text{Im} F(s)}{\pi(s-t-i\epsilon)}, \quad \rho(r) = \frac{1}{(2\pi)^3} \int d^3 q e^{-i\vec{q}\cdot\vec{r}} F(q) = \frac{1}{4\pi^2} \frac{1}{r} \int_{4m_\pi^2}^{\infty} ds e^{-\sqrt{s}r} \text{Im} F(s)$$

This is the first report on gravitational radii of hadrons from actual experimental measurements.

$$\sqrt{\langle r^2 \rangle_{\text{mass}}} = 0.32 \sim 0.39 \text{ fm}, \quad \sqrt{\langle r^2 \rangle_{\text{mech}}} = 0.82 \sim 0.88 \text{ fm} \leftarrow$$

First finding on gravitational radius from actual experimental measurements

$$\Leftrightarrow \sqrt{\langle r^2 \rangle_{\text{charge}}} = 0.672 \pm 0.008 \text{ fm}$$



Possible studies on GPDs at hadron accelerator facilities

**SK, M. Strikman, K. Sudoh,
PRD 80 (2009) 074003;**

**T. Sawada, W.-C. Chang, SK, J.-C. Peng, S. Sawada, and K. Tanaka,
PRD 93 (2016) 114034.**

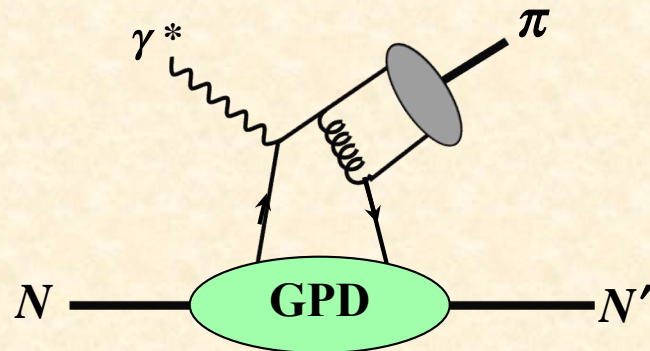
J-PARC LoI 2019-07, J.-K. Ahn *et al.* (2019).

J-PARC proposal under preparation,

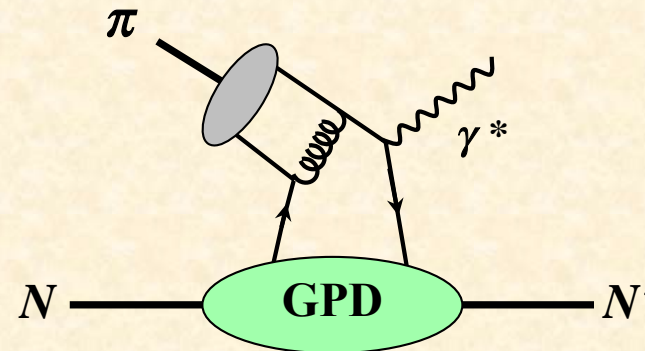
**Please get in touch with W.-C. Chang, N. Tomida
if you are interested in this project.**

GPD projects at JLab /EIC and J-PARC

JLab / EIC

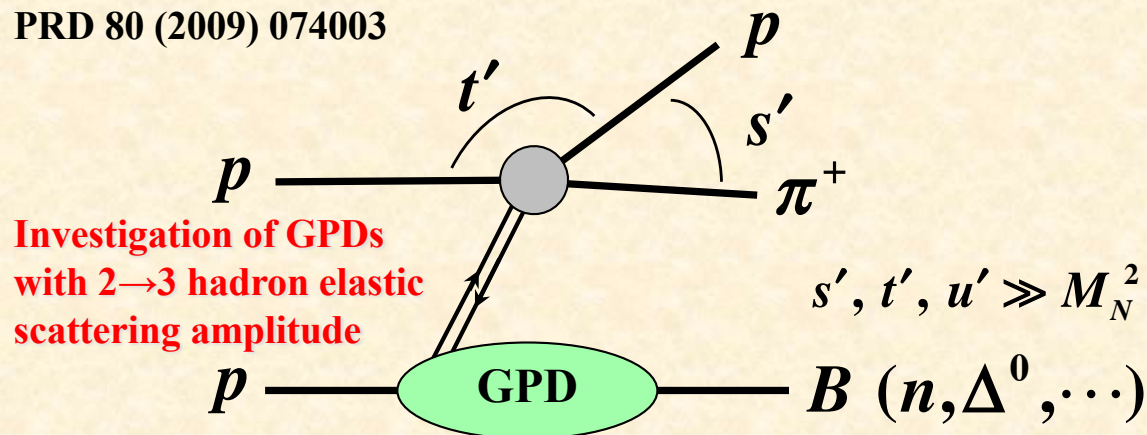


J-PARC



$$\int \frac{dz^-}{4\pi} e^{ixP^+z^-} \langle p' | \bar{\psi}(-z/2) \gamma^+ \gamma_5 \psi(z/2) | p \rangle \Big|_{z^+=0, \vec{z}_\perp=0} = \frac{1}{2P^+} \left[\tilde{H}(x, \xi, t) \bar{u}(p') \gamma^+ \gamma_5 u(p) + \tilde{E}(x, \xi, t) \bar{u}(p') \frac{\gamma_5 \Delta^+}{2M} u(p) \right]$$

SK, M. Strikman, K. Sudoh,
PRD 80 (2009) 074003



J-W. Qiu and Z. Yu,
JHEP 08 (2022) 103;
PRD 107 (2023) 014007.

$$\begin{aligned} \pi + N &\rightarrow \gamma + \gamma + N' \\ h + M_B &\rightarrow h' + \gamma + M_D \\ h + M_B &\rightarrow h' + M_C + M_D \end{aligned}$$

Cross section estimate (ξ dependence)

Skewness parameter: $\xi = \frac{p_N^+ - p_B^+}{p_N^+ + p_B^+}$

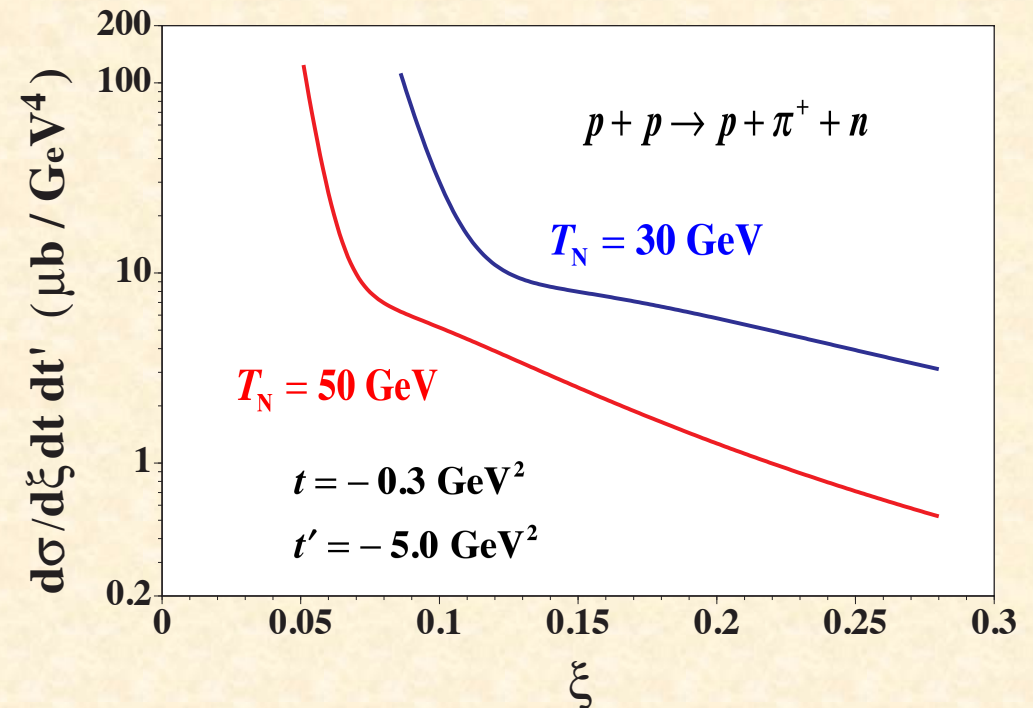
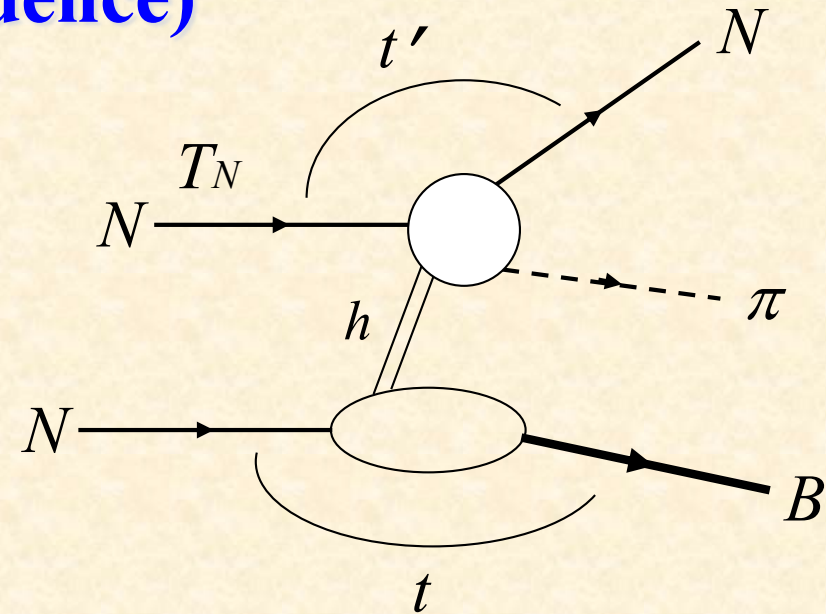
$\frac{d\sigma}{d\xi dt dt'} \left(\frac{\mu\text{b}}{\text{GeV}^2} \right)$ as a function of ξ

at fixed $T_N = 30$ (50) GeV,

$t = -0.3 \text{ GeV}^2$, $t' = -5 \text{ GeV}^2$.

At this stage, our numerical results are for rough order of magnitude estimates on cross sections by assuming π - and ρ -like intermediate states.

For the details, please look at our PRD paper in 2009.



Exclusive Drell-Yan $\pi^- + p \rightarrow \mu^+ \mu^- + n$ and GPDs

$$\frac{d\sigma_L}{dQ'^2 dt} = \frac{4\pi\alpha^2}{27} \frac{\tau^2}{Q'^2} f_\pi^2 \left[(1 - \xi^2) |\tilde{H}^{du}(-\xi, \xi, t)|^2 - 2\xi^2 \text{Re}\{\tilde{H}^{du}(-\xi, \xi, t)^* \tilde{E}^{du}(-\xi, \xi, t)\} - \xi^2 \frac{t}{4m_N^2} |\tilde{E}^{du}(-\xi, \xi, t)|^2 \right]$$

$$Q'^2 = q'^2, \quad t = (p - p')^2, \quad \tau = \frac{Q'^2}{2p \cdot q_\pi} \approx \frac{Q'^2}{s - m_\pi^2}$$

$$\int \frac{dz^-}{4\pi} e^{ixP^+z^-} \langle p(p') | \bar{q}(-z/2) \gamma^+ \gamma_5 q(z/2) | p(p) \rangle_{z^+=0, \vec{z}_\perp=0} = \frac{1}{2P^+} \left[\tilde{H}_p^q(x, \xi, t) \bar{u}(p') \gamma^+ \gamma_5 u(p) + \tilde{E}_p^q(x, \xi, t) \bar{u}(p') \frac{\gamma_5 \Delta^+}{2M} u(p) \right]$$

$$\int \frac{dz^-}{4\pi} e^{ixP^+z^-} \langle n(p') | \bar{q}_d(-z/2) \gamma^+ \gamma_5 q_u(z/2) | p(p) \rangle_{z^+=0, \vec{z}_\perp=0} = \frac{1}{2P^+} \left[\tilde{H}_{p \rightarrow n}^{du}(x, \xi, t) \bar{u}(p') \gamma^+ \gamma_5 u(p) + \tilde{E}_{p \rightarrow n}^{du}(x, \xi, t) \bar{u}(p') \frac{\gamma_5 \Delta^+}{2M} u(p) \right]$$

$$\tilde{H}^{du}(x, \xi, t) = \frac{8}{3} \alpha_s \int_{-1}^1 dz \frac{\phi_\pi(z)}{1-z^2} \int_{-1}^1 dx' \left[\frac{e_d}{x-x'-i\epsilon} - \frac{e_u}{x+x'-i\epsilon} \right] [\tilde{H}^d(x', \xi, t) - \tilde{H}^u(x', \xi, t)]$$

$$\tilde{E}^{du}(x, \xi, t) = \frac{8}{3} \alpha_s \int_{-1}^1 dz \frac{\phi_\pi(z)}{1-z^2} \int_{-1}^1 dx' \left[\frac{e_d}{x-x'-i\epsilon} - \frac{e_u}{x+x'-i\epsilon} \right] [\tilde{E}^d(x', \xi, t) - \tilde{E}^u(x', \xi, t)]$$

**T. Sawada, W.-C. Chang, SK, J.-C. Peng,
S. Sawada, and K. Tanaka, PRD93 (2016) 114034.**

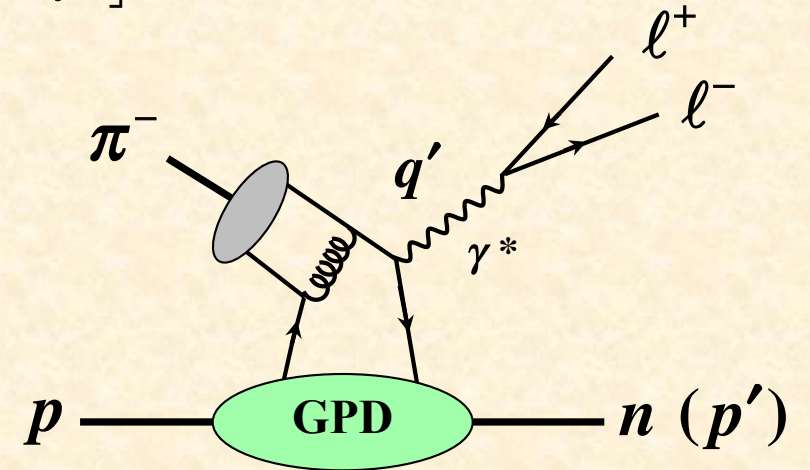
LETTER OF INTENT

Studying Generalized Parton Distributions with Exclusive Drell-Yan process

at J-PARC

JungKeun Ahn,¹ Sakiko Ashikaga,² Wen-Chen Chang,^{3,*} Seonho Choi,⁴ Stefan Diehl,⁵ Yuji Goto,⁶ Kenneth Hicks,⁷ Youichi Igarashi,⁸ Kyungseon Joo,⁵ Shunzo Kumano,^{9,10} Yue Ma,⁶ Kei Nagai,³ Kenichi Nakano,¹¹ Masayuki Niiyama,¹² Hiroyuki Nomi,^{13,8,†} Hiroaki Ohnishi,¹⁴ Jen-Chieh Peng,¹⁵ Hiroyuki Sako,¹⁶ Shin'ya Sawada,^{8,‡} Takahiro Sawada,¹⁷ Kotaro Shirotori,¹³ Kazuhiro Tanaka,^{18,10} and Natsuki Tomida¹³

LoI for a J-PARC experiment

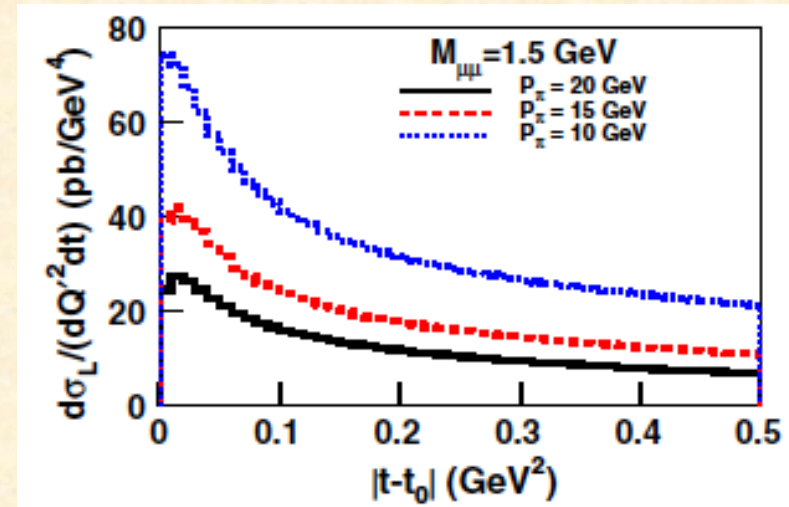
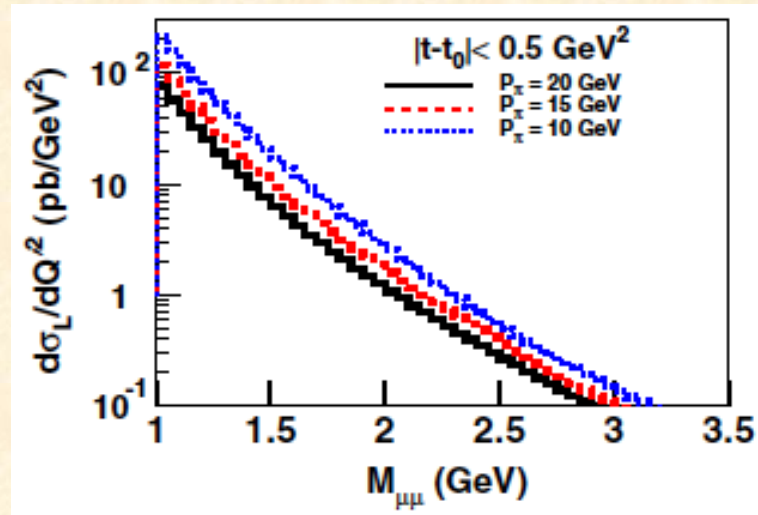


$$\pi^- (\bar{u}d) + p(uud) \rightarrow n(udd) + \gamma^* (\rightarrow \ell^+ \ell^-)$$

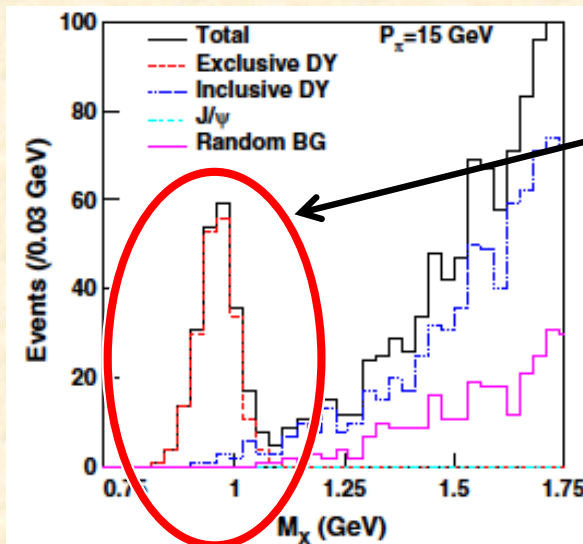
Expected Drell-Yan events at J-PARC

$$Q'^2 = q'^2, \quad t = (p - p')^2, \quad \tau = \frac{Q'^2}{2p \cdot q_\pi} \approx \frac{Q'^2}{s - m_N^2}$$

$$\frac{d\sigma_L}{dQ'^2 dt} = \frac{4\pi\alpha^2}{27} \frac{\tau^2}{Q'^2} f_\pi^2 \left[(1 - \xi^2) |\tilde{H}^{du}(-\xi, \xi, t)|^2 - 2\xi^2 \text{Re}\{\tilde{H}^{du}(-\xi, \xi, t)^* \tilde{E}^{du}(-\xi, \xi, t)\} - \xi^2 \frac{t}{4m_N^2} |\tilde{E}^{du}(-\xi, \xi, t)|^2 \right]$$



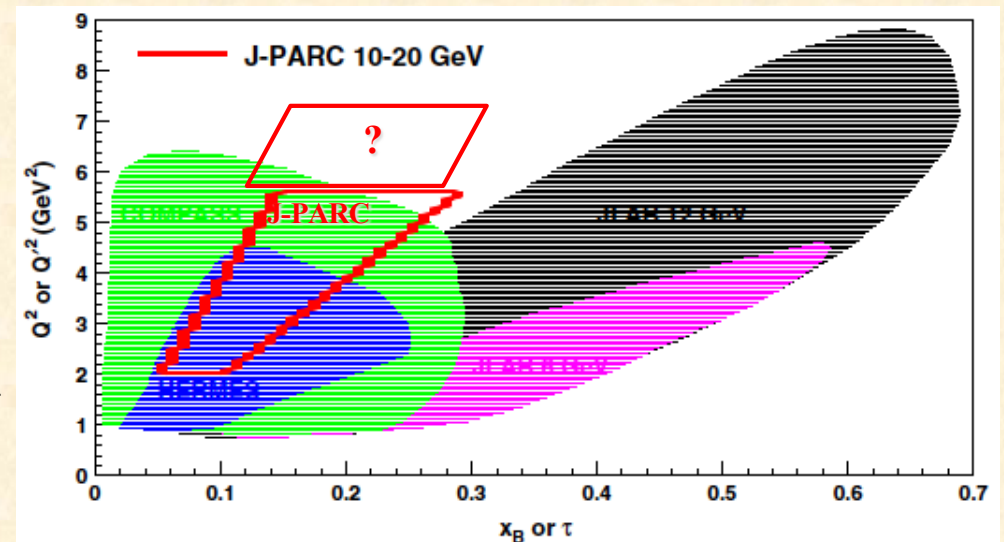
Missing mass



**Exclusive
Drell-Yan**

$$M_X^2 = (q + p - q')^2$$

$$q = p_\pi, \quad p = p_p, \quad q = p_{\mu^+\mu^-}$$



Physics of J-PAC high-momentum beamline

2020

2025

**E16: Hadron masses
in nuclear medium**

E50: Charmed baryons

E16

E50

Proposal

Electron pair spectrometer at the J-PARC
50-GeV PS to explore the chiral symmetry in
QCD

April 28, 2006
June 07, 2006 rev.1

S. Yokkaichi¹, H. En'yo, M. Naruki, R. Muto, T. Tabaru
RIKEN

K. Ozawa, H. Hamagaki
Center for Nuclear Study, Graduate School of Science, University of Tokyo

K. Shigaki
Graduate School of Science, Hiroshima University

S. Sawada, M. Sekimoto
High Energy Accelerator Research Organization (KEK)

F. Sakuma, K. Aoki
Department of Physics, Kyoto University

KEK/J-PARC-PAC 2012-19
Charmed Baryon Spectroscopy via the (π, D^{*-}) reaction

Y. Morino, T. Nakano,* H. Noumi[†],* K. Shirotori, Y. Sugaya, and T. Yamaga
*Research Center for Nuclear Physics (RCNP), Osaka University,
10-1, Mihogaoka, Ibaraki, Osaka, 567-0047, Japan*

K. Ozawa[‡]
*Institute of Particle and Nuclear Studies (IPNS),
High Energy Accelerator Research Organization (KEK),
1-1 Oho, Tsukuba, Ibaraki 305-0801, Japan*

T. Ishikawa
*Research Center for Electron Photon Science,
Tohoku University, 1-2-1, Mikamine,
Taihaku-ku, Sendai, Miyagi 982-0826, Japan*

Y. Miyachi
*Physics Department, Yamagata University, 1-4-12,
Kojirakawa-machi, Yamagata 990-8560, Japan*

K. Tanida
*Department of Physics and Astronomy,
Seoul National University, Seoul 151-747, Korea*

**High-energy hadron
(LoI)**

There is a possibility
for high-energy hadron physics,
including nucleon structure, ...

LETTER OF INTENT

Studying Generalized Parton Distributions with Exclusive Drell-Yan process
at J- PARC

JungKeun Ahn,¹ Sakiko Ashikag,² Wen-Chen Chang,^{3,*} Seonho Choi,⁴ Stefan
Diehl,⁵ Yuji Goto,⁶ Kenneth Hicks,⁷ Youichi Igarashi,⁸ Kyungseon Joo,⁵ Shunzo
Kumano,^{9,10} Yue Ma,⁶ Kei Nagai,³ Kenichi Nakano,¹¹ Masayuki Niiyama,¹² Hiroyuki
Noumi,^{13,8,†} Hiroaki Ohnishi,¹⁴ Jen-Chieh Peng,¹⁵ Hiroyuki Sako,¹⁶ Shin'ya Sawada,^{8,‡}
Takahiro Sawada,¹⁷ Kotaro Shirotori,¹³ Kazuhiro Tanaka,^{18,10} and Natsuki Tomida¹³

**J-PARC proposal under preparation (2023),
Please get in touch with
W.-C. Chang (Academia Sinica, Taiwan)
if you are interested in this project.
Japanese scientists on this project, e.g.
Natsuki Tomida (Kyoto University)**

Possible GPD studies at neutrino facilities

**X. Chen, SK, R. Kunitomo, S. Wu, Y.-P. Xie,
Euro. Phys. J. A 60 (2024) 208.**

See also

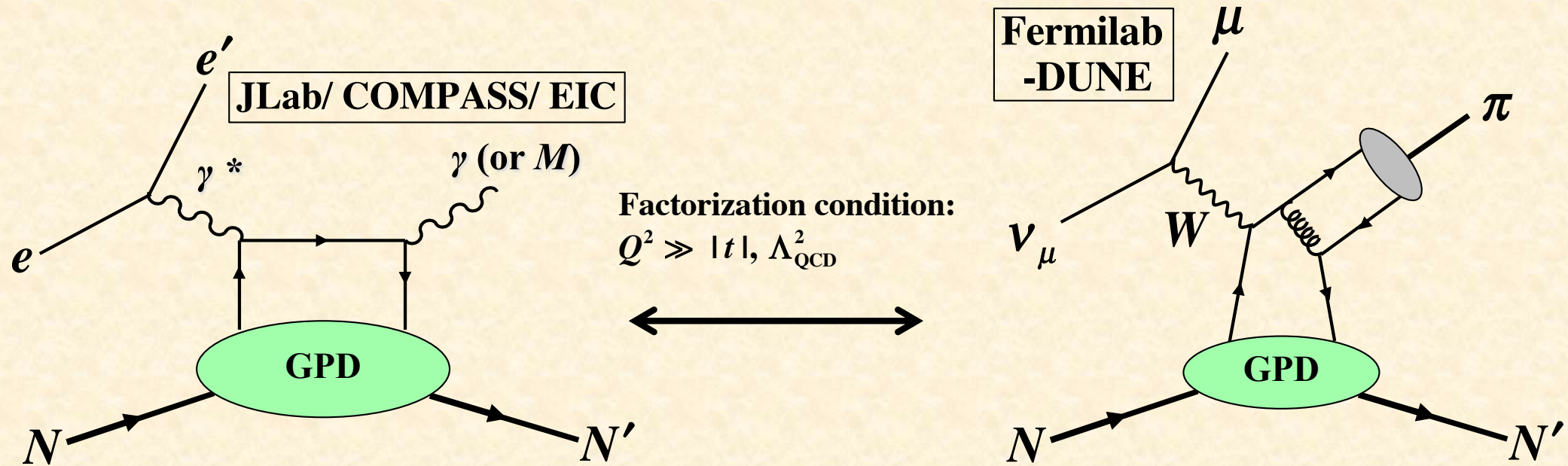
SK, EPJ Web Conf. 208 (2019) 07003.

**EIC yellow report, R. Abdul Khalek *et al.*, arXiv:2103.05419,
Sec. 7.5.2, Neutrino physics by SK and R. Petti.**

SK and R. Petti, PoS (NuFact2021) 092.

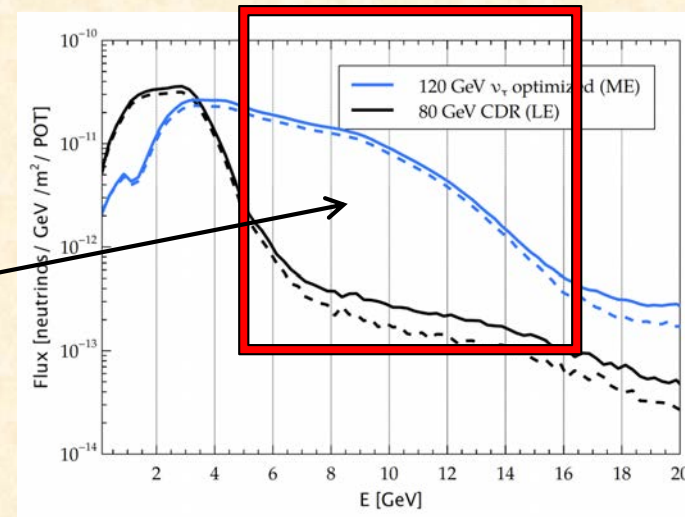
Neutrino reactions for gravitational form factors @Fermilab-DUNE

(Origins of hadron masses and pressures)



**Deep Underground Neutrino Experiment (DUNE)
at Long-Baseline Neutrino Facility (LBNF)**

**High-energy part of the LBNF ν beam
can be used for the GPD studies.**



J. Rout *et al.*, PRD 102 (2020) 116018

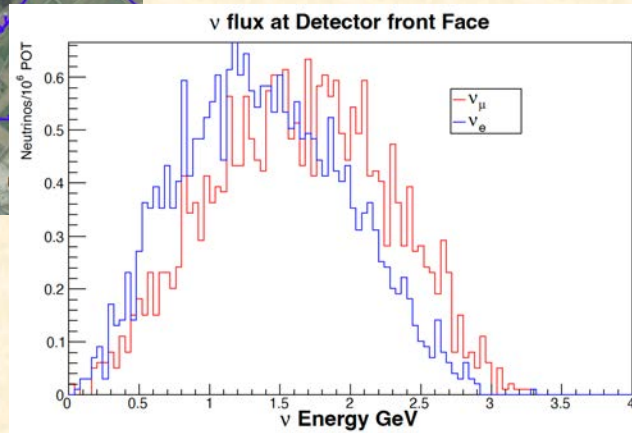
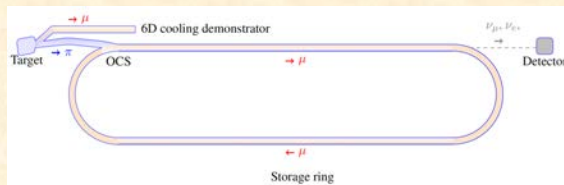
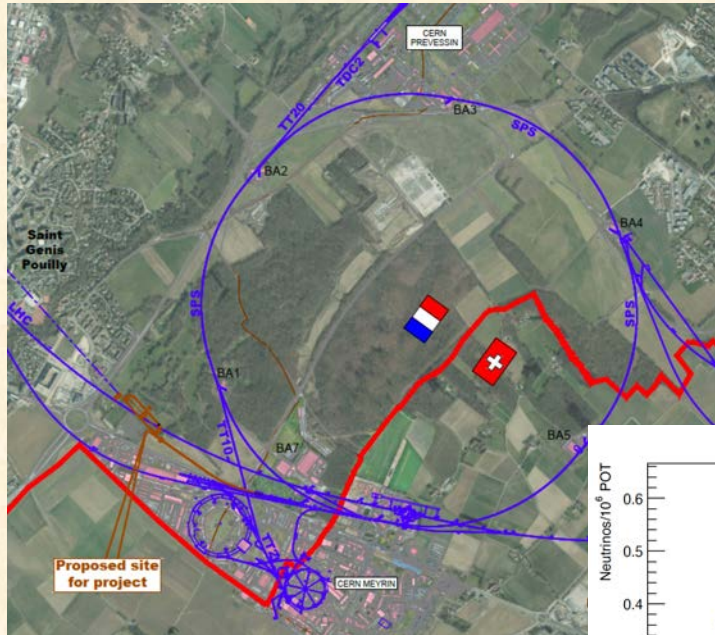
nuSTORM (Neutrinos from Stored Muons)

Fermilab

Feasibility Study, C. C. Ahdida *et al.*, (2020); L. A. Ruso *et al.*, arXiv:2203.07545.

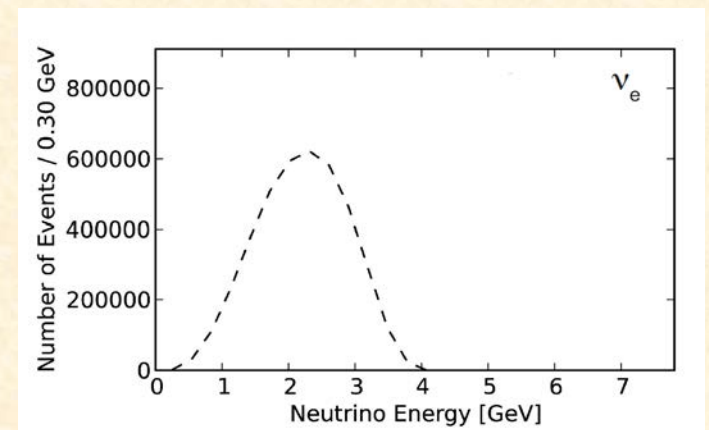
CERN

**Letter of Intent, arXiv:1206.0294,
P. Kyberd *et al.* (2012);
Proposal, D. Adey *et al.*, arXiv:1308.6822.
No recent update.**



At this stage, the considered beam energy is not high enough for structure-function studies; however, high-energy option could be possible. (personal communications: Xianguo Lu)
→ SK's talk at the nuSTORM-collaboration meeting on July 15, 2024

They could be interested in the higher-energy possibility.



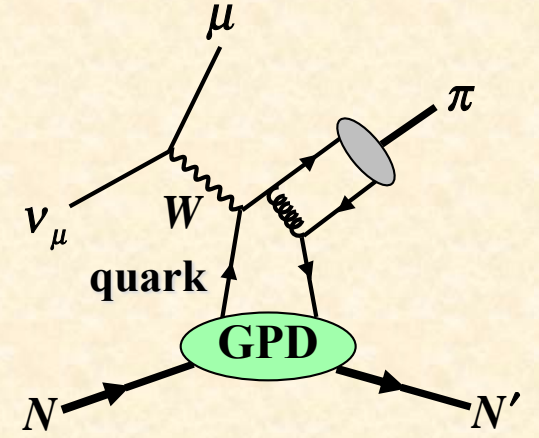
Cross section formalism

B. Pire, L. Szymanowski, J. Wagner,
Phys. Rev. D 95, 114029 (2017).

Cross section

$$\frac{d\sigma(\nu_\ell N \rightarrow \ell^- N' \pi)}{dy dQ^2 dt d\phi} = \Gamma \varepsilon \sigma_L, \quad \varepsilon \simeq \frac{1-y}{1-y+y^2/2}, \quad \Gamma = \frac{G_F^2 Q^2}{32 (2\pi)^4 (s - m_N^2)^2 y (1-\varepsilon) \sqrt{1+4x^2 m_N^2 / Q^2}}$$

$$\sigma_L = \varepsilon_L^* W_{\mu\nu} \varepsilon_L^\nu = \frac{1}{Q^2} \left[(1-\xi^2) \left\{ |C_q \mathcal{H}_q + C_g \mathcal{H}_g|^2 + |C_q \tilde{\mathcal{H}}_q|^2 \right\} + \frac{\xi^4}{1-\xi^2} \left\{ |C_q \mathcal{E}_q + C_g \mathcal{E}_g|^2 + |C_q \tilde{\mathcal{E}}_q|^2 \right\} \right. \\ \left. - 2\xi^2 \operatorname{Re} \left\{ (C_q \mathcal{H}_q + C_g \mathcal{H}_g)(C_q \mathcal{E}_q + C_g \mathcal{E}_g)^* \right\} - 2\xi^2 \operatorname{Re} \left\{ C_q \tilde{\mathcal{H}}_q (C_q \tilde{\mathcal{E}}_q)^* \right\} \right]$$



Quark contributions

$$T_q = -i \frac{C_q}{2Q} N(p') \left[\mathcal{H}_q \hat{n} + \mathcal{E}_q \frac{i\sigma^{\mu\nu} n_\mu \Delta_\nu}{2m_N} - \tilde{\mathcal{H}}_q \hat{n} \gamma_5 - \tilde{\mathcal{E}}_q \frac{\gamma_5 n \cdot \Delta}{2m_N} \right] N(p)$$

$$\mathcal{F}_q = 2f_\pi \int \frac{dz \phi_\pi(z)}{1-z} \int dx \frac{F_q(x, \xi, t)}{x - \xi + i\varepsilon} \\ = (\text{pion distribution amplitude}) \cdot (\text{quark GPD})$$

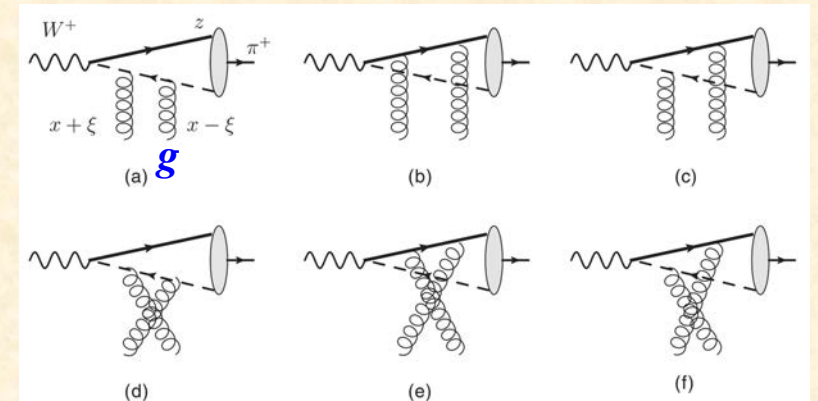
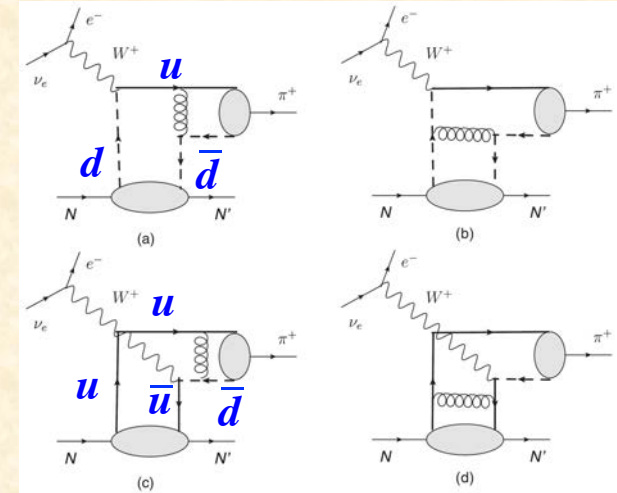
$$F_q(x, \xi, t) \equiv F_d(x, \xi, t) - F_u(-x, \xi, t)$$

$$F = H, E, \tilde{H}, \tilde{E}$$

Gluon contributions

$$T_g = -i \frac{C_g}{2Q} N(p') \left[\mathcal{H}^g \hat{n} + \mathcal{E}^g \frac{i\sigma^{\mu\nu} n_\mu \Delta_\nu}{2m_N} \right] N(p)$$

$$\mathcal{F}_g = \frac{8f_\pi}{\xi} \int \frac{dz \phi_\pi(z)}{z(1-z)} \int dx \frac{F_g(x, \xi, t)}{x - \xi + i\varepsilon}$$



Cross sections

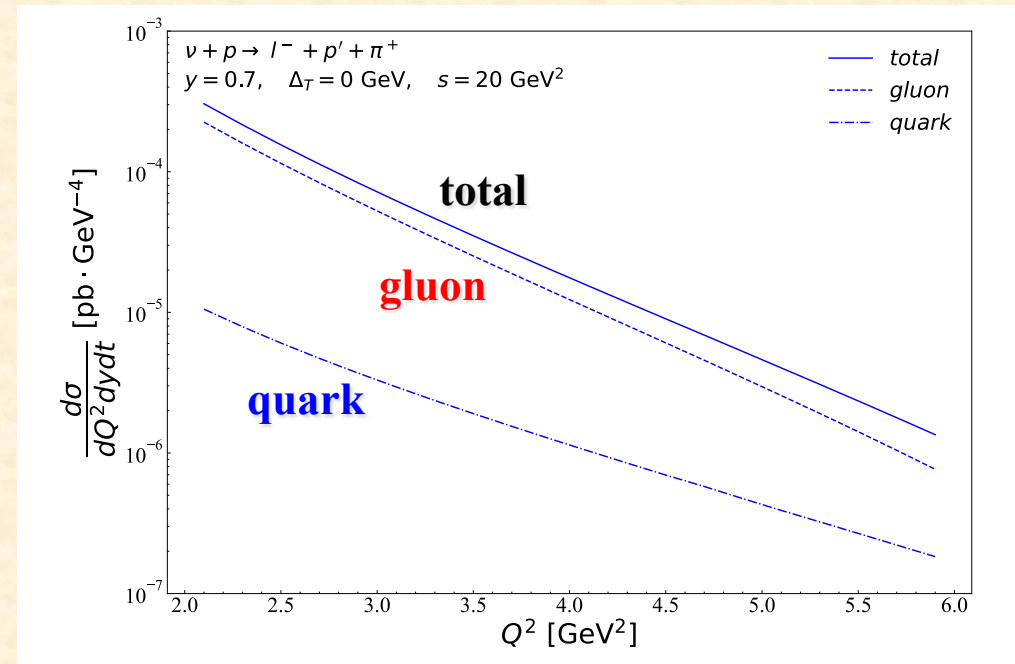
π^+ production: $\nu p \rightarrow \ell^- \pi^+ p$

$$\mathcal{F}_q = 2f_\pi \int \frac{dz \phi_\pi(z)}{1-z} \int dx \frac{F_q(x, \xi, t)}{x - \xi + i\varepsilon} \quad \text{gluon} \gg \text{quark}$$

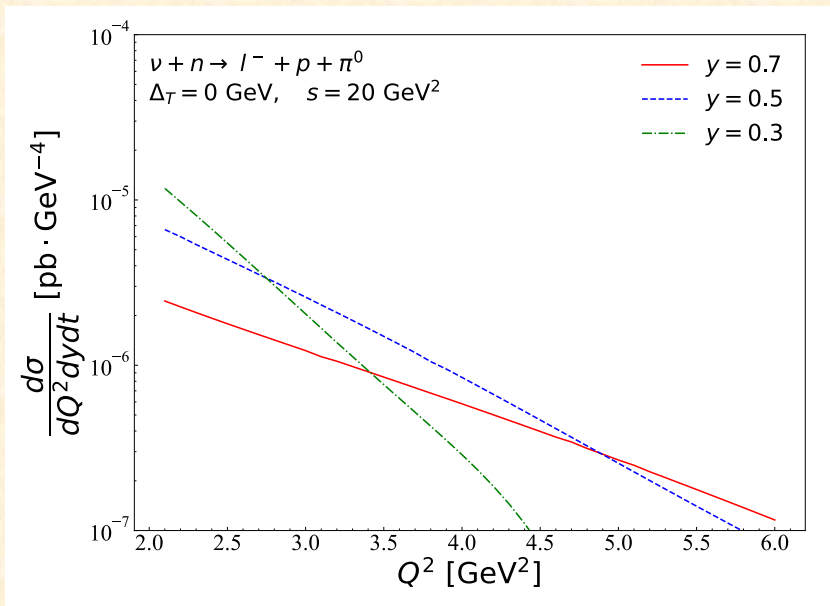
$$\mathcal{F}_g = \frac{8f_\pi}{\xi} \int \frac{dz \phi_\pi(z)}{z(1-z)} \int dx \frac{F_g(x, \xi, t)}{x - \xi + i\varepsilon}$$

$$\frac{\mathcal{F}_q}{\mathcal{F}_g} \sim \frac{\xi}{8} = \frac{0.1 \sim 0.3}{8} = 0.01 \sim 0.04$$

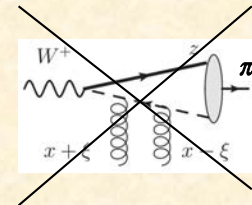
= a few % $\ll 1$



π^0 production: $\nu n \rightarrow \ell^- \pi^0 p$



no gluon



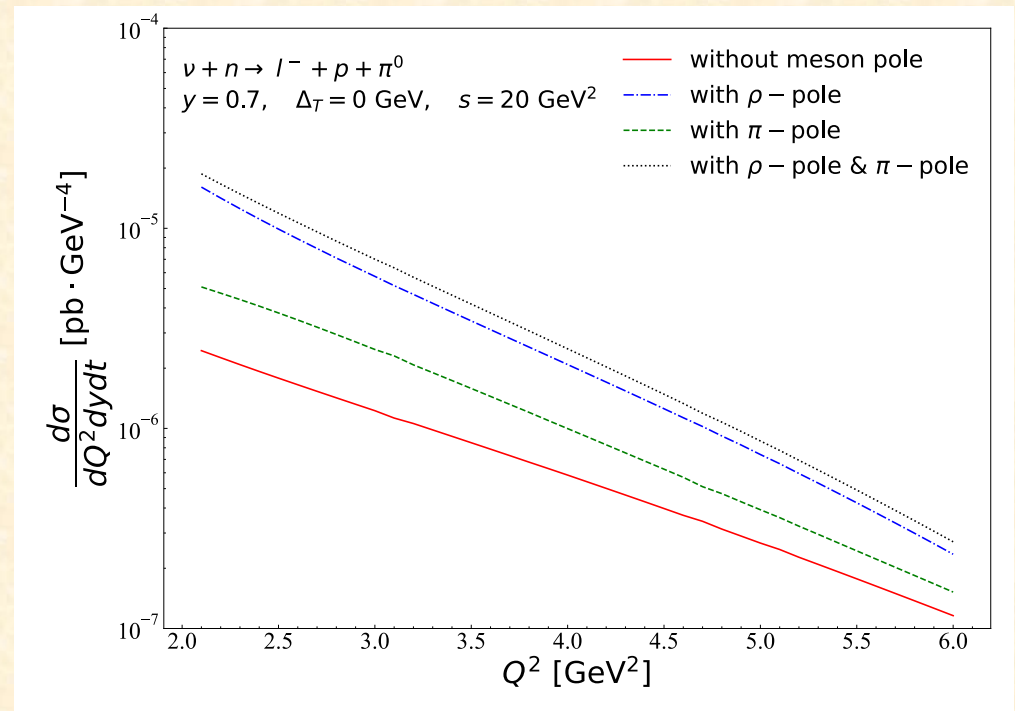
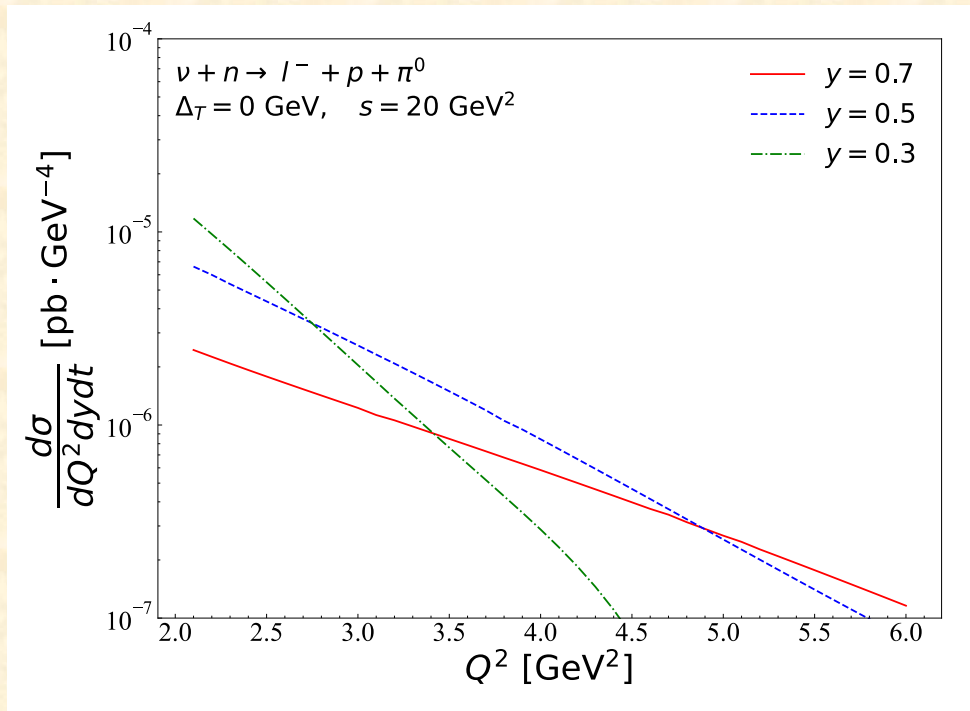
no gluon for π^0

Neutrino GPD studies are complementary to the charged-lepton projects.

- Gluon GPDs could be probed in charged-pion production.
- Quark GPDs could be probed in π^0 production.
- Flavor dependence of quark GPDs could be investigated.

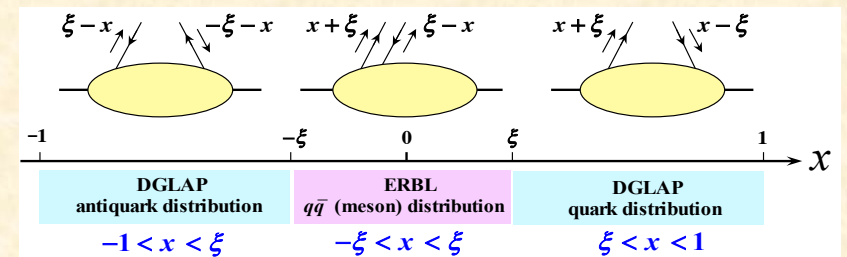
Contribution of each term to the π^0 -production cross section

$$\frac{d\sigma(\nu_\ell N \rightarrow \ell^- N' \pi)}{dy dQ^2 dt d\phi} \propto \frac{1}{Q^2} \left[(1 - \xi^2) \left\{ \left| C_q \mathcal{H}_q + \cancel{C_s \mathcal{H}_s} \right|^2 + \left| C_q \tilde{\mathcal{H}}_q \right|^2 \right\} + \frac{\xi^4}{1 - \xi^2} \left\{ \left| C_q \mathcal{E}_q + \cancel{C_s \mathcal{E}_s} \right|^2 + \left| C_q \tilde{\mathcal{E}}_q \right|^2 \right\} \right. \\ \left. - 2\xi^2 \operatorname{Re} \left\{ (C_q \mathcal{H}_q + \cancel{C_s \mathcal{H}_s}) (C_q \mathcal{E}_q + \cancel{C_s \mathcal{E}_s})^* \right\} - 2\xi^2 \operatorname{Re} \left\{ C_q \tilde{\mathcal{H}}_q (C_q \mathcal{E}_q)^* \right\} \right]$$



$$\cancel{H_s} > H_q > \tilde{H}_q > E_q, \tilde{E}_q, \cancel{E_s}$$

- π^0 production is sensitive to quark $\mathcal{H}_q$.
- GPDs in the ERBL (Efremov-Radyushkin-Brodsky-Lepage) region could be probed.



Structure functions of spin-3/2 Δ (Ω)

**D.-Y. Fu, Y.-B. Dong, S. Kumano,
Phys. Rev. D 109 (2024) 096006;
arXiv:2508.15245 (Phys. Rev. D in press).**

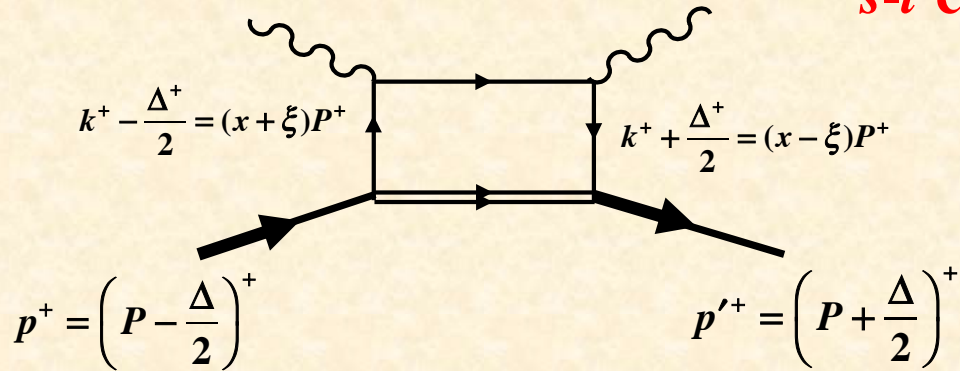
GPD $H_q^h(x, \xi, t)$ and GDA(= timelike GPD) $\Phi_q^{hh}(z, \zeta, W^2)$

GPD: $H_q(x, \xi, t) = \int \frac{dy^-}{4\pi} e^{ixP^+y^-} \langle h(p') | \bar{\psi}(-y/2) \gamma^+ \psi(y/2) | h(p) \rangle \Big|_{y^+=0, \vec{y}_\perp=0}, \quad P^+ = \frac{(p+p')^+}{2}$

GDA: $\Phi_q(z, \zeta, s) = \int \frac{dy^-}{2\pi} e^{izP^+y^-} \langle h(p) \bar{h}(p') | \bar{\psi}(-y/2) \gamma^+ \psi(y/2) | 0 \rangle \Big|_{y^+=0, \vec{y}_\perp=0}$

DA: $\Phi_q^\pi(z, \zeta, s) = \int \frac{dy^-}{2\pi} e^{izP^+y^-} \langle \pi(p) | \bar{\psi}(-y/2) \gamma^+ \gamma_5 \psi(y/2) | 0 \rangle \Big|_{y^+=0, \vec{y}_\perp=0}$

$H_q^h(x, \xi, t)$



$P = \frac{p+p'}{2}, \quad \Delta = p' - p$

Bjorken variable: $x = \frac{Q^2}{2p \cdot q}$

Momentum transfer squared: $t = \Delta^2$

Skewness parameter: $\xi = \frac{p^+ - p'^+}{p^+ + p'^+} = -\frac{\Delta^+}{2P^+}$

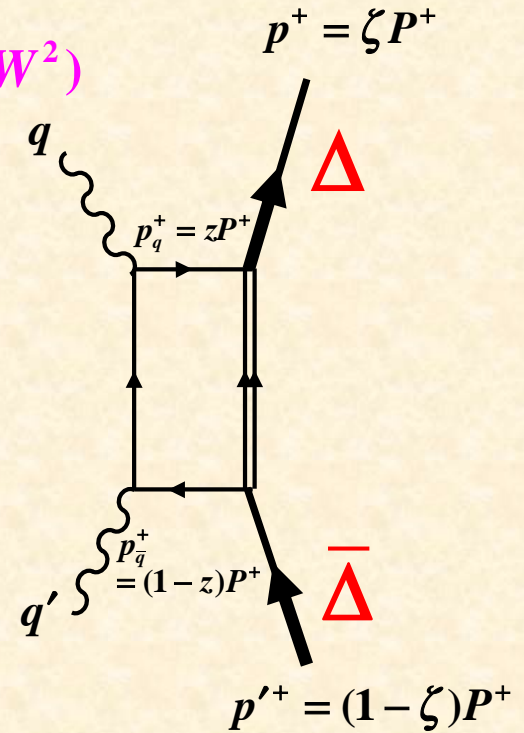
JLab / COMPASS

$\longleftrightarrow$
s-t crossing

$z \Leftrightarrow \frac{1-x/\xi}{2}$
 $\zeta \Leftrightarrow \frac{1-1/\xi}{2}$
 $W^2 \Leftrightarrow t$

KEKB

$\Phi_q^{hh}(z, \zeta, W^2)$



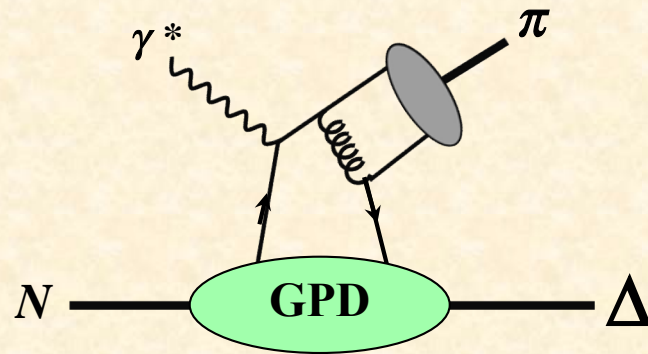
Bjorken variable for $\gamma\gamma^*$: $z = \frac{Q^2}{2q \cdot q'}$

Light-cone momentum ratio for a hadron in $h\bar{h}$: $\zeta = \frac{p^+}{p'^+} = \frac{1+\beta \cos \theta}{2}$

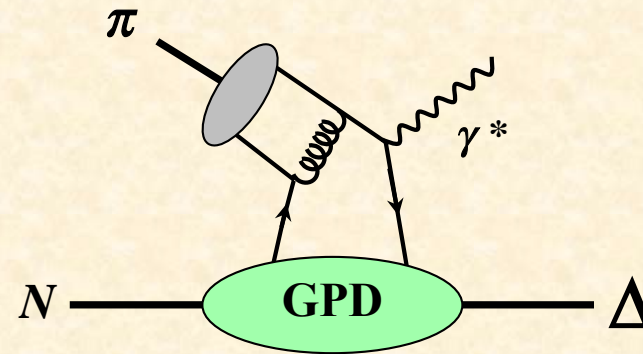
Invariant mass of $h\bar{h}$: $W^2 = (p+p')^2$

Transition GPDs from N to Δ

JLab / EIC



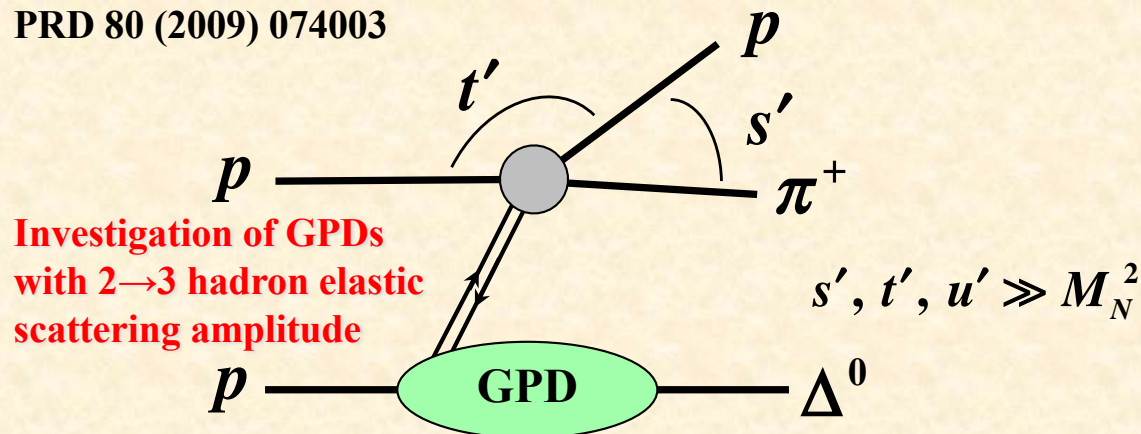
J-PARC



In future

$$K^- + p \rightarrow \Lambda_{1405} + \gamma^* ?$$

SK, M. Strikman, K. Sudoh,
PRD 80 (2009) 074003



J-W. Qiu and Z. Yu,
JHEP 08 (2022) 103;
PRD 107 (2023) 014007.

$$\pi + N \rightarrow \gamma + \gamma + N'$$

$$h + M_B \rightarrow h' + \gamma + M_D$$

$$h + M_B \rightarrow h' + M_C + M_D$$

Quark GPDs of spin-3/2 hadrons

may skip

The quark GPDs can be defined by the maxtrix element

$$A_{\lambda', \mu', \lambda \mu}^q = \int \frac{dz^-}{2\pi} e^{ix(P \cdot z)} \left\langle p', \lambda' \left| O_{\mu' \mu}^q(z) \right| p, \lambda \right\rangle_{z^+=0, \vec{z}_\perp=0}, \quad \mu, \mu': \text{parton helicities}$$

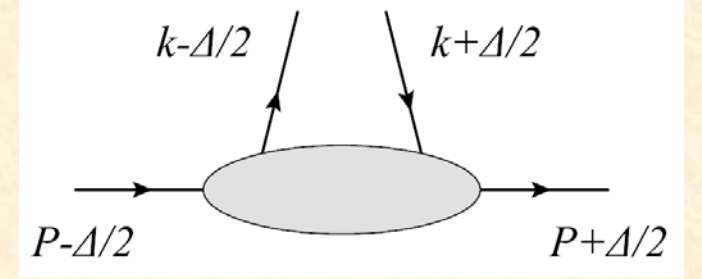
and the quark transversity GPDs are defined by

$$T_{\lambda' \lambda}^{qi} = \frac{1}{2} \int \frac{dz^-}{2\pi} e^{ix(P \cdot z)} \left\langle p', \lambda' \left| \bar{\psi} \left(-\frac{1}{2} z \right) i \sigma^{ni} \psi \left(\frac{1}{2} z \right) \right| p, \lambda \right\rangle_{z^+=0, \vec{z}_\perp=0} = -\bar{u}_\alpha(p', \lambda') H^{qT, i, \alpha', \alpha}(x, \xi, t) u_\alpha(p, \lambda)$$

$$\sigma^{\mu\nu} = \frac{i}{2} [\gamma^\mu, \gamma^\nu]$$

$u_\alpha(p', \lambda')$: Rarita-Schwinger spinor

$H^{qT, i, \alpha', \alpha}(x, \xi, t)$: tensor function which can be decomposed into 16 terms.



See also

D.-Y. Fu, B.-D. Sun, and Y.-B. Dong,
Phys. Rev. D 106 (2022) 116012.

$$\begin{aligned} H^{qT, i, \alpha', \alpha} = & H_1^{qT} \frac{i \sigma^{ni}}{P \cdot n} g^{\alpha' \alpha} + H_2^{qT} \frac{n^{[\alpha'} g^{\alpha]i}}{P \cdot n} + H_3^{qT} \frac{(\not{P} P^i - P \cdot n \gamma^i)}{M P \cdot n} g^{\alpha' \alpha} + H_4^{qT} \frac{(\not{P} P^i - P \cdot n \gamma^i)}{M^3 P \cdot n} P^{\alpha'} P^\alpha \\ & + H_5^{qT} \frac{(\not{P} \Delta^i - \Delta \cdot n \gamma^i)}{M P \cdot n} g^{\alpha' \alpha} + H_6^{qT} \frac{(\not{P} \Delta^i - \Delta \cdot n \gamma^i)}{M^3 P \cdot n} P^{\alpha'} P^\alpha + H_7^{qT} \frac{(\Delta^i + 2\xi P^i)}{M^2} g^{\alpha' \alpha} + H_8^{qT} \frac{(\Delta^i + 2\xi P^i)}{M^4} P^{\alpha'} P^\alpha \\ & + H_9^{qT} \frac{(\Delta \cdot n n^{[\alpha'} g^{\alpha]i} - 2n^{\alpha'} n^\alpha \Delta^i)}{(P \cdot n)^2} + H_{10}^{qT} \frac{(P \cdot n n^{[\alpha'} g^{\alpha]i} - 2n^{\alpha'} n^\alpha P^i)}{(P \cdot n)^2} \\ & + H_{11}^{qT} \frac{(\Delta \cdot n P^{[\alpha'} g^{\alpha]i} - P^{[\alpha'} n^{\alpha]} \Delta^i)}{M^2 P \cdot n} + H_{12}^{qT} \frac{(P \cdot n P^{[\alpha'} g^{\alpha]i} - P^{[\alpha'} n^{\alpha]} P^i)}{M^2 (P \cdot n)^2} \\ & + H_{13}^{qT} \frac{M \not{P} (\Delta \cdot n n^{[\alpha'} g^{\alpha]i} - 2n^{\alpha'} n^\alpha \Delta^i)}{(P \cdot n)^3} + H_{14}^{qT} \frac{M \not{P} (P \cdot n n^{[\alpha'} g^{\alpha]i} - 2n^{\alpha'} n^\alpha P^i)}{(P \cdot n)^3} \\ & + H_{15}^{qT} \frac{\not{P} (\Delta \cdot n P^{[\alpha'} g^{\alpha]i} - P^{[\alpha'} n^{\alpha]} \Delta^i)}{M (P \cdot n)^2} + H_{16}^{qT} \frac{\not{P} (P \cdot n P^{[\alpha'} g^{\alpha]i} - P^{[\alpha'} n^{\alpha]} P^i)}{M (P \cdot n)^2} \end{aligned}$$

$$a^{[\mu} b^{\nu]} = a^\mu b^\nu - a^\nu b^\mu, \quad a^{\{\mu} b^{\nu\}} = a^\mu b^\nu + a^\nu b^\mu$$

Even and odd properties with respect to the skewness ξ

$$H_i^{qT}(x, \xi, t) = H_i^{qT}(x, -\xi, t) \quad i = 1, 2, 5 - 9, 12, 13, 16$$

$$H_i^{qT}(x, \xi, t) = -H_i^{qT}(x, -\xi, t) \quad i = 3, 4, 10, 11, 14, 15$$

Gluon GPDs of spin-3/2 hadrons

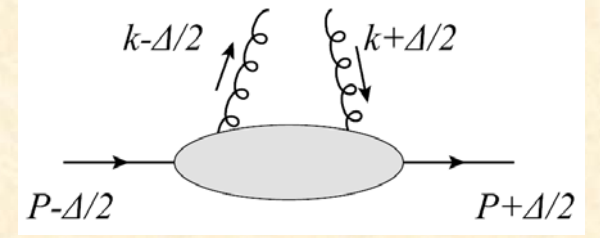
may skip

The gluon GPDs can be defined by the maxtrix element

$$A_{\lambda'\mu',\lambda\mu}^g = \frac{1}{P^+} \int \frac{dz^-}{2\pi} e^{ix(P \cdot z)} \left\langle p', \lambda' | O_{\mu'\mu}^g(z) | p, \lambda \right\rangle_{z^+=0, \vec{z}_\perp=0}$$

and the gluon transversity GPDs are defined by

$$T_{\lambda'\lambda}^{qi} = \frac{1}{2P^+} \int \frac{dz^-}{2\pi} e^{ix(P \cdot z)} \left\langle p', \lambda' | G^{ni} \left(-\frac{1}{2} z \right) G^{jn} \left(\frac{1}{2} z \right) | p, \lambda \right\rangle_{z^+=0, \vec{z}_\perp=0} = -\bar{u}_{\alpha'}(p', \lambda') H^{gT, ij, \alpha', \alpha}(x, \xi, t) u_\alpha(p, \lambda)$$



$$\begin{aligned} H^{gT, ij, \alpha', \alpha} = & \hat{S}_{ij} \left[H_1^{gT} \frac{(\Delta^i + 2\xi P^i) i \sigma^{nj}}{M P \cdot n} g^{\alpha\alpha} + H_2^{gT} \frac{n^{[\alpha'} g^{\alpha]i} (\not{P} P^j - P \cdot n \gamma^j)}{(P \cdot n)^2} \right. \\ & + H_3^{gT} \frac{(\Delta^i + 2\xi P^i) n^{[\alpha'} g^{\alpha]j}}{M P \cdot n} + H_4^{gT} \frac{(\Delta^i + 2\xi P^i)(\Delta^j + 2\xi P^j) P^{\alpha'} P^\alpha}{M^5} \\ & + H_5^{gT} \frac{(P \cdot n g^{\alpha'i} - n^{\alpha'} P^i)(P \cdot n g^{\alpha'j} - n^\alpha P^j)}{M (P \cdot n)^2} + H_6^{gT} \frac{(\Delta \cdot n g^{\alpha'i} - n^{\alpha'} \Delta^i)(\Delta \cdot n g^{\alpha'j} - n^\alpha \Delta^j)}{M (P \cdot n)^2} \\ & + H_7^{gT} \frac{(\Delta^i + 2\xi P^i)(\Delta \cdot n n^{[\alpha'} g^{\alpha]j} - 2n^{\alpha'} n^\alpha \Delta^j)}{M (P \cdot n)^2} + H_8^{gT} \frac{(\Delta^i + 2\xi P^i)(P \cdot n n^{[\alpha'} g^{\alpha]j} - 2n^{\alpha'} n^\alpha P^j)}{M (P \cdot n)^2} \\ & + H_9^{gT} \frac{(\Delta^i + 2\xi P^i)(\Delta \cdot n P^{[\alpha'} g^{\alpha]j} - P^{[\alpha'} n^{\alpha] \Delta^j})}{M^3 P \cdot n} + H_{10}^{gT} \frac{(\Delta^i + 2\xi P^i)(P \cdot n P^{[\alpha'} g^{\alpha]j} - P^{[\alpha'} n^{\alpha] P^j})}{M^3 P \cdot n} \\ & + H_{11}^{gT} \frac{(P \cdot n g^{\alpha'i} - n^{\alpha'} P^i) \not{n} (P \cdot n g^{\alpha'j} - n^\alpha P^j)}{(P \cdot n)^3} + H_{12}^{gT} \frac{(\Delta \cdot n g^{\alpha'i} - n^{\alpha'} \Delta^i) \not{n} (\Delta \cdot n g^{\alpha'j} - n^\alpha \Delta^j)}{(P \cdot n)^3} \\ & + H_{13}^{gT} \frac{(\Delta^i + 2\xi P^i) \not{n} (\Delta \cdot n n^{[\alpha'} g^{\alpha]j} - 2n^{\alpha'} n^\alpha \Delta^j)}{(P \cdot n)^3} + H_{14}^{gT} \frac{(\Delta^i + 2\xi P^i) \not{n} (P \cdot n n^{[\alpha'} g^{\alpha]j} - 2n^{\alpha'} n^\alpha P^j)}{(P \cdot n)^3} \\ & \left. + H_{15}^{gT} \frac{(\Delta^i + 2\xi P^i) \not{n} (\Delta \cdot n P^{[\alpha'} g^{\alpha]j} - P^{[\alpha'} n^{\alpha] \Delta^j})}{M^2 (P \cdot n)^2} + H_{16}^{gT} \frac{(\Delta^i + 2\xi P^i) \not{n} (P \cdot n P^{[\alpha'} g^{\alpha]j} - P^{[\alpha'} n^{\alpha] P^j})}{M^2 (P \cdot n)^2} \right] \end{aligned}$$

Even and odd properties with respect to the skewness ξ

$$H_i^{gT}(x, \xi, t) = H_i^{gT}(x, -\xi, t) \quad i = 1, 3 - 7, 10 - 13, 16$$

$$H_i^{gT}(x, \xi, t) = -H_i^{gT}(x, -\xi, t) \quad i = 2, 8, 9, 14, 15$$

Quark transversity GPDs of spin-3/2 Δ by diquark spectator model

may skip

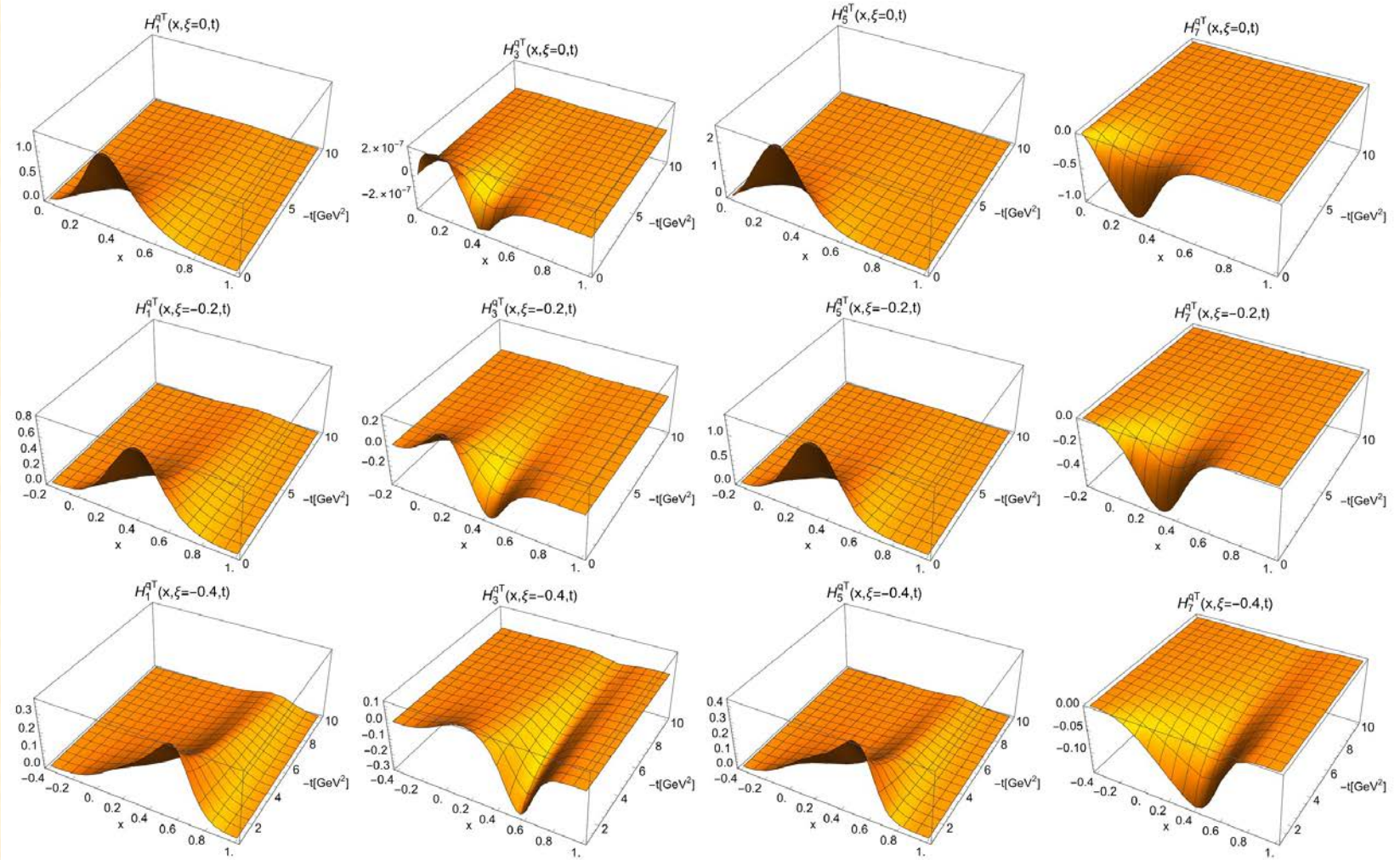
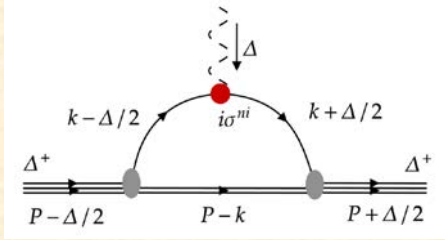
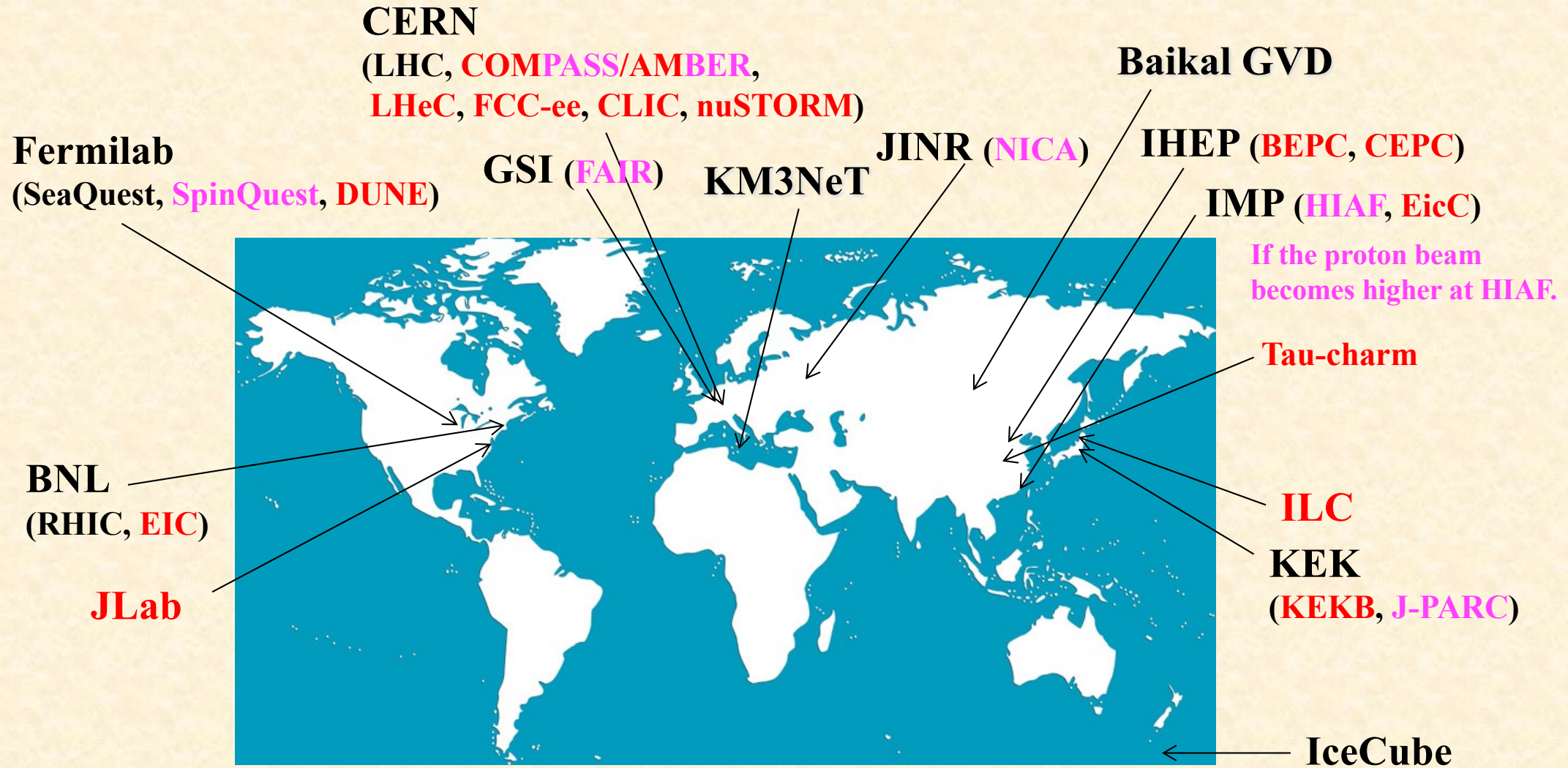


Figure 4: The 3D d quark transversity GPDs of Δ^+ $H_{1,3,5,7}^{qT}$ at $\xi = 0, -0.2, -0.4$.

Prospects on GPD projects

High-energy hadron physics experiments



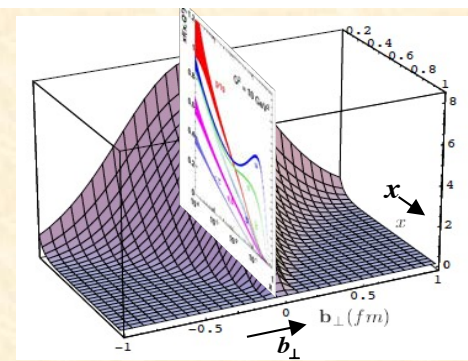
Facilities on hadron structure functions on GPDs including future possibilities.

Hadron accelerator facilities. Lepton accelerator facilities.

By hadron tomography

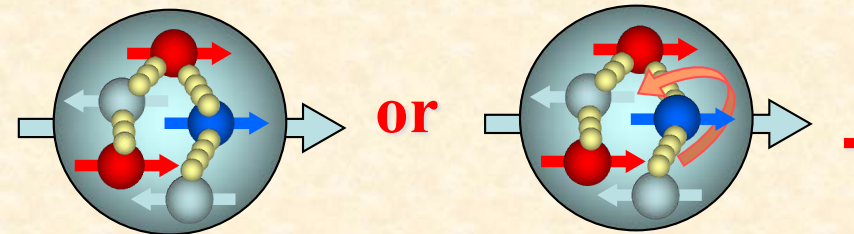


3D view
of hadrons

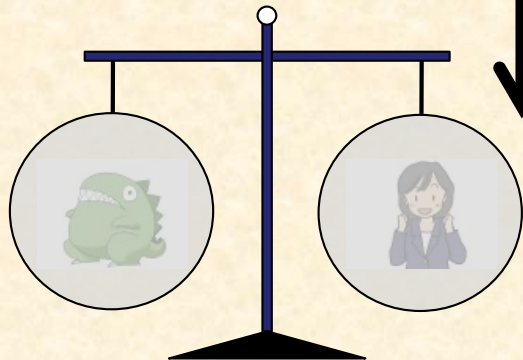


Origin of nucleon spin

By the tomography, we determine



Exotic hadrons



By tomography,
we determine

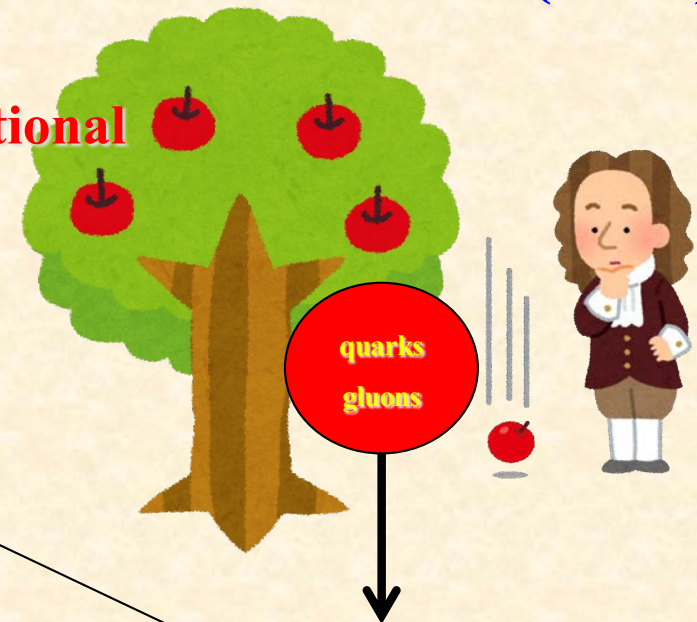


or



Origin of gravitational source (mass)

By tomography,
we determine gravitational
sources in terms of
quarks and gluons.



Part I. Summary on the GPDs

Hadron-tomography and gravitational form factors by the GPDs.

- **Puzzle to find the origin of hadron masses and pressures in terms of quark and gluon degrees of freedom**
- **Puzzle to find the origin of nucleon spin**
- **Exotic hadron candidates could be studied in the same tomography method.**
- **In addition to the electron scattering projects, the GPD studies are possible by neutrino- and hadron-beam facilities and e^+e^- colliders.**
- **transversity GPDs for spin-3/2 hadrons:
16 quark GPDs and 16 gluon GPDs**
- **Δ (Ω) GPDs could be investigated at KEKB (BES) “in principle”.**
- **There is already an experimental study at JLab on $N \rightarrow \Delta$ GPDs.**

Structure function of spin-1 hadrons

Contents

1. Introduction

- JLab experiment on spin-1 structure functions (**fully approved in 2023!**)

2. Tensor-polarized structure function b_1

- b_1 sum rule
- b_1 by “standard” deuteron model
- Tensor-polarized PDFs at Fermilab (hadron accelerator facilities by Drell-Yan)

may skip

3. Gluon transversity

- Gluon transversity at JLab and Fermilab

4. TMDs, PDFs, fragmentation functions of spin-1 hadrons

- TMDs, PDFs, and fragmentation functions up to twist 4
- Their useful relations
- Other works, semi-inclusive DIS, twist-3 PDF f_{LT}

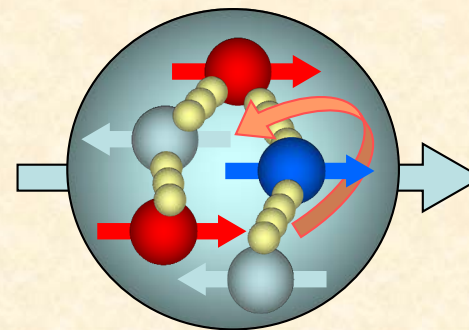
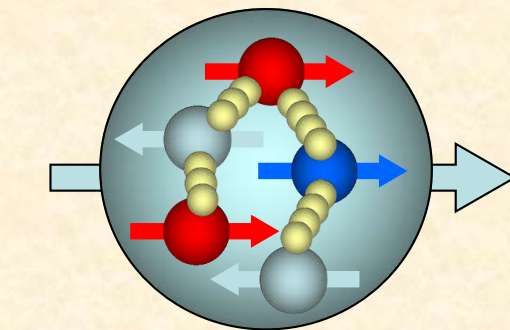
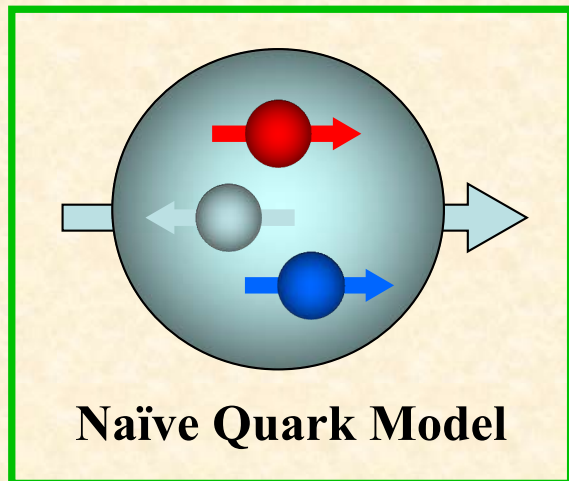
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5. Future prospects and summary

Nucleon spin

Almost none of nucleon spin is carried by quarks!

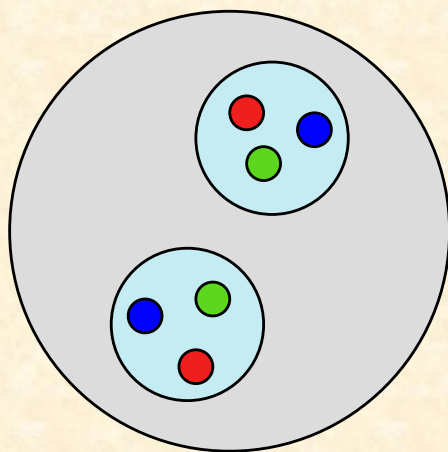
→ Nucleon spin puzzle!?



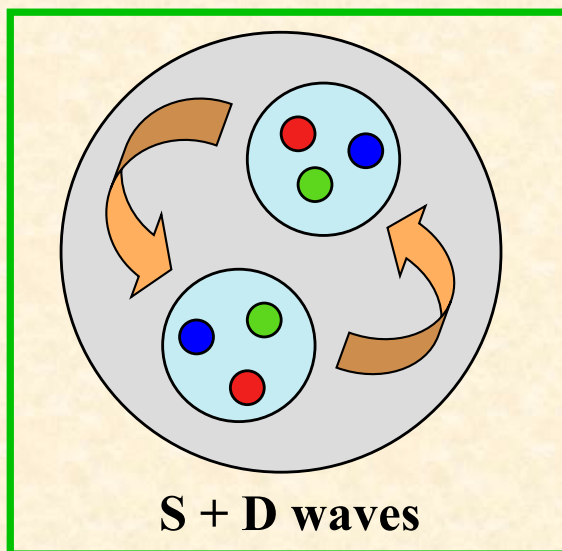
“old” standard model

Tensor structure b_1 (e.g. deuteron)

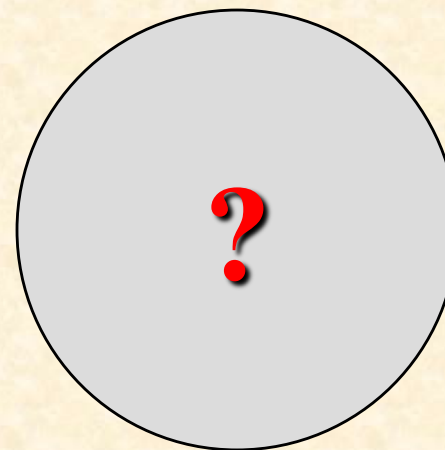
Tensor-structure puzzle!?



$b_1 = 0$



standard model $b_1 \neq 0$



$b_1^{\text{experiment}} \neq b_1^{\text{“standard model”}}$

Gluon transversity $\Delta_T g$

Note on our notations:

Tensor-polarized gluon distribution: $\delta_T g$

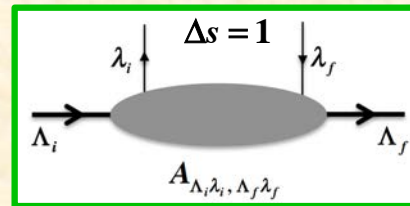
Gluon transversity: $\Delta_T g$

Helicity amplitude $A(\Lambda_i, \lambda_i, \Lambda_f, \lambda_f)$, conservation $\Lambda_i - \lambda_i = \Lambda_f - \lambda_f$

Longitudinally-polarized quark in nucleon: $\Delta q(x) \sim A\left(+\frac{1}{2} + \frac{1}{2}, +\frac{1}{2} + \frac{1}{2}\right) - A\left(+\frac{1}{2} - \frac{1}{2}, +\frac{1}{2} - \frac{1}{2}\right)$

Quark transversity in nucleon:

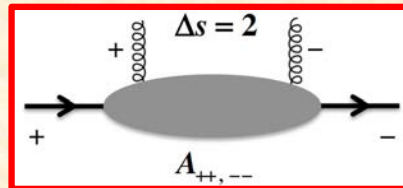
$\Delta_T q(x) \sim A\left(+\frac{1}{2} + \frac{1}{2}, -\frac{1}{2} - \frac{1}{2}\right)$, $\lambda_i = +\frac{1}{2} \rightarrow \lambda_f = -\frac{1}{2}$ quark spin flip ($\Delta s = 1$)



Gluon transversity in deuteron:

$\Delta_T g(x) \sim A(+1 + 1, -1 - 1)$,

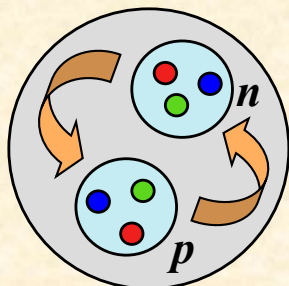
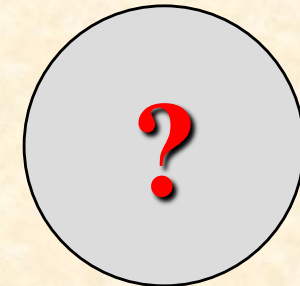
$A\left(+\frac{1}{2} + 1, -\frac{1}{2} - 1\right)$ not possible for nucleon



Note: Gluon transversity does not exist for spin-1/2 nucleons.

$b_1 (\delta_T q, \delta_T g) \neq 0 \Leftrightarrow$ still $\Delta_T g = 0$

What would be the mechanism(s)
for creating $\Delta_T g \neq 0$?



S + D waves

Physics beyond “the standard model” in nuclear physics?
(Physics beyond the standard model in particle physics???)

JLab PAC-38 (2011) proposal, PR12-11-110

Full approval in 2023

The Deuteron Tensor Structure Function b_1

2011

A Proposal to Jefferson Lab PAC-38.
(Update to LOI-11-003)

J.-P. Chen (co-spokesperson), P. Solvignon (co-spokesperson),
K. Allada, A. Camsonne, A. Deur, D. Gaskell,
C. Keith, S. Wood, J. Zhang
Thomas Jefferson National Accelerator Facility, Newport News, VA 23606

N. Kalantarians (co-spokesperson), O. Rondon (co-spokesperson)
Donal B. Day, Hovhannes Baghdasaryan, Charles Hanretty
Richard Lindgren, Blaine Norum, Zhihong Ye
University of Virginia, Charlottesville, VA 22903

PR12-13-011

The Deuteron Tensor Structure Function b_1

2023

A Proposal to Jefferson Lab PAC-40
(Update to PR12-11-110)

K. Allada, A. Camsonne, J.-P. Chen,[†]
A. Deur, D. Gaskell, M. Jones, C. Keith, J. Pierce,
P. Solvignon,[†] S. Wood, J. Zhang
Thomas Jefferson National Accelerator Facility, Newport News, VA 23606

Rui
Seon

O. Rondon Aramayo,[‡] D. Crabb, D. B. Day,
C. Hanretty, D. Keller,[‡] R. Lindgren, S. Liuti, B. Norum,
Zhihong Ye, X. Zheng
University of Virginia, Charlottesville, VA 22903

N. Kalantarians[‡]
Hampton University, Hampton VA 23668

T. Badman, J. Calarco, J. Dawson,
S. Phillips, E. Long,[‡] K. Slifer^{‡,§}, R. Zielinski
University of New Hampshire, Durham, NH 03861

G. Ron
Hebrew University of Jerusalem, Jerusalem

W. Bertozzi, S. Gilad, J. Huang
A. Kelleher, V. Sulkosky
Massachusetts Institute of Technology, Cambridge, MA 02139

b_1 experiment

Gluon transversity

A Letter of Intent to Jefferson Lab PAC 44, June 6, 2016
Search for Exotic Gluonic States in the Nucleus

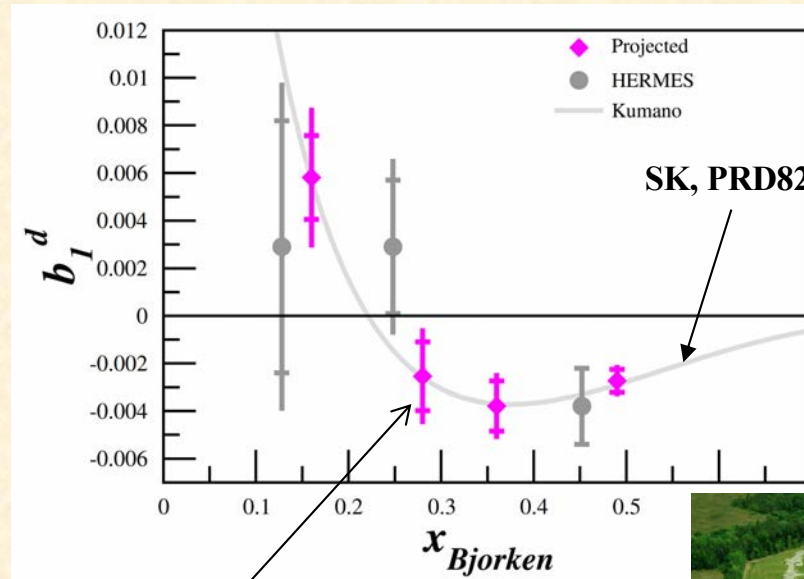
M. Jones, C. Keith, J. Maxwell*, D. Meekins
Thomas Jefferson National Accelerator Facility, Newport News, VA 23606

W. Detmold, R. Jaffe, R. Milner, P. Shanahan
Laboratory for Nuclear Science, MIT, Cambridge, MA 02139

D. Crabb, D. Day, D. Keller, O. A. Rondon
University of Virginia, Charlottesville, VA 22904

J. Pierce

Oak Ridge National Laboratory, Oak Ridge, TN 37831



Expected errors by JLab

See J. Poudel, A. Bacchetta, J.-P. Chen, N. Santiesteban, EPJA 61 (2025) 81 for updated information.



Eur. Phys. J. A (2025) 61:81
<https://doi.org/10.1140/epja/s10050-025-01558-w>

Review

THE EUROPEAN
PHYSICAL JOURNAL A



Experimental study of tensor structure function of deuteron

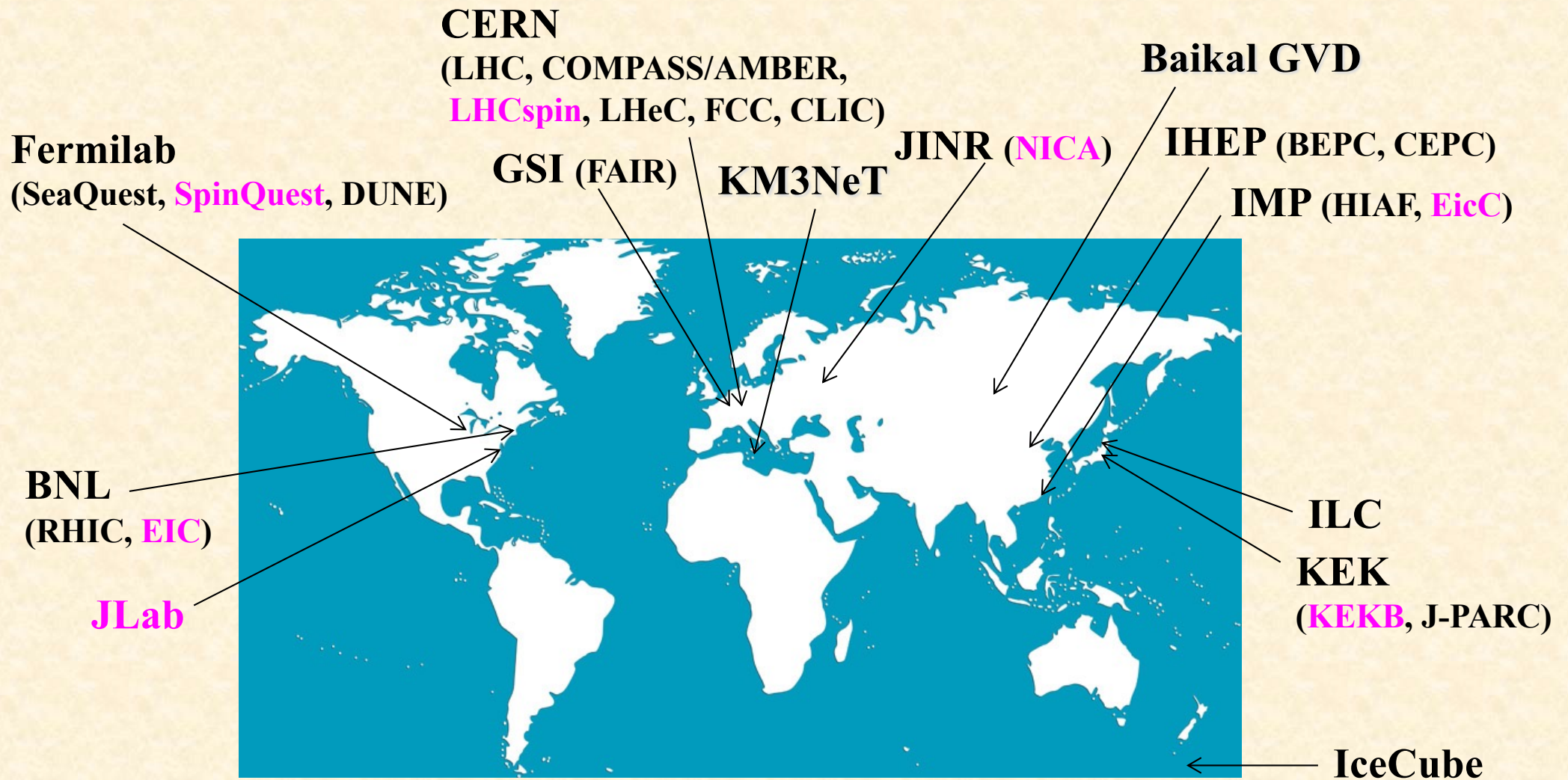
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High-energy hadron physics experiments



Facilities on spin-1 hadron structure functions including future possibilities.

Tensor-polarized structure function b_1

Electron scattering from a spin-1 hadron

P. Hoodbhoy, R. L. Jaffe, and A. Manohar, NP B312 (1989) 571.

[L. L. Frankfurt and M. I. Strikman, NP A405 (1983) 557.]

$$W_{\mu\nu} = \boxed{-F_1 g_{\mu\nu} + F_2 \frac{p_\mu p_\nu}{\nu} + g_1 \frac{i}{\nu} \epsilon_{\mu\nu\lambda\sigma} q^\lambda s^\sigma + g_2 \frac{i}{\nu^2} \epsilon_{\mu\nu\lambda\sigma} q^\lambda (p \cdot q s^\sigma - s \cdot q p^\sigma)} \quad \text{spin-1/2, spin-1}$$

$$\boxed{-b_1 r_{\mu\nu} + \frac{1}{6} b_2 (s_{\mu\nu} + t_{\mu\nu} + u_{\mu\nu}) + \frac{1}{2} b_3 (s_{\mu\nu} - u_{\mu\nu}) + \frac{1}{2} b_4 (s_{\mu\nu} - t_{\mu\nu})} \quad \text{spin-1 only}$$

Note: Obvious factors from $q^\mu W_{\mu\nu} = q^\nu W_{\mu\nu} = 0$ are not explicitly written.

$E^\mu =$ polarization vector

$$\nu = p \cdot q, \quad \kappa = 1 + M^2 Q^2 / \nu^2, \quad E^2 = -M^2, \quad s^\sigma = -\frac{i}{M^2} \epsilon^{\sigma\alpha\beta\tau} E_\alpha^* E_\beta p_\tau$$

$b_1, \dots, b_4$ terms are defined so that they vanish by spin average.

$$r_{\mu\nu} = \frac{1}{\nu^2} \left(q \cdot E^* q \cdot E - \frac{1}{3} \nu^2 \kappa \right) g_{\mu\nu}, \quad s_{\mu\nu} = \frac{2}{\nu^2} \left(q \cdot E^* q \cdot E - \frac{1}{3} \nu^2 \kappa \right) \frac{p_\mu p_\nu}{\nu}$$

b_1, b_2 terms are defined to satisfy $2xb_1 = b_2$ in the Bjorken scaling limit.

$$t_{\mu\nu} = \frac{1}{2\nu^2} \left(q \cdot E^* p_\mu E_\nu + q \cdot E^* p_\nu E_\mu + q \cdot E p_\mu E_\nu^* + q \cdot E p_\nu E_\mu^* - \frac{4}{3} \nu p_\mu p_\nu \right)$$

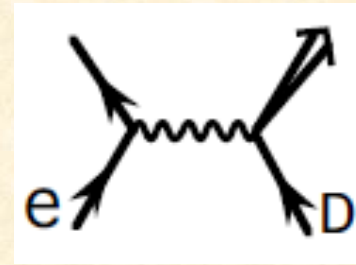
$$u_{\mu\nu} = \frac{1}{\nu} \left(E_\mu^* E_\nu + E_\nu^* E_\mu + \frac{2}{3} M^2 g_{\mu\nu} - \frac{2}{3} p_\mu p_\nu \right)$$

$2xb_1 = b_2$ in the scaling limit $\sim O(1)$

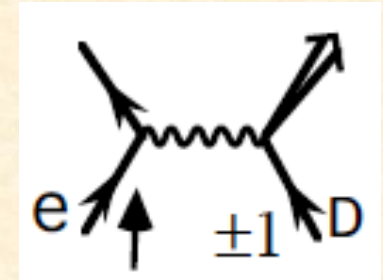
$b_3, b_4 = \text{twist-4} \sim \frac{M^2}{Q^2}$

Structure Functions

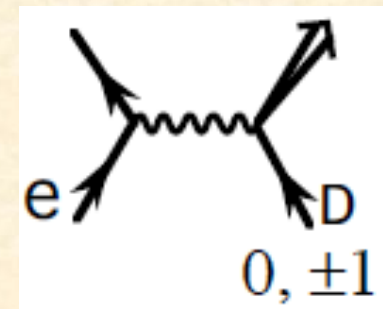
$$F_1 \propto \langle d\sigma \rangle$$



$$g_1 \propto d\sigma(\uparrow, +1) - d\sigma(\uparrow, -1)$$



$$b_1 \propto d\sigma(0) - \frac{d\sigma(+1) + d\sigma(-1)}{2}$$



note: $\sigma(0) - \frac{\sigma(+1) + \sigma(-1)}{2} = 3\langle\sigma\rangle - \frac{3}{2}[\sigma(+1) + \sigma(-1)]$

Parton Model

$$F_1 = \frac{1}{2} \sum_i e_i^2 (q_i + \bar{q}_i)$$

$$q_i = \frac{1}{3} (q_i^{+1} + q_i^0 + q_i^{-1})$$

$$g_1 = \frac{1}{2} \sum_i e_i^2 (\Delta q_i + \Delta \bar{q}_i)$$

$$\Delta q_i = q_{i\uparrow}^{+1} - q_{i\downarrow}^{+1} \quad [q_{\uparrow}^H(x, Q^2)]$$

$$b_1 = \frac{1}{2} \sum_i e_i^2 (\delta_T q_i + \delta_T \bar{q}_i)$$

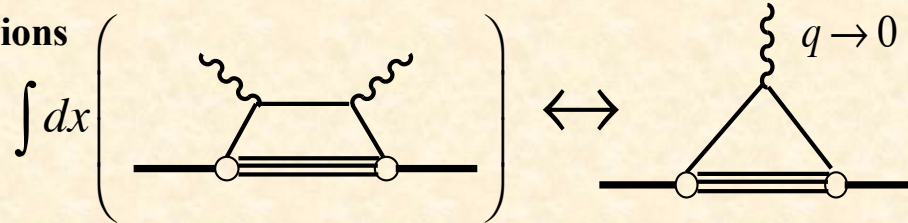
$$\delta_T q_i = q_i^0 - \frac{q_i^{+1} + q_i^{-1}}{2}$$

Sum rule for b_1

Constraint on valence-tensor polarization (sum rule)

may skip

Folow Feynman's book on
Photon-Hadron Interactions



F.E.Close and SK,
PRD42, 2377 (1990)

Intuitive derivation without calculation:

$$\int dx b_1(x) = \text{dimensionless quantity} \\ = (\text{mass})^2 \cdot (\text{quadrupole moment})$$

$$\int dx b_1^D(x) = \frac{5}{36} \int dx [\delta_T u_v(x) + \delta_T d_v(x)] + \sum_i e_i^2 \int dx \delta_T \bar{q}_{i,D}(x)$$

Elastic amplitude in a parton model

$$\Gamma_{H,H} = \langle p, H | J_0(0) | p, H \rangle = \sum_i e_i \int dx [q_i^H(x) - \bar{q}_i^H(x)]$$

$$\Gamma_{0,0} - \frac{\Gamma_{1,1} + \Gamma_{-1,-1}}{2} = \frac{1}{3} \int dx [\delta_T u_v(x) + \delta_T d_v(x)]$$

$$b_1 = \frac{1}{2} \sum_i e_i^2 (\delta_T q_i + \delta_T \bar{q}_i)$$

$$\delta_T q_i = q_i^0 - \frac{q_i^{+1} + q_i^{-1}}{2}$$

$$\delta_T q_v \equiv \delta_T q - \delta_T \bar{q}$$

Macroscopically $\Gamma_{0,0} = \lim_{t \rightarrow 0} \left[F_c(t) - \frac{t}{3M^2} F_Q(t) \right], \quad \Gamma_{+1,+1} = \Gamma_{-1,-1} = \lim_{t \rightarrow 0} \left[F_c(t) + \frac{t}{6M^2} F_Q(t) \right]$

$$\Gamma_{0,0} - \frac{1}{2} (\Gamma_{1,1} + \Gamma_{-1,-1}) = - \lim_{t \rightarrow 0} \frac{t}{2M^2} F_Q(t)$$

$$\begin{aligned} \int dx b_1^D(x) &= \frac{5}{36} 3 \left[\Gamma_{0,0} - \frac{1}{2} (\Gamma_{1,1} + \Gamma_{-1,-1}) \right] + \sum_i e_i^2 \int dx \delta_T \bar{q}_{i,D}(x) \\ &= - \frac{5}{24} \lim_{t \rightarrow 0} \frac{t}{M^2} F_Q(t) + \sum_i e_i^2 \int dx \delta_T \bar{q}_{i,D}(x) \\ &= 0 \text{ (valence)} + \sum_i e_i^2 \int dx \delta_T \bar{q}_{i,D}(x) \end{aligned}$$

$$\int dx b_1^D(x) = - \frac{5}{24} \lim_{t \rightarrow 0} \frac{t}{M^2} F_Q(t) + \sum_i e_i^2 \int dx \delta_T \bar{q}_{i,D}(x)$$

Constraint on tensor-polarized

valence quarks: $\int dx \delta_T q_v(x) = 0$

Similarity to the Gottfried sum rule

SK, Phys. Rept. 303 (1998) 183.

may skip

$$\begin{aligned}
 S_G &= \int_0^1 \frac{dx}{x} [F_2^{\mu p}(x) - F_2^{\mu n}(x)] \\
 &= \frac{1}{3} + \frac{2}{3} \int_0^1 dx [\bar{u}(x) - \bar{d}(x)] \\
 &= \frac{1}{3} \quad \text{if } \bar{u} = \bar{d}
 \end{aligned}$$

(Gottfried sum rule)

$$\begin{aligned}
 F_2^{\mu p}(x)_{\text{LO}} &= x \left[\frac{4}{9} \{u(x) + \bar{u}(x)\} + \frac{1}{9} \{d(x) + \bar{d}(x)\} + \frac{1}{9} \{s(x) + \bar{s}(x)\} \right] \\
 F_2^{\mu n}(x)_{\text{LO}} &= x \left[\frac{4}{9} \{u(x) + \bar{u}(x)\} + \frac{1}{9} \{d(x) + \bar{d}(x)\} + \frac{1}{9} \{s(x) + \bar{s}(x)\} \right]_n \\
 &= x \left[\frac{4}{9} \{d(x) + \bar{d}(x)\} + \frac{1}{9} \{u(x) + \bar{u}(x)\} + \frac{1}{9} \{s(x) + \bar{s}(x)\} \right] \\
 \frac{1}{x} [F_2^{\mu p}(x)_{\text{LO}} - F_2^{\mu n}(x)_{\text{LO}}] &= \frac{3}{9} \{u(x) + \bar{u}(x)\} - \frac{3}{9} \{d(x) + \bar{d}(x)\} \\
 \int_0^1 \frac{dx}{x} [F_2^{\mu p}(x)_{\text{LO}} - F_2^{\mu n}(x)_{\text{LO}}] &= \int_0^1 dx \left[\frac{1}{3} \{u_v(x) + 2\bar{u}(x)\} - \frac{1}{3} \{d_v(x) + 2\bar{d}(x)\} \right] \\
 &= \frac{2}{3} - \frac{1}{3} + \frac{2}{3} \int_0^1 dx [\bar{u}(x) - \bar{d}(x)]
 \end{aligned}$$

NMC measurement (PRL 66 (1991) 2712; PRD 50 (1994) R1)

$$\int_{0.004}^{0.8} \frac{dx}{x} [F_2^{\mu p}(x) - F_2^{\mu n}(x)] = 0.221 \pm 0.008 \pm 0.019$$

Extrapolating the NMC data, they obtained

$$S_G = 0.235 \pm 0.026$$

30% is missing! $\Rightarrow \bar{u} < \bar{d}$?

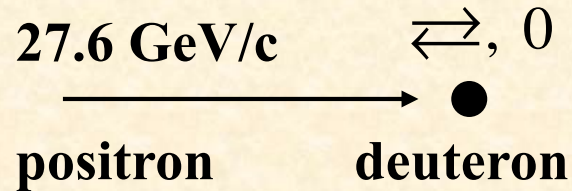
$$\int_0^1 \frac{dx}{x} [F_2^p(x) - F_2^n(x)] = \frac{1}{3} + \frac{2}{3} \int_0^1 dx [\bar{u}(x) - \bar{d}(x)]$$

$$\int dx b_1^D(x) = \lim_{t \rightarrow 0} -\frac{5}{24} \frac{t}{M^2} F_Q(t) + \sum_i e_i^2 \int dx \delta_T \bar{q}_i(x)$$

As the Gottfried-sum-rule violation indicated $\bar{u} < \bar{d}$, the b_1 -sum-rule violation suggests a finite tensor polarization for antiquarks ($\delta_T \bar{u} \neq 0$).

HERMES results on b_1

A. Airapetian *et al.* (HERMES), PRL 95 (2005) 242001.



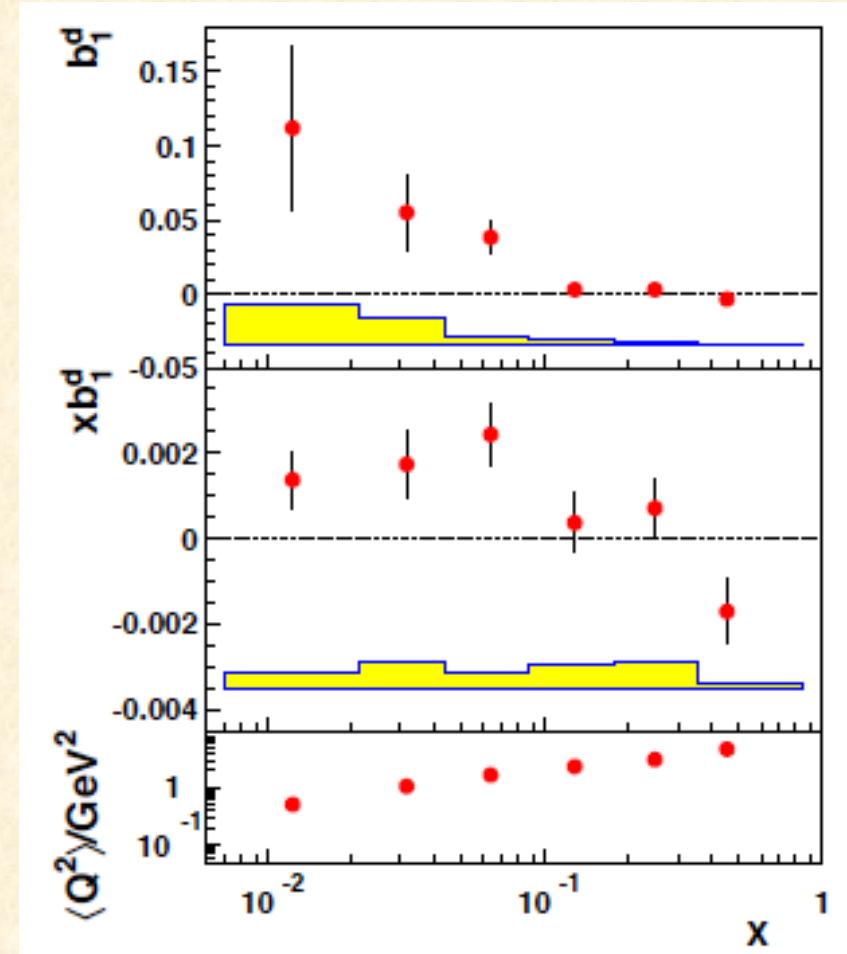
b_1 measurement in the kinematical region

$$0.01 < x < 0.45, \quad 0.5 \text{ GeV}^2 < Q^2 < 5 \text{ GeV}^2$$

b_1 sum in the restricted Q^2 range $Q^2 > 1 \text{ GeV}^2$

$$\int_{0.02}^{0.85} dx b_1(x) = [0.35 \pm 0.10(\text{stat}) \pm 0.18(\text{sys})] \times 10^{-2}$$

at $Q^2 = 5 \text{ GeV}^2$



$$\int dx b_1^D(x) = \lim_{t \rightarrow 0} -\frac{5}{24} \frac{t}{M^2} F_Q(t) + \sum_i e_i^2 \int dx \delta_T \bar{q}_i(x) = 0 ?$$

$$\int \frac{dx}{x} [F_2^p(x) - F_2^n(x)] = \frac{1}{3} \int dx [u_v - d_v] + \frac{2}{3} \int dx [\bar{u} - \bar{d}] \neq 1/3$$

Drell-Yan experiments probe these antiquark distributions.

**“Standard” deuteron model
prediction for b_1**

Basic convolution approach

may skip

Convolution model: $A_{hH,hH}(x,Q^2) = \int \frac{dy}{y} \sum_s f_s^H(y) \hat{A}_{hs,hs}(x/y, Q^2) \equiv \sum_s f_s^H(y) \otimes \hat{A}_{hs,hs}(y, Q^2)$

$$A_{hH,h'H'} = \varepsilon_{h'}^{*\mu} W_{\mu\nu}^{H'H} \varepsilon_h^\nu, \quad b_1 = A_{+0,+0} - \frac{A_{++,++} + A_{+,-,+}}{2}$$

$$\hat{A}_{+\uparrow,+\uparrow} = F_1 - g_1, \quad \hat{A}_{+\downarrow,+\downarrow} = F_1 + g_1$$

Momentum distribution: $f^H(y) = \int d^3 p \, y |\phi^H(\vec{p})|^2 \delta\left(y - \frac{E - p_z}{M_N}\right)$

$$y = \frac{Mp \cdot q}{M_N P \cdot q} \simeq \frac{2p^-}{P^-}, \quad f^H(y) \equiv f_{\uparrow}^H(y) + f_{\downarrow}^H(y)$$

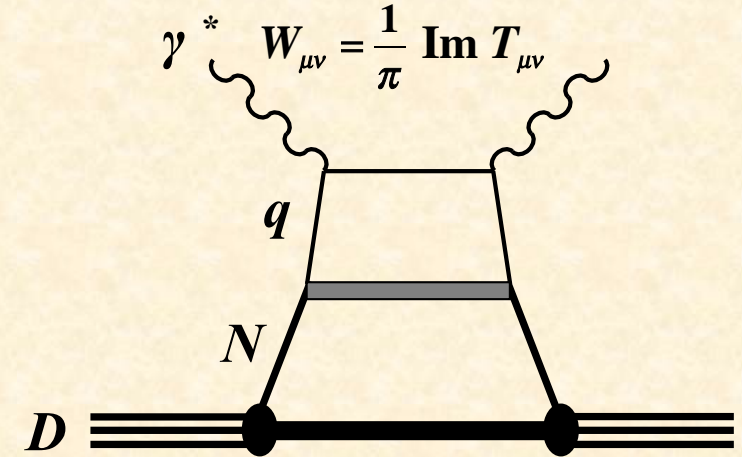
D-state admixture: $\phi^H(\vec{p}) = \phi_{\ell=0}^H(\vec{p}) + \phi_{\ell=2}^H(\vec{p})$

↓

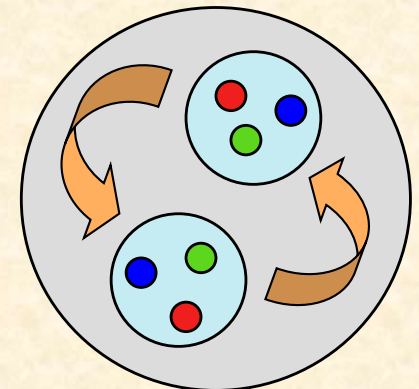
$$b_1(x) = \int \frac{dy}{y} \delta_T f(y) F_1^N(x/y, Q^2)$$

$$\delta_T f(y) = f^0(y) - \frac{f^+(y) + f^-(y)}{2}$$

$$= \int d^3 p \, y \left[-\frac{3}{4\sqrt{2}\pi} \phi_0(p) \phi_2(p) + \frac{3}{16\pi} |\phi_2(p)|^2 \right] (3 \cos^2 \theta - 1) \delta\left(y - \frac{p \cdot q}{M_N \nu}\right)$$



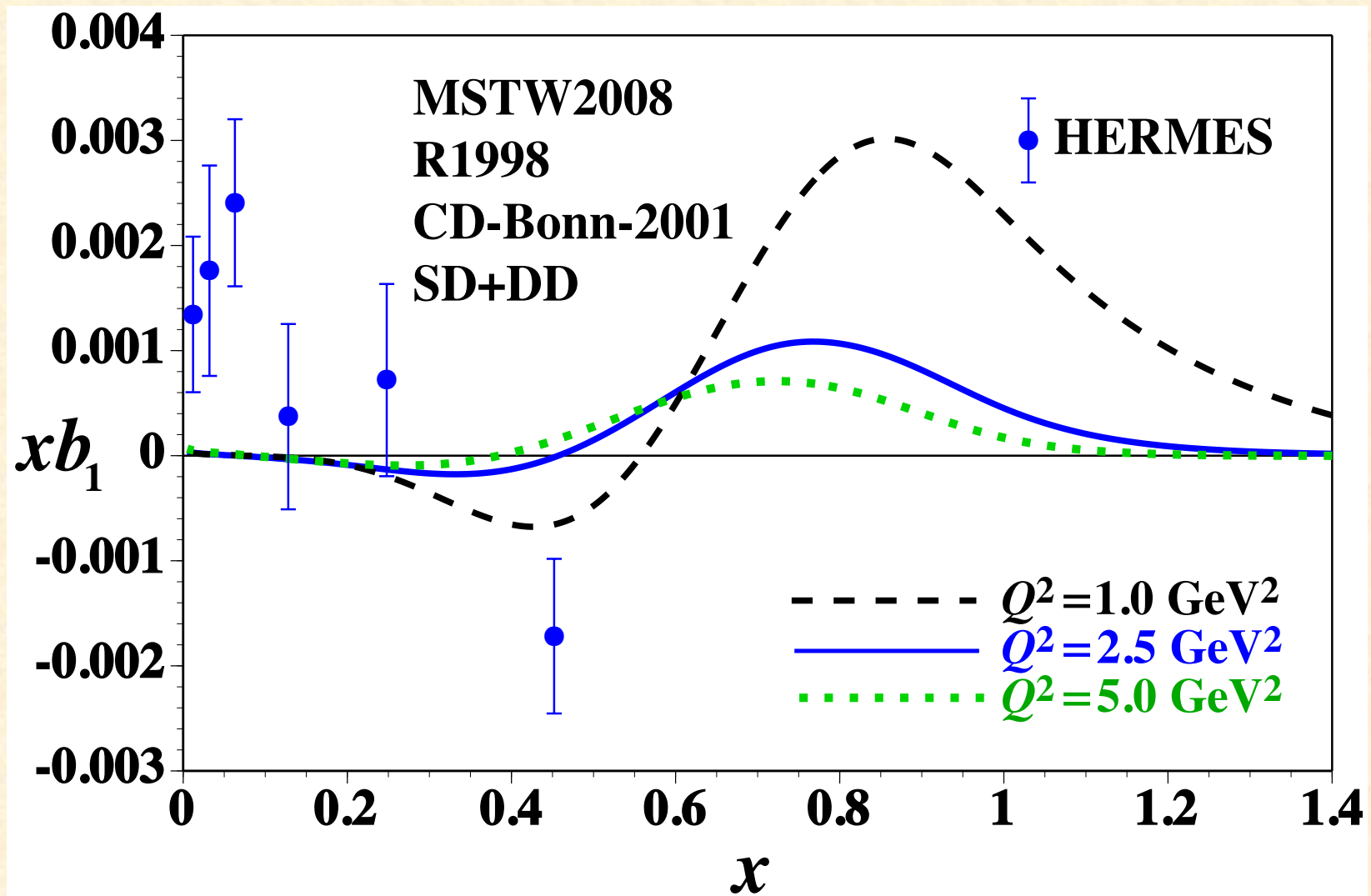
**Standard model
of the deuteron**



S + D waves

Comparison with HERMES measurements

may skip



“Standard-model” prediction for b_1 of deuteron

$$b_1(x) = \int \frac{dy}{y} \delta_T f(y) F_1^N(x/y, Q^2), \quad y = \frac{Mp \cdot q}{M_N P \cdot q} \simeq \frac{2p^-}{P^-}$$

$$\delta_T f(y) = f^0(y) - \frac{f^+(y) + f^-(y)}{2}$$

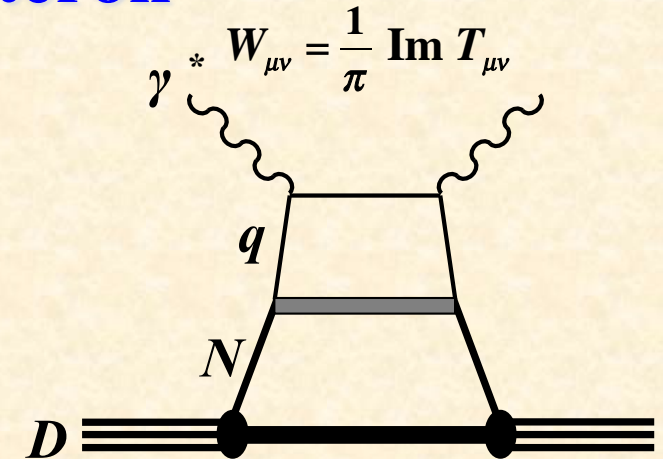
$$= \int d^3 p y \left[-\frac{3}{4\sqrt{2}\pi} \phi_0(p) \phi_2(p) + \frac{3}{16\pi} |\phi_2(p)|^2 \right] (3 \cos^2 \theta - 1) \delta\left(y - \frac{p \cdot q}{M_N v}\right)$$

S-D term **D-D term**

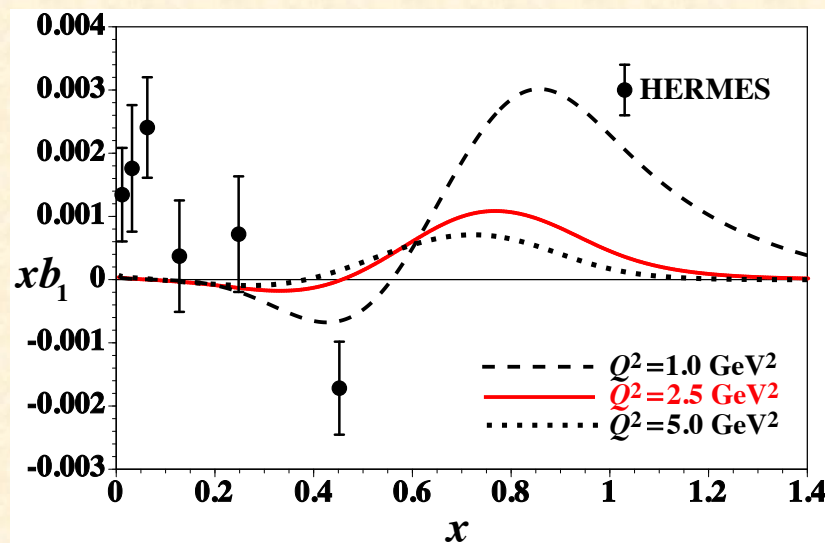
Nucleon momentum distribution:

$$f^H(y) \equiv f_{\uparrow}^H(y) + f_{\downarrow}^H(y) = \int d^3 p y |\phi^H(\vec{p})|^2 \delta\left(y - \frac{E - p_z}{M_N}\right)$$

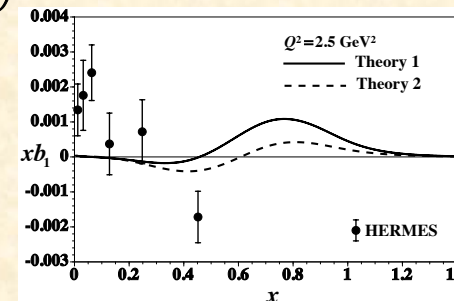
D-state admixture: $\phi^H(\vec{p}) = \phi_{\ell=0}^H(\vec{p}) + \phi_{\ell=2}^H(\vec{p})$



**Standard model
of the deuteron**



W. Cosyn, Yu-Bing Dong, SK, M. Sargsian,
Phys. Rev. D 95 (2017) 074036.



$|b_1(\text{theory})| \ll |b_1(\text{HERMES})|$
at $x < 0.5$

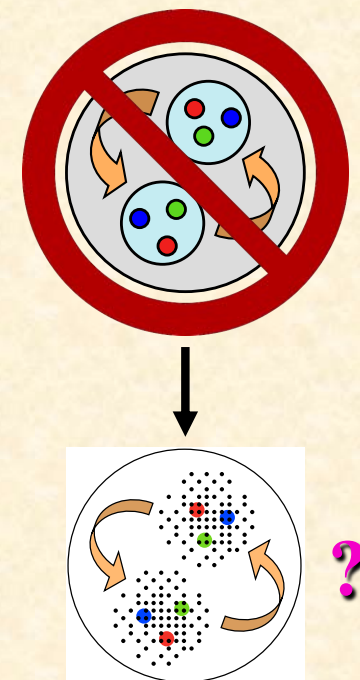
**Standard convolution model does not
work for the deuteron tensor structure!?**

G. A. Miller, PRC 89 (2014) 045203,

Interesting suggestions:

hidden-color, 6-quark, $\dots$

$|6q\rangle = |NN\rangle + |\Delta\Delta\rangle + |CC\rangle + \dots$



Tensor-polarized PDFs
at hadron accelerator facilities
(*e.g.* Fermilab)

Spin asymmetries in the parton model

unpolarized: q_a ,

longitudinally polarized: Δq_a ,

transversely polarized: $\Delta_T q_a$,

tensor polarized: δq_a

Unpolarized cross section

$$\left\langle \frac{d\sigma}{dx_A dx_B d\Omega} \right\rangle = \frac{\alpha^2}{4Q^2} (1 + \cos^2 \theta) \frac{1}{3} \sum_a e_a^2 [q_a(x_A) \bar{q}_a(x_B) + \bar{q}_a(x_A) q_a(x_B)]$$

Spin asymmetries

$$A_{LL} = \frac{\sum_a e_a^2 [\Delta q_a(x_A) \Delta \bar{q}_a(x_B) + \Delta \bar{q}_a(x_A) \Delta q_a(x_B)]}{\sum_a e_a^2 [q_a(x_A) \bar{q}_a(x_B) + \bar{q}_a(x_A) q_a(x_B)]}$$

M. Hino and SK,
PRD 59 (1999) 094026;
60 (1999) 054018.

$$A_{TT} = \frac{\sin^2 \theta \cos(2\phi)}{1 + \cos^2 \theta} \frac{\sum_a e_a^2 [\Delta_T q_a(x_A) \Delta_T \bar{q}_a(x_B) + \Delta_T \bar{q}_a(x_A) \Delta_T q_a(x_B)]}{\sum_a e_a^2 [q_a(x_A) \bar{q}_a(x_B) + \bar{q}_a(x_A) q_a(x_B)]}$$

$$A_{UQ_0} = \frac{\sum_a e_a^2 [q_a(x_A) \delta_T \bar{q}_a(x_B) + \bar{q}_a(x_A) \delta_T q_a(x_B)]}{2 \sum_a e_a^2 [q_a(x_A) \bar{q}_a(x_B) + \bar{q}_a(x_A) q_a(x_B)]}$$

$$\begin{aligned} A_{LT} &= A_{TL} = A_{UT} = A_{TU} = A_{TQ_0} = A_{UQ_1} \\ &= A_{LQ_1} = A_{TQ_1} = A_{UQ_2} = A_{LQ_2} = A_{TQ_2} = 0 \end{aligned}$$

Advantage of the hadron reaction ($\delta \bar{q}$ measurement)

$$A_{UQ_0} (\text{large } x_F) \approx \frac{\sum_a e_a^2 q_a(x_A) \delta_T \bar{q}_a(x_B)}{2 \sum_a e_a^2 q_a(x_A) \bar{q}_a(x_B)}$$

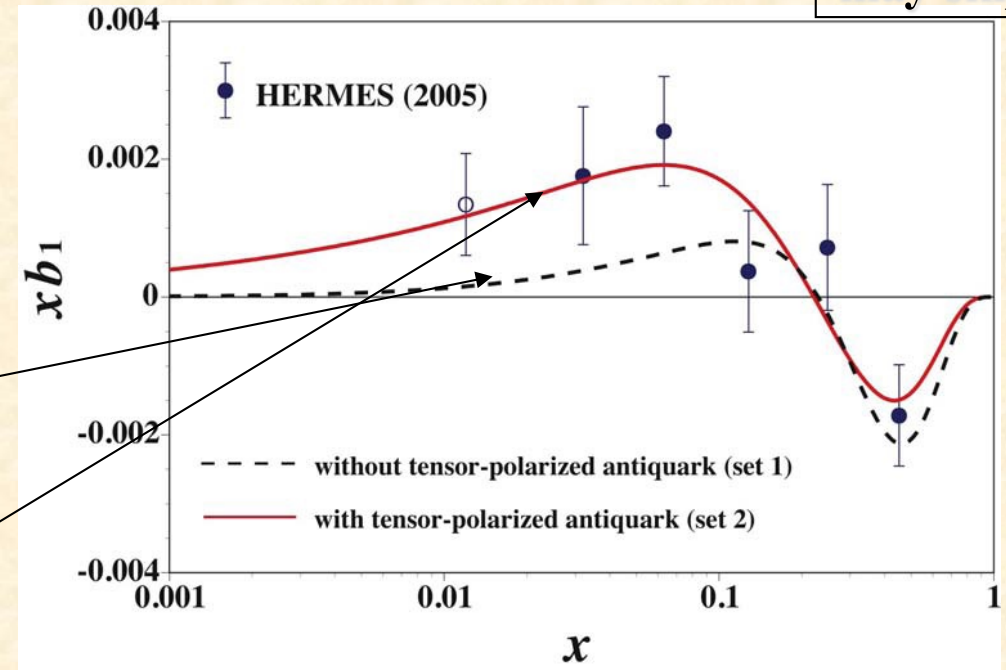
Note: $\delta \neq$ transversity in my notation

Tensor-polarized PDFs

SK, PRD 82 (2010) 017501.

Two-types of fit results:

- set-1 ($\delta_T \bar{q} = 0$): $\chi^2 / \text{d.o.f.} = 2.83$
Without $\delta_T \bar{q}$, the fit is not good enough.
- **set-2 ($\delta_T \bar{q} \neq 0$): $\chi^2 / \text{d.o.f.} = 1.57$**
With finite $\delta_T \bar{q}$, the fit is reasonably good.



Obtained tensor-polarized distributions

$\delta_T q(x)$, $\delta_T \bar{q}(x)$ from the HERMES data.

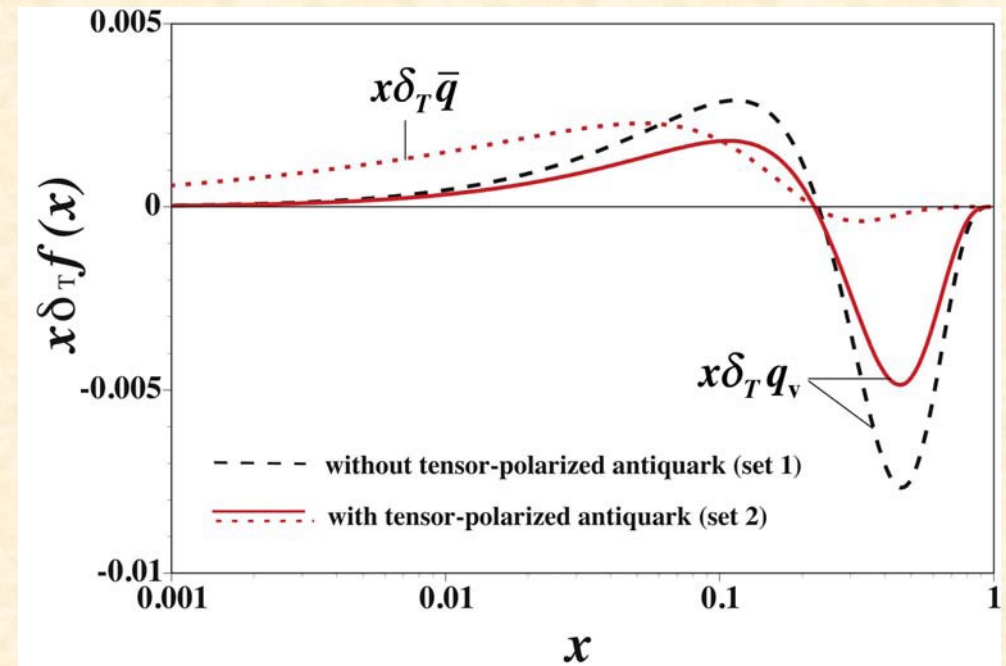
→ They could be used for

- experimental proposals,
- comparison with theoretical models.

Finite tensor polarization for antiquarks:

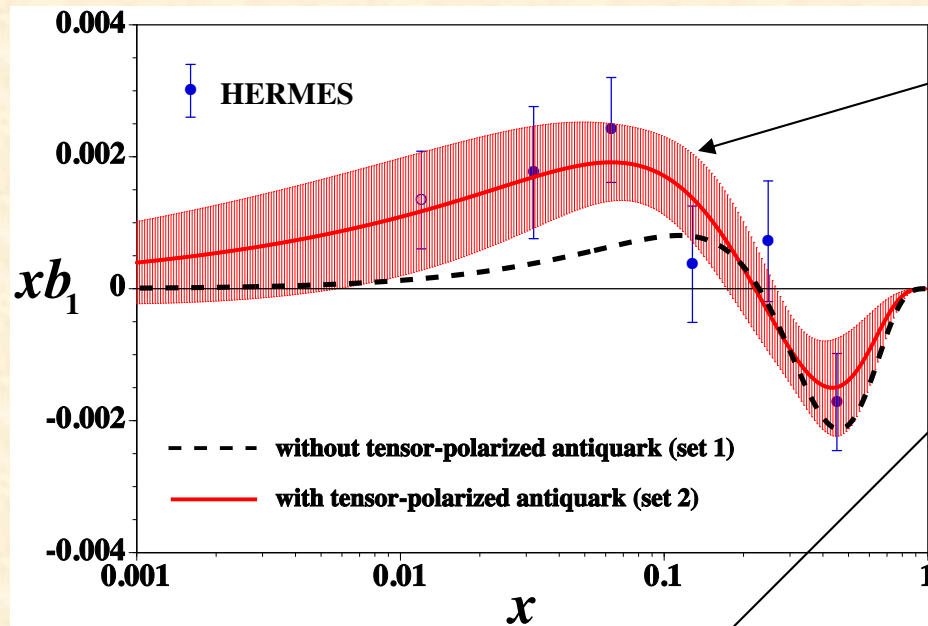
$$\int_0^1 dx b_1(x) = 0.058$$

$$= \frac{1}{9} \int_0^1 dx [4\delta_T \bar{u}(x) + \delta_T \bar{d}(x) + \delta_T \bar{s}(x)]$$



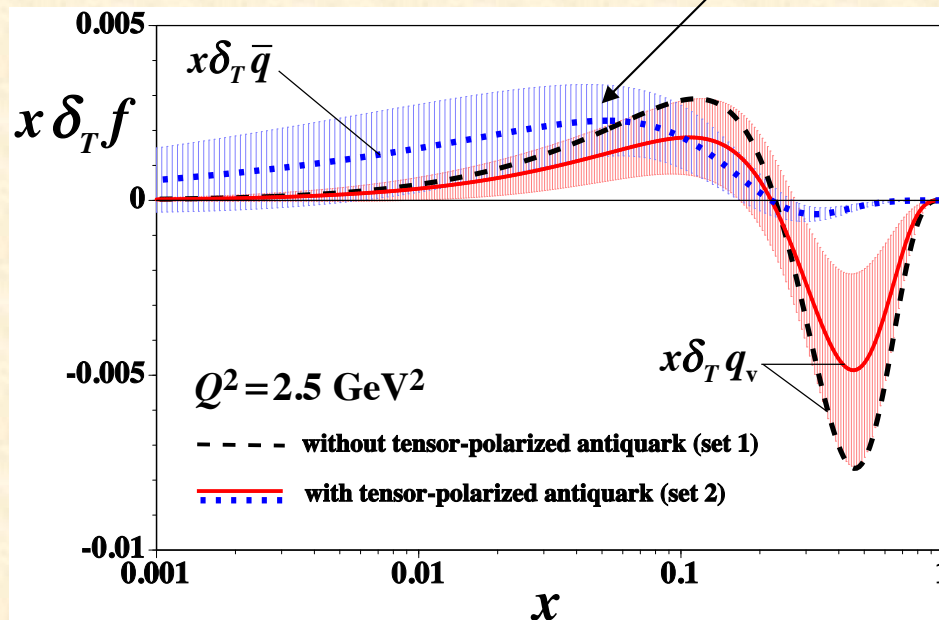
Tensor-polarized PDFs with errors

may skip



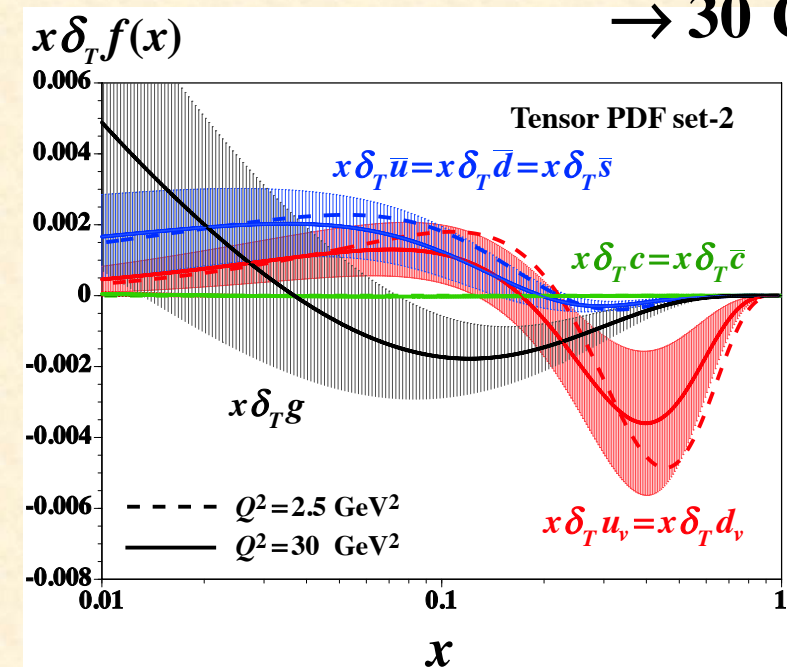
still large errors,
need experimental improvement
→ JLab, EIC, ...

experimental measurement
for antiquark distributions
→ Fermilab, ...



Q^2 evolution

$Q^2 = 2.5 \text{ GeV}^2$
→ 30 GeV^2



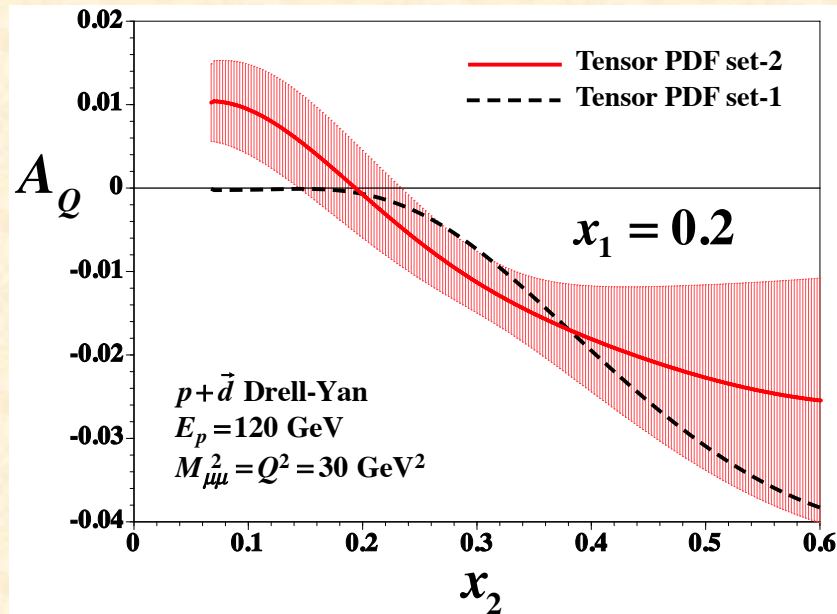
Tensor-polarized spin asymmetry at Fermilab

Spin asymmetry in proton-deuteron Drell-Yan process with tensor-polarized deuteron

$$A_{UQ_0} = \frac{\sum_a e_a^2 \left[q_a(x_A) \delta_T \bar{q}_a(x_B) + \bar{q}_a(x_A) \delta_T q_a(x_B) \right]}{2 \sum_a e_a^2 \left[q_a(x_A) \bar{q}_a(x_B) + \bar{q}_a(x_A) q_a(x_B) \right]}$$

Polarized fixed-target experiments
at the Fermilab Main Injector

E1039-SpinQuest



SK and Qin-Tao Song,
PRD 94 (2016) 054022

Drell-Yan experiment with a polarized proton target

Co-Spokespersons: A. Klein, X. Jiang, Los Alamos National Laboratory

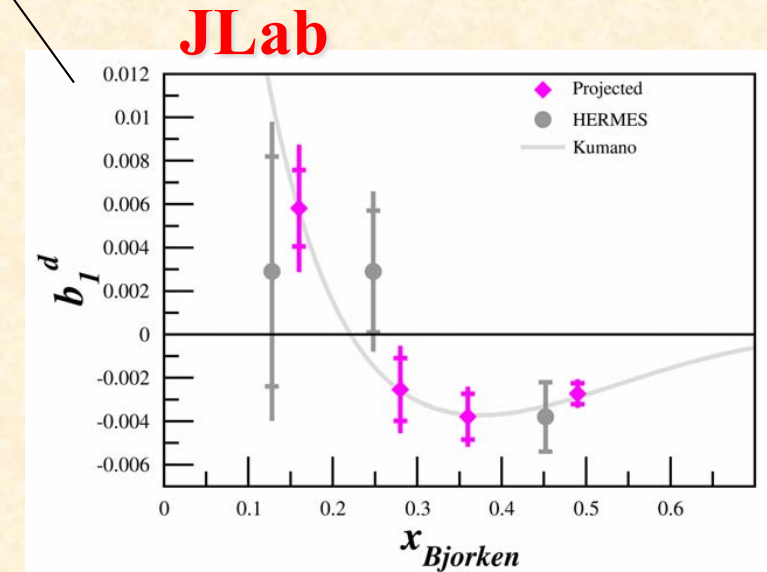
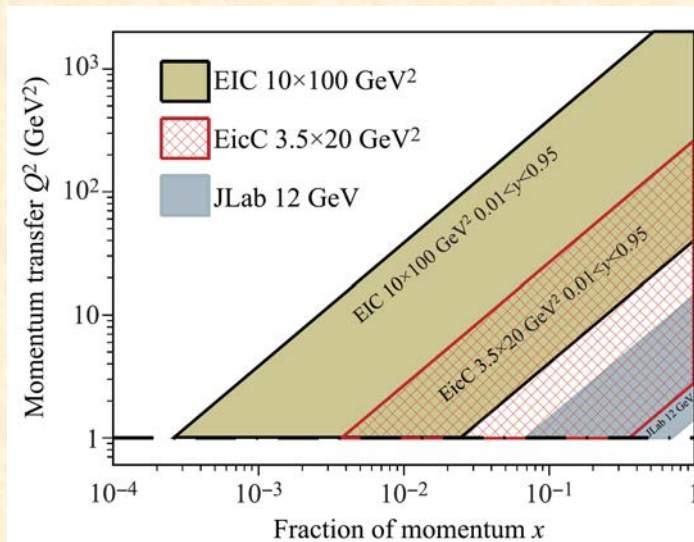
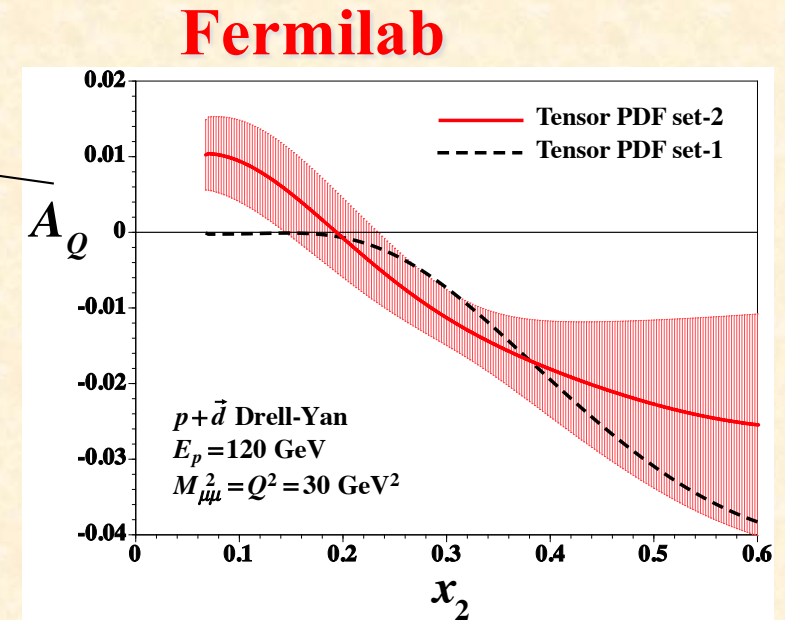
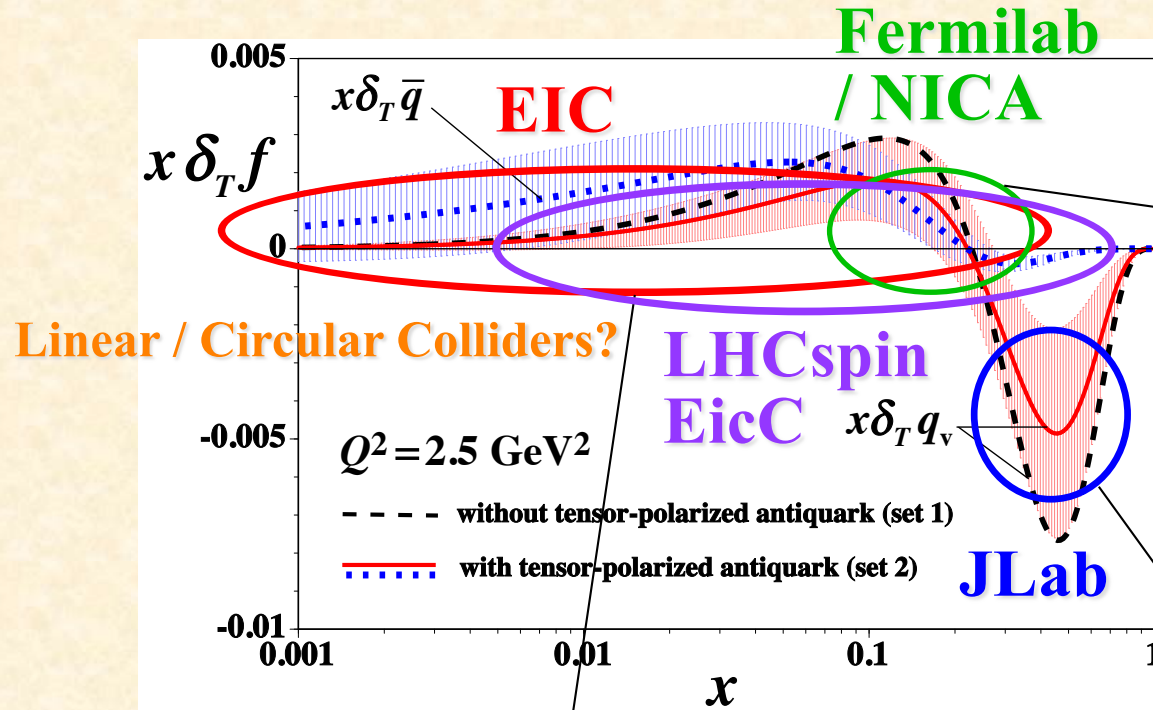
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x regions of b_1 in 2020's and 2030's

may skip



Gluon transversity

Gluon transversity $\Delta_T g$

Note on our notations:

Tensor-polarized gluon distribution: $\delta_T g$

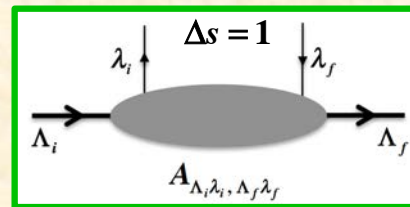
Gluon transversity: $\Delta_T g$

Helicity amplitude $A(\Lambda_i, \lambda_i, \Lambda_f, \lambda_f)$, conservation $\Lambda_i - \lambda_i = \Lambda_f - \lambda_f$

Longitudinally-polarized quark in nucleon: $\Delta q(x) \sim A\left(+\frac{1}{2} + \frac{1}{2}, +\frac{1}{2} + \frac{1}{2}\right) - A\left(+\frac{1}{2} - \frac{1}{2}, +\frac{1}{2} - \frac{1}{2}\right)$

Quark transversity in nucleon:

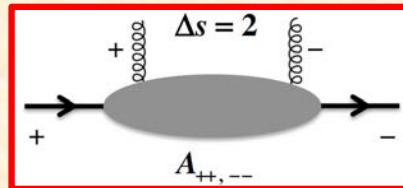
$\Delta_T q(x) \sim A\left(+\frac{1}{2} + \frac{1}{2}, -\frac{1}{2} - \frac{1}{2}\right)$, $\lambda_i = +\frac{1}{2} \rightarrow \lambda_f = -\frac{1}{2}$ quark spin flip ($\Delta s = 1$)



Gluon transversity in deuteron:

$\Delta_T g(x) \sim A(+1 + 1, -1 - 1)$,

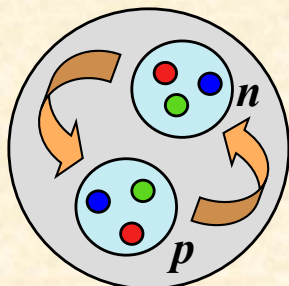
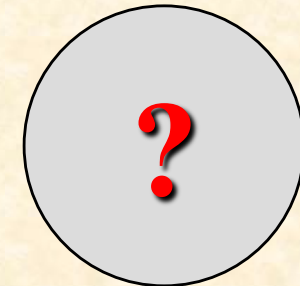
$A\left(+\frac{1}{2} + 1, -\frac{1}{2} - 1\right)$ not possible for nucleon



Note: Gluon transversity does not exist for spin-1/2 nucleons.

$b_1 (\delta_T q, \delta_T g) \neq 0 \Leftrightarrow$ still $\Delta_T g = 0$

What would be the mechanism(s)
for creating $\Delta_T g \neq 0$?



S + D waves

Physics beyond “the standard model” in nuclear physics?
(Physics beyond the standard model in particle physics???)

Letter of Intent at Jefferson Lab

Jefferson Lab,
Electron accelerator ~12 GeV



LoI, arXiv:1803.11206

A Letter of Intent to Jefferson Lab PAC 44, June 6, 2016
Search for Exotic Gluonic States in the Nucleus

M. Jones, C. Keith, J. Maxwell*, D. Meekins

Thomas Jefferson National Accelerator Facility, Newport News, VA 23606

W. Detmold, R. Jaffe, R. Milner, P. Shanahan

Laboratory for Nuclear Science, MIT, Cambridge, MA 02139

D. Crabb, D. Day, D. Keller, O. A. Rondon

University of Virginia, Charlottesville, VA 22904

J. Pierce

Oak Ridge National Laboratory, Oak Ridge, TN 37831

Electron scattering with polarized-deuteron target

$$\left. \frac{d\sigma}{dx dy d\phi} \right|_{Q^2 \gg M^2} = \frac{e^4 ME}{4\pi^2 Q^4} \left[xy^2 F_1(x, Q^2) + (1-y) F_2(x, Q^2) - \frac{1}{2} x(1-y) \Delta(x, Q^2) \cos(2\phi) \right]$$

$$\Delta(x, Q^2) = \frac{\alpha_s}{2\pi} \sum_q e_q^2 x^2 \int_x^1 \frac{dy}{y^3} \Delta_T g(y, Q^2)$$

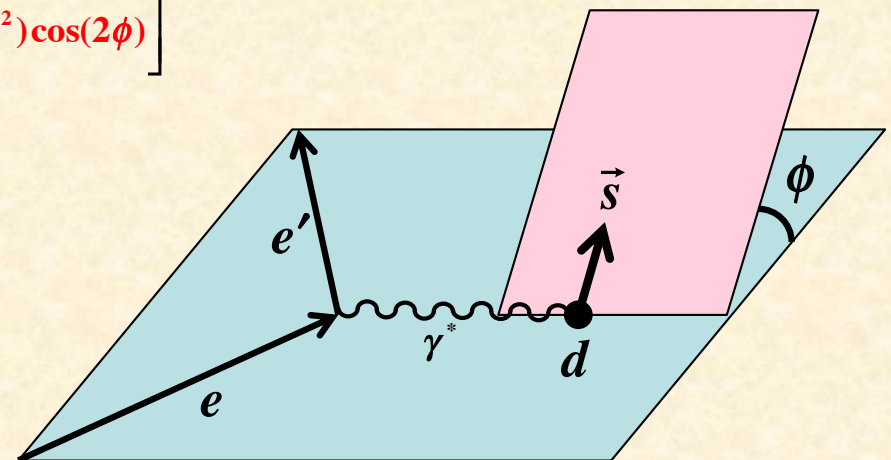
By looking at the deuteron-polarization angle ϕ ,
the quark transversity $\Delta_T g$ can be measured.

Lattice QCD estimates:

W. Detmold and P. E. Shanahan,

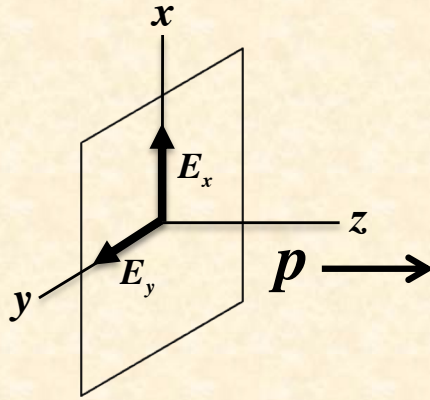
PRD 94 (2016) 014507; 95 (2017) 079902.

For development of polarized deuteron target,
see D. Keller, D. Crabb, D. Day
Nucl. Inst. Meth. Phys. Res. A981 (2020) 164504.



Gluon transversity distribution in deuteron

may skip



Linear-polarization difference: $d\sigma(E_x - E_y) \propto \Delta_T g$

$$\Delta_T g(x) = \int \frac{d\xi^-}{2\pi} x p^+ e^{i x p^+ \xi^-} \left\langle p E_x \left| A^x(0) A^x(\xi) - A^y(0) A^y(\xi) \right| p E_x \right\rangle_{\xi^+ = \xi_T = 0}$$

$$= g_{\hat{x}/\hat{x}} - g_{\hat{y}/\hat{x}}$$

$g_{\hat{y}/\hat{x}}$ = gluon distribution with the gluon linear polarization ε_y in the deuteron linear polarization E_x

Polarization vectors $\vec{E}_x = \vec{\varepsilon}_x = (1, 0, 0)$, $\vec{E}_y = \vec{\varepsilon}_y = (0, 1, 0)$

Spin and tensor of the deuteron

$$S^\mu = \frac{1}{M} \varepsilon^{\mu\nu\alpha\beta} p_\nu \text{Im}(E_\alpha^* E_\beta), \quad T^{\mu\nu} = -\frac{1}{3} \left(g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2} \right) - \text{Re}(E^{\mu*} E^\nu)$$

$$E^\mu = (0, \vec{E}), \quad \vec{E}_\pm = \frac{1}{\sqrt{2}} (\mp 1, -i, 0), \quad \vec{E}_0 = (0, 0, 1)$$

- $\vec{E}_+, \vec{E}_0, \vec{E}_-$: Spin states with z -components of spin $s_z = +1, 0, -1$
- $\vec{E}_x = (1, 0, 0), \vec{E}_y = (0, 1, 0)$: **Linear polarizations**
→ to measure gluon transversity

(1) Prepare $s_x = 0$ [$\vec{E}_x = (1, 0, 0)$] by taking the quantization axis x and $s_y = 0$ [$\vec{E}_y = (0, 1, 0)$] by taking the quantization axis y .

(2) Combination of transverse polarizations.

Transverse polarization

Linear polarization

$$S = (S_T^x, S_T^y, S_L),$$

$$T = \frac{1}{2} \begin{pmatrix} -\frac{2}{3} S_{LL} + S_{TT}^{xx} & S_{TT}^{xy} & S_{LT}^x \\ S_{TT}^{xy} & -\frac{2}{3} S_{LL} - S_{TT}^{xx} & S_{LT}^y \\ S_{LT}^x & S_{LT}^y & \frac{4}{3} S_{LL} \end{pmatrix}$$

$$S_{TT}^{xy} = S_{LT}^x = S_{LT}^y = 0$$

Polarizations	$\vec{E}$	S_T^x	S_T^y	S_L	S_{LL}	S_{TT}^{xx}
Longitudinal $+z$	$\frac{1}{\sqrt{2}}(-1, -i, 0)$	0	0	+1	$+\frac{1}{2}$	0
Longitudinal $-z$	$\frac{1}{\sqrt{2}}(+1, -i, 0)$	0	0	-1	$+\frac{1}{2}$	0
Transverse $+x$	$\frac{1}{\sqrt{2}}(0, -1, -i)$	+1	0	0	$-\frac{1}{4}$	$+\frac{1}{2}$
Transverse $-x$	$\frac{1}{\sqrt{2}}(0, +1, -i)$	-1	0	0	$-\frac{1}{4}$	$+\frac{1}{2}$
Transverse $+y$	$\frac{1}{\sqrt{2}}(-i, 0, -1)$	0	+1	0	$-\frac{1}{4}$	$-\frac{1}{2}$
Transverse $-y$	$\frac{1}{\sqrt{2}}(-i, 0, +1)$	0	-1	0	$-\frac{1}{4}$	$-\frac{1}{2}$
Linear x	$(1, 0, 0)$	0	0	0	$+\frac{1}{2}$	-1
Linear y	$(0, 1, 0)$	0	0	0	$+\frac{1}{2}$	+1

Proton-deuteron Drell-Yan cross section

SK and Qin-Tao Song,
PRD 101 (2020) 054011 & 094013.

Drell-Yan cross section

$$\frac{d\sigma_{pd \rightarrow \mu^+ \mu^- X}(E_x - E_y)}{d\tau dq_T^2 d\phi dy} = \frac{\alpha^2 \alpha_s C_F q_T^2}{6\pi s^3} \cos(2\phi) \int_{\min(x_a)}^1 dx_a \frac{1}{(x_a x_b)^2 (x_a - x_1)(\tau - x_a x_2)^2} \sum_q e_q^2 x_a [q_A(x_a) + \bar{q}_A(x_a)] x_b \Delta_T g_B(x_b)$$

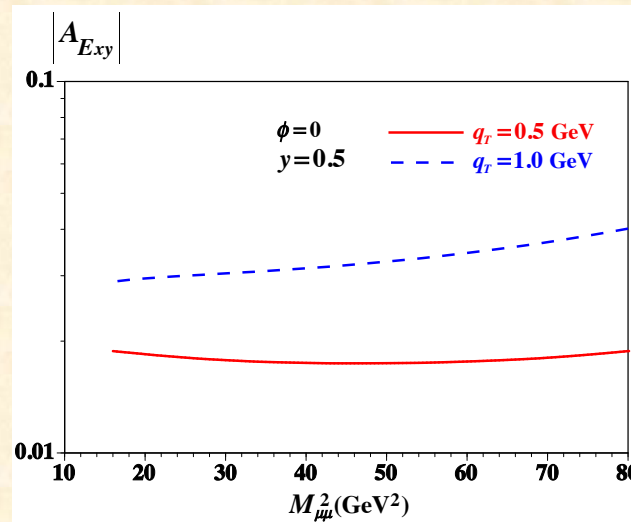
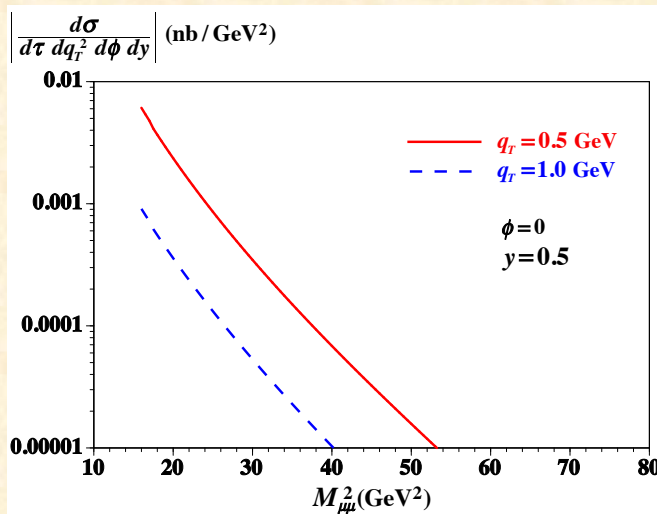
$$C_F = \frac{N_c^2 - 1}{2N_c}, \quad \min(x_a) = \frac{x_1 - \tau}{1 - x_2}, \quad x_b = \frac{x_a x_2 - \tau}{x_a - \tau}$$

= (unpolarized PDFs of proton) * (gluon transversity distribution in the deuteron)

- Consider the Fermilab-E1039 experiment with the proton beam of $p = 120$ GeV
- No available $\Delta_T g$, so we may tentatively assume $\Delta_T g = \Delta g_p + \Delta g_n$ $\left(\text{or } \frac{\Delta g_p + \Delta g_n}{2}, \frac{\Delta g_p + \Delta g_n}{4} \right)$
- CTEQ14 for $q(x) + \bar{q}(x)$, NNPDFpol1.1 for $\Delta g(x)$

Cross section: Dimuon mass squared ($M_{\mu\mu}^2 = Q^2$) dependence

Spin asymmetry: $A_{E_{xy}} = \frac{\frac{d\sigma_{pd \rightarrow \mu^+ \mu^- X}}{d\tau dq_T^2 d\phi dy}(E_x) - \frac{d\sigma_{pd \rightarrow \mu^+ \mu^- X}}{d\tau dq_T^2 d\phi dy}(E_y)}{\frac{d\sigma_{pd \rightarrow \mu^+ \mu^- X}}{d\tau dq_T^2 d\phi dy}(E_x) + \frac{d\sigma_{pd \rightarrow \mu^+ \mu^- X}}{d\tau dq_T^2 d\phi dy}(E_y)}$



Proposal at Fermilab-PAC (D. Keller)

Experimental possibility at Fermilab in 2020's

**Polarized fixed-target experiments
at the Main Injector,
Proton beam = 120 GeV** © Fermilab



Fermilab-E1039 (SpinQuest)

Drell-Yan experiment with a polarized proton target

Co-Spokespersons: A. Klein, X. Jiang, Los Alamos National Laboratory

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**Fermilab experimentalists are interested
in the gluon transversity by replacing
the E1039 proton target for the deuteron one.
(Spokesperson of E1039: D. Keller)
However, there was no theoretical formalism
until our work.**

**SK and Q.-T. Song,
PRD 101 (2020) 054011 & 094013**

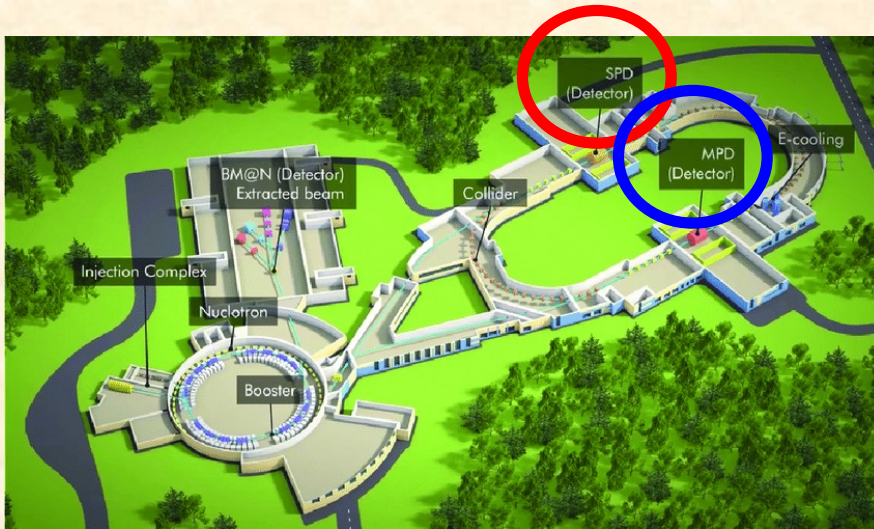
The Transverse Structure of the Deuteron with Drell-Yan

D. Keller¹

¹University of Virginia, Charlottesville, VA 22904

Proposal for a Fermilab-PAC in 2023.

Nuclotron-based Ion Collider fAcility (NICA)



SPD (Spin Physics Detector for physics with polarized beams)

MPD (MultiPurpose Detector for heavy ion physics)

$$\vec{p} + \vec{p}: \sqrt{s_{pp}} = 12 \sim 27 \text{ GeV}$$

$$\vec{d} + \vec{d}: \sqrt{s_{NN}} = 4 \sim 14 \text{ GeV}$$

$\vec{p} + \vec{d}$ is also possible.

On the physics potential to study the gluon content of proton and deuteron at NICA SPD, A. Arbuzov *et al.* (NICA project), Nucl. Part. Phys. 119 (2021) 103858.

Unique opportunity in high-energy spin physics,
especially on the deuteron spin physics.

→ Theoretical formalisms need to be developed.



**TMDs, PDFs,
and fragmentation functions
for spin-1 hadrons up to twist 4**

TMD correlation functions for spin-1 hadrons

Correlation functions

Spin vector: $S^\mu = S_L \frac{P^+}{M} \bar{n}^\mu - S_L \frac{M}{2P^+} n^\mu + S_T^\mu$

Tensor: $T^{\mu\nu} = \frac{1}{2} \left[\frac{4}{3} S_{LL} \frac{(P^+)^2}{M^2} \bar{n}^\mu \bar{n}^\nu + \frac{P^+}{M} \bar{n}^{(\mu} S_{LT}^{\nu)} - \frac{2}{3} S_{LL} (\bar{n}^{(\mu} n^{\nu)} - g_T^{\mu\nu}) + S_{TT}^{\mu\nu} - \frac{M}{2P^+} n^{(\mu} S_{LT}^{\nu)} + \frac{1}{3} S_{LL} \frac{M^2}{(P^+)^2} n^\mu n^\nu \right]$

Tensor part (twist-2): **Bacchetta, Mulders, PRD 62 (2000) 114004**

$$\Phi(k, P, T) = \left(\frac{A_{13}}{M} I + \frac{A_{14}}{M^2} \not{P} + \frac{A_{15}}{M^2} \not{K} + \frac{A_{16}}{M^3} \sigma_{\rho\sigma} P^\rho k^\sigma \right) k_\mu k_\nu T^{\mu\nu} + \left[A_{17} \gamma_\nu + \left(\frac{A_{18}}{M} P^\rho + \frac{A_{19}}{M} k^\rho \right) \sigma_{\nu\rho} + \frac{A_{20}}{M^2} \varepsilon_{\nu\rho\sigma} P^\rho k^\sigma \gamma^\tau \gamma_5 \right] k_\mu T^{\mu\nu}$$

Tensor part (twist-2, 3, 4): n^μ dependent terms are added for up to twist 4.

[For the spin-1/2 nucleon: **Goeke, Metzand, Schlegel, PLB 618 (2005) ,90; Metz, Schweitzer, Teckentrup, PLB 680 (2009) 141.**]

Kumano-Song-2021, for the details see PRD 103 (2021) 014025

$$\Phi(k, P, T | n) = \left(\frac{A_{13}}{M} I + \frac{A_{14}}{M^2} \not{P} + \frac{A_{15}}{M^2} \not{K} + \frac{A_{16}}{M^3} \sigma_{\rho\sigma} P^\rho k^\sigma \right) k_\mu k_\nu T^{\mu\nu} + \left[A_{17} \gamma_\nu + \left(\frac{A_{18}}{M} P^\rho + \frac{A_{19}}{M} k^\rho \right) \sigma_{\nu\rho} + \frac{A_{20}}{M^2} \varepsilon_{\nu\rho\sigma} P^\rho k^\sigma \gamma^\tau \gamma_5 \right] k_\mu T^{\mu\nu}$$

**Bacchetta
-Mulders (2000)**

**New terms
in our paper
(2021)**

$$\begin{aligned} & + \left(\frac{B_{21}M}{P \cdot n} k_\mu + \frac{B_{22}M^3}{(P \cdot n)^2} n_\mu \right) n_\nu T^{\mu\nu} + i \gamma_5 \varepsilon_{\mu\rho\sigma} P^\rho \left(\frac{B_{23}}{(P \cdot n)M} k^\tau n^\sigma k_\nu + \frac{B_{24}M}{(P \cdot n)^2} k^\tau n^\sigma n_\nu \right) T^{\mu\nu} \\ & + \left[\frac{B_{25}}{P \cdot n} \not{n} k_\mu k_\nu + \left(\frac{B_{26}M^2}{(P \cdot n)^2} \not{n} + \frac{B_{28}}{P \cdot n} \not{P} + \frac{B_{30}}{P \cdot n} \not{K} \right) k_\mu n_\nu + \left(\frac{B_{27}M^4}{(P \cdot n)^3} \not{n} + \frac{B_{29}M^2}{(P \cdot n)^2} \not{P} + \frac{B_{31}M^2}{(P \cdot n)^2} \not{K} \right) n_\mu n_\nu + \frac{B_{32}M^2}{P \cdot n} \gamma_\mu n_\nu \right] T^{\mu\nu} \\ & - \left[\varepsilon_{\mu\rho\sigma} \gamma^\tau P^\rho \left(\frac{B_{34}}{P \cdot n} n^\sigma k_\nu + \frac{B_{33}}{P \cdot n} k^\sigma n_\nu + \frac{B_{35}M^2}{(P \cdot n)^2} n^\sigma n_\nu \right) + \varepsilon_{\lambda\rho\sigma} k^\lambda \gamma^\tau P^\rho n^\sigma \left(\frac{B_{36}}{P \cdot n M^2} k_\mu k_\nu + \frac{B_{37}}{(P \cdot n)^2} k_\mu n_\nu + \frac{B_{38}M^2}{(P \cdot n)^3} n_\mu n_\nu \right) \right] \gamma_5 T^{\mu\nu} \\ & + \varepsilon_{\mu\rho\sigma} k^\tau P^\rho n^\sigma \left(\frac{B_{39}}{(P \cdot n)^2} k_\nu + \frac{B_{40}M^2}{(P \cdot n)^3} n_\nu \right) \not{n} \gamma_5 T^{\mu\nu} \\ & + \sigma_{\rho\sigma} \left[P^\rho k^\sigma \left(\frac{B_{41}}{(P \cdot n)M} k_\mu n_\nu + \frac{B_{42}M}{(P \cdot n)^2} n_\mu n_\nu \right) + P^\rho n^\sigma \left(\frac{B_{43}}{(P \cdot n)M} k_\mu k_\nu + \frac{B_{44}M}{(P \cdot n)^2} k_\mu n_\nu + \frac{B_{45}M^3}{(P \cdot n)^3} n_\mu n_\nu \right) \right] T^{\mu\nu} \\ & + \sigma_{\rho\sigma} \left[k^\rho n^\sigma \left(\frac{B_{46}}{(P \cdot n)M} k_\mu k_\nu + \frac{B_{47}M}{(P \cdot n)^2} k_\mu n_\nu + \frac{B_{48}M^3}{(P \cdot n)^3} n_\mu n_\nu \right) \right] T^{\mu\nu} + \sigma_{\mu\sigma} \left[n^\sigma \left(\frac{B_{49}M}{P \cdot n} k_\nu + \frac{B_{50}M^3}{(P \cdot n)^2} n_\nu \right) + \left(\frac{B_{51}M}{P \cdot n} P^\sigma + \frac{B_{52}M}{P \cdot n} k^\sigma \right) n_\nu \right] T^{\mu\nu} \end{aligned}$$

From this correlation function, new tensor-polarized TMDs are defined in twist-3 and 4 in addition to twist-2 ones.

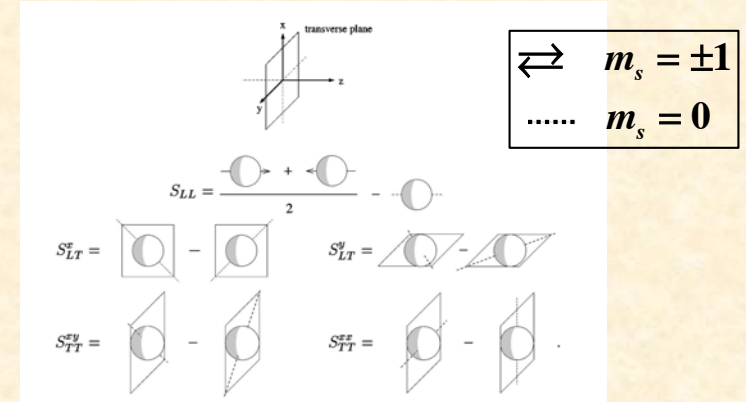
Terms associated with
 $n = \frac{1}{\sqrt{2}}(1, 0, 0, -1)$

TMDs and their sum rules for spin-1 hadrons

see our PRD paper
for the details

Twist-2 TMDs Bacchetta-Mulders, PRD 62 (2000) 114004.

Quark Hadron	U (γ^+)		L ($\gamma^+\gamma_5$)		T ($i\sigma^{i+}\gamma_5 / \sigma^{i+}$)	
	T-even	T-odd	T-even	T-odd	T-even	T-odd
U	f_1					$[h_1^\perp]$
L			g_{1L}		$[h_{1L}^\perp]$	
T		$f_{1T}^\perp$	g_{1T}		$[h_1], [h_{1T}^\perp]$	
LL	f_{1LL}					$[h_{1LL}^\perp]$
LT	f_{1LT}			g_{1LT}		$[h_{1LT}^\perp], [h_{1LT}^\perp]$
TT	f_{1TT}			g_{1TT}		$[h_{1TT}^\perp], [h_{1TT}^\perp]$



Time-reversal invariance in collinear correlation functions (PDFs)

$$\int d^2 k_T \Phi_{T\text{-odd}}(x, k_T^2) = 0$$

Sum rules for the TMDs of spin-1 hadrons

$$\int d^2 k_T h_{1LT}(x, k_T^2) = 0, \quad \int d^2 k_T g_{LT}(x, k_T^2) = 0,$$

$$\int d^2 k_T h_{LL}(x, k_T^2) = 0, \quad \int d^2 k_T h_{3LT}(x, k_T^2) = 0$$

Twist-3 TMDs SK and Qin-Tao Song, PRD 103 (2021) 014025.

Quark Hadron	$\gamma^i, 1, i\gamma_5$		$\gamma^+\gamma_5$		σ^{ij}, σ^{i+}	
	T-even	T-odd	T-even	T-odd	T-even	T-odd
U	$f_1^\perp$ [e]			$g^\perp$		[h]
L		$f_L^\perp$ [e_L]	$g_L^\perp$		$[h_L]$	
T		$f_T, f_T^\perp$ [e_T, e_T^\perp]	$g_T, g_T^\perp$		$[h_T], [h_T^\perp]$	
LL	$f_{LL}^\perp$ [e_{LL}]			$g_{LL}^\perp$		$[h_{LL}^\perp]$
LT	$f_{LT}, f_{LT}^\perp$ [e_{LT}, e_{LT}^\perp]			$g_{LT}, g_{LT}^\perp$		$[h_{LT}^\perp], [h_{LT}^\perp]$
TT	$f_{TT}, f_{TT}^\perp$ [e_{TT}, e_{TT}^\perp]			$g_{TT}, g_{TT}^\perp$		$[h_{TT}^\perp], [h_{TT}^\perp]$

Twist-4 TMDs

Quark Hadron	γ^-		$\gamma^-\gamma_5$		σ^{i-}	
	T-even	T-odd	T-even	T-odd	T-even	T-odd
U	f_3					$[h_3^\perp]$
L			g_{3L}		$[h_{3L}^\perp]$	
T		$f_{3T}^\perp$	g_{3T}		$[h_{3T}^\perp], [h_{3T}^\perp]$	
LL	f_{3LL}					$[h_{3LL}^\perp]$
LT	f_{3LT}			g_{3LT}		$[h_{3LT}^\perp], [h_{3LT}^\perp]$
TT	f_{3TT}			g_{3TT}		$[h_{3TT}^\perp], [h_{3TT}^\perp]$

PDFs for spin-1 hadrons

Twist-2 PDFs

Quark Hadron	$U (\gamma^+)$		$L (\gamma^+ \gamma_5)$		$T (i\sigma^{i+} \gamma_5 / \sigma^{i+})$	
	T-even	T-odd	T-even	T-odd	T-even	T-odd
U	f_1					
L			$g_{1L}(g_1)$			
T					$[h_1]$	
LL	$f_{1LL}(b_1)$					
LT						*1
TT						

***1: $h_{1LT}(x)$, *2: $g_{LT}(x)$, *3: $h_{LL}(x)$, *4: $h_{3LT}(x)$**

Because of the time-reversal invariance, the collinear PDF vanishes.

However, since the time-reversal invariance cannot be imposed in the fragmentation functions, we should note that the corresponding fragmentation function should exist as a collinear fragmentation function.

[] = chiral odd

Twist-3 PDFs

Quark Hadron	$\gamma^i, 1, i\gamma_5$		$\gamma^+ \gamma_5$		σ^{ij}, σ^{-+}	
	T-even	T-odd	T-even	T-odd	T-even	T-odd
U	$[e]$					
L					$[h_L]$	
T			g_T			
LL	$[e_{LL}]$					*3
LT	f_{LT}			*2		
TT						

Twist-4 PDFs

Quark Hadron	γ^-		$\gamma^- \gamma_5$		σ^{i-}	
	T-even	T-odd	T-even	T-odd	T-even	T-odd
U	f_3					
L			g_{3L}			
T					$[h_{3T}]$	
LL	f_{3LL}					
LT						*4
TT						

New collinear PDFs

SK and Qin-Tao Song, PRD 103 (2021) 014025.

New fragmentation functions (FFs) for spin-1 hadrons

see arXiv:2201.05397

Corresponding fragmentation functions exist for the spin-1 haddrons
simply by changing function names and kinematical variables.

Collinear FFs:
X. Ji, PRD 49, 114 (1994).

TMD distribution functions: $f, g, h, e; x, k_T, S, T, M, n, \gamma^+, \sigma^{i+}$
 $\Downarrow$

TMD fragmentation functions: $D, G, H, E; z, k_T, S_h, T_h, M_h, \bar{n}, \gamma^-, \sigma^{i-}$

Shandong group:
K.-B Chen, W.-H. Ynag,
S.-Y. Wei, Z.-T. Liang,
PRD 94 (2016) 034003.

Collinear FFs, twist 2

Quark Hadron	U (γ^+)		L ($\gamma^+\gamma_s$)		T ($i\sigma^{i+}\gamma_s / \sigma^{i+}$)	
	T-even	T-odd	T-even	T-odd	T-even	T-odd
U	D_1					
L			G_{1L}			
T					$[H_1]$	
LL	D_{1LL}					
LT						$[H_{1LT}]$
TT						

Collinear FFs, twist 3

Quark Hadron	$\gamma^i, 1, i\gamma_s$		$\gamma^i\gamma_s$		σ^{ij}, σ^{-+}	
	T-even	T-odd	T-even	T-odd	T-even	T-odd
U	$[E]$					
L					$[H_L]$	
T			G_T			
LL	$[E_{LL}]$					$[H_{LL}]$
LT	D_{LT}			G_{LT}		
TT						

Collinear FFs, twist 4

Quark Hadron	γ^-		$\gamma^-\gamma_s$		σ^{i-}	
	T-even	T-odd	T-even	T-odd	T-even	T-odd
U	D_3					
L			G_{3L}			
T					$[H_{3T}]$	
LL	D_{3LL}					
LT						$[H_{3LT}]$
TT						

TMD FFs, twist 2 [] = chiral odd

Quark Hadron	U (γ^+)		L ($\gamma^+\gamma_s$)		T ($i\sigma^{i+}\gamma_s / \sigma^{i+}$)	
	T-even	T-odd	T-even	T-odd	T-even	T-odd
U	D_1					$[H_1^+]$
L			G_{1L}		$[H_{1L}^+]$	
T		D_{1T}^+	G_{1T}		$[H_1], [H_{1T}^+]$	
LL	D_{1LL}					$[H_{1LL}^+]$
LT	D_{1LT}			G_{1LT}		$[H_{1LT}], [H_{1LT}^+]$
TT	D_{1TT}			G_{1TT}		$[H_{1TT}], [H_{1TT}^+]$

TMD FFs, twist 3

Quark Hadron	$\gamma^i, 1, i\gamma_s$		$\gamma^i\gamma_s$		σ^{ij}, σ^{-+}	
	T-even	T-odd	T-even	T-odd	T-even	T-odd
U	D^+ $[E]$			G^+		$[H]$
L		D_L^+ $[E_L]$	G_L^+		$[H_L]$	
T		D_T, D_T^+ $[E_T, E_T^+]$	G_T, G_T^+		$[H_T], [H_T^+]$	
LL	D_{LL}^+ $[E_{LL}]$			G_{LL}^+		$[H_{LL}]$
LT	D_{LT}, D_{LT}^+ $[E_{LT}, E_{LT}^+]$			G_{LT}, G_{LT}^+		$[H_{LT}], [H_{LT}^+]$
TT	D_{TT}, D_{TT}^+ $[E_{TT}, E_{TT}^+]$			G_{TT}, G_{TT}^+		$[H_{TT}], [H_{TT}^+]$

TMD FFs, twist 4

Quark Hadron	γ^-		$\gamma^-\gamma_s$		σ^{i-}	
	T-even	T-odd	T-even	T-odd	T-even	T-odd
U	D_3					$[H_3^+]$
L			G_{3L}		$[H_{3L}^+]$	
T		D_{3T}^+	G_{3T}		$[H_{3T}], [H_{3T}^+]$	
LL	D_{3LL}					$[H_{3LL}^+]$
LT	D_{3LT}			G_{3LT}		$[H_{3LT}], [H_{3LT}^+]$
TT	D_{3TT}			G_{3TT}		$[H_{3TT}], [H_{3TT}^+]$

New TMD FFs

Analogous relations to Wandzura-Wilczek relation and Burkhardt-Cottingham sum rule

may skip

Twist-3 PDFs

Quark Hadron	U (γ^+)		L ($\gamma^+\gamma_5$)		T ($i\sigma^{i+}\gamma_5 / \sigma^{i+}$)	
	T-even	T-odd	T-even	T-odd	T-even	T-odd
U	f_1					
L			$g_{1L}(g_1)$			
T					$[h_1]$	
LL		$f_{1LL}(b_1)$				
LT						*1
TT						

Twist-2 PDFs

Quark Hadron	$\gamma^i, 1, i\gamma_5$		$\gamma^+\gamma_5$		σ^{ij}, σ^{-+}	
	T-even	T-odd	T-even	T-odd	T-even	T-odd
U	$[e]$					
L					$[h_L]$	
T			g_T			
LL	$[e_{LL}]$					*3
LT		f_{LT}		*2		
TT						

[] = chiral odd

We derived analogous relations to Wandzura-Wilczek relation and Burkhardt-Cottingham sum rule for f_{LT} and f_{1LL} .

SK and Qin-Tao Song,
JHEP 09 (2021) 141.

For spin-1/2 nucleons,

$$g_2(x) = -g_1(x) + \int_x^1 \frac{dy}{y} g_1(y) \text{ (Wandzura-Wilczek relation),} \quad \int_0^1 dx g_2(x) = 0 \text{ (Burkhardt-Cottingham sum rule)}$$

For tensor-polarized spin-1 hadrons, we obtained

$$f_{2LT}^+(x) = -f_{1LL}^+(x) + \int_x^1 \frac{dy}{y} f_{1LL}^+(y), \quad \int_0^1 dx f_{2LT}^+(x) = 0, \quad f_{2LT}(x) \equiv \frac{2}{3} f_{LT}(x) - f_{1LL}(x)$$

$$\int_0^1 dx f_{LT}^+(x) = 0 \text{ if } \int_0^1 dx f_{1LL}^+(x) = \frac{2}{3} \int_0^1 dx b_1^+(x) = 0$$

Existence of multiparton distribution functions: $F_{G,LT}(x_1, x_2)$, $G_{G,LT}(x_1, x_2)$, $H_{G,LL}^\perp(x_1, x_2)$, $H_{G,TT}(x_1, x_2)$

Relations from equation of motion and Lorentz-invariance relation for spin-1 hadrons

SK and Qin-Tao Song,
PLB 826 (2022) 136908.

may skip

$$\bullet \ x f_{LT}(x) - \int_{-1}^{+1} dy [F_{D,LT}(x,y) + G_{D,LT}(x,y)] = 0, \quad x f_{LT}(x) - f_{1LT}^{(1)}(x) - \mathcal{P} \int_{-1}^{+1} dy \frac{F_{G,LT}(x,y) + G_{G,LT}(x,y)}{x-y} = 0$$

$$\bullet \ x e_{LL}(x) - 2 \int_{-1}^{+1} dy H_{D,LL}^{\perp}(x,y) - \frac{m}{M} f_{1LL}(x) = 0, \quad x e_{LL}(x) - 2 \mathcal{P} \int_{-1}^{+1} dy \frac{H_{G,LL}^{\perp}(x,y)}{x-y} - \frac{m}{M} f_{1LL}(x) = 0$$

and the Lorentz-invariance relation

$$\bullet \ \frac{df_{1LT}^{(1)}(x)}{dx} - f_{LT}(x) + \frac{3}{2} f_{1LL}(x) - 2 \mathcal{P} \int_{-1}^{+1} dy \frac{F_{G,LT}(x,y)}{(x-y)^2} = 0$$

Lorentz invariance

= frame independence of twist-3 observables

transverse-momentum moment of TMD: $f^{(1)}(x) = \int d^2 k_T \frac{\vec{k}_T^2}{2M^2} f(x, k_T^2)$

Twist-2 PDFs

Quark Hadron	U (γ^+)		L ($\gamma^+ \gamma_5$)		T ($i\sigma^{i+} \gamma_5 / \sigma^{i+}$)	
	T-even	T-odd	T-even	T-odd	T-even	T-odd
U	f_1					
L			$g_{1L}(g_1)$			
T					$[h_1]$	
LL	$f_{1LL}(b_1)$					
LT						
TT						

Twist-3 PDFs

Quark Hadron	$\gamma^i, 1, i\gamma_5$		$\gamma^+ \gamma_5$		σ^{ij}, σ^{i+}	
	T-even	T-odd	T-even	T-odd	T-even	T-odd
U	$[e]$					
L					$[h_L]$	
T			g_T			
LL	$[e_{LL}]$					
LT	f_{LT}					$*1$
TT						

Twist-3 TMDs

Quark Hadron	U (γ^+)		L ($\gamma^+ \gamma_5$)		T ($i\sigma^{i+} \gamma_5 / \sigma^{i+}$)	
	T-even	T-odd	T-even	T-odd	T-even	T-odd
U	f_1					$[h_1^{\perp}]$
L			g_{1L}		$[h_{1L}^{\perp}]$	
T		$f_{1T}^{\perp}$	g_{1T}		$[h_1], [h_{1T}^{\perp}]$	
LL	f_{1LL}					$[h_{1LL}^{\perp}]$
LT	f_{1LT}		g_{1LT}		$[h_{1LT}], [h_{1LT}^{\perp}]$	
TT	f_{1TT}		g_{1TT}		$[h_{1TT}], [h_{1TT}^{\perp}]$	

[] = chiral odd

Relations on fragmentation functions

Qin-Tao Song,
PRD 108 (2023) 094041.

may skip

- $E_{LL}(z) + iH_{LL}(z) - \frac{m_q}{M} z D_{1LL}(z) = 2z \left[-iH_{1LL}^{\perp(1)}(z) + \mathcal{P} \int_z^\infty \frac{dz_1}{(z_1)^2} \frac{H_{G,LL}^\perp(z, z_1)}{1/z - 1/z_1} \right]$
- $D_{LT}(z) + iG_{LT}(z) + i\frac{m_q}{M} z H_{1LT}(z) = -z \left[iG_{1LT}^{(1)}(z) - \int_z^\infty \frac{dz_1}{(z_1)^2} \frac{G_{G,LL}(z, z_1)}{1/z - 1/z_1} \right] - z \left[D_{1LT}^{(1)}(z) + \int_z^\infty \frac{dz_1}{(z_1)^2} \frac{D_{G,LT}(z, z_1)}{1/z - 1/z_1} \right]$
- $iH_{1TT}^{(1)}(z) + \int_z^\infty \frac{dz_1}{(z_1)^2} \frac{H_{G,TT}(z, z_1)}{1/z - 1/z_1} = 0$
- $\frac{3}{2} D_{1LL}(z) - D_{LT}(z) - z \left(1 - z \frac{d}{dz} \right) D_{1LT}^{(1)}(z) = -2 \int_z^\infty \frac{dz_1}{(z_1)^2} \frac{\text{Re}[D_{G,LT}(z, z_1)]}{(1/z - 1/z_1)^2}$
- $H_{LL}(z) + 2H_{1LT}(z) + z \left(1 - z \frac{d}{dz} \right) H_{1LL}^{\perp(1)}(z) = -2 \int_z^\infty \frac{dz_1}{(z_1)^2} \frac{\text{Im}[H_{G,LL}^\perp(z, z_1)]}{(1/z - 1/z_1)^2}$
- $G_{LT}(z) + z \left(1 - z \frac{d}{dz} \right) G_{1LT}^{(1)}(z) = -2 \int_z^\infty \frac{dz_1}{(z_1)^2} \frac{\text{Im}[H_{G,LT}(z, z_1)]}{(1/z - 1/z_1)^2}$

Twist-2 TMD FFs

Twist-2 FFs [] = chiral odd

Twist-3 FFs

Twist-2 TMD FFs

Quark Hadron	U (γ^+)		L ($\gamma^+ \gamma_s$)		T ($i\sigma^{i+} \gamma_s / \sigma^{i+}$)	
	T-even	T-odd	T-even	T-odd	T-even	T-odd
U	D_1					
L			G_{1L}			
T					$[H_1]$	
LL	D_{1LL}					
LT					$[H_{1LT}]$	
TT						

Quark Hadron	$\gamma^i, 1, i\gamma_s$		$\gamma^i \gamma_s$		σ^{ij}, σ^{-+}	
	T-even	T-odd	T-even	T-odd	T-even	T-odd
U	$[E]$					
L					$[H_L]$	
T			G_T			
LL	$[E_{1L}]$				$[H_{LL}]$	
LT	D_{1LT}		G_{1LT}			
TT						

Quark Hadron	U (γ^+)		L ($\gamma^+ \gamma_s$)		T ($i\sigma^{i+} \gamma_s / \sigma^{i+}$)	
	T-even	T-odd	T-even	T-odd	T-even	T-odd
U	D_1					$[H_1^\perp]$
L			G_{1L}		$[H_{1L}^\perp]$	
T		$D_{1T}^\perp$	G_{1T}		$[H_1], [H_{1T}^\perp]$	
LL	D_{1LL}				$[H_{1LL}^\perp]$	
LT	D_{1LT}		G_{1LT}		$[H_{1LT}], [H_{1TT}^\perp]$	
TT	D_{1TT}		G_{1TT}		$[H_{1TT}], [H_{1TT}^\perp]$	

Other works on spin-1

I may miss your papers.

GPDs

E.R. Berger, F. Cano, M. Diehl, B. Pire, Phys. Rev. Lett. 87, 142302 (2001);
W. Cosyn, B. Pire, Phys. Rev. D 98, 074020 (2018).

Lightcone models on ρ and D

B.D. Sun, Y.B. Dong, Phys. Rev. D 96, 036019 (2017); 99, 016023 (2019); 101, 096008 (2020);
N. Kumar, Phys. Rev. D 99, 014039 (2019);
S. Kaur, C. Mondal, X. Zhao, C.-R. Ji, arXiv:2507.09886.

Lightcone models on TMDs

C. Shi, et al., Phys. Rev. D 106, 014026 (2022);
S. Kaur, et al., Phys. Lett. B 851, 138563 (2024).

Spin-3/2

J. Zhao, Z. Zhang, Z.-T. Liang, T. Liu, Y.-J Zhou, Phys. Rev. D 106 (2022) 094006;
Dongyan Fu, Bao-Dong Sun, and Yubing Dong, Phys. Rev. D, 106 (2022) 116012; 10 (2023) 116021;
Dongyan Fu, Yubing Dong, S. Kumano, Phys. Rev. D 109 (2024) 096006;
arXiv:2508.15245 (Phys. Rev. D in press); ...

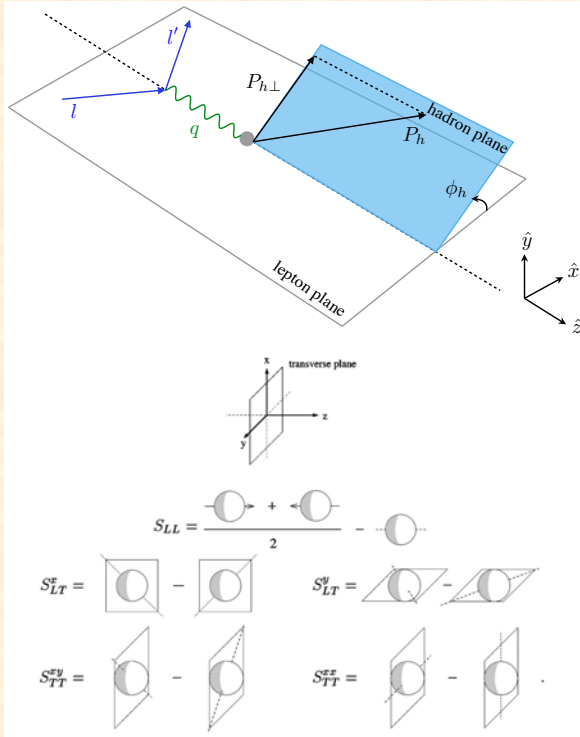
Semi-inclusive DIS with a spin-1 hadron

may skip

J. Zhao, A. Bacchetta, S. Kumano,
T. Liu, and Y.-J. Zhou, arXiv:2508.06134.

$$\frac{d\sigma_{\text{Tens}}}{dx_d dy dz d\phi_h d\psi dP_{h\perp}^2}$$

$$\begin{aligned} &= \frac{\alpha^2}{x_d y Q^2} \frac{y^2}{2(1-\varepsilon)} \left(1 + \frac{\gamma^2}{2x_d} \right) \left\{ S_{LL} \left[F_{U(LL),T} + \varepsilon F_{U(LL),L} + \sqrt{2\varepsilon(1+\varepsilon)} \cos \phi_h F_{U(LL)}^{\cos \phi_h} \right. \right. \\ &\quad \left. \left. + \varepsilon \cos(2\phi_h) F_{U(LL)}^{\cos 2\phi_h} + \lambda_e \sqrt{2\varepsilon(1-\varepsilon)} \sin \phi_h F_{L(LL)}^{\sin \phi_h} \right] \right. \\ &\quad + |S_{LT}| \left[\cos(\phi_h - \phi_{LT}) \left(F_{U(LT),T}^{\cos(\phi_h - \phi_{LT})} + \varepsilon F_{U(LT),L}^{\cos(\phi_h - \phi_{LT})} \right) \right. \\ &\quad + \sqrt{2\varepsilon(1+\varepsilon)} \left(\cos \phi_{LT} F_{U(LT)}^{\cos \phi_{LT}} + \cos(2\phi_h - \phi_{LT}) F_{U(LT)}^{\cos(2\phi_h - \phi_{LT})} \right) \\ &\quad + \varepsilon \left(\cos(\phi_h + \phi_{LT}) F_{U(LT)}^{\cos(\phi_h + \phi_{LT})} + \cos(3\phi_h - \phi_{LT}) F_{U(LT)}^{\cos(3\phi_h - \phi_{LT})} \right) \\ &\quad + \lambda_e \sqrt{2\varepsilon(1-\varepsilon)} \left(\sin \phi_{LT} F_{L(LT)}^{\sin \phi_{LT}} + \sin(2\phi_h - \phi_{LT}) F_{L(LT)}^{\sin(2\phi_h - \phi_{LT})} \right) \\ &\quad \left. \left. + \lambda_e \sqrt{1-\varepsilon^2} \sin(\phi_h - \phi_{LT}) F_{L(LT)}^{\sin(\phi_h - \phi_{LT})} \right] \right. \\ &\quad + |S_{TT}| \left[\cos(2\phi_h - 2\phi_{TT}) \left(F_{U(TT),T}^{\cos(2\phi_h - 2\phi_{TT})} + \varepsilon F_{U(TT),L}^{\cos(2\phi_h - 2\phi_{TT})} \right) \right. \\ &\quad + \sqrt{2\varepsilon(1+\varepsilon)} \left(\cos(\phi_h - 2\phi_{TT}) F_{U(TT)}^{\cos(\phi_h - 2\phi_{TT})} + \cos(3\phi_h - 2\phi_{TT}) F_{U(TT)}^{\cos(3\phi_h - 2\phi_{TT})} \right) \\ &\quad + \varepsilon \left(\cos(2\phi_{TT}) F_{U(TT)}^{\cos(2\phi_{TT})} + \cos(4\phi_h - 2\phi_{TT}) F_{U(TT)}^{\cos(4\phi_h - 2\phi_{TT})} \right) \\ &\quad + \lambda_e \sqrt{2\varepsilon(1-\varepsilon)} \left(\sin(\phi_h - 2\phi_{TT}) F_{L(TT)}^{\sin(\phi_h - 2\phi_{TT})} + \sin(3\phi_h - 2\phi_{TT}) F_{L(TT)}^{\sin(3\phi_h - 2\phi_{TT})} \right) \\ &\quad \left. \left. + \lambda_e \sqrt{1-\varepsilon^2} \sin(2\phi_h - 2\phi_{TT}) F_{L(TT)}^{\sin(2\phi_h - 2\phi_{TT})} \right] \right\} \end{aligned}$$



$w \cdot g \cdot F_{U(LT), T} : F_{\text{incoming lepton spin (target spin), photon polarization}}$

$$\mathcal{C}[wfD] = x \sum_a e_a^2 \int d^2 \mathbf{k}_T d^2 \mathbf{p}_T \delta^{(2)}(\mathbf{k}_T - \mathbf{p}_T - \mathbf{P}_{h\perp}/z) w(\mathbf{k}_T, \mathbf{p}_T) f^a(x, k_T^2) D^a(z, p_T^2)$$

$$F_{U(LL),T} = \mathcal{C}[f_{1LL}D_1],$$

$$F_{U(LL),L} = 0,$$

$$F_{U(LL)}^{\cos \phi_h} = \frac{2M}{Q} \mathcal{C} \left[-\frac{\hat{\mathbf{h}} \cdot \mathbf{p}_T}{M} \left(x f_{LL}^\perp D_1 + \frac{M_h}{M} h_{1LL}^\perp \frac{\tilde{H}}{z} \right) - \frac{\hat{\mathbf{h}} \cdot \mathbf{k}_T}{M_h} \left(x h_{LL} H_1^\perp + \frac{M_h}{M} f_{1LL} \frac{\tilde{D}^\perp}{z} \right) \right],$$

$$F_{U(LL)}^{\cos 2\phi_h} = \mathcal{C} \left[-\frac{2(\hat{\mathbf{h}} \cdot \mathbf{k}_T)(\hat{\mathbf{h}} \cdot \mathbf{p}_T) - \mathbf{k}_T \cdot \mathbf{p}_T}{MM_h} h_{1LL}^\perp H_1^\perp \right],$$

$$F_{L(LL)}^{\sin \phi_h} = \frac{2M}{Q} \mathcal{C} \left[-\frac{\hat{\mathbf{h}} \cdot \mathbf{k}_T}{M_h} \left(x e_{LL} H_1^\perp + \frac{M_h}{M} f_{1LL} \frac{\tilde{G}^\perp}{z} \right) + \frac{\hat{\mathbf{h}} \cdot \mathbf{p}_T}{M} \left(x g_{LL}^\perp D_1 + \frac{M_h}{M} h_{1LL}^\perp \frac{\tilde{E}}{z} \right) \right],$$

$$F_{U(LT),T}^{\cos(\phi_h - \phi_{LT})} = \mathcal{C} \left[\frac{\hat{\mathbf{h}} \cdot \mathbf{p}_T}{M} f_{1LT} D_1 \right],$$

$$F_{U(LT),L}^{\cos(\phi_h - \phi_{LT})} = 0,$$

$$F_{U(LT)}^{\cos \phi_{LT}} = \frac{2M}{Q} \mathcal{C} \left\{ - \left[\left(x f_{LT} D_1 + \frac{M_h}{M} h_{1LT} \frac{\tilde{H}}{z} \right) + \frac{\mathbf{k}_T \cdot \mathbf{p}_T}{2MM_h} \left[\left(x h_{LT} H_1^\perp + \frac{M_h}{M} g_{1LT} \frac{\tilde{G}^\perp}{z} \right) + \left(x h_{LT}^\perp H_1^\perp - \frac{M_h}{M} f_{1LT} \frac{\tilde{D}^\perp}{z} \right) \right] \right] \right\},$$

$$F_{U(LT)}^{\cos(2\phi_h - \phi_{LT})} = \frac{2M}{Q} \mathcal{C} \left\{ \frac{\mathbf{p}_T^2 - 2(\hat{\mathbf{h}} \cdot \mathbf{p}_T)^2}{2M^2} \left(x f_{LT}^\perp D_1 + \frac{M_h}{M} h_{1LT}^\perp \frac{\tilde{H}}{z} \right) - \frac{2(\hat{\mathbf{h}} \cdot \mathbf{k}_T)(\hat{\mathbf{h}} \cdot \mathbf{p}_T) - \mathbf{k}_T \cdot \mathbf{p}_T}{2MM_h} \left[\left(x h_{LT}^\perp H_1^\perp + \frac{M_h}{M} f_{1LT} \frac{\tilde{D}^\perp}{z} \right) - \left(x h_{LT} H_1^\perp - \frac{M_h}{M} g_{1LT} \frac{\tilde{G}^\perp}{z} \right) \right] \right\},$$

$$F_{U(LT)}^{\cos(\phi_h + \phi_{LT})} = \mathcal{C} \left[-\frac{\hat{\mathbf{h}} \cdot \mathbf{k}_T}{M_h} h_{1LT} H_1^\perp \right],$$

$$F_{U(LT)}^{\cos(3\phi_h - \phi_{LT})} = \mathcal{C} \left[-\frac{4(\hat{\mathbf{h}} \cdot \mathbf{k}_T)(\hat{\mathbf{h}} \cdot \mathbf{p}_T)^2 - (\hat{\mathbf{h}} \cdot \mathbf{k}_T) \mathbf{p}_T^2 - 2(\hat{\mathbf{h}} \cdot \mathbf{p}_T)(\mathbf{k}_T \cdot \mathbf{p}_T)}{2M^2 M_h} h_{1LT}^\perp H_1^\perp \right],$$

$$F_{L(LT)}^{\sin \phi_{LT}} = \frac{2M}{Q} \mathcal{C} \left\{ \left(x g_{LT} D_1 + \frac{M_h}{M} h_{1LT} \frac{\tilde{E}}{z} \right) + \frac{\mathbf{k}_T \cdot \mathbf{p}_T}{2MM_h} \left[\left(x e_{LT} H_1^\perp - \frac{M_h}{M} g_{1LT} \frac{\tilde{D}^\perp}{z} \right) - \left(x e_{LT}^\perp H_1^\perp + \frac{M_h}{M} f_{1LT} \frac{\tilde{G}^\perp}{z} \right) \right] \right\},$$

$$F_{L(LT)}^{\sin(2\phi_h - \phi_{LT})} = \frac{2M}{Q} \mathcal{C} \left\{ -\frac{2(\hat{\mathbf{h}} \cdot \mathbf{k}_T)(\hat{\mathbf{h}} \cdot \mathbf{p}_T) - \mathbf{k}_T \cdot \mathbf{p}_T}{2MM_h} \left[\left(x e_{LT} H_1^\perp + \frac{M_h}{M} f_{1LT} \frac{\tilde{G}^\perp}{z} \right) + \left(x e_{LT}^\perp H_1^\perp - \frac{M_h}{M} g_{1LT} \frac{\tilde{D}^\perp}{z} \right) \right] \right\},$$

may skip

$$F_{L(LT)}^{\sin(\phi_h - \phi_{LT})} = \mathcal{C} \left[-\frac{\hat{\mathbf{h}} \cdot \mathbf{p}_T}{M} g_{1LT} D_1 \right],$$

$$F_{U(TT),T}^{\cos(2\phi_h - 2\phi_{LT})} = \mathcal{C} \left[-\frac{2(\hat{\mathbf{h}} \cdot \mathbf{p}_T)^2 - \mathbf{p}_T^2}{M^2} f_{1TT} D_1 \right],$$

$$F_{U(TT),L}^{\cos(2\phi_h - 2\phi_{LT})} = 0,$$

$$F_{U(TT)}^{\cos(\phi_h - 2\phi_{TT})} = \frac{2M}{Q} \mathcal{C} \left\{ \frac{\hat{\mathbf{h}} \cdot \mathbf{p}_T}{M} (x f_{TT} D_1 - \frac{M_h}{M} h_{1TT} \frac{\tilde{H}}{z}) + \frac{(\hat{\mathbf{h}} \cdot \mathbf{k}_T) \mathbf{p}_T^2 - 2(\hat{\mathbf{h}} \cdot \mathbf{p}_T)(\mathbf{k}_T \cdot \mathbf{p}_T)}{2M^2 M_h} \left[\left(x h_{TT} H_1^\perp - \frac{M_h}{M} g_{1TT} \frac{\tilde{G}^\perp}{z} \right) + \left(x h_{TT}^\perp H_1^\perp - \frac{M_h}{M} f_{1TT} \frac{\tilde{D}^\perp}{z} \right) \right] \right\},$$

$$F_{U(TT)}^{\cos(3\phi_h - 2\phi_{TT})} = \frac{2M}{Q} \mathcal{C} \left\{ \frac{3(\hat{\mathbf{h}} \cdot \mathbf{p}_T)(2(\hat{\mathbf{h}} \cdot \mathbf{p}_T)^2 - \mathbf{p}_T^2)}{2M^3} \left(x f_{TT}^\perp D_1 + \frac{M_h}{M} h_{1TT}^\perp \frac{\tilde{H}}{z} \right) + \frac{4(\mathbf{h} \cdot \mathbf{k}_T)(\mathbf{h} \cdot \mathbf{p}_T)^2 - 2(\mathbf{h} \cdot \mathbf{p}_T)(\mathbf{k}_T \cdot \mathbf{p}_T) - (\mathbf{h} \cdot \mathbf{k}_T) \mathbf{p}_T^2}{2M^2 M_h} \times \left[\left(x h_{TT}^\perp H_1^\perp + \frac{M_h}{M} f_{1TT} \frac{\tilde{D}^\perp}{z} \right) - \left(x h_{TT} H_1^\perp + \frac{M_h}{M} g_{1TT} \frac{\tilde{G}^\perp}{z} \right) \right] \right\},$$

$$F_{U(TT)}^{\cos(2\phi_{TT})} = \mathcal{C} \left[\frac{\mathbf{k}_T \cdot \mathbf{p}_T}{MM_h} h_{1TT} H_1^\perp \right],$$

$$F_{U(TT)}^{\cos(4\phi_h - 2\phi_{TT})} = \mathcal{C} \left[-\left(\frac{4(\hat{\mathbf{h}} \cdot \mathbf{k}_T)(\hat{\mathbf{h}} \cdot \mathbf{p}_T) \mathbf{p}_T^2 - 8(\hat{\mathbf{h}} \cdot \mathbf{k}_T)(\hat{\mathbf{h}} \cdot \mathbf{p}_T)^3}{2M^3 M_h} + \frac{4(\mathbf{k}_T \cdot \mathbf{p}_T)(\hat{\mathbf{h}} \cdot \mathbf{p}_T)^2 - (\mathbf{k}_T \cdot \mathbf{p}_T) \mathbf{p}_T^2}{2M^3 M_h} \right) h_{1TT}^\perp H_1^\perp \right],$$

$$F_{L(TT)}^{\sin(\phi_h - 2\phi_{TT})} = \frac{2M}{Q} \mathcal{C} \left\{ \frac{\hat{\mathbf{h}} \cdot \mathbf{p}_T}{M} \left(x g_{TT} D_1 + \frac{M_h}{M} h_{1TT} \frac{\tilde{E}}{z} \right) - \frac{(\hat{\mathbf{h}} \cdot \mathbf{k}_T) \mathbf{p}_T^2 - 2(\hat{\mathbf{h}} \cdot \mathbf{p}_T)(\mathbf{k}_T \cdot \mathbf{p}_T)}{2M^2 M_h} \left[\left(x e_{TT} H_1^\perp - \frac{M_h}{M} f_{1TT} \frac{\tilde{G}^\perp}{z} \right) - \left(x e_{TT}^\perp H_1^\perp - \frac{M_h}{M} g_{1TT} \frac{\tilde{D}^\perp}{z} \right) \right] \right\},$$

$$F_{L(TT)}^{\sin(3\phi_h - 2\phi_{TT})} = \frac{2M}{Q} \mathcal{C} \left\{ -\frac{3(\hat{\mathbf{h}} \cdot \mathbf{p}_T)(2(\hat{\mathbf{h}} \cdot \mathbf{p}_T)^2 - \mathbf{p}_T^2)}{2M^3} \left(x g_{TT}^\perp D_1 + \frac{M_h}{M} h_{1TT}^\perp \frac{\tilde{E}}{z} \right) + \frac{4(\mathbf{h} \cdot \mathbf{k}_T)(\mathbf{h} \cdot \mathbf{p}_T)^2 - 2(\mathbf{h} \cdot \mathbf{p}_T)(\mathbf{k}_T \cdot \mathbf{p}_T) - (\mathbf{h} \cdot \mathbf{k}_T) \mathbf{p}_T^2}{2M^2 M_h} \times \left[\left(x e_{TT} H_1^\perp + \frac{M_h}{M} f_{1TT} \frac{\tilde{G}^\perp}{z} \right) + \left(x e_{TT}^\perp H_1^\perp + \frac{M_h}{M} g_{1TT} \frac{\tilde{D}^\perp}{z} \right) \right] \right\},$$

$$F_{L(TT)}^{\sin(2\phi_h - 2\phi_{TT})} = \mathcal{C} \left[-\frac{2(\hat{\mathbf{h}} \cdot \mathbf{p}_T)^2 - \mathbf{p}_T^2}{M^2} g_{1TT} D_1 \right],$$

Twist 3 PDF f_{LT}

may skip

Twist-3 PDFs

Quark Hadron	$\gamma^i, 1, i\gamma_5$		$\gamma^+ \gamma_5$		$\sigma^{\mu\nu}, \sigma^{+-}$	
	T-even	T-odd	T-even	T-odd	T-even	T-odd
U	[e]					
L					[h _L]	
T			g _T			
LL	[e _{LL}]					
LT	f _{LT}					
TT						

- Drell-Yan: Si-Yi Qiao and Qin-Tao Song, PRD 111 (2025) 054026
- Semi-inclusive DIS: J. Zhao, A. Bacchetta, S. Kumano, T. Liu, and Y.-J. Zhou, arXiv:2508.06134.

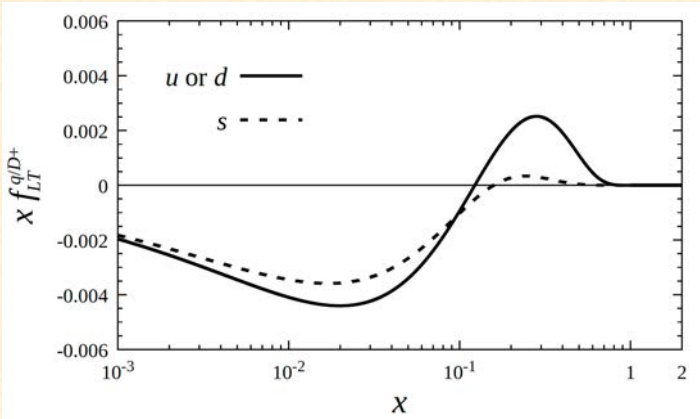
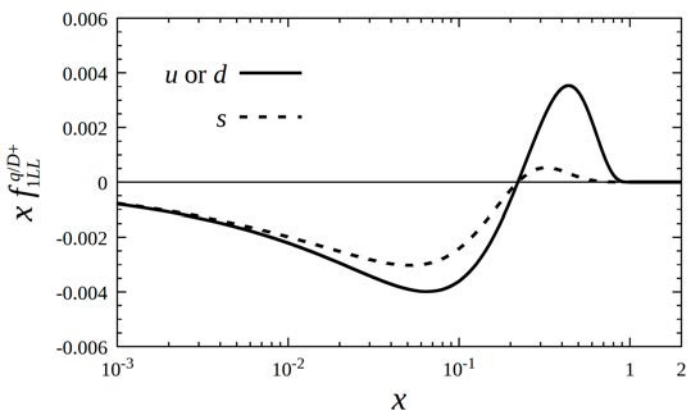
Twist-2 TMDs

Quark Hadron	U (γ^+)		L ($\gamma^+ \gamma_5$)		T ($i\sigma^{i+} \gamma_5 / \sigma^{i+}$)	
	T-even	T-odd	T-even	T-odd	T-even	T-odd
U	f ₁					[h ₁ ⁺]
L			g _{1L}			[h _{1L} ⁺]
T		f _{1T} ⁺	g _{1T}		[h ₁], [h _{1T} ⁺]	
LL	f _{1LL}					[h _{1LL} ⁺]
LT	f _{1LT}		g _{1LT}			[h _{1LT}], [h _{1LT} ⁺]
TT	f _{1TT}		g _{1TT}			[h _{1TT}], [h _{1TT} ⁺]

- $f_{LT}(x)$ estimate: S. Kumano, K. Kuroki, arXiv:2509.05046.

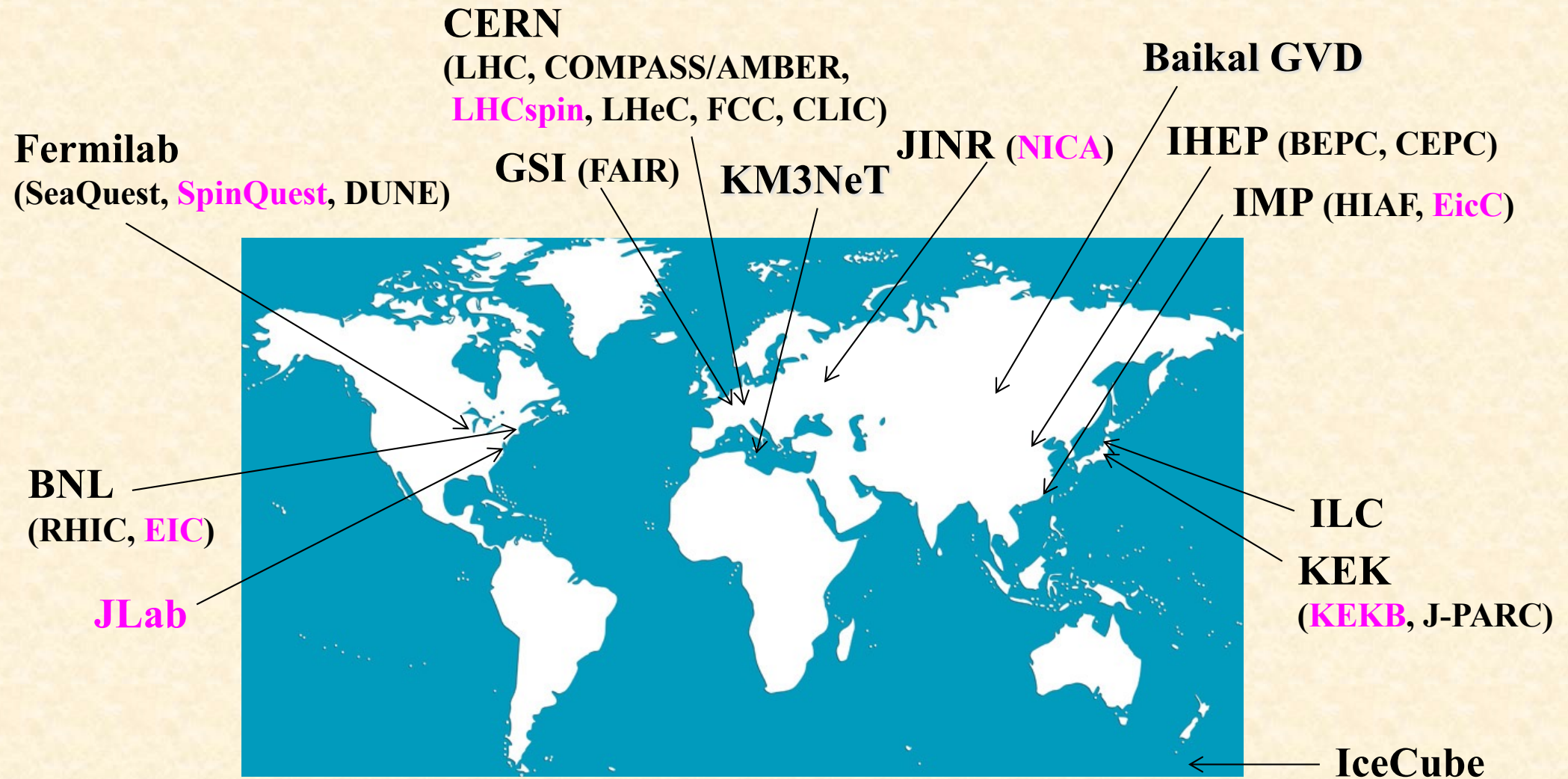
Wandzura-Wilczek-like relation : $f_{LT}^{q+}(x) = \frac{3}{2} \int_x^2 \frac{dy}{y} f_{1LL}^{q+}(y)$

$$f_{1LL}^{q+} = f_{1LL}^q + f_{1LL}^{\bar{q}} = -\frac{2}{3}(\delta_T q + \delta_T \bar{q}) = -\frac{2}{3}(b_1^q + b_1^{\bar{q}})$$



Future prospects and summary

High-energy hadron physics experiments



Facilities on spin-1 hadron structure functions including future possibilities.

Spin-1 deuteron experiments from the middle of 2020's

JLab



The Deuteron Tensor Structure Function b_1

A Proposal to Jefferson Lab PAC-38.
(Update to LCB 11-003)

J.-P. Chen (co-spokesperson), P. Solvignon (co-spokesperson),
K. Allada, A. Cammarone, A. Durr, D. Gaskell,
C. Keith, S. Wood, J. Zhang
Thomas Jefferson National Accelerator Facility, Newport News, VA 23606

N. Kalantarians (co-spokesperson), O. Roudon (co-spokesperson)
Donal B. Day, Hovhannes Bagdasaryan, Charles Hanretty
Richard Ludgren, Blaine Norum, Zhilong Ye
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K. Sitar (co-spokesperson), A. Atkins, T. Badman,
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R. Gilman
Rutgers, The State University of New Jersey, Piscataway, NJ 08854

Seonho Choi, Hoyoung Kang, Hyeok Kang, Yoonmin Oh
Seoul National University, Seoul 151-747 Korea

**Proposal (approved),
Experiment: late 2020's**

A Letter of Intent to Jefferson Lab PAC 44, June 6, 2016
Search for Exotic Gluonic States in the Nucleus

M. Jones, C. Keith, J. Maxwell*, D. Meekins
Thomas Jefferson National Accelerator Facility, Newport News, VA 23606

W. Detmold, R. Jaffe, R. Milner, P. Shanahan
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D. Crabb, D. Day, D. Keller, O. A. Roudon
University of Virginia, Charlottesville, VA 22904

J. Pierce
Oak Ridge National Laboratory, Oak Ridge, TN 37831

Fermilab



The Transverse Structure of the Deuteron with Drell-Yan

D. Keller¹

¹University of Virginia, Charlottesville, VA 22904

**Proposal,
Fermilab-PAC: 2022
Experiment: 2020's**

NICA



Progress in Particle and Nuclear Physics 119 (2021) 103858

Contents lists available at ScienceDirect

Progress in Particle and Nuclear Physics

journal homepage: www.elsevier.com/locate/ppnp

Review

On the physics potential to study the gluon content of proton and deuteron at NICA SPD

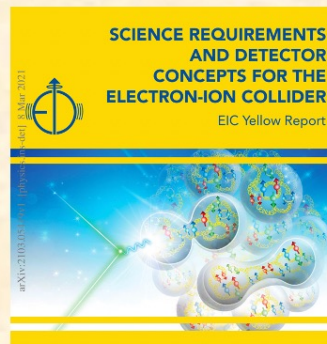
A. Arbutov^a, A. Bacchetta^{b,c}, M. Butenschoen^d, F.G. Celiberto^{b,c,d},
U. D'Alesio^{b,h}, M. Deka^e, I. Denisenko^e, M.G. Echevarria^a, A. Efremov^a,
N.Ya. Ivanov^{a,d}, A. Guskov^{a,b,e}, A. Karpishkov^{a,d}, Ya. Klopov^{a,d}, B.A. Kniehl^a,
A. Kotzinian^{a,d}, S. Kumano^a, J.P. Lansberg^a, Keh-Fei Liu^f, F. Murgia^h,
M. Nefedov^g, B. Parsamyan^{a,d,g}, C. Pisano^{a,b,h}, M. Radici^g, A. Rymbekova^a,
V. Saleev^a, A. Shipilova^{a,d}, Qin-Tao Song^a, O. Teryaev^a

Prog. Nucl. Part. Phys.

119 (2021) 103858,

Experiment: middle of 2020's

2030's EIC/EicC



R. Abdul Khalek et al.

Nucl. Phys. A 1026 (2022) 122447.

LHCspin



CERN-ESPP-Note-2018-111

The LHCSpin Project

C. A. Aidala^a, A. Bacchetta^{a,b}, M. Boglione^{c,d}, G. Bozzi^{e,f}, V. Caracciolo^{g,h}, M. Chiosso^{i,j}, R. Cimino^g,
G. Ciullo^{g,h}, M. Contalbrigo^{g,h}, U. D'Alesio^{g,h}, P. Di Nezza^g, R. Engels^g, K. Grigoryev^h, D. Keller^h,
P. Lenise^{g,h}, S. Liuti^h, A. Metz^g, P.J. Mulders^{h,i}, F. Murgia^g, A. Nass^h, D. Panzieri^{h,i},
L. L. Pappalardo^{g,h}, B. Pascaud^{g,h}, C. Pisano^{g,h}, M. Radici^g, F. Rathmann^h, D. Reggiani^h, M. Schlegel^h,
S. Scopetta^{g,h}, E. Steffens^h, A. Vasiliev^h

**arXiv:1901.08002,
Experiment: ~2028**

**D. P. Anderle et al.,
Front. Phys. 16 (2021) 64701.**



Frontiers of Physics

<https://doi.org/10.1007/s11467-021-1062-0>

Front. Phys.
16(6), 64701 (2021)

REVIEW ARTICLE

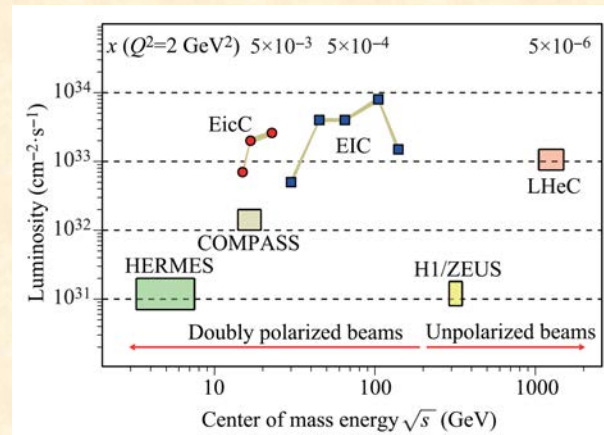
Electron-ion collider in China

Daniela P. Anderle¹, Valerio Bertone², Xu Cao^{3,4}, Lei Chang⁵, Ningbo Chang⁶, Gu Chen⁷,
Xurong Chen^{8,9}, Zhaojun Chen⁸, Zhaofeng Cui¹⁰, Lingyun Dai¹¹, Weitian Dong¹², Minghui Ding¹³,
Xu Feng¹⁴, Chang Gong¹⁵, Longcheng Guo¹⁶, Feng-Kun Guo^{1,17}, Chengdong Han^{18,19}, Jun He²⁰,
Tie-Juan Hou²¹, Hongxia Huang²², Yin Huang²³, Krešimir Kumerik²⁴, L. P. Kaptari^{25,26},
Demin Li²⁷, Henghe Li²⁸, Minxiang Li^{29,30}, Xueqian Li³¹, Yutao Liang³², Zuoqiang Liang³³, Chen Liu³⁴,
Chuan Liu³⁵, Guoming Liu³⁶, Jie Liu³⁷, Liuning Liu³⁸, Xiang Liu³⁹, Tianbo Liu⁴⁰, Xiaofeng Luo⁴¹,
Zhun Lyu⁴², Boqiang Ma⁴³, Fu Ma⁴⁴, Jianping Ma^{45,46}, Yuguang Ma^{47,48}, Lijun Mao⁴⁹,
Cédric Mezrag⁵⁰, Hervé Moutarde⁵¹, Jialun Ping⁵², Sixue Qin⁵³, Hang Ren⁵⁴, Craig D. Roberts⁵⁵,
Juan Rojo^{56,57}, Guodong Shen⁵⁸, Chao Shi⁵⁹, Qintao Song⁶⁰, Hao Sun⁶¹, Paweł Szumard⁶²,
Enke Wang⁶³, Fan Wang⁶⁴, Qian Wang⁶⁵, Rong Wang⁶⁶, Ruiru Wang⁶⁷, Taofeng Wang⁶⁸, Wei Wang⁶⁹,
Xiaoyu Wang⁷⁰, Xiaoyun Wang⁷¹, Jiajun Wu⁷², Xinggang Wu⁷³, Lei Xia⁷⁴, Bowen Xiao^{75,76},
Guoqing Xiao⁷⁷, Ju-Jun Xie⁷⁸, Yaping Xie⁷⁹, Hongxi Xing⁸⁰, Huihui Xu⁸¹, Xu Xu^{82,83},
Shusheng Xu⁸⁴, Mengshi Yan⁸⁵, Wenbiao Yan⁸⁶, Wencheng Yao⁸⁷, Xihui Yao⁸⁸, Jiancheng Yang⁸⁹,
Yi-Bo Yang^{90,91}, Zhi Yang⁹², Deliang Yao⁹³, Zhibong Ye⁹⁴, Peilin Yin⁹⁵, C.-P. Yuan⁹⁶, Wenlong Zhan⁹⁷,
Jianhui Zhang⁹⁸, Jinlong Zhang⁹⁹, Pengming Zhang¹⁰⁰, Yifeng Zhang¹⁰¹, Chao-Hsi Chang¹⁰²,
Zhenyu Zhang¹⁰³, Hongwei Zhao¹⁰⁴, Kuang-Da Zhao¹⁰⁵, Qing Zhao¹⁰⁶, Yuxiang Zhao¹⁰⁷,
Zhengguo Zhao¹⁰⁸, Liang Zheng¹⁰⁹, Jian Zhou¹¹⁰, Xiang Zhou¹¹¹, Xiaorong Zhou¹¹²,
Bingsong Zou^{113,114}, Liping Zou¹¹⁵

Electron-ion collider projects in the world

CERN

may skip



BNL



EIC-US

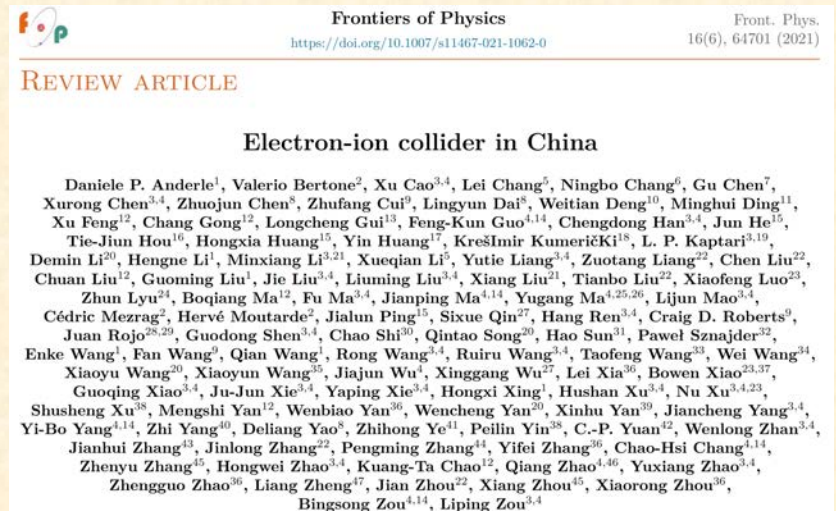
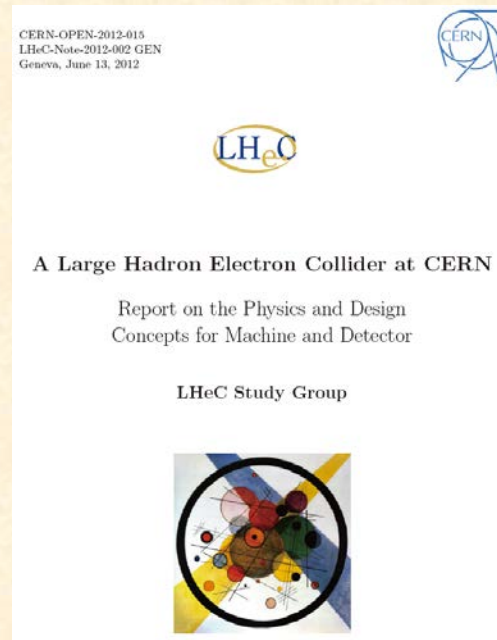
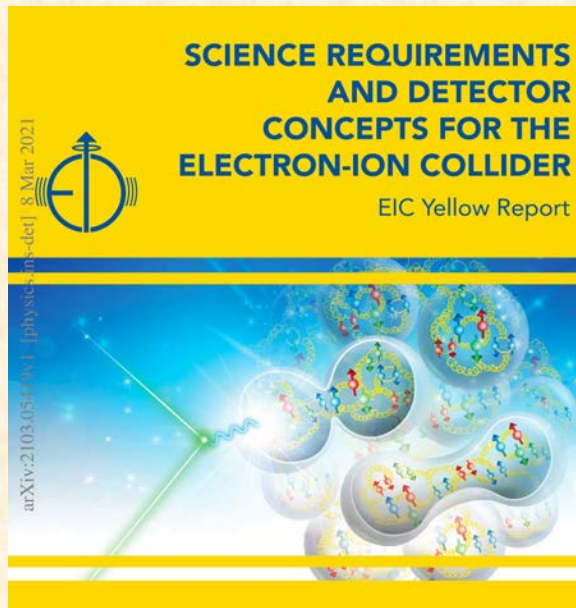
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arXiv:2103.05419.

LHeC

J. L. Abelleira Fernandez *et al.*,
J. Phys. G: Nucl. Part. Phys.
39 (2012) 075001.

Institute of Modern Physics,
High Intensity Heavy Ion
Accelerator Facility (HIAF)
→ Electron-ion collider in China (EicC)

D. P. Anderle *et al.*, Front. Phys. 16 (2021) 64701.



Part II. Summary

Spin-1 structure functions of the deuteron (additional spin structure to nucleon spin)

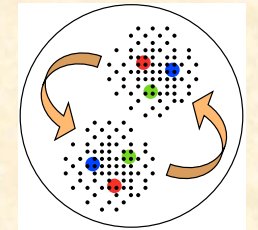
- Tensor structure in quark-gluon degrees of freedom
- Tensor-polarized structure function b_1 and PDFs, gluon transversity

Experiments at JLab, Fermilab, NICA, LHCspin/AMBER, EIC/EicC, ...

- New signature beyond “standard” hadron physics?

(beyond the standard model in particle physics???)

- TMDs up to twist 4
- Higher-twist effects could be sizable at a few GeV^2 Q^2



standard model

There are various experimental projects on the polarized spin-1 deuteron in 2020's and 2030', and “exotic” hadron structure could be found by focusing on the spin-1 nature.

Comment: There is no nuclear effect in ρ and ϕ mesons, so that the gluon transversity, for example, could be sensitive to new physics?!

The End

The End