中国科学院大学 University of Chinese Academy of Sciences | 高能核物理理论组 HIGH ENERGY NUCLEAR PHYSICS THEORY GROUP

Deconfinement transition in SU(N) theories from perturbation theory

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$SU(2)$ pure-gluon Lagrangian density with background field reads:

$$\mathcal{L}_{\mathsf{G}} = -\frac{1}{4} G^{\mu\nu,a} G^{a}_{\mu\nu}, \tag{1}$$

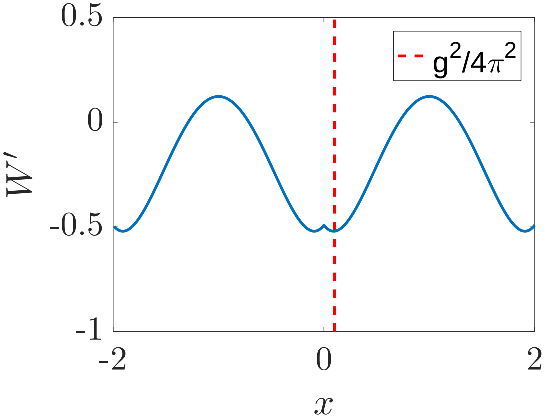
$$G^{a}_{\mu\nu}(x) = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + f^{abc} A^b_\mu A^c_\nu, \tag{2}$$

devide the field into two parts:

$$A^a_\mu = \bar{A}^a_\mu + a^a_\mu,\, A^3_0 = \bar{A}_0, \tag{3}$$

$x = \frac{g\bar{A}_0}{\pi T}$

$$W' = \pi^2 T^4 \left[-\frac{1}{15} + \frac{1}{12} x^2 (x-2)^2 + \frac{g^2}{24\pi^2} \left(1 + 2x(x-2) + \frac{3}{4} x^2 (x-2)^2 \right) \right], \tag{4}$$



$$\langle L\rangle = \frac{1}{N_c} tr\left(\exp[ig\int_0^{1/T}\bar{A}_0 d\tau]\right)$$
$$= \frac{1}{2} tr\begin{pmatrix}\exp[ig\frac{\bar{A}_0}{2T}] & 0 \\ 0 & \exp[-ig\frac{\bar{A}_0}{2T}]\end{pmatrix} \qquad (5)$$
$$= \cos(\frac{g\bar{A}_0}{2T}) = \cos(\frac{\pi x}{2}),$$

$$x_{min} = g^2/4\pi^2. \qquad (6)$$

- $\langle L\rangle$=const.?

massive gauge field

$$\mathcal{L} = \frac{1}{4} G^a_{\mu\nu} G^{\mu\nu,a} + \frac{m^2}{2} a_\mu a^\mu + \bar{D}_\mu \bar{c} D^\mu c + ih\bar{D}_\mu a^\mu, \tag{7}$$

charge $\eta = + - 0$,

$$S_{a,h} = \int \frac{d^4 p}{(2\pi)^4} \left[\frac{1}{2} a_\mu (\Delta^{-1}) a^\mu + ih\bar{D}_\mu a^\mu \right] \tag{8}$$

Redefine a new field $\mathcal{A}_\mu = \tilde{A}_\mu + i\frac{\bar{D}_\mu}{\bar{D}(m)} h_a$

$$S_{a,h} = \int \frac{d^4 p}{(2\pi)^4} \frac{1}{2} \left[\mathcal{A}_\mu \Delta^{-1} \mathcal{A}^\mu + h \frac{\bar{D}_\mu D^\mu}{\Delta^{-1}} h \right], \tag{9}$$

$$\frac{1}{\beta V} \frac{1}{2} \mathrm{Tr} \ln \Delta^{-1}_{ah} = \frac{1}{2} (3 f_m(\eta x) + f_0(\eta)), \tag{10}$$

massive gauge field

$$\mathcal{L} = \frac{1}{4} G^a_{\mu\nu} G^{\mu\nu,a} + \frac{m^2}{2} a_\mu a^\mu + \bar{D}_\mu \bar{c} D^\mu c + ih \bar{D}_\mu a^\mu, \tag{11}$$

LDW gauge, as a background field generalization of the Landau gauge, has a core gauge condition given by

$$\bar{D}_\mu a^a_\mu = 0 \tag{12}$$

if a local SU(N) transformation is applied to the background field as

$$\bar{A}^U_\mu = U \bar{A}_\mu U^{-1} + \frac{i}{g} U \partial_\mu U^{-1}$$

at the same time, the transformation $\varphi \to U \varphi U^{-1}$ is imposed on the fluctuation field, ghost fields, and Nakanishi-Lautrup field $\varphi = (a, c, \bar{c}, h)$, then the action $S_{\bar{A}}$ satisfies the symmetry

$$S_{\bar{A}}[\varphi] = S_{\bar{A}^U}[U \varphi U^{-1}]$$

The one-loop background field potential can be written as

$$V^{(1)}(T,r) = \frac{3}{2}\mathcal{F}_m(T,r) - \frac{1}{2}\mathcal{F}_0(T,r). \tag{13}$$

$$V^{(1)}_{T\gg m}(T,r) \approx \mathcal{F}_0(T,r). \tag{14}$$

The massive gluon contribution can be ignored.$r_{min} = 0$

$$V^{(1)}_{T\ll m}(T,r) \approx -\frac{1}{2}\mathcal{F}_0(T,r). \tag{15}$$

The massive gluon contribution is exponentially suppressed, massless modes dominate,with the minimum located at $r = \pi$.

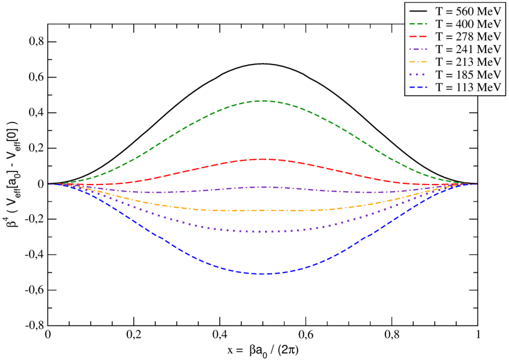


图: from M. Quandt.1603.08058v1,$r = \pi x$

$SU(3)$ case

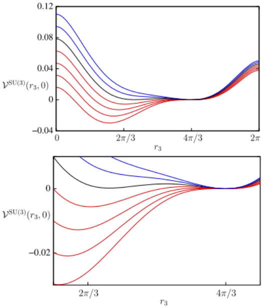


Fig. 2. The SU(3) background field potential in the $r_8 = 0$ direction in $d = 4$, for decreasing values of $\bar{m}$ (increasing temperatures) from top to bottom. The bottom figure is a close-up view around T_c. (Color online: $T = T_c$ (black), $T < T_c$ (blue), $T > T_c$ (red).)

$$\langle L \rangle = \frac{1}{3}\left[e^{-i\frac{r_8}{\sqrt{3}}} + 2e^{i\frac{r_8}{2\sqrt{3}}}\cos(r_3/2)\right] =$$

$$\ell = \frac{1+2\cos(r_3/2)}{3}$$

Polyakov loop

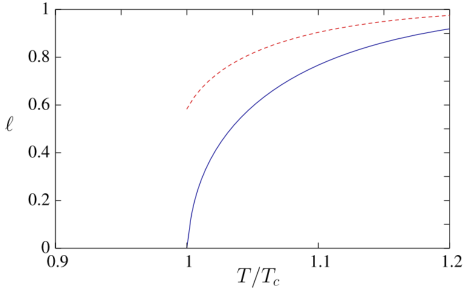
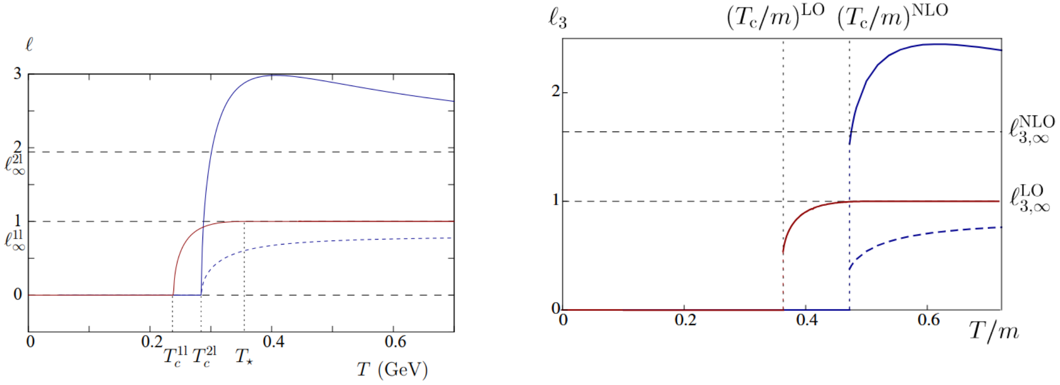


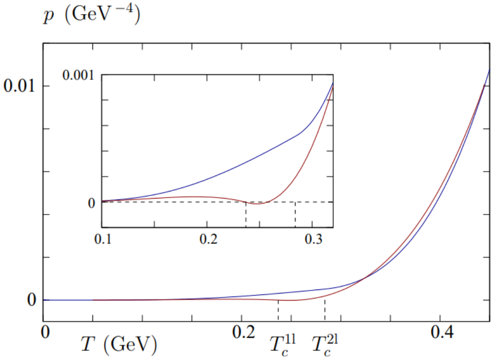
Fig. 3. The Polyakov loop as a function of the temperature normalized to the transition temperature for $N = 2$ (solid line) and $N = 3$ (dashed line). (Color online.)

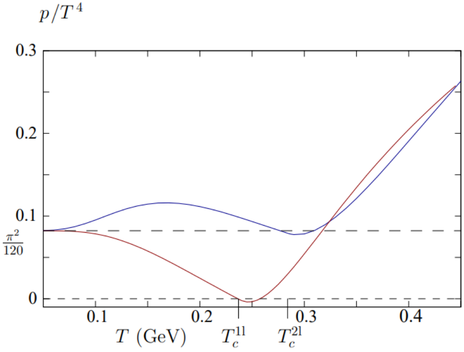
- **SU(2) (solid line):** the Polyakov loop increases continuously from a value "close to zero" at low temperatures to a non-zero value .
- **SU(3) (dashed line):** Polyakov loop exhibits a "jump" from a value "close to zero" at low temperatures directly to a non-zero value of approximately 0.6..



left $SU(2)$ right $SU(3)$

pressure





$$s = \frac{\partial P}{\partial T}$$

Conclusion

T_c (MeV)	one loop	two loop	lattice	FRG/DSE
SU(2)	238	284	295	230 (300)
SU(3)	185	254	270	275

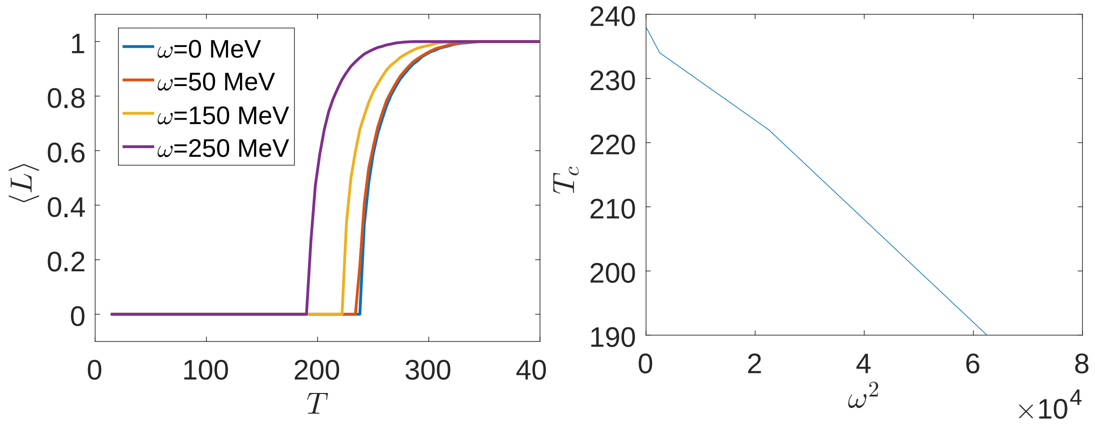
- The one-loop calculation can capture the main features of the deconfinement phase transition.
- The two-loop calculation gives better results in three ways: the critical temperature matches lattice QCD data more closely, it removes an unphysical problem with the Polyakov loop, and it corrects the issue of negative entropy.
- This approach is easy to use in calculations and can be systematically improved to higher orders.

$$\Omega = \sum_{\eta} \Omega^{\{\eta\}}(\eta x) = \sum_{\eta} [\frac{3}{2} f_m(\eta x) - \frac{1}{2} f_0(\eta x)], \tag{16}$$

$k_t r \to 0$

$$\Omega^{(1)} = \sum_{\eta} \Omega^{\{\eta\}}(\eta x) = \sum_{\eta} \frac{1}{2} [f_{m,+\omega}(\eta x) + f_{m,-\omega}(\eta x) + f_m(\eta x) - f_0(\eta x)], \tag{17}$$

$$f_{m,-\omega}(\eta x) = \int \frac{d^3k}{2 \times (2\pi)^3} \ln\left(1 - 2\cos(\eta \pi x) e^{-\beta(\sqrt{k^2+m^2}-\omega)} + e^{-2\beta(\sqrt{k^2+m^2}-\omega)}\right), \tag{18}$$



ω=0 MeV
ω=50 MeV
ω=150 MeV
ω=250 MeV
⟨L⟩
1
0.8
0.6
0.4
0.2
0
0
100
200
300
40
T
240
230
220
210
200
190
T_c
0
2
4
6
8
ω^2
×10^4

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谢谢

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