# Lattice QCD study of hadron scattering based on CLQCD ensembles

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CLQCD

味物理前沿研讨会暨味物理讲座100期特别活动

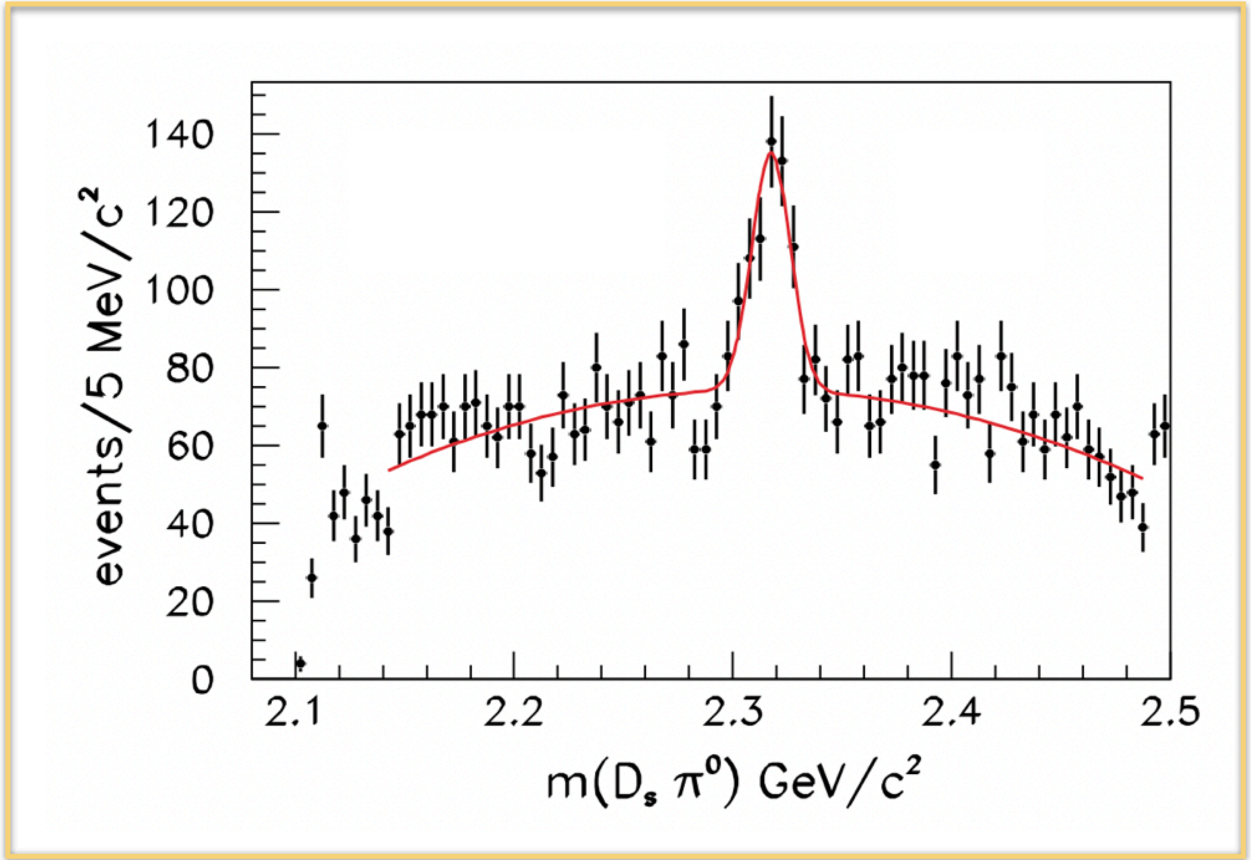
2026年1月30日-2月4日，三亚

# Hadron scattering

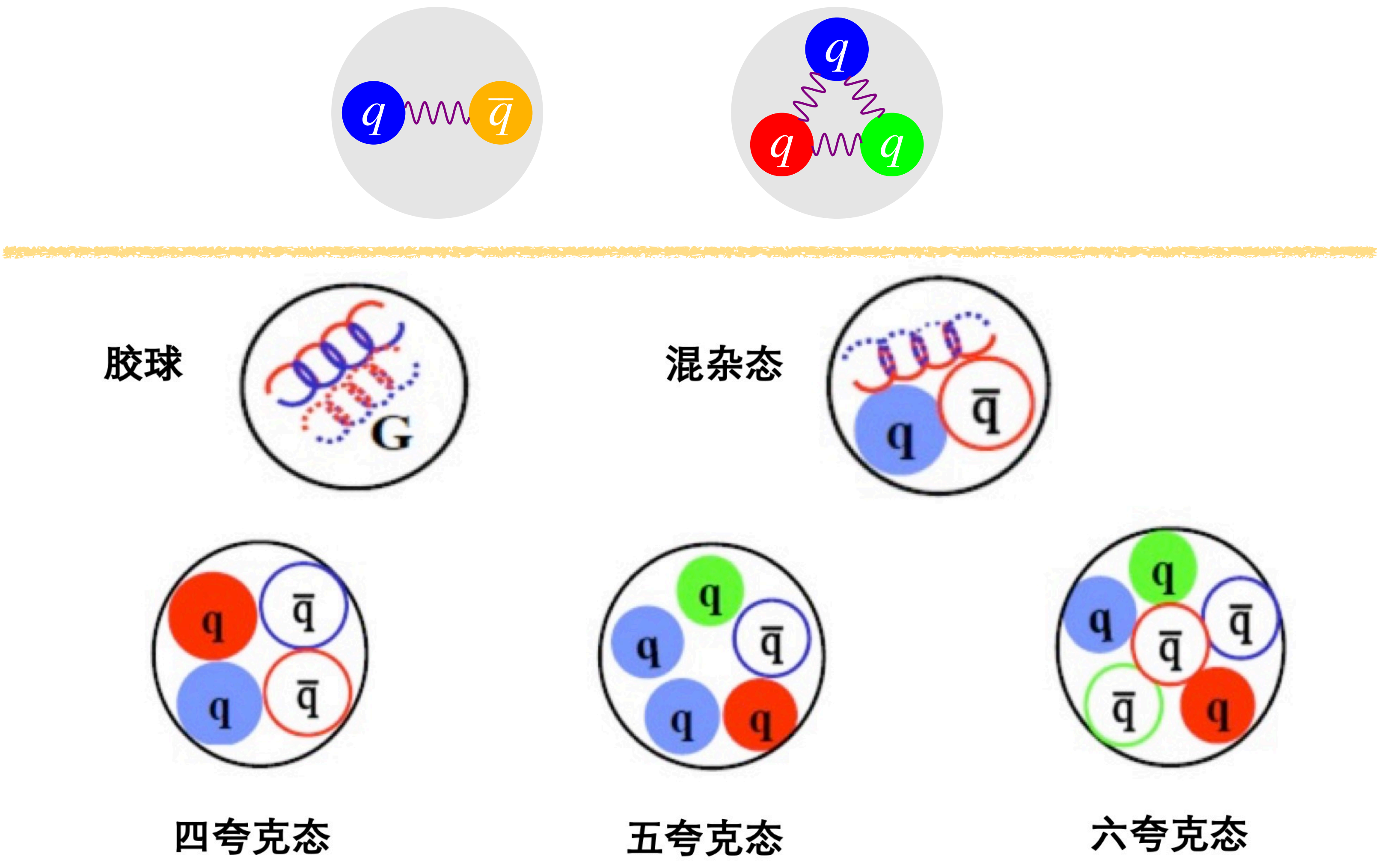
✦ Spectroscopy:

- Most of the hadrons observed in experiments are unstable.
- Understand how hadrons are built from quarks and gluons.

✦ New experimental discoveries provide opportunities and challenges for lattice studies.




Quark model and beyond



# CLQCD ensembles

- QCD Lagrangian:

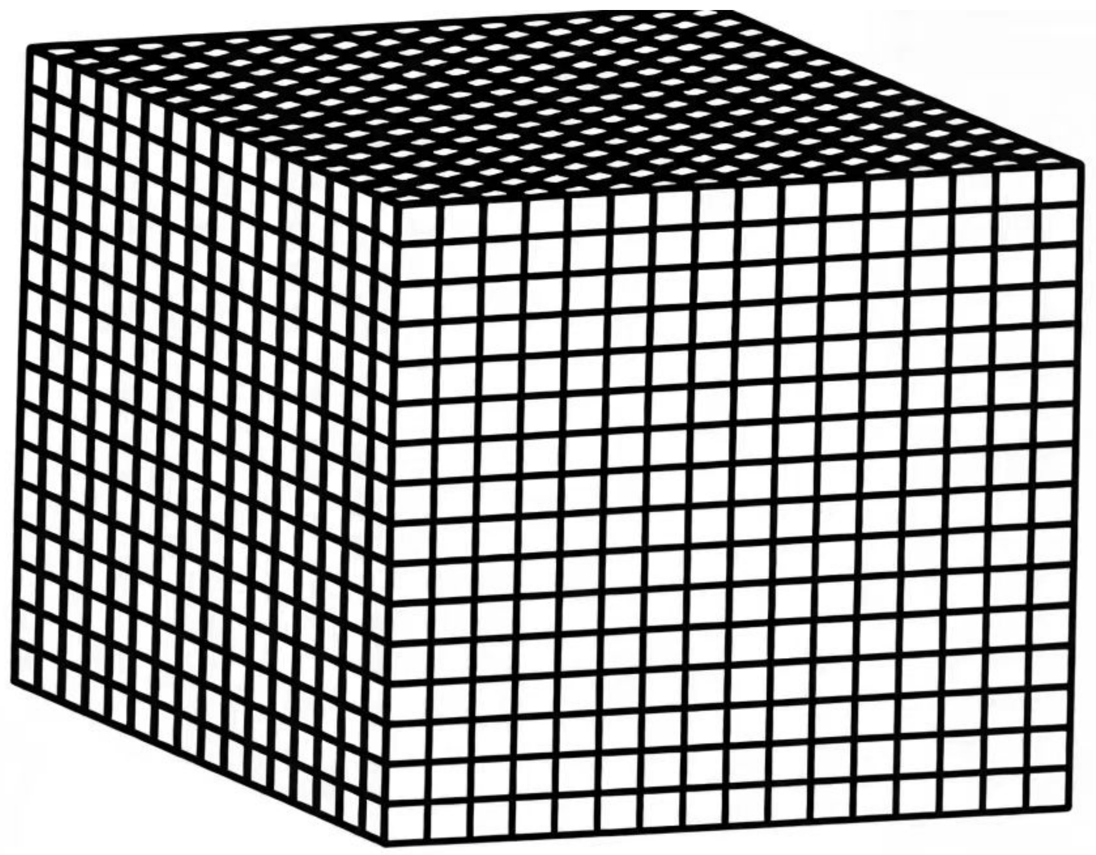
$$L_{QCD} = \bar{\psi}(i\gamma^{\mu}D_{\mu} - m_i)\psi - \frac{1}{4}F^{i}_{\mu\nu}F_{\mu\nu,i}$$

- An observable $\mathcal{O}$ is evaluated by:

$$\langle \mathcal{O} \rangle = \frac{\int D\bar{\psi}D\psi DU \mathcal{O}(\bar{\psi}, \psi, U)e^{-S(\bar{\psi},\psi,U)}}{Z}$$

- A configuration is one possible state of the gluon and quark fields, i.e. one path in the path integral.

- Parameters: quark masses, lattice spacing $a$, volume $L^3 \times T$.



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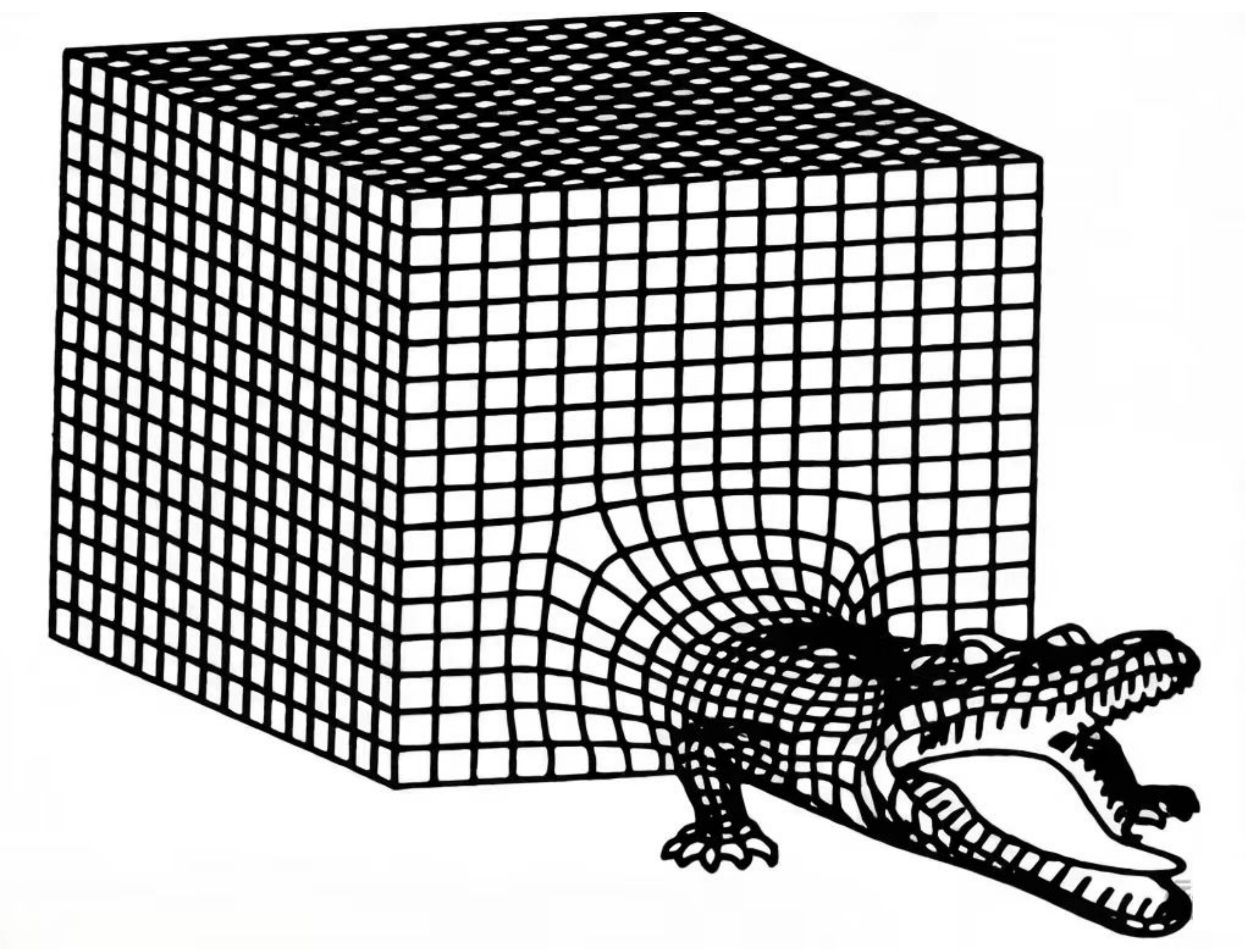


- Unphysical light quark mass
- Finite lattice spacing
- Finite volume

# CLQCD ensembles

| Lattice spacing | Volume($L^3 \times T$) | $M_\pi$ (MeV) | # of confs |
|---|---|---|---|
| 0.105 fm | $24^3 \times 72$ | 292 | 1000 |
| | $32^3 \times 64$ | | 1000 |
| | $48^3 \times 96$ | | 800 |
| | $64^3 \times 128$ | | 70 |
| | $32^3 \times 64$ | 225 | 450 |
| | $48^3 \times 96$ | | 700 |
| | $48^3 \times 96$ | 135 | 700 |
| | $64^3 \times 128$ | | 200 |
| 0.090 fm | $32^3 \times 64$ | 350 | 900 |
| 0.077 fm | $24^3 \times 72$ | 300 | 250 |
| | $32^3 \times 96$ | | 480 |
| | $48^3 \times 96$ | | 200 |
| | $32^3 \times 64$ | 210 | 460 |
| | $48^3 \times 96$ | | 200 |
| | $64^3 \times 128$ | 135 | 180 |
| 0.069 fm | $36^3 \times 108$ | 300 | 700 |
| 0.052 fm | $48^3 \times 144$ | 317 | 1000 |
| | $64^3 \times 128$ | | 100 |

- $N_f = 2 + 1$
- Symanzik-improved gauge action.
- Wilson-clover quark action.
- Lattice spacing determined by gradient flow.
- Quark masses $m_u, m_d, m_s$ are computed based on these ensembles and the results agree with FLAG average value.
- Quark propagators using distillation smearing.

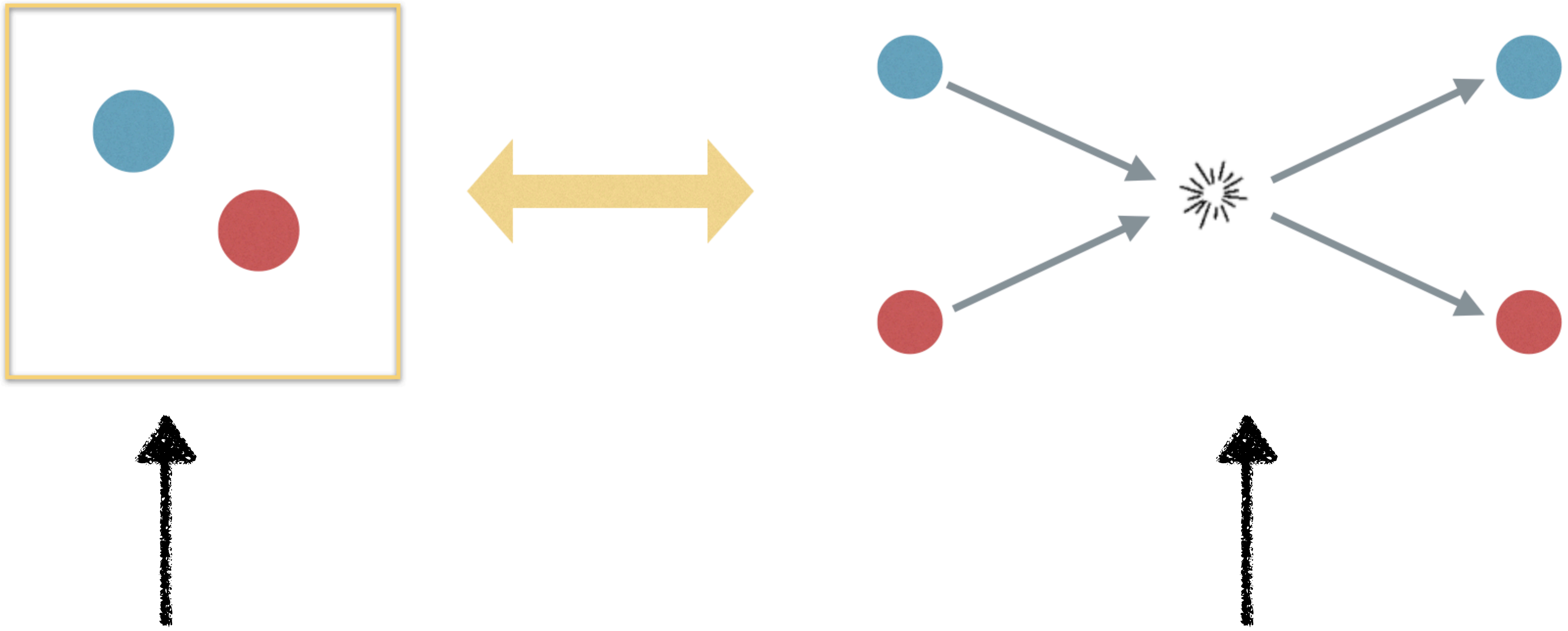
Z.-C.Hu el al., (CLQCD), PRD109(2024)5,054507

# Outline

- Hadron scattering on Lattice
- Results
  - Meson-meson scattering: $\rho, K^*, D_0^*(2300)$
  - Meson-baryon scattering: pentaquarks
  - Baryon-baryon scattering: $NN, N\Lambda, N\Xi, \Lambda\Lambda$
  - Three-body scattering
- Summary and discussions

# Scattering on lattice

Lüscher's finite volume method: M. Lüscher, Nucl. Phys. B354, 531(1991)



Finite volume spectrum

Infinite volume scattering parameters

# Scattering on lattice

Excited states:

✦ build large basis of operators $\{\mathcal{O}_1, \mathcal{O}_2, \cdots\}$ with desired quantum numbers, construct the matrix of correlation function:

$$C_{ij} = < 0 | \mathcal{O}_i \mathcal{O}_j^\dagger | 0 > = \sum_n Z_i^n Z_j^{n*} e^{-E_n t}$$

✦ Solve the generalized eigenvalue problem(GEVP):

$$C_{ij} v_j^n(t) = \lambda_n(t) C_{ij}^0 v_j^n(t)$$

✦ Eigenvalues: $\lambda_n(t) \sim e^{-E_n t}(1 + e^{-\Delta E\, t})$

✦ Optimal linear combinations of the operators to overlap on the n'th state:

$$\Omega_n = \sum_i v_i^n \mathcal{O}_i$$

# Scattering on lattice

CLQCD

- General Lüscher's formula for two-body scattering:

$$\det[\mathbf{1} + i\rho \cdot \mathbf{t} \cdot (1 + i\mathbf{M})] = 0$$

Diagonal matrix of phase-space factors $\rho_{ij} = \delta_{ij}\frac{2k_i}{E_{cm}}$

Infinite-volume scattering matrix

Finite volume information $M(E_{cm}, L)$

- Resonances/bound states are formally defined as poles in scattering amplitudes.

# Meson-meson scattering

- $\pi\pi$ P-wave scattering and the $\rho$ resonance
- $K\pi$ P-wave scattering and the $K^*$ resonnace
- $D\pi$ scattering and $D_0^*(2300)$

# $\pi\pi$ P-wave scattering and the $\rho$ resonance

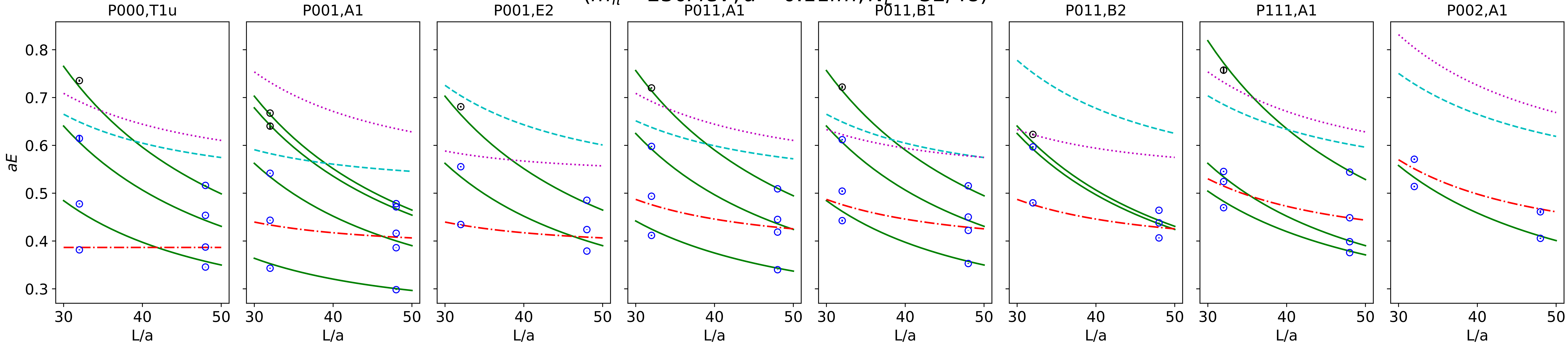
Zhengli Wang et al., JHEP08(2025)064

Interpolating operators:

$$\mathcal{O}_{\rho,i}(\vec{x},t) = \frac{1}{\sqrt{2}}\left(\bar{u}\gamma_i u(\vec{x},t) - \bar{d}\gamma_i d(\vec{x},t)\right)$$

$$(\pi\pi)^{[\vec{k}_1,\vec{k}_2]}_{\vec{P},\Lambda,\mu} = \sum_{\substack{\vec{k}_1\in\{\vec{k}_1\}^\star \\ \vec{k}_2\in\{\vec{k}_2\}^\star \\ \vec{k}_1+\vec{k}_2=\vec{P}}} \mathcal{C}\left(\vec{P},\Lambda,\mu;\vec{k}_1;\vec{k}_2\right) \times \left(\pi^+(\vec{k}_1)\pi^-(\vec{k}_2) - \pi^-(\vec{k}_1)\pi^+(\vec{k}_2)\right)$$

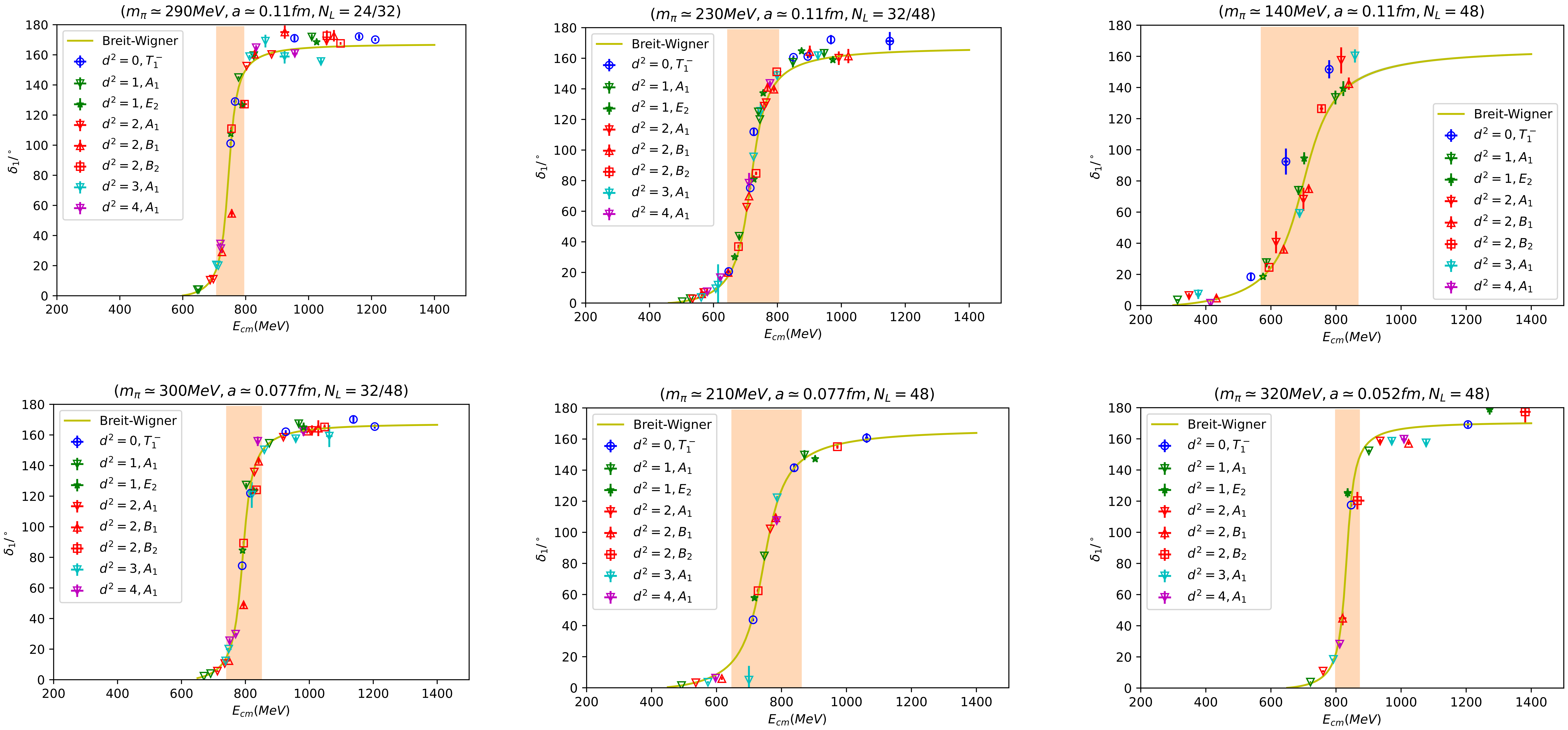
$(m_\pi \simeq 230MeV, a \simeq 0.11fm, N_L = 32/48)$



# $\pi\pi$ P-wave scattering and the $\rho$ resonance

CLQCD

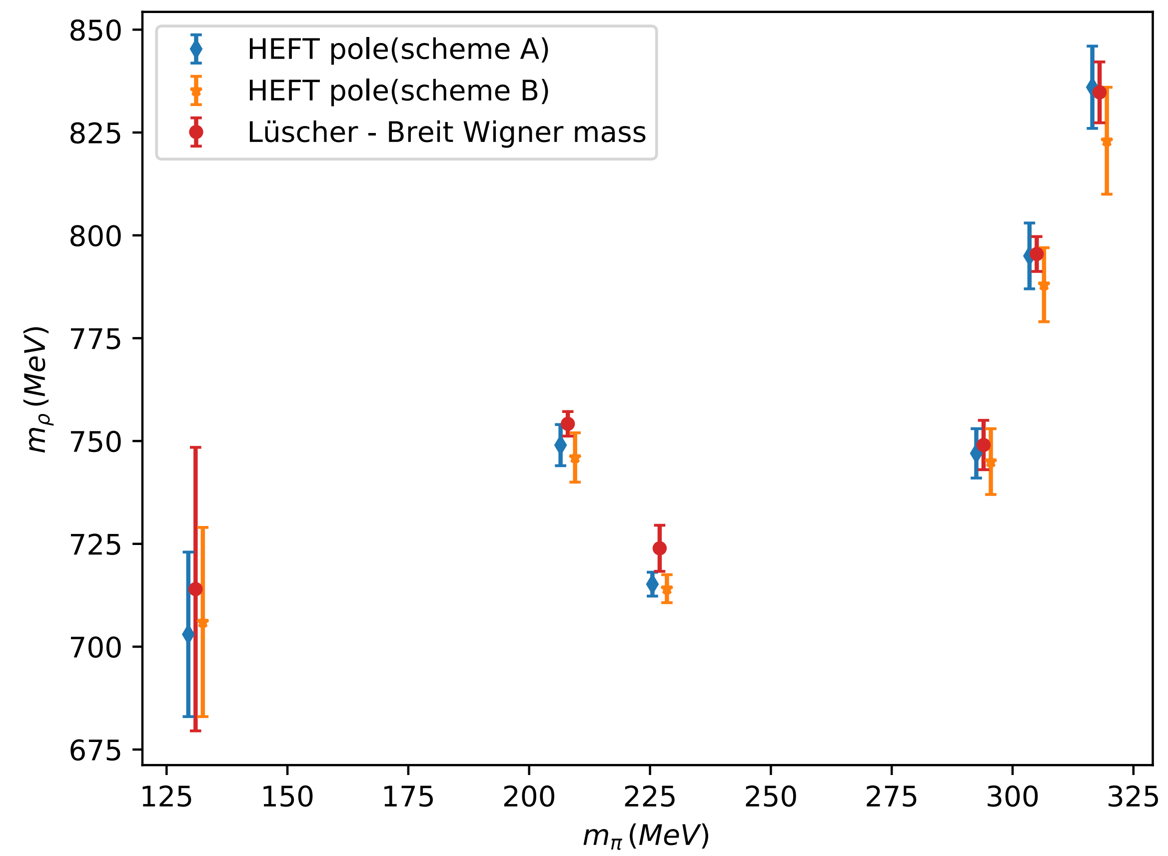
Zhengli Wang et al., JHEP08(2025)064

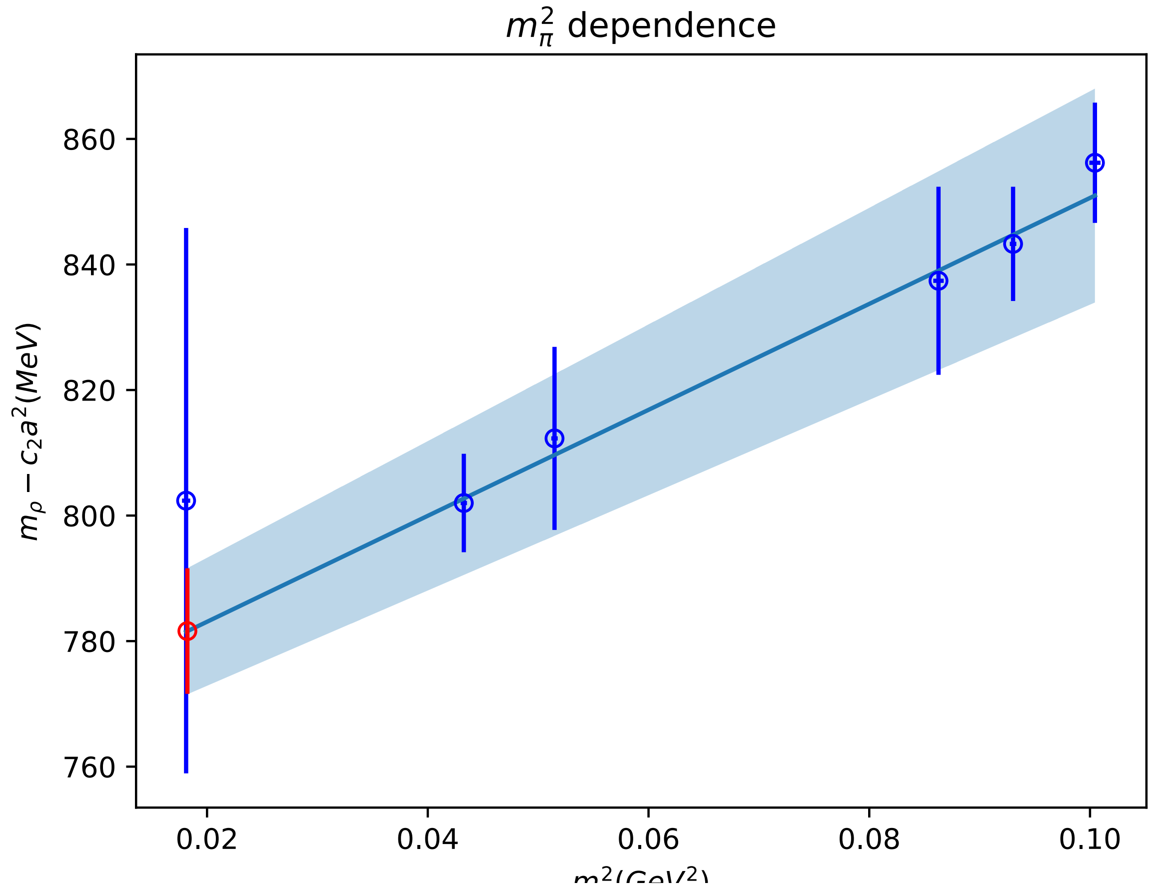


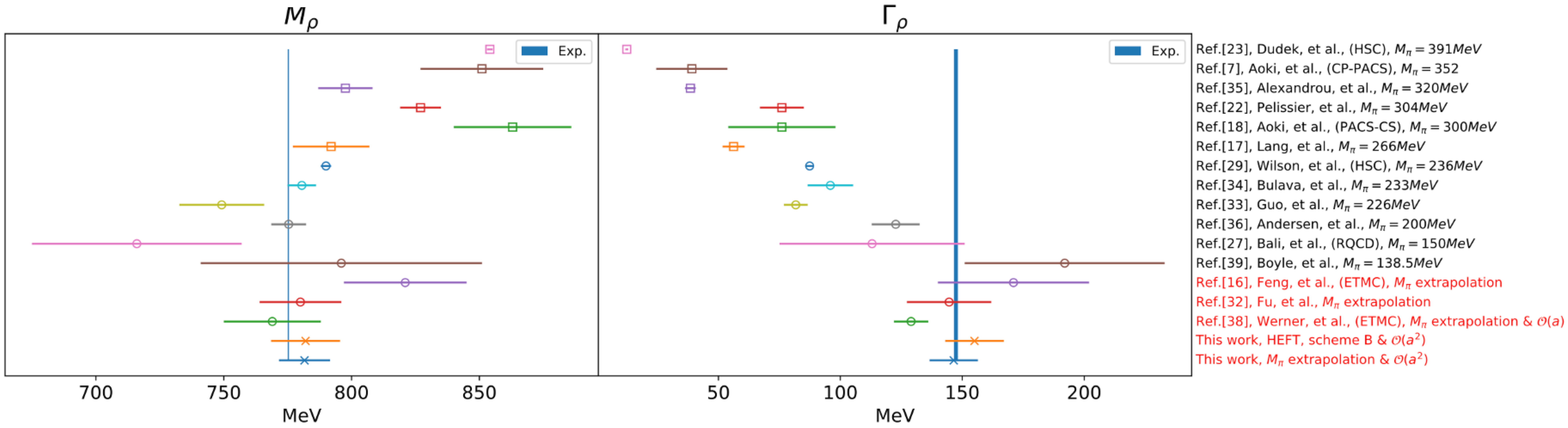
# $\pi\pi$ P-wave scattering and the $\rho$ resonance

Zhengli Wang et al., JHEP08(2025)064

Pion mass and continuum extrapolation: $m_\rho = c_0 + c_1 m_\pi^2 + c_2 a^2$



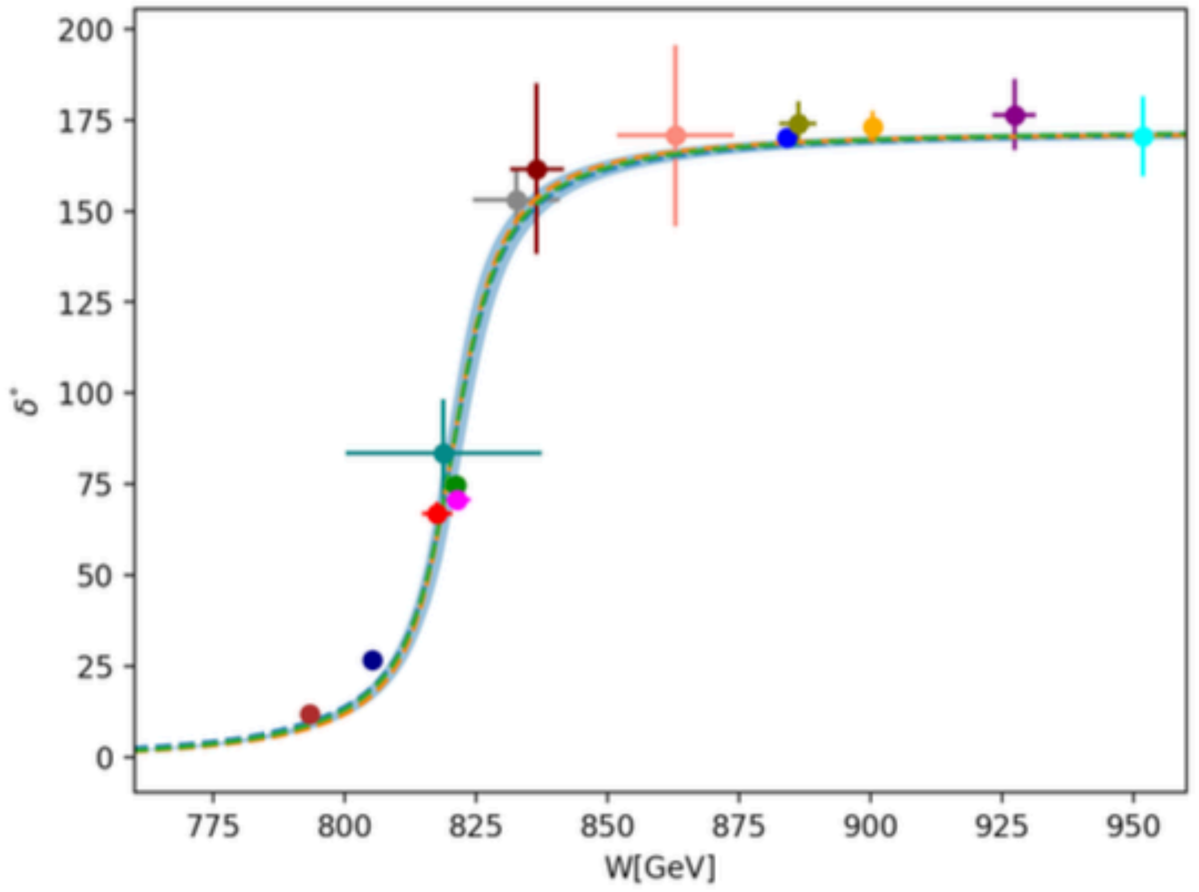



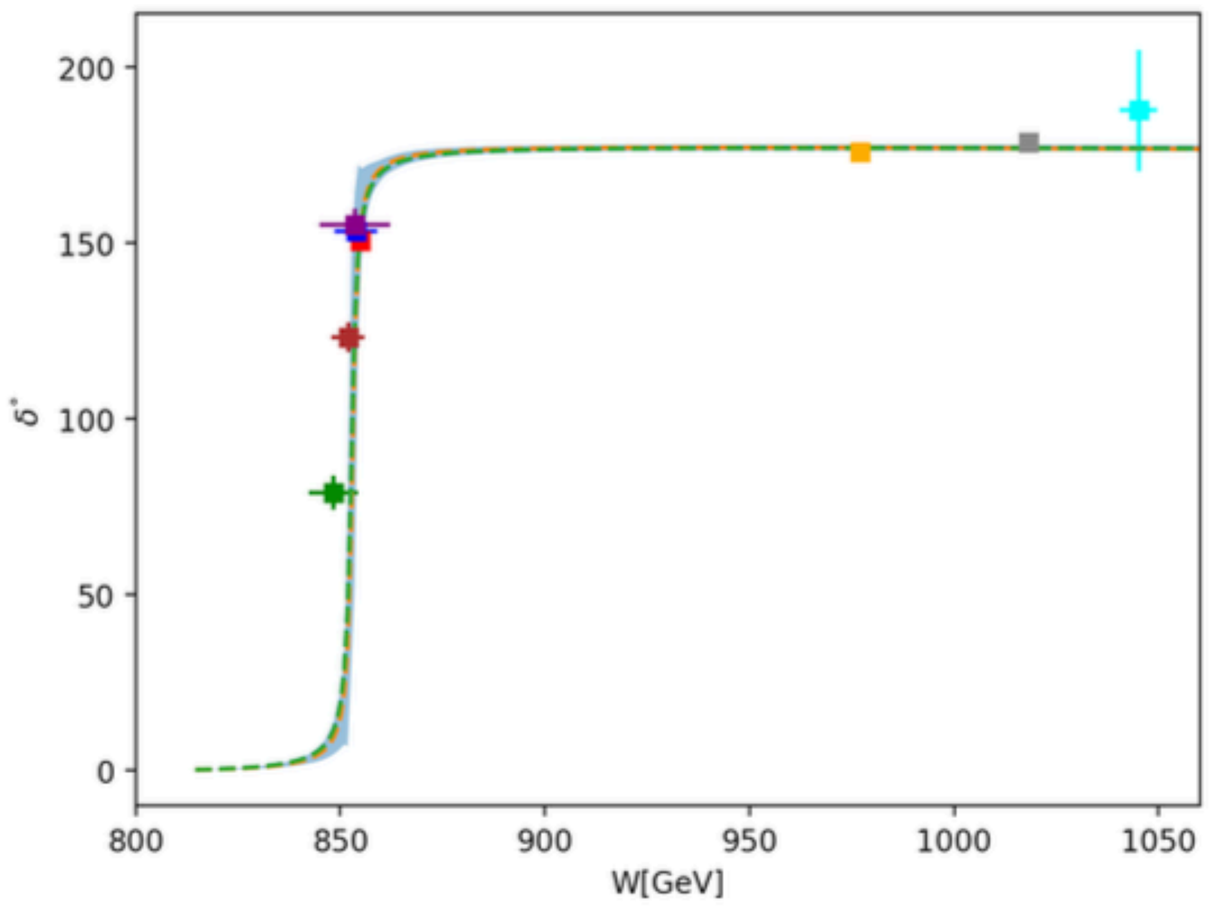
# $K\pi$ P-wave scattering and the $\rho$ resonance

Qu-Zhi Li et al., work in progress , arXiv:2602.xxxx

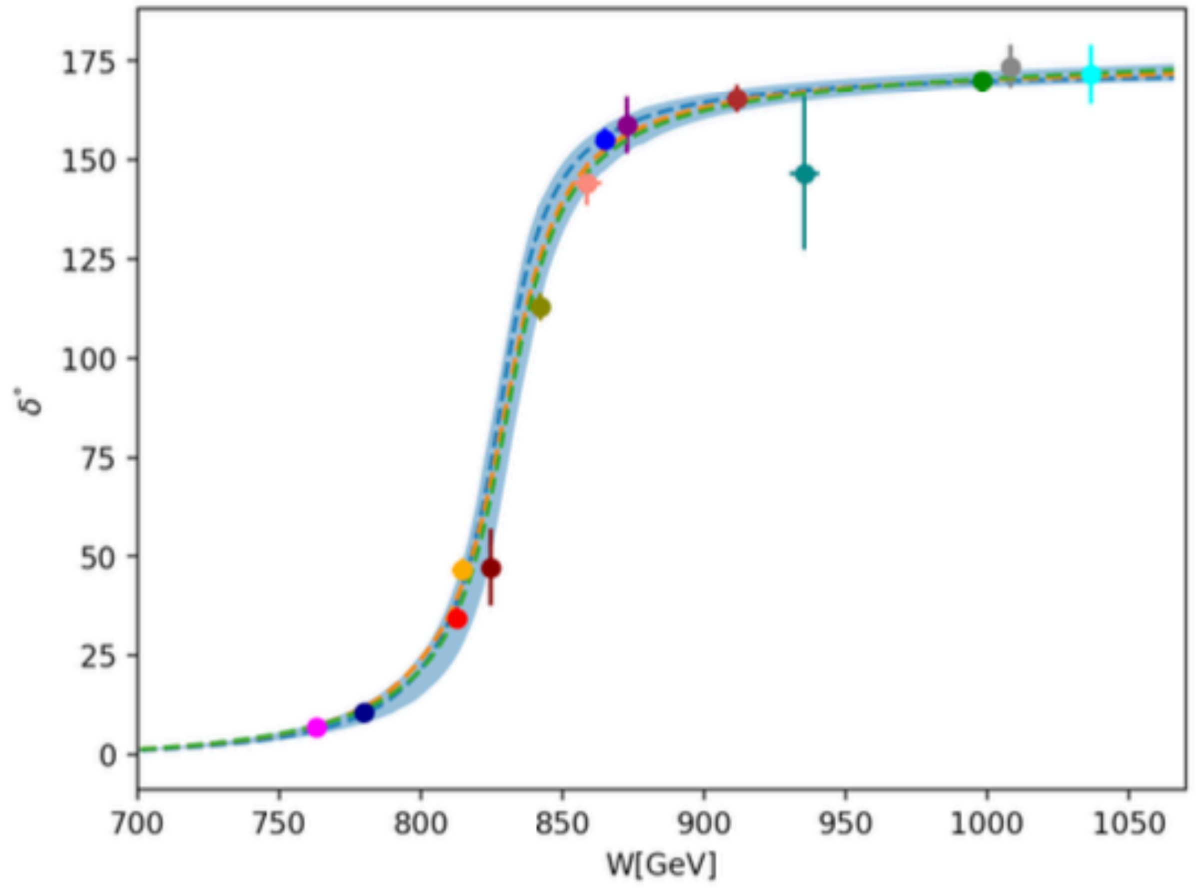
$m_\pi = 230\text{MeV}, \text{a} = 0.105\text{fm}$



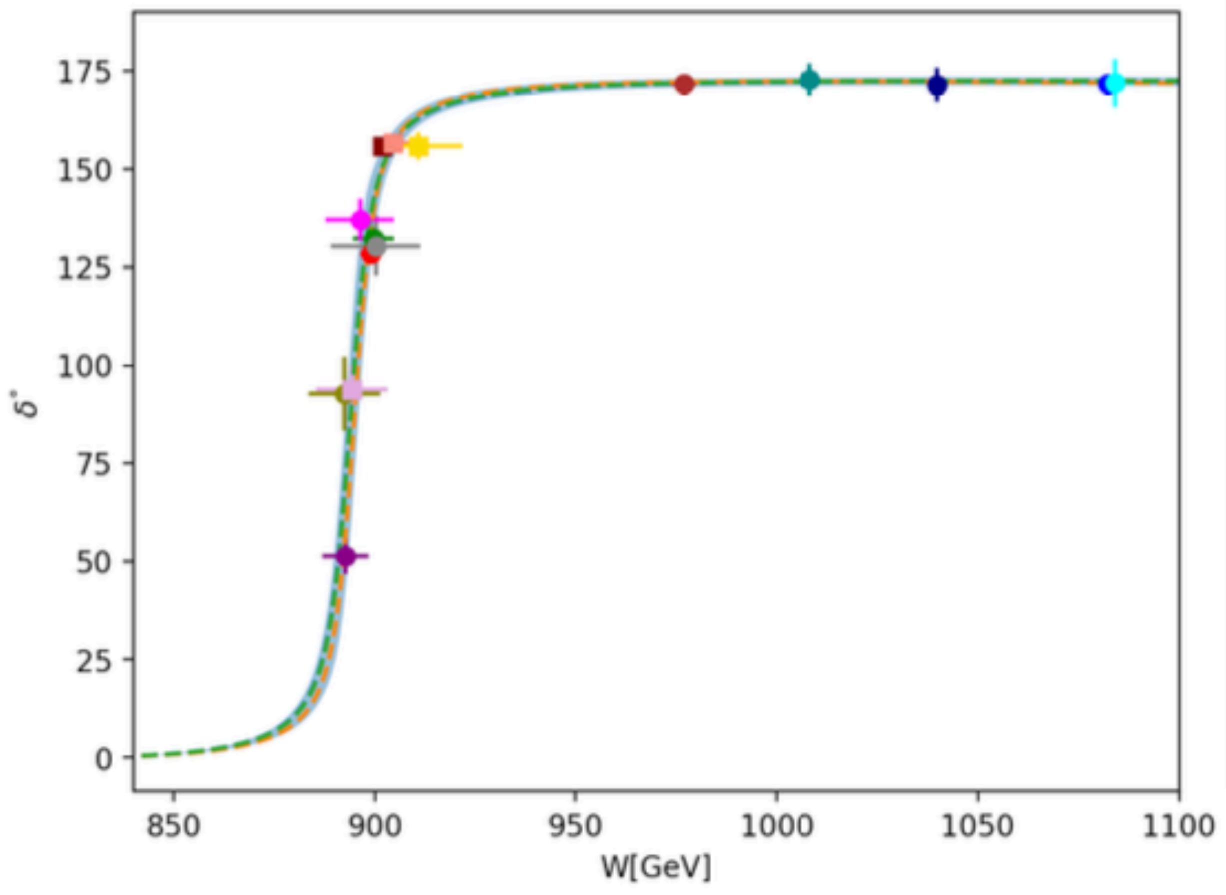

$m_\pi = 290\text{MeV}, \text{a} = 0.105\text{fm}$



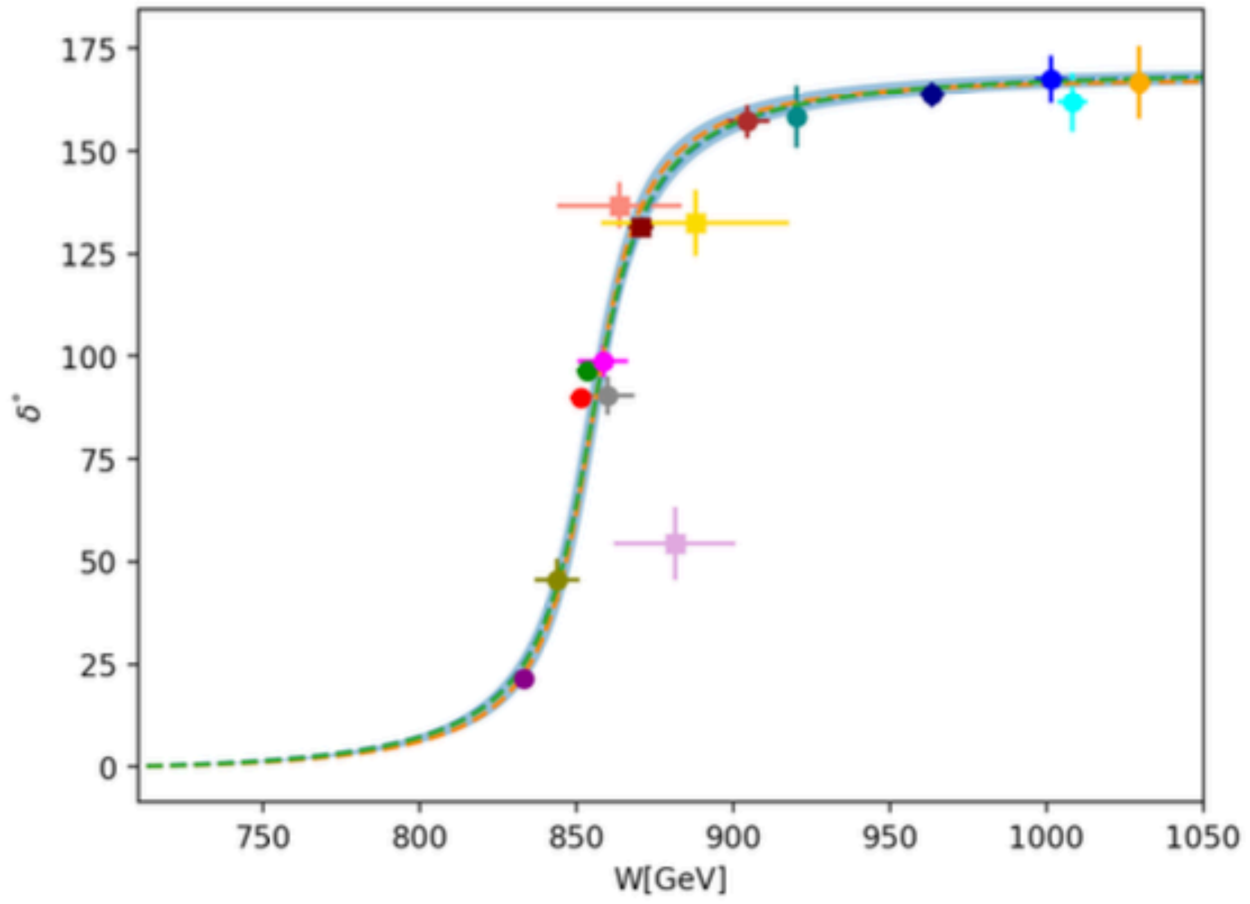

$m_\pi = 135\text{MeV}, \text{a} = 0.105\text{fm}$



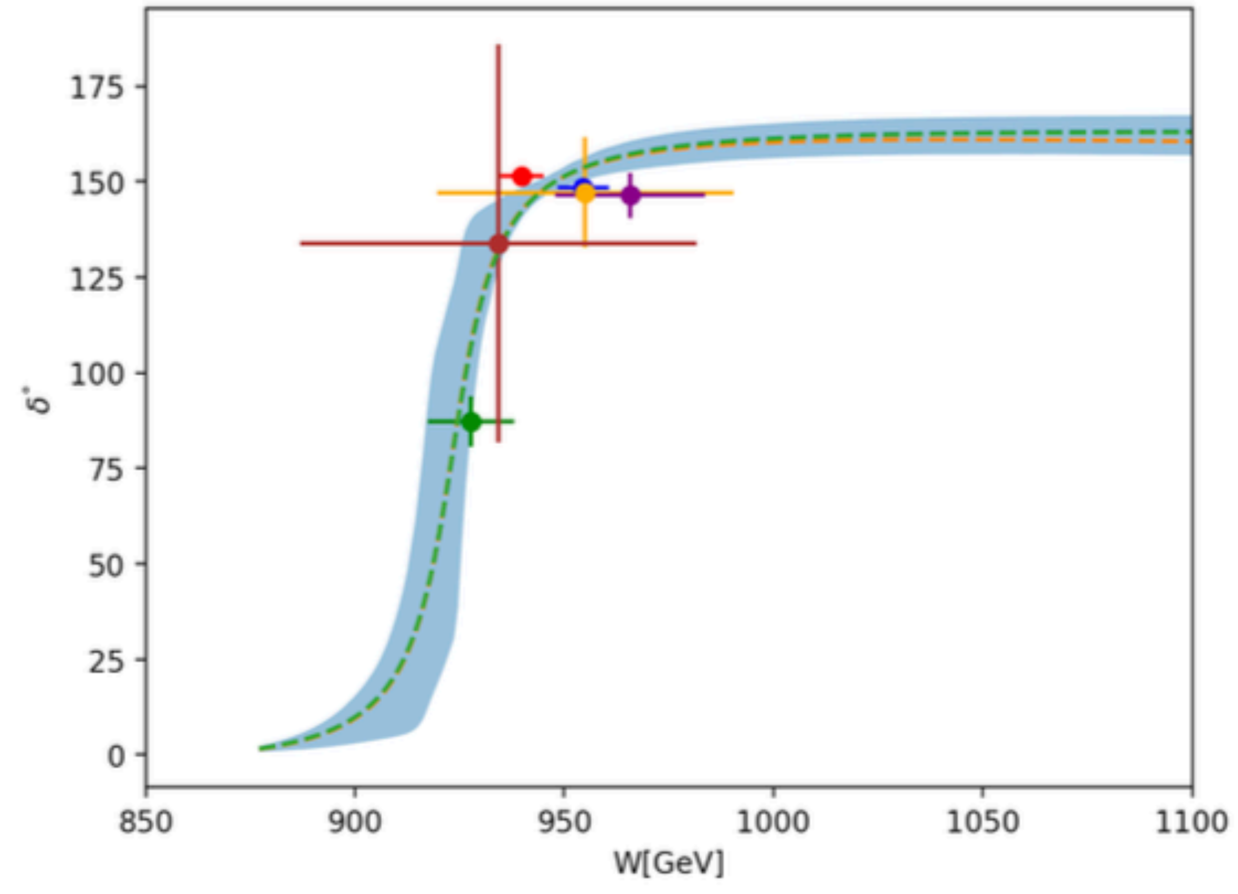

$m_\pi = 300\text{MeV}, \text{a} = 0.077\text{fm}$




$m_\pi = 210\text{MeV}, \text{a} = 0.077\text{fm}$




$m_\pi = 320\text{MeV}, \text{a} = 0.052\text{fm}$



# $K\pi$ P-wave scattering and the $\rho$ resonance

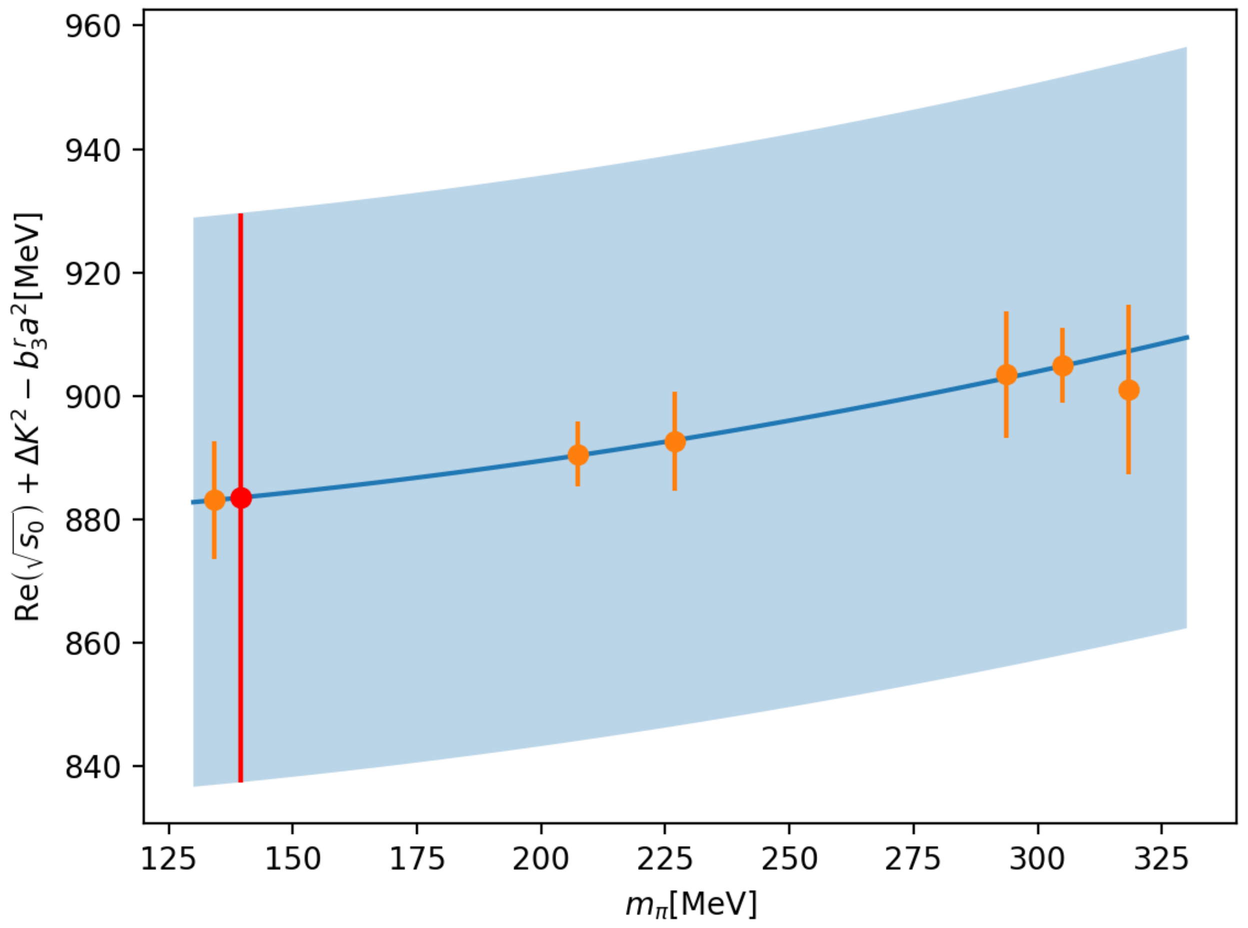
CLQCD

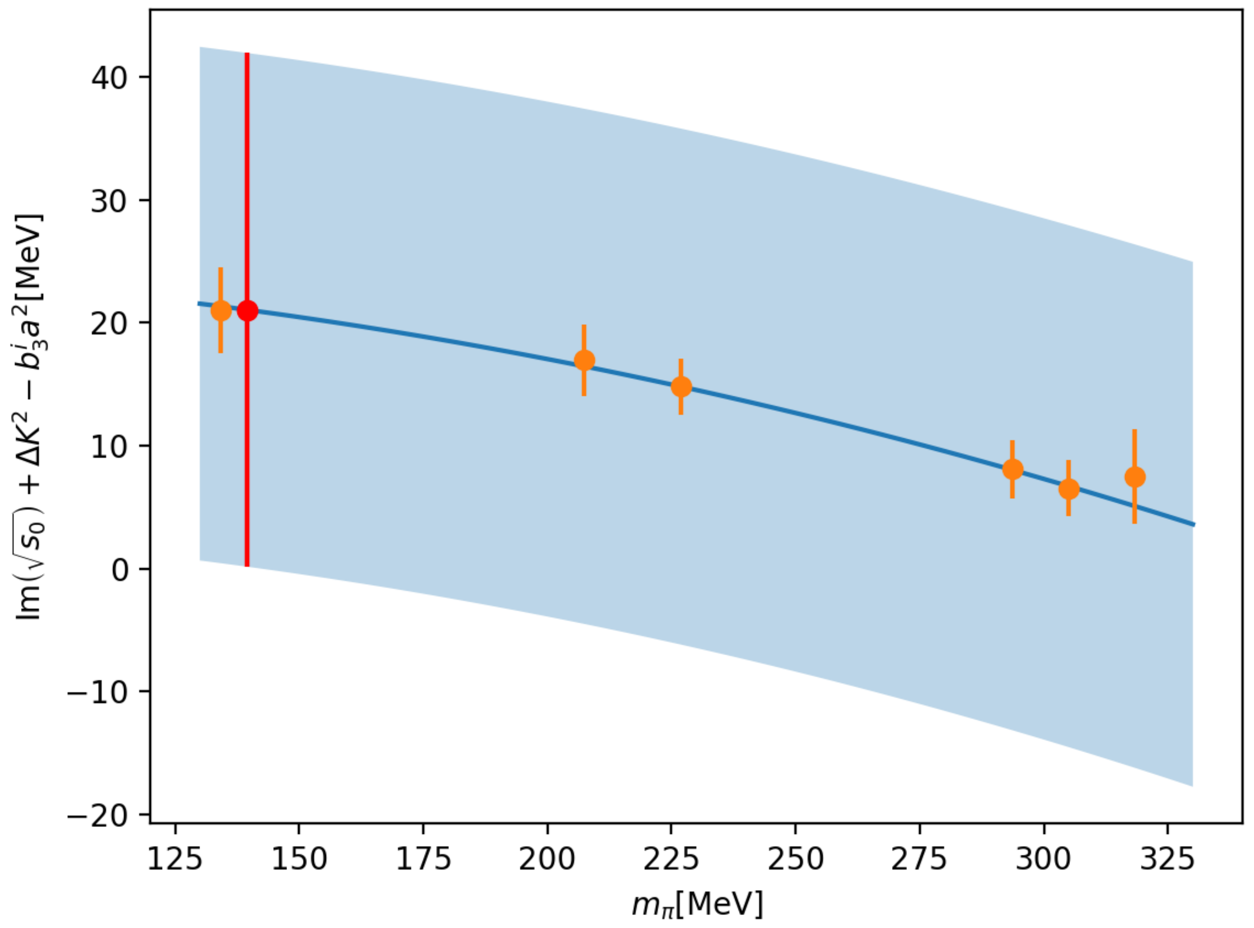
Qu-Zhi Li et al., work in progress , arXiv:2602.xxxx

Pion mass and continuum extrapolation:

$$\mathrm{Re}\left(\sqrt{s_0}\right) = b_0^r + b_1^r m_\pi^2 + b_2^r m_K^2 + b_3^r a^2$$

$$\mathrm{Im}\left(\sqrt{s_0}\right) = b_0^i + b_1^i m_\pi^2 + b_2^i m_K^2 + b_3^i a^2$$




| | pole position (MeV) |
|---|---|
| This work | $884(46) - i21(21)$ |
| PDG | $890(14) - i26(6)$ |

# Scalar charmed mesons $D_0^*(2300)$ & $D_{s0}{}^*(2317)$

CLQCD

**Charm-light**

$D_0^*(2300)$ $\quad I(J^P) = \frac{1}{2}(0^+)$

Mass $m = 2343 \pm 10$ MeV

Width $\Gamma = 229 \pm 16$ MeV

**Charm-strange**

$D_{s0}^*(2317)$ $\quad I(J^P) = 0(0^+)$

Mass $m = 2317.8 \pm 0.5$ MeV

Width $\Gamma < 3.8$ MeV

- Mass ordering puzzle: $c\bar{l}$ is heavier than $c\bar{s}$
- $D_{s0}^*(2317)$ is much lighter than quark model predictions
- Very different widths of $D_0^*(2300)$ and $D_{s0}^*(2317)$

# $D_0^*(2300)$

- D. Mohler, S. Prelovsek and R. Woloshyn, Phys. Rev. D 87, 034501(2013)

  $D\pi(I = 1/2)$ scattering at $m_\pi = 266$MeV, resonance pole.

- G. Moir et. al., JHEP10(2016)011

  $D\pi, D\eta, D_s\bar{K}$ coupled channel scattering at $m_\pi = 391$MeV, bound state pole.

- L. Gayer et. al., JHEP 07 (2021) 123

  $D\pi(I = 1/2)$ single channel scattering at $m_\pi = 239$MeV, resonance pole .

- The pole positions do not agree with the experimental values, and do not show a monotonic movement with the change of $m_\pi$ .

# $D_0^*(2300)$

H. Yan, C. Liu, L. Liu, Y. Meng and H. Xing, PRD111(2025)1,014503

- Pion mass depencence of the pole position of $D_0^*(2300)$
- Six ensembles with four different pion masses and three lattice spacings.
- Large number of interpolating operators are constructed to reliably extract the full spectrum.

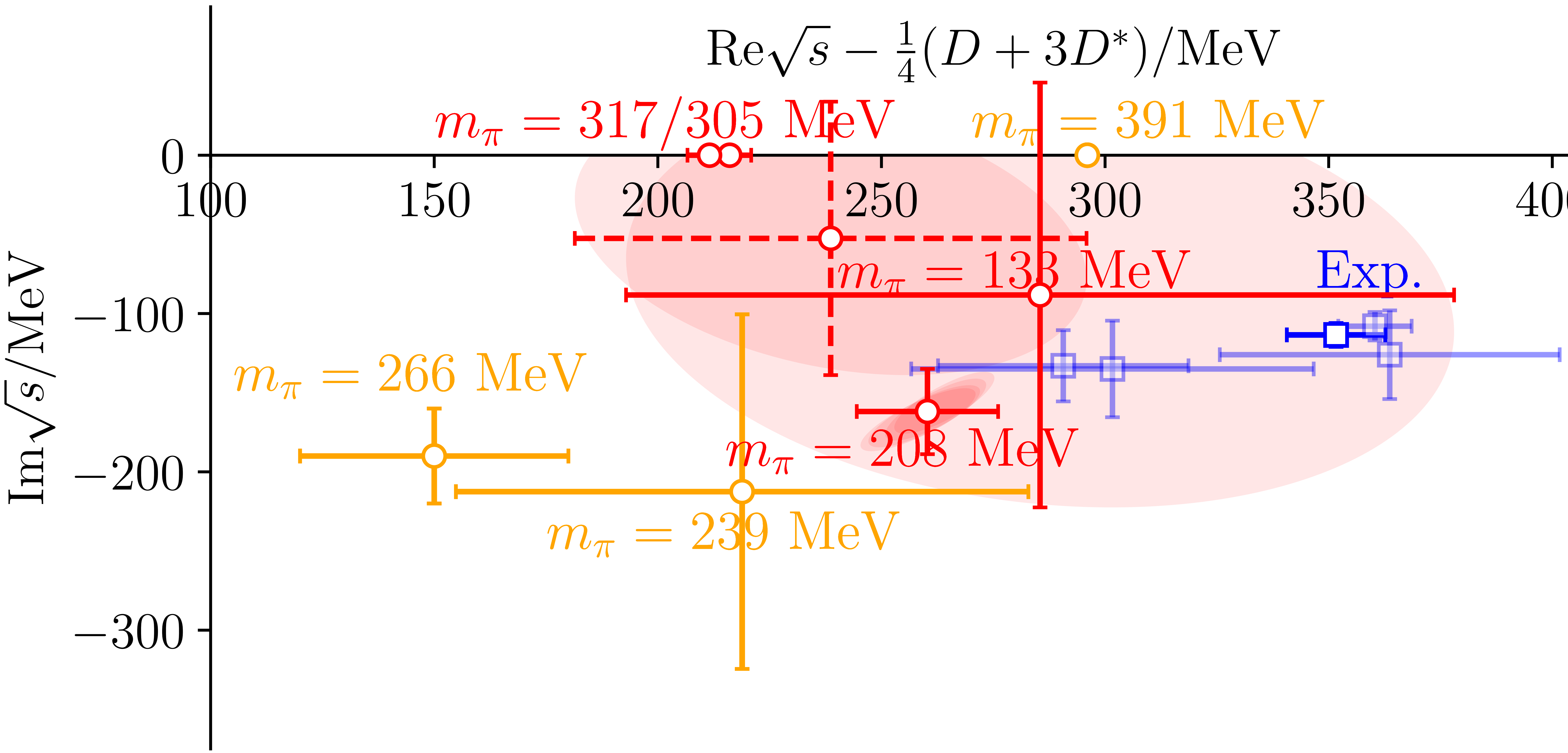
| Lattice spacing | Volume$(L^3 \times T)$ | $M_\pi$ (MeV) | # of confs |
|---|---|---|---|
| ~0.105fm | $48^3 \times 96$ | 135 | 259 |
| ~0.077fm | $32^3 \times 96$ | 305 | 566 |
| | $48^3 \times 96$ | | 200 |
| | $32^3 \times 64$ | 208 | 460 |
| | $48^3 \times 96$ | | 250 |
| ~0.052fm | $48^3 \times 144$ | 317 | 270 |

$$O_{D\pi}^{I=\frac{1}{2}}(P) = \sqrt{2}D^0(p_1)\pi^+(p_2) - D^+(p_1)\pi^0(p_2), (P = p_1 + p_2, \quad P^2 = 0,1,2,3,4)$$

$$\mathcal{O}_1(P) = \bar{d}\Gamma c(P)$$

# $D_0^*(2300)$

**Pole position**




- At $m_\pi \sim 300$MeV, there is a virtual state pole. When pion mass decreases, it becomes a resonance and the pole position gets closer to the experimental value.

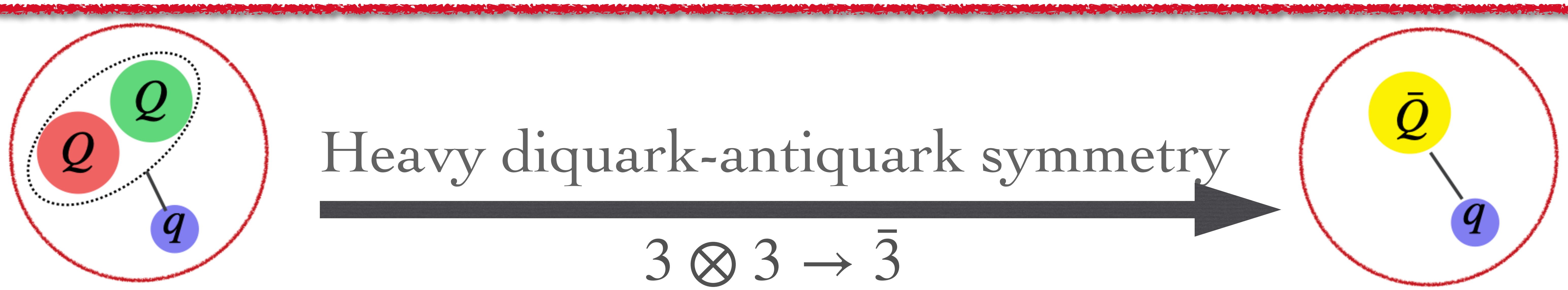
# Meson-baryon scattering

- Hidden-charm pentaquarks: $\Sigma_c D, \Sigma_c D^*$ scattering
- Double-charm pentaquarks: $(\Xi_{cc}, \Omega_{cc}) - (\pi, K)$ scattering

# Meson-baryon scattering

- Hidden-charm pentaquarks: $\Sigma_c D, \Sigma_c D^*$ scattering
- Double-charm pentaquarks: $(\Xi_{cc}, \Omega_{cc}) - (\pi, K)$ scattering

# Doubly charmed Pentaquarks




| | | |
|---|---|---|
| $I = 1/2$ | $\Omega_{cc}\bar{K}$ | $D_s K$ |
| $I = 1$ | $\Xi_{cc}K$ | $D\bar{K}$ |
| $I = 0$ | $\Xi_{cc}K$ | $D\bar{K}$ |
| $I = 3/2$ | $\Xi_{cc}\pi$ | $D\pi$ |
| $I = 0$ | $\Xi_{cc}\bar{K}$, $\Omega_{cc}\eta$ | $DK$, $D_s\eta$ |
| $I = 1$ | $\Xi_{cc}\bar{K}$, $\Omega_{cc}\pi$, | $DK$, $D_s\pi$ |
| $I = 1/2$ | $\Xi_{cc}\pi$, $\Xi_{cc}\eta$, $\Omega_{cc}K$ | $D\pi$, $D\eta$, $D_s\bar{K}$ |

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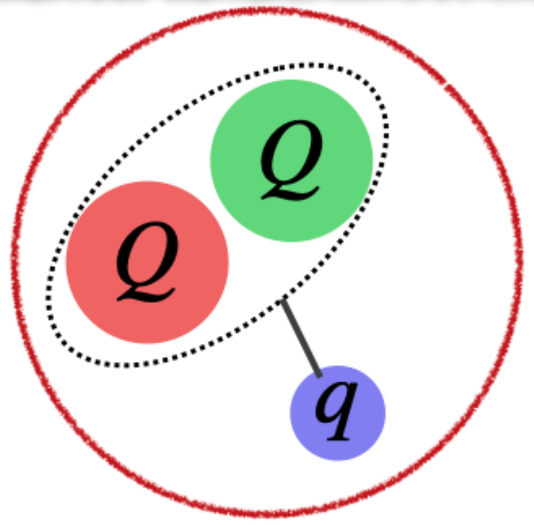
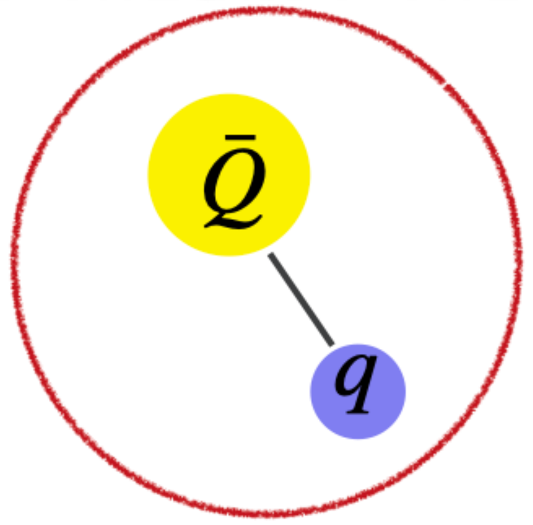



Heavy diquark-antiquark symmetry

$3 \otimes 3 \rightarrow \bar{3}$




| | | |
|---|---|---|
| $I = 1/2$ | $\Omega_{cc}\bar{K}$ | $D_s K$ |
| $I = 1$ | $\Xi_{cc}K$ | $D\bar{K}$ |
| $I = 0$ | $\Xi_{cc}K$ | $D\bar{K}$ |
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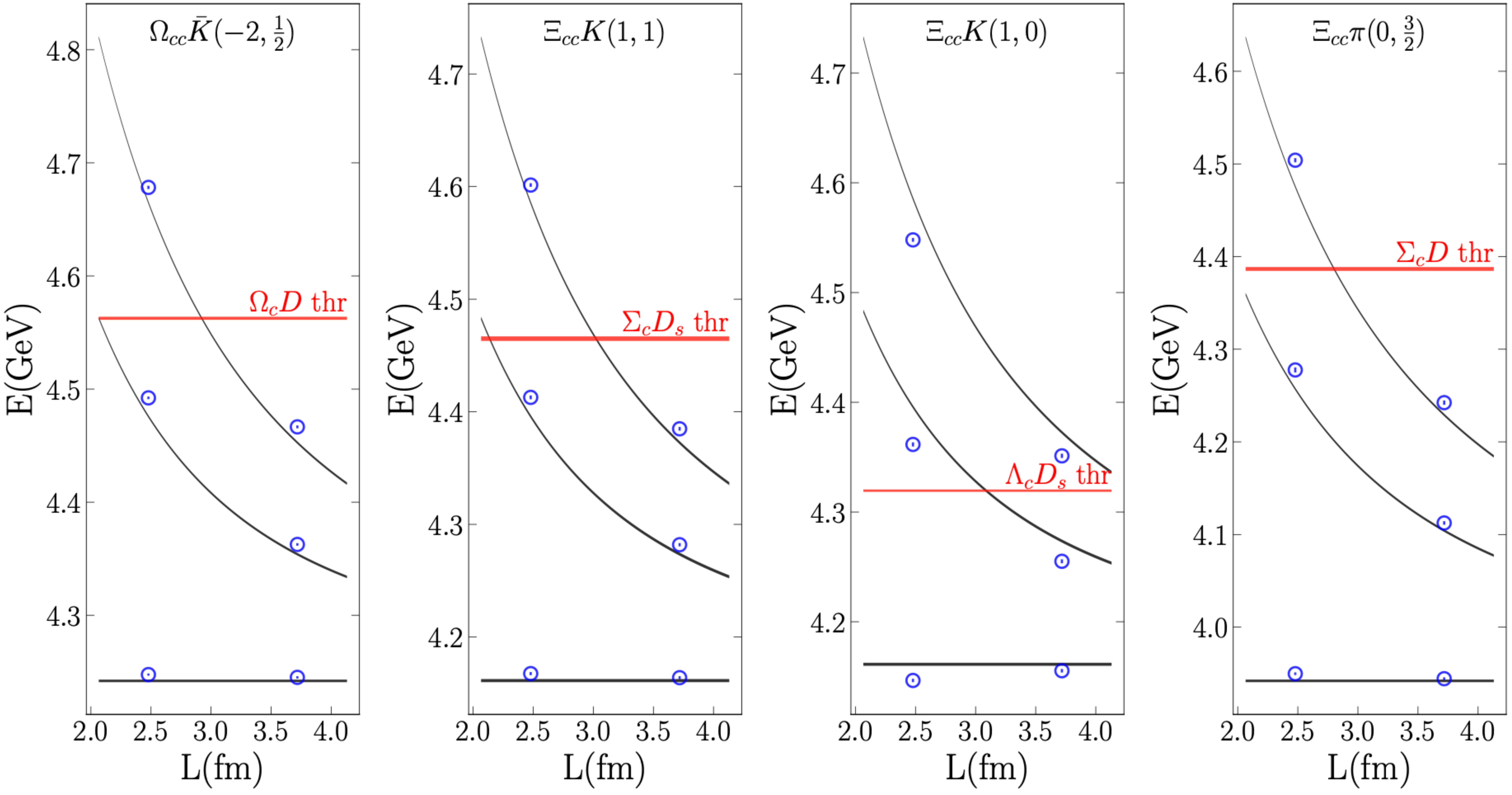
# Doubly charmed Pentaquarks

J.-Y. Yi, Z.-R. Liang, L. Liu and D.-L. Yao, arXiv:2511.12611, JHEP accepted



# Double-charm Pentaquarks

CLQCD

- Scattering lengths from lattice QCD and BChPT

| $(S, I)$ | Processes | $M_\pi \sim 300$ MeV | $M_\pi \sim 210$ MeV | EOMS | HB |
|---|---|---|---|---|---|
| $(-2, \frac{1}{2})$ | $\Omega_{cc}\bar{K} \to \Omega_{cc}\bar{K}$ | $-0.161(20)$ | $-0.136(12)$ | $-0.09^{+0.12}_{-0.13}$ | -0.20(1) |
| $(1, 1)$ | $\Xi_{cc}K \to \Xi_{cc}K$ | $-0.177(23)$ | $-0.212(14)$ | $-0.60 \pm 0.13$ | $-0.25(1)$ |
| $(1, 0)$ | $\Xi_{cc}K \to \Xi_{cc}K$ | $0.63(10)$ | $0.694(90)$ | $1.03 \pm 0.19$ | $0.92(2)$ |
| $(0, \frac{3}{2})$ | $\Xi_{cc}\pi \to \Xi_{cc}\pi$ | $-0.140(15)$ | $-0.143(24)$ | $-0.16 \pm 0.02$ | $-0.10(2)$ |

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# Double-charm Pentaquarks

Scattering amplitude:

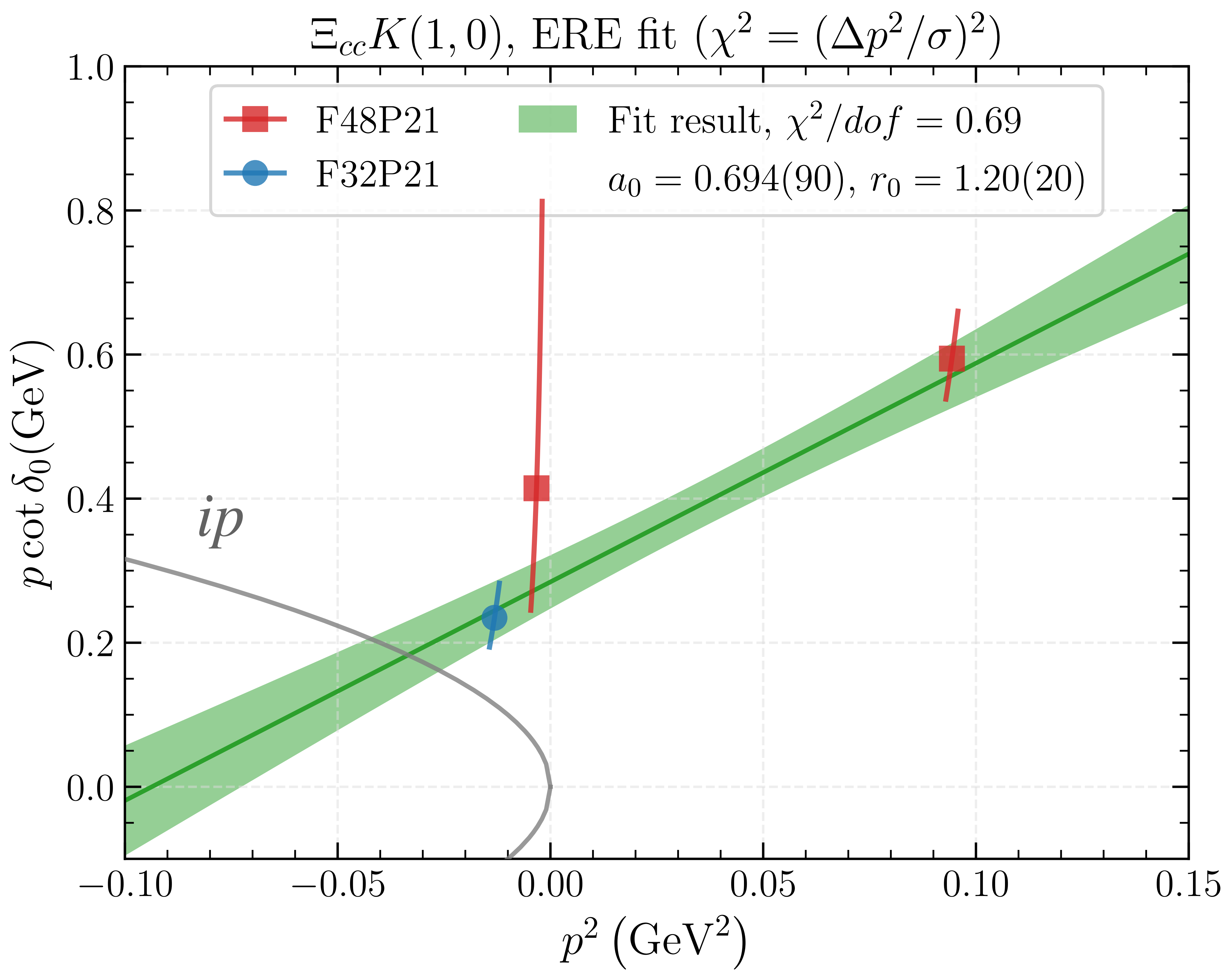
$$T \sim \frac{1}{pcot\delta - ip}$$

Effective range expansion:

$$pcot\delta(p) = \frac{1}{a_0} + \frac{1}{2} r_0 p^2 + \cdots$$

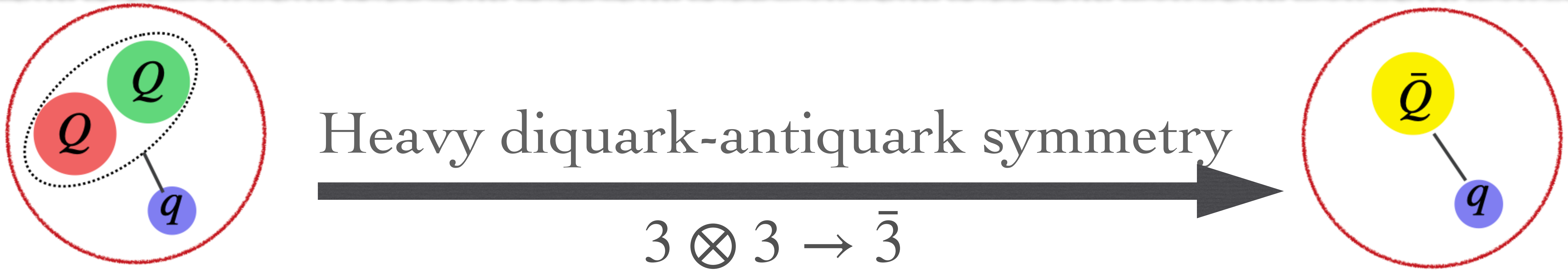
Luscher's formula:

$$pcot\delta(p) = \frac{2Z_{00}(1; (\frac{pL}{2\pi})^2)}{L\sqrt{\pi}}$$




$m_\pi = 210\,\text{MeV}$

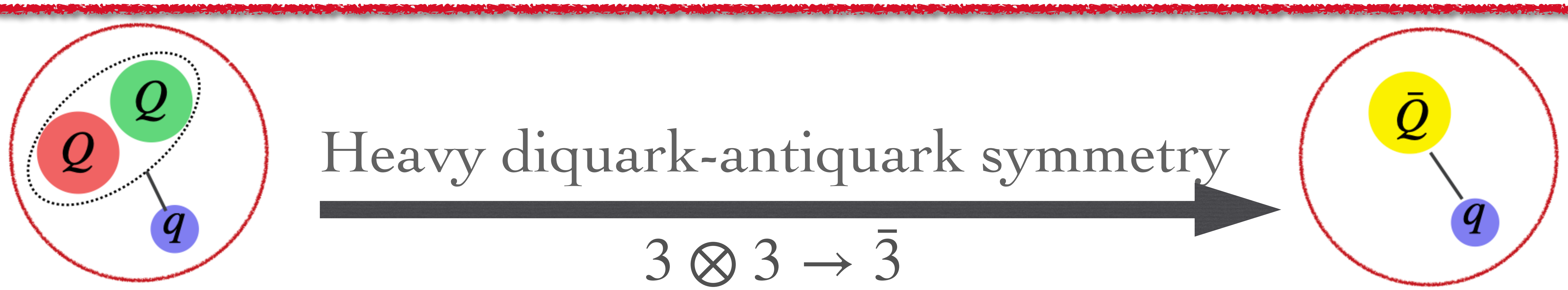
# Doubly charmed Pentaquarks




| | | | |
|---|---|---|---|
| $I = 1/2$ | $\Omega_{cc}\bar{K}$ | **Repulsive** | $D_s K$ |
| $I = 1$ | $\Xi_{cc}K$ | **Repulsive** | $D\bar{K}$ |
| $I = 0$ | $\Xi_{cc}K$ | **Attractive, virtual pole** | $D\bar{K}$ |
| $I = 3/2$ | $\Xi_{cc}\pi$ | **Repulsive** | $D\pi$ |
| $I = 0$ | $\Xi_{cc}\bar{K}$, $\Omega_{cc}\eta$ | | $DK$, $D_s\eta$ |
| $I = 1$ | $\Xi_{cc}\bar{K}$, $\Omega_{cc}\pi$, | | $DK$, $D_s\pi$ |
| $I = 1/2$ | $\Xi_{cc}\pi$, $\Xi_{cc}\eta$, $\Omega_{cc}K$ | | $D\pi$, $D\eta$, $D_s\bar{K}$ |

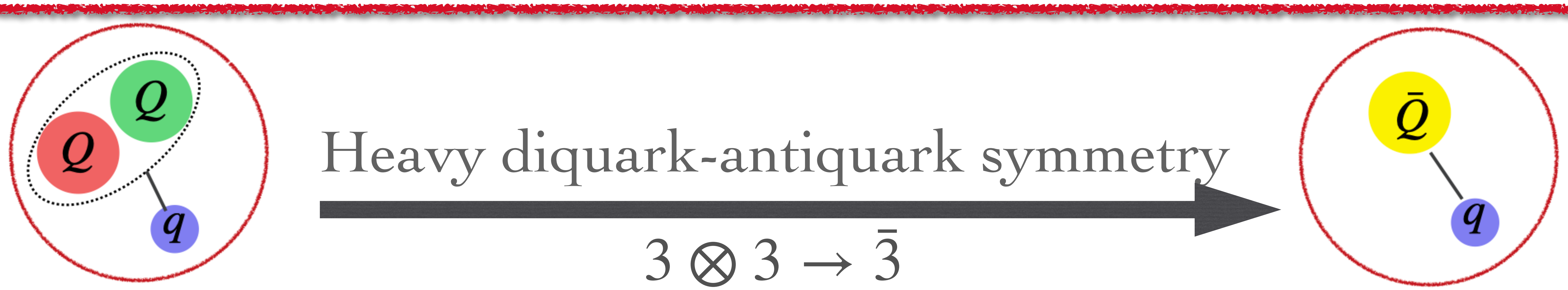
# Doubly charmed Pentaquarks

CLQCD




| | | | | |
|---|---|---|---|---|
| $I = 1/2$ | $\Omega_{cc}\bar{K}$ | Repulsive | $D_s K$ | |
| $I = 1$ | $\Xi_{cc}K$ | Repulsive | $D\bar{K}$ | |
| $I = 0$ | $\Xi_{cc}K$ | Attractive, virtual pole | $D\bar{K}$ | |
| $I = 3/2$ | $\Xi_{cc}\pi$ | Repulsive | $D\pi$ | |
| $I = 0$ | $\Xi_{cc}\bar{K},\ \Omega_{cc}\eta$ ?? | | $DK,\ D_s\eta$ | $D^*_{s0}(2317)$ |
| $I = 1$ | $\Xi_{cc}\bar{K},\ \Omega_{cc}\pi,$ | | $DK,\ D_s\pi$ | |
| $I = 1/2$ | $\Xi_{cc}\pi,\ \Xi_{cc}\eta,\ \Omega_{cc}K$ ?? | | $D\pi,\ D\eta,\ D_s\bar{K}$ | $D^*_0(2300)$ |

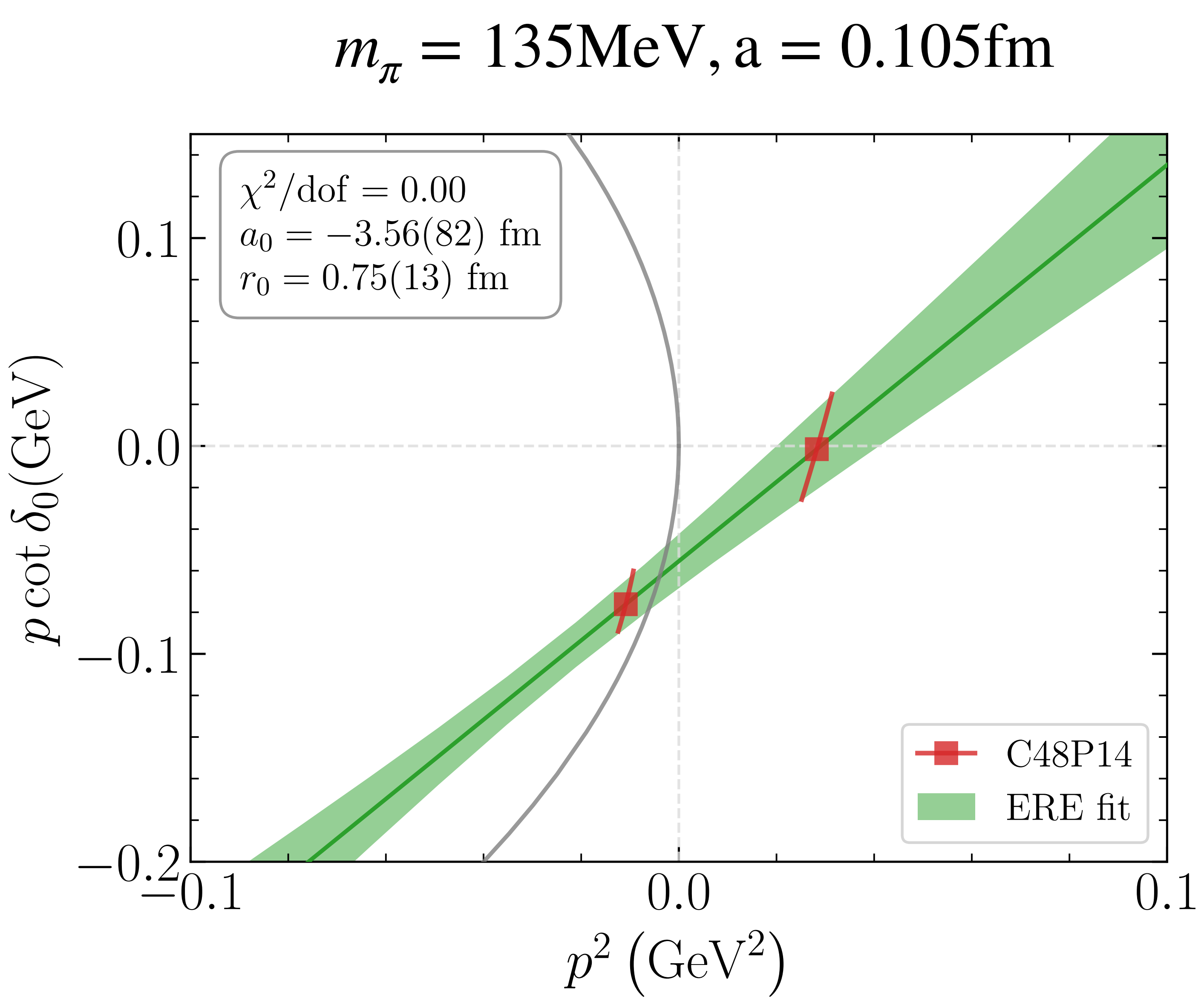
# Doubly charmed Pentaquarks

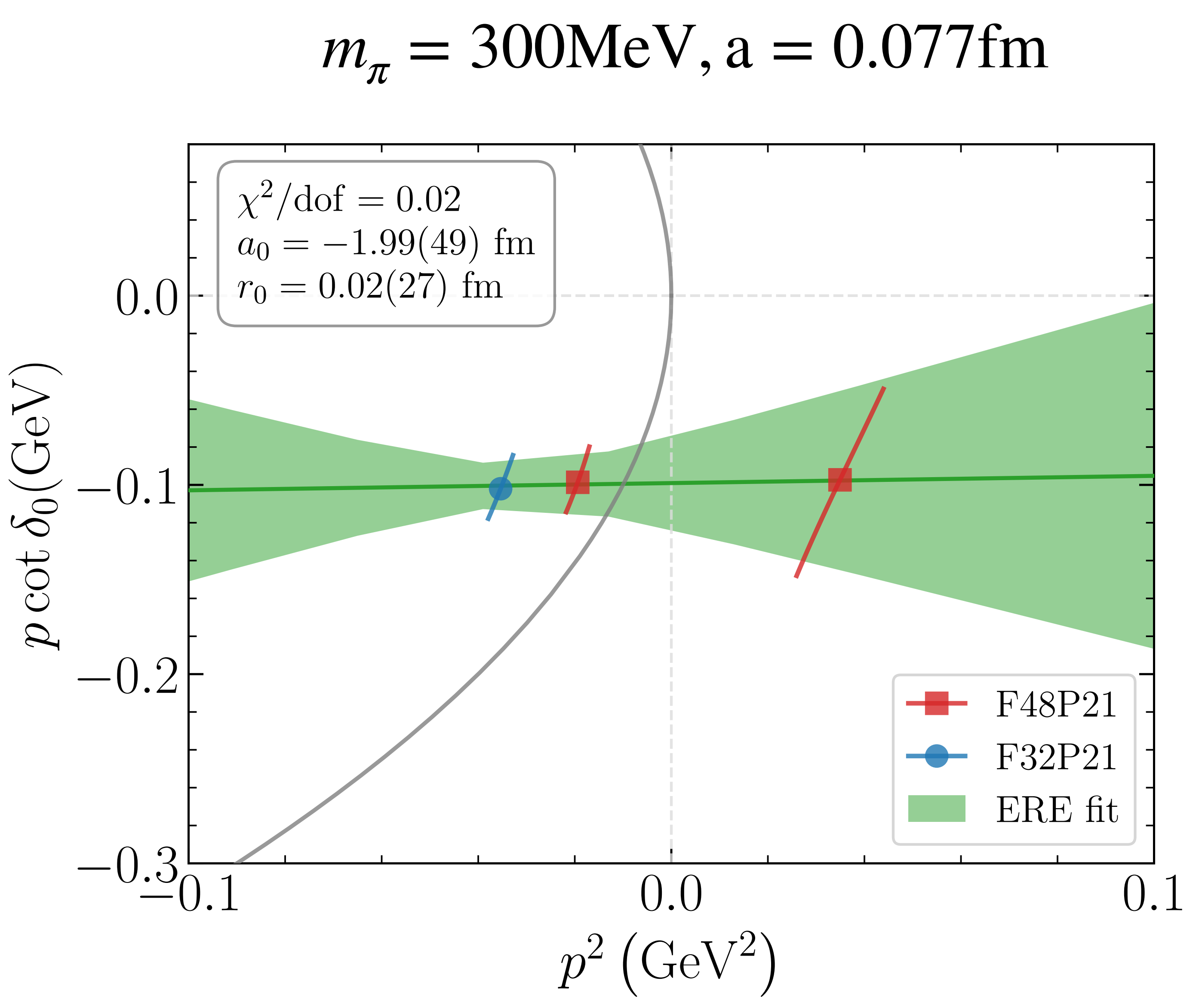


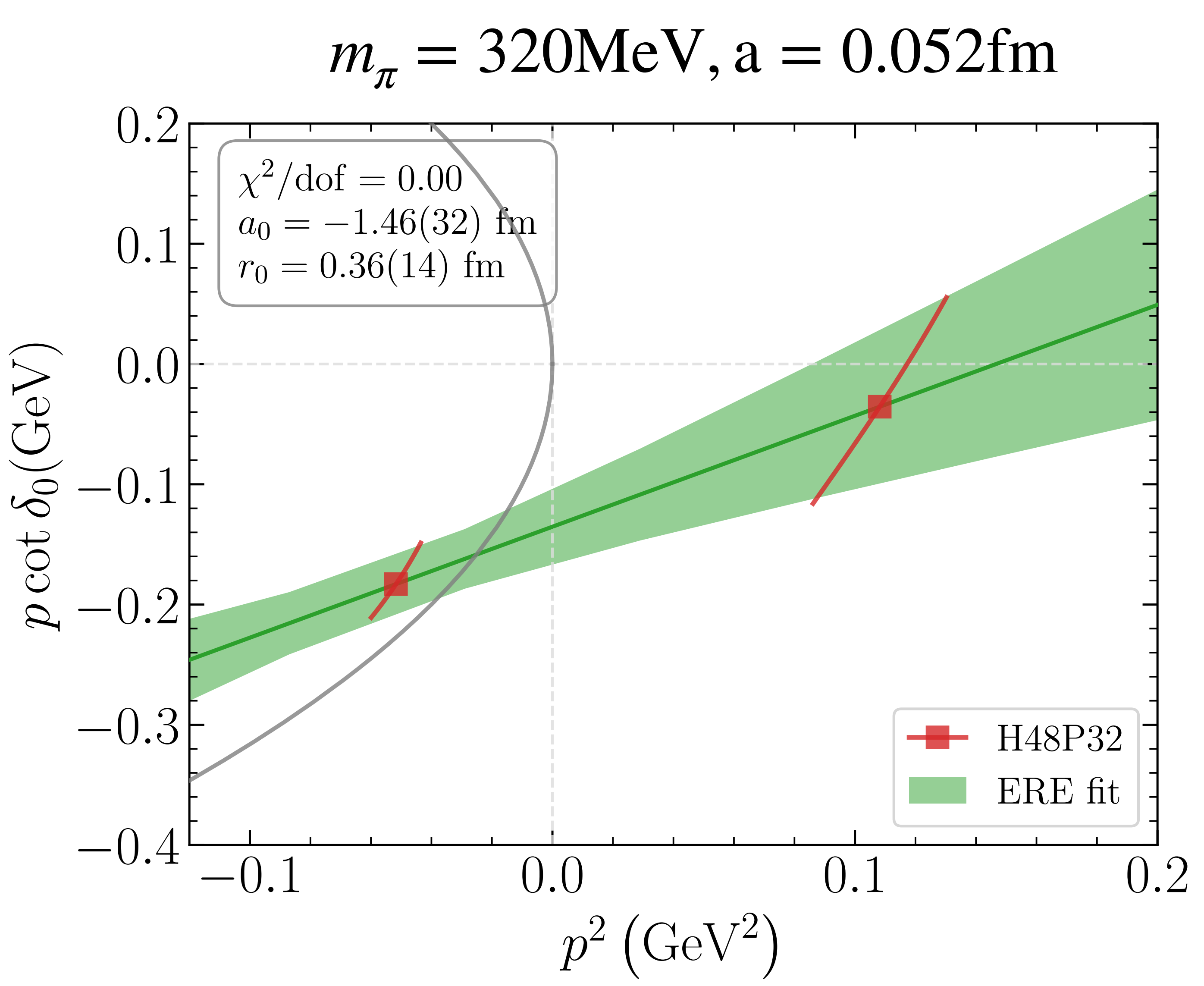

| | | | | |
|---|---|---|---|---|
| $I = 1/2$ | $\Omega_{cc}\bar{K}$ | Repulsive | $D_s K$ | |
| $I = 1$ | $\Xi_{cc}K$ | Repulsive | $D\bar{K}$ | |
| $I = 0$ | $\Xi_{cc}K$ | Attractive, virtual pole | $D\bar{K}$ | |
| $I = 3/2$ | $\Xi_{cc}\pi$ | Repulsive | $D\pi$ | |
| $I = 0$ | $\Xi_{cc}\bar{K}$, $\Omega_{cc}\eta$ ?? | | $DK$, $D_s\eta$ | $D^*_{s0}(2317)$ |
| $I = 1$ | $\Xi_{cc}\bar{K}$, $\Omega_{cc}\pi$, | | $DK$, $D_s\pi$ | |
| $I = 1/2$ | $\Xi_{cc}\pi$, $\Xi_{cc}\eta$, $\Omega_{cc}K$ ?? | | $D\pi$, $D\eta$, $D_s\bar{K}$ | $D^*_0(2300)$ |

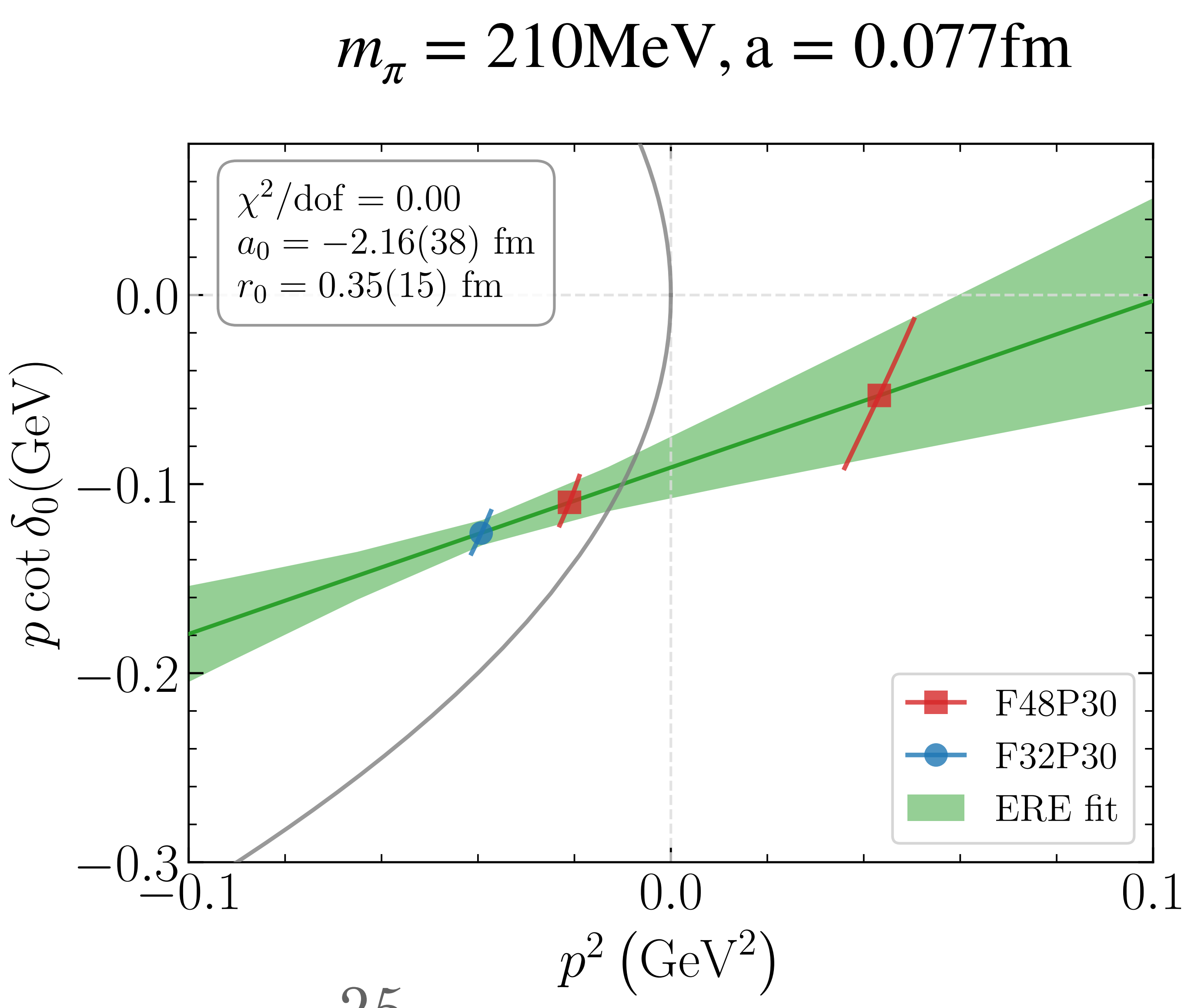
# Double-charm Pentaquarks






$I = 0 \;\; \Xi_{cc}\bar{K}$

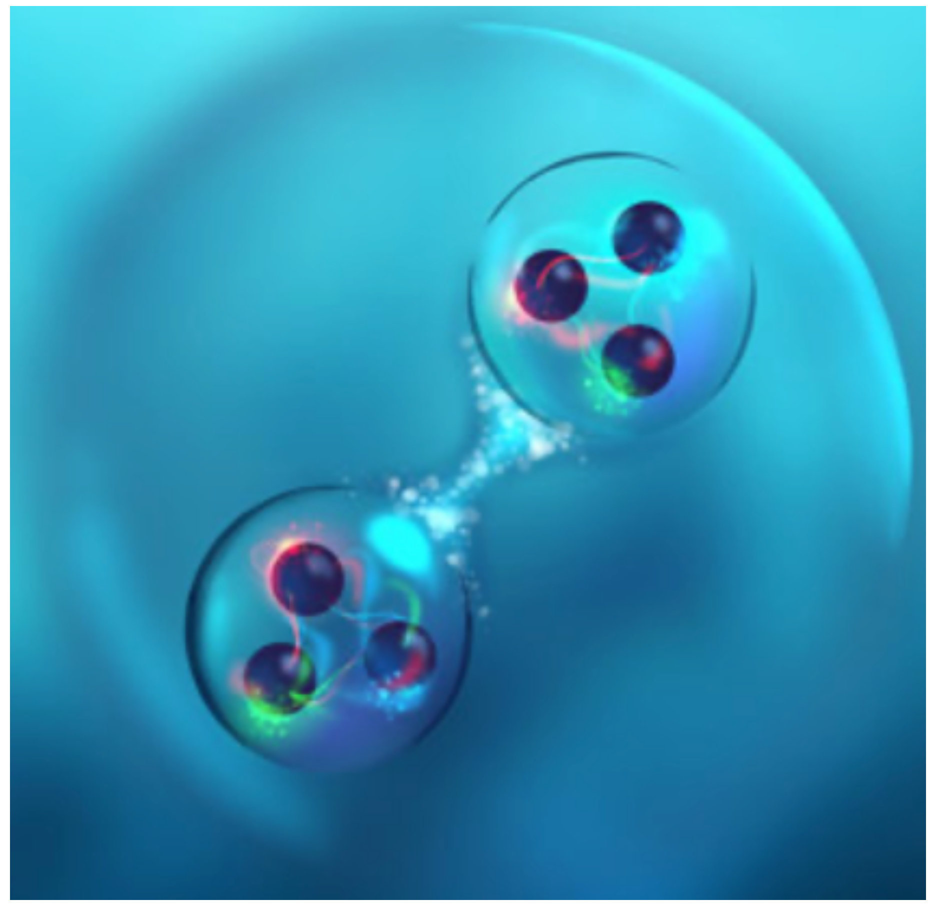
A bound state pole is found at various pion masses and lattice spacings.

# Baryon-baryon scattering

- Deuteron: $NN$ scattering
- $\Lambda\Lambda - N\Xi, N\Lambda$ scattering

# *NN* scattering

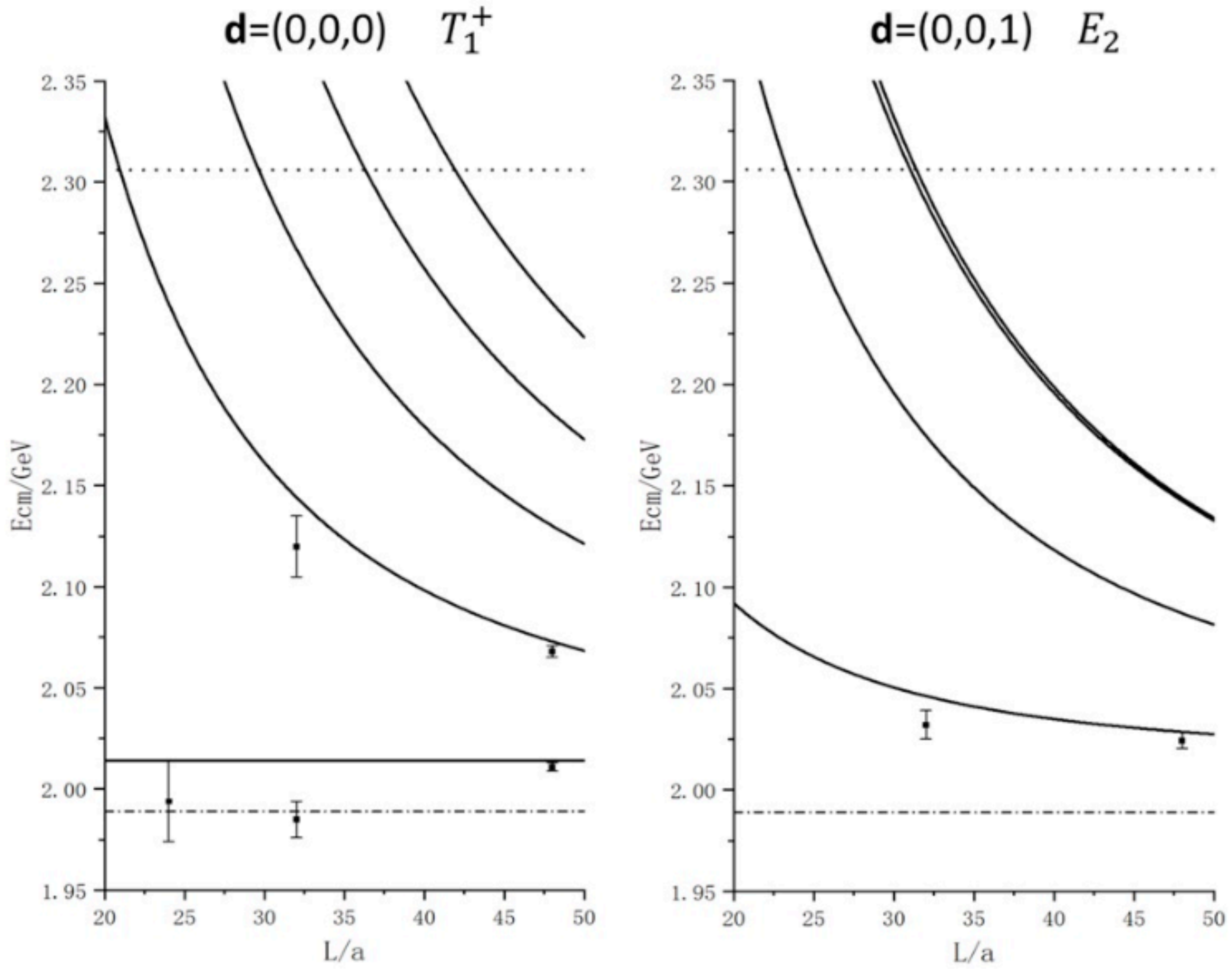
- The only known stable dibaryon — Deuteron.
- Lattice calculation is difficult due to poor signal.
- Most of the previous lattice studies are performed at very large pion masses and the results are controversial.



| ID | $L^3 \times T$ | $a$(fm) | $m_\pi$(MeV) | $m_\pi L$ | $m_N$(MeV) | $N_{cfg}$ |
|---|---|---|---|---|---|---|
| C24P29 | $24^3 \times 72$ | 0.1053 | 292 | 3.75 | 1026(4) | 838 |
| C32P29 | $32^3 \times 64$ | 0.1053 | 292 | 5.01 | 1005(3) | 981 |
| C48P29 | $48^3 \times 96$ | 0.1053 | 292 | 7.52 | 1007(1) | 731 |

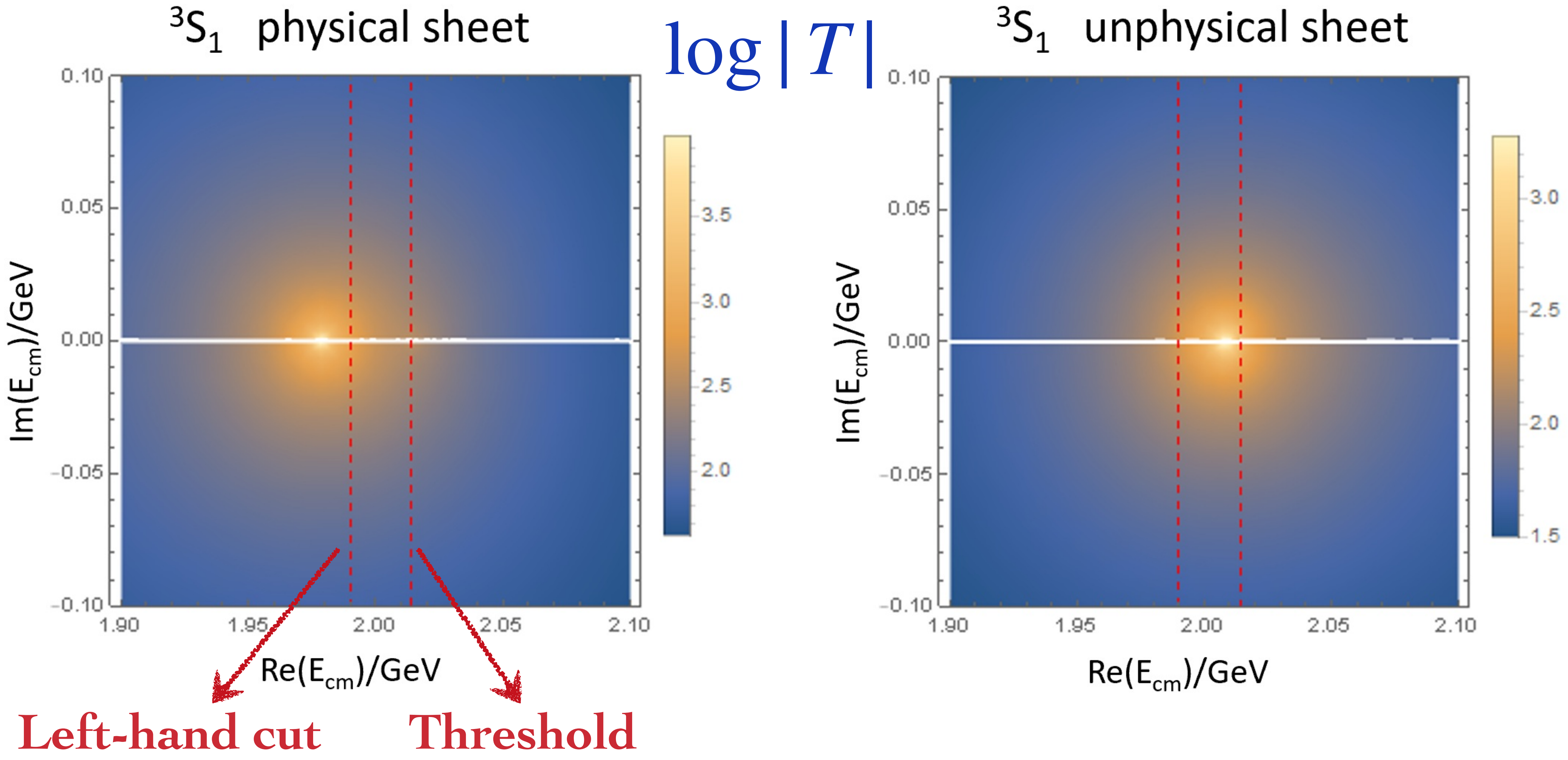
# *NN* scattering

Finite-volume spectrum




Parametrize the scattering amplitude as:

$$T^{-1} = c_0 + c_1 s - i\rho$$




- Virtual pole is found at both $^3S_1$ and $^1S_0$ channels, $E_B(^3S_1) = 6^{+5}_{-3}$, $E_B(^1S_0) = 11^{+6}_{-5}$ .
- Finite-volume Hamiltonian method considering the left-hand cut effects gives the same conclusions.

# $\Lambda\Lambda - N\Xi$ scattering

- H-dibaryon$(I = 0, J^P = 0^+)$ was predicted long time ago. (Jaffe 1977)
- Has not been observed in experiments.
- Lattice QCD calculations give different results.

| Lattice spacing | Volume$(L^3 \times T)$ | $M_\pi$ (MeV) | $M_\pi L$ | # of confs |
|---|---|---|---|---|
| ~0.105fm | $24^3 \times 72$ | 290 | 3.7 | 1000 |
| | $32^3 \times 64$ | 290 | 4.9 | 1000 |
| | $32^3 \times 64$ | 230 | 3.9 | 450 |
| | $48^3 \times 96$ | 230 | 5.9 | 260 |
| ~0.077fm | $32^3 \times 96$ | 300 | 3.7 | 780 |
| | $48^3 \times 96$ | 300 | 5.6 | 360 |
| | $48^3 \times 96$ | 210 | 4.0 | 220 |
| ~0.052fm | $48^3 \times 144$ | 320 | 4.0 | 430 |

# ΛΛ scattering

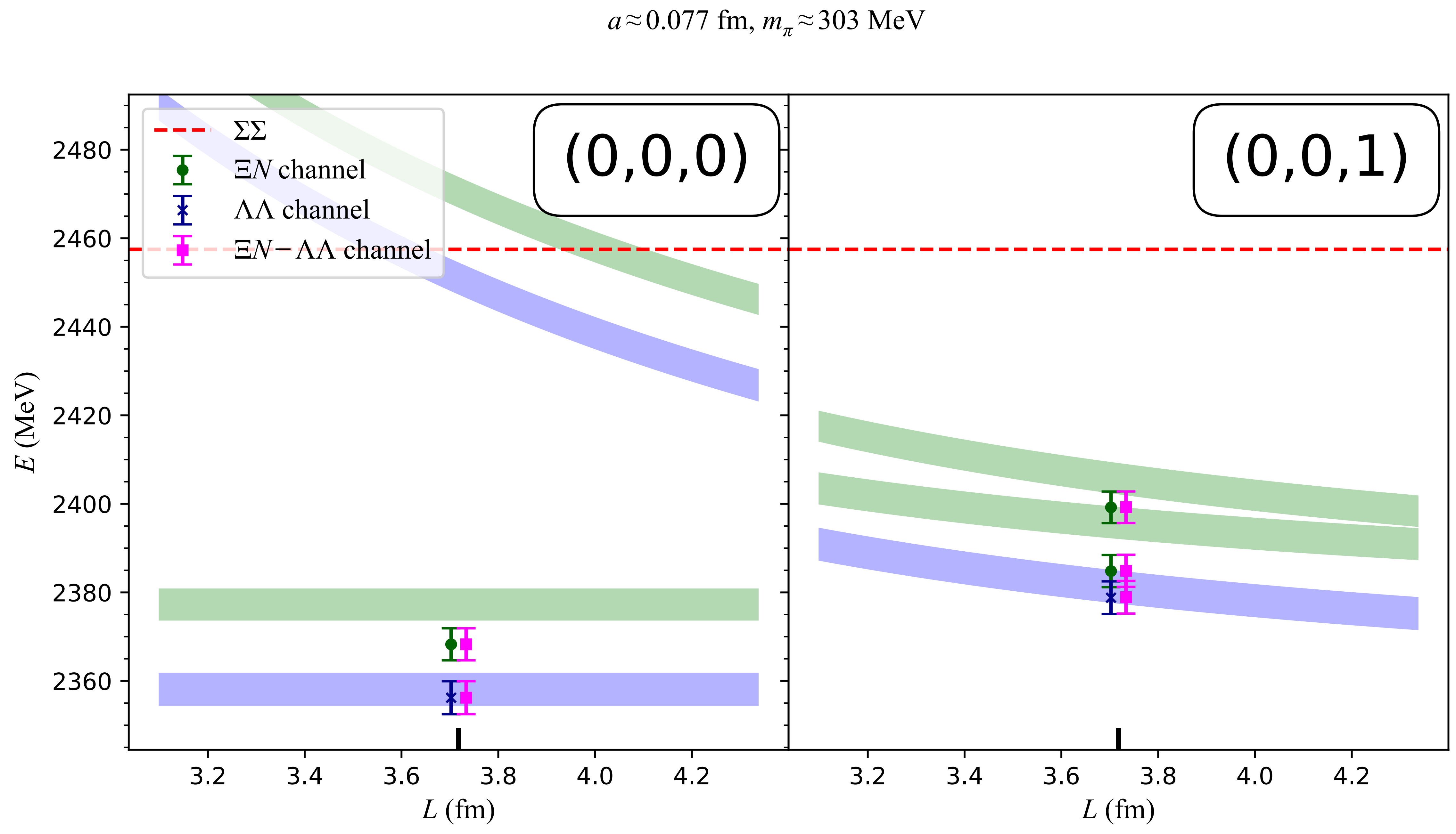
- We are intrested in the $I(J^P) = 0(0^+)$ ΛΛ scattering
- coupled channels: $\Lambda\Lambda, N\Xi, \Sigma\Sigma$
- To avoid the complexity of three coupled channels, we will keep the energy range below $\Sigma\Sigma$ threshold, and consider only $\Lambda\Lambda$ and $N\Xi$.

$$\mathcal{O}^{\mathbf{P}}(\Lambda\Lambda) = \Lambda(\mathbf{p}_1)\Lambda(\mathbf{p}_2)$$

$$\mathcal{O}^{\mathbf{P}}(N\Xi) = p(\mathbf{p}_1)\Xi^-(\mathbf{p}_2) - n(\mathbf{p}_1)\Xi^0(\mathbf{p}_2)$$

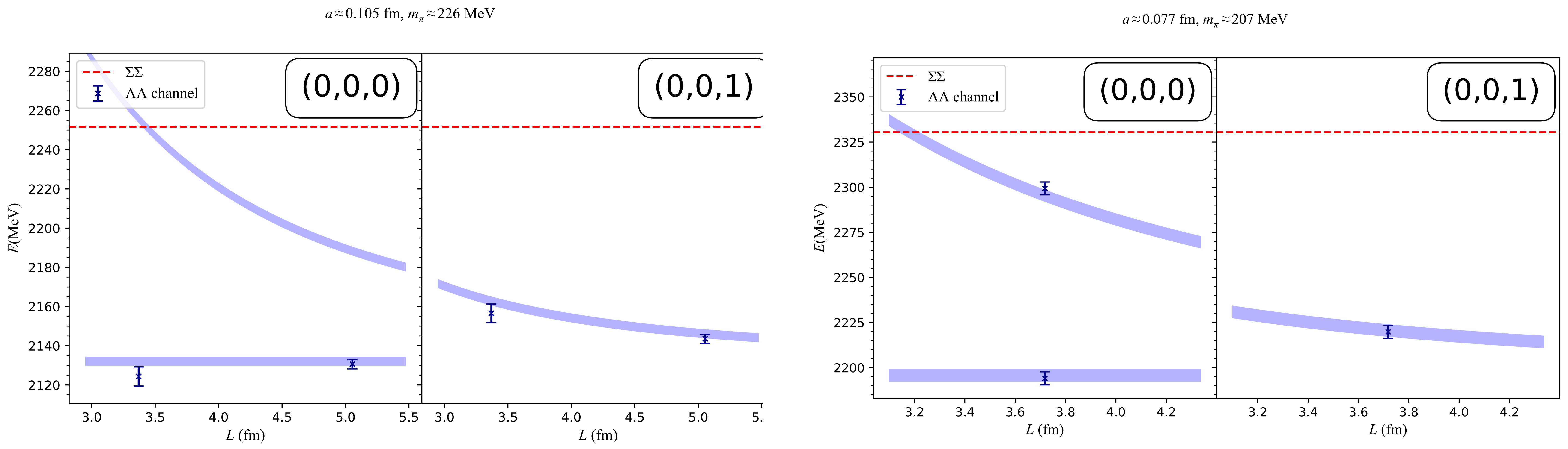
$$\mathbf{P} = \mathbf{p}_1 + \mathbf{p}_2, \quad \mathbf{P} = (0,0,0), (0,0,1)$$

The energy levels obtained from GEVP using both $\Lambda\Lambda$ and $N\Xi$ operators are almost the same as using them separately.



# ΛΛ scattering

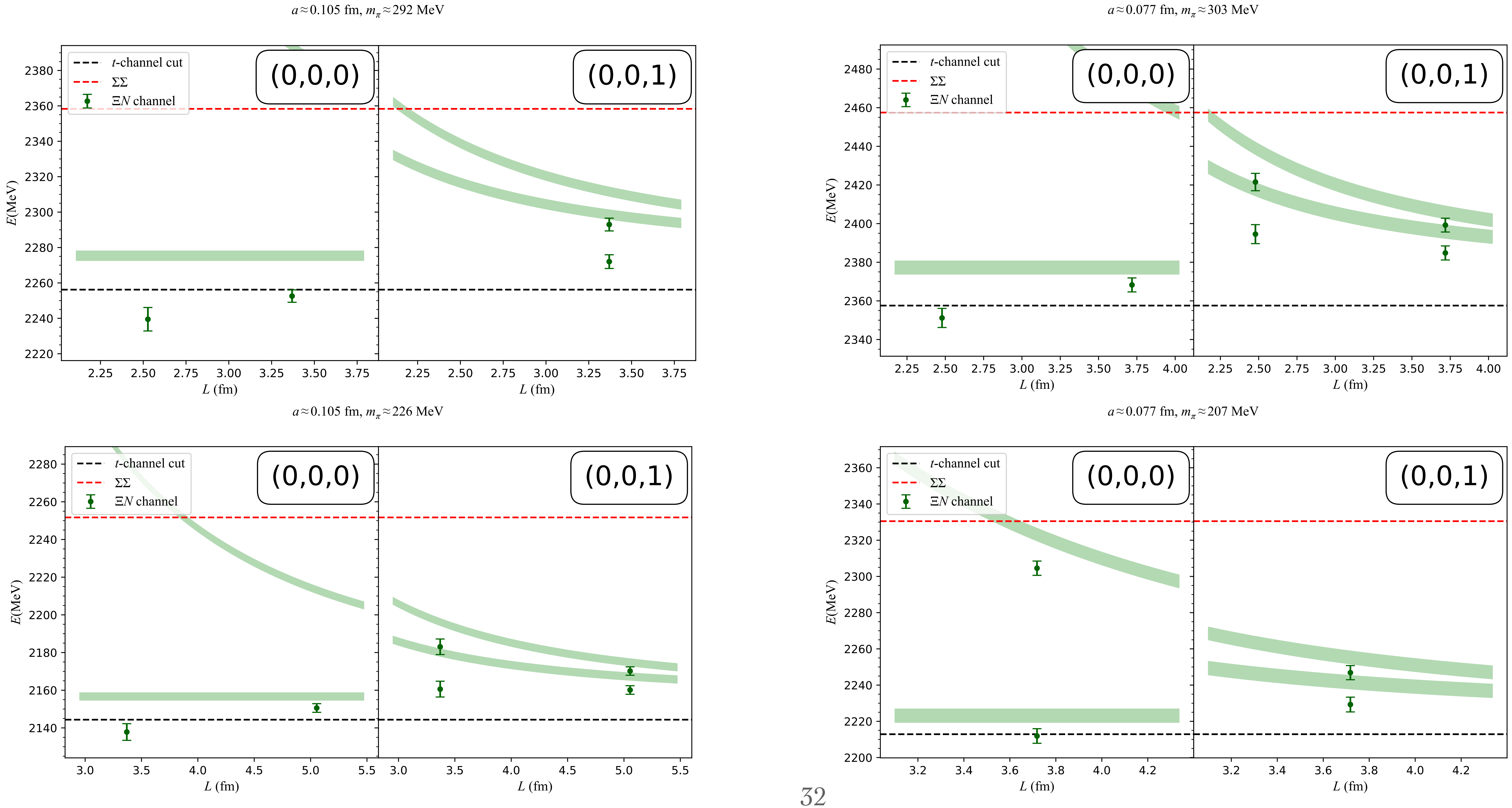
## ΛΛ finite volume spectrum


- Except for $a = 0.105\text{fm}$, $m_\pi = 292\text{MeV}$, the interacting enengies are all very close to the non-interacting energies.

# ΛΛ scattering

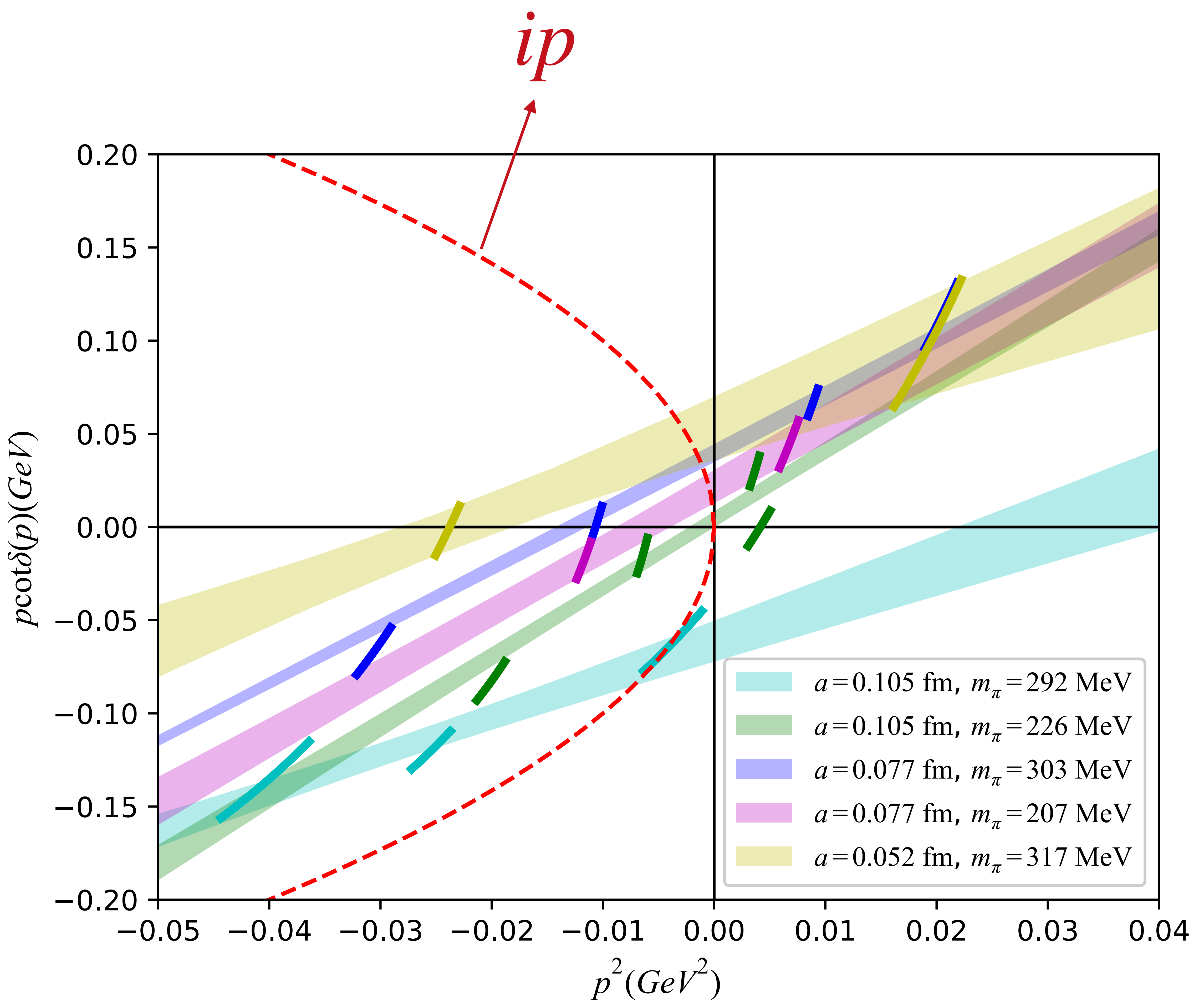
## $N\Xi$ finite volume spectrum



# ΛΛ scattering

## $N\Xi$ scattering amplitude

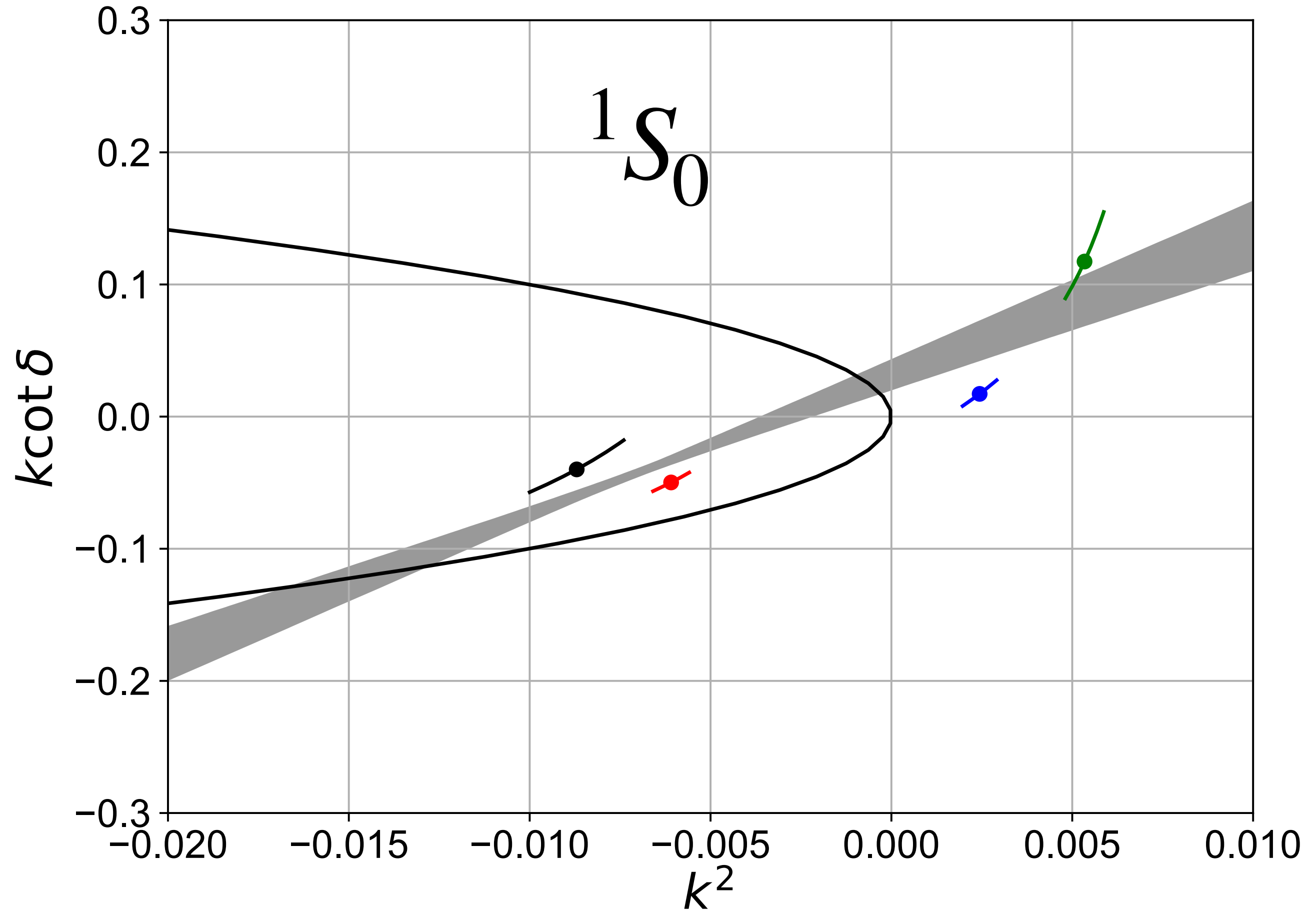
- The scattering amplitude is paramerized by effective range expansion: $p\cot\delta(p) = \frac{1}{a_0} + \frac{1}{2} r_0 p^2$
- At $a = 0.105$ fm, $m_\pi = 292$ MeV, there is a bound state pole, when the lattice spacing becomes smaller and pion mass gets closer to the physical value, the pole becomes a virtual state pole.
- The effects of the left-hand cut has not been considered.

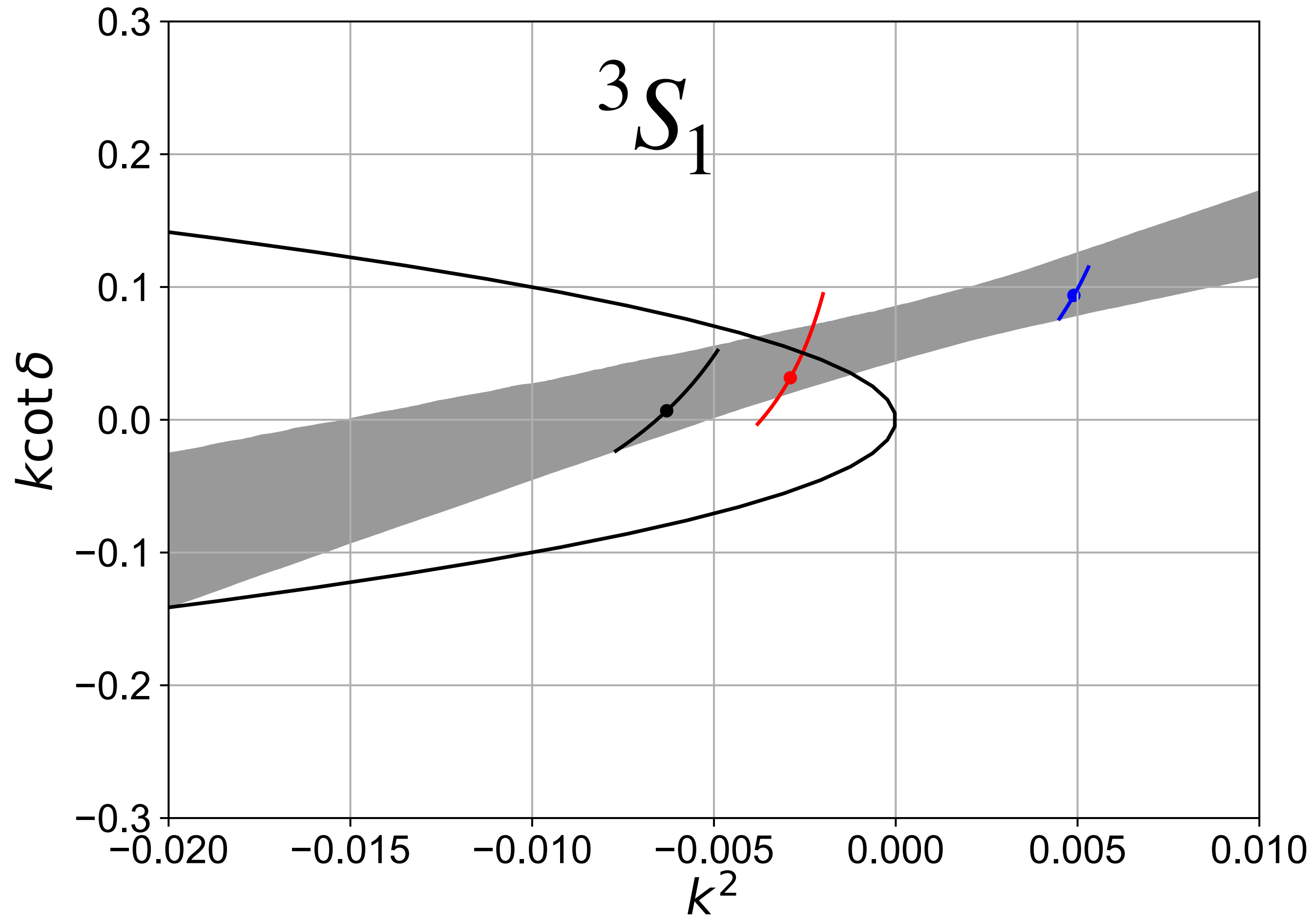


# $N\Lambda$ scattering

Hang Liu et al., work in progress , arXiv:2602.xxxx

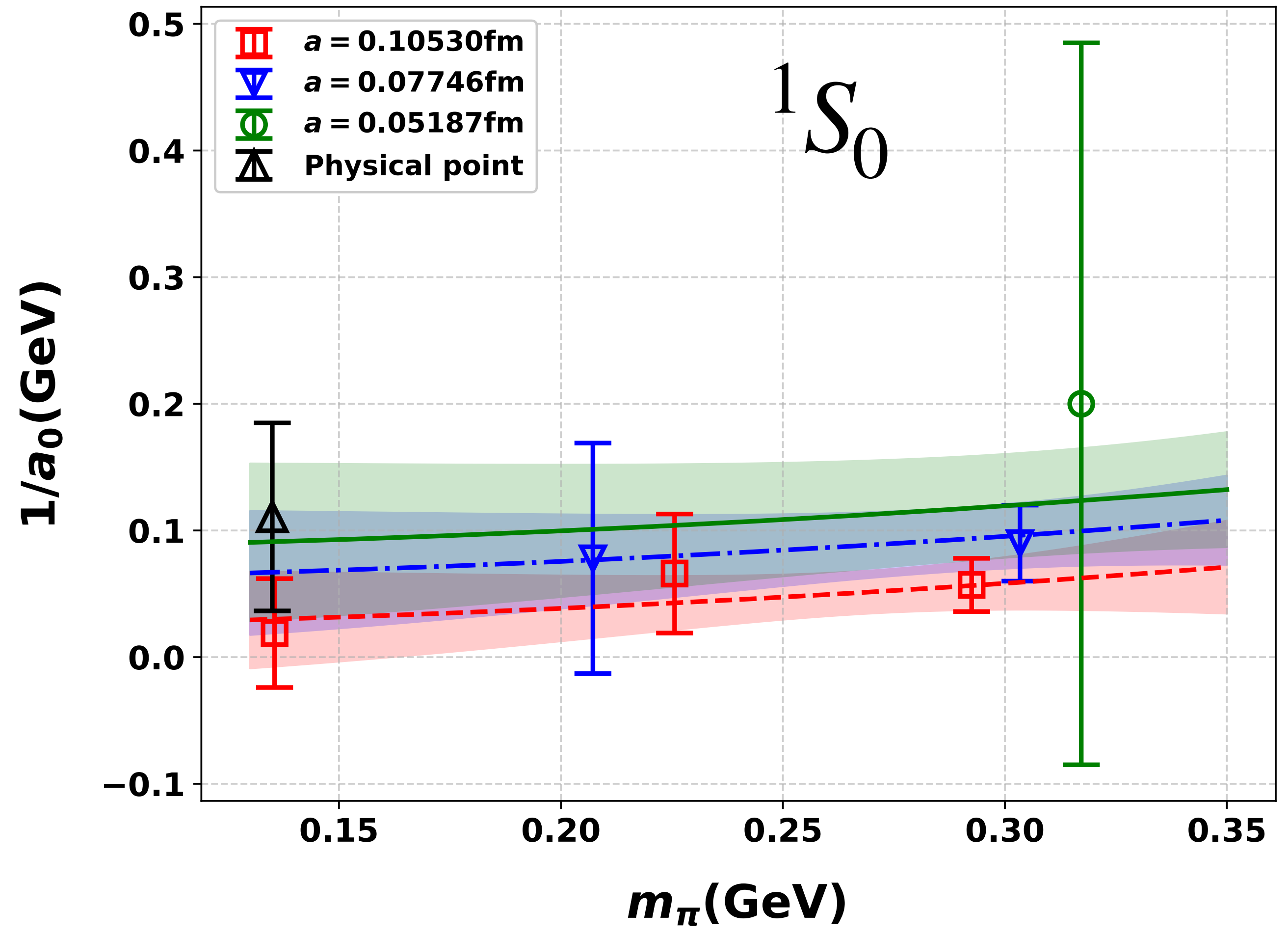
$m_\pi = 290\text{MeV}, \text{a} = 0.105\text{fm}$

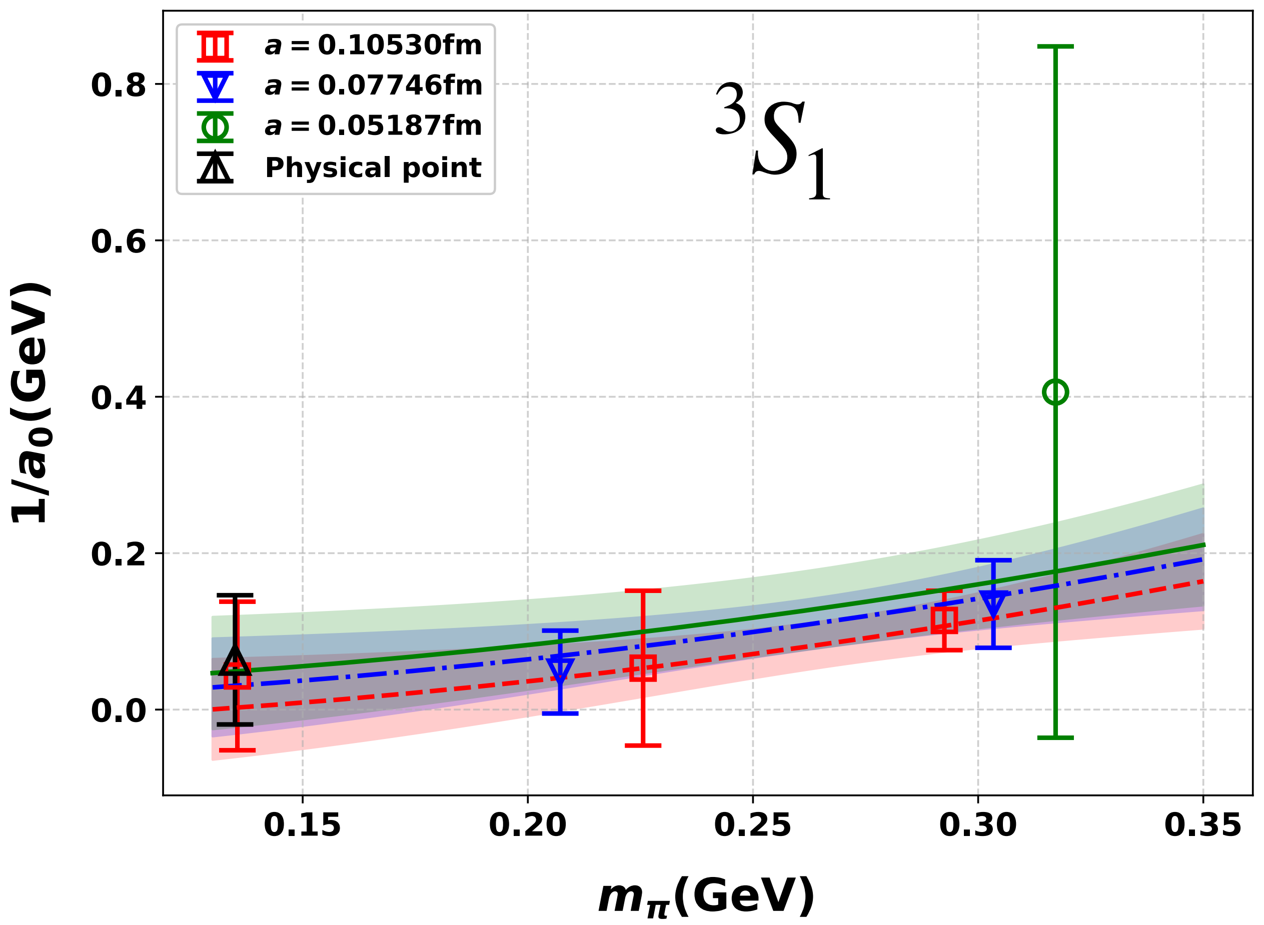



Pion mass and continuum extrapolation:

$$a_0^{-1} = a_{0,\text{phys}}^{-1} + c_1(m_\pi^2 - m_{\pi,\text{phys}}^2) + c_2 a^2$$




$$a_{0,\text{phys}}^{-1} = 0.111(74)\,\text{GeV}, \quad r_{0,\text{phys}} = 3.7(1.2)\,\text{fm}.$$

$$a_{0,\text{phys}}^{-1} = 0.064(83)\,\text{GeV}, \quad r_{0,\text{phys}} = 2.8(1.6)\,\text{fm}.$$

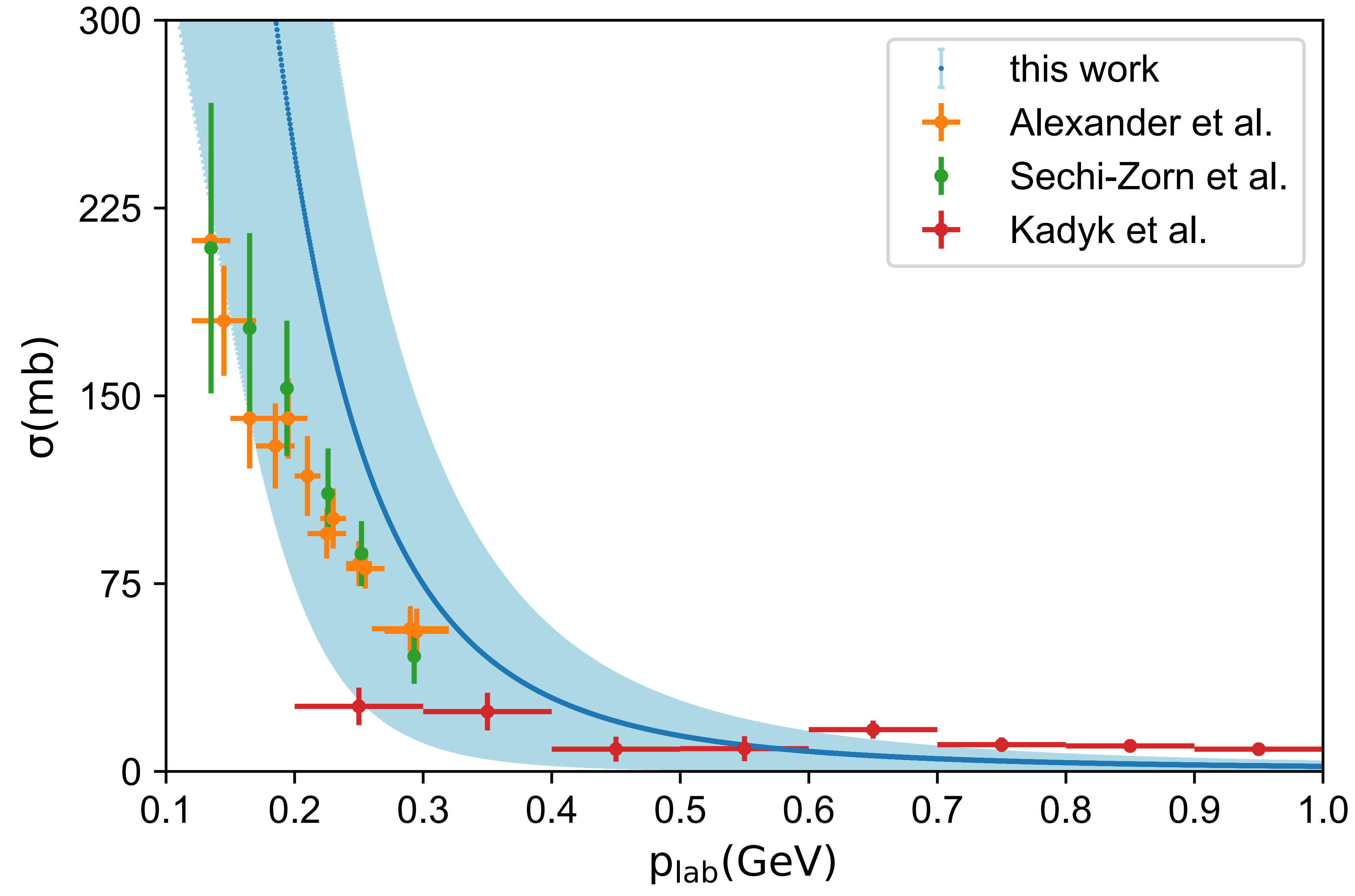
Recent Exp. measurement(STAR):

$$a_0 = 2.32^{+0.12}_{-0.11}\,\text{fm}, \quad r_0 = 3.5^{+2.7}_{-1.3}\,\text{fm}.$$

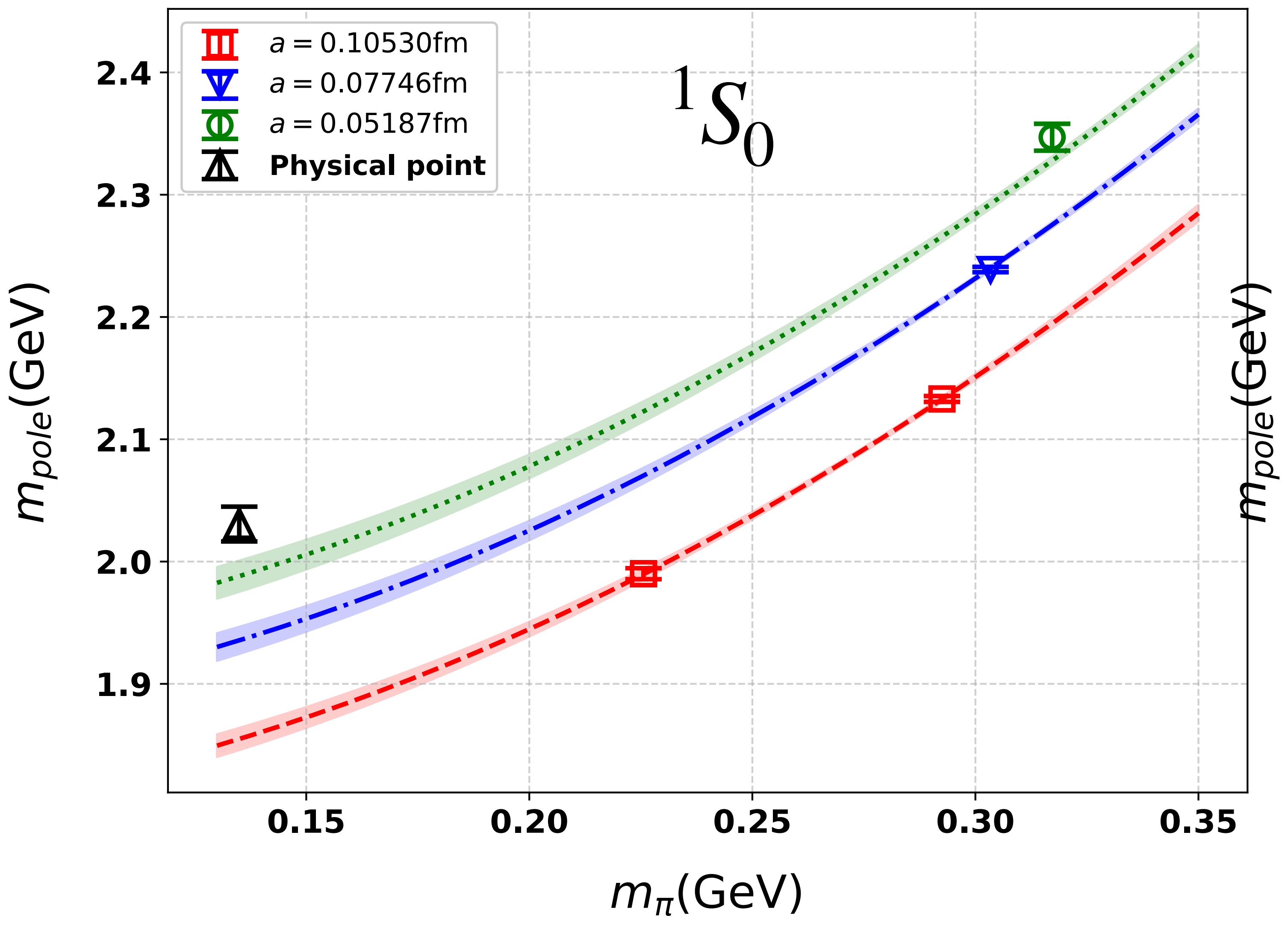
# $N\Lambda$ scattering

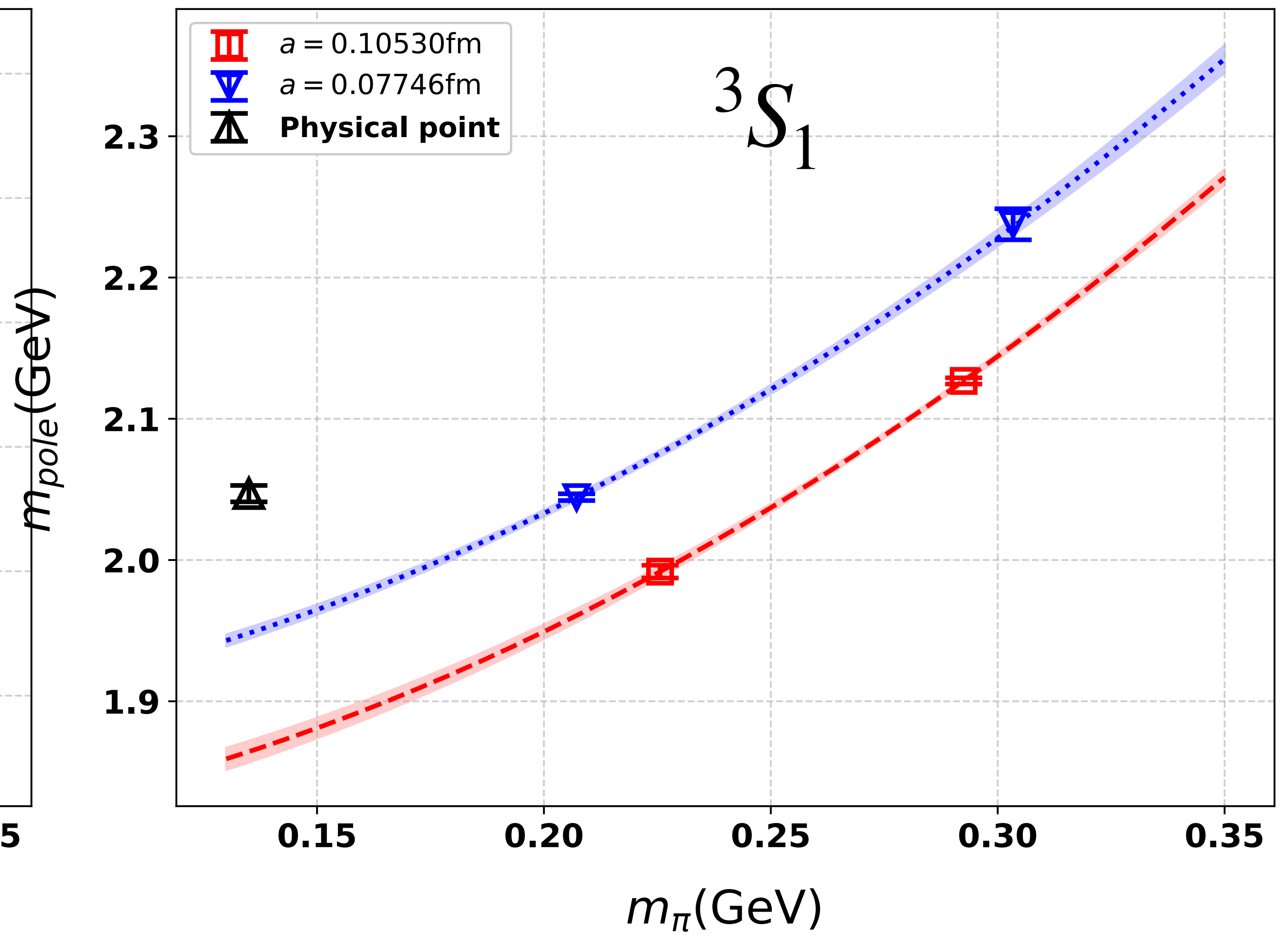
Hang Liu et al., work in progress , arXiv:2602.xxxx

Corss section




Pole position extrapolation




Binding energy:

$$E_B(^1S_0) = 17(17)\text{MeV}$$

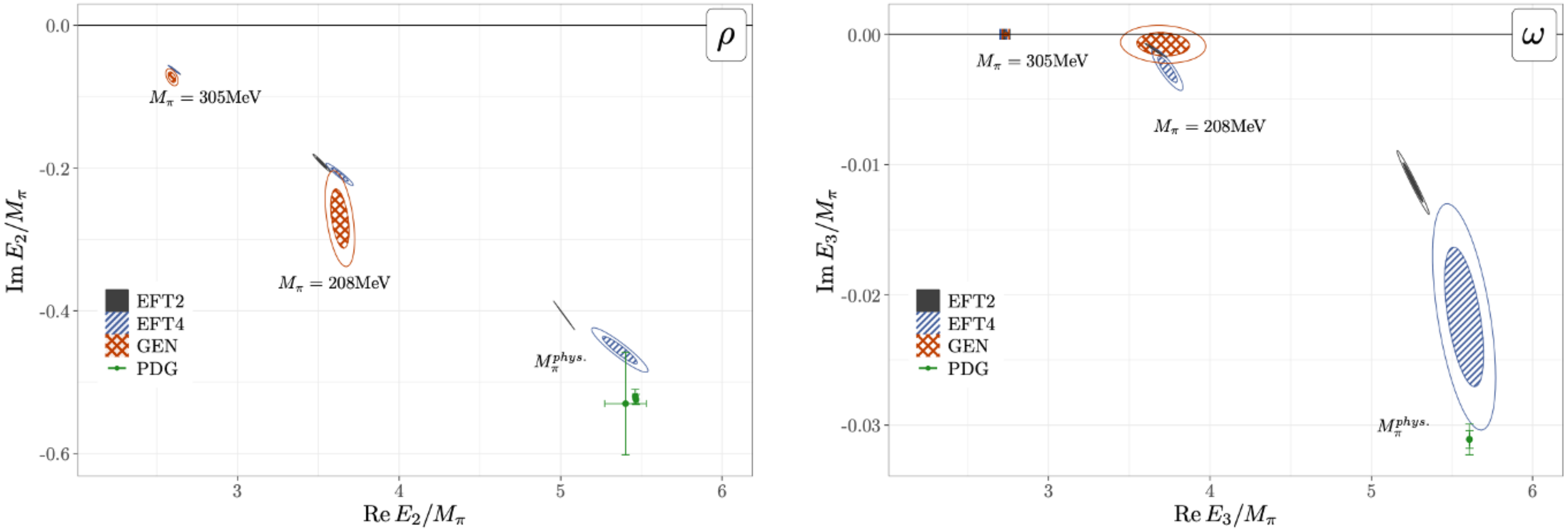
$$E_B(^3S_1) = 14(11)\text{MeV}$$

# Three body scattering

- $\omega$ meson: $\pi\pi\pi$ $I = 0$ scattering

# $\pi\pi\pi$ $I = 1$ scattering

Haobo Yan et al., PRL133(2024)21,211906 (Editor's suggestion)




- First determination of $\omega$-meson resonance from three-body scattering in lattice QCD

$$\sqrt{s_\rho} = (748.9(10.0) - i\,63.5(1.8))\,\text{MeV}$$
$$\sqrt{s_\omega} = (778.0(11.2) - i\,3.0(5))\quad\text{MeV}$$

# Summary and discussions

CLQCD

- **Meson-meson scattering:**
  - The resonance $\rho$ and $K^*$ are determined with full control of systematics.
  - The pion mass dependence of $D_0^*(2300)$ has a non-monotonic behavior.
- **Meson-baryon scattering:**
  - The scattering of the $(\Xi_{cc}, \Omega_{cc}) - (\pi, K, \bar{K})$ has similar behaviour as $(D, D_s) - (\pi, K, \bar{K})$.
  - A double-charm pentaquark is predicted as the $I = 0$ $\Xi_{cc}\bar{K}$ channel.
- **Baryon-baryon scattering:**
  - Deuteron is not bound at $m_\pi \sim 290$MeV
  - $\Lambda\Lambda$ interaction is two weak to form a bound state. $N\Xi/N\Lambda$ has attractive interaction, but are not bound either.
  - Neet to improve precision.
- **Three-body scattering:**
  - First determination of the $\omega$ meson from $\pi\pi\pi$ scattering.
  - Other three-body scattering process: $DD\pi, DDK$

# Thanks!