

1) ν Oscillation & Leptonic CP Phases

2) Dirac CP Phase

3) Majorana CP Phases

4) Summary

Georg G. Raffelt

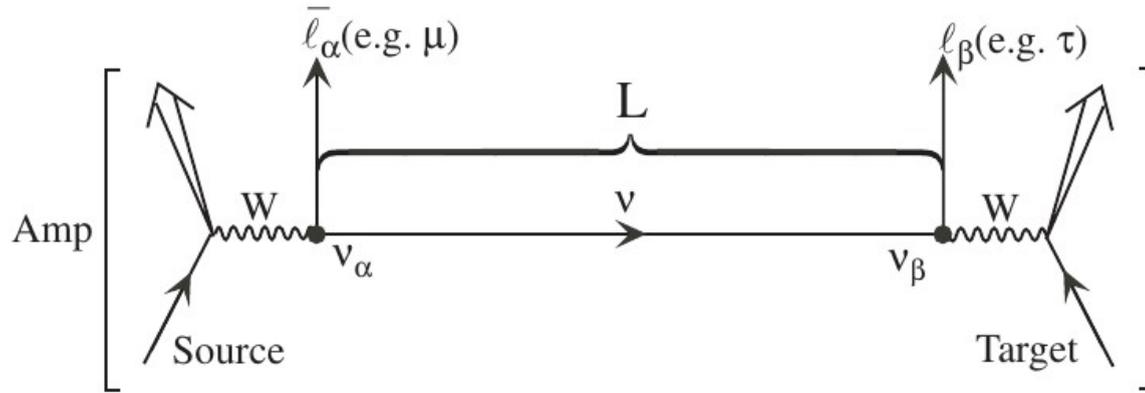
Stars as Laboratories for Fundamental Physics

The Astrophysics of Neutrinos, Axions, and Other
Weakly Interacting Particles

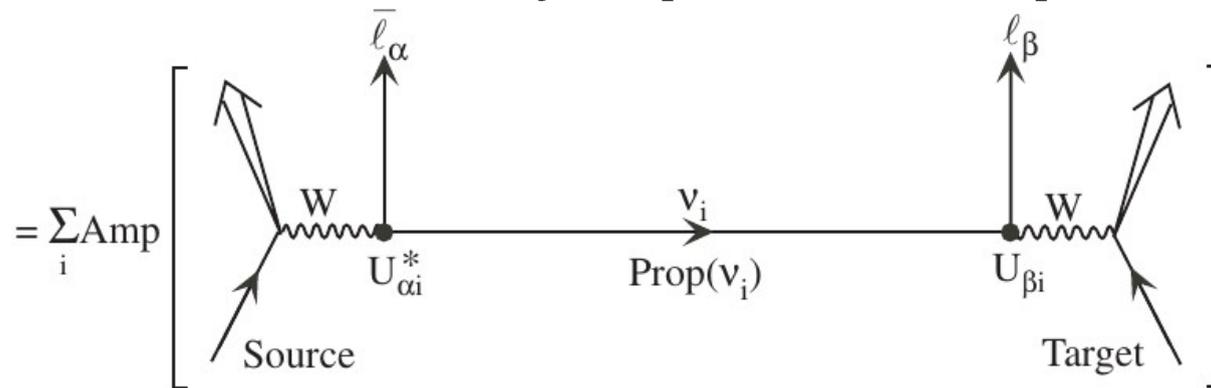
In the standard model, neutrinos have been assigned the most minimal properties compatible with experimental data: zero mass, zero charge, zero dipole moments, zero decay rate, zero almost everything.

Why neutrino physics is so important?

ν Oscillation as 1st New Physics



B. Kayser [[hep-ph/0506165](https://arxiv.org/abs/hep-ph/0506165)]



$$\nu_\alpha = \sum_i U_{\alpha i} \nu_i \rightarrow \sum_i U_{\alpha i} e^{i(E_i t - \vec{P}_i \cdot \vec{x})} \nu_i$$

$$= \sum_i U_{\alpha i} P_i U_{\beta i}^\dagger \nu_\beta \equiv \sum_\beta A_{\alpha\beta} \nu_\beta$$

Reactor: $O(1\sim 10)$ km

Accelerator: $O(100\sim 1000)$ km

Atmospheric: $O(10^3\sim 10^4)$ km

Solar: $O(10^5)$ km

ν oscillation is essentially quantum interference @ macroscopic scales!

$$\delta m^2 = (10^{-5} \sim 10^{-3}) \text{ eV}^2$$

$$P_{\alpha\beta} |_{\alpha \neq \beta} = \sin^2 2\theta \sin^2 \left(\delta m^2 \frac{L}{4E} \right)$$

Why Tiny ν Masses are Important?

- **The world seems not affected by the tiny neutrino mass!**
- **Higgs boson** \Rightarrow electroweak symmetry breaking & mass. $\sim O(100)\text{GeV}$
 - Neutrino mass \Rightarrow Mixing
 - 3 Neutrino \Rightarrow possible **CP violation**
 - CP violation \Rightarrow **Leptogenesis**
 - \Rightarrow **Matter-Antimatter Asymmetry**
 - There is something left in the Universe.
 - **EW Baryogenesis** is not enough.



Daya Bay @ **March 8, 2012**



LHC @ **July 4, 2012**

Seesaw & Leptogenesis

With decreasing temperature, heavy N decays to light SM particles.

$$N_k \rightarrow \begin{cases} \ell_j + \bar{\phi} \\ \bar{\ell}_j + \phi \end{cases}$$

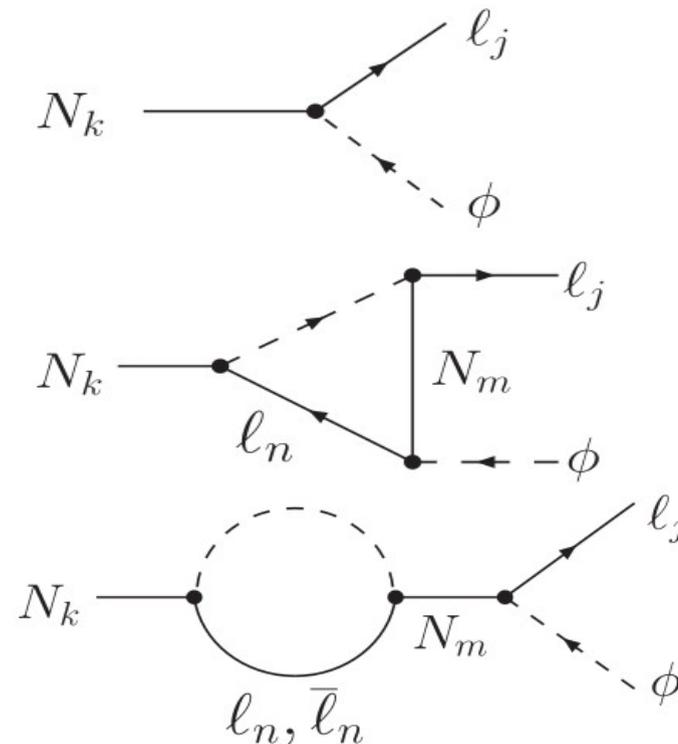


Matter-Antimatter Asymmetry

- Interference between tree & loop diagrams

$$\begin{aligned} \Gamma &= \Gamma_{\text{tree}} + \Gamma_{\text{loop}}(+\delta_D, +\delta_M) \\ \bar{\Gamma} &= \Gamma_{\text{tree}} + \Gamma_{\text{loop}}(-\delta_D, -\delta_M) \end{aligned}$$

The matter-antimatter asymmetry needs Dirac/Majorana CP phases.



Yanagida

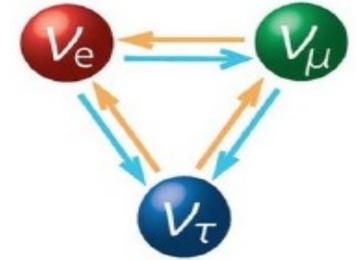


Fukugita

- PMNS Matrix

$$U_{\text{PMNS}} = \mathcal{P} \begin{pmatrix} c_s c_r & s_s c_r & s_r e^{-i\delta_D} \\ -s_s c_a - c_s s_a s_r e^{i\delta_D} & +c_s c_a - s_s s_a s_r e^{i\delta_D} & s_a c_r \\ +s_s s_a - c_s c_a s_r e^{i\delta_D} & -c_s s_a - s_s c_a s_r e^{i\delta_D} & c_a c_r \end{pmatrix} \mathcal{Q}$$

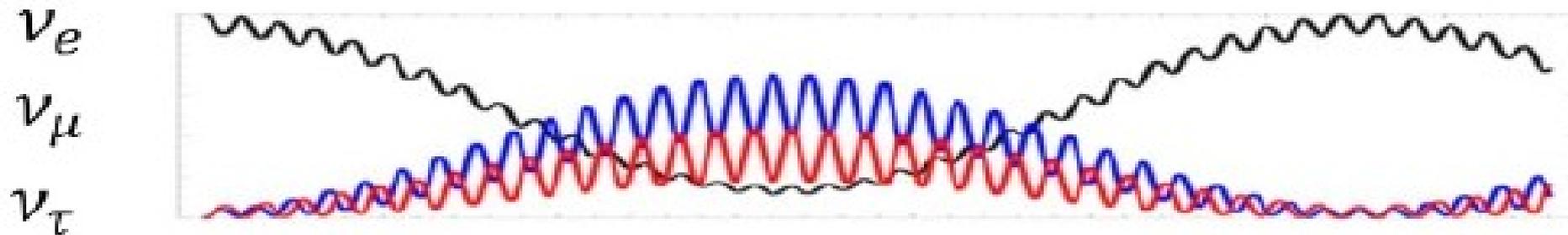
Dirac Phase



$(s, a, r) \equiv (12, 23, 13)$ for (**solar, atmospheric, reactor**) angles

$$\mathcal{P} \equiv \text{diag}(e^{i\phi_1}, e^{i\phi_2}, e^{i\phi_3}) \quad \& \quad \mathcal{Q} \equiv \text{diag}(e^{i\delta_{M1}}, 1, e^{i\delta_{M3}})$$

Majorana Phases



质量差 + 混合 → 中微子振荡

Daya Bay heralded a new era of precision measurement in 2012!

Kam-Biu Luk @ Neutrino 2022

	Normal Ordering ($\Delta\chi^2 = 0.6$)		Inverted Ordering (best fit)	
	bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range
$\sin^2 \theta_{12}$	$0.307^{+0.012}_{-0.011}$	$0.275 \rightarrow 0.345$	$0.308^{+0.012}_{-0.011}$	$0.275 \rightarrow 0.345$
$\theta_{12}/^\circ$	$33.68^{+0.73}_{-0.70}$	$31.63 \rightarrow 35.95$	$33.68^{+0.73}_{-0.70}$	$31.63 \rightarrow 35.95$
$\sin^2 \theta_{23}$	$0.561^{+0.012}_{-0.015}$	$0.430 \rightarrow 0.596$	$0.562^{+0.012}_{-0.015}$	$0.437 \rightarrow 0.597$
$\theta_{23}/^\circ$	$48.5^{+0.7}_{-0.9}$	$41.0 \rightarrow 50.5$	$48.6^{+0.7}_{-0.9}$	$41.4 \rightarrow 50.6$
$\sin^2 \theta_{13}$	$0.02195^{+0.00054}_{-0.00058}$	$0.02023 \rightarrow 0.02376$	$0.02224^{+0.00056}_{-0.00057}$	$0.02053 \rightarrow 0.02397$
$\theta_{13}/^\circ$	$8.52^{+0.11}_{-0.11}$	$8.18 \rightarrow 8.87$	$8.58^{+0.11}_{-0.11}$	$8.24 \rightarrow 8.91$
$\delta_{CP}/^\circ$	177^{+19}_{-20}	$96 \rightarrow 422$	285^{+25}_{-28}	$201 \rightarrow 348$
$\frac{\Delta m_{21}^2}{10^{-5} \text{ eV}^2}$	$7.49^{+0.19}_{-0.19}$	$6.92 \rightarrow 8.05$	$7.49^{+0.19}_{-0.19}$	$6.92 \rightarrow 8.05$
$\frac{\Delta m_{3\ell}^2}{10^{-3} \text{ eV}^2}$	$+2.534^{+0.025}_{-0.023}$	$+2.463 \rightarrow +2.606$	$-2.510^{+0.024}_{-0.025}$	$-2.584 \rightarrow -2.438$

IC19 without SK atmospheric data

Esteban, Gonzalez-Garcia, Maltoni, Martinez-Soler, Pinheiro & Schwetz [[2410.05380](#), [2601.09791](#)]

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- Full flavor symmetries

$$\begin{pmatrix} \nu_L \\ \ell_L \end{pmatrix} \in \left(2, -\frac{1}{2} \right)$$

Same doublet



$$U_\ell^\dagger U_\nu = I$$

1. Full symmetry **HAS TO** be **Broken!**

2. Many other factors (**Yukawa couplings**, **VEVs**) may enter.

- Residual flavor symmetries

If mixing is **TRUELY determined by symmetry**, it has to be the **residual symmetry**

Lam, PRL101, 121602(2008), PRD78, 073015(2008)

$$\mathcal{H} \equiv \mathcal{G} \times \mathcal{F}$$

$$S_4$$

$$\{G_1, G_3, F\}$$

Top
down



Bottom
up



\mathcal{G}

\mathcal{F}

$$\mathbb{Z}_2^S \times \mathbb{Z}_2^{\mu T}$$

$$\mathbb{Z}_3 \equiv \{I, F, F^2\}$$

$$G_1(G_2), G_3$$

$$F \equiv \text{diag}(1, \omega, \omega^2)$$

See also Smirnov et. al., [1204.0445, 1212.2149, 1510.00344]

Residual symmetry directly applies on the mass matrix!

- Invariance

$$\mathbf{G}_\nu^T \mathbf{M}_\nu \mathbf{G}_\nu = \mathbf{M}_\nu$$

- Diagonalization

$$\mathbf{V}_\nu^T \mathbf{M}_\nu \mathbf{V}_\nu = \mathbf{D}_\nu$$

- Rephasing

$$\mathbf{D}_\nu = \mathbf{d}_\nu^T \mathbf{D}_\nu \mathbf{d}_\nu$$

$$\begin{aligned} \mathbf{M}_\nu &= \mathbf{G}_\nu^T \mathbf{M}_\nu \mathbf{G}_\nu = \mathbf{G}_\nu^T \mathbf{V}_\nu^* \mathbf{D}_\nu \mathbf{V}_\nu^\dagger \mathbf{G}_\nu \\ &= \mathbf{V}_\nu^* \mathbf{D}_\nu \mathbf{V}_\nu^\dagger = \mathbf{V}_\nu^* \mathbf{d}_\nu^T \mathbf{D}_\nu \mathbf{d}_\nu \mathbf{V}_\nu^\dagger \end{aligned}$$



$$\mathbf{V}_\nu^\dagger \mathbf{G}_\nu = \mathbf{d}_\nu \mathbf{V}_\nu^\dagger$$

$$\begin{aligned} \mathbf{G}_\nu &= \mathbf{V}_\nu \mathbf{d}_\nu \mathbf{V}_\nu^\dagger & \nu_i: \mathcal{G} &\equiv \mathbb{Z}_2^S (\overline{\mathbb{Z}}_2^S) \times \mathbb{Z}_2^{\mu\tau} & d_\nu^i &= \text{diag} (\pm 1, \pm 1, \pm 1) \\ \mathbf{F}_\ell &= \mathbf{V}_\ell \mathbf{d}_\ell \mathbf{V}_\ell^\dagger & \ell_i: \mathcal{F} &\in U(1) \times U(1) & d_\ell^i &= \text{diag} (e^{i\beta_1}, e^{i\beta_2}, e^{i\beta_3}) \end{aligned}$$

Direct connection between residual symmetry & mixing pattern!

- Although $Z_2^{\mu T}$, represented by \mathbf{G}_3 , is Broken!
- **No particular reason** for Z_2^S or \overline{Z}_2^S to be Broken!

$$Z_2^S : \mathbf{G}_1(\mathbf{k}) = \frac{1}{2+k^2} \begin{pmatrix} 2-k^2 & 2k & 2k \\ & k^2 & -2 \\ & & k^2 \end{pmatrix}$$

$$\overline{Z}_2^S : \mathbf{G}_2(\mathbf{k}) = \frac{1}{2+k^2} \begin{pmatrix} 2-k^2 & 2k & 2k \\ & -2 & k^2 \\ & & -2 \end{pmatrix}$$

- Z_2^S & \overline{Z}_2^S are **Dependent**

$$\mathbf{G}_1(\mathbf{k}) = \mathbf{G}_2(\mathbf{k})\mathbf{G}_3$$

- **DIFFERENT Consequences!!!**

SFG, Dicus, Repko
PRD 2011 [1012.2571]
PLB 2011 [1104.0602]
PRL 2012 [1108.0964]

$$G_i = V_\nu d_\nu^{(i)} V_\nu^\dagger$$

$$\mathbb{Z}_2^S (G_1)$$

$$\overline{\mathbb{Z}}_2^S (G_2)$$

$$\cos \delta_D = \frac{(s_s^2 - c_s^2 s_r^2)(c_a^2 - s_a^2)}{4c_a s_a c_s s_s s_r}$$

$$\cos \delta_D = \frac{(s_s^2 s_r^2 - c_s^2)(c_a^2 - s_a^2)}{4c_a s_a c_s s_s s_r}$$

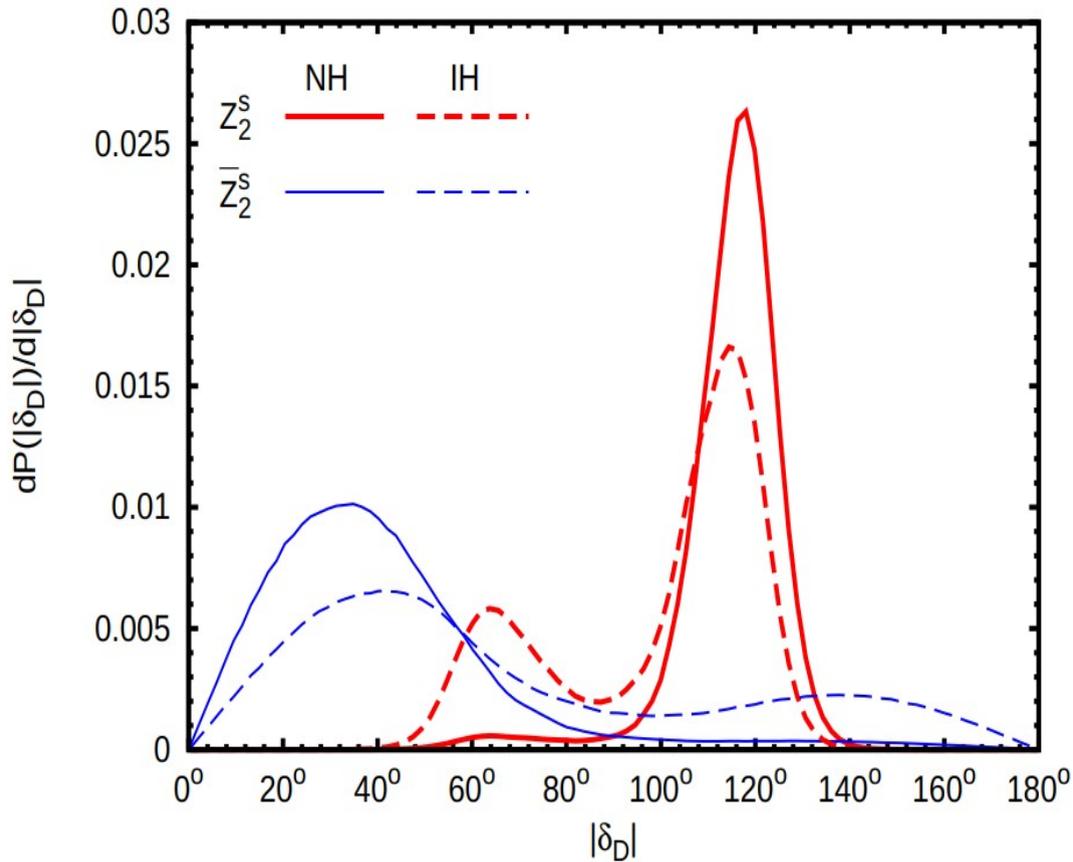
Residual Symmetry vs Custodial Symmetry

- Gauge symmetry has to be broken. Otherwise, no mixing.
- **Weak mixing angle** is a function of gauge couplings, which cannot be dictated by gauge symmetry (and VEV).
- **Weak mixing angle is related to** the physical observables, the **gauge boson masses** by **custodial symmetry**

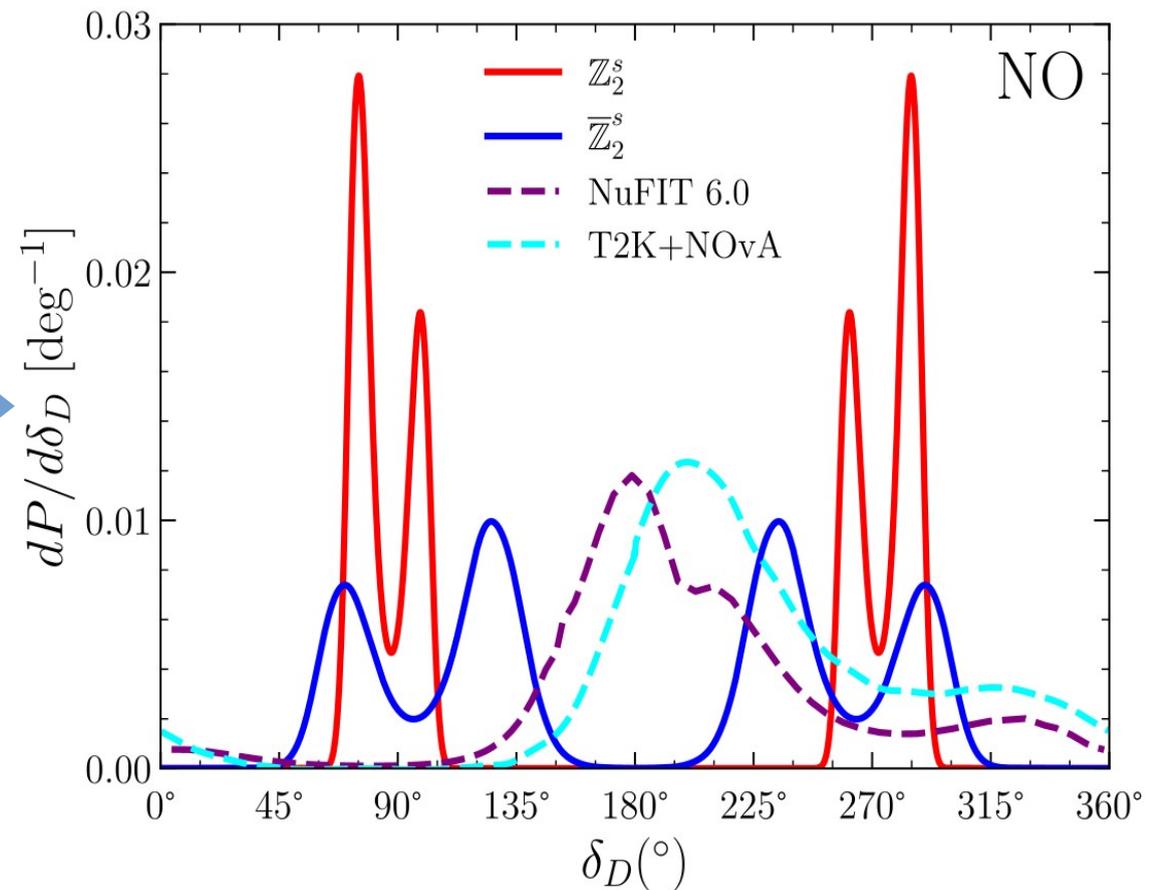
Prediction of Large CP

$$\cos \delta_D = \frac{(s_s^2 - c_s^2 s_r^2)(c_a^2 - s_a^2)}{4c_a s_a c_s s_s s_r}$$

$$\cos \delta_D = \frac{(s_s^2 s_r^2 - c_s^2)(c_a^2 - s_a^2)}{4c_a s_a c_s s_s s_r}$$



+JUNO



A. Hanlon, **SFG**, W. Repko [1308.6522]

SFG, Chui-Fan Kong & Joao Pinheiro [2511.15391]

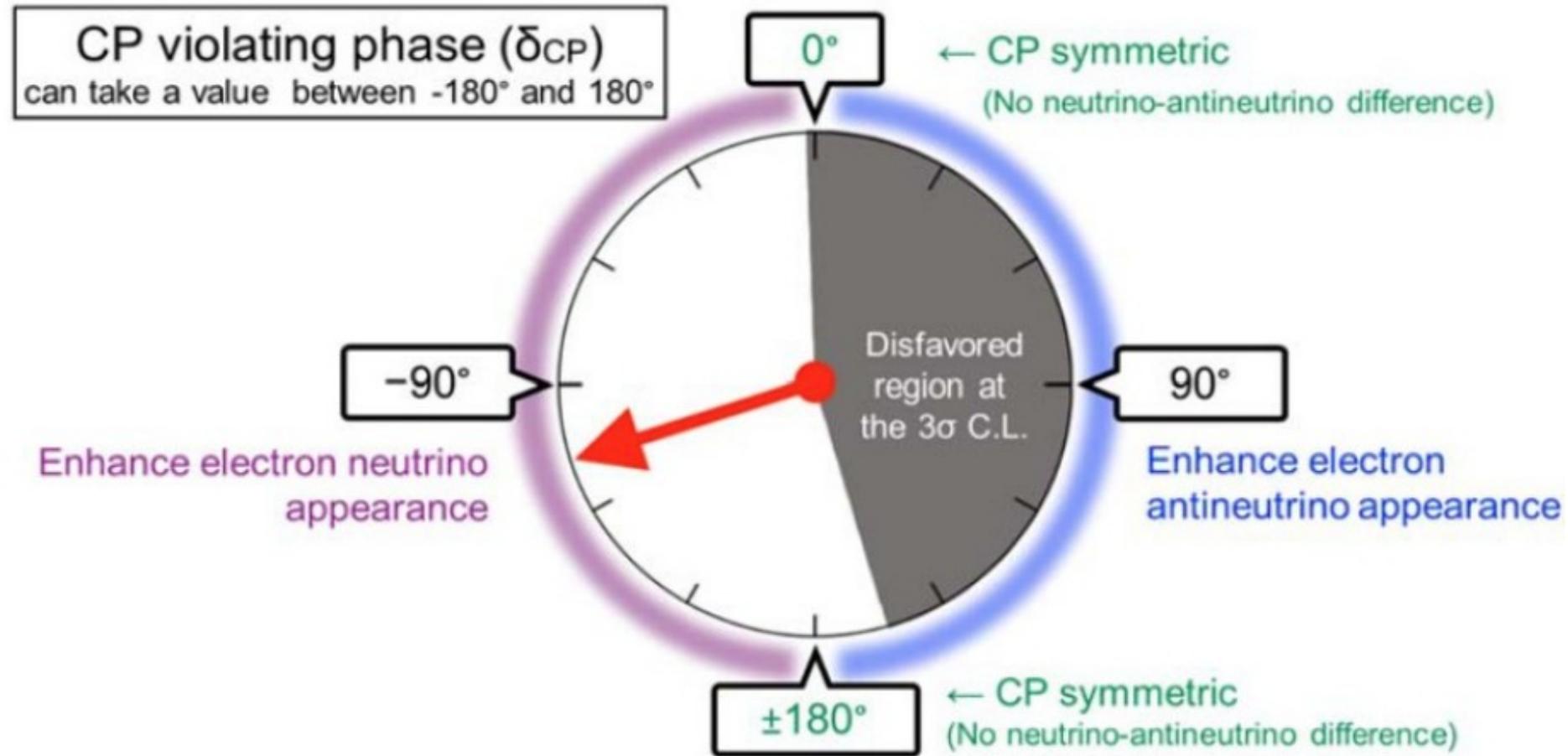


Fig.1 The arrow indicates the value most compatible with the data. The gray region is disfavored at 99.7% (3σ) confidence level. Nearly half of the possible values are excluded.

https://www.kek.jp/en/newsroom/attic/PR20200416_T2K_E.pdf

Nature vol. 580, pages 339-344(2020)

- Non-Standard Interactions

$$\mathcal{H} \equiv \frac{1}{2\mathbf{E}_\nu} U \begin{pmatrix} 0 & & \\ & \Delta m_s^2 & \\ & & \Delta m_a^2 \end{pmatrix} U^\dagger + V_{cc} \begin{pmatrix} 1 + \epsilon_{ee} & \epsilon_{e\mu} & \epsilon_{e\tau} \\ \epsilon_{e\mu}^* & \epsilon_{\mu\mu} & \epsilon_{\mu\tau} \\ \epsilon_{e\tau}^* & \epsilon_{\mu\tau}^* & \epsilon_{\tau\tau} \end{pmatrix}$$

Scalar NSI: SFG & Parke [1812.08376]

Smirnov & Xu [1909.07505]

Du, Li, Tang, Vihonen & Yu [2011.14292, 2106.15800]

Tang & Zhang [1705.09500]

Du, Li, Tang, Vihonen & Yu [2106.15800]

- Non-Unitary Mixing

$$N = N^{NP} U = \begin{pmatrix} \alpha_{11} & 0 & 0 \\ |\alpha_{21}| e^{i\phi} & \alpha_{22} & 0 \\ \alpha_{31} & \alpha_{32} & \alpha_{33} \end{pmatrix} U$$

SFG, Pasquini, Tortola & Valle [1605.01670]

Tang, Zhang & Li [1708.04909]

Hu, Ling, Tang & Wang [2008.09730]

- Sterile neutrinos

Chatterjee & Palazzo [2005.10338]

- Lorentz violation

Rahaman, Razzaque & Sankar [2201.03250]

- Dark NSI

Berlin [1608.01307]

Zhao [1701.02735]

Brdar, Kopp, Liu, Prass & Wang [1705.09455]

Liao, Marfatia & Whisnant [1803.01773]

Chao, Hu, Jiang & Jin [2009.14703]

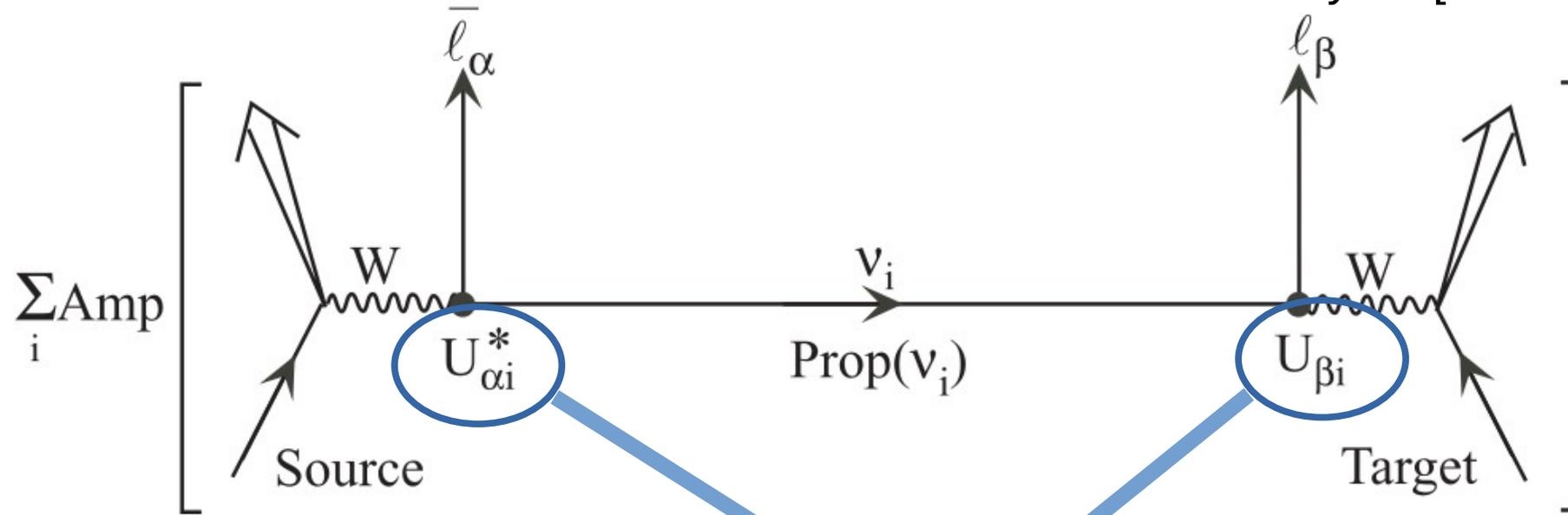
SFG, Murayama [1904.02518]

SFG, PoS NuFact2019 (2020) 108

SFG, J.Phys.Conf.Ser. 1468 (2020) 1, 012125

Two Mixing Matrices in ν Oscillation

B. Kayser [[arXiv:hep-ph/0506165](https://arxiv.org/abs/hep-ph/0506165)]



$$A_{\beta\alpha}^{\text{OSC}} \equiv \sum_i U_{\beta i} e^{-i \frac{m_i^2}{2E_\nu} L} U_{\alpha i}^*$$

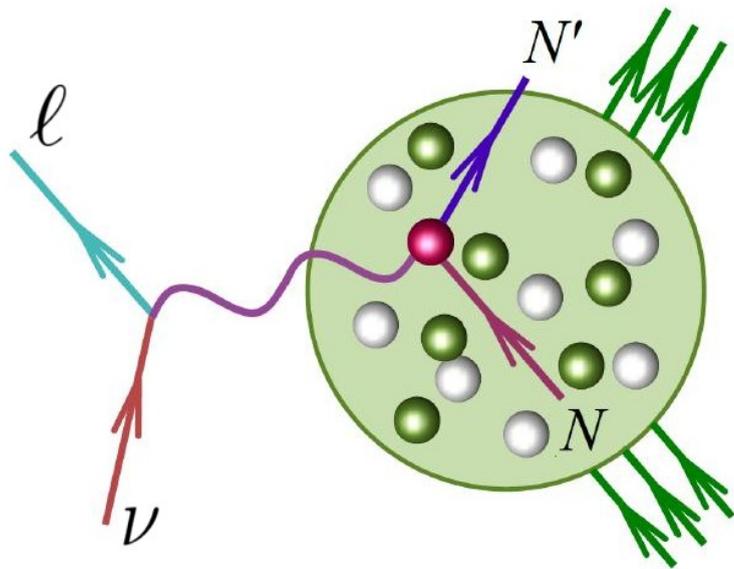
Should they be the same?

Production vs Detection

$$A_{\beta\alpha}^{\text{OSC}} \equiv \sum_i U_{\beta i} e^{-i \frac{m_i^2}{2E_\nu} L} U_{\alpha i}^*$$

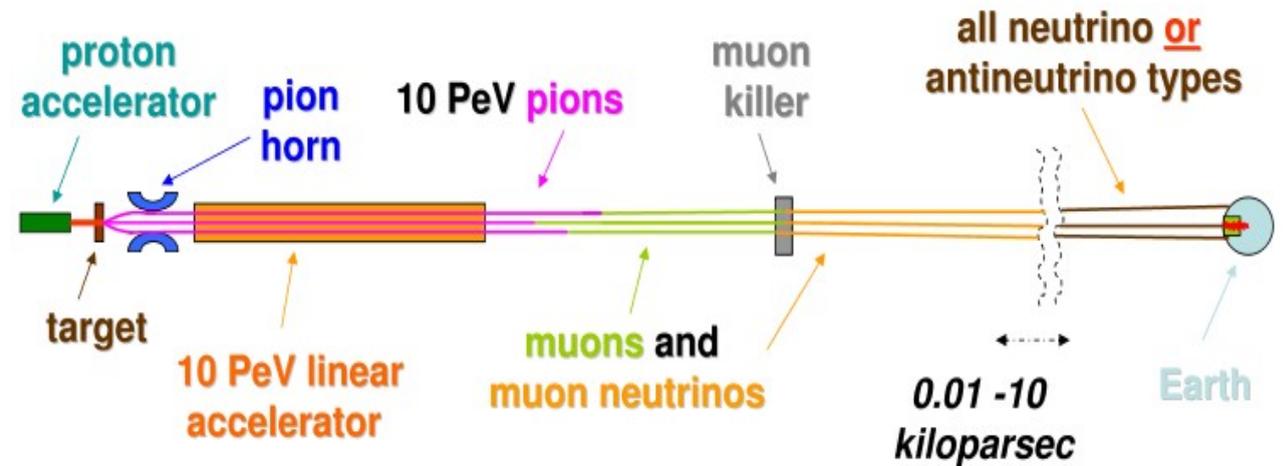
$Q_d^2 \approx ?$

$Q_p^2 \approx (100 \text{ MeV})^2$



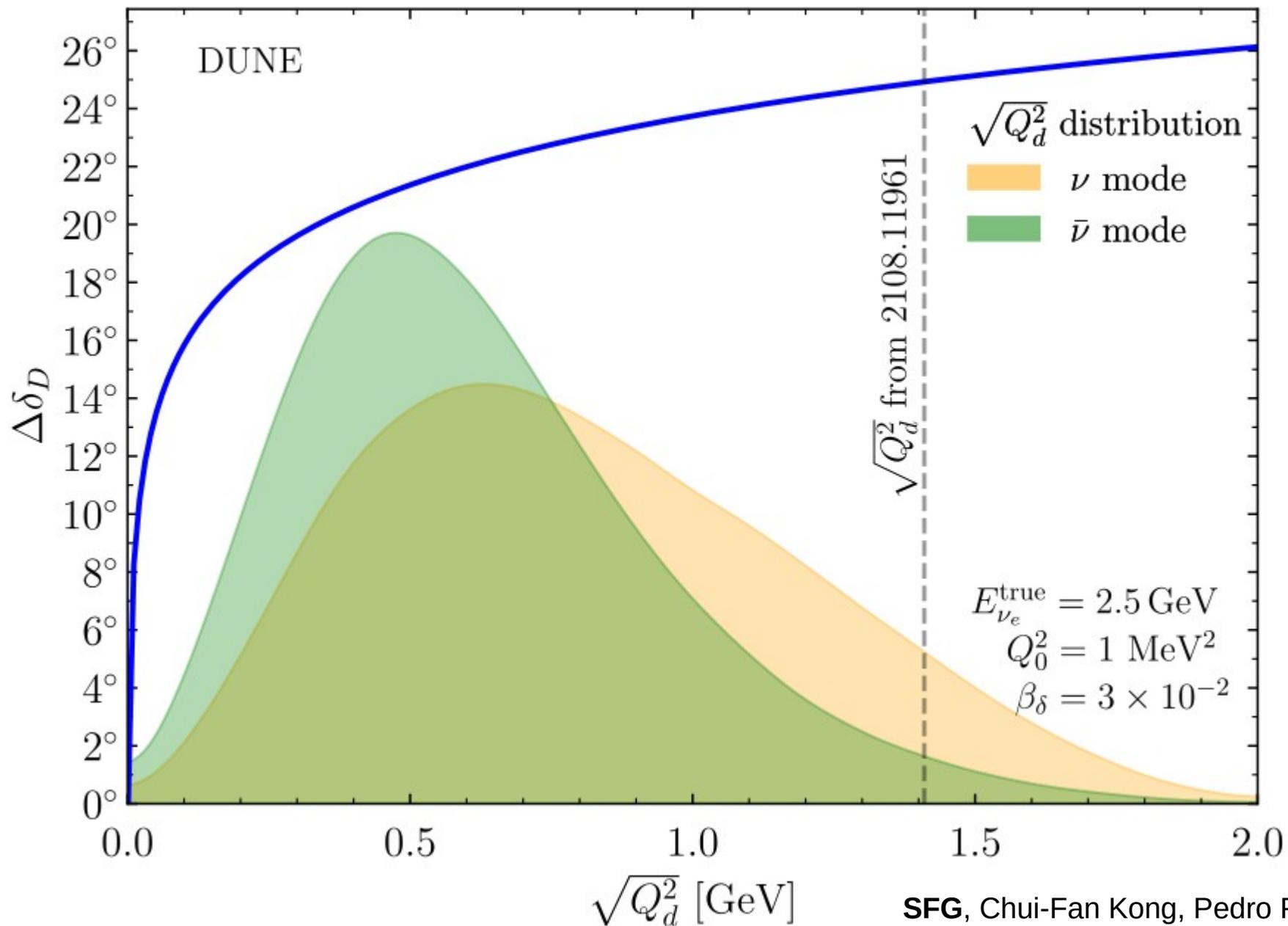
Detection: ES, RES, DIS

Pion Accelerator Neutrino Beam Concept



Production: π, μ, K decays

Mismatched Momentum Transfer



SFG, Chui-Fan Kong, Pedro Pasquini [PRD, 2310.04077]

$$\mathcal{A}_{\beta\alpha}^{\text{OSC}} \equiv \sum_i U_{\beta i} e^{-i\frac{m_i^2}{2E\nu}L} U_{\alpha i}^*$$

$$\theta_{ij}(Q^2) \equiv \theta_{ij}(Q_0^2) + \beta_{ij} \ln \left(\left| \frac{Q^2}{Q_0^2} \right| \right)$$

$$\delta_D(Q^2) \equiv \delta_D(Q_0^2) + \beta_\delta \ln \left(\left| \frac{Q^2}{Q_0^2} \right| \right)$$

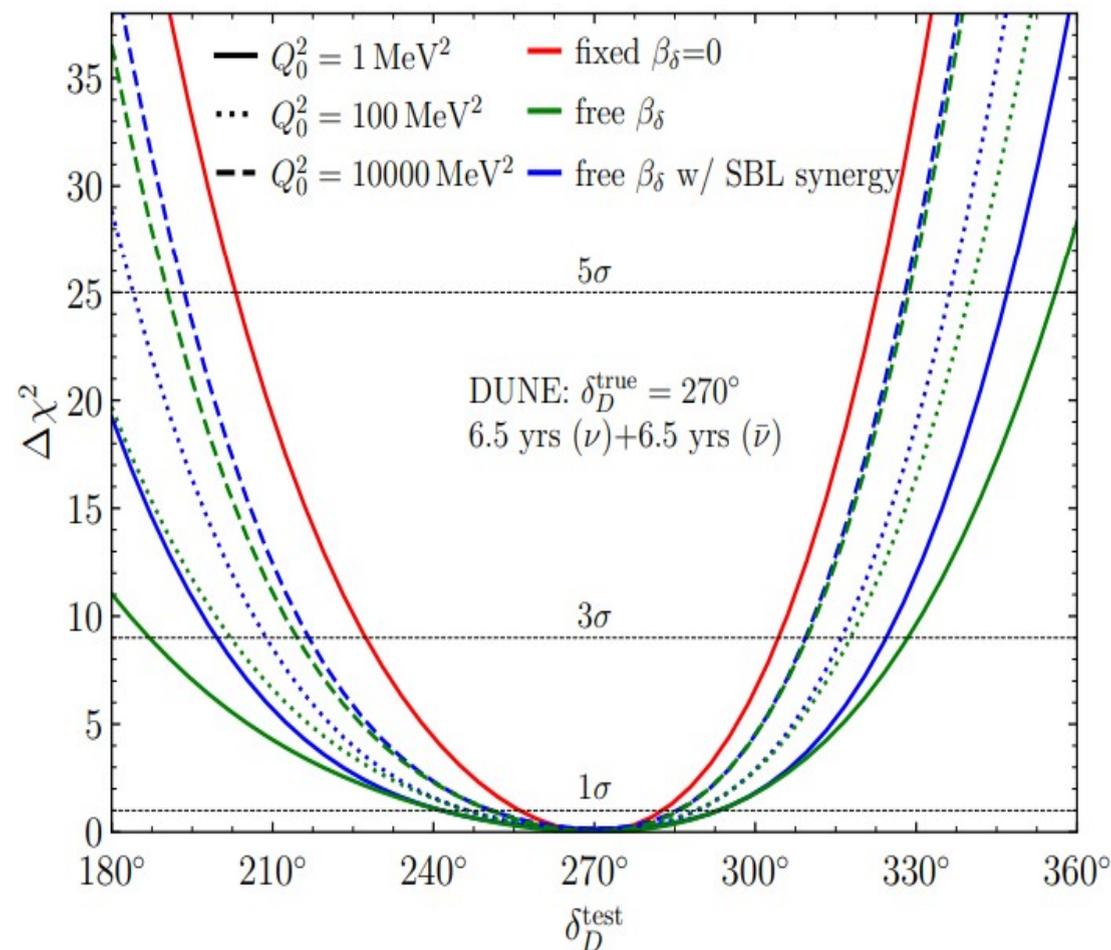
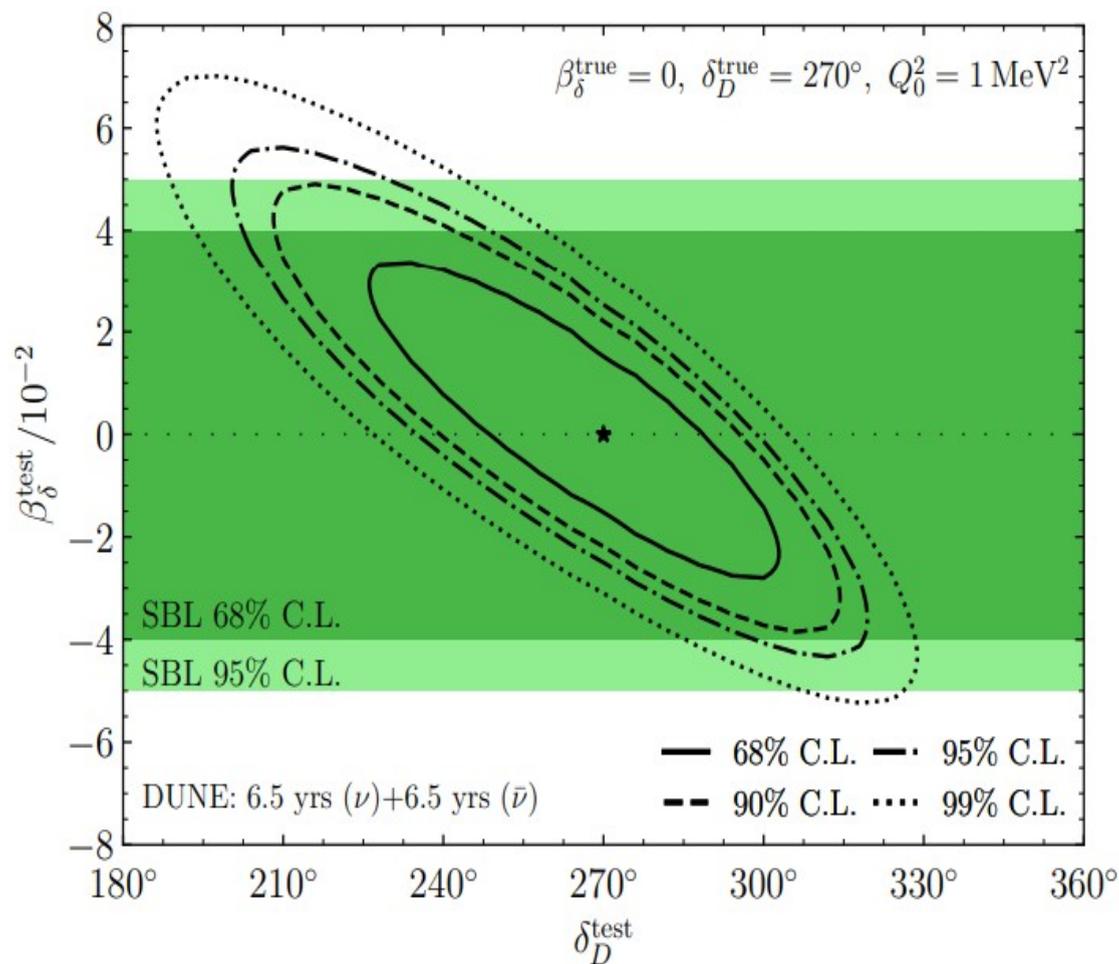
The mixing matrices needs not to be the same!

$$\mathcal{A}_{\beta\alpha}^{\text{OSC}} = U_{\beta i}(Q_d^2) e^{-iL\frac{m_i^2}{2E\nu}} U_{\alpha i}^*(Q_p^2)$$

Degeneracy with CP Phase

$$\mathcal{A}_{\beta\alpha}^{\text{osc}} = U_{\beta i}(Q_d^2) e^{-iL \frac{m_i^2}{2E\nu}} U_{\alpha i}^*(Q_p^2)$$

$$\delta_D(Q^2) \equiv \delta_D(Q_0^2) + \beta_\delta \ln \left(\left| \frac{Q^2}{Q_0^2} \right| \right)$$



SFG, Chui-Fan Kong, Pedro Pasquini [[PRD, 2310.04077](#)]

$$\mathcal{A}_{\beta\alpha}^{\text{osc}} = U_{\beta i}(Q_d^2) e^{-iL \frac{m_i^2}{2E_\nu}} U_{\alpha i}^*(Q_p^2)$$

$$\frac{1}{E_\nu} \ll L \ll \frac{2E_\nu}{\Delta m_{ij}^2}$$

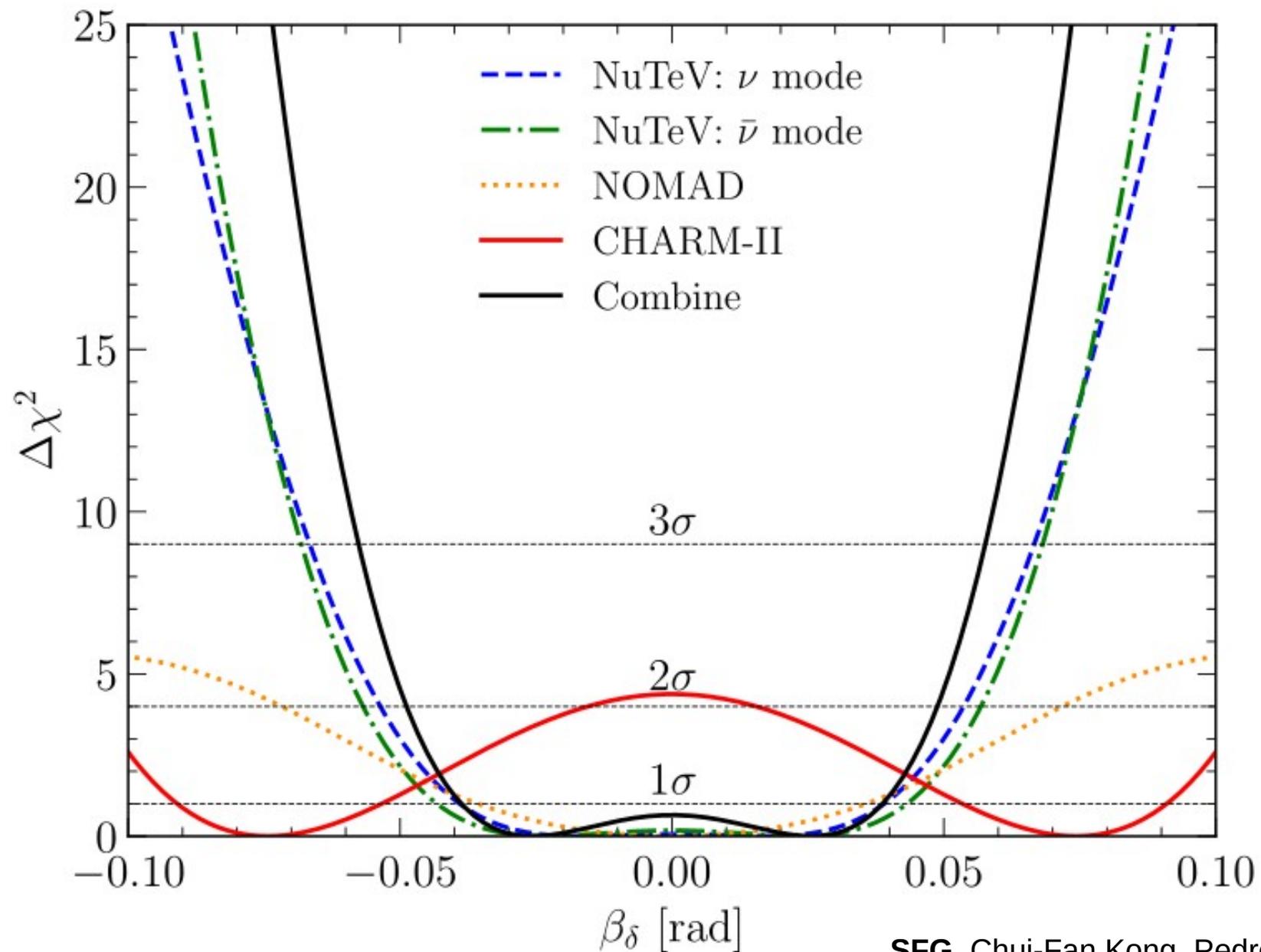
Non-zero transition even in the zero distance limit!

$$P_{\alpha\beta} = \left| [U(Q_d^2)U^\dagger(Q_p^2)]_{\beta\alpha} \right|^2 \neq 0$$

Effective non-unitary mixing!

$$P_{ee}(Q_{d,p}^2) = 1 - \sin^2 \left(\frac{\Delta\delta_D}{2} \right) \sin^2 2\theta_{13} \quad P_{\mu e}(Q_{d,p}^2) = \sin^2 \left(\frac{\Delta\delta_D}{2} \right) s_{23}^2 \sin^2 2\theta_{13}$$

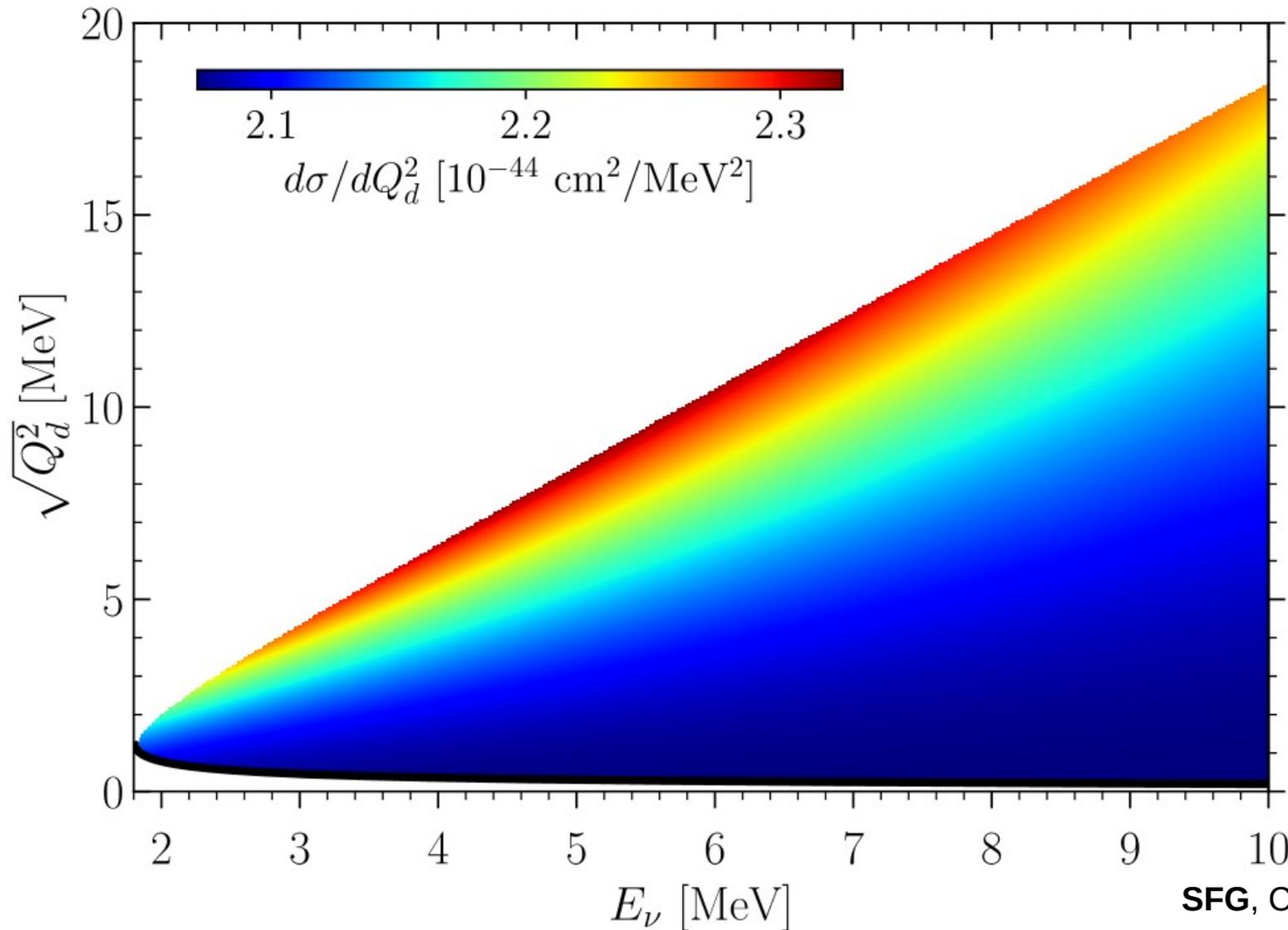
Constraint @ Short-Baseline Experiments



SFG, Chui-Fan Kong, Pedro Pasquini [[PRD, 2310.04077](#)]

$$Q_p^2 \approx 1.67 \text{ MeV}^2$$

$$Q_d^2 \approx \mathcal{O}(10^{-2} \sim 10^2) \text{ MeV}^2$$



- Reactor ν oscillation does not involve CP phase

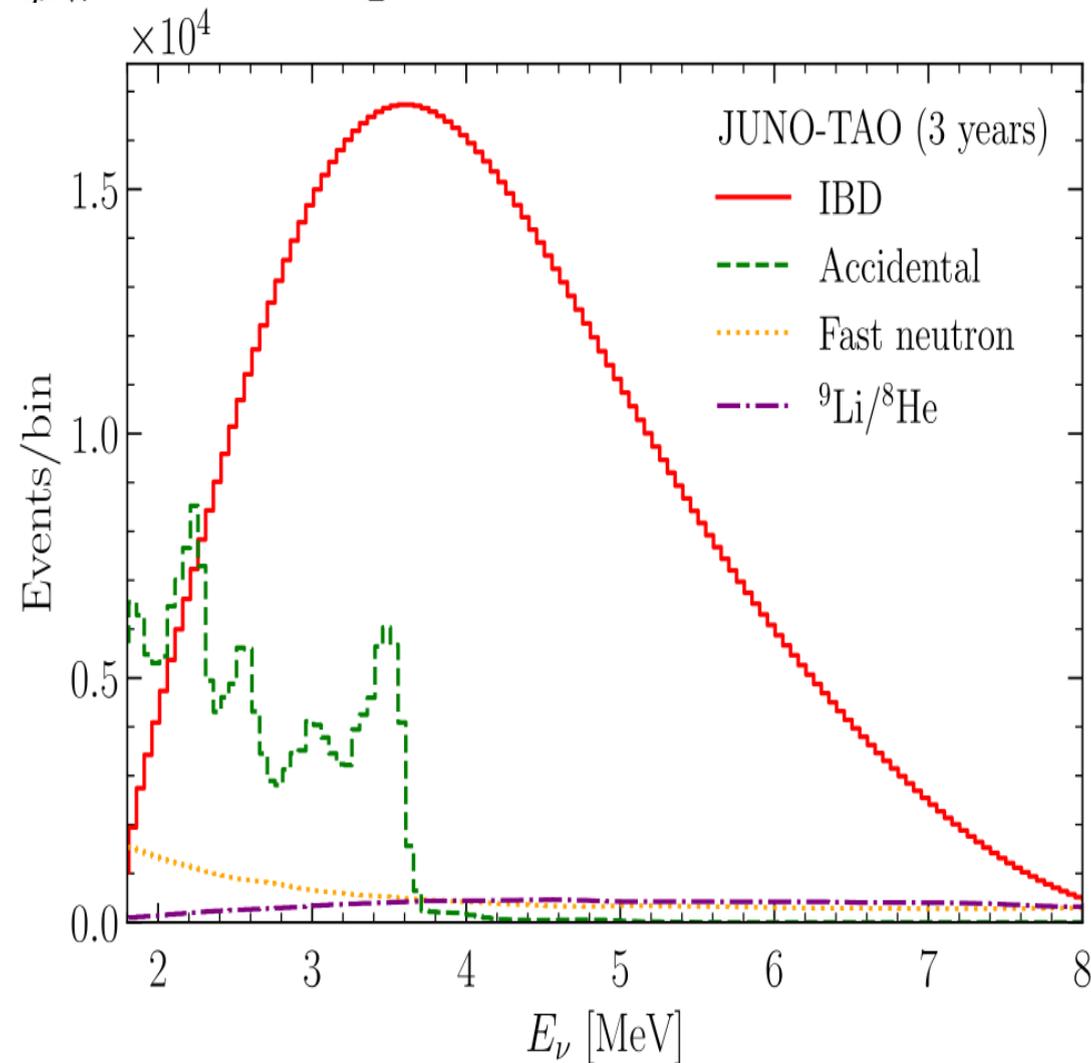
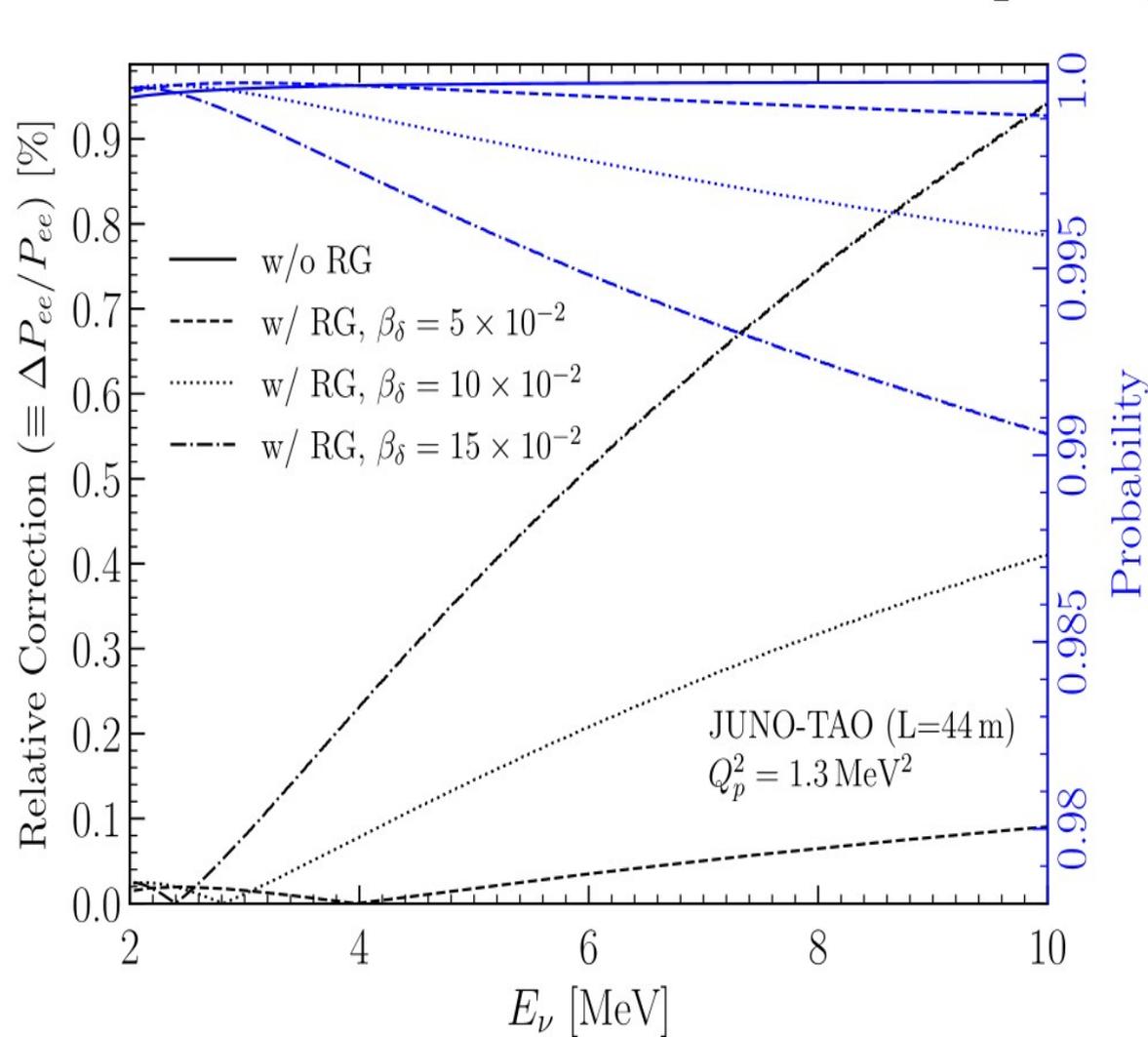
$$P_{ee} = 1 - \cos^4 \theta_{13} \sin^2 2\theta_{12} \sin^2 (\Delta_{21}) - \cos^2 \theta_{12} \sin^2 2\theta_{13} \sin^2 (\Delta_{31}) - \sin^2 \theta_{12} \sin^2 2\theta_{13} \sin^2 (\Delta_{32})$$

- But it can probe the RG running of Dirac CP phase

SFG, Chui-Fan Kong, Pedro Pasquini [PRD, 2411.18251]

CP RG Running for Reactor ν Osc

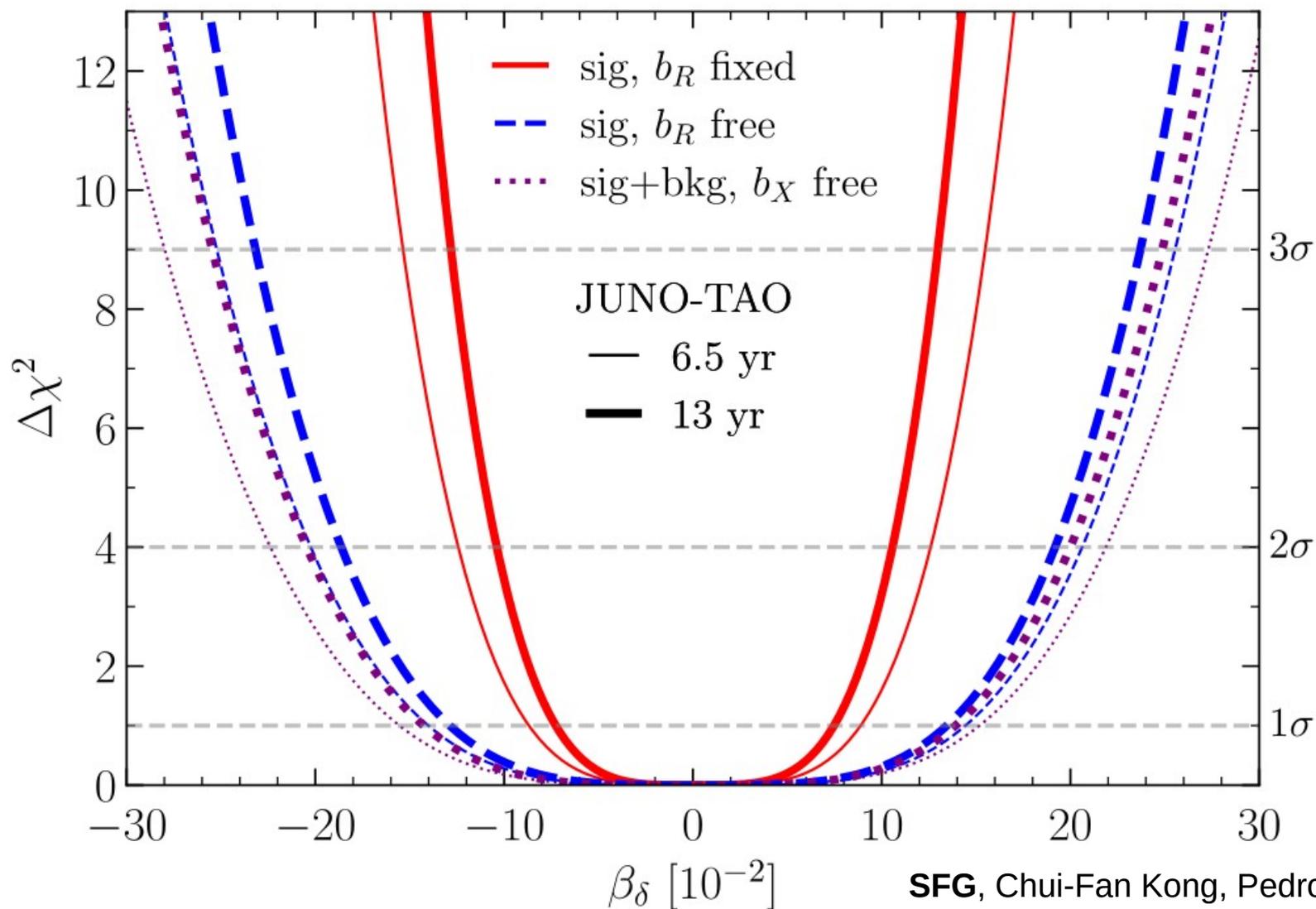
$$1 - P_{ee}(Q_{d,p}^2) \approx \left[\frac{1}{2} \ln \left(\left| \frac{Q_d^2}{Q_p^2} \right| \right) \sin 2\theta_{13} \beta_\delta \right]^2$$



SFG, Chui-Fan Kong, Pedro Pasquini [PRD, 2411.18251]

Zero Distance Effect

$$\bar{P}_{ee}(E_\nu) \equiv \int P_{ee}(Q_{p,d}^2) \frac{1}{\sigma} \frac{d\sigma}{dQ_d^2} dQ_d^2$$



1) ν Oscillation & Leptonic CP Violation

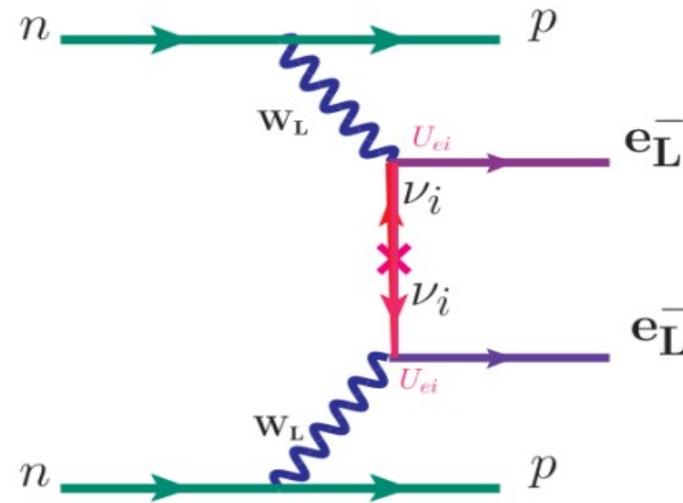
2) Dirac CP Phase

3) Majorana CP Phases

4) Summary

Neutrinoless Double Beta Decay

- Mediated by **Majorana Neutrino**



Lepton # Violation!

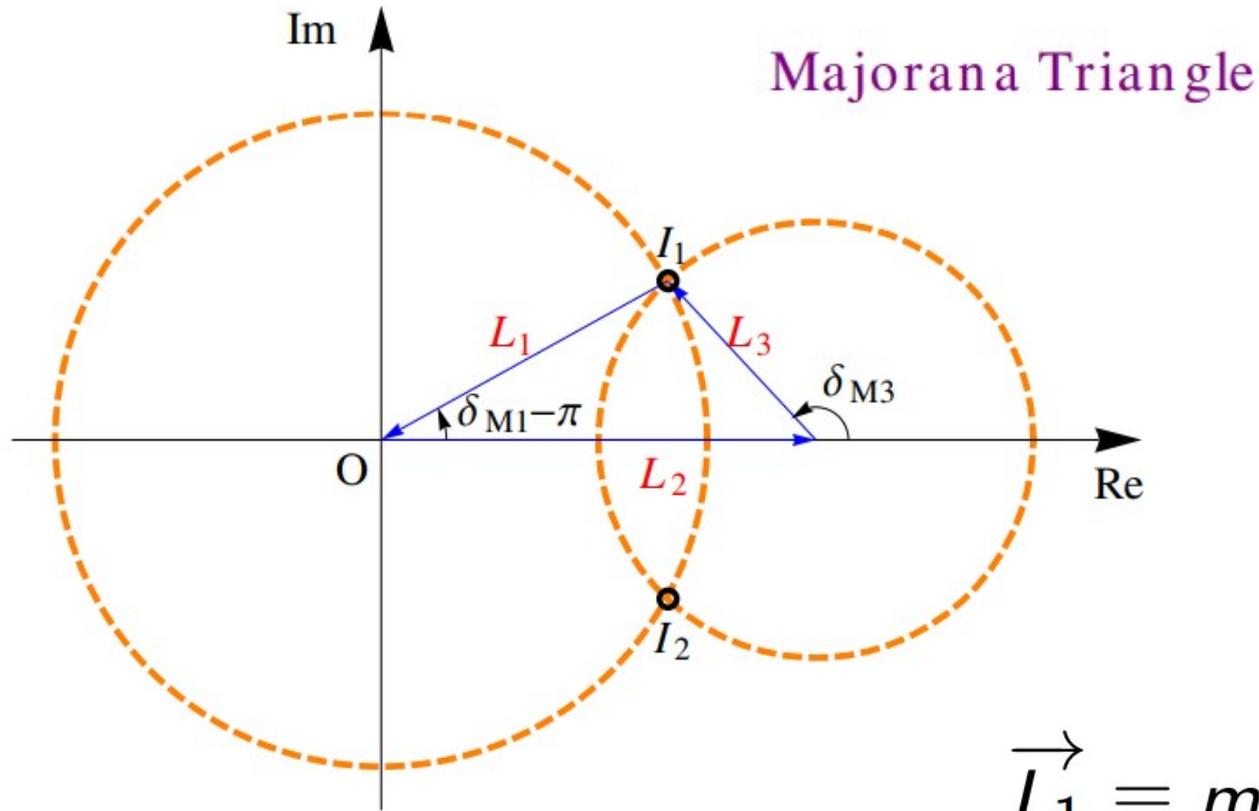
- Mass Suppression**

$$\mathcal{M} \propto \sum_i U_{ei} \frac{i}{p \not{\partial} - m_i} U_{ei} \approx \sum_i U_{ei} \frac{m_i}{p^2} U_{ei}$$

- Effective Mass**

$$\langle m \rangle_{ee} \equiv \left| \sum_i m_i U_{ei}^2 \right| = \left| c_s^2 c_r^2 m_1 e^{i\delta_{M1}} + s_s^2 c_r^2 m_2 + s_r^2 m_3 e^{i\delta_{M3}} \right|$$

Majorana Triangle



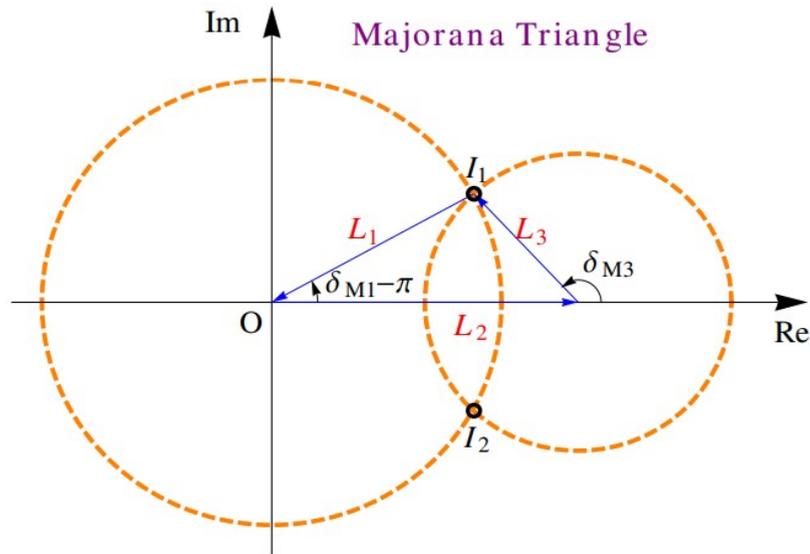
$$\langle m \rangle_{ee} \equiv \vec{L}_1 + \vec{L}_2 + \vec{L}_3 = 0$$

SFG & Manfred Lindner, PRD 95 (2017) No.3, 033003 [[1608.01618](#)]

Xing & Zhou, Chin.Phys.C 39 (2015) [[1404.7001](#)]

$$\begin{aligned} \vec{L}_1 &\equiv m_1 U_{e1}^2 = m_1 c_r^2 c_s^2 e^{i\delta_{M1}}, \\ \vec{L}_2 &\equiv m_2 U_{e2}^2 = \sqrt{m_1^2 + \Delta m_s^2} c_r^2 s_s^2, \\ \vec{L}_3 &\equiv m_3 U_{e3}^2 = \sqrt{m_1^2 + \Delta m_a^2} s_r^2 e^{i\delta_{M3}}. \end{aligned}$$

Catching 2 Majorana CP Phases



$$\cos \delta_{M1} = - \frac{m_1^2 c_r^4 c_s^4 + m_2^2 c_r^4 s_s^4 - m_3^2 s_r^4}{2m_1 m_2 c_r^4 c_s^2 s_s^2}$$

$$\cos \delta_{M3} = + \frac{m_1^2 c_r^4 c_s^4 - m_2^2 c_r^4 s_s^4 - m_3^2 s_r^4}{2m_2 m_3 c_r^2 s_r^2 s_s^2}$$

- Nonzero \rightarrow only 1 dof

$$|\mathbf{m}_{ee}| = \mathbf{f}$$

- Zero \rightarrow 2 Majorana CP phases

$$|\mathbf{m}_{ee}| = 0 \Rightarrow \mathbb{R}(\mathbf{m}_{ee}) = \mathbb{I}(\mathbf{m}_{ee}) = 0$$

$$|\mathbf{m}_{ee}| < \mathbf{f} \Rightarrow \mathbb{R}(\mathbf{m}_{ee}) < \mathbf{f} \quad \mathbb{I}(\mathbf{m}_{ee}) < \mathbf{f}$$

Xing, Zhao & Zhou [[1504.05820](#)]

Xing & Zhao [[1612.08538](#)]

Cao, Huang, Li, Wang, Wen, Xing, Zhao & Zhou [[1908.08355](#)]

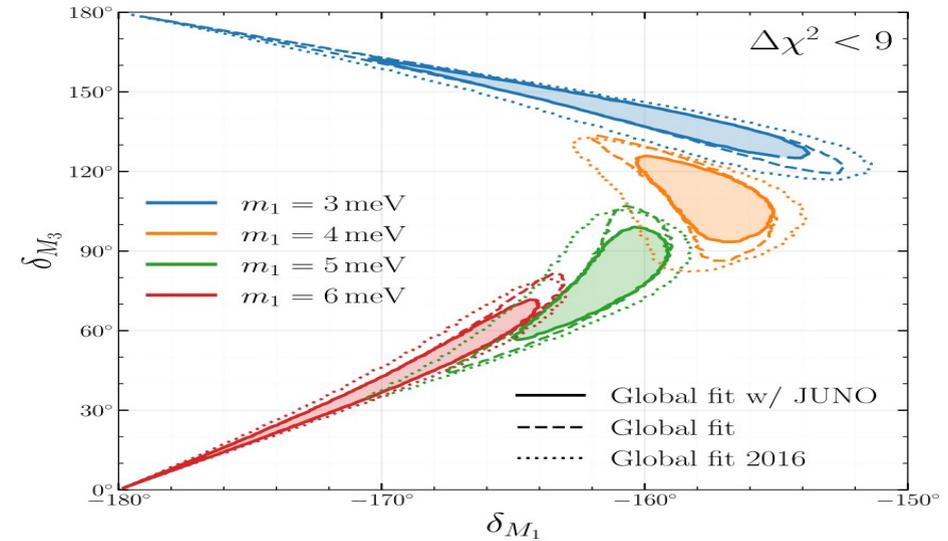
SFG & Manfred Lindner, [[1608.01618](#)]

SFG, Chui-Fan Kong, Manfred Lindner, Joao Pinheiro [[2509.14534](#)]

JUNO for Catching 2 Majorana CP Phases

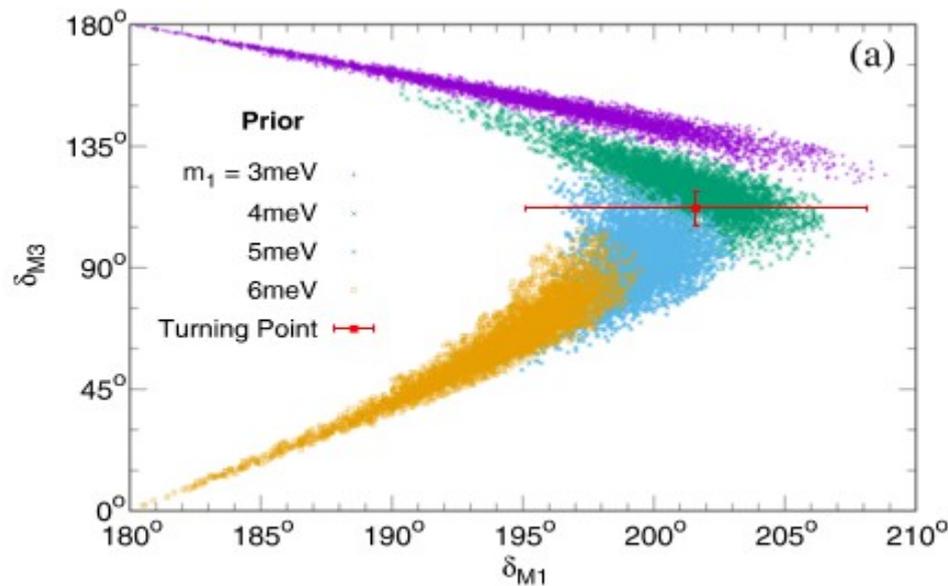
$$\cos \delta_{M1} = - \frac{m_1^2 c_r^4 c_s^4 + m_2^2 c_r^4 s_s^4 - m_3^2 s_r^4}{2m_1 m_2 c_r^4 c_s^2 s_s^2}$$

$$\cos \delta_{M3} = + \frac{m_1^2 c_r^4 c_s^4 - m_2^2 c_r^4 s_s^4 - m_3^2 s_r^4}{2m_2 m_3 c_r^2 s_r^2 s_s^2}$$

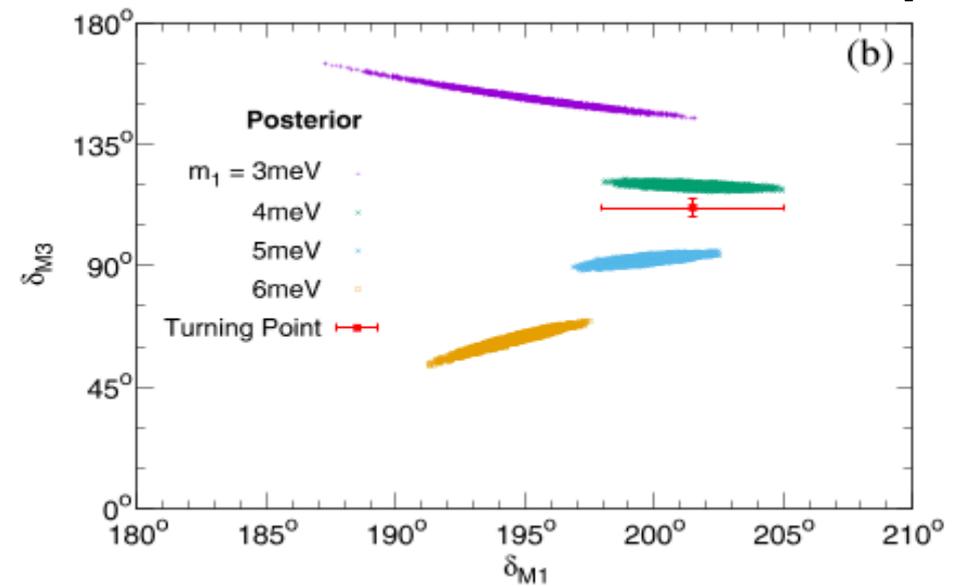


SFG, Chui-Fan Kong, Manfred Lindner, Joao Pinheiro
[2509.14534]

- JUNO can significantly reduce uncertainties

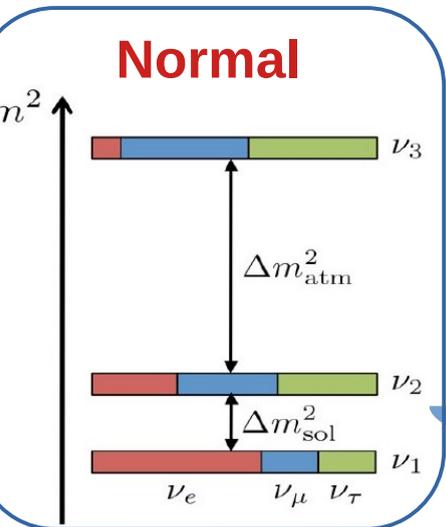
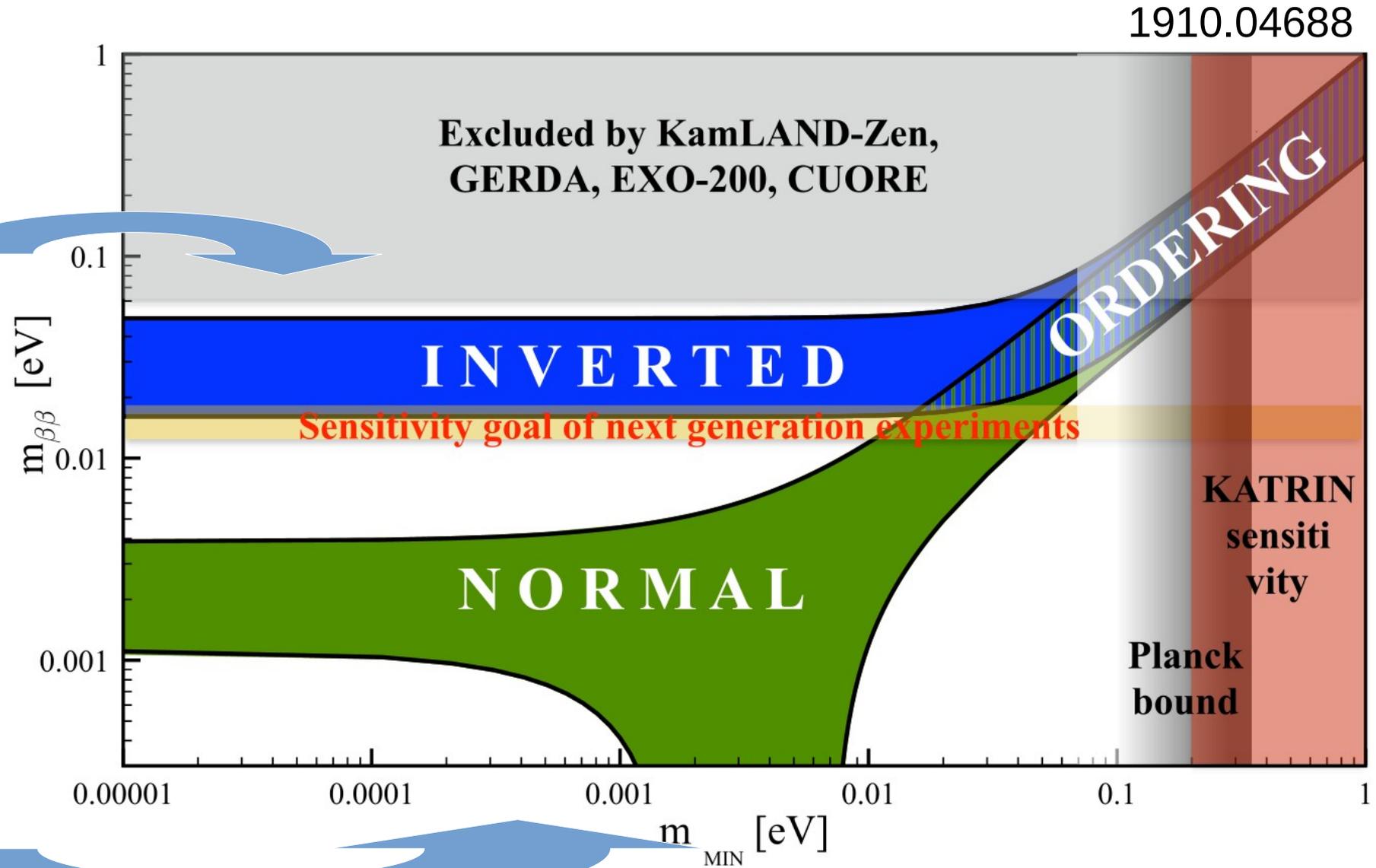
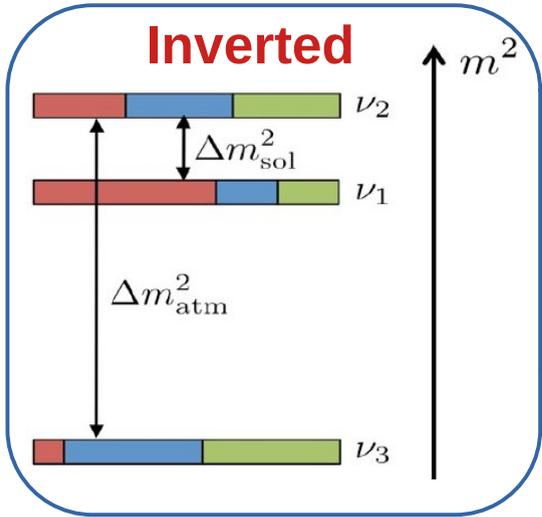


SFG & Manfred Lindner, [1608.01618]



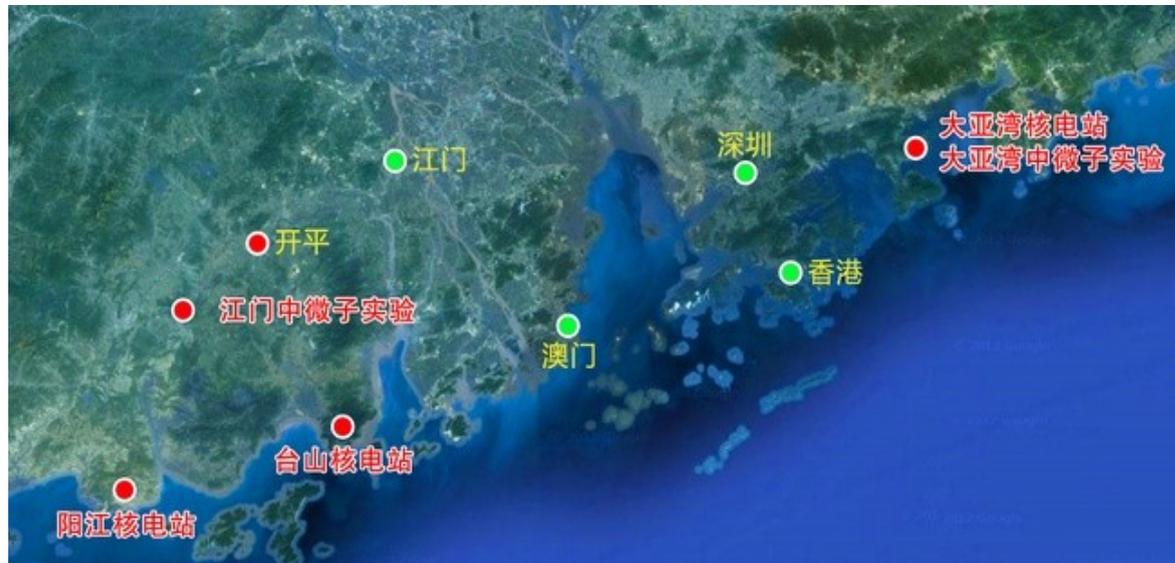
see also SFG & Rodejohann [1507.05514]

Normal vs Inverted

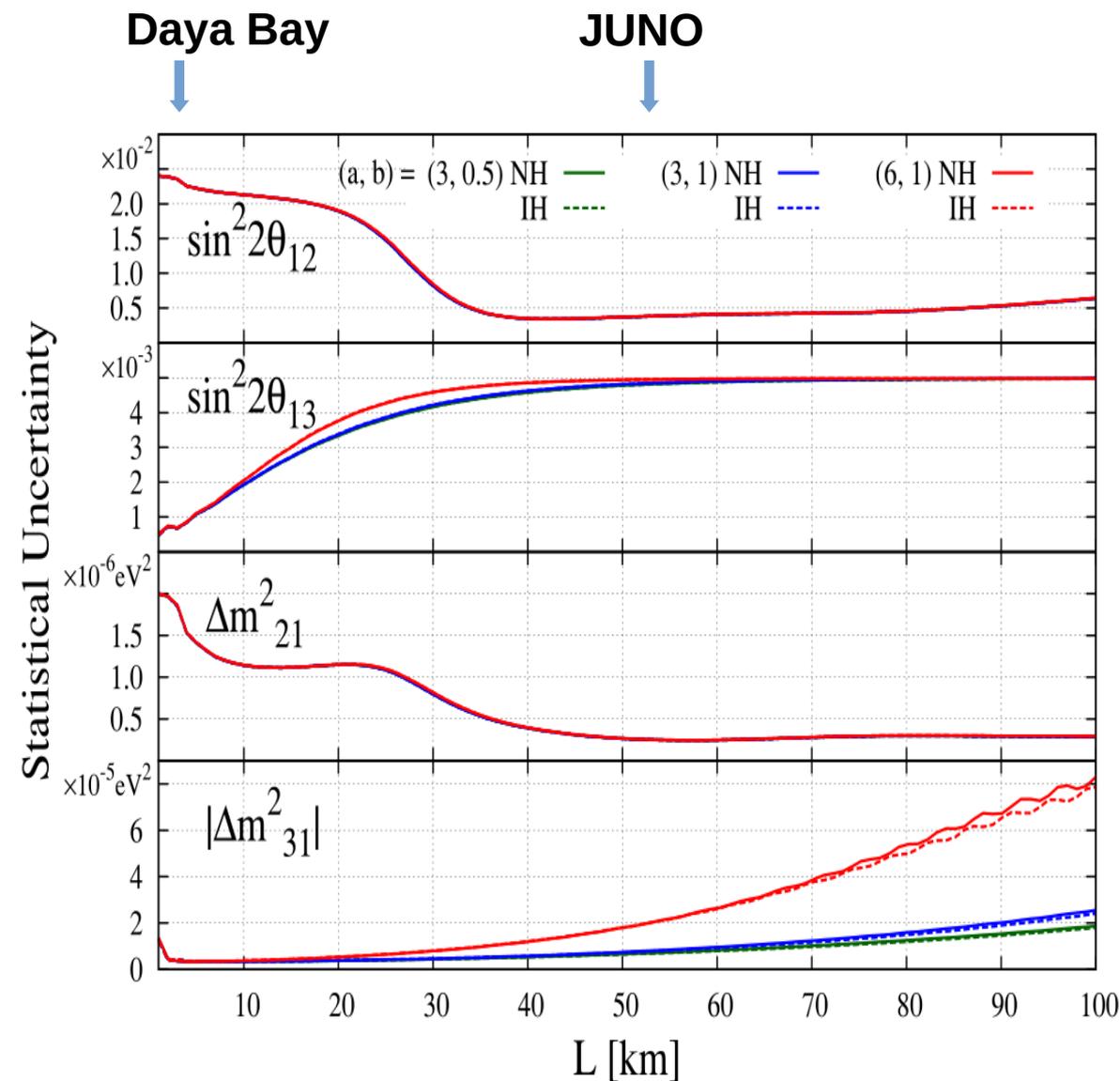
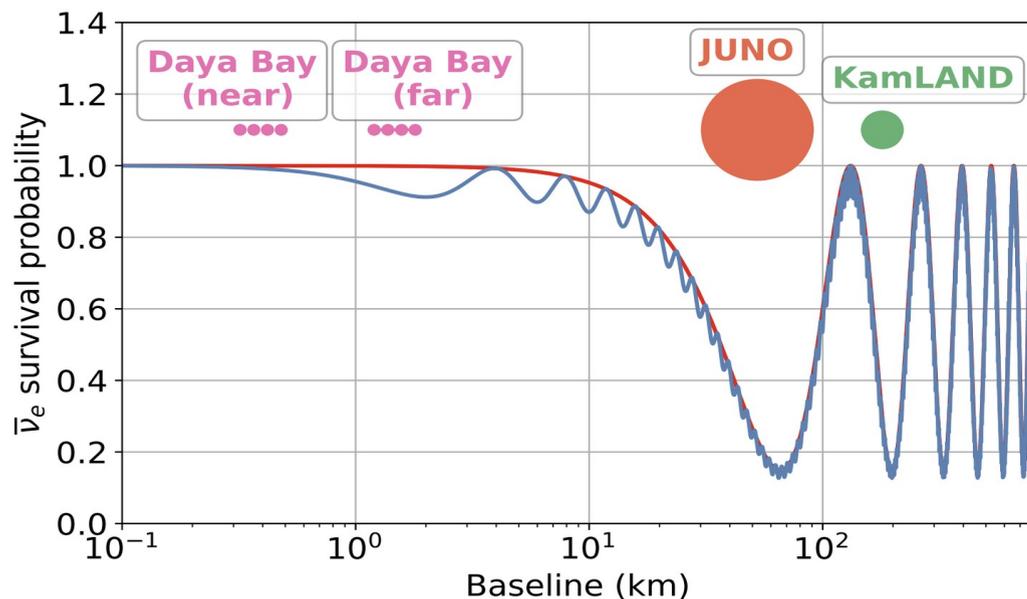


What else?

Daya Bay + JUNO → Precision Era



$$P_{ee} \approx 1 - \sin^2(2\theta_r) \sin^2\left(\frac{\Delta m_a^2 L}{4E_\nu}\right) - \sin^2(2\theta_s) \sin^2\left(\frac{\Delta m_s^2 L}{4E_\nu}\right)$$



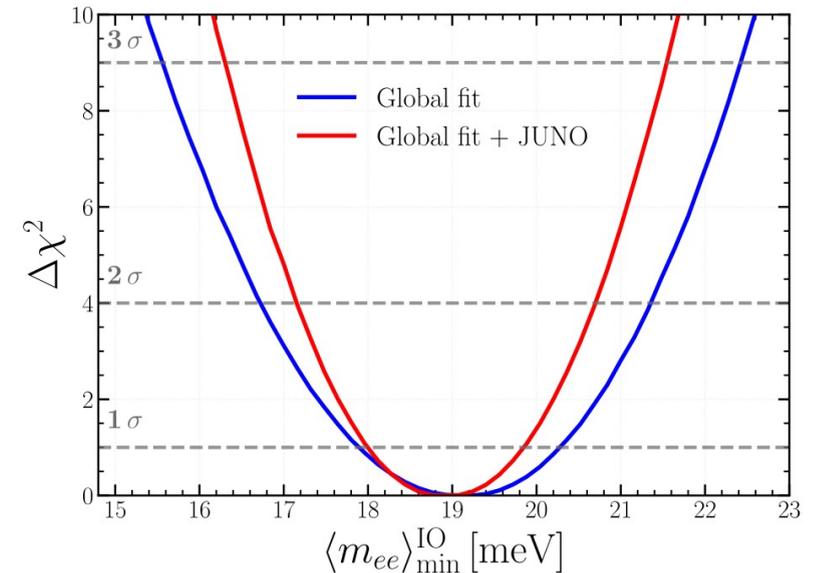
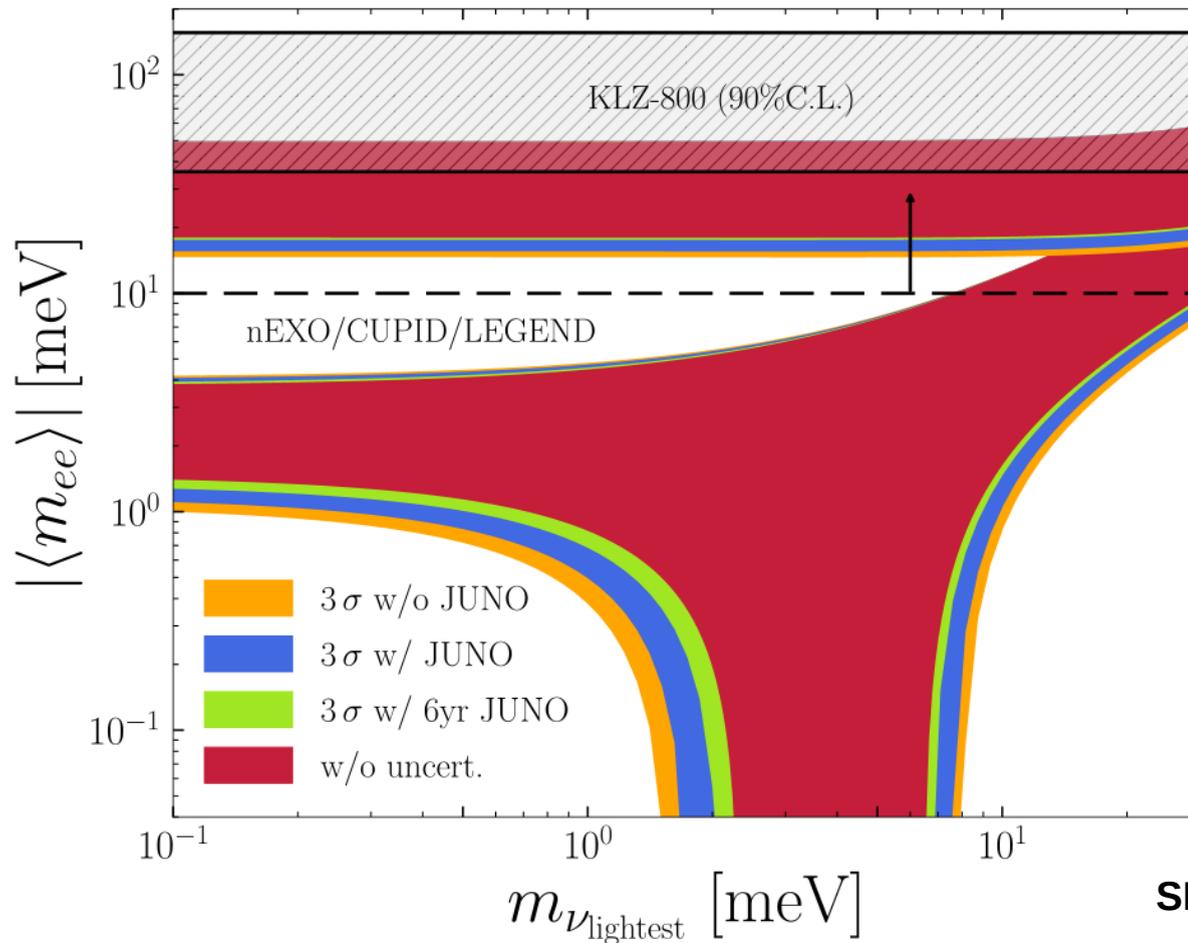
SFG, Hagiwara, Okamura & Takaesu [[1210.8141](#)]

$$\left(T_{1/2}^{0\nu}\right)^{-1} \equiv \underbrace{G^{0\nu}}_{\text{Atomic}} \underbrace{|M^{0\nu}|^2}_{\text{Nuclear}} \underbrace{|\langle m_{ee} \rangle|^2}_{\text{Particle}}$$

$$T_{1/2}^{0\nu} \propto \frac{1}{|\langle m_{ee} \rangle|^2}$$

$$M \times t \propto \left(T_{1/2}^{0\nu}\right)^2 \propto \frac{1}{|\langle m_{ee} \rangle|^4}$$

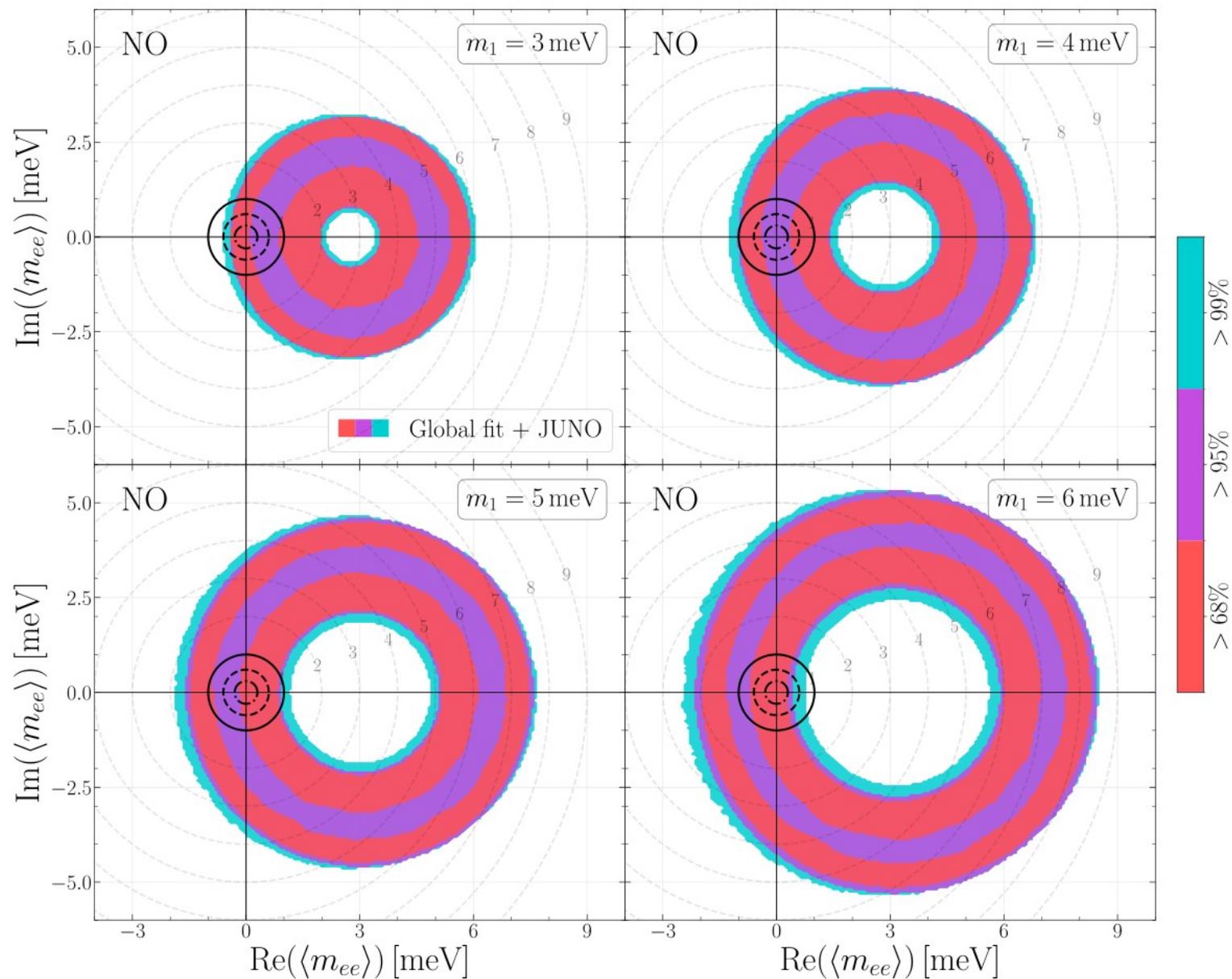
Vary by 4.3 for IO



SFG, Chui-Fan Kong, Manfred Lindner, Joao Pinheiro [2509.14534]

Uncertainties by particle physics can be eliminated!

Possibility of Vanishing m_{ee}



SFG, Chui-Fan Kong, Manfred Lindner, Joao Pinheiro [2509.14534]

Prey of Leptonic CP Phases

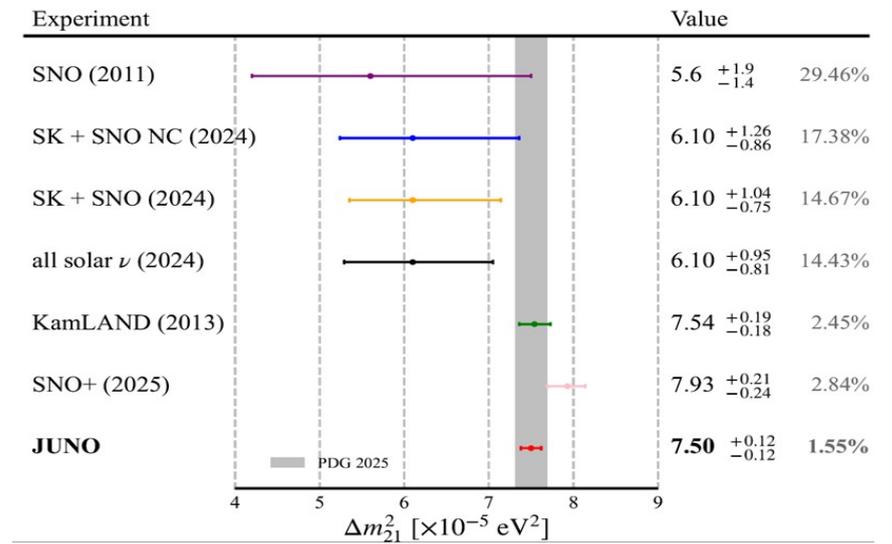
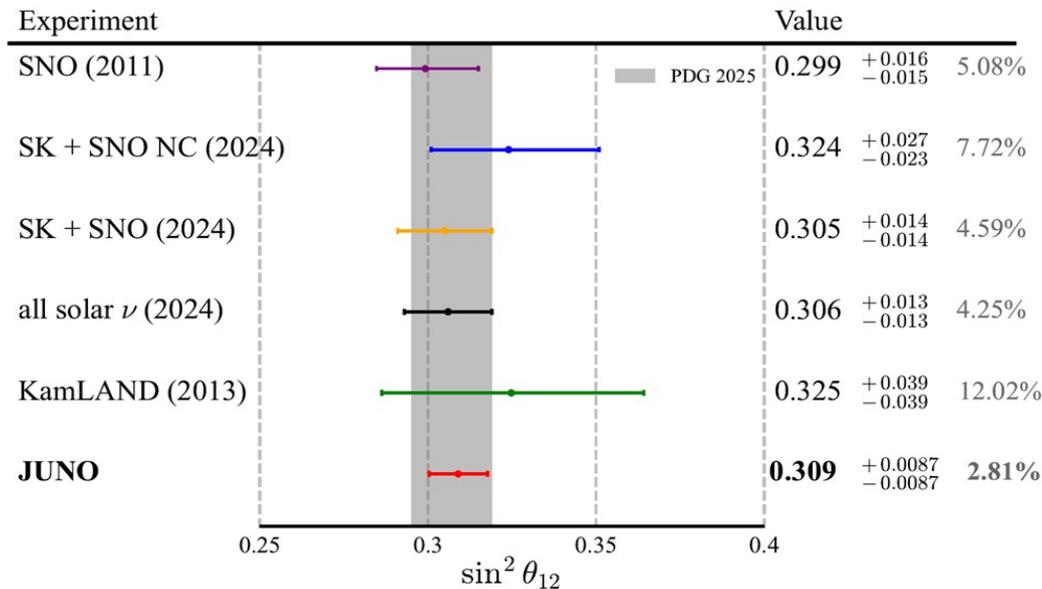
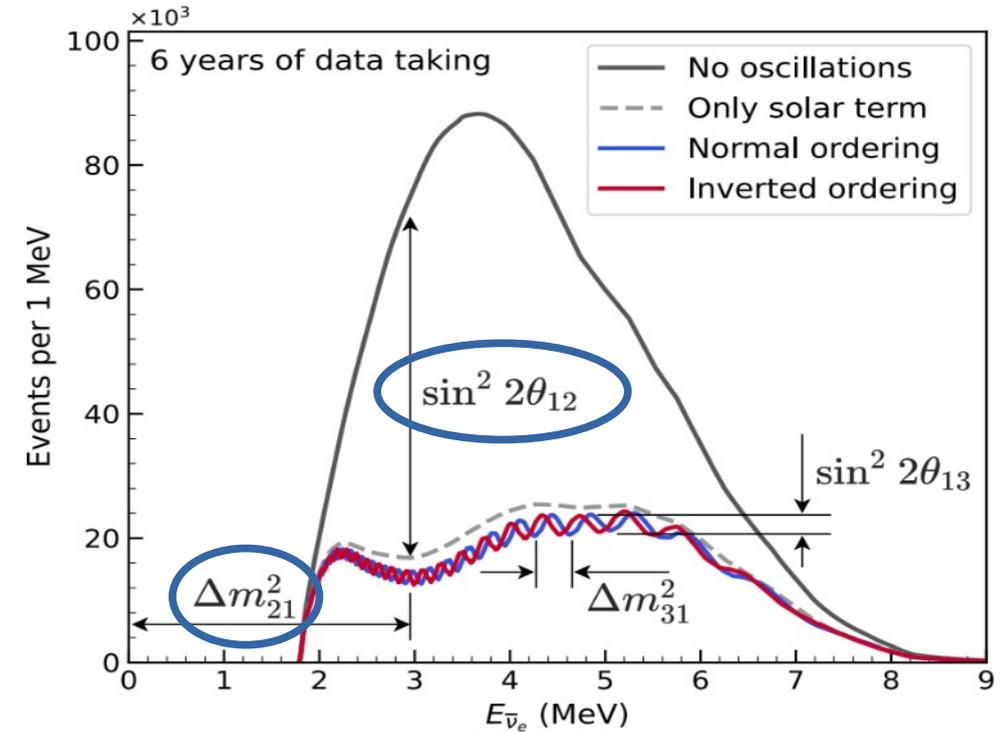


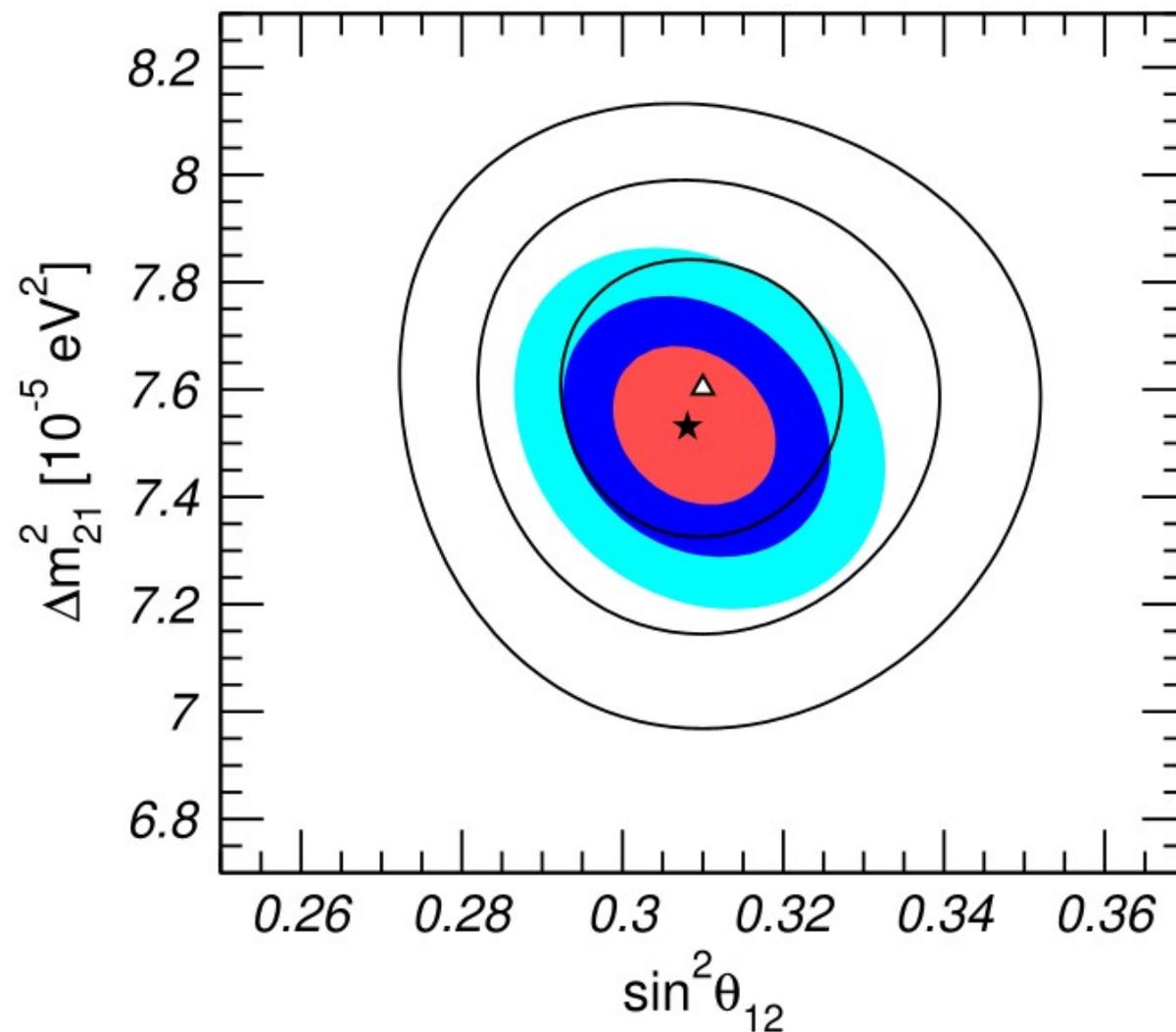
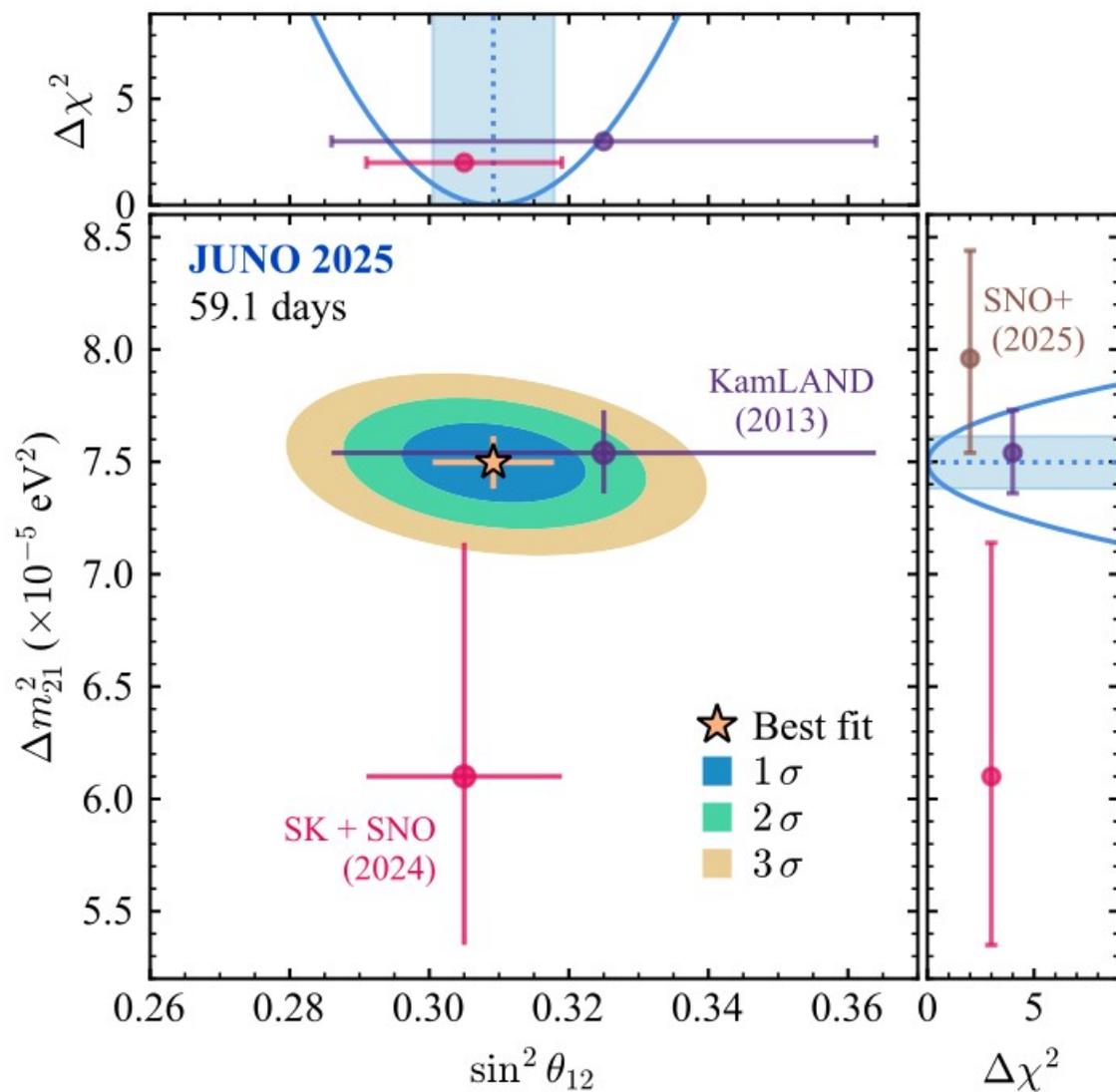


李政道研究所
Tsung-Dao Lee Institute

Thank You

JUNO Experiment & 1st Data





JUNO Collaboration [[2511.14593](#)]

NuFit [[2601.09791](#)]

- **Two Nontrivial Independent** possibilities of \mathbf{d}_ν :

$$\mathbf{d}_\nu^{(1)} = \text{diag}(-1, 1, 1), \quad \mathbf{d}_\nu^{(2)} = \text{diag}(1, -1, 1), \quad \mathbf{d}_\nu^{(3)} = -\mathbf{d}_\nu^{(1)} \mathbf{d}_\nu^{(2)}.$$

- θ_s parameterized in terms of \mathbf{k} : $\tan \theta_s = \sqrt{2}/k$

$$V_\nu(k) = \begin{pmatrix} \frac{k}{\sqrt{2+k^2}} & \frac{\sqrt{2}}{\sqrt{2+k^2}} & 0 \\ \frac{1}{\sqrt{2+k^2}} & \frac{k}{\sqrt{2(2+k^2)}} & \frac{-1}{\sqrt{2}} \\ \frac{1}{\sqrt{2+k^2}} & \frac{k}{\sqrt{2(2+k^2)}} & \frac{1}{\sqrt{2}} \end{pmatrix} \quad \begin{array}{ll} \mathbf{k} = 2 & \theta_s = 35.3^\circ \text{ [TBM]} \\ \mathbf{k} = \frac{3}{\sqrt{2}} & \theta_s = 33.7^\circ \\ \mathbf{k} = \sqrt{6} & \theta_s = 30.0^\circ \end{array}$$

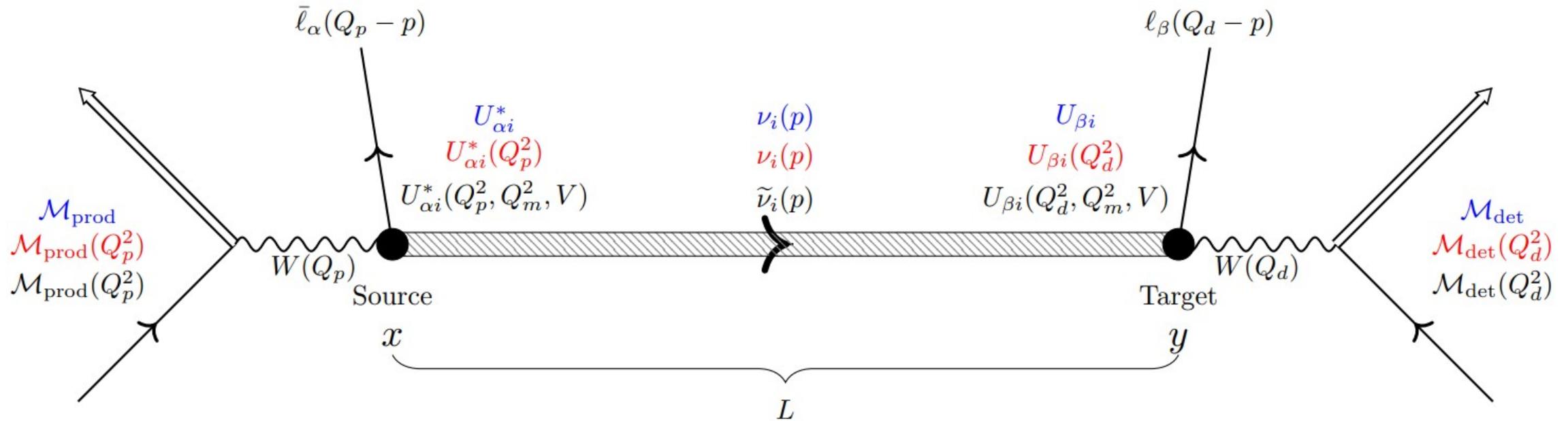
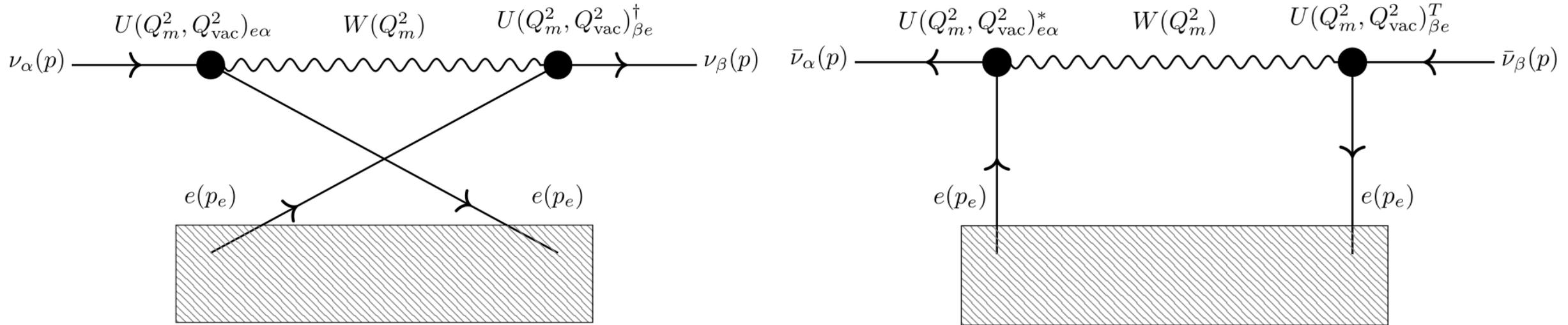
- **Two Independent Symmetry Transformations** $\mathbf{G}_i = \mathbf{V}_\nu \mathbf{d}_\nu^{(i)} \mathbf{V}_\nu^\dagger$

$$\mathbf{G}_1 = \frac{1}{2+k^2} \begin{pmatrix} 2-k^2 & 2k & 2k \\ & k^2 & -2 \\ & & k^2 \end{pmatrix}, \quad \mathbf{G}_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

- $\mathbb{Z}_2^S (\times \overline{\mathbb{Z}}_2^S) \times \mathbb{Z}_2^{\mu\tau} \equiv \mathcal{G} = \{\mathbf{E}, \mathbf{G}_1, \mathbf{G}_2 (\equiv \mathbf{G}_1 \mathbf{G}_3), \mathbf{G}_3\}$

Lam, PRL101:121602(2008), PRD78:073015(2008)

Momentum Transfer in Matter Effect



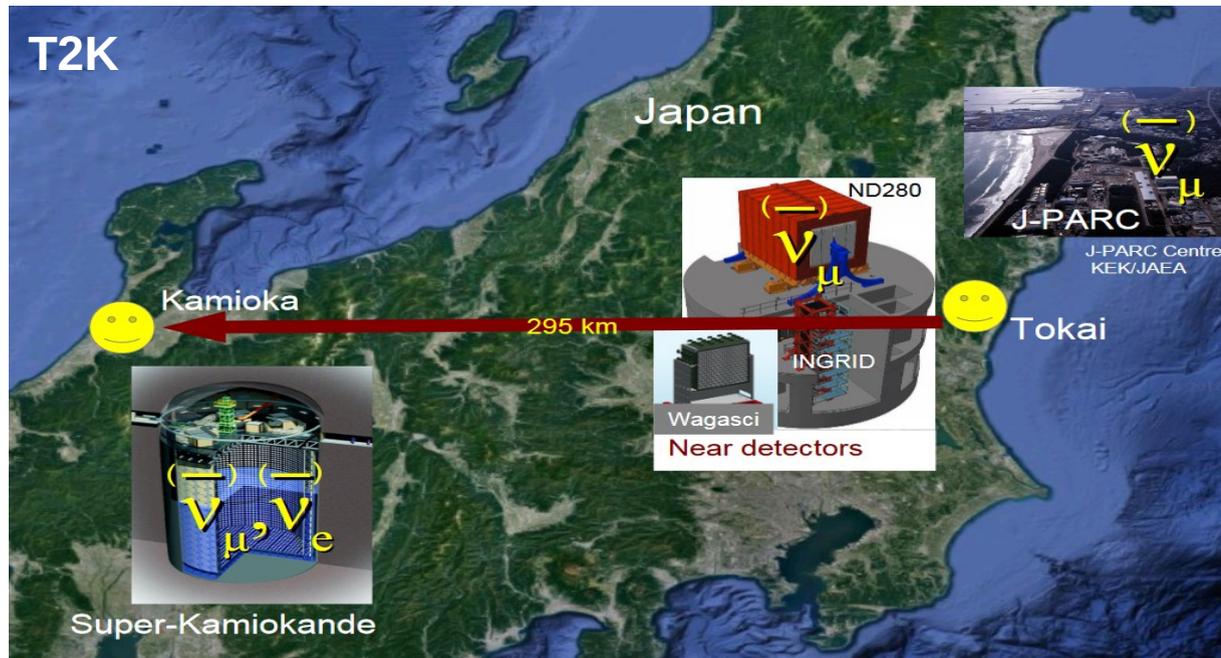
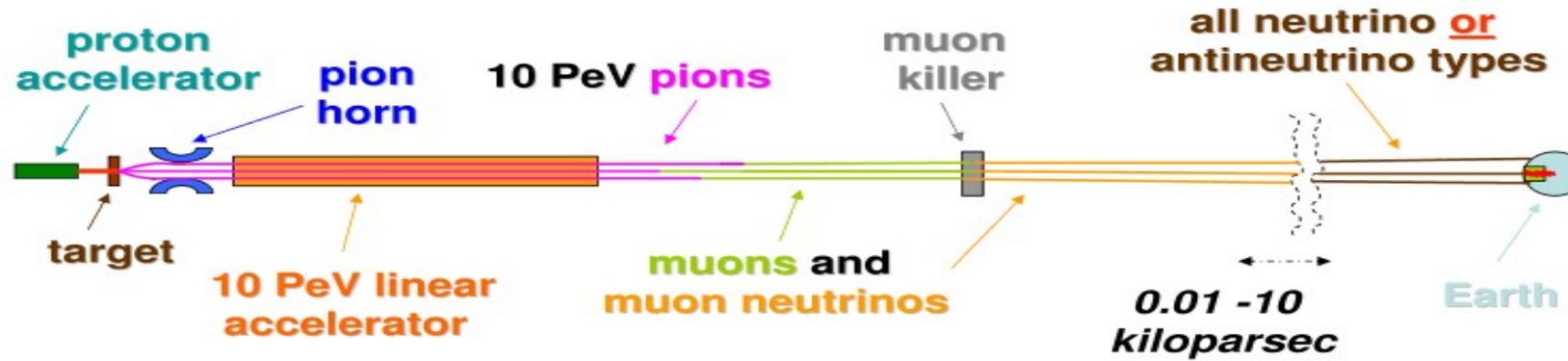
blue : vacuum w/o RG running

red : vacuum w/ RG running

black : w/ matter effect & RG running

Accelerator ν Experiments

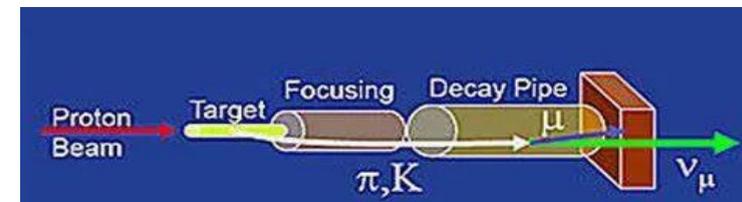
Pion Accelerator Neutrino Beam Concept



K. Engman, Science 345, 6204

PHYSICAL REVIEW LETTERS

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Feasibility of Using High-Energy Neutrinos to Study the Weak Interactions

M. Schwartz
Phys. Rev. Lett. **4**, 306 – Published 15 March 1960

Theoretical Discussions on Possible High-Energy Neutrino Experiments

T. D. Lee and C. N. Yang
Phys. Rev. Lett. **4**, 307 – Published 15 March 1960

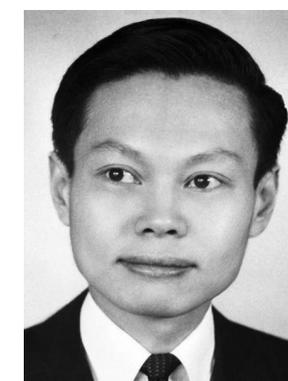
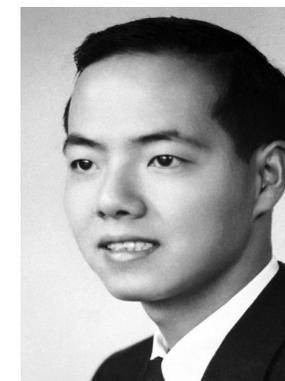
历史印记——60年前高能加速器中微子实验原理诞生

🕒 2021-01-04 👁 50

作者：葛韶锋

这项60年前的工作，今天还在为粒子物理研究指引着前进的方向

中微子是粒子物理标准模型中非常独特而重要的基本粒子。中微子只参与弱相互作用，能够穿透几光年的铅板，是非常难以探测的鬼魅粒子。然而，恰恰是中微子率先给出了超出标准模型的第一个新物理现象——中微子振荡——这已经得到大量实验的证实，并于2015年获诺贝尔物理学奖，成为指引超出标准模型新物理研究的重要线索。在上世纪五六十年代粒子物理发展的黄金期，施瓦兹、李政道与杨振宁提出的高能加速器中微子实验原理，是一个非常革命性的思想，一直在有力地推动着包括中微子在内的粒子物理不断向前发展。



何小刚，李政道先生和现代中微子物理，《现代物理知识杂志》2021

<https://tdli.sjtu.edu.cn/EN/customize/436?columnId=35>

*** TD Lee 1926-2024

<https://www.nature.com/articles/d41586-024-02585-1> Among his many other accomplishments, T.D. Lee is given credit for the idea of neutrino beams. Following a suggestion in the theory paper by Lee and Yang, Lederman, Schwartz, and Steinberger verified that there are 2 different neutrinos in 1962 and won their Nobel Prize in 1988. In his Nobel Lecture, Schwartz recalled: “The Columbia University Physics Department had a tradition of a coffee hour At one of these Professor T. D. Lee was leading such a discussion of the possibilities for investigating weak interactions at high energies. That evening the key notion came to me I called T. D. Lee at home with the news and his enthusiasm was overwhelming. The next day planning for the experiment began in earnest”. Schwartz also gratefully mentioned in Nobel Prize webpage: “T.D. Lee was the inspirer of this experiment”. And in Lederman's Nobel Banquet Speech: “We are also agreed that we owe much to many others, and I would like to mention Professor T. D. Lee, our Columbia colleague, for his guidance and inspiration.”

Long-Baseline Neutrino Oscillation Newsletters

<https://www.hep.anl.gov/ndk/longbnews/>

Maintained by Maury Goodman

- Oscillation probabilities @ Accelerator Neutrino Exps

$$P_{\nu_\mu \rightarrow \nu_e} \approx 4s_a^2 c_r^2 s_r^2 \sin^2 \phi_{31} - 8c_a s_a c_r^2 s_r c_s s_s \sin \phi_{21} \sin \phi_{31} [\cos \delta_D \cos \phi_{31} \pm \sin \delta_D \sin \phi_{31}]$$

$$P_{\bar{\nu}_\mu \rightarrow \bar{\nu}_e}$$

for ν & $\bar{\nu}$, respectively. $[\phi_{ij} \equiv \frac{\delta m_{ij}^2 L}{4E_\nu}]$

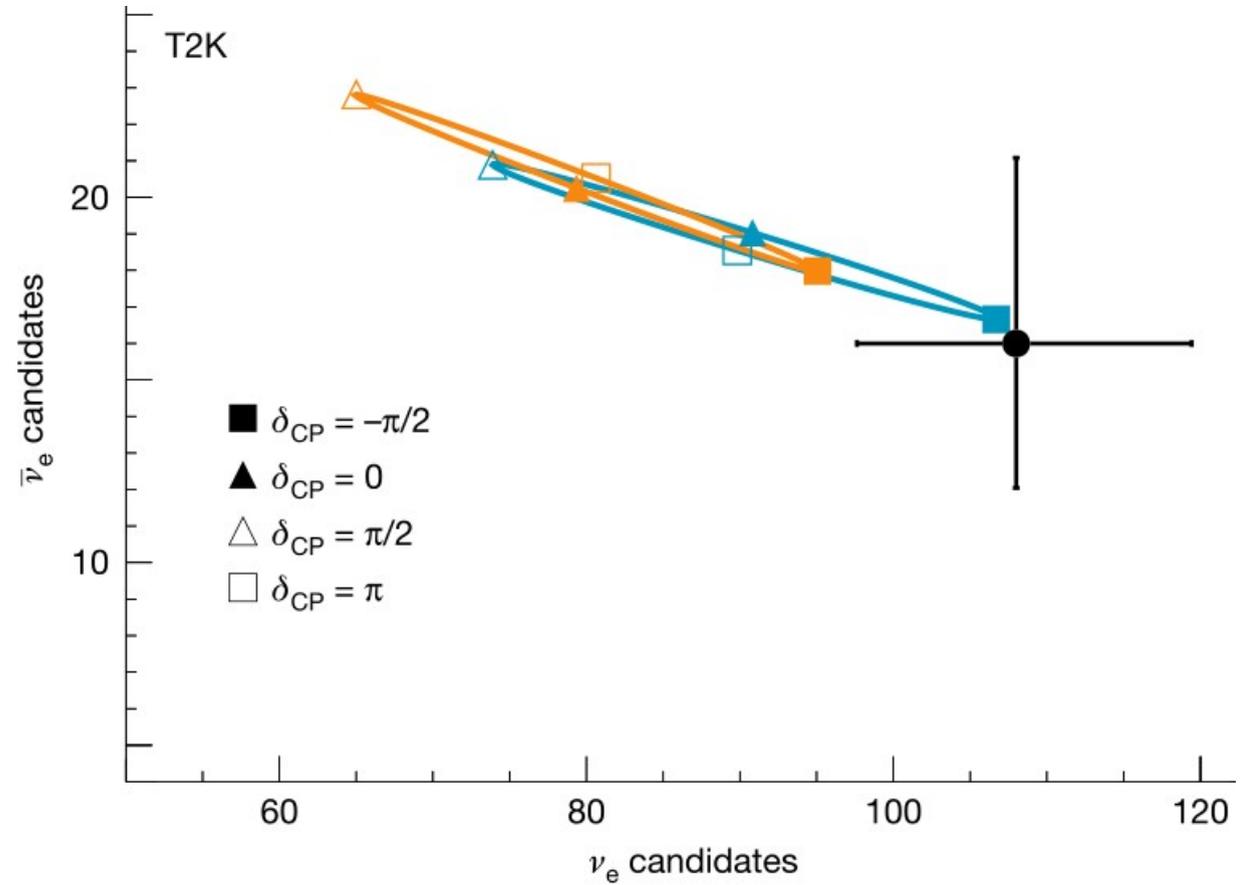
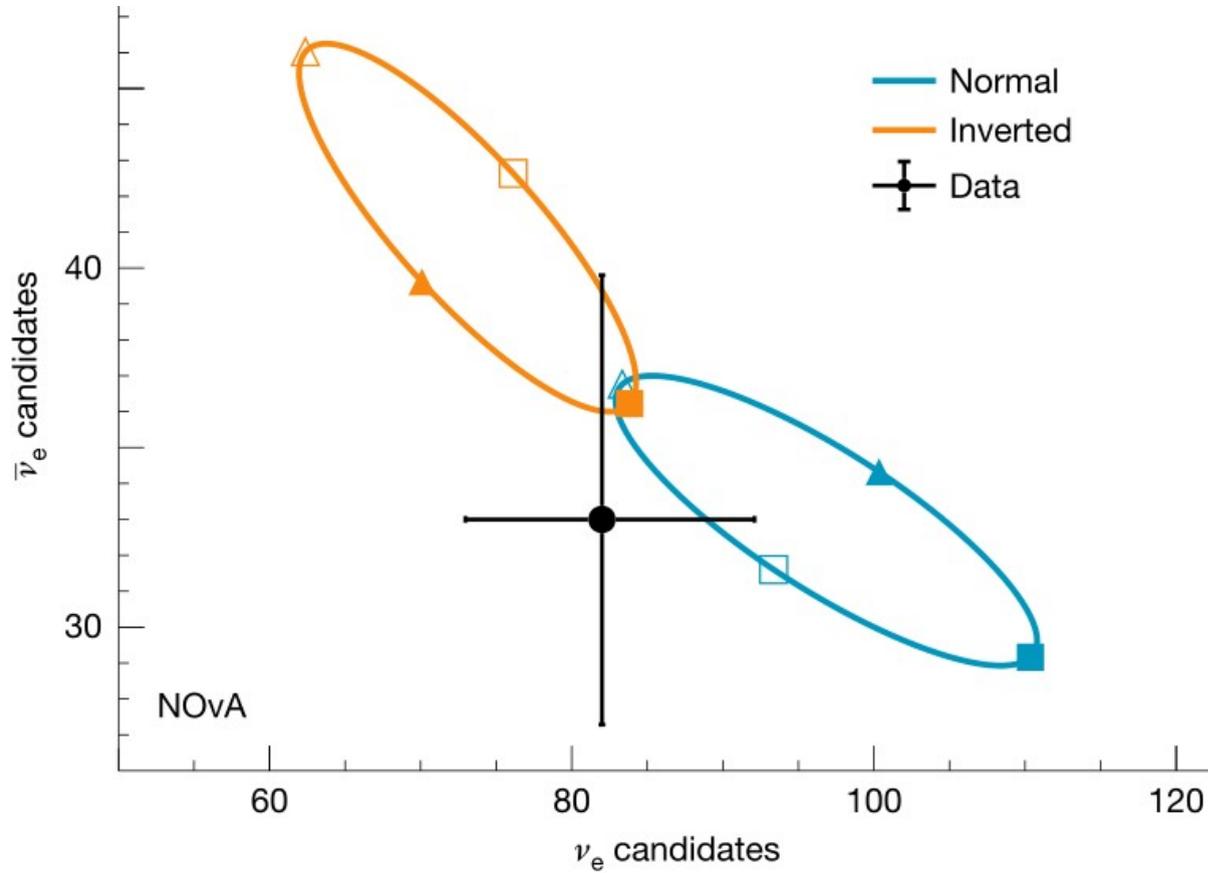
- Run both ν & $\bar{\nu}$ modes @ first peak $[\phi_{31} = \frac{\pi}{2}, \phi_{21} = \alpha \frac{\pi}{2}]$, $\alpha = \frac{\delta M_{21}^2}{|\delta M_{31}^2|} \sim 3\%$

$$P_{\bar{\nu}_\mu \rightarrow \bar{\nu}_e} + P_{\nu_\mu \rightarrow \nu_e} = 2s_a^2 c_r^2 s_r^2,$$

$$P_{\bar{\nu}_\mu \rightarrow \bar{\nu}_e} - P_{\nu_\mu \rightarrow \nu_e} = \alpha \pi \sin(2\theta_s) \sin(2\theta_r) \sin(2\theta_a) \cos \theta_r \sin \delta_D.$$

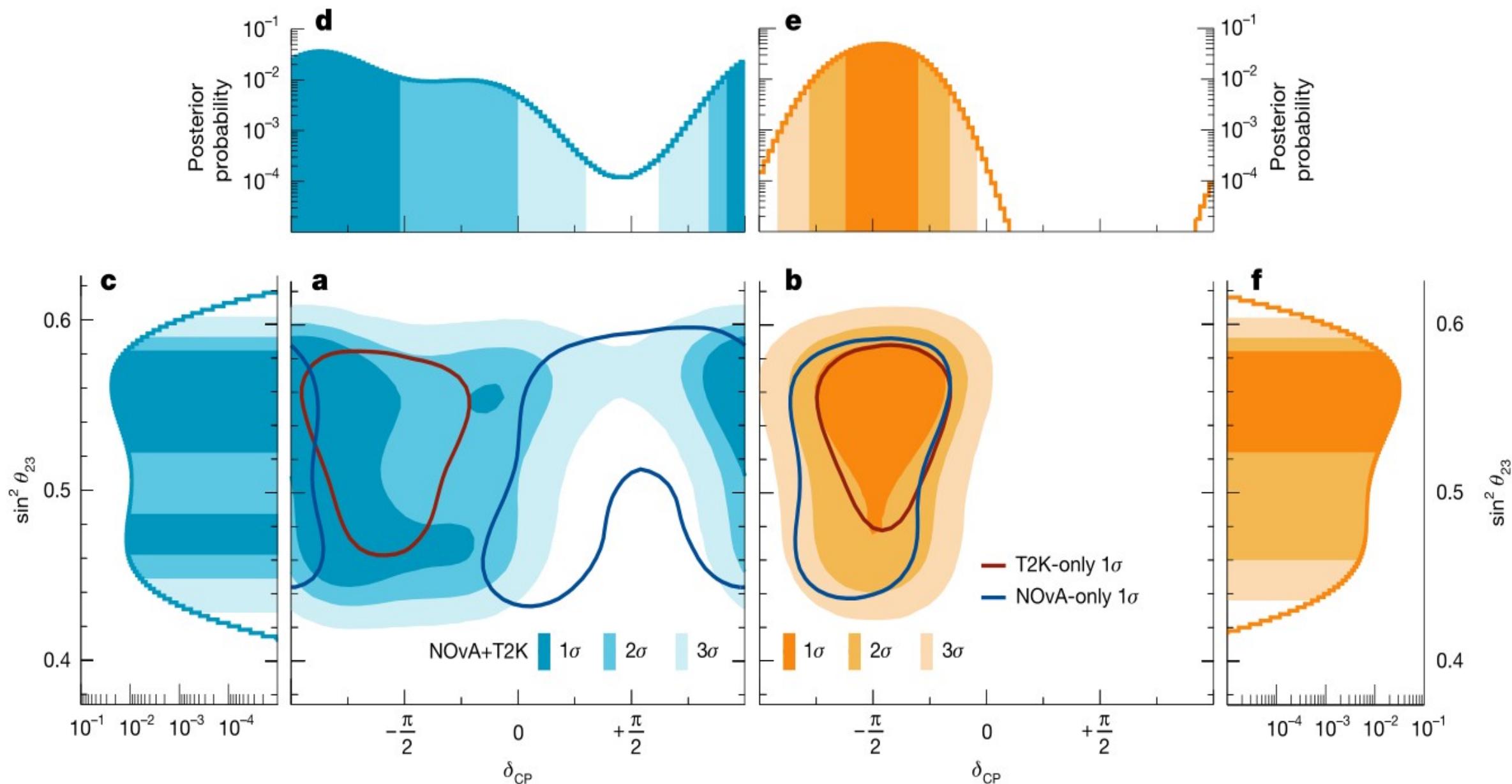
Difference in ν & $\bar{\nu}$ osc. probabilities \rightarrow infer CP phase δ_D

T2K vs NOvA



Nature Vol. 646, pp.818-824, on October 22, 2025

T2K+NOvA Joint Analysis

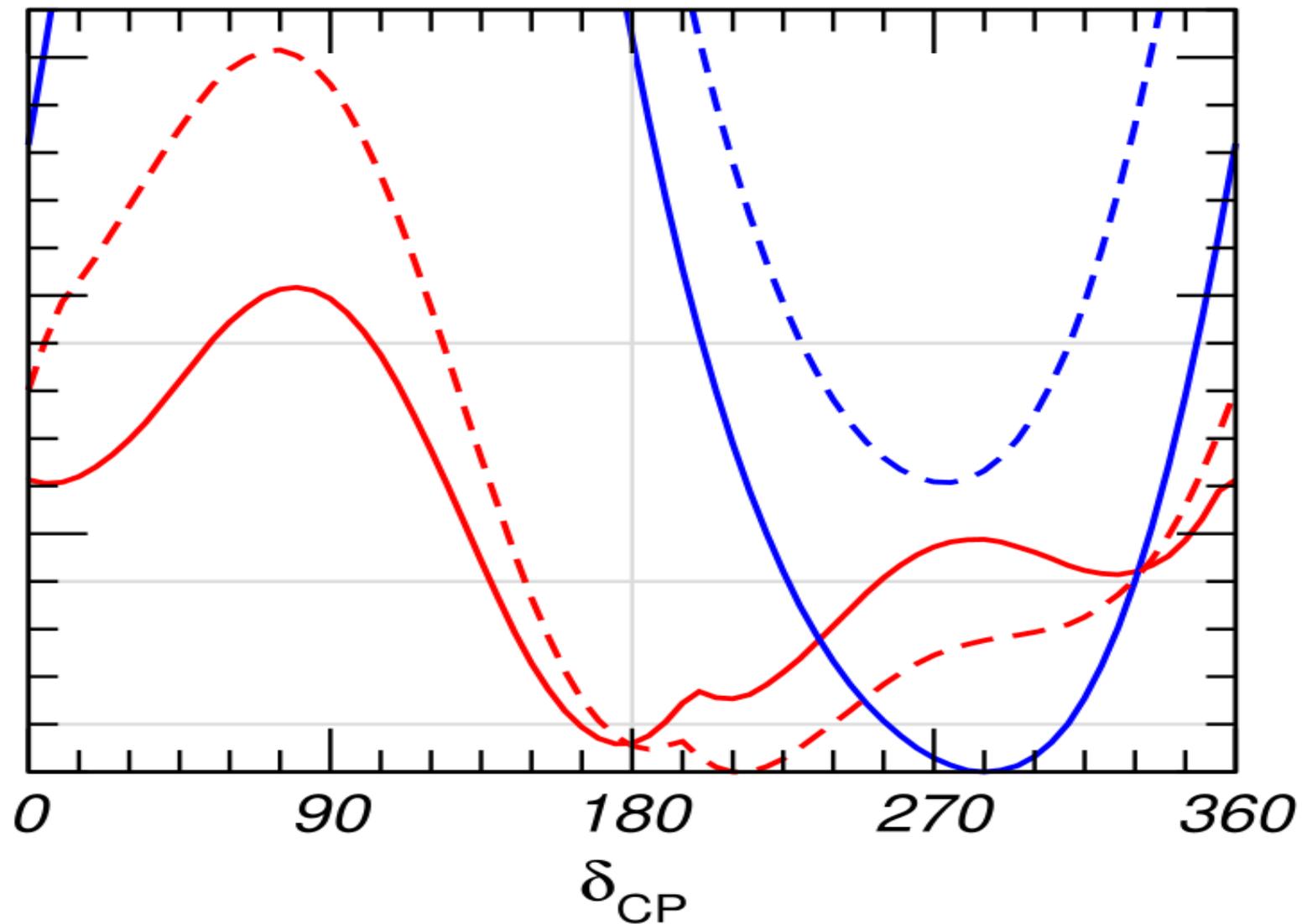


Nature Vol. 646, pp.818-824, on October 22, 2025

CP from Global Fit

— NO, IO (IC19 w/o SK-atm)
- - - NO, IO (IC24 with SK-atm)

NuFIT 6.0 (2024)



$$P_{\bar{\nu}_\mu \rightarrow \bar{\nu}_e} - P_{\nu_\mu \rightarrow \nu_e} = \alpha\pi \sin(2\theta_s) \sin(2\theta_r) \sin(2\theta_a) \cos\theta_r \sin\delta_D$$

@ 1st oscillation peak

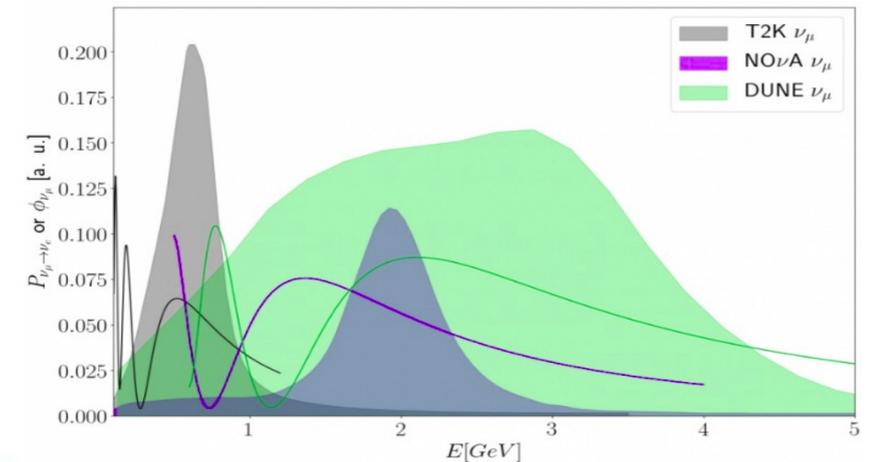
● Efficiency:

- Proton accelerators produce ν more efficiently than $\bar{\nu}$ ($\sigma_\nu > \sigma_{\bar{\nu}}$).
- The $\bar{\nu}$ mode needs more beam time [$T_{\bar{\nu}} : T_\nu = 2 : 1$].
- Undercut statistics \Rightarrow Difficult to reduce the uncertainty.

● Degeneracy:

- Only $\sin\delta_D$ appears in $P_{\nu_\mu \rightarrow \nu_e}$ & $P_{\bar{\nu}_\mu \rightarrow \bar{\nu}_e}$.
- Cannot distinguish δ_D from $\pi - \delta_D$.

- CP Uncertainty $\frac{\partial P_{\mu e}}{\partial \delta_D} \propto \cos\delta_D \Rightarrow \Delta(\delta_D) \propto 1/\cos\delta_D$



SFG [1704.08518, PoS NuFact2019 (2020) 108]

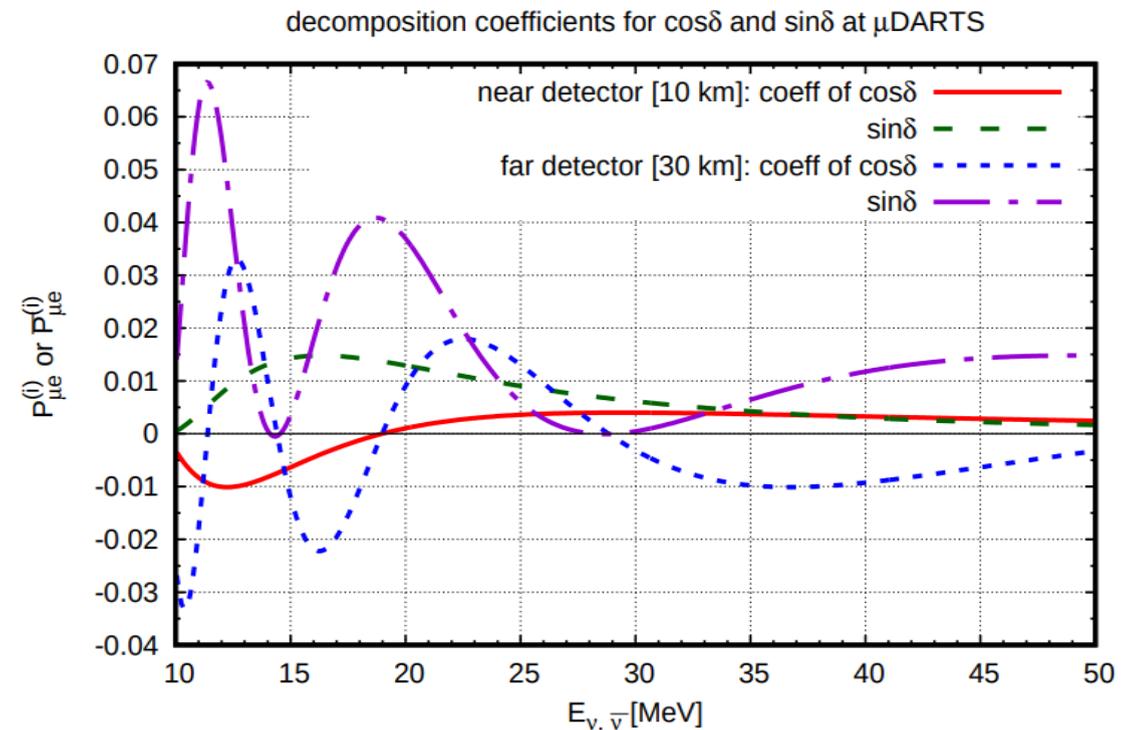
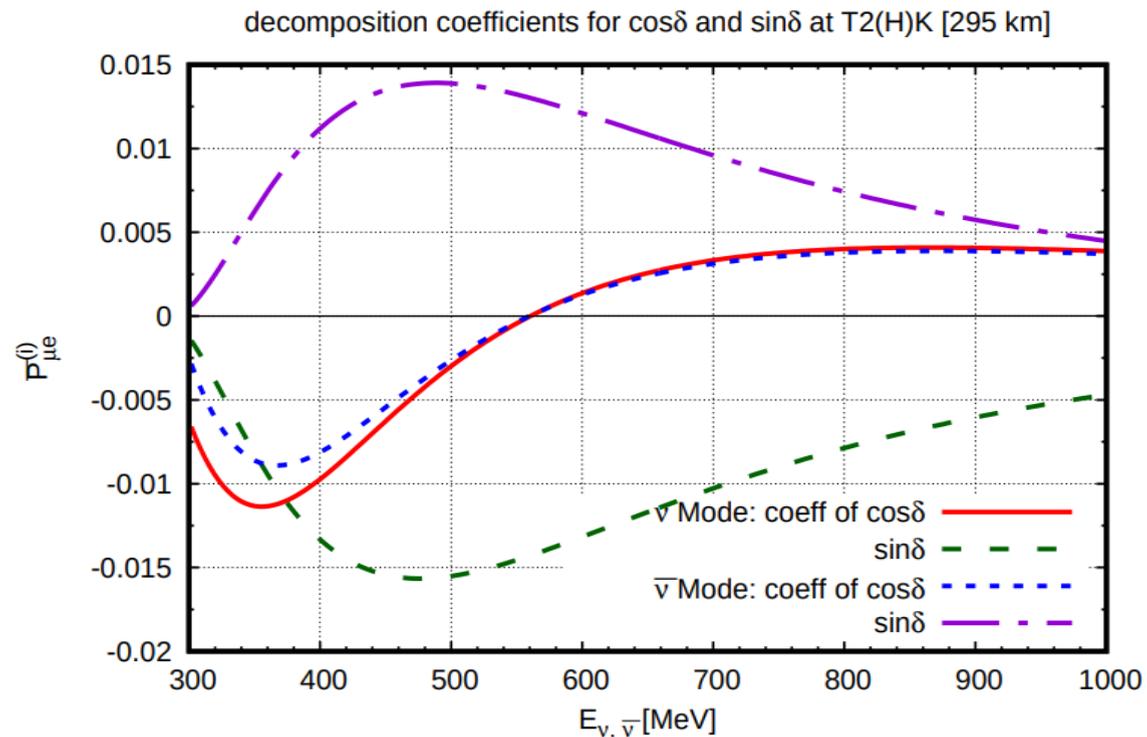
μ DAR & Accelerator ν

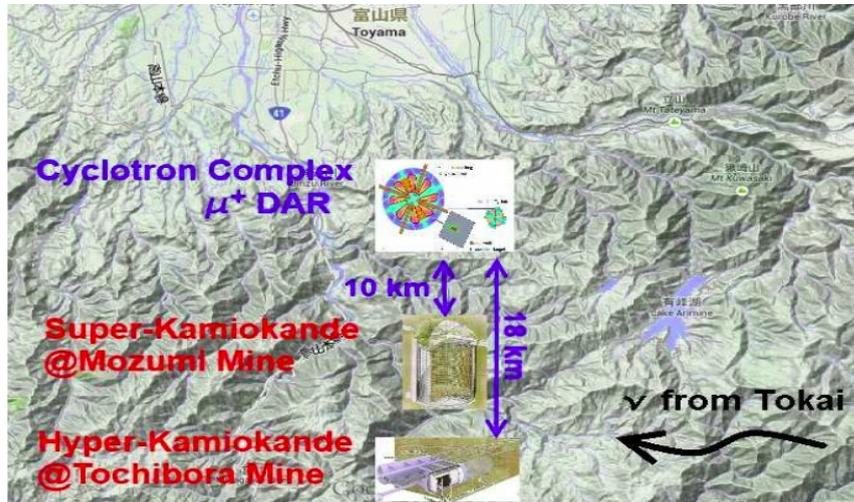
Combining $\nu_\mu \rightarrow \nu_e$ @ accelerator [narrow peak @ 550 MeV] & $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ @ μ DAR [wide peak \sim 45 MeV] solves the 2 problems:

- **Efficiency:**

- $\bar{\nu}$ @ high intensity, μ DAR is plentiful enough.
- Accelerator Exps can devote all run time to the ν mode. With same run time, the statistical uncertainty drops by $\sqrt{3}$.

- **Degeneracy: (decomposition in propagation basis [1309.3176])**





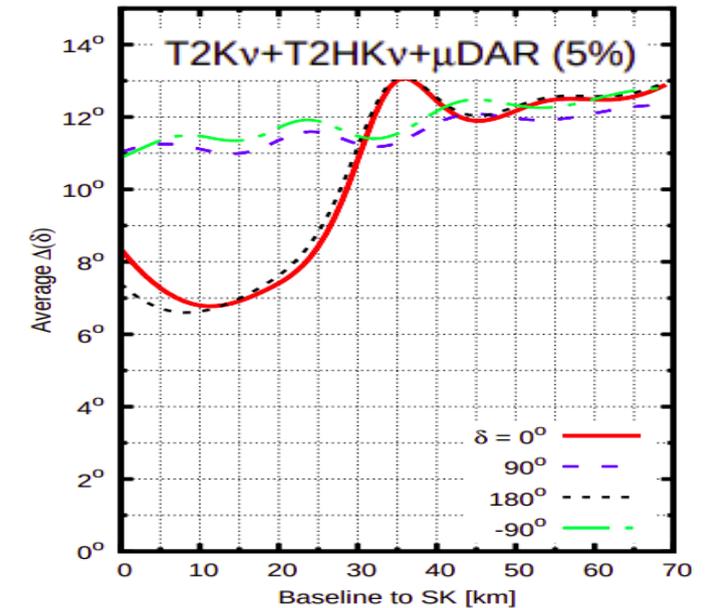
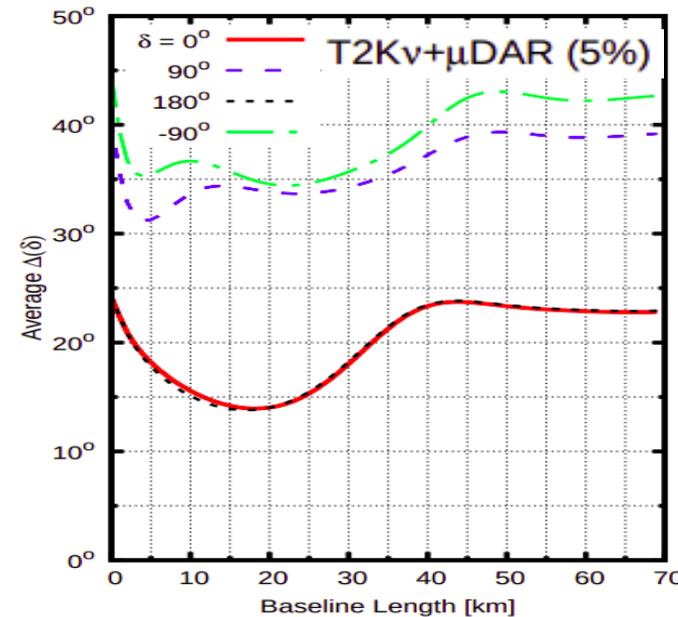
1 μ DAR source + 2 detectors

Advantages

- Full (**100%**) duty factor!
- **Lower** intensity: $\sim 9\text{mA}$ [$\sim 4\times$ lower than DAE δ ALUS]
- Not far beyond the current state-of-art technology of cyclotron [**2.2mA** @ Paul Scherrer Institute]
- MUCH **cheaper** & technically **easier**.
 - Only one cyclotron.
 - Lower intensity.

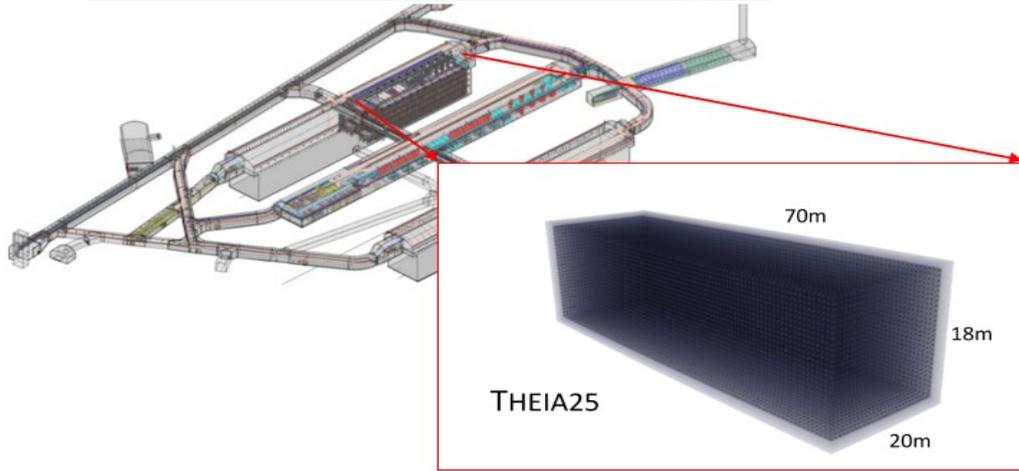
Disadvantage: A second detector!

Harnik, Kelly & Machado [1911.05088]



Evslin, SFG, Hagiwara [1506.05023]
SFG, Pasquini, Tortola, Valle [1605.01670]
SFG, Smirnov [1607.08513]

μ THEIA+DUNE



THEIA [1911.03501, 2202.12839]

SFG, Kong, Pasquini, Eur.Phys.J.C 82 (2022) 6, 572 [2202.05038]

