Theory Overview of Heavy-Quark Hadron Decays in QCD

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味物理前沿研讨会

Milestones in heavy flavor physics since 2000

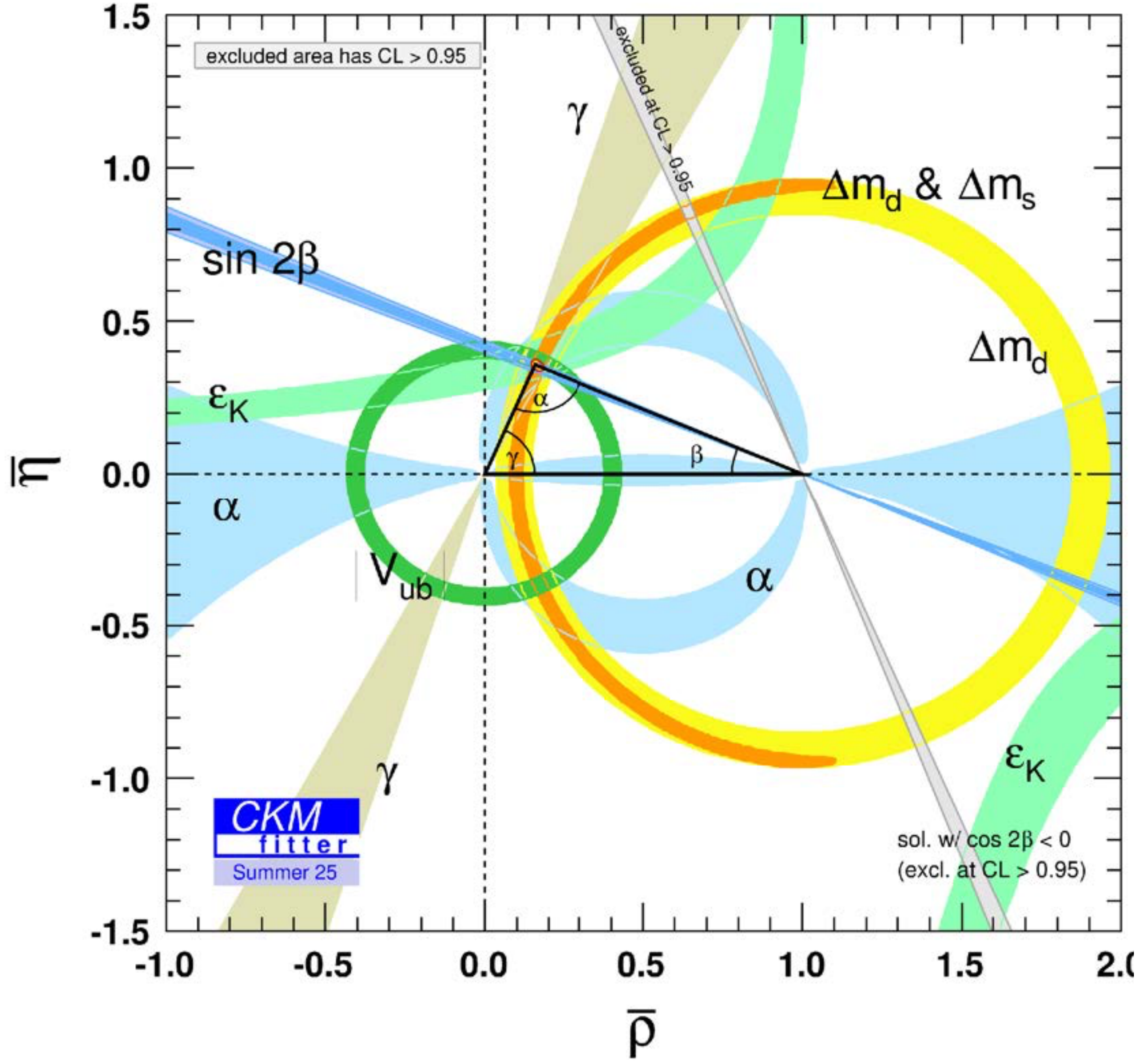
Experimental progresses	Year
First observation of CPV in baryon decays	2025
First observation of CPV in D-meson decays	2019
First observation of doubly heavy baryons	2017
First observation of heavy pentaquark	2015
First observation of direct CPV	2004
CPV in B-meson system	2001
	2000

Theoretical progresses

QCDF PQCD SCET

High Precision

Precision Test of the Standard Model



- CKM Phases

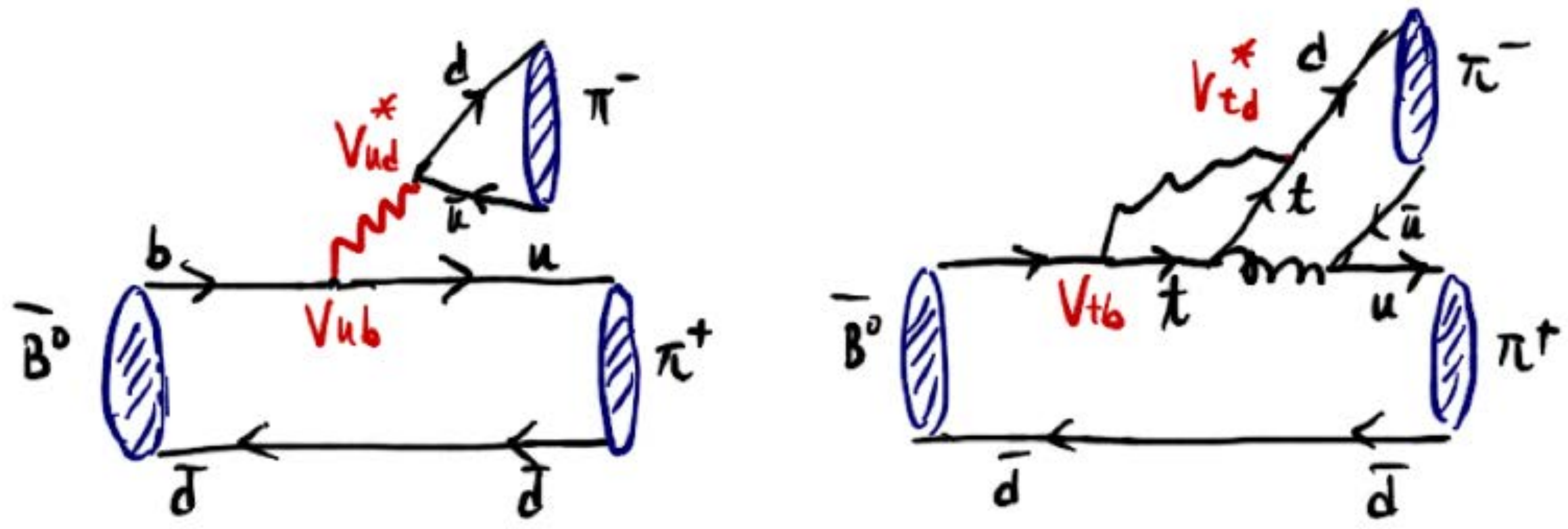
$$\alpha = \arg\left(-\frac{V_{tb}^* V_{td}}{V_{ub}^* V_{ud}}\right)$$ CPV of $B \to \pi\pi$

$$\beta = \arg\left(-\frac{V_{cb}^* V_{cd}}{V_{tb}^* V_{td}}\right)$$ CPV of $B \to J/\psi K_S$

$$\gamma = \arg\left(-\frac{V_{ub}^* V_{ud}}{V_{cb}^* V_{cd}}\right)$$ Interference of $B \to DK$

Heavy Flavour Physics is an Important Topic

- We take $B \to \pi\pi$ as an example



$$\lambda_{\pi\pi} = \frac{V_{tb}^* V_{td}}{V_{tb} V_{td}^*} \left[\frac{V_{ub} V_{ud}^* T_{\pi\pi} e^{i\delta_1} - V_{tb} V_{td}^* P_{\pi\pi} e^{i\delta_2}}{V_{ub}^* V_{ud} T_{\pi\pi} e^{i\delta_1} - V_{tb}^* V_{td} P P_{\pi\pi} e^{i\delta_2}} \right] = e^{2i\alpha} \left[\frac{1 + r e^{i(\delta - \alpha)}}{1 + r e^{i(\delta + \alpha)}} \right]$$

$$r = \left| \frac{P_{\pi\pi}}{T_{\pi\pi}} \right| \left| \frac{V_{tb} V_{td}^*}{V_{ub} V_{ud}^*} \right|, \quad \delta = \delta_2 - \delta_1,$$

- It is important for us to calculate the parameters r, δ_1 and δ_2 accurately as possible as we can in the theoretical side.

Puzzles in nonleptonic two-body B decays

- $B \to \pi^0\pi^0$ puzzle

$Br(B^0 \to \pi^0\pi^0) = (1.55 \pm 0.16) \times 10^{-6}$

Much larger than expected.

- $B \to K\pi$ puzzle

$\Delta A_{CP} = A_{CP}(B^- \to K^-\pi^0) - A_{CP}(B^0 \to K^-\pi^+) = (11.3 \pm 1.2)\%$

9σ deviation from isospin limit.

- $B \to K^{(*)}K^{(*)}$ puzzle

$$R_{KK}^{sd} = \left|\frac{V_{td}}{V_{ts}}\right|^2 \frac{\Gamma(B_s \to K^0\bar{K}^0)}{\Gamma(B_d \to K^0\bar{K}^0)} = 0.62 \pm 0.14$$

Over 3σ deviation from SM prediction.

- $B \to D^{(*)}L$ puzzle

$$R_{s(L)}^{(*)} = \frac{\Gamma\left(\bar{B}_{(s)}^0 \to D_{(s)}^{(*)+} L^-\right)}{\mathrm{d}\Gamma\left(\bar{B}_{(s)}^0 \to D_{(s)}^{(*)+} \ell^- \nu_\ell\right)/dq^2|_{q^2=m_L^2}}$$

$R_\pi^{(*)}, R_K, R_{s\pi}$: Over 4σ deviation from SM prediction.

- Could these anomalies be attributed to effects of new physics?

The Multi-scale Problem: Factorization

- Many scales are involved in the hadronic decays of heavy flavor hadrons

NP scale	EW scale	Heavy quark scale	Intermediate scale	Hadronization scale
TeV or beyond	m_W	$m_b(m_c)$	$\sqrt{m_b\Lambda}, m_c$	Λ

Effective theory+RG

$$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}}\sum_i C_i O_i + \sum_i C_i' O_i'$$

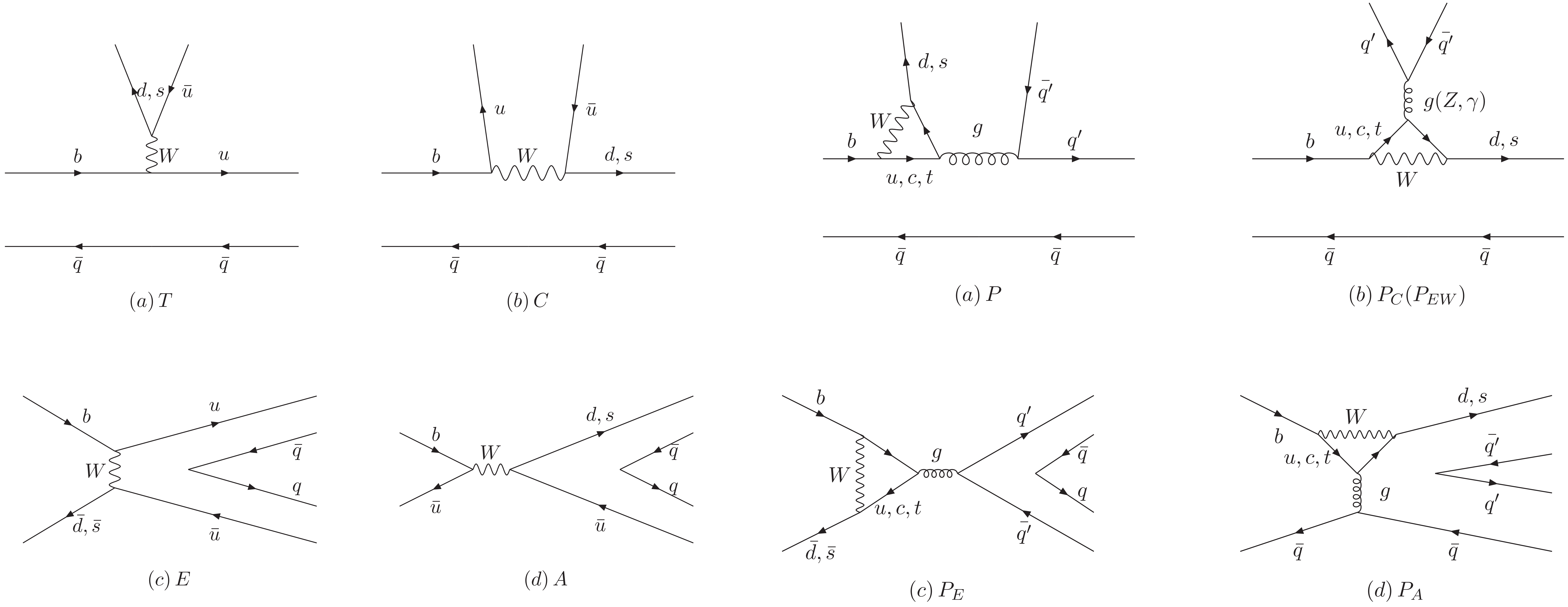
Nonperturbative

$$\langle f|O_i|B\rangle = \mathcal{A}_{LP} + O\left(\frac{\Lambda}{m_b}, \frac{\Lambda}{E_f}\right)$$

- Wilson coefficients C_i: perturbatively calculable. (3-loops)
- Hadronic matrix elements $<f|O_i|B>$: essentially nonperturbative quantities.

The Multi-scale Problem: Factorization

- Within the formula of semi-leptonic B decays, we factorize the amplitude of Tree operators



$$T^{P_1P_2} = i\frac{G_F}{\sqrt{2}}V_{ub}V_{uq'}a_1(\mu)f_{p_2}(m_B^2 - m_{p_1}^2)F_0^{BP_1}(m_{p_2}^2),$$

$$a_1 = C_2 + \frac{1}{N_c}C_1 \qquad N_C \to N_C^*$$

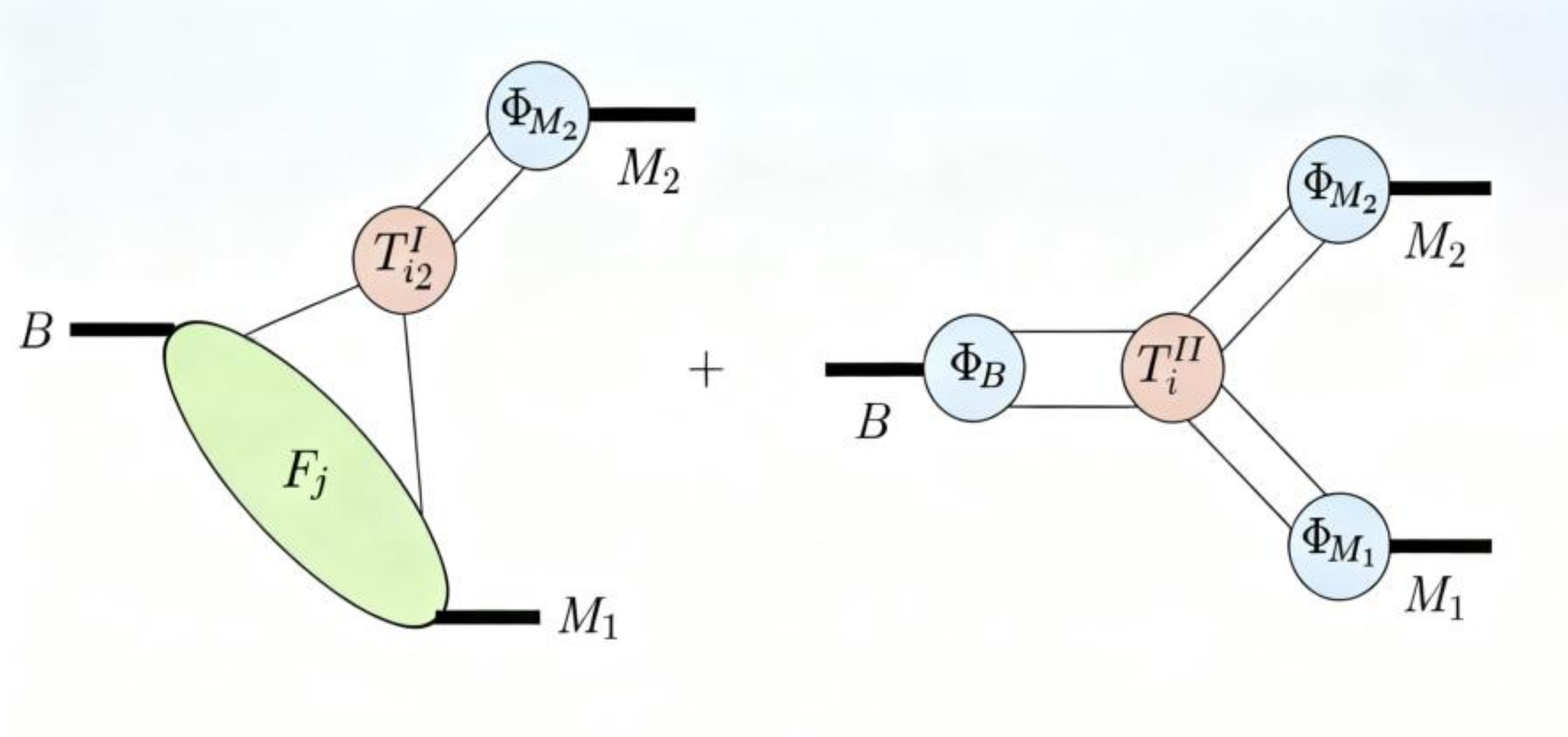
- Small strong phases from QCD corrections of WCs.

The Multi-scale Problem: Factorization

- **QCD factorization and SCET**

  [Bauer et.al, (2001)] [Beneke, Buchalla, Neubert, Sachrajda (1999)]

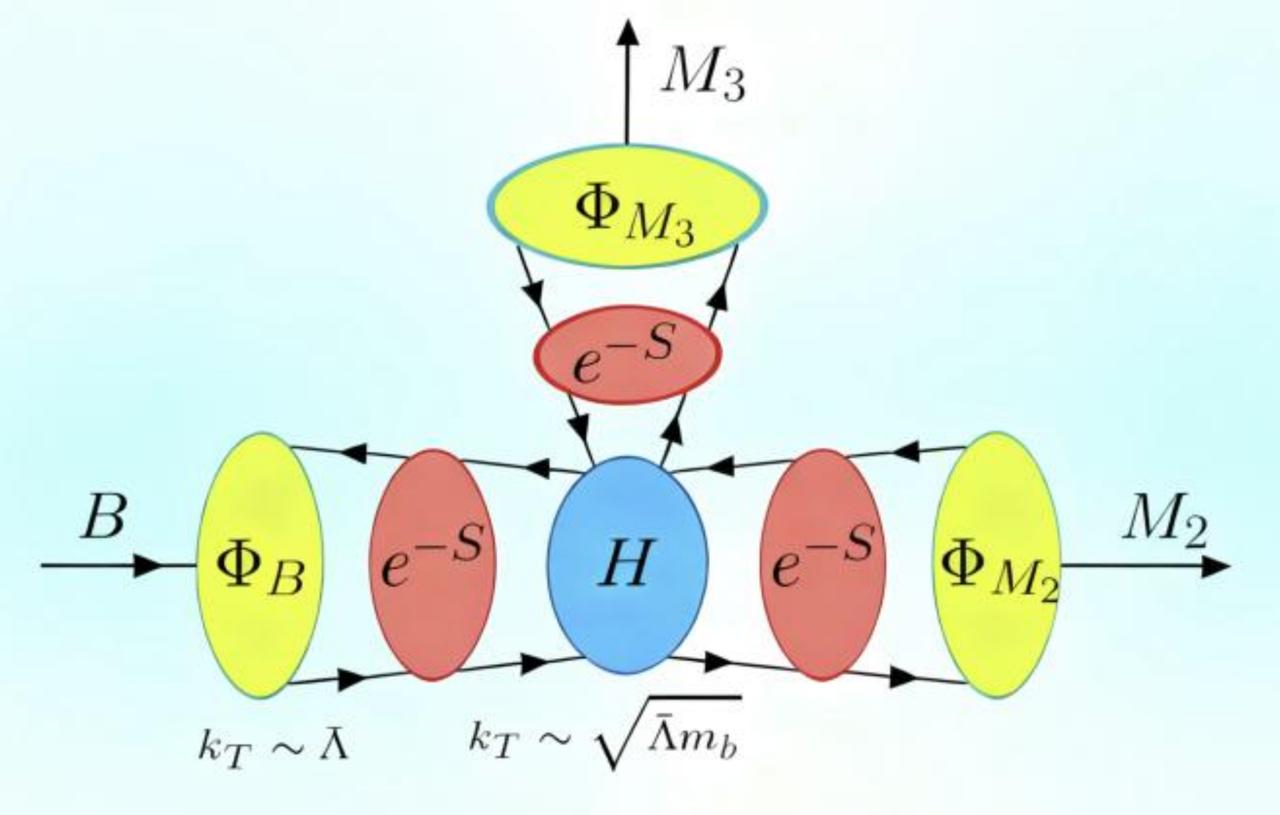
$$\langle M_1 M_2 | O_i | \bar{B} \rangle = \mathrm{F}^{\mathrm{B}\to M_1} \mathrm{T_i^I} \otimes \phi_{M_2} + \mathrm{T_i^{II}} \otimes \phi_B \otimes \phi_{M_1} \otimes \phi_{M_2}$$



- **Perturbative QCD (PQCD)**

  [Li, Lü, Xiao etc.]

$$\langle M_1 M_2 | O_i | \bar{B} \rangle = H \otimes \phi_B \otimes \phi_{M_1} \otimes \phi_{M_2} \otimes \mathrm{e}^{-\sum S_i}$$



QCDF & SCET

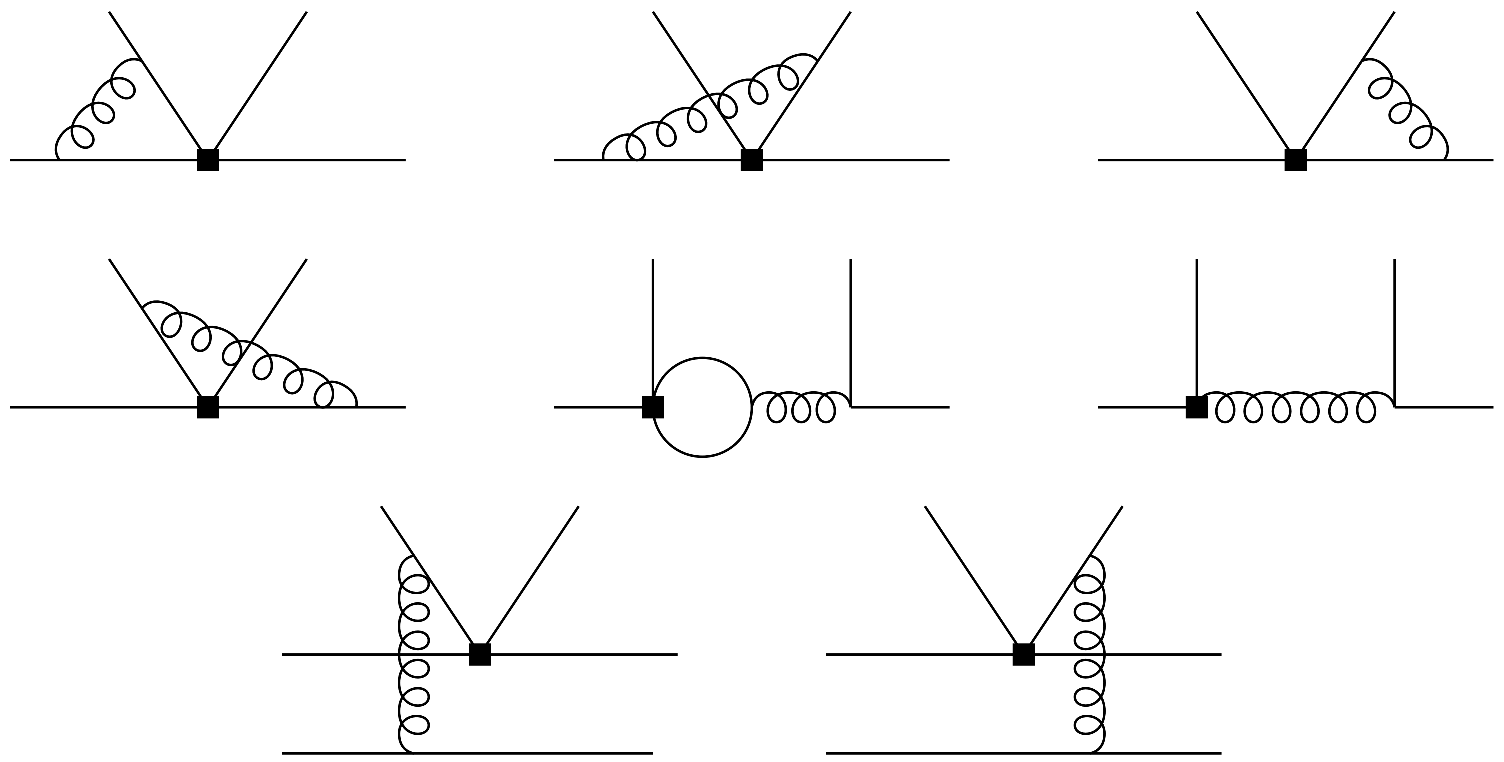
- At leading order in the heavy-quark expansion (Λ/m_b)

$$\langle M_1 M_2 | C_i O_i | \bar{B} \rangle_{\mathcal{L}_{\rm eff}} = \sum_{\rm terms} C(\mu_h) \times \Big\{ F_{B\to M_1} \times \underbrace{T^{\rm I}(\mu_h,\mu_s)}_{1+\alpha_s+\ldots} \star f_{M_2}\Phi_{M_2}(\mu_s) + f_B \Phi_B(\mu_s) \star \Big[\underbrace{T^{\rm II}(\mu_h,\mu_I)}_{1+\ldots} \star \underbrace{J^{\rm II}(\mu_I,\mu_s)}_{\alpha_s+\ldots} \Big] \star f_{M_1}\Phi_{M_1}(\mu_s) \star f_{M_2}\Phi_{M_2}(\mu_s) \Big\}$$

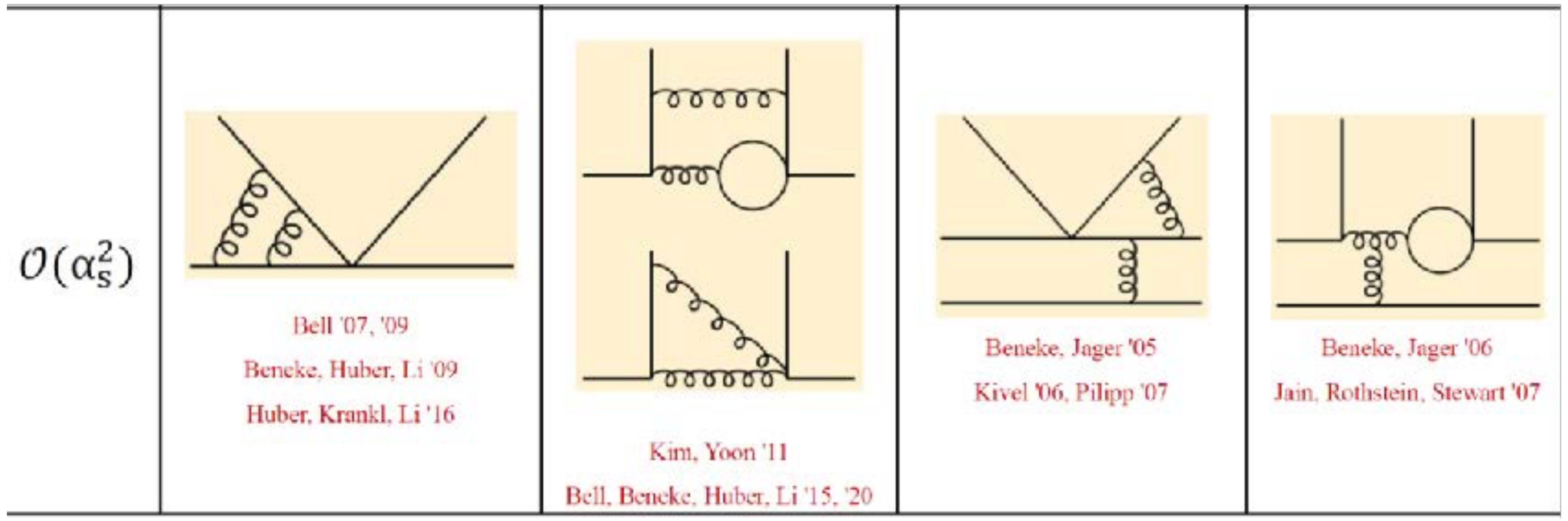
- Form factors and LCDAs universal non-perturbative objects
  - Vertex corrections $T^I = 1 + O(\alpha_s/\pi)$
  - Spectator scattering $T^{II} = O(\alpha_s)$ and real

QCDF: Two-loop Corrections to $T^{I,II}$

- NLO Corrections [BBNS, 99-04]



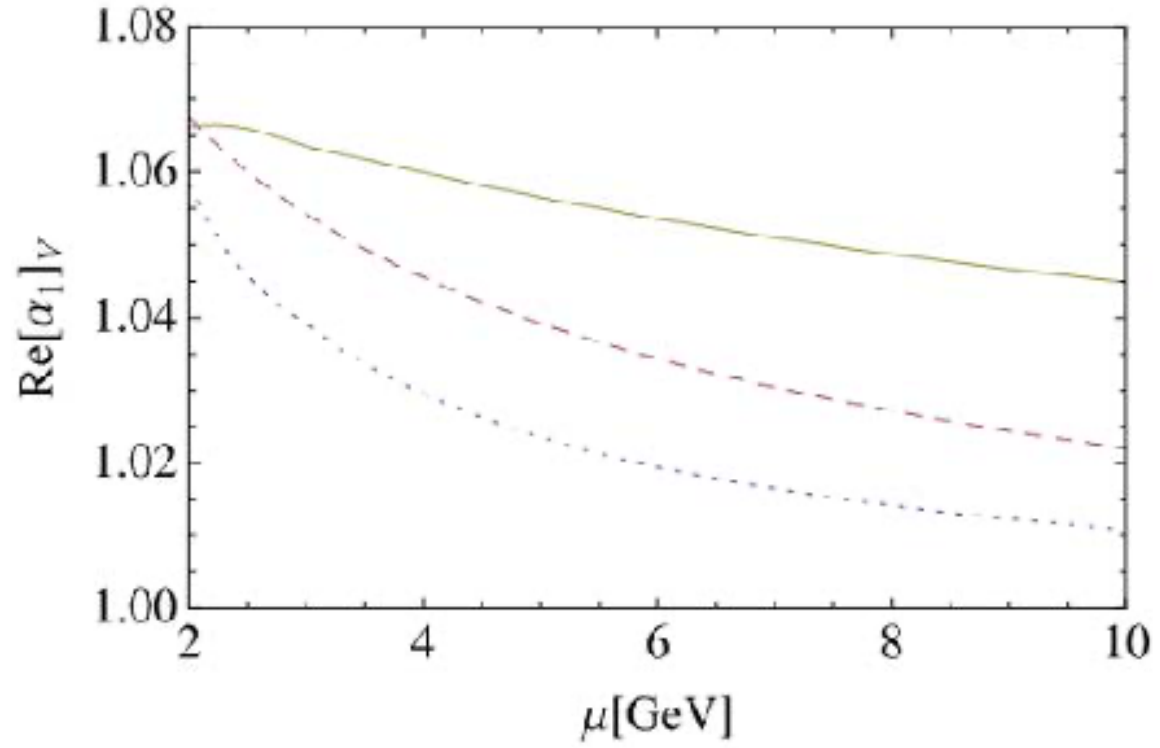
- NNLO Corrections [05-20]

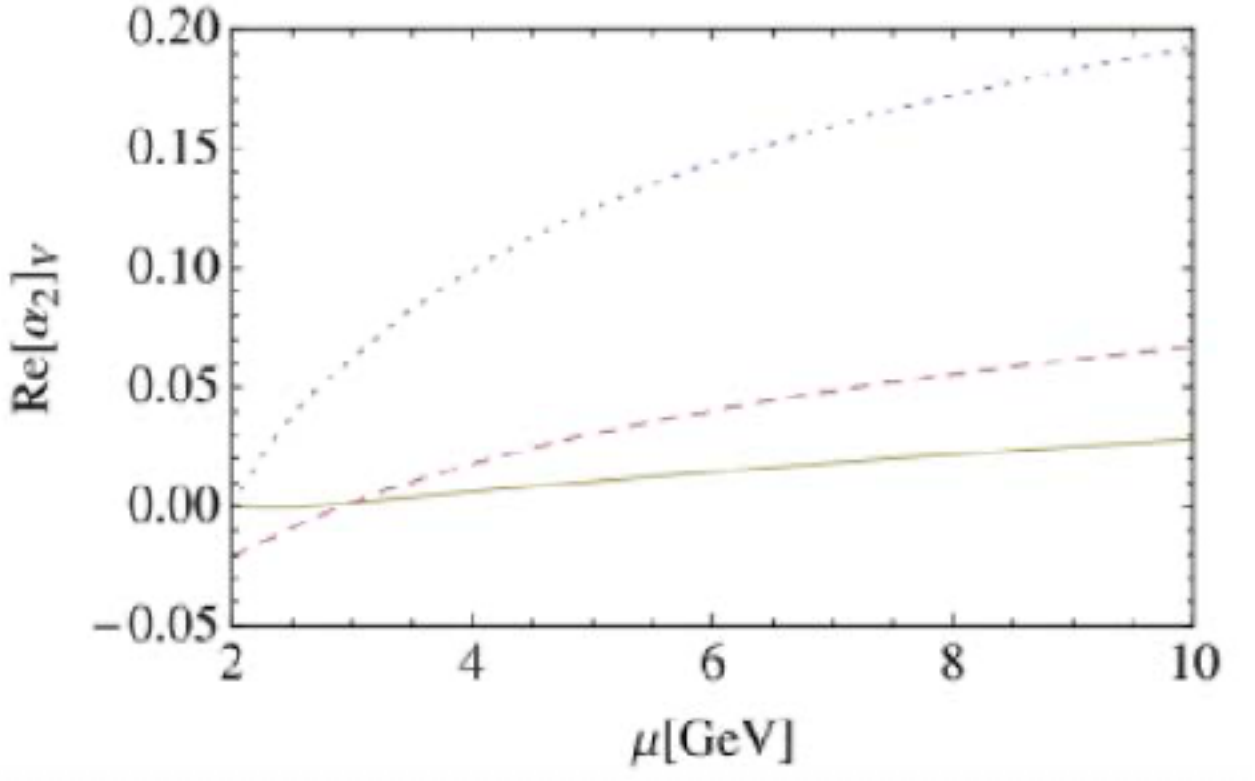


- Results

$$\alpha_1(\pi\pi) = 1.009 + [0.023 + 0.010\,i]_{\rm NLO} + [0.026 + 0.028\,i]_{\rm NNLO} - \left[\frac{r_{\rm sp}}{0.445}\right]\left\{[0.014]_{\rm LOsp} + [0.034 + 0.027i]_{\rm NLOsp} + [0.008]_{\rm tw3}\right\} = 1.000^{+0.029}_{-0.069} + (0.011^{+0.023}_{-0.050})i$$

$$\alpha_2(\pi\pi) = 0.220 - [0.179 + 0.077\,i]_{\rm NLO} - [0.031 + 0.050\,i]_{\rm NNLO} + \left[\frac{r_{\rm sp}}{0.445}\right]\left\{[0.114]_{\rm LOsp} + [0.049 + 0.051i]_{\rm NLOsp} + [0.067]_{\rm tw3}\right\} = 0.240^{+0.217}_{-0.125} + (-0.077^{+0.115}_{-0.078})i$$





- Comments

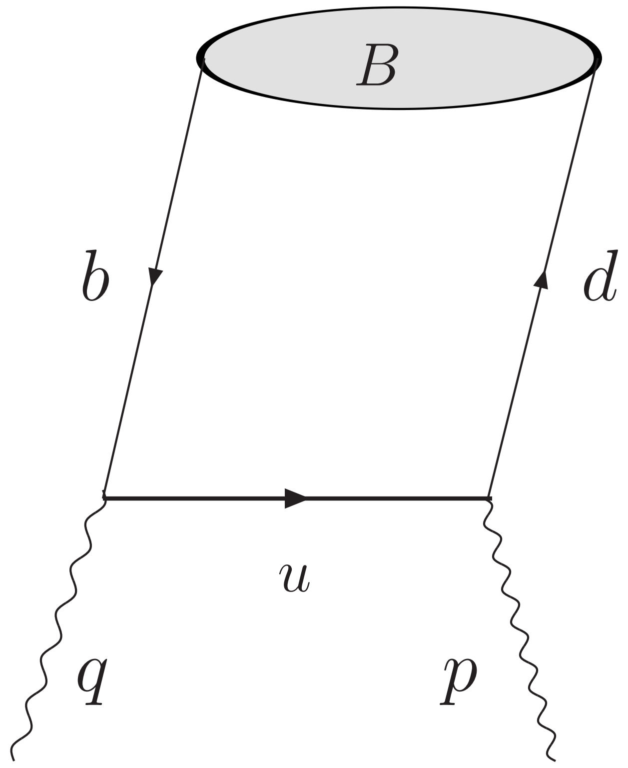
Small impact on $B \to \pi^0\pi^0$: strong cancellation between T_i^I and T_i^{II}

Small impact on $B \to \pi K$: strong cancellation between tree and penguin operators

Small impact on $B \to D_q^{(*)}L$: small real part

Precision Calculations of Form Factors

- LCSR is a QCD inspired method for the large recoil region of the heavy-to-light form factors.



The correlation function

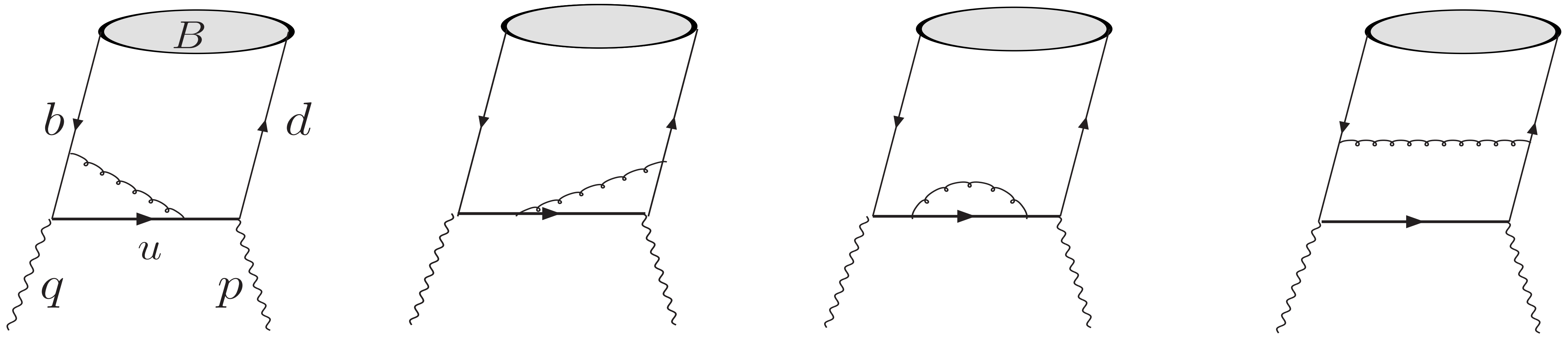
$\langle 0|T\{J_{\text{weak}}(0), J_B(x)\}|\bar{B}(P)\rangle$

$\langle \bar{M}(P)|T\{J_{\text{weak}}(0), J_M(x)\}|0\rangle$

- Light meson LCSR
- Heavy meson LCSR

Quark Level ⇔ Quark hadron duality ⇔ Hadron Level

- Higher-order corrections



(a) (b) (c) (d)

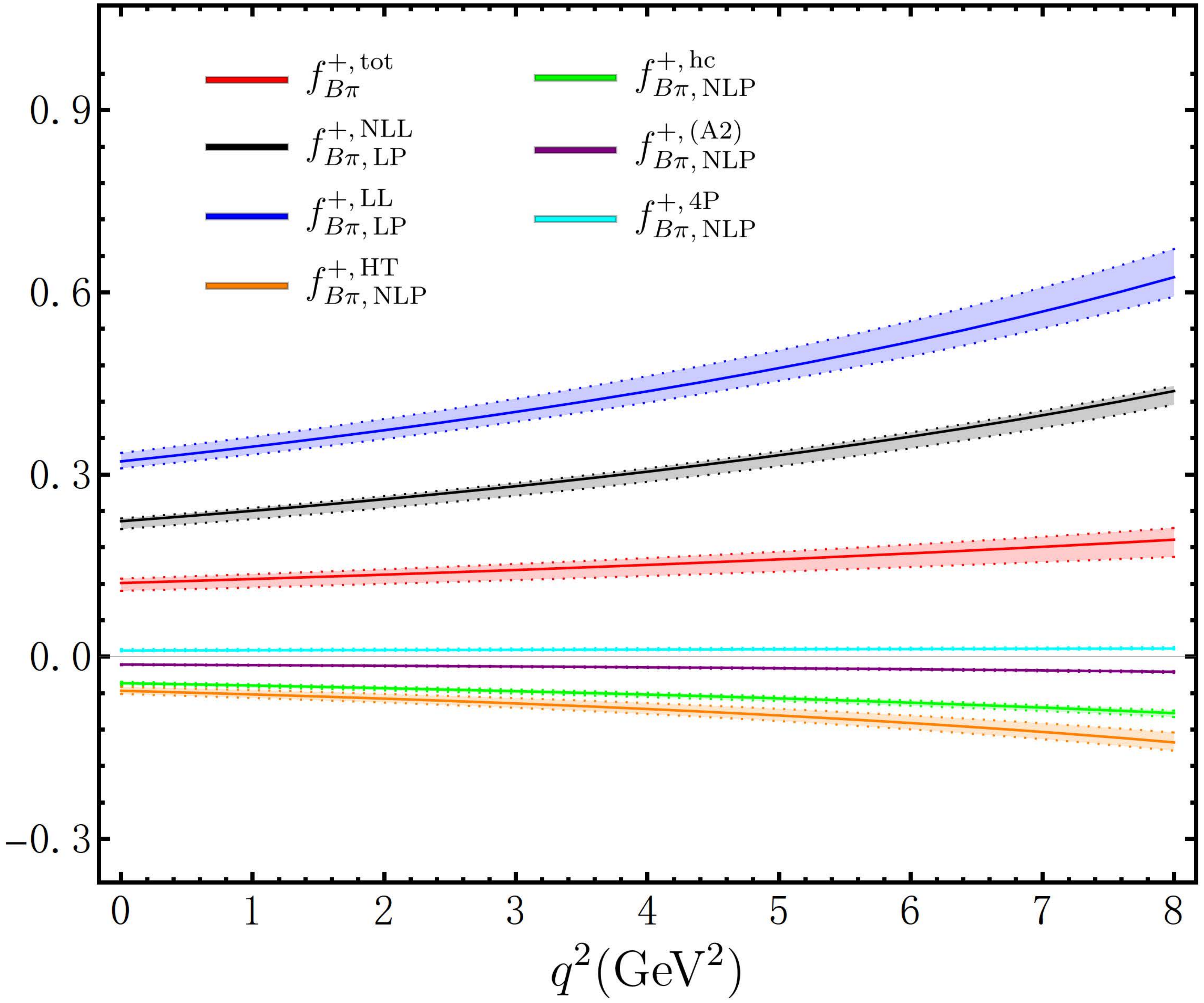
- Higher-power corrections

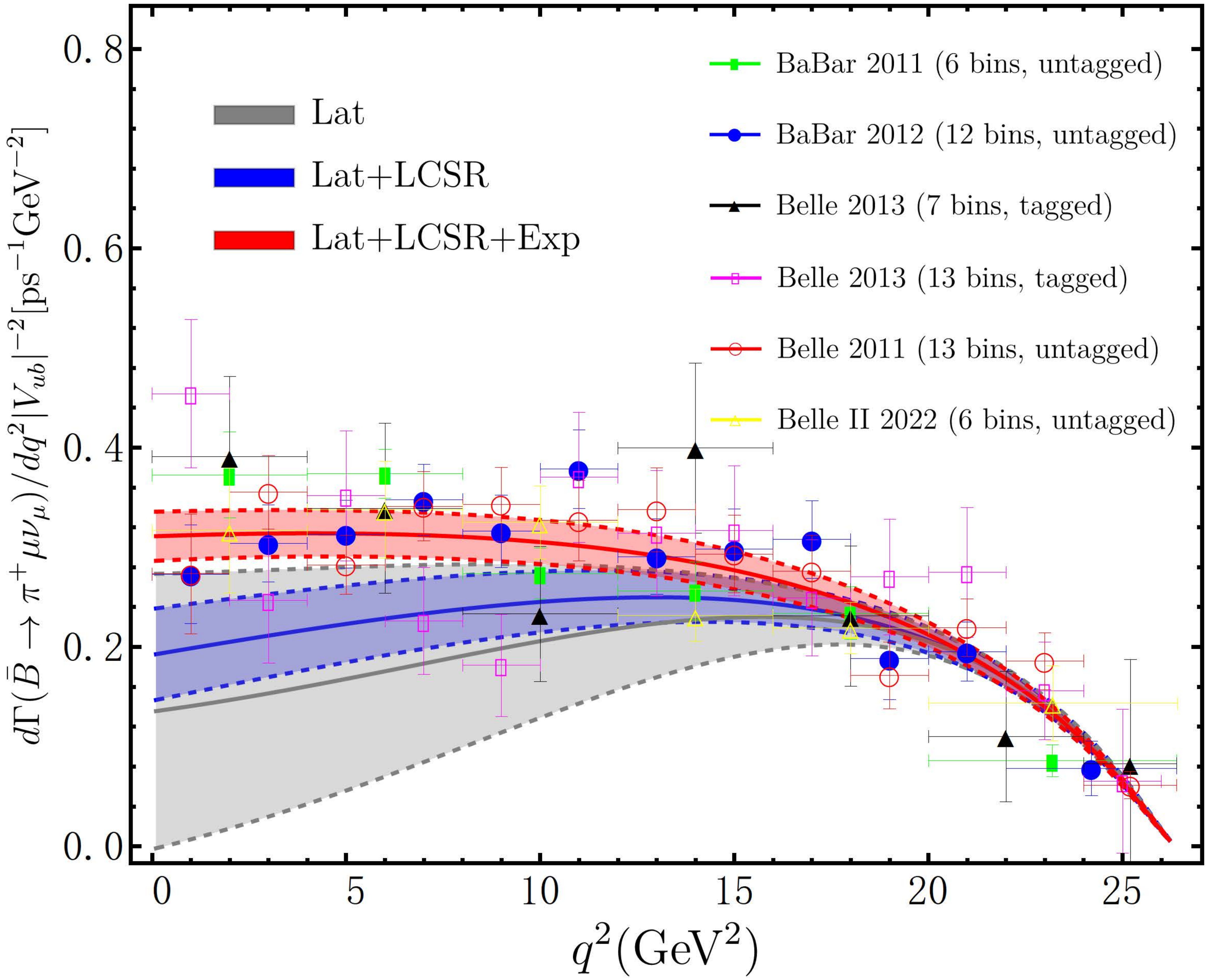
  - Hard-collinear propagator
  - Subleading effective current
  - Higher-twist two-particle LCDA
  - Three-particle LCDA

- $B \to \pi, K$ form factors [Cui, Huang, Shen, Wang, Wang, 2022]
- $B \to D$ form factors [Gao, Huber, Ji, Wang, Wang, Wei, 2022]
- $B \to D^*$ form factors [Cui, huang, Wang, Zhao, 2023]
- $B \to V$ form factors [Gao, Lu, Shen, Wang, 2019]

Precision Calculations of Form Factors

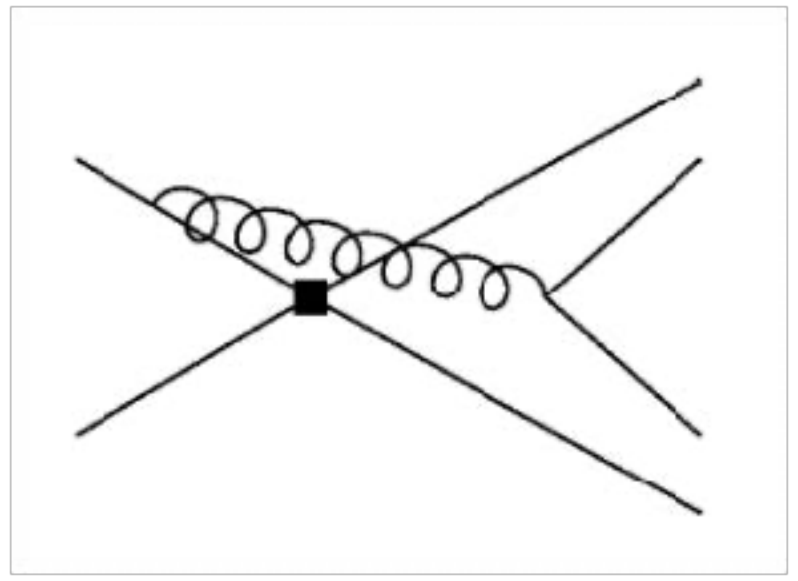
- Cui, Huang, Shen, Wang, Wang, 2022

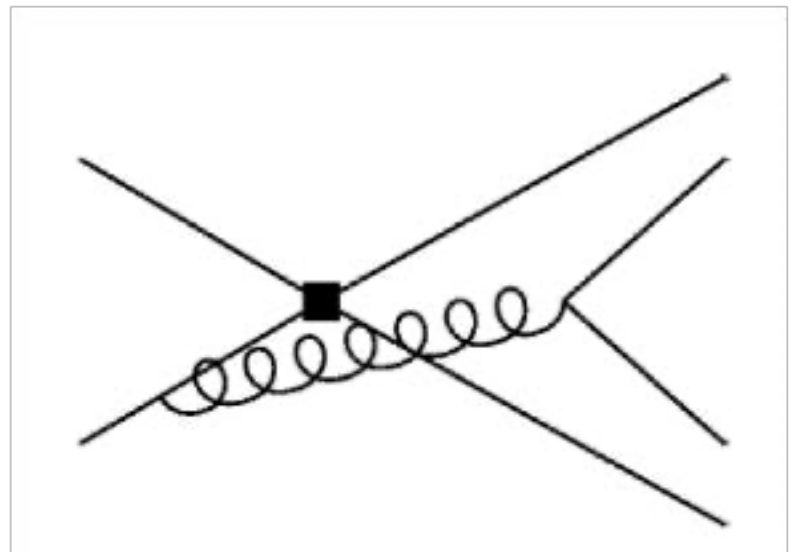




$$|V_{ub}|_{B\to\pi\ell\bar{\nu}_\ell} = (3.72 \pm 0.14) \times 10^{-3}$$

Annihilation Topology





- Weak annihilation (WA) crucial for
  - strong phases
  - direct CP violation

- QCDF
  - WA is power suppressed
  - but numerically non-negligible
  - endpoint divergences → phenomenological parameters

$$A_1^{ib} \supset \int_0^1 dxdy \frac{\phi_1(x)\phi_2(y)}{\bar{x}^2 y}$$

$$X_A = \ln \frac{m_b}{\Lambda}(1 + \rho_A e^{i\phi_A})$$

- WA arises only from subleading SCET operators

One gluon is not enough: we need two independent soft–collinear conversions to form two opposite collinear color-singlet mesons.

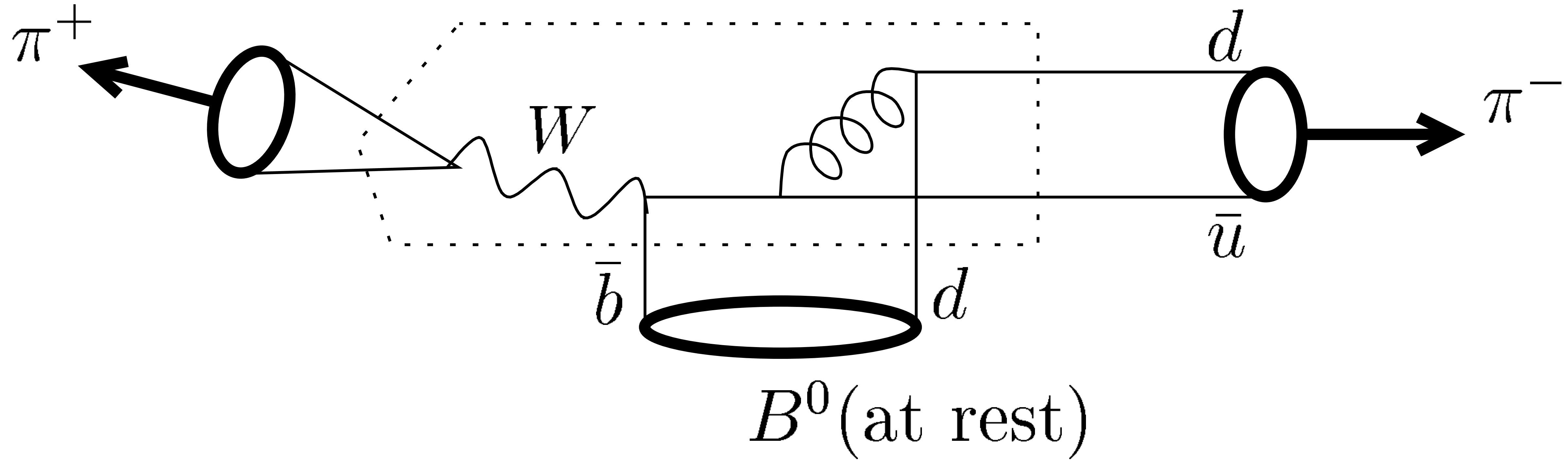
- Power Counting

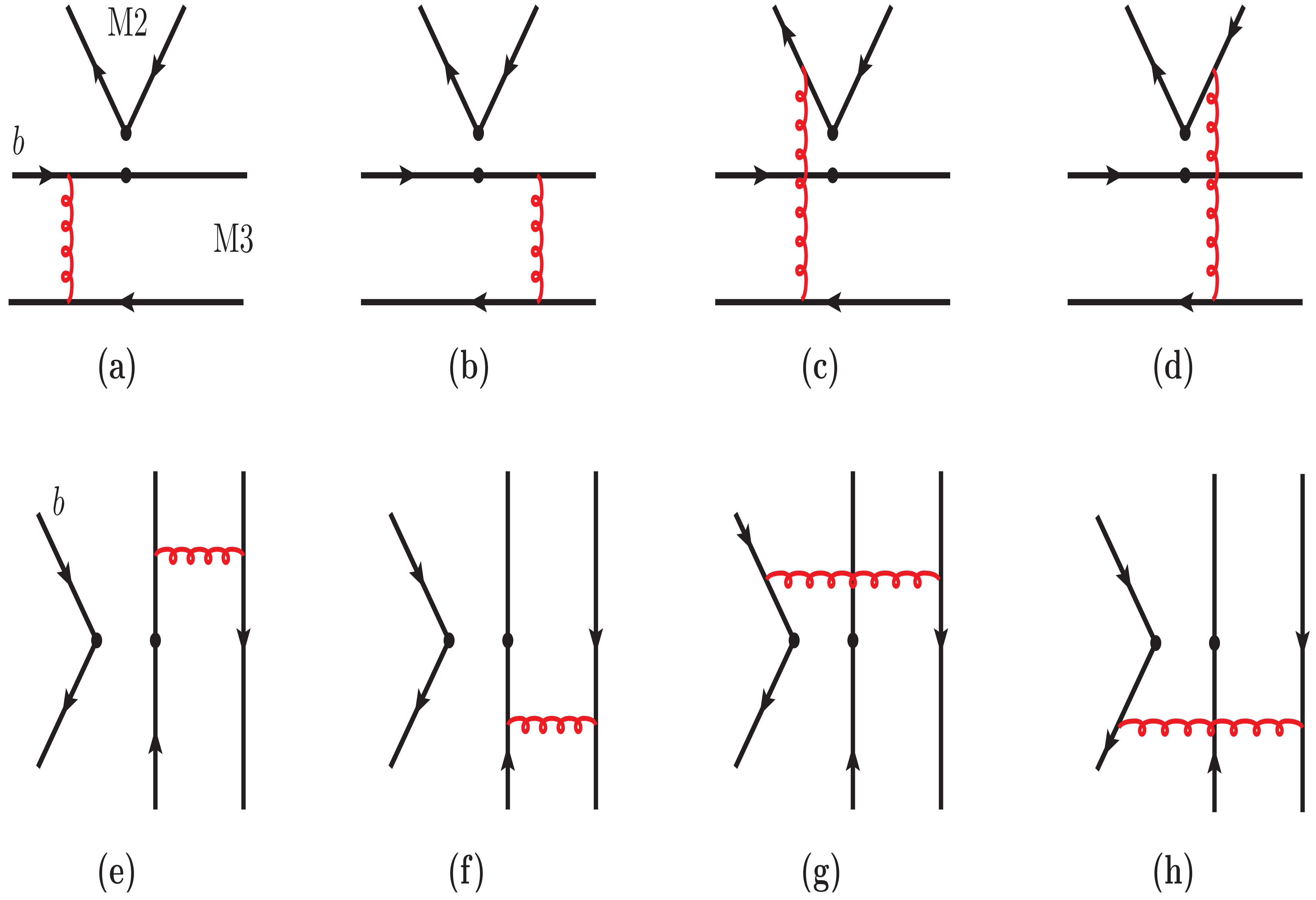
$$\mathscr{A}_{\rm WA} \sim \lambda^2 \sim \frac{\Lambda_{\rm QCD}}{m_b} \quad \Rightarrow \quad \text{power suppressed}$$

- WA absent at leading power
  - WA is power suppressed
  - Endpoints Parameterization
  - Large Wilson Coefficients

Lü, Shen, Wang, Wang, 2023

PQCD： kT Factorization





$$\mathscr{A} \sim \int d^4k_1 d^4k_2 d^4k_3 Tr[C(t)\Phi_B(k_1)\Phi_2(k_2)\Phi_3(k_3)H(k_1,k_2,k_3,t)]\exp[-S(t)]$$

Annihilations in PQCD

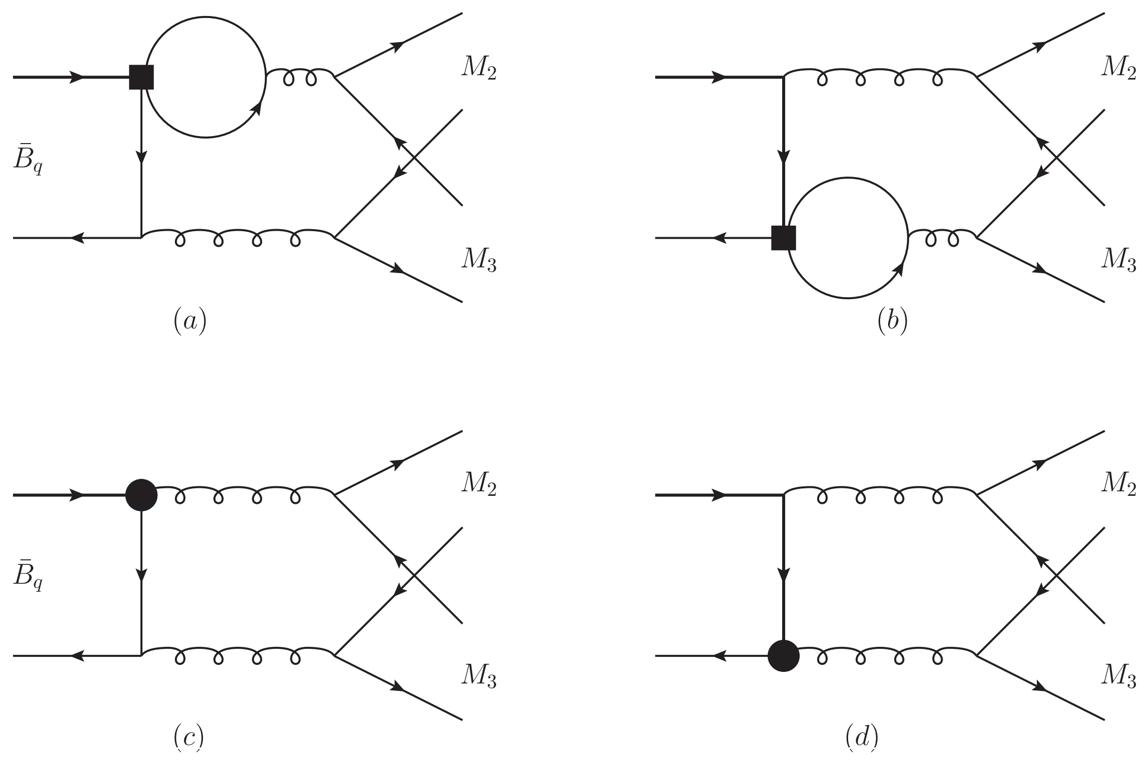
- In PQCD, annihilation amplitudes are treated as perturbatively calculable and are not power-suppressed numerically once Sudakov effects are included.

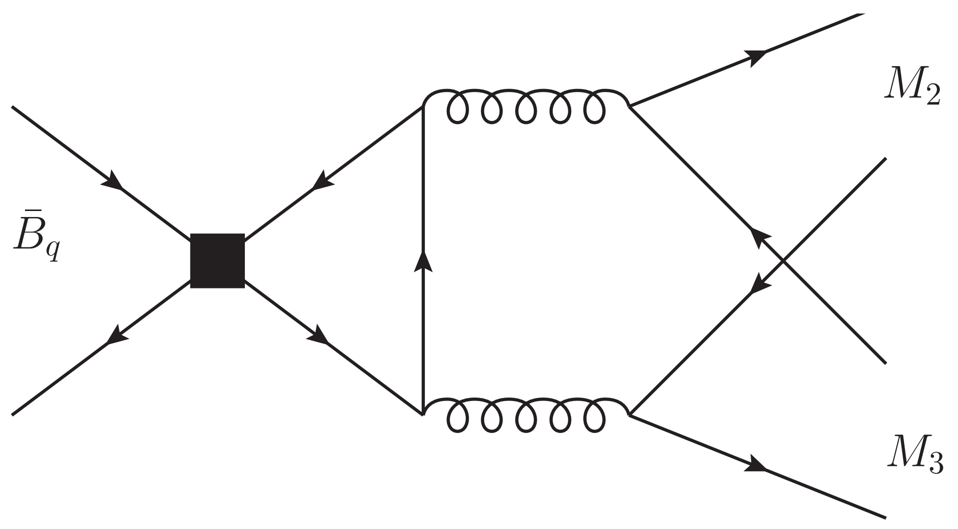
$$\frac{1}{xm_b^2} \longrightarrow \frac{1}{xm_b^2 + k_T^2}$$

- Large calculable strong phases
- CP asymmetries naturally sizable

  - Direct CP asymmetry of $B \to K\pi$
  - Pure annihilation decays $B \to K^+K^-, B_s \to \pi^+\pi^-$

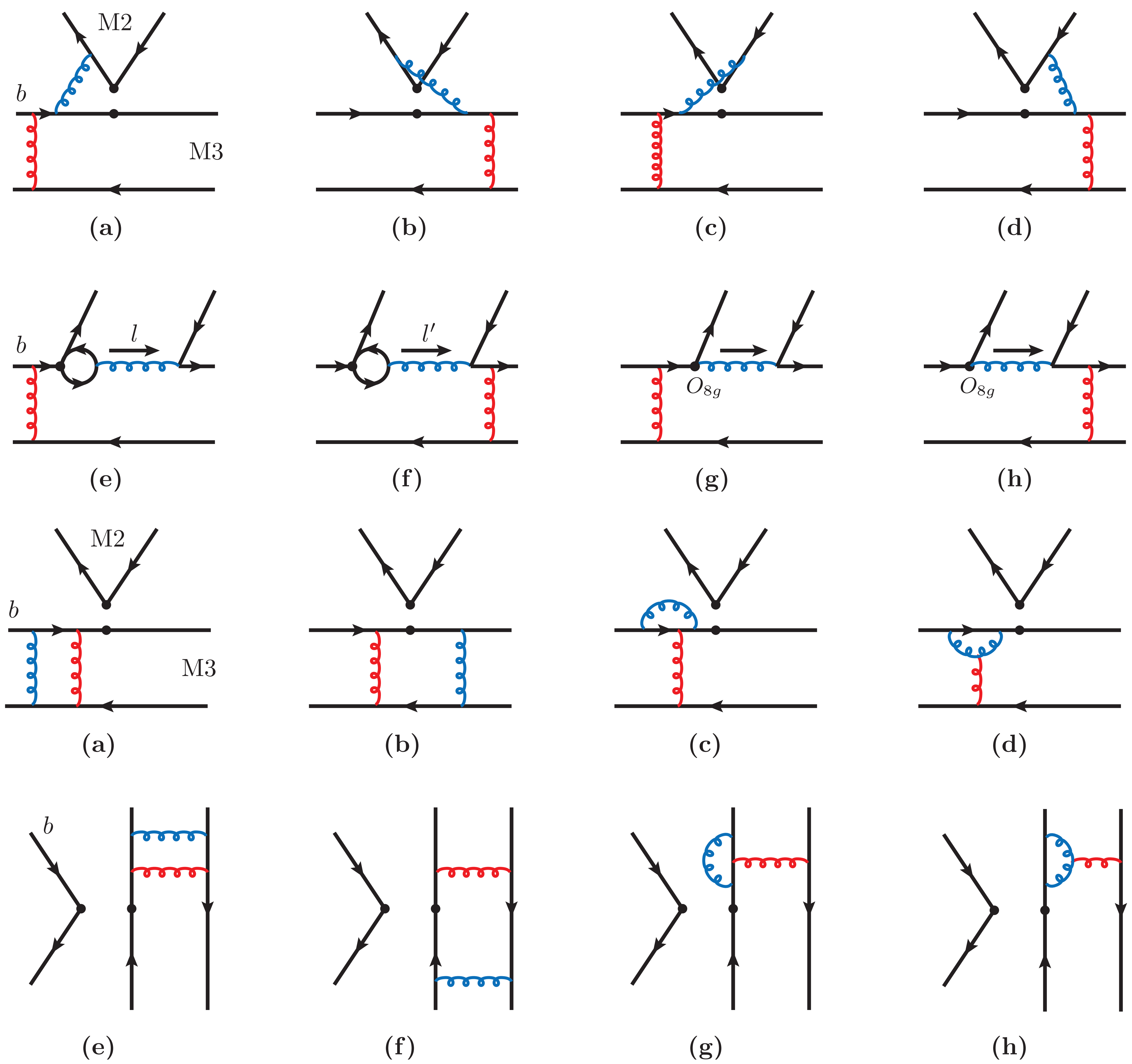
- High order contributions Sheng, Chu, Shen, Zou, 2025

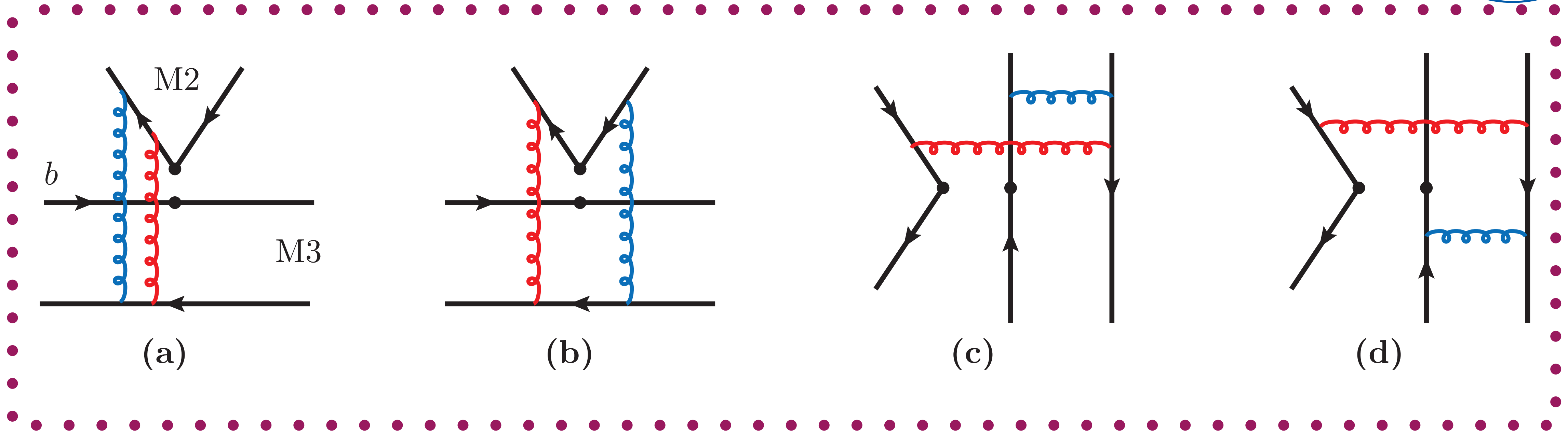




	$10^6\mathcal{B}\|_{\text{Theory}}$	$10^6\mathcal{B}\|_{\text{Exp.}}$	f_L
$\bar{B}_s \to \pi^+\pi^-$,	$0.39^{+0.19}_{-0.18}$ $(0.36^{+0.21}_{-0.18})$	0.72 ± 0.10	-
$\bar{B}_s \to \pi^0\pi^0$	$0.19^{+0.10}_{-0.09}$ $(0.18^{+0.11}_{-0.10})$	< 7.7	1.0 (1.0)
$\bar{B}_s \to \rho^0\rho^0$	$0.89^{+0.19}_{-0.17}$ $(0.82^{+0.19}_{-0.16})$	< 320	~ 1.0 (~ 1.0)
$\bar{B}_s \to \rho^+\rho^-$,	$1.71^{+0.36}_{-0.32}$ $(1.58^{+0.36}_{-0.30})$	-	~ 1.0 (~ 1.0)
$\bar{B}_s \to \omega\omega$	$0.62^{+0.27}_{-0.25}$ $(0.55^{+0.31}_{-0.25})$	-	~ 1.0 (~ 1.0)
$\bar{B}_s \to \rho\omega$	~ 0 (~ 0)	-	~ 1.0 (~ 1.0)
$\bar{B}_d \to K^+K^-$	$0.12^{+0.05}_{-0.03}$ $(0.11^{+0.04}_{-0.03})$	0.078 ± 0.015	-
$\bar{B}_d \to K^{*+}K^{*-}$	$0.14^{+0.06}_{-0.05}$ $(0.12^{+0.06}_{-0.04})$	< 0.4	~ 1.0 (~ 1.0)
$\bar{B}_d \to \phi\phi$	$0.029^{+0.010}_{-0.010}$ $(0.015^{+0.007}_{-0.005})$	< 0.027	0.99 (0.97)

PQCD： NLO Corrections

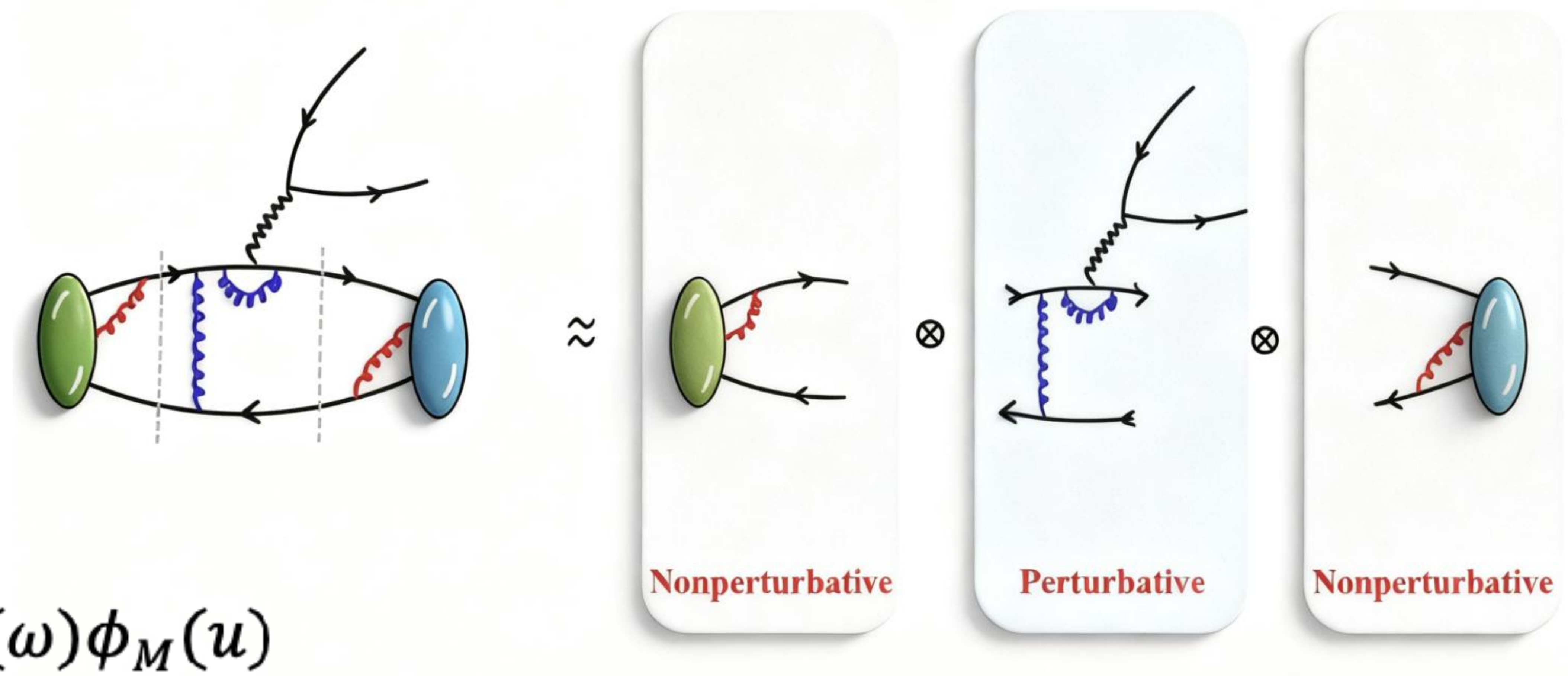




Mode	LO	+VC	+QL	+MP	$+\mathcal{F}^{\rm NLO}$	PDG [108]
$\mathcal{B}(B^+\to\pi^+\pi^0)$	3.58	3.89	$\cdots$	$\cdots$	$4.18^{+1.32}_{-0.97}$	5.5 ± 0.4
$\mathcal{A}_{\rm CP}$	−0.05	0.09	$\cdots$	$\cdots$	$0.08^{+0.09}_{-0.09}$	3 ± 4
$\mathcal{B}(B^0\to\pi^+\pi^-)$	6.97	6.82	6.92	6.76	$7.31^{+2.38}_{-1.72}$	5.12 ± 0.19
$C_{\pi^+\pi^-}$	−23.4	−27.6	−13.8	−13.3	$-12.8^{+3.5}_{-3.3}$	-32 ± 4
$S_{\pi^+\pi^-}$	−31.1	−35.5	−46.4	−37.0	$-36.4^{+1.5}_{-1.5}$	-65 ± 4
$\mathcal{B}(B^0\to\pi^0\pi^0)$	0.14	0.29	0.30	0.22	$0.23^{+0.07}_{-0.05}$	1.59 ± 0.26
$C_{\pi^0\pi^0}$	−3.1	60.1	73.6	77.6	$80.2^{+5.2}_{-6.7}$	33 ± 22
$\mathcal{B}(B^+\to\pi^+K^0)$	17.0	20.8	28.0	19.4	$20.3^{+6.3}_{-4.4}$	23.7 ± 0.8
$\mathcal{A}_{\rm CP}$	−1.19	−0.95	−0.06	−0.08	$-0.08^{+0.08}_{-0.09}$	-1.7 ± 1.6
$\mathcal{B}(B^+\to\pi^0K^+)$	10.0	12.75	16.76	11.92	$12.3^{+3.8}_{-2.7}$	12.9 ± 0.5
$\mathcal{A}_{\rm CP}$	−10.9	−5.20	2.26	2.48	$2.28^{+1.61}_{-1.74}$	3.7 ± 2.1
$\mathcal{B}(B^0\to\pi^-K^+)$	14.3	18.0	23.9	16.4	$17.1^{+5.2}_{-3.7}$	19.6 ± 0.5
$\mathcal{A}_{\rm CP}$	−15.2	−14.2	−4.16	−5.42	$-5.43^{+2.24}_{-2.34}$	-8.3 ± 0.4
$\mathcal{B}(B^0\to\pi^0K^0)$	5.90	8.12	10.4	6.99	$7.38^{+2.11}_{-1.50}$	9.9 ± 0.5
$C_{\pi^0K^0}$	−2.62	−7.31	−6.57	−7.97	$-7.70^{+0.21}_{-0.13}$	0 ± 13
$S_{\pi^0K^0}$	70.1	73.5	71.6	71.9	$71.9^{+0.6}_{-0.6}$	58 ± 17

Chai, Cheng, Ju, Yan, Lu, Xiao, 2022

Precision Calculations of Wave Function



- High precision predictions rely on reliable knowledge on nonperturbative inputs in both QCDF and PQCD

- Form factors of $B \to P$ and $B \to V$ transitions

$$F_{i,LP}^{B\to M}(E) = C_i^{(A0)}(E)\zeta_a(E) + \int_0^\infty \frac{d\omega}{\omega}\int_0^1 du\, C_i^{(B1)}(E,u)J_i(E,\omega)\phi_B^+(\omega)\phi_M(u)$$

- LCDA of light-meson: ϕ_M

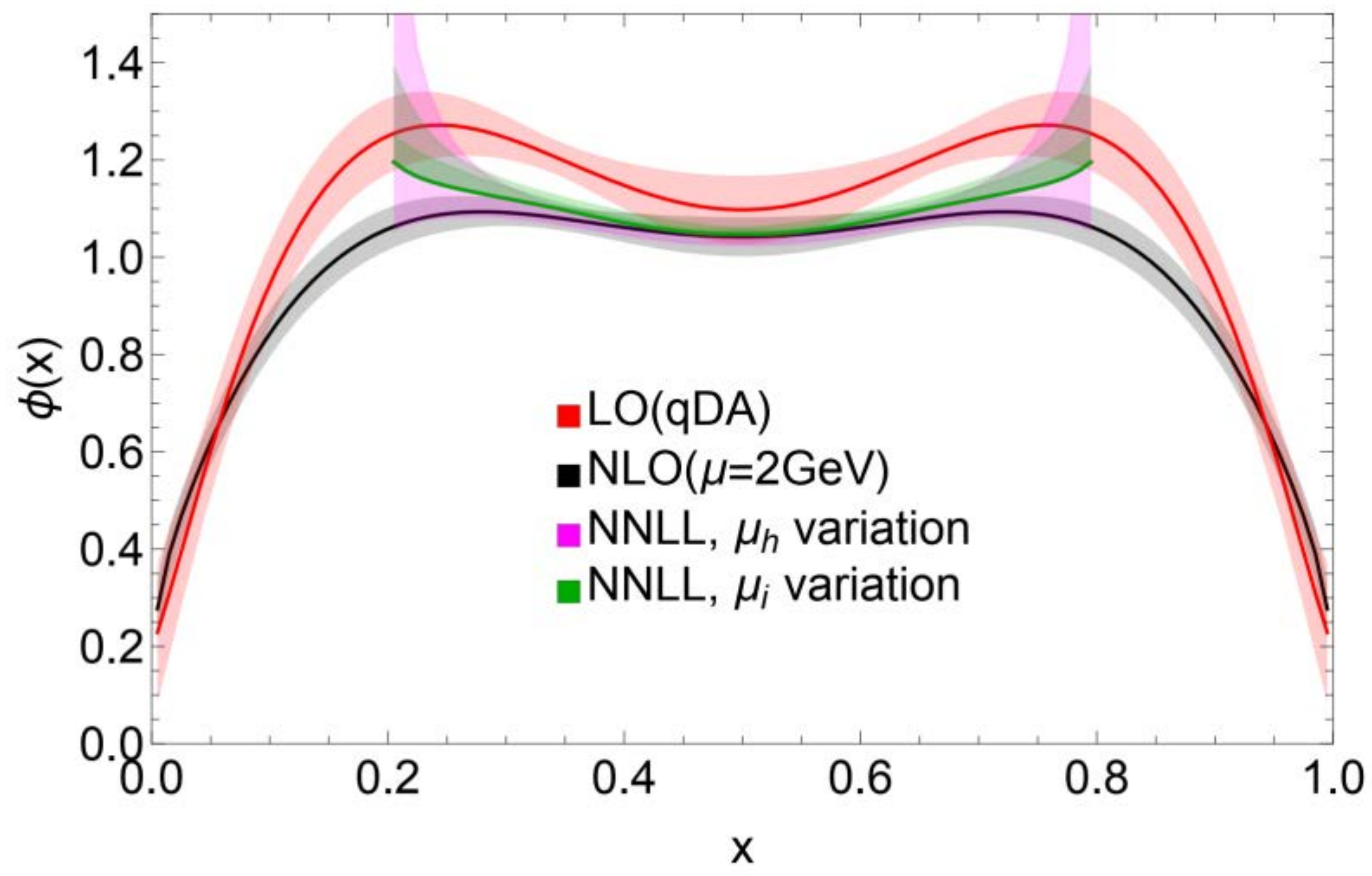
The leading twist LCDAs of light mesons are expanded in Gegenbauer polynomials: [QCDSR，LQCD]

$$\phi_\pi(x) = 6x(1-x)\Sigma a_n C_n^{3/2}(2x-1)$$

RQCD: $a_2(2\ \mathrm{GeV}) = 0.116^{+0.019}_{-0.020}$

LPC: $a_2(2\ \mathrm{GeV}) = 0.258 \pm 0.087,\ a_4(2\ \mathrm{GeV}) = 0.122 \pm 0.056,$
$a_6(2\ \mathrm{GeV}) = 0.068 \pm 0.038$

Hua, Wang, Xu, Zhang； Chen, Feng, Jia; Back, Zhao, …..



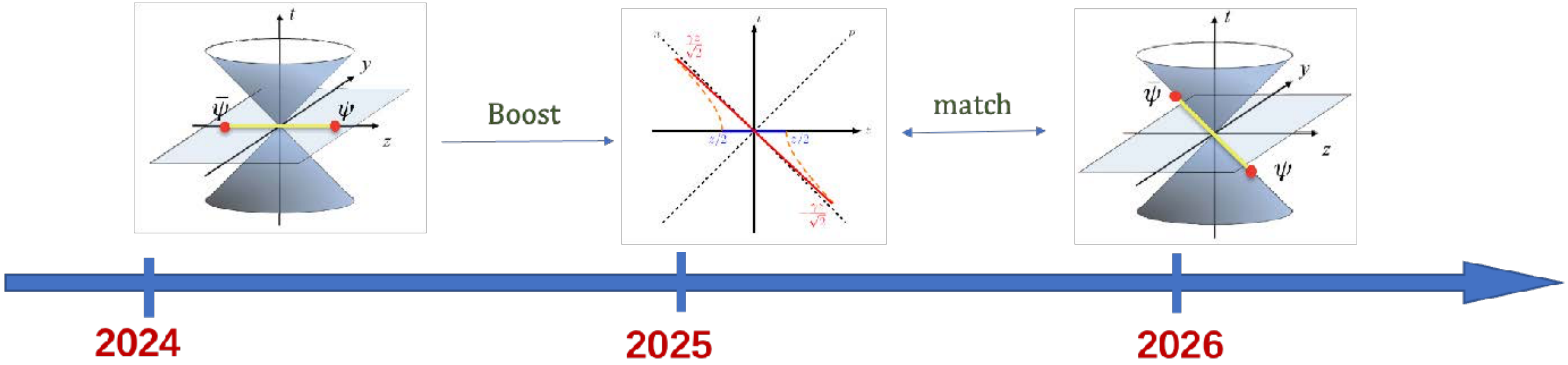
Hua, Wang, Xu, Zhang et.al, PRL 129 (2022)

Precision Calculations of WF

- [see Wang's talk]

- A heavy flavor meson consists of a pair of heavy and light quarks
- The leading twist light-cone distribution amplitudes [Wang, Wang, Xu, Zhao, 2020]

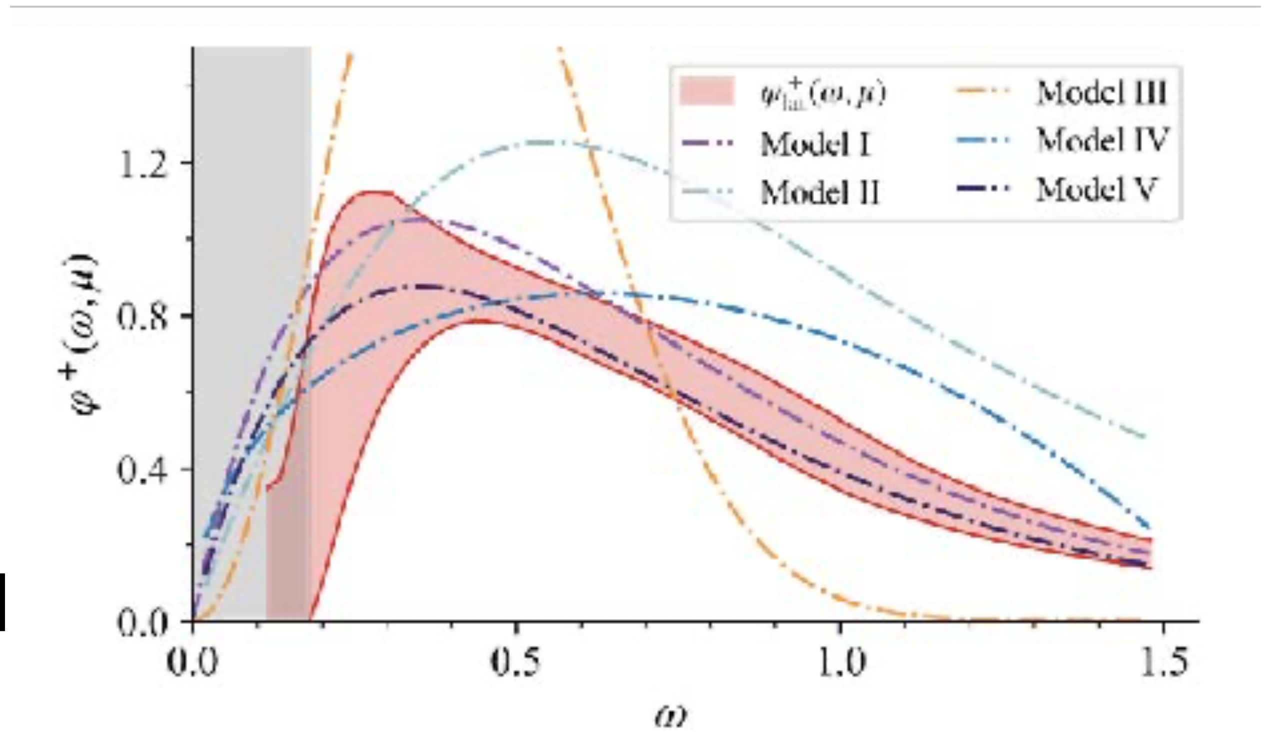
$$iF_B m_B\, \varphi_B^+(\xi,\mu) = \int_{-\infty}^{+\infty} \frac{d\tau}{2\pi} e^{in_z\cdot v\xi\tau} \langle 0|(\bar{q}_s Y_s)(\tau n_z) n_z\cdot \gamma\gamma_5 (Y_s^\dagger h_v)(0)|\bar{B}(v)\rangle$$

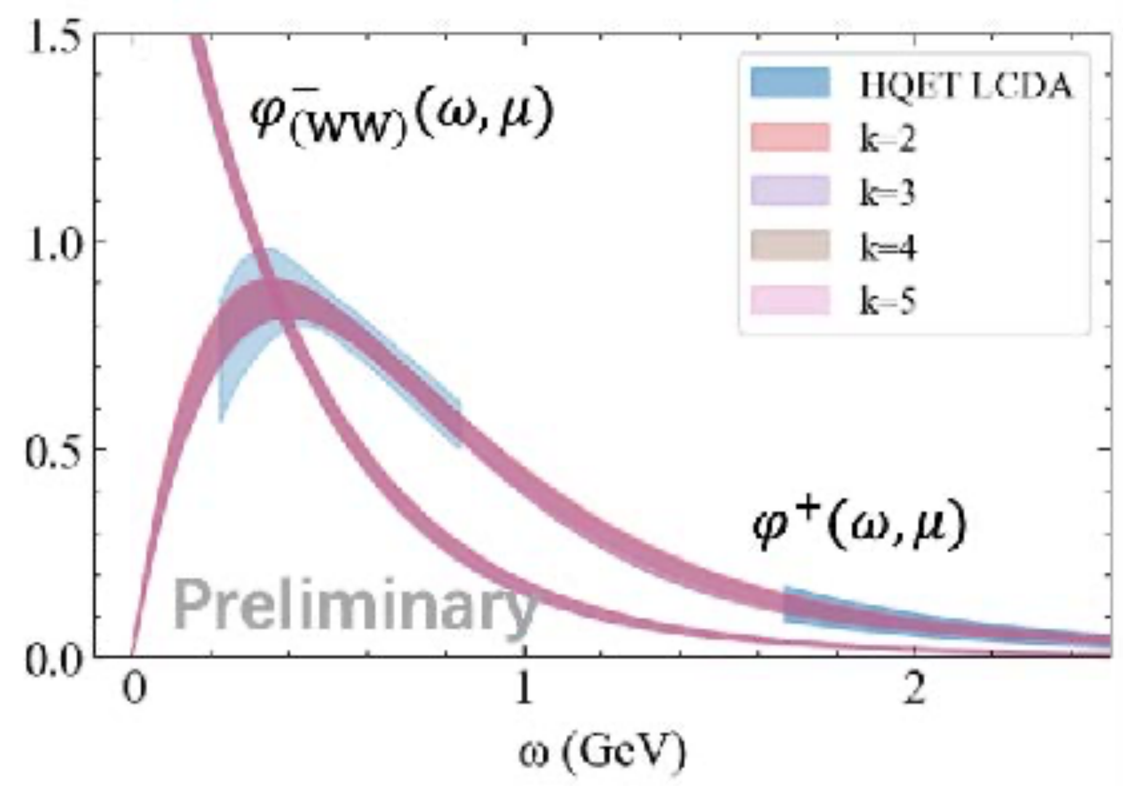


First numerical result

A more precise calculation

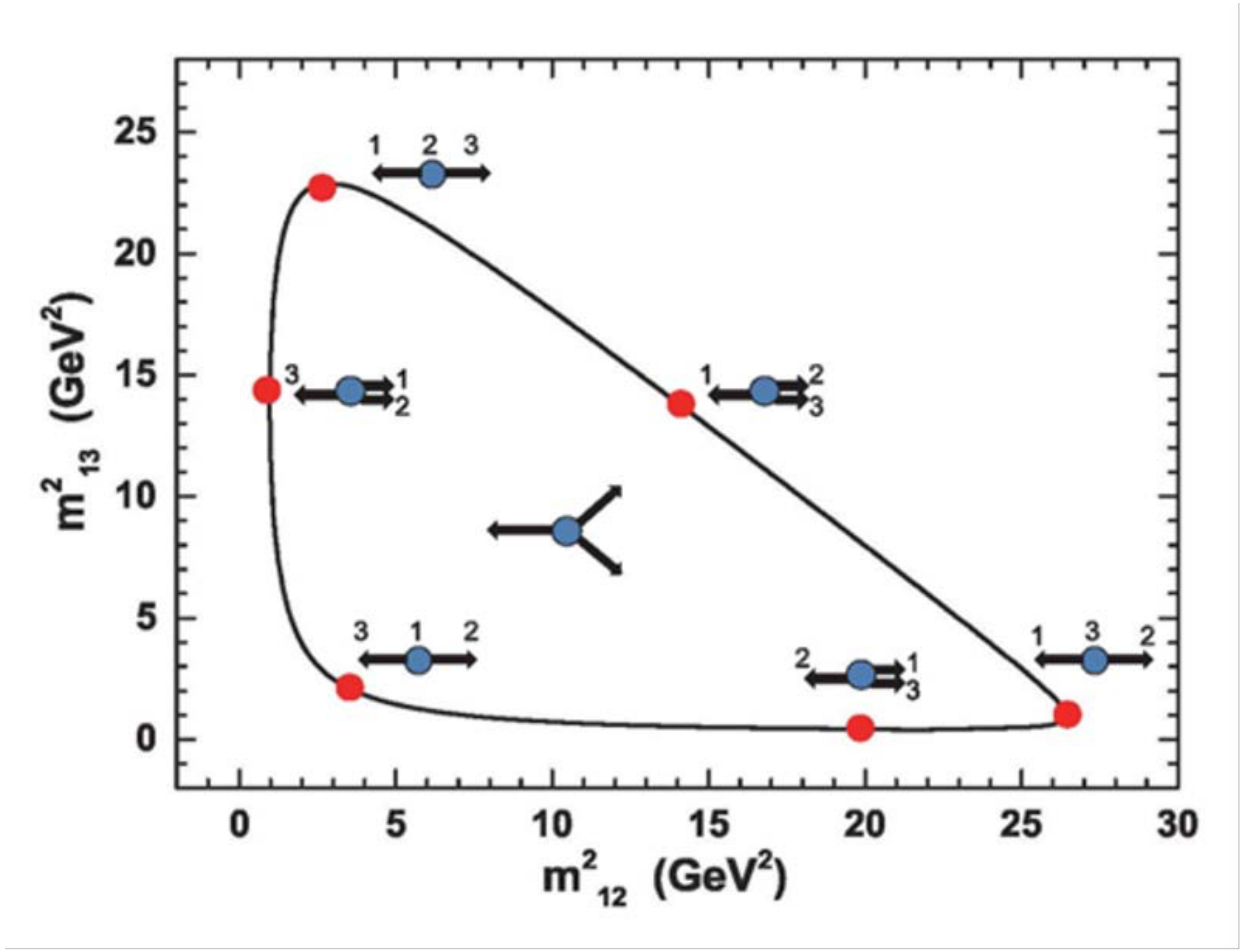
[Wang, Han, Hua, Ji, Lu, 2025]





[Wang, Han, Hua, Ji, Lu, 2026]

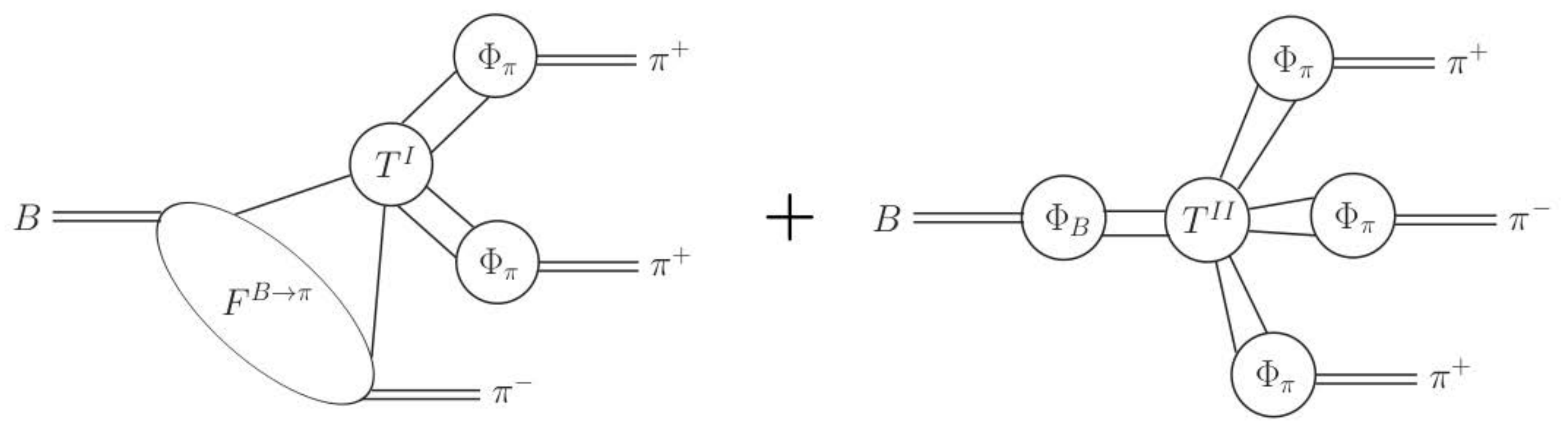
Multi-body decays of B meson

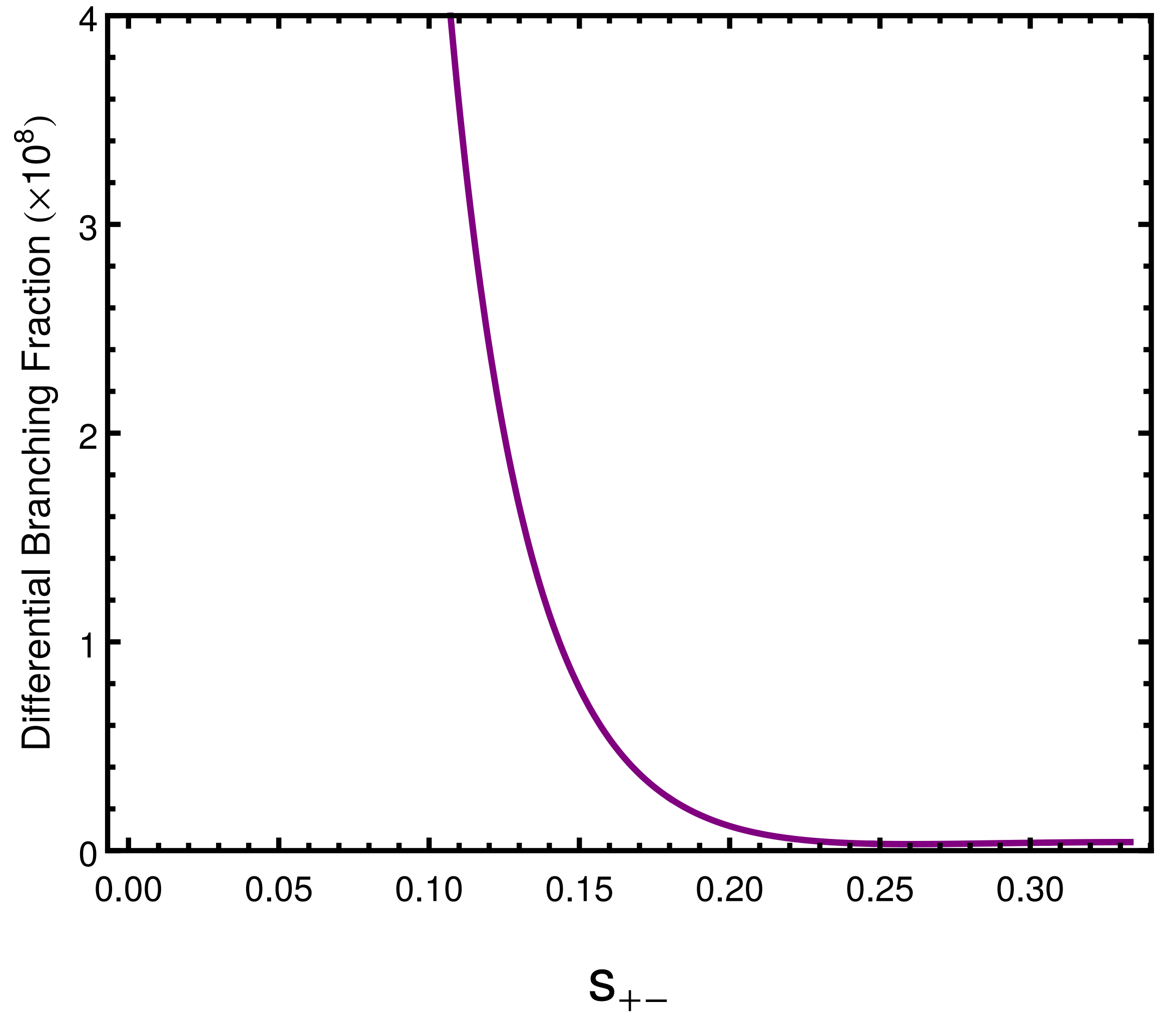


Multi-body decays of B meson: QCDF

Kränkl, Mannel, Virto, 2016

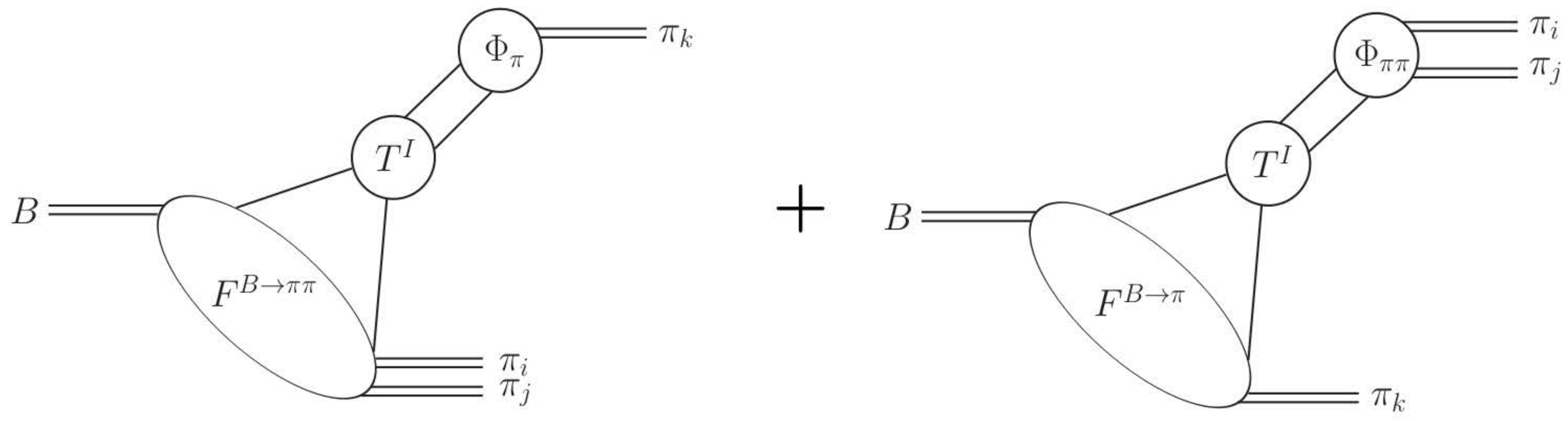
- Center Region

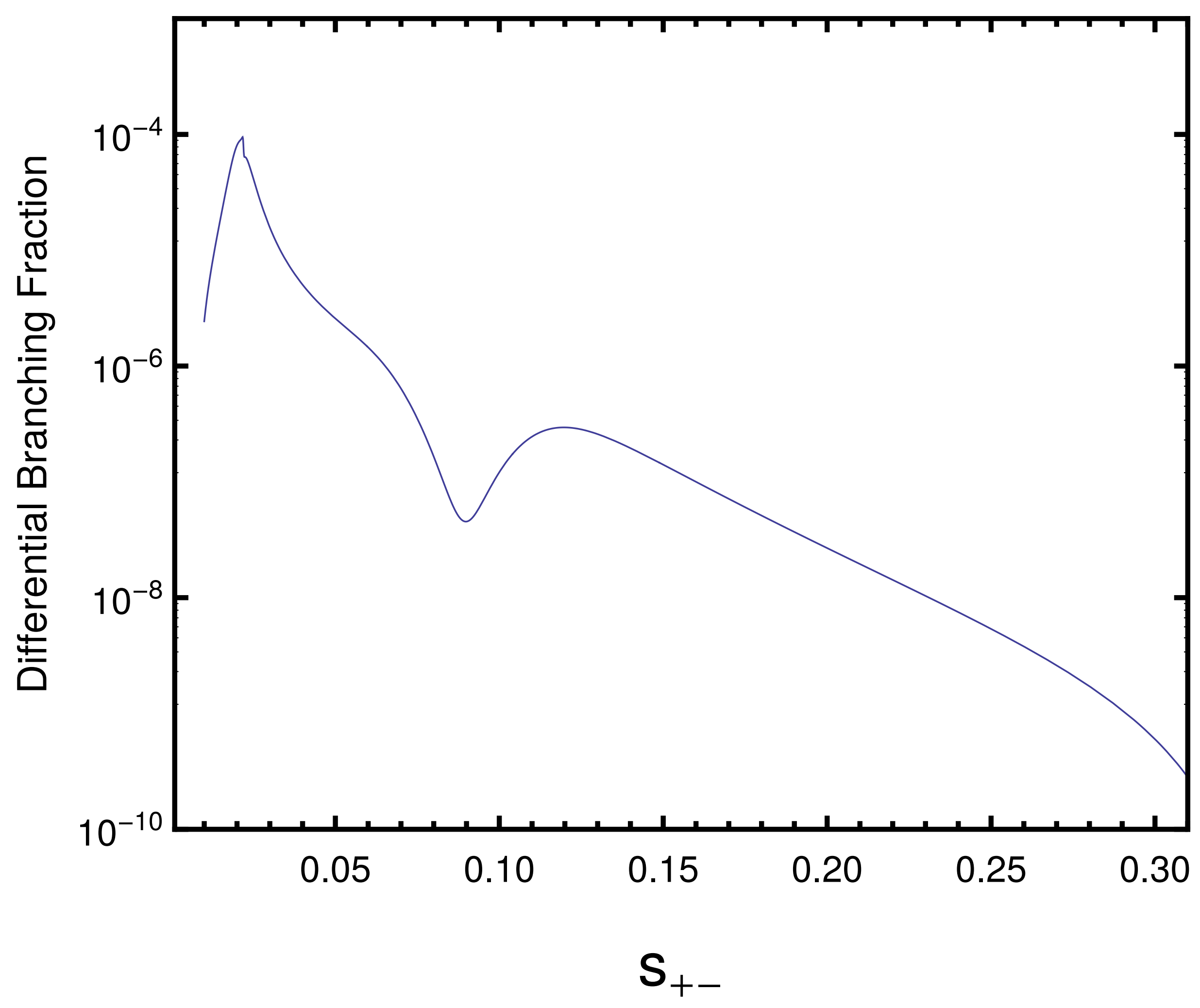




$$\langle \pi^+\pi^+\pi^- | \mathcal{O}_i | \bar{B} \rangle = F^{B\to\pi} \int du\, dv T_i^I(u,v)\Phi_\pi(u)\Phi_\pi(v)$$
$$+ \int du\, dv\, dz\, d\omega\, T_i^{II}(u,v,z,\omega)\Phi_B(\omega)\Phi_\pi(u)\Phi_\pi(v)\Phi_\pi(z)$$

- Edge Region

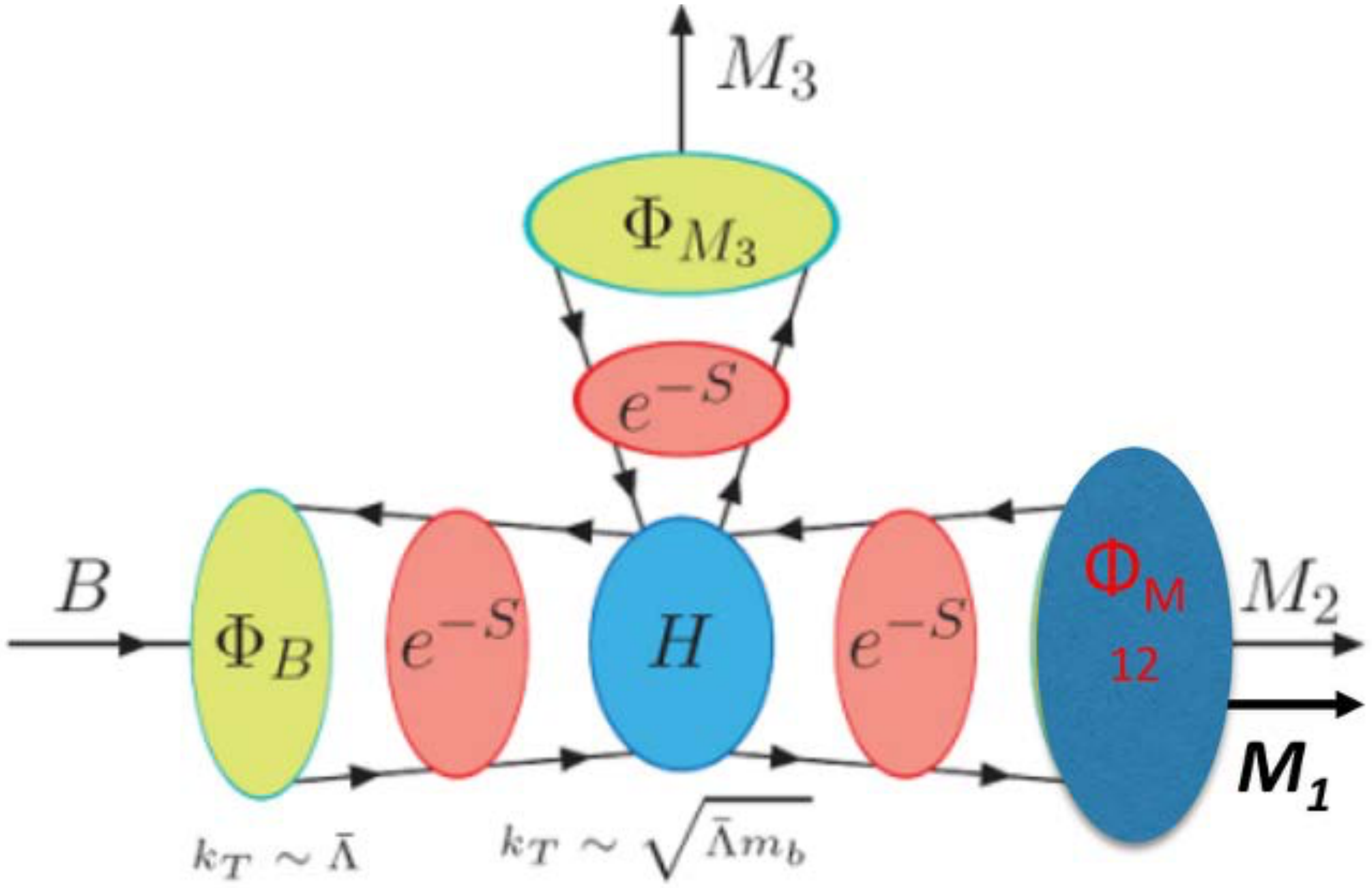




$$\langle \pi^+\pi^+\pi^- | \mathcal{Q}_i | B \rangle_{s_{+-}\ll 1} = T_i^I \otimes F^{B\to\pi^+} \otimes \Phi_{\pi^+\pi^-} + T_i^I \otimes F^{B\to\pi^+\pi^-} \otimes \Phi_{\pi^+}$$

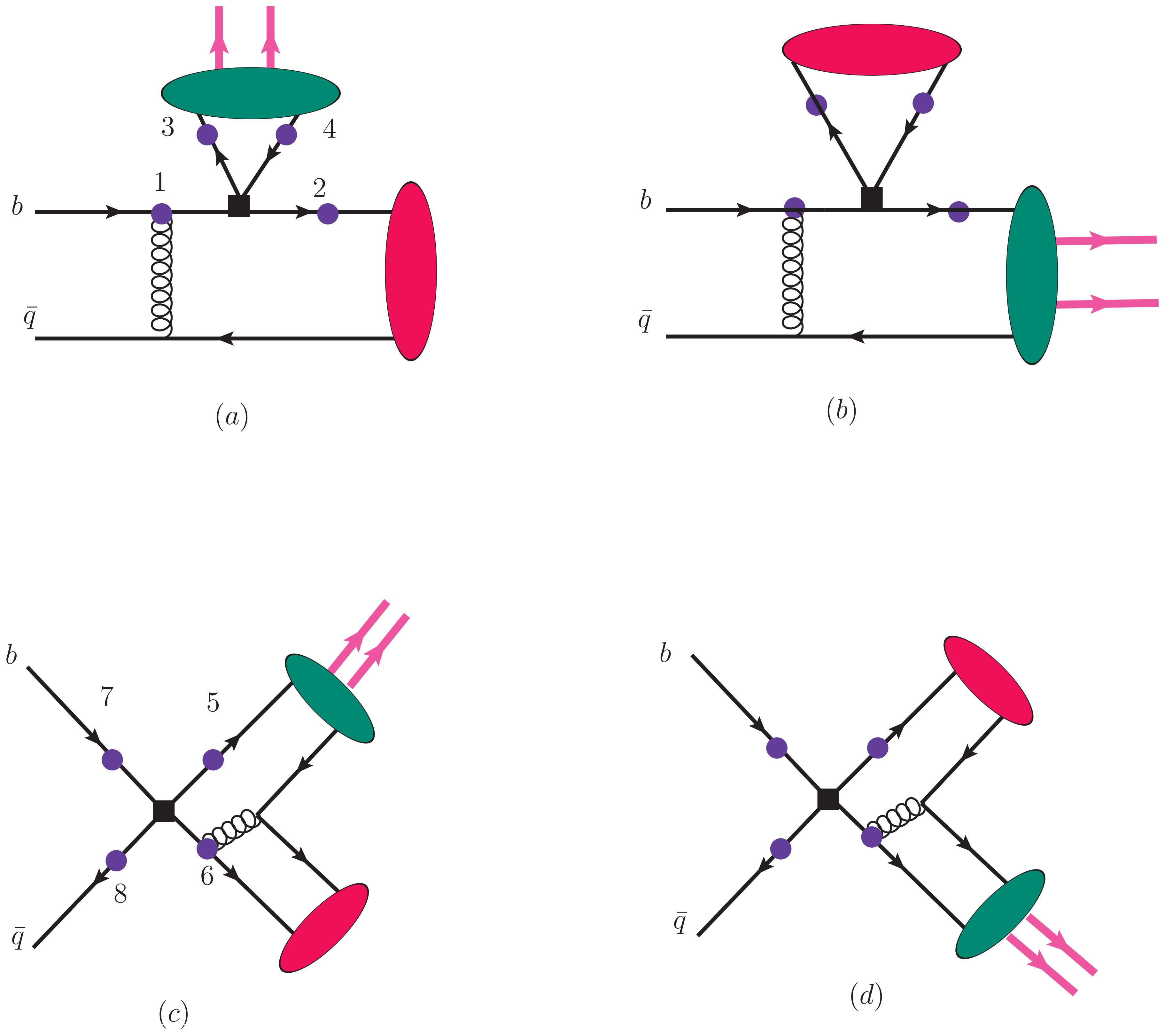
Multi-body decays of B meson: PQCD

- Framework   Wang, Wang, Li, Li, 2016



$$\Phi_B \otimes \mathcal{H} \otimes \Phi_{M_1 M_2} \otimes \Phi_{M_3} \left\{ -s(s,p) - 2\int_{1/b}^{t} \frac{d\mu}{\mu} \gamma_q(\alpha_s(\mu)) \right\}$$

- Feynman Diagram   Wang, Wang, Li, Li, Zou, YL, 2016



Multi-body decays of B meson: PQCD

- Two-meson DAs Zou,YL, Li,Liu, 2020

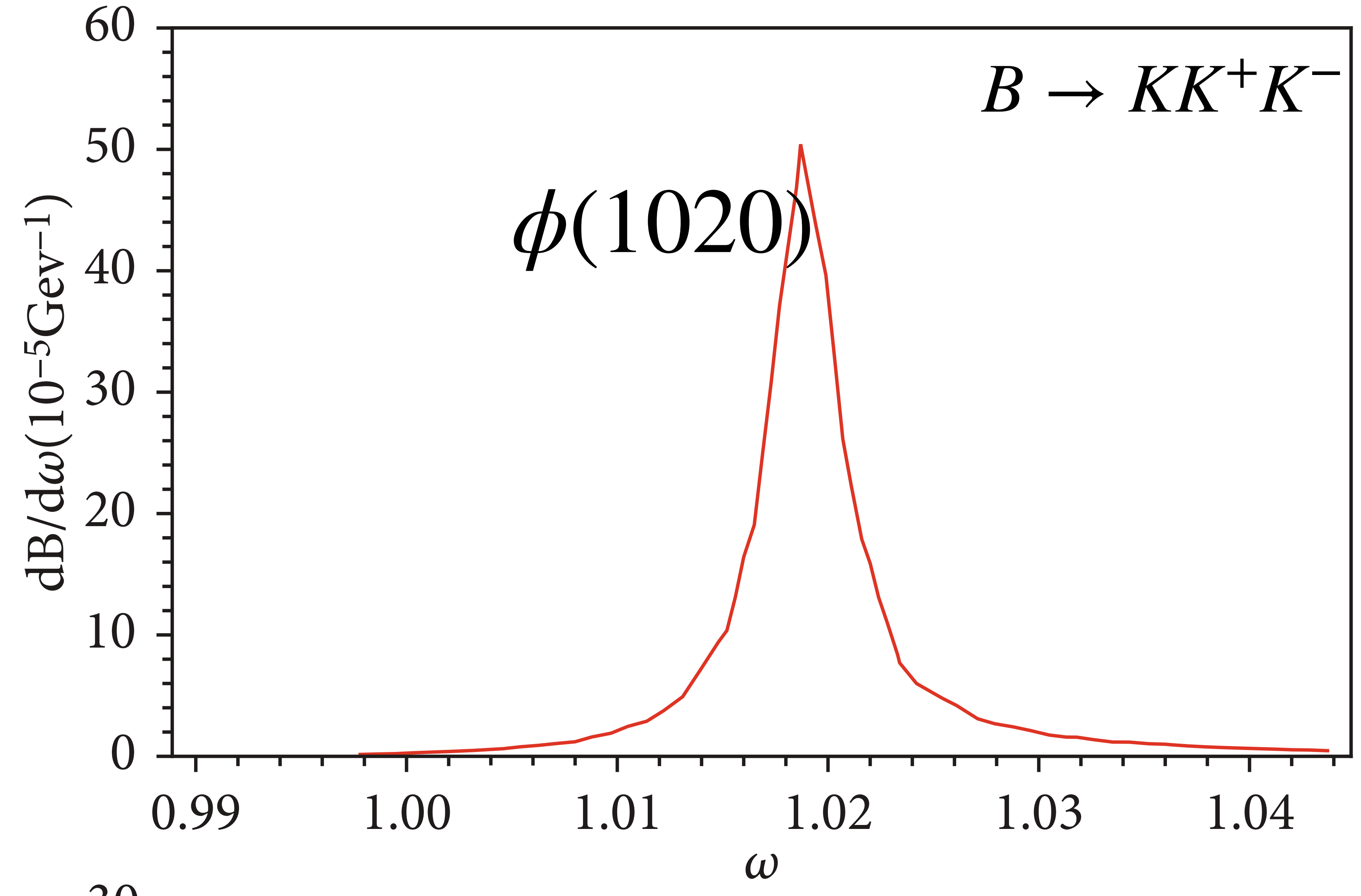
$$\Phi_P^L(z,\zeta,s)=\frac{1}{\sqrt{2N_C}}\left[\omega\not\epsilon_P\phi_P^0(z,\zeta,s)+\omega\phi_P^s(z,\zeta,s)+\frac{\not p_1\not p_2-\not p_2\not p_1}{\omega(2\zeta-1)}\phi_P^t(z,\zeta,s)\right]$$

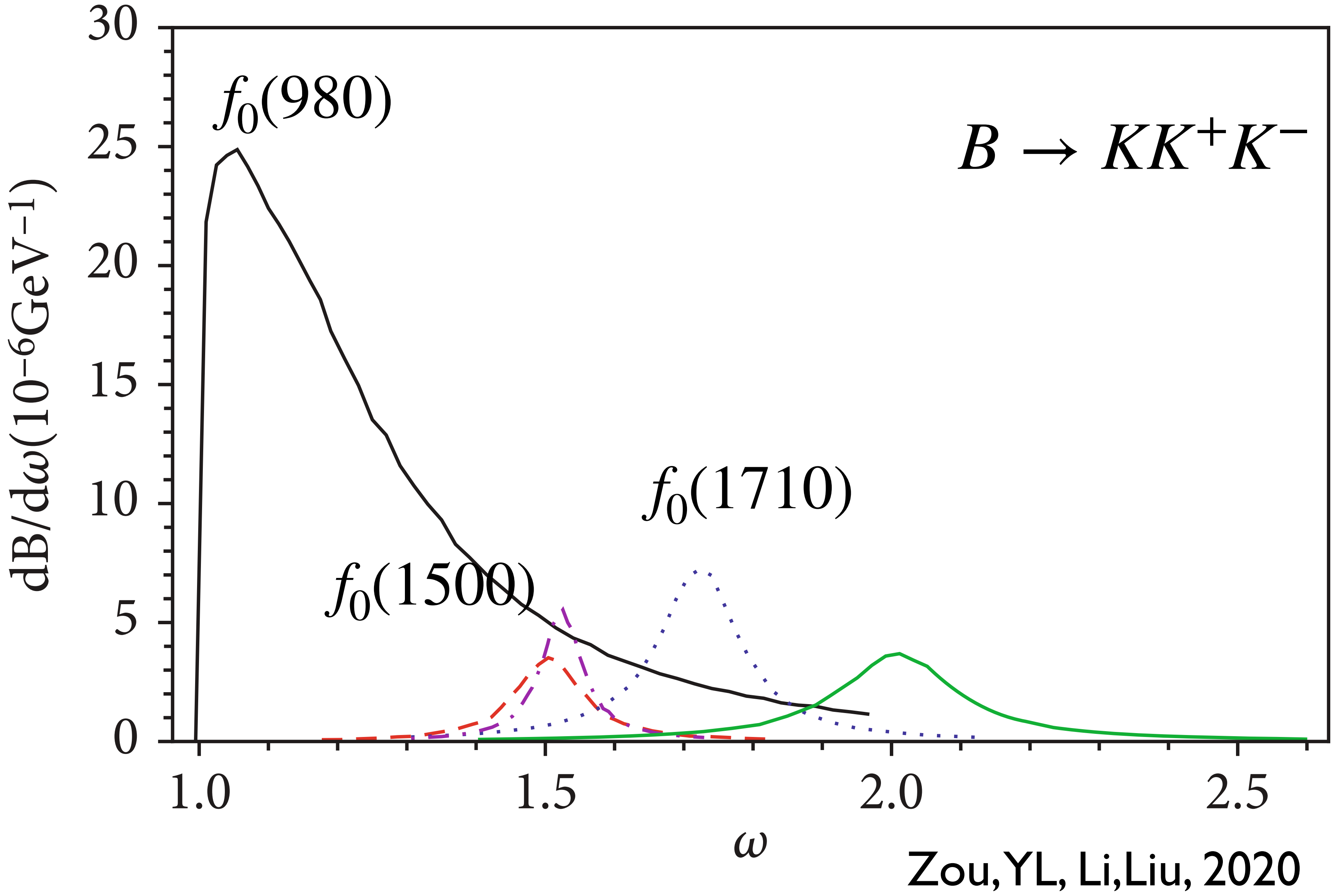
- Two-meson LCDAs

$$\begin{aligned}\phi_P^0(z,\zeta,s)&=\frac{3F_P^{\parallel}(s)}{\sqrt{2N_C}}z(1-z)\left[1+a_p^0C_2^{3/2}(1-2z)\right]P_1(2\zeta-1),\\ \phi_P^s(z,\zeta,s)&=\frac{3F_P^{\perp}(s)}{2\sqrt{2N_C}}z(1-2z)\left[1+a_p^s\left(1-10z+10z^2\right)\right]P_1(2\zeta-1),\\ \phi_P^t(z,\zeta,s)&=\frac{3F_P^{\perp}(s)}{2\sqrt{2N_C}}z(1-2z)\left[1+a_p^tC_2^{3/2}(1-2z)\right]P_1(2\zeta-1),\end{aligned}$$

- Time-like Form Factors

$$F_P(\omega^2)=\frac{m_j^2}{m_j^2-\omega^2-im_j\Gamma_j(\omega)}$$

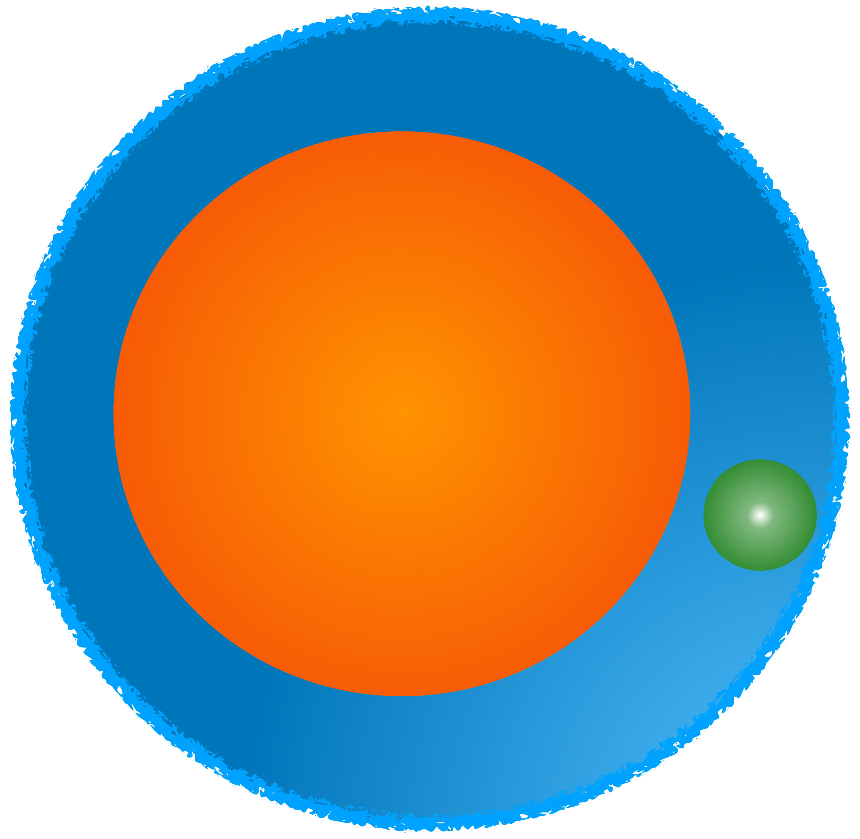


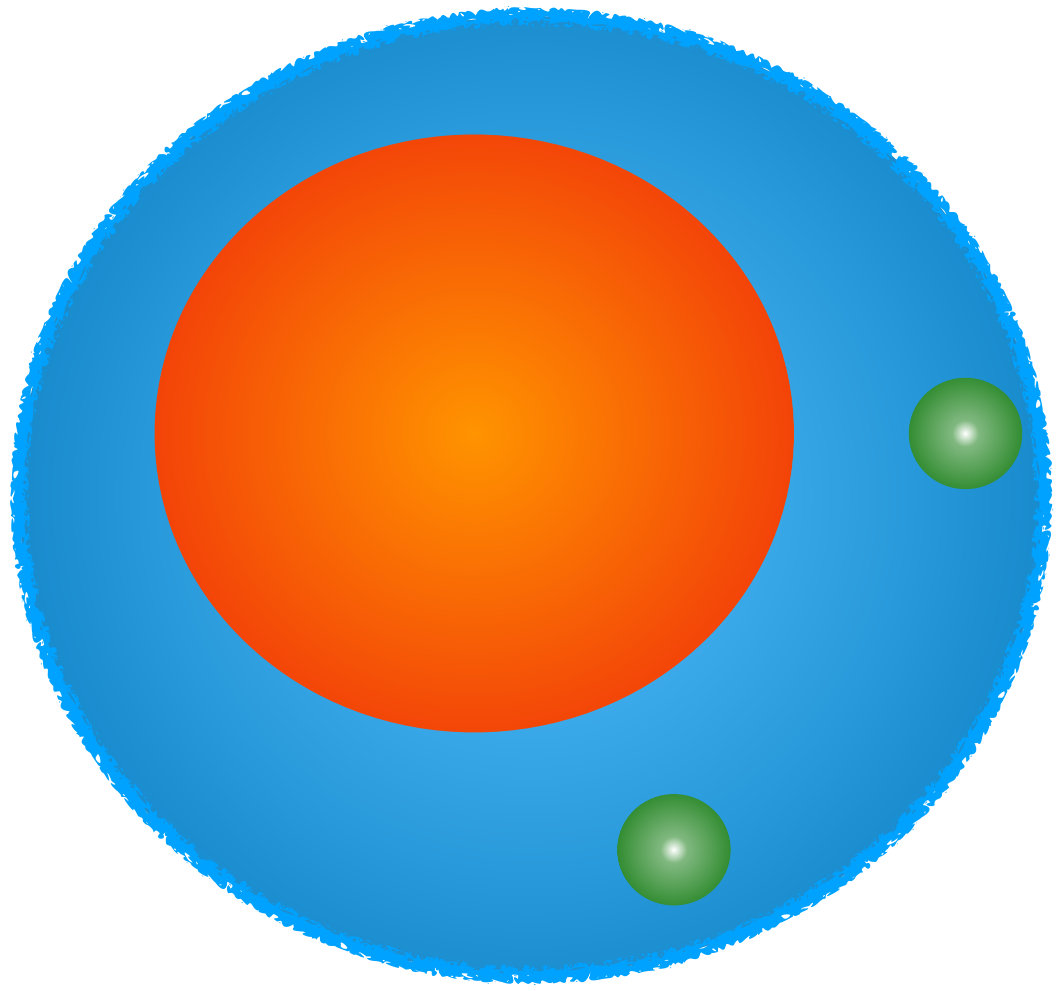


Zou,YL, Li,Liu, 2020

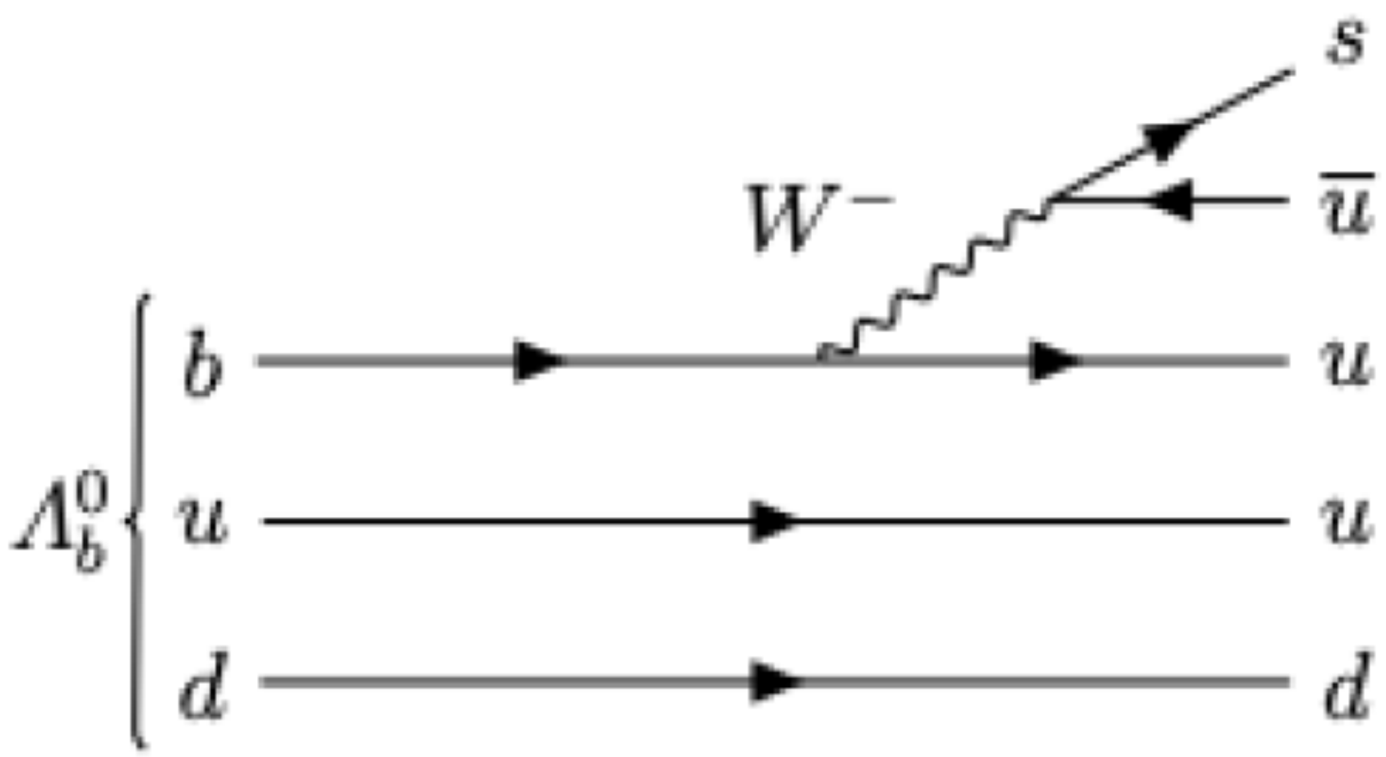
Hadronic Decays of Λ_b Baryon

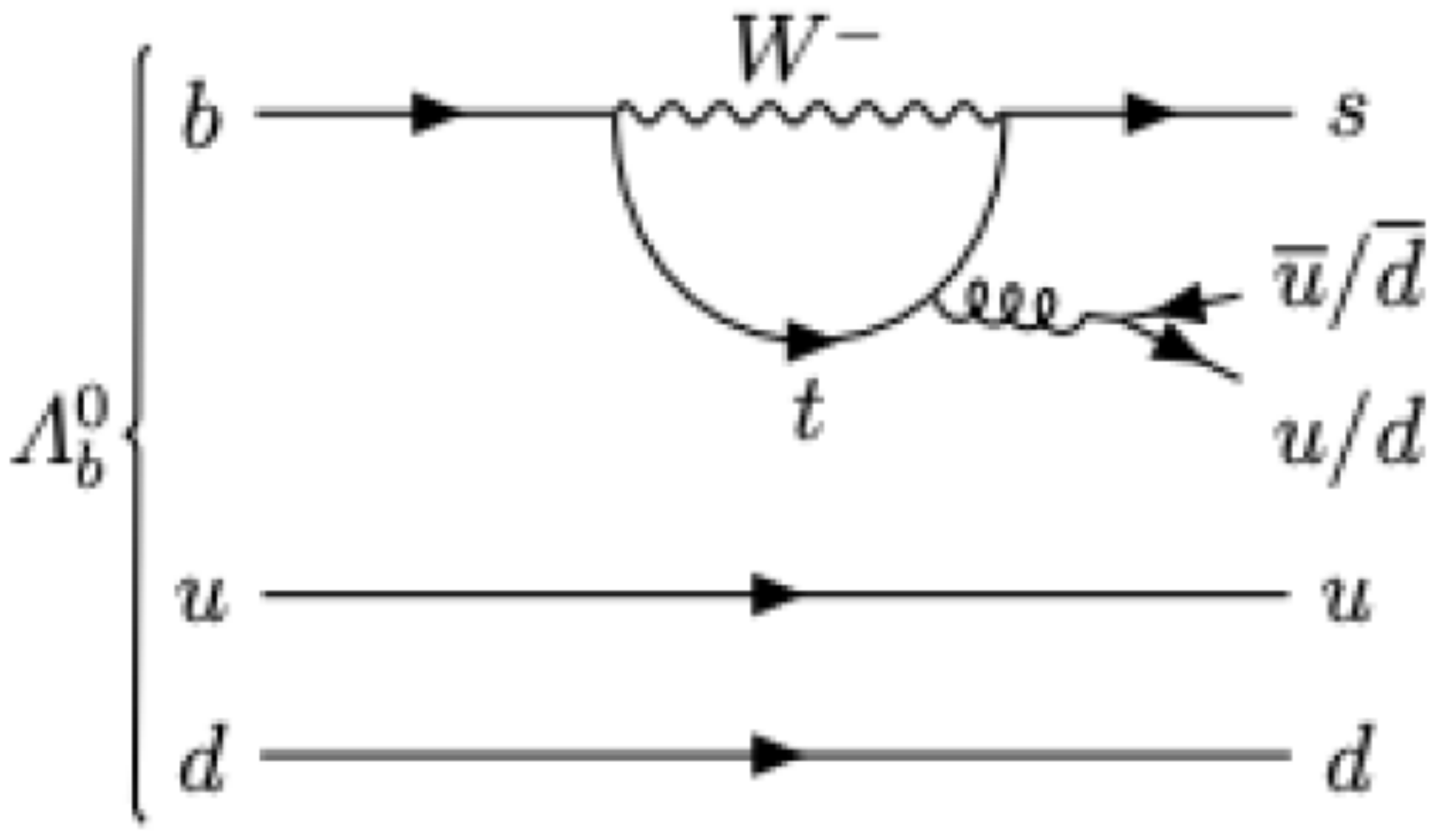
- From meson to baryon





- First Observation of CP Violation in Baryon [Fu-sheng Yu's talk]



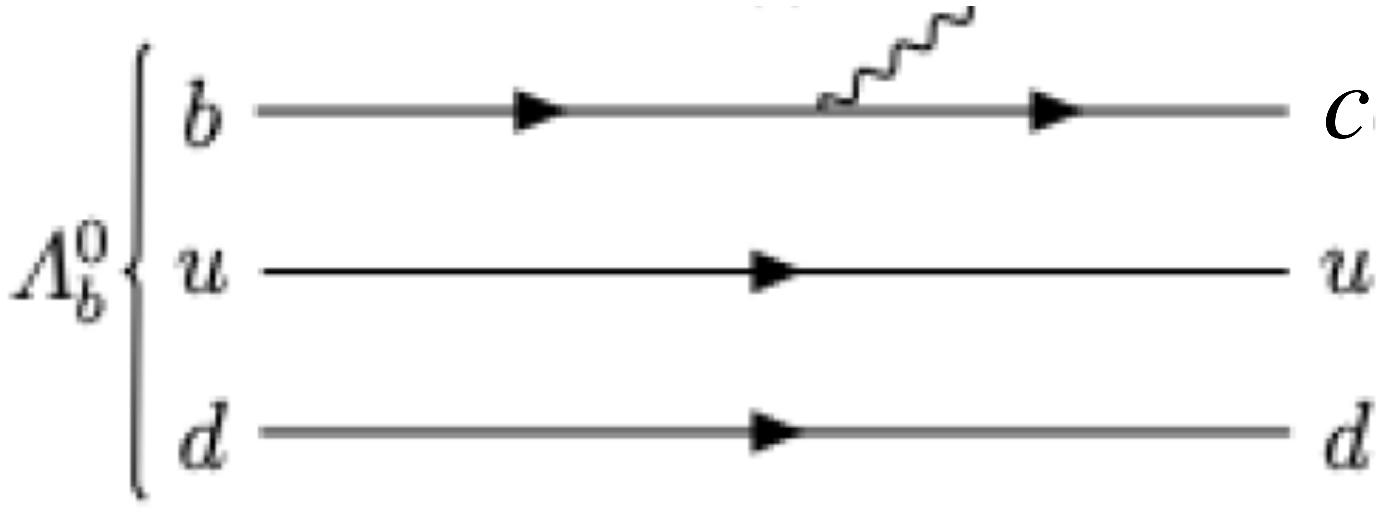


Hadronic Decays of Λ_b Baryon

- Λ_b-Baryon weak decay: $\Lambda_b \to \Lambda_c$

[Miao, Deng, Huang, Gao, Shen, 2022]

[YL, Chen, Wang, Zou, 2025]

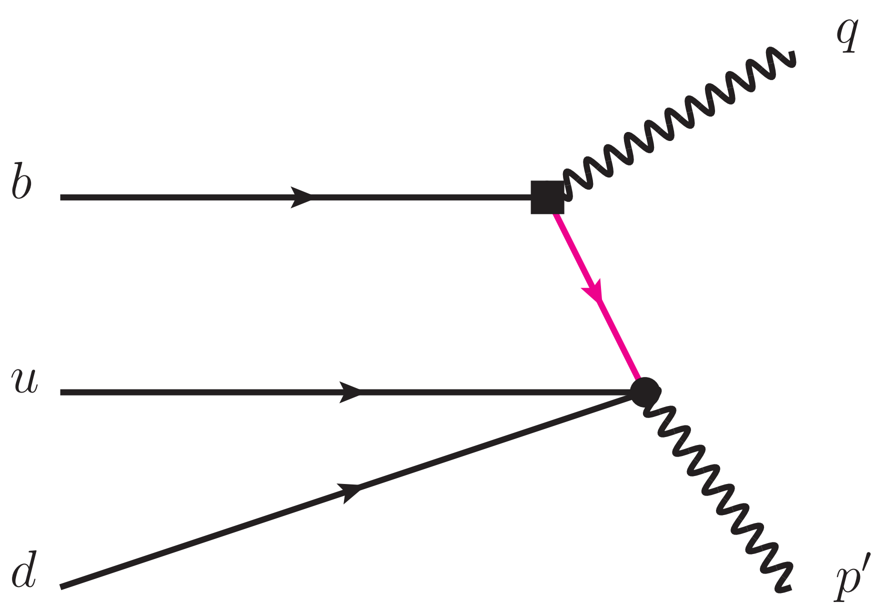


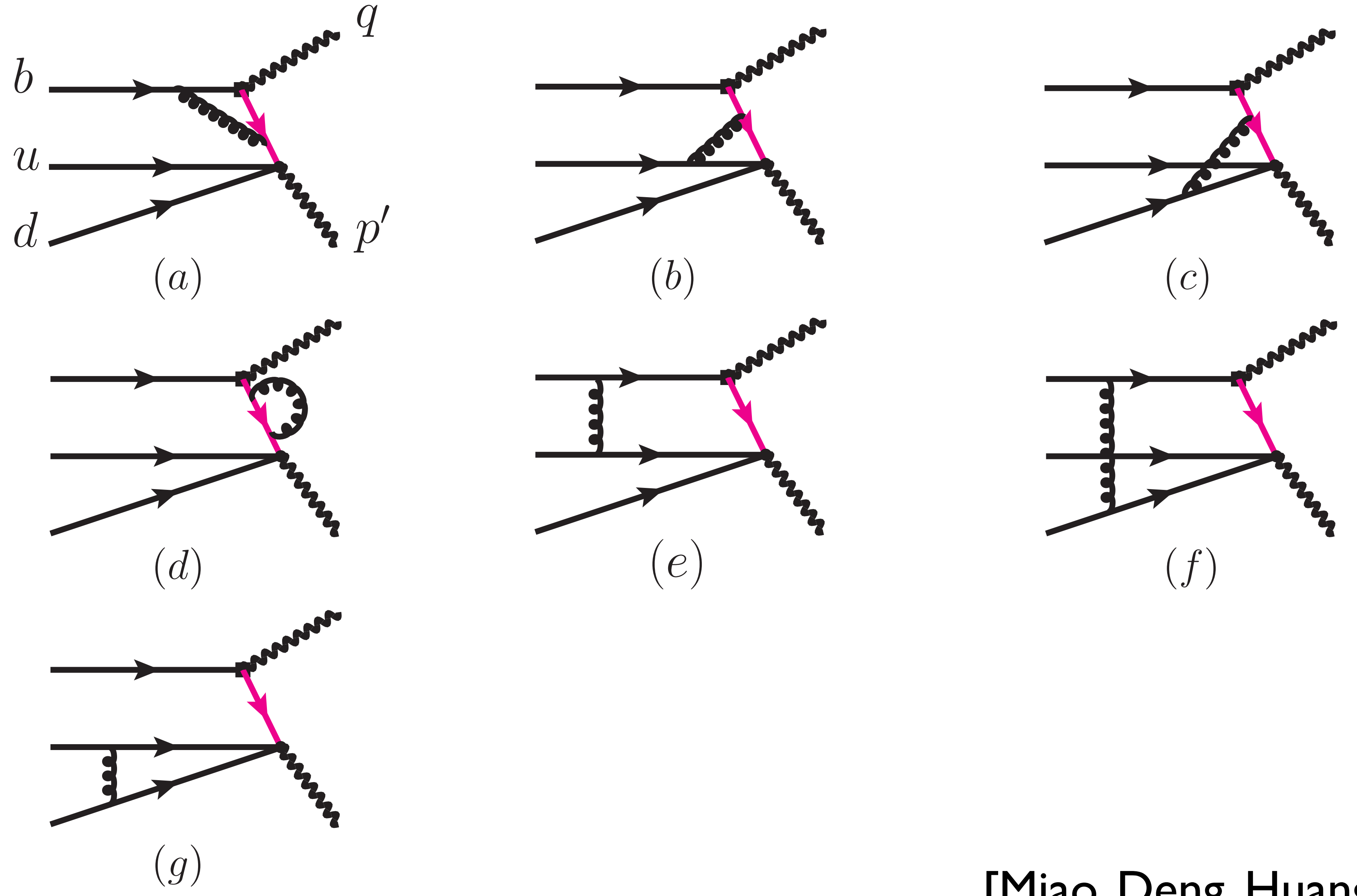
$$\langle \Lambda_c(p', s')|\bar{c}\gamma^\mu b|\Lambda_b(p, s)\rangle = \bar{u}_{\Lambda_c}(p', s')\left[f_1(q^2)\gamma^\mu - \frac{f_2(q^2)}{m_{\Lambda_b}} i\sigma^{\mu\nu}q_\nu + \frac{f_3(q^2)}{m_{\Lambda_b}} q^\mu\right] u_{\Lambda_b}(p, s),$$

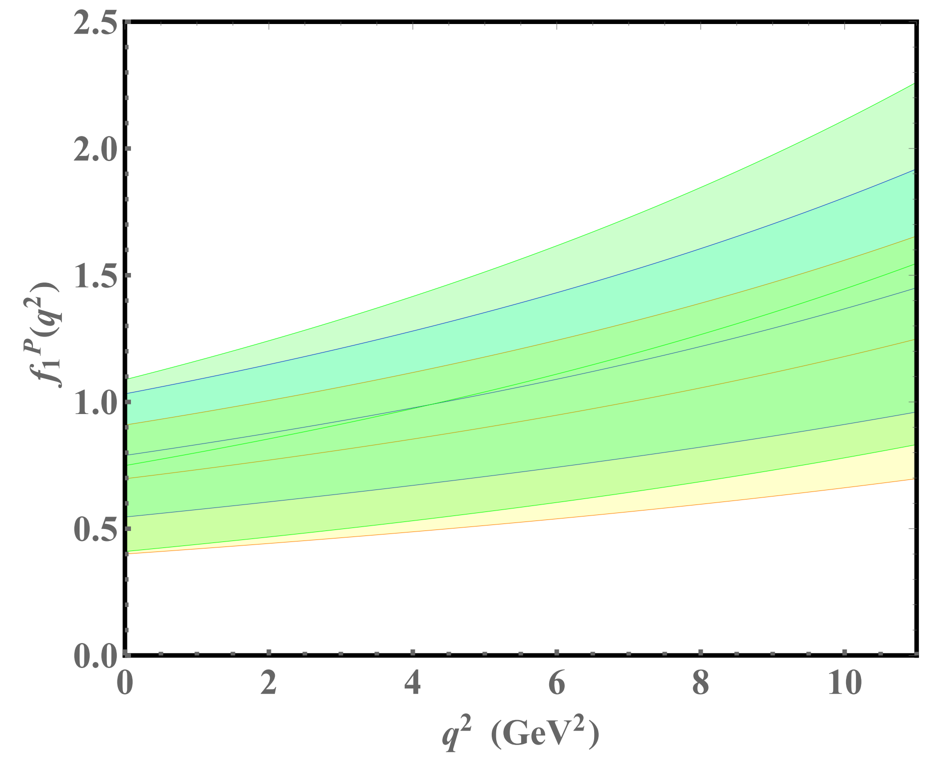
$$\langle \Lambda_c(p', s')|\bar{c}\gamma^\mu\gamma_5 b|\Lambda_b(p, s)\rangle = \bar{u}_{\Lambda_c}(p', s')\left[g_1(q^2)\gamma^\mu - \frac{g_2(q^2)}{m_{\Lambda_b}} i\sigma^{\mu\nu}q_\nu + \frac{g_3(q^2)}{m_{\Lambda_b}} q^\mu\right] \gamma_5 u_{\Lambda_b}(p, s),$$

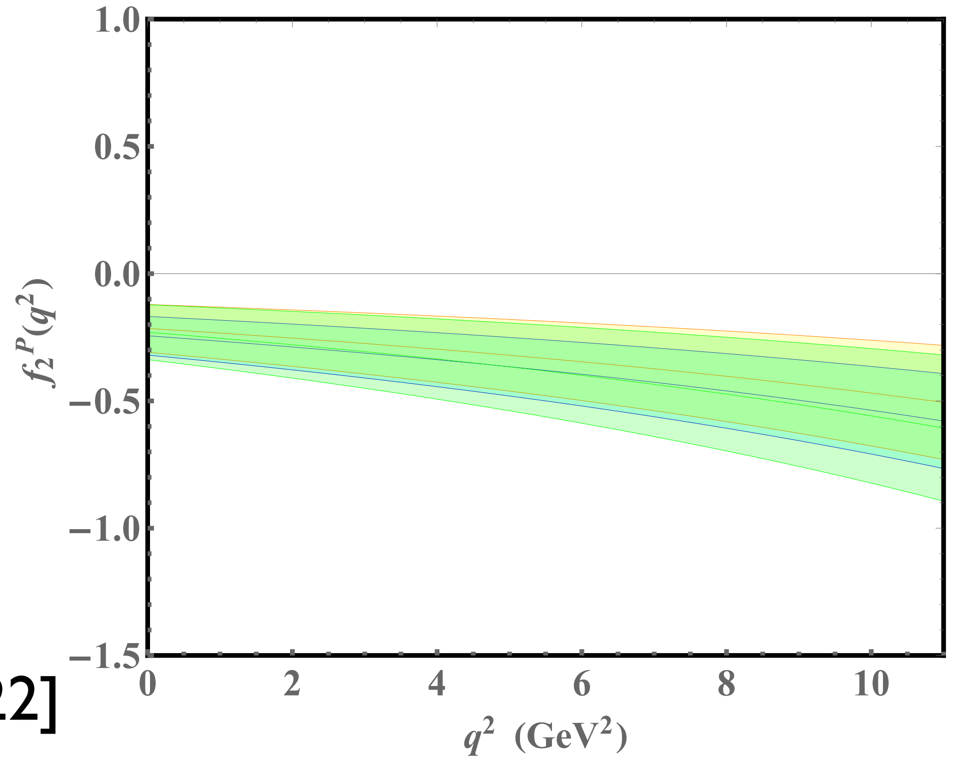
- Form Factors in LCSR

[Miao, Deng, Huang, Gao, Shen, 2022]





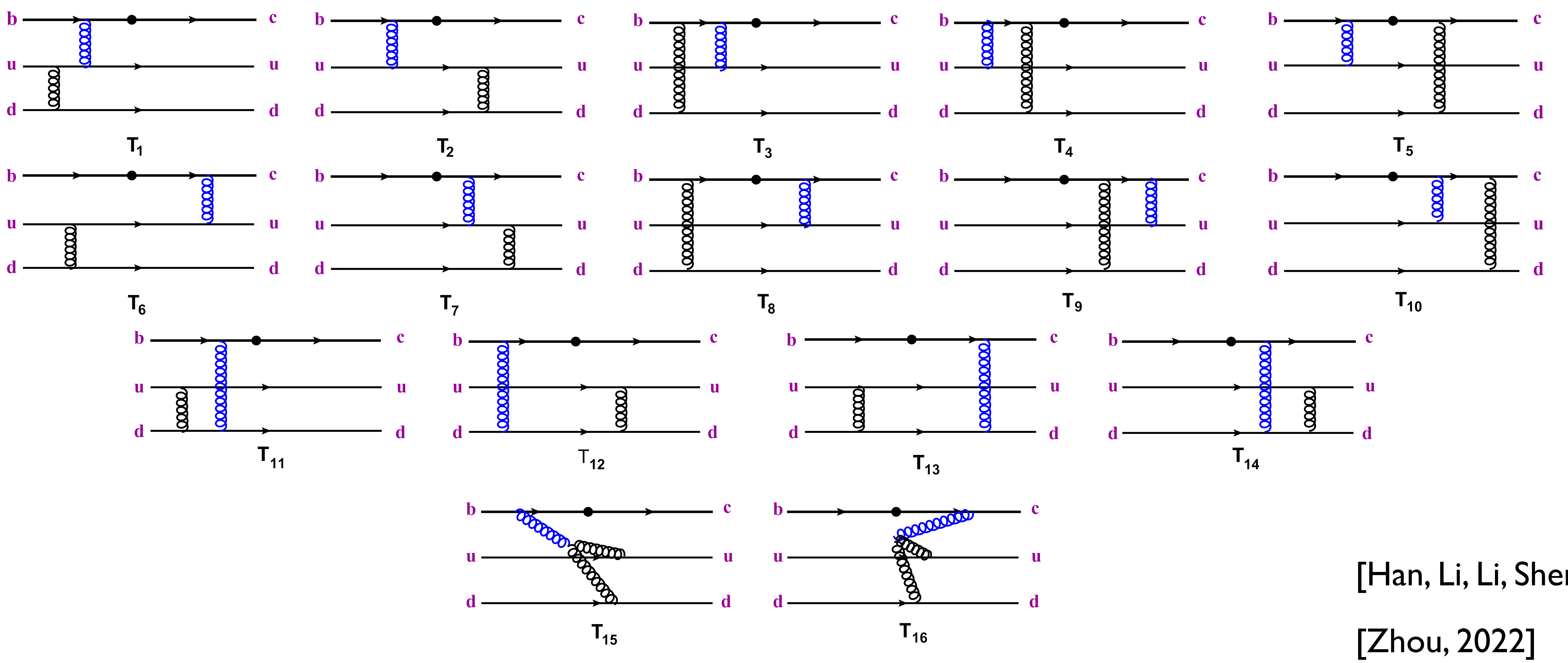




[Miao, Deng, Huang, Gao, Shen, 2022]

Hadronic Decays of Λ_b Baryon

- Form Factors in PQCD

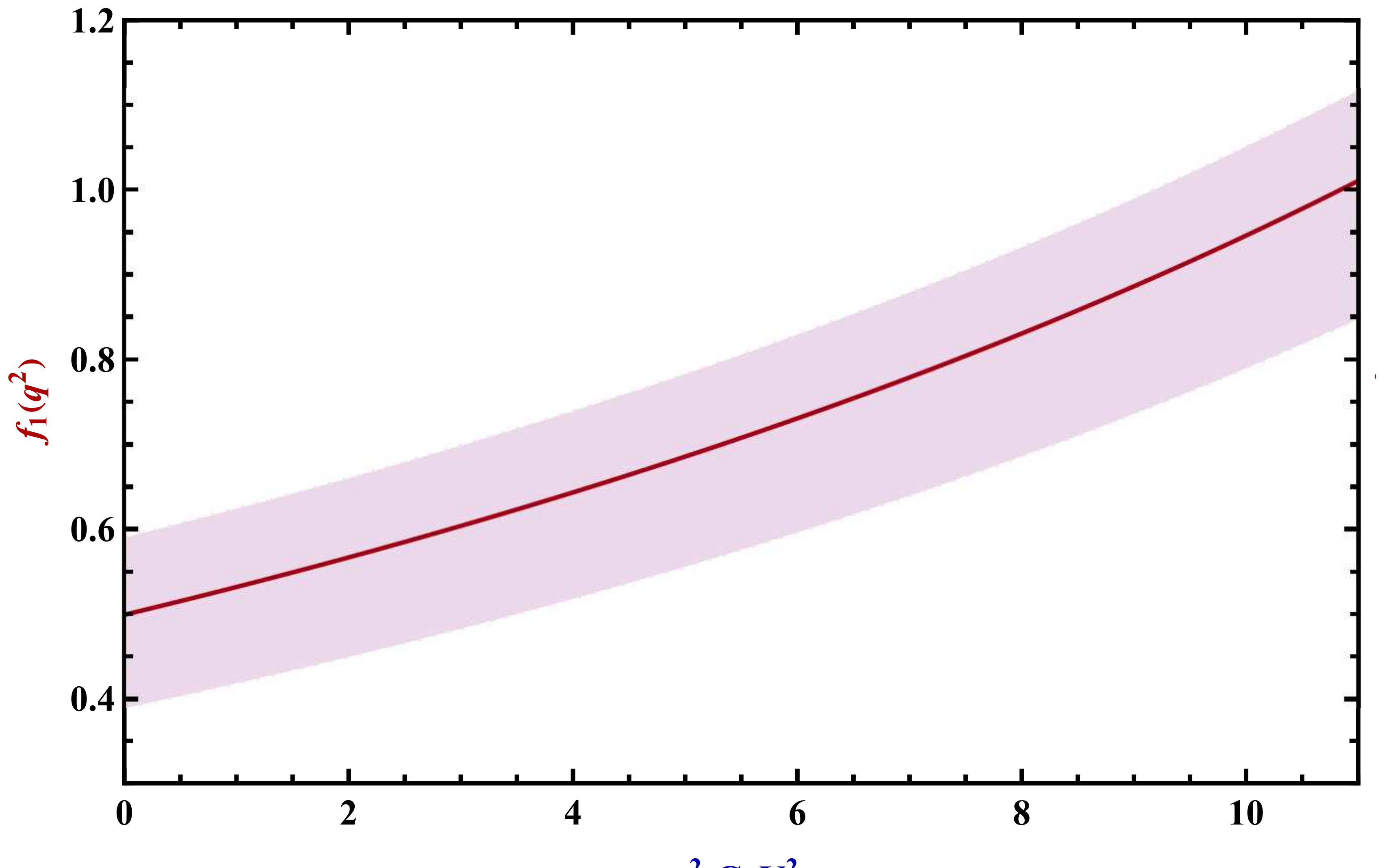


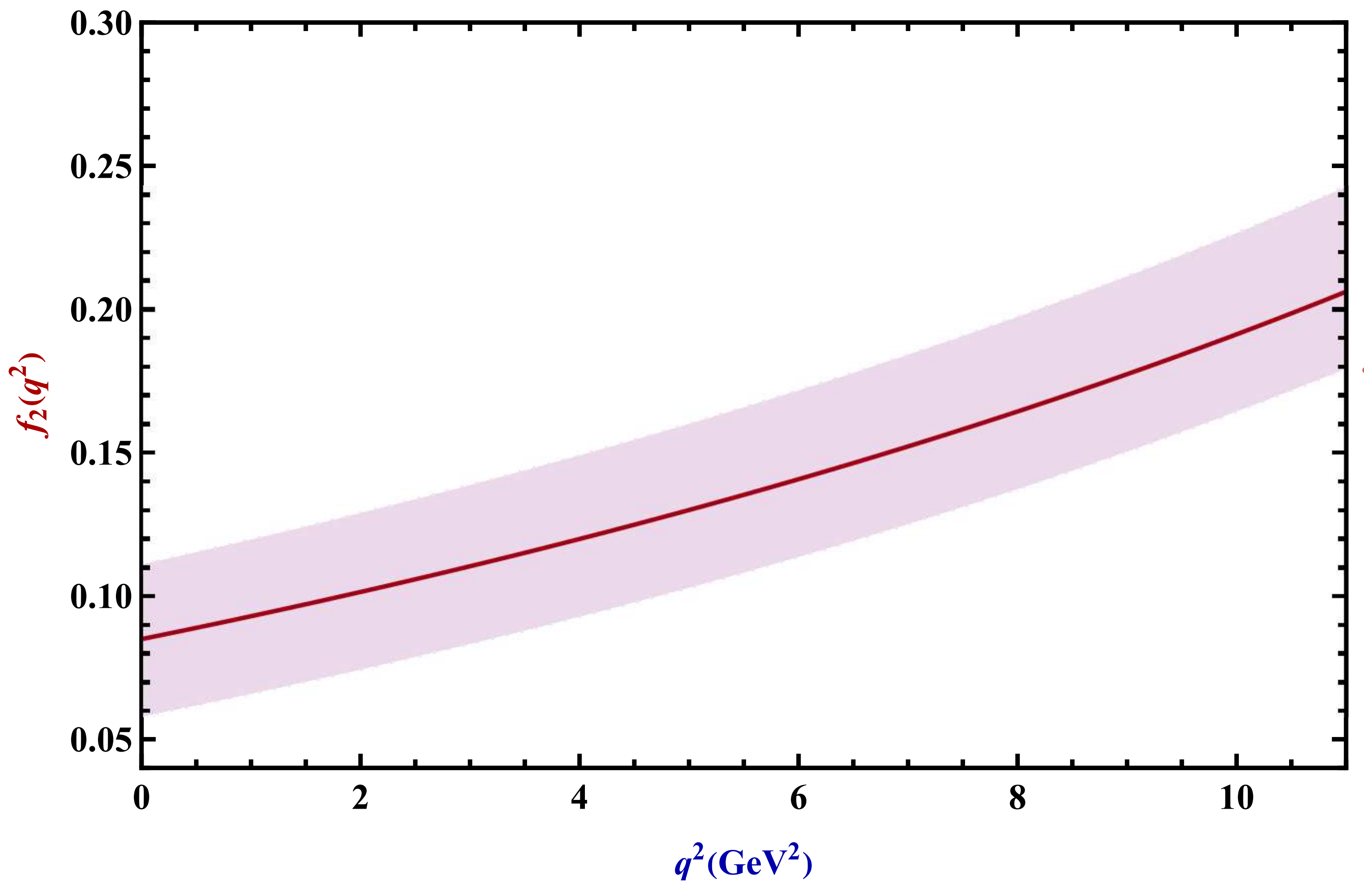
$$\mathcal{M} \sim \Psi_{\Lambda_b}(x_i, b_i, \mu) \otimes H(x_i, b_i, x'_i, b'_i, \mu) \otimes \Psi_{\Lambda_c}(x'_i, b'_i, \mu)$$

[Han, Li, Li, Shen, Xiao, Yu, 2022]

[Zhou, 2022]

[YL, Chen, Wang, Zou, 2025]





Hadronic Decays of $\Lambda_b \to p\pi$

T C E B PE

$$\mathcal{A} = \int_0^1 [dx] \int [d^2b] \Psi_{\Lambda_b} T_H \Psi_p \Psi_h e^{-S}$$

Han, Yu, Li, Li, Wang, Xiao, Yu, 2025

	$Br(\times 10^{-6})$
$\Lambda_b \to p\pi^-$	$3.34^{+2.53+1.33+0.10+0.47}_{-1.30-1.10-0.11-0.14}$
$\Lambda_b \to pK^-$	$2.83^{+2.17+1.17+0.49+2.19}_{-1.05-0.92-0.37-0.66}$

Topics not included

- Heavy hadron spectroscopy
- Inclusive decays of heavy hadrons
- Four-body B decays
- Symmetry analysis
- Charm physics
- $B \to \mathbf{BB}$
- ……

Theoretical Wishlist

- Systematic understanding of the (higher-twist) B-meson distribution amplitudes.
  - Renormalization properties beyond the one-loop approximation [conformal symmetry].
  - Perturbative constraints at large ω_i [OPE technique].
  - Rapidity/Ultraviolet subtraction to get rid of the radiative tail (short-distance effect).
  - Nonperturbative determinations from the lattice QCD simulation.

✓力耕不欺

- QCD factorization for the subleading power corrections.
  - SCET analysis for the pion-photon form factor as the first step [operator structures, symmetry constraints, etc].
  - General treatment of the rapidity divergences in the (naïve)-factorization formulae.
  - Rigorous factorization proof taking into account the Glauber gluons.
  - Novel resummation techniques for collecting enhanced logarithms.

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- Future phenomenological applications in preparation.
  - Subleading power corrections to the radiative leptonic $B \to \gamma \ell \bar{\nu}_\ell$ decays.
  - Nonfactorizable quark-loop effects for $B \to V \ell \ell$ and $B \to V \gamma$.
  - QED factorization for the exclusive bottom-meson decays.

✓探骊得珠

- Very promising future for QCD aspects of heavy-quark physics!