

半轻衰变与 CKM 矩阵元的理论研究

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Overview

I: 半轻衰变的核心问题

II: 一些最新进展

i: $|V_{cb}|$ from $\bar{B} \rightarrow X_c l \bar{\nu}_l$

ii: $|V_{ub}|$ from $B \rightarrow \pi \pi l^+ \nu_l$

III: 总结和展望

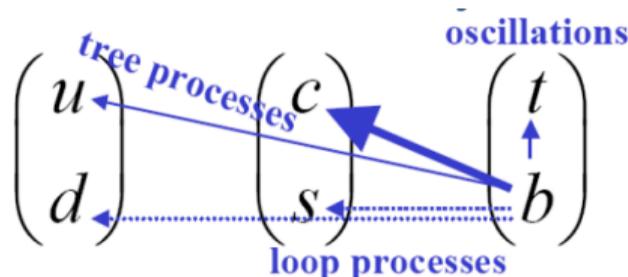
- rotation between the weak interaction eigenstates and the mass eigenstates

$$V_{\text{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

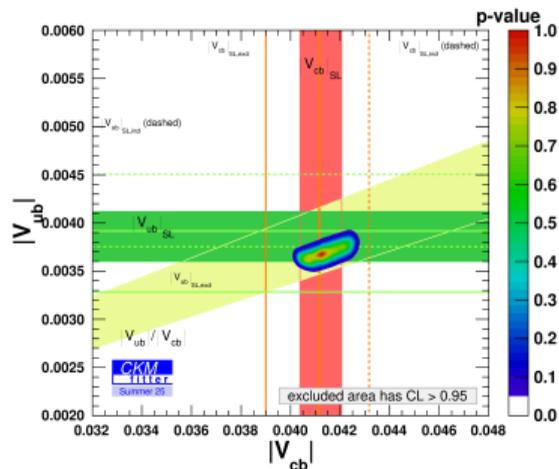
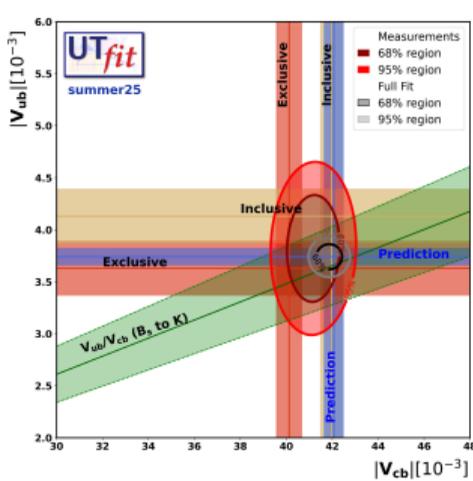
β 衰变 K 介子衰变 B 介子衰变
 D 介子衰变 K, B 介子混合/FCNC

- $VV^\dagger = V^\dagger V = I_3$ in the Standard Model
- $VV^\dagger \neq V^\dagger V \neq I_3$ New Physics
- $|V_{ub}|/|V_{cb}|$ contributes to CPV measurement in B decays

- CKM matrix elements are mainly determined via the **charged current processes**, i.e.,
 $b \rightarrow u l^- \bar{\nu}$, $b \rightarrow c l^- \bar{\nu}$, $c \rightarrow s l^+ \nu$
- Flavor changing neutral current processes** are sensitive to new physical contributions, i.e.,
 $b \rightarrow s l^+ l^-$, $b \rightarrow d l^+ l^-$



核心问题: $|V_{cb}|$, $|V_{ub}|$ 疑难

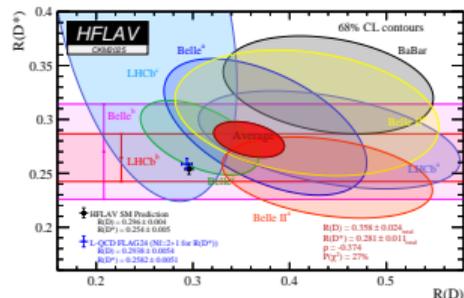


- **$|V_{cb}|$ tension** $|V_{cb}| = (41.1 \pm 1.2) \times 10^{-3}$: 2.5σ tension between $(42.2 \pm 0.5) \times 10^{-3}$ and $(39.8 \pm 0.6) \times 10^{-3}$ determined via the $B \rightarrow X_c l \bar{\nu}$ and $B \rightarrow D^{(*)} l \bar{\nu}$ processes, respectively
- **$|V_{ub}|$ tension** $|V_{ub}| = (3.82 \pm 0.20) \times 10^{-3}$: 2.5σ tension between $(4.13 \pm 0.25) \times 10^{-3}$ and $(3.67 \pm 0.15) \times 10^{-3}$ determined via the $B \rightarrow X_u l \bar{\nu}$ and $B \rightarrow \pi l \bar{\nu}$ processes, respectively

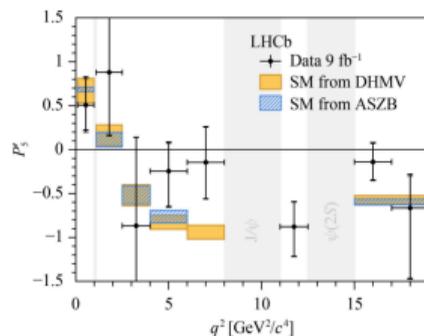
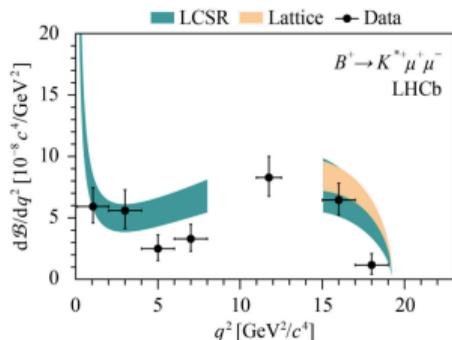
[PDG 2024, 25 update]

核心问题: 反常现象

- **LFU** $R_{D^{(*)}} = \frac{\mathcal{B}(B \rightarrow D^{(*)} \tau^- \bar{\nu})}{\mathcal{B}(B \rightarrow D^{(*)} \mu^- \bar{\nu})}$: exceed the SM predictions by 2.5σ and 2.3σ [HFCPV 2025]
- make the CKM determinations more complicated if confirmed



- **Anomalies** in FCNC processes $B \rightarrow K^* \mu^+ \mu^-$: 3.6σ derivation from SM of $d\mathcal{B}(B \rightarrow K^* \mu^+ \mu^-)/dq^2$ in $q^2 \in [1, 6] \text{ GeV}^2$, 1.9σ derivation from SM of $p'_5 = S_5/\sqrt{F_L(1-F_L)}$ in $q^2 \in [4, 8] \text{ GeV}^2$



[Heavy Flavour Physics and CP Violation at LHCb: a Ten-Year Review, Front. Phys.18.44601(2023)], see Xing-bo's talk for the updates

解决方案 在传统过程继续奋斗

- 更精确的测量和格点计算 see Zhao-feng and Bai-qian's talks

$|V_{cs}| = 0.975 \pm 0.006$: 0.972 ± 0.007 and 0.984 ± 0.012 measured via the $D \rightarrow Kl\nu$ and $D_s \rightarrow \mu^+ \nu_\mu$ processes
 $\sim 1.5\sigma$ [PDG 2024, 25 update] $\leftarrow 3\sigma$ in [PDG 2021]

- 更全面更系统的物理分析方法 see Jun-hao's talk

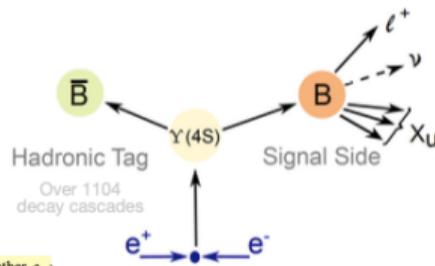
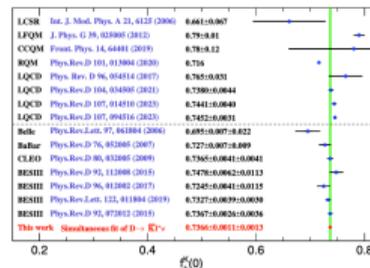
$|V_{ub}|$ result from Belle collaboration with Simultaneous Determination in excl. and incl. processes [Belle PRL131, 211801 (2023)]
 $(3.78 \pm 0.23 \pm 0.16 \pm 0.14) \times 10^{-3}$ and
 $(3.88 \pm 0.20 \pm 0.31 \pm 0.09) \times 10^{-3}$

- 精细结构、丰富的 QCD 效应

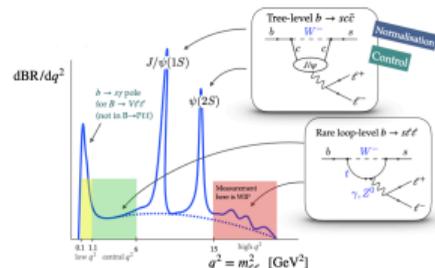
high order QCD corrections, more structures

[AK, TM, YMW, JHEP 02 (2013) 010, AK, TM, AAP, YMW, JHEP 09 (2010) 089]

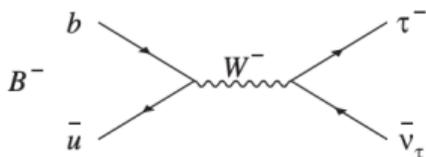
[BESIII 2023,24, Fermi Lattice and MILC 23]



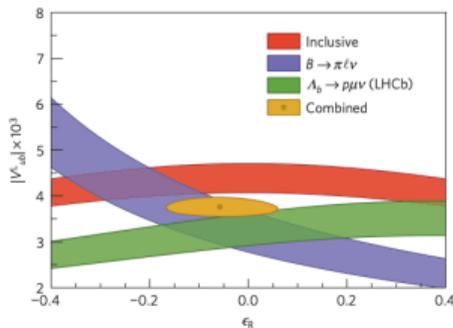
$$\mathcal{B}(B \rightarrow \pi^0 \ell \nu) + \mathcal{B}(B \rightarrow \pi^+ \ell \nu) + \mathcal{B}(B \rightarrow X_u^{\text{other}} \ell \nu) = \mathcal{B}(B \rightarrow X_u \ell \nu)$$



解决方案 寻找新的增长极



- $|V_{ub}|$ from **pure leptonic decay** $(4.11 \pm 0.39) \times 10^{-3}$:
 $|V_{ub}|f_B = 0.72 \pm 0.09$ MeV from Belle, 1.01 ± 0.14 MeV from BABAR,
 0.77 ± 0.12 MeV average [FLAG, PRD 107. 052008 (2023)]



- $|V_{ub}|$ from **baryon decay** see Xiao-kang's talk

$$\frac{|V_{ub}|^2}{|V_{cb}|^2} = \frac{\mathcal{B}(\Lambda_b^0 \rightarrow p \mu^- \bar{\nu})}{\mathcal{B}(\Lambda_b^0 \rightarrow \Lambda_c^+ \mu^- \bar{\nu})} R_{\text{FF}} = 0.68 \pm 0.07 \downarrow$$

$$\frac{|V_{ub}|}{|V_{cb}|} = 0.079 \pm 0.006 \xrightarrow{|V_{cb}|} |V_{ub}| = (3.97 \pm 0.26) \times 10^{-3}$$

consistent with the determinations in exclusive $B \rightarrow \pi l \bar{\nu}$ decay
 [LHCb, Nature Physics 11. 743-747 (2015)]

- $|V_{ub}|/|V_{cb}|_{\text{high}} = 0.098 \pm 0.008$ via $\mathcal{B}(B_s \rightarrow K^- \mu^+ \nu)/\mathcal{B}(B_s \rightarrow D_s^- \mu^+ \nu)$ [LHCb, PRL 126. 081804 (2021)]
- $|V_{cb}| = (42.3 \pm 1.7) \times 10^{-3}$ from $B_s \rightarrow D_s \mu^+ \nu$ [LHCb, PRD 101.072004 (2020)]
- $|V_{ub}| = 3.58(9) \times 10^{-3}$ from $B_s \rightarrow K l \nu$ [PRD104. 114041 (2021)]
- $d\mathcal{B}(B \rightarrow K^* \mu^+ \mu^-)/dq^2$ and p'_5 in $\Lambda_b^0 \rightarrow \Lambda \mu^+ \mu^-$ [LHCb, JHEP 06 (2015) 115]

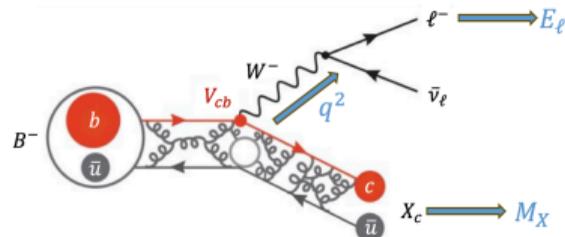
一些最新进展

- i 在单举过程 $\bar{B} \rightarrow X_c l \bar{\nu}$ 中确定 $|V_{cb}|$ 的最新精度
- ii 在 B 介子四体半轻衰变中确定 $|V_{ub}|$ 的探索

I apologize for not covering many of the latest developments, such as the baryonic decays and the outstanding performance of lattice QCD.

$|V_{cb}|$ from $\bar{B} \rightarrow X_c \ell \bar{\nu}_\ell$

- $m_b \gg \Lambda_{\text{QCD}} \Rightarrow p_b = m_b v + k$, expand in $k \sim i\mathcal{D} \Downarrow$
heavy quark expansion (HQE)
- Optical Theorem \Rightarrow **operator product expansion (OPE)**



[picture from Matteo Fael (CERN)]

$$d\Gamma = \sum_i \frac{d\Gamma_i}{(m_b)^i}, \quad d\Gamma_i = \sum_k C_i^{(k)} \langle B | O_i^{(k)} | B \rangle$$

non-perturbative matrix elements string of $(i\mathcal{D})^k$
perturbative Wilson coefficients \Downarrow
parameters μ_π^2, μ_G^2 and ρ_D^2, ρ_{LS}^2

- the inclusive semi-decay decay rate (SM)

$$\Gamma(\bar{B} \rightarrow X_c \ell \bar{\nu}_\ell) = |V_{cb}|^2 G_F^2 \frac{m_b^5}{16\pi^3} f(m_b, m_c, \mu_\pi^2, \mu_G^2, \rho_D^3, \rho_{LS}^3, \dots)$$

$$f = f^{\text{LP}} + f^{\text{NLO},\pi} \frac{\mu_\pi^2}{m_b^2} + f^{\text{NLO},G} \frac{\mu_G^2}{m_b^2} + f^{\text{NNLO},D} \frac{\rho_D^3}{m_b^3} + f^{\text{NNLO},LS} \frac{\rho_{LS}^3}{m_b^3} + \mathcal{O}\left(\frac{\Lambda_{\text{QCD}}^4}{m_B^4}\right)$$

- extract $|V_{cb}|$ provided we know the parameters μ_π^2, μ_G^2 and ρ_D^2, ρ_{LS}^2

$|V_{cb}|$ from $\bar{B} \rightarrow X_c l \bar{\nu}_l$

- the inclusive decay spectrum depends on 3 kinematical variables

$$\frac{d^3\Gamma}{dE_l dq^2 dm_X^2} = |V_{cb}|^2 G_F^2 \frac{m_b^5}{16\pi^3} \frac{d^3 f(m_b, m_c, \mu_\pi^2, \mu_G^2, \rho_D^3, \rho_{LS}^3, \dots)}{dE_l dq^2 dm_X^2}$$

- define the **spectral moment** to confront the experimental data

$$M_{ijk} = \int dE_l dq^2 dm_X^2 [E_l]^{(i)} [q^2]^{(j)} [m_X^2]^{(k)} \frac{d^3\Gamma}{dE_l dq^2 dm_X^2}$$

- extract **HQE parameters** μ_π^2, μ_G^2 and ρ_D^2, ρ_{LS}^2 by global fitting of the **normalized moments**
 $\hat{M}_{ijk} \equiv \frac{M_{ijk}}{M_{000}}$ in (E_l, q^2, m_X^2) with $|V_{cb}|^2$ drops out

- i.e., **q^2 moments** are measured at several low cuts

$$M_n(q_{\text{cut}}^2) = \int_{q_{\text{cut}}^2}^{q_{\text{max}}^2} dq^2 [q^2]^n \frac{d\Gamma}{dq^2}, \quad \langle q^{2n} \rangle = \frac{M_n(q_{\text{cut}}^2)}{M_0(q_{\text{cut}}^2)}$$

- define central moments to reduce correlations

$$Q_1(q_{\text{cut}}^2) = \langle q^2 \rangle, \quad Q_2(q_{\text{cut}}^2) = \langle (q^2 - \langle q^2 \rangle)^2 \rangle, \quad Q_3(q_{\text{cut}}^2) = \langle (q^2 - \langle q^2 \rangle)^3 \rangle$$

- measurements only available at Belle (2021) and Belle II (2022)

$|V_{cb}|$ from $\bar{B} \rightarrow X_c \ell \bar{\nu}_\ell$

| | dE_ℓ | dm_X^2 | dq^2 | Γ |
|------------------------------------|---|---------------------------------------|---|--|
| 1 | α_s^2 [Melnikov 2008] α_s^2 [Pak, Czarnecki 2008] | α_s^2 [Fael, Herren 2024] | α_s^2 NEW | α_s^3 [Fael, Schönwald, Steinhäuser 2020] |
| $1/m_b^2$ | α_s [Alberti, Ewerth, Gambino, Nandi 2012, 2013] | α_s | α_s | α_s |
| $1/m_b^3$ | 1 Need improvement! [Gremm, Kapustin 1997] | 1 | α_s [Mannel, Moreno Pivovarov 2021] | α_s [Mannel, Pivovarov 2019] |
| $1/m_b^{4,5}$ $1/(m_b^3 m_c^2)$ | Proliferation of non-perturbative parameters 1 RPI can reduce them, but restricted to dq^2 | | | |
| | α_s [Fael, Mannel, Vos 2018] | α_s [Mannel, Mitrin, Vos 2023] | α_s [Mannel, Turocyk, Uraltsev 2010] | α_s [Mannel, Turocyk, Uraltsev 2010] |

state-of-the-art calculation [picture from Gael Finauri (TUM)]

| m_b^{kin} | $\bar{m}_c(2 \text{ GeV})$ | μ_π^2 | μ_G^2 | ρ_D^3 | ρ_{LS}^3 | $\text{BR}_{c\ell\nu}$ | $10^3 V_{cb} $ |
|--------------------|----------------------------|-------------|-----------|------------|---------------|------------------------|-----------------|
| 4.572 | 1.090 | 0.430 | 0.282 | 0.161 | -0.091 | 10.61 | 41.83 |
| 0.012 | 0.010 | 0.040 | 0.048 | 0.018 | 0.089 | 0.15 | 0.47 |

1.1% precision

[screenshot from Gael Finauri, "Challenges in Semileptonic B decays" 2024, Vienna]

- $(42.2 \pm 0.5) \times 10^{-3}$ and $(39.8 \pm 0.6) \times 10^{-3}$ from $B \rightarrow X_c \Gamma \bar{\nu}$ and $B \rightarrow D^{(*)} \Gamma \bar{\nu}$

- take kinematical scheme to avoid renormalon ambiguities and badly converging perturbative series ($\mu_k = 1 \text{ GeV}$, $\alpha_s^{(4)}(m_b) = 0.2185$)

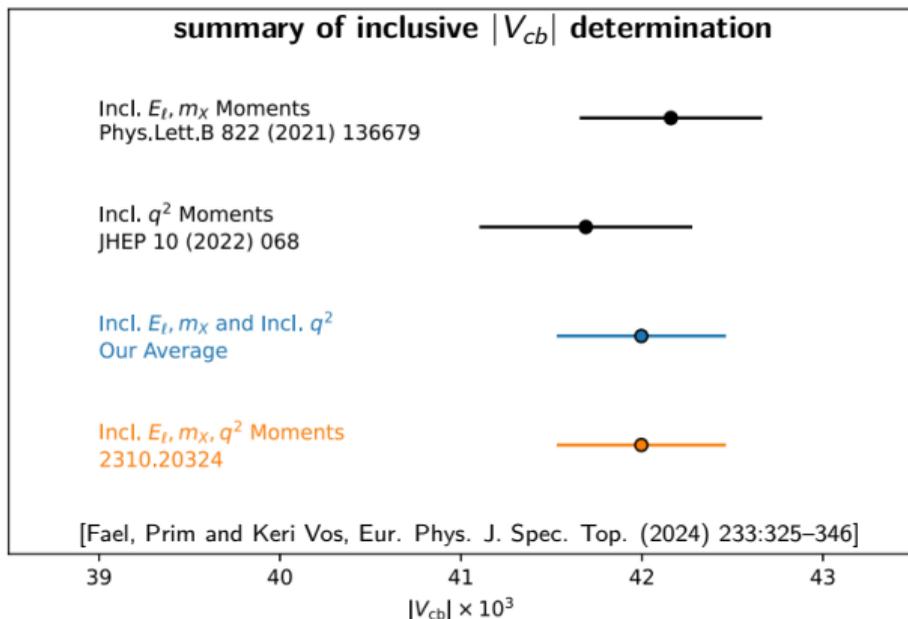
$$m_b^{\text{OS}} = m_b^{\text{kin}} \left[1 + 0.029\alpha_s + 0.027\alpha_s^2 \right],$$

$$(\mu_\pi^2)^{\text{kin}}, \quad (\rho_D^3)^{\text{kin}},$$

$$m_c^{\text{OS}} = \bar{m}_c(2\text{GeV}) \left[1 + 0.18\alpha_s + 0.14\alpha_s^2 \right]$$

- fit to (E_ℓ, q^2, m_X^2) moments [2501.09090, 2310.20324]

$$|V_{cb}| \text{ from } \bar{B} \rightarrow X_c \ell \bar{\nu}_\ell$$



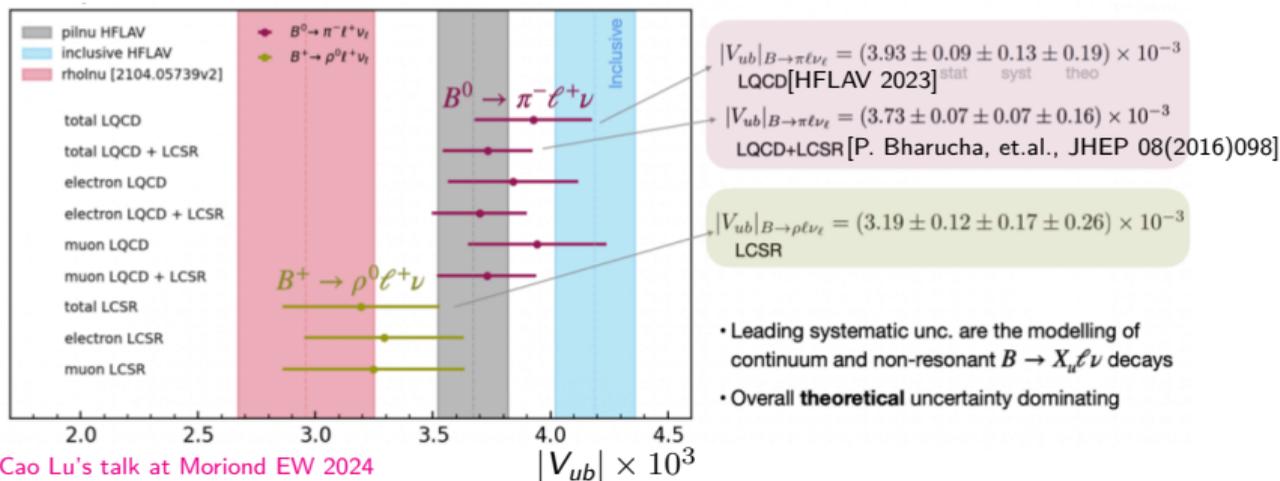
- open source Python package (kolya) for inclusive $|V_{cb}|$ determination [Fael, Milutin and Keri Vos, 2409.15007]
- recent advances in inclusive charm decay [see Qin Qin's talk at "HFPCV 2025", PKU]

$|V_{ub}|$ from $B \rightarrow \pi\pi\ell^+\nu$

- 含有激发态粒子的过程也可以提供独立的测量 the same $b \rightarrow u$ transition
- $B \rightarrow V$ transitions updated via B -meson LCSRs [Gao, et.al., PRD 101.074035(2020)]

$$|V_{ub}| \times 10^3 \xrightarrow{B \rightarrow \rho\ell\nu} 3.05^{+1.34}_{-1.30}|_{\text{th}} \quad {}^{+0.19}_{-0.20}|_{\text{da}}, \quad \xrightarrow{B \rightarrow \omega\ell\nu} 2.54^{+1.09}_{-1.05}|_{\text{th}} \quad {}^{+0.18}_{-0.19}|_{\text{da}}$$

- simultaneous measurements of dB/dq^2 for $B \rightarrow \pi^-\ell^+\nu_\ell$ and $B \rightarrow \rho^0\ell^+\nu_\ell$ [Belle-II, PRD 111.112009(2025)]

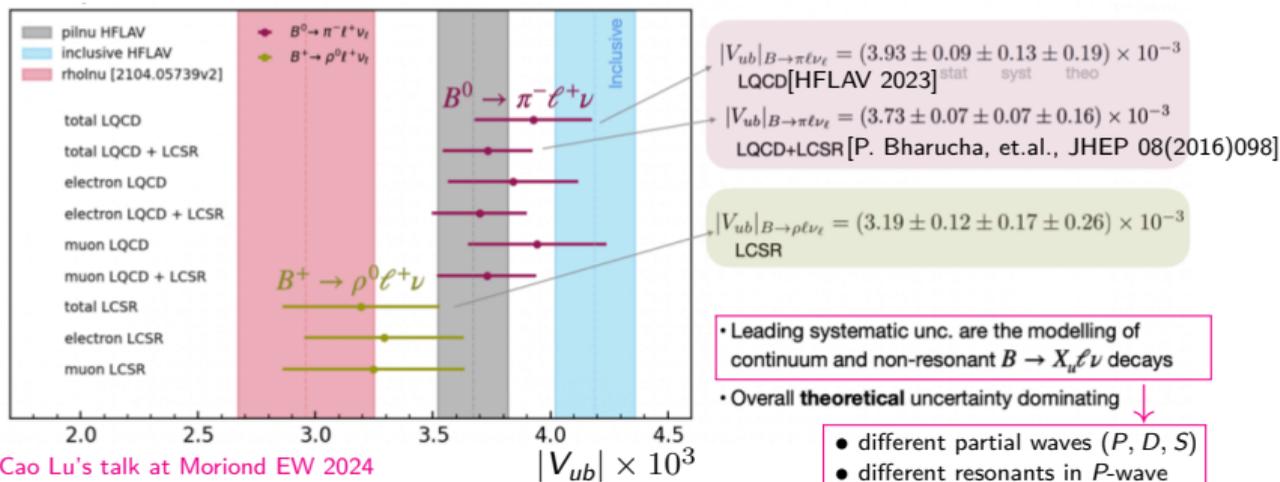


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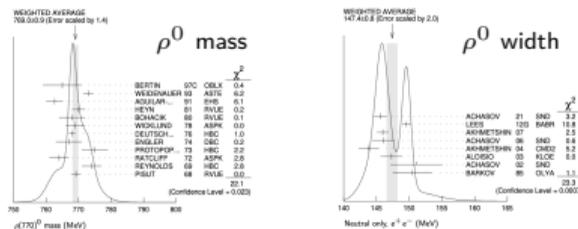
- simultaneous measurements of dB/dq^2 for $B \rightarrow \pi^- l^+ \nu_l$ and $B \rightarrow \rho^0 l^+ \nu_l$ [Belle-II, PRD 111.112009(2025)]



Cao Lu's talk at Moriond EW 2024

$|V_{ub}|$ from $B \rightarrow \pi\pi l^+ \nu$

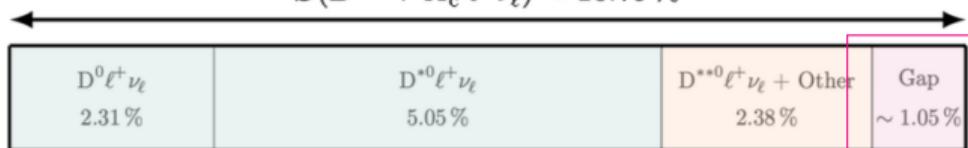
- the $\pi\pi$ spectra in $[0.554, 0.996]$ GeV serves as the candidate region for ρ
- ρ is usually treated as a stable single particle in the QCD studies of $B \rightarrow \rho$ transitions (LQCD, LCSRs)



- the signal channel in a $B \rightarrow \rho l \nu$ -type decay is $B \rightarrow \pi\pi l \nu$ (B_{lA})
- How to accurately describe the **width effects** of unstable intermediate particles, the contributions and **interference effects** of different partial waves, and the **QCD backgrounds** from non-resonant states
- Dipion LCDAs (2π **DA**s) provide a most general description of $\pi\pi$ spectral

- opportunities for $|V_{cb}|$ from $B \rightarrow D^* l \nu, X_c l \bar{\nu}$ and $B \rightarrow K^* l^+ l^-$ anomalies

$$\mathcal{B}(B^+ \rightarrow X_c^0 \ell^+ \nu_\ell) \approx 10.79\%$$



careful study of $D\pi, D\pi\pi$ and $D\pi\pi\pi$ spectral

...or is it even bigger?

2πDAs

- Chiral-even LC expansion with gauge factor $[x, 0]$

$$\langle \pi^a(k_1) \pi^b(k_2) | \bar{q}_f(zn) \gamma_\mu \tau q_f(0) | 0 \rangle = \kappa_{ab} k_\mu \int dx e^{iuz(k \cdot n)} \Phi_{\parallel}^{ab,ff}(\mu, u, \zeta, k^2)$$

$$\langle 0 | \bar{u}(x) \gamma_\mu \gamma_5 d(-x) | \rho^-(p) \rangle = f_\rho m_\rho \int_0^1 du e^{i(2u-1)p \cdot x} \left[p_\mu \frac{\epsilon^{(\lambda)} \cdot x}{p \cdot x} (\phi_{\parallel}(u, \mu) - \phi_{\perp}^3(u, \mu)) + \dots \right]$$

- 2πDAs is decomposed in terms of $C_n^{3/2}(2u-1)$ and $C_\ell^{1/2}(2\zeta-1)$

$$\Phi^{l=1}(u, \zeta, k^2, \mu) = 6u(1-u) \sum_{n=0, \text{even}}^{\infty} \sum_{l=1, \text{odd}}^{n+1} B_{n\ell}^{l=1}(k^2, \mu) C_n^{3/2}(2u-1) C_\ell^{1/2}(2\zeta-1)$$

$$\Phi^{l=0}(u, \zeta, k^2, \mu) = 6u(1-u) \sum_{n=1, \text{odd}}^{\infty} \sum_{l=0, \text{even}}^{n+1} B_{n\ell}^{l=0}(k^2, \mu) C_n^{3/2}(2u-1) C_\ell^{1/2}(2\zeta-1)$$

- Evolution from $4m_\pi^2$ to large k^2** via the Watson theorem of $\pi\pi$ scattering amplitudes

$$B_{n\ell}^l(k^2) = B_{n\ell}^l(0) \text{Exp} \left[\sum_{m=1}^{N-1} \frac{k^{2m}}{m!} \frac{d^m}{dk^{2m}} \ln B_{n\ell}^l(0) + \frac{k^{2N}}{\pi} \int_{4m_\pi^2}^{\infty} ds \frac{\delta_\ell^l(s)}{s^N(s-k^2-i0)} \right]$$

2πDAs in a wide range of energies is given by δ_ℓ^l and a few subtraction constants

2πDAs

- The subtraction constants of $B_{n\ell}(k^2)$ at low k^2 (around the threshold)

| (nl) | $B_{n\ell}^{\parallel}(0)$ | $c_1^{\parallel,(nl)}$ | $\frac{d}{dk^2} \ln B_{n\ell}^{\parallel}(0)$ | $B_{n\ell}^{\perp}(0)$ | $c_1^{\perp,(nl)}$ | $\frac{d}{dk^2} \ln B_{n\ell}^{\perp}(0)$ |
|------|----------------------------|------------------------|---|---------------------------|--------------------|---|
| (01) | 1 | 0 | 1.46 \rightarrow 1.80 | 1 | 0 | 0.68 \rightarrow 0.60 |
| (21) | -0.113 \rightarrow 0.218 | -0.340 | 0.481 | 0.113 \rightarrow 0.185 | -0.538 | -0.153 |
| (23) | 0.147 \rightarrow -0.038 | 0 | 0.368 | 0.113 \rightarrow 0.185 | 0 | 0.153 |
| (10) | -0.556 | - | 0.413 | - | - | - |
| (12) | 0.556 | - | 0.413 | - | - | - |

△ firstly studied in the effective low-energy theory based on instanton vacuum [Polyakov 1999]

△ updated with the kinematical constraints and the new a_2^{π}, a_2^{ρ} [SC 2019, 2023]

- 2πDAs were introduced at leading twist [Polyakov 1999, Diehl 1998]

$$\langle \pi^a(k_1)\pi^b(k_2)|\bar{q}_f(zn)\gamma_{\mu}\tau_{q_f'}(0)|0\rangle = \kappa_{ab}k_{\mu} \int dx e^{iuz(k\cdot n)} \Phi_{\parallel}^{ab,ff'}(u, \zeta, k^2)$$

- improved to twist-three level recently [SC, PRD 112. L111301(2025)]

$$\langle \pi(k_1)\pi(k_2)|\bar{q}(0)q(x)|0\rangle = \int du e^{i\bar{u}k\cdot x} \frac{ik^2(k\cdot x)}{2f_{2\pi}^{\perp}} \Phi_{\parallel}^{(s)},$$

$$\langle \pi(k_1)\pi(k_2)|\bar{q}(0)\sigma^{\mu\nu}q(x)|0\rangle = -\frac{i}{f_{2\pi}^{\perp}} \int du e^{i\bar{u}k\cdot x} \left[\frac{k_{\mu}\bar{k}_{\nu} - k_{\nu}\bar{k}_{\mu}}{2\zeta - 1} \Phi_{\perp} - k^2 \frac{k_{\mu}x_{\nu} - k_{\nu}x_{\mu}}{k\cdot x} \Phi_{\parallel}^{(t)} \right].$$

$B \rightarrow \pi\pi$ form factors

- **QCD dynamics of B_{I4} decays is incorporated in $B \rightarrow \pi\pi$ form factors**, ~~the $B \rightarrow \rho$ ffs followed by $\rho \rightarrow \pi\pi$ in the cascade decay~~ [X.-W. Kang, et.,al, PRD 89. 053015 (2014), S. Faller, et.,al, PRD 89. 014015 (2014)]

$$\begin{aligned}
 i\langle \pi^+(k_1)\pi^-(k_2)|\bar{u}\gamma_\nu(1-\gamma_5)b|\bar{B}^0(p)\rangle &= F_\perp(q^2, k^2, \zeta) \frac{2}{\sqrt{k^2}\sqrt{\lambda_B}} i\epsilon_{\nu\alpha\beta\gamma} q^\alpha k^\beta \bar{k}^\gamma \\
 &+ F_t(q^2, k^2, \zeta) \frac{q_\nu}{\sqrt{q^2}} + F_0(q^2, k^2, \zeta) \frac{2\sqrt{q^2}}{\sqrt{\lambda_B}} \left(k_\nu - \frac{k \cdot q}{q^2} q_\nu\right) \\
 &+ F_\parallel(q^2, k^2, \zeta) \frac{1}{\sqrt{k^2}} \left(\bar{k}_\nu - \frac{4(q \cdot k)(q \cdot \bar{k})}{\lambda_B} k_\nu + \frac{4k^2(q \cdot \bar{k})}{\lambda_B} q_\nu\right)
 \end{aligned}$$

- * $\lambda = \lambda(m_B^2, k^2, q^2)$ is the Källén function,
 $q \cdot k = (m_B^2 - q^2 - k^2)/2$, $q \cdot \bar{k} = \sqrt{\lambda}\beta_\pi(k^2) \cos\theta_\pi/2 = \sqrt{\lambda}(2\zeta - 1)$, $\beta_\pi(k^2) = \sqrt{1 - 4m_\pi^2/k^2}$

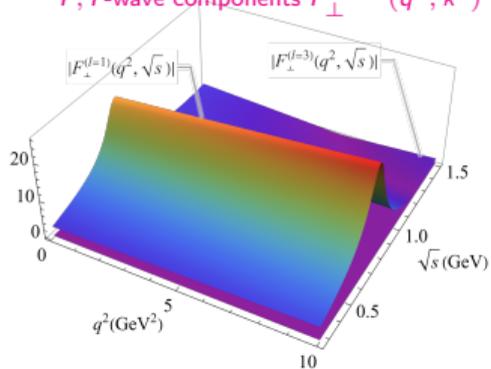
- **SU(3) flavor symmetry/breaking** with the intermediate resonance
 - [R.M. Wang, Y.G. Xu, J.H. Sheng, X.D. Cheng, et.al., 2301.00090, PRD 112, 033002 (2025)]
- **QCDF** (QCD factorization) in the large two-pion mass
 - [P. Böer, T. Feldmann and D. van Dyk, JHEP02, 133(2017)] $T_I \propto F_{B \rightarrow \pi} \otimes \phi_\pi$,

$B \rightarrow \pi\pi$ form factors

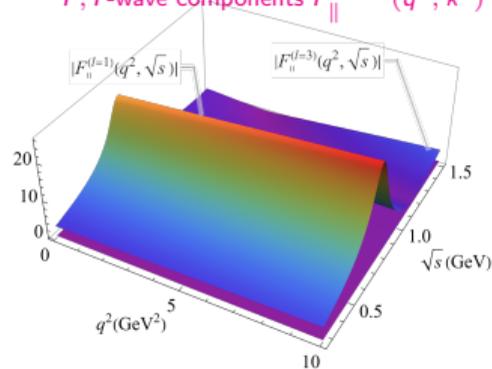
- **LQCD** (Lattice QCD) in the ρ resonance region with a simple BW model
 - [L. Leskovec, et.al, PRL 134.161901 (2025), **Editors' Suggestion**]
- **HChPT** (Heavy-meson Chiral Perturbative Theory) in the large q^2 by taking dispersive methods in terms of Omnés functions
 - [X.-W. Kang, B. Kubis, C. Hanhart, and U.-G. Meißner, PRD 89. 053015 (2014)]
 - in the full phase-space by a novel parameterization with unitarity
 - [F. Herren, B. Kubis and R. van Tonder, PRD 112, 014037 (2025), **Editors' Suggestion**]
- **LCSRs** (Light-cone sum rules) in the small and intermediate q^2
 - [SC, A. Khodjamirian and J. Virto, JHEP 05(2017)157] B -meson LCSRs , [S. Descotes-Genon, A. Khodjamirian, J. Virto and K.K. Vos, JHEP 12(2019)083, 06(2023)034] $B \rightarrow K\pi$
 - [C. Hambrock and A. Khodjamirian, NPB 905. 379-390(2016)] 2π DAs LCSRS of $F_{\parallel, \perp}$
 - [SC, A. Khodjamirian and J. Virto, PRD(R) 96. 051901(2017)] timelike-helicity FF F_{τ} and F_0
 - [SC, PRD 99. 053005(2019)] 2π DAs updates and $B \rightarrow [\pi\pi]_{S,P}$ FFs
 - [SC and J.M Shen, EPJC 6:554(2020), SC and S.L Zhang, EPJC 84:379(2024)] Pheno
 - [SC, PRD 112. L111301(2025)] twist-three 2π DAs and $|V_{ub}|$ extraction
 - [SC, L.Y. Dai, J.M. Shen and S.L. Zhang, arXiv:2509.15659] $D_s \rightarrow [\pi\pi]_S e\nu$, minor $q\bar{q}$ contribution

$B \rightarrow \pi\pi$ form factors

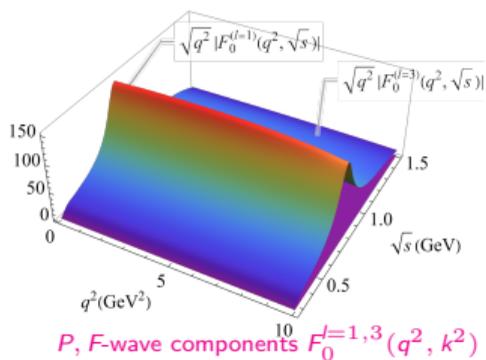
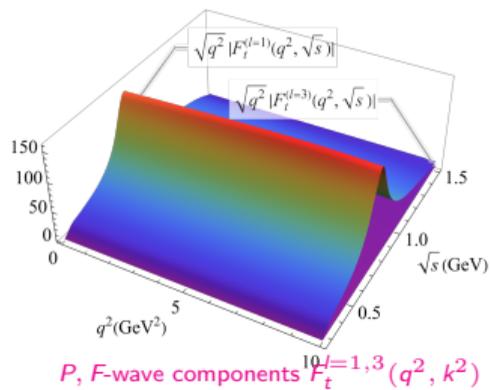
P, F -wave components $F_{\perp}^{J=1,3}(q^2, k^2)$



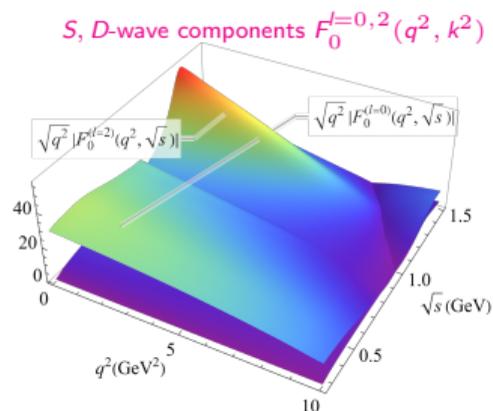
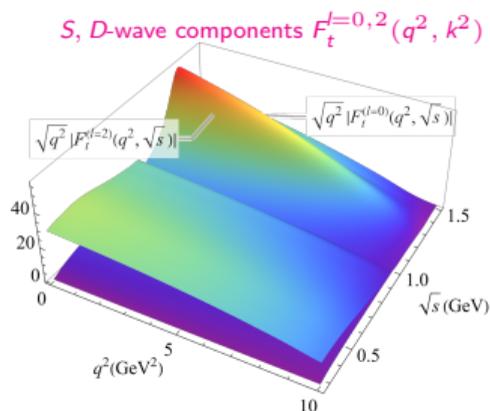
P, F -wave components $F_{\parallel}^{J=1,3}(q^2, k^2)$



[SC, PRD 112. L111301(2025)]



$B \rightarrow \pi\pi$ form factors



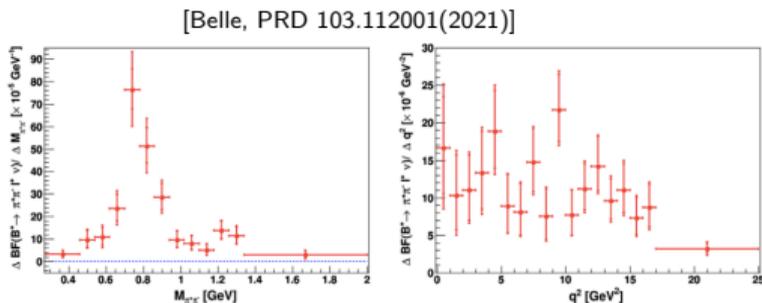
- twist-three contributions vanish in $F_{\perp, \parallel}^{(l=1,3,\dots)}$, while persist in $F_{t,0}^{(l=0,1,\dots)}$, give a significant correction $\sim 40\%$ to leading twist result
- ρ resonance is dominate in $F_{\perp, \parallel, t, 0}^{(l=1)}(q^2, k^2)$, F-wave contributions are negligible
- S-wave dominates in the small k^2 regions, D-wave is sizable in the large k^2 and small q^2 regions

$|V_{ub}|$ from $B \rightarrow \pi\pi l^+\nu$

- 2D partial differential decay width

$$\frac{d^2\Gamma}{dq^2 dk^2} = G_F^2 |V_{ub}|^2 \frac{\beta_\pi \sqrt{\lambda} q^2}{3 (4\pi)^5 m_B^3} \left[(|F_0^{(S)}|^2 + |F_0^{(P)}|^2) + \beta_\pi^2 (|F_{\parallel}^{(P)}|^2 + |F_{\perp}^{(P)}|^2) + \dots \right]$$

- 2D measurement of $\Delta\mathcal{B}^i$ (10^{-5}) in different bins



take the PDG value: $|V_{ub}| \times 10^3 = 3.82 \pm 0.20 \Rightarrow$

a good consistent in ρ, f_0 regions

| Bins | \sqrt{s} | q^2 | $\Delta\mathcal{B}^i$ | $\Delta\mathcal{B}^i$ [20] |
|------|-------------------|----------|--------------------------|---------------------------------|
| 1 | $[4m_\pi^2, 0.6]$ | $[0, 8]$ | $0.27 \pm 0.03 \pm 0.06$ | $0.84^{+0.39}_{-0.32} \pm 0.18$ |
| 2 | $(0.6, 0.9]$ | $[0, 4]$ | $1.91 \pm 0.21 \pm 0.38$ | $2.39^{+0.53}_{-0.47} \pm 0.32$ |
| 3 | $(0.6, 0.9]$ | $(4, 8]$ | $1.54 \pm 0.17 \pm 0.27$ | $2.16^{+0.47}_{-0.42} \pm 0.23$ |
| 4 | $(0.9, 1.2]$ | $[0, 4]$ | $0.65 \pm 0.07 \pm 0.12$ | $0.70^{+0.32}_{-0.25} \pm 0.20$ |
| 5 | $(0.9, 1.2]$ | $(4, 8]$ | $0.41 \pm 0.04 \pm 0.08$ | $0.64^{+0.28}_{-0.22} \pm 0.11$ |
| 6 | $(1.2, 1.5]$ | $[0, 4]$ | $0.57 \pm 0.05 \pm 0.10$ | $0.91^{+0.35}_{-0.28} \pm 0.12$ |
| 7 | $(1.2, 1.5]$ | $(4, 8]$ | $0.16 \pm 0.02 \pm 0.02$ | $0.64^{+0.32}_{-0.26} \pm 0.08$ |

$|V_{ub}| \times 10^3 = 4.27 \pm 0.49|_{\text{Data}} \pm 0.55|_{\text{LCSR}}, 3.96 \pm 0.47|_{\text{Data}} \pm 0.52|_{\text{LCSR}}$ in ρ, f_0 regions of $B_{1/4}$ decays

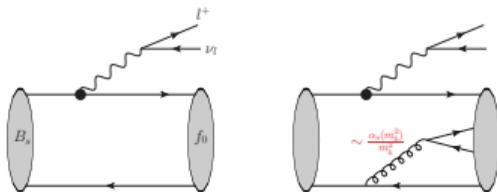
The controversy of $f_0(980)$ structure

- From the quantum theory, it is a superposition of all possible Fock states

$$|f_0(980)\rangle, \quad |[\pi\pi]_S\rangle = \psi_{q\bar{q}}|q\bar{q}\rangle + \psi_{q\bar{q}g}|q\bar{q}g\rangle + \psi_{q\bar{q}q\bar{q}}|q\bar{q}q\bar{q}\rangle + \dots$$

- Hadron spectroscopy** provides clear evidence for the **complex config**
- The underlying partonic dynamics can be extracted directly from **Semileptonic B, D decays** in terms of scale-dependent **LCDAs**

- color transparency mechanism** in $B_{(s)} \rightarrow f_0 l^+ \nu_l$ decays



* high Fock states' contribution is doubly suppressed by α_s and $\mathcal{O}(1/Q^2)$, FSI is weak

- the mechanism fails in $D_s \rightarrow f_0 l^+ \nu_l$ decays** (the produced $q\bar{q}$ is nonrelativistic)
- while the energy-dependent partonic configurations is a QCD result, the **cascade decay analyses of $D_s(\rightarrow f_0) \rightarrow \pi\pi e\nu$** under $q\bar{q}$ ansatz **agrees with data** [SC, Zhang, EPJC 84. 379(2024), Hu, et.al., PRD 112.

The controversy of $f_0(980)$ structure

- cascade decay analyses of $D_s(\rightarrow f_0) \rightarrow \pi\pi e\nu$ under $q\bar{q}$ ansatz agrees with data

$$\frac{d\Gamma(D_s^+ \rightarrow [\pi\pi]_S l^+ \nu)}{dsdq^2} = \frac{1}{\pi} \frac{G_F^2 |V_{cs}|^2}{192\pi^3 m_{D_s}^3} |f_+(q^2)|^2 \frac{\lambda^{3/2}(m_{D_s}^2, s, q^2) g_1 \beta_\pi(s)}{|m_S^2 - s + i(g_1 \beta_\pi(s) + g_2 \beta_K(s))|^2}$$

- * $D_s \rightarrow f_0$ FFs have revealed the scale dependence, the seemingly agreement arises from the Flatté parameterization with unsound use of f_0 LCDAs under the single-meson assumption, a fine-tuned delicate interplay ?
- a model-independent study of the signal channel $D_s \rightarrow [\pi\pi]_S e^+ \nu$

$$\frac{d^2\Gamma(D_s^+ \rightarrow [\pi\pi]_S l^+ \nu)}{dsdq^2} = \frac{G_F^2 |V_{cs}|^2 \beta_\pi \pi(s) \sqrt{\lambda_{D_s} q^2}}{3(4\pi)^5 m_{D_s}^3} \sum_{\ell=0}^{\infty} |F_0^{(\ell)}(q^2, s)|^2$$

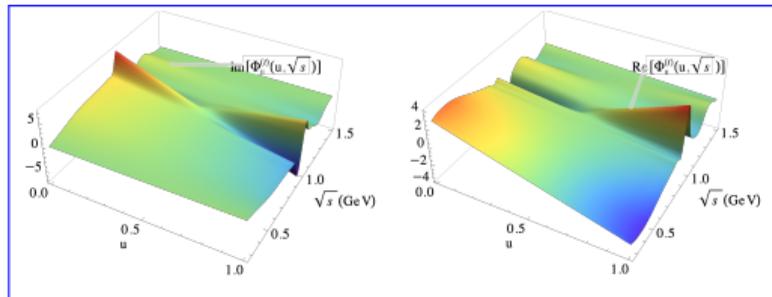
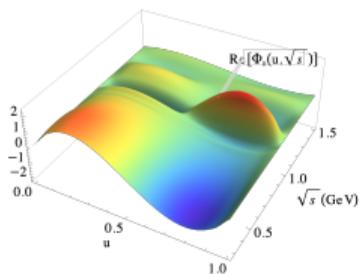
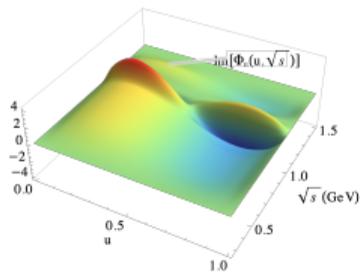
- * calculate directly the $D_s \rightarrow [\pi\pi]_S$ FFs

- Twist-2 and twist-3 contributions to $D_s \rightarrow \pi\pi, f_0$ FFs at $q^2 = 0$

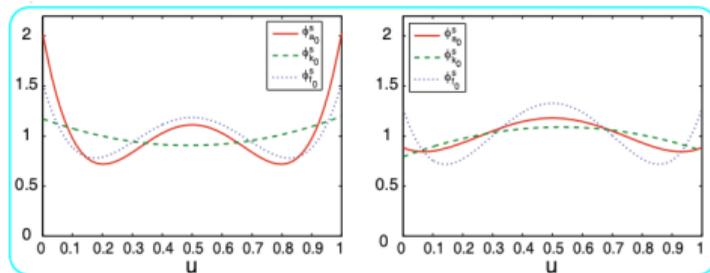
under the $q\bar{q}$ approximation

| Form Factors | Twist-2 | Twist-3 | Total |
|---|---|---|---|
| $\sqrt{q^2} F_0^{(l=0)}(0) = \sqrt{q^2} F_t^{(l=0)}(0)$ | $0.20_{-0.02}^{+0.02} - i0.24_{-0.02}^{+0.02}$ | $-0.41_{-0.05}^{+0.04} + i0.51_{-0.04}^{+0.02}$ | $-0.21_{-0.01}^{+0.02} + i0.27_{-0.02}^{+0.03}$ |
| $\sqrt{q^2} F_0^{(l=2)}(0) = \sqrt{q^2} F_t^{(l=2)}(0)$ | $0.27_{-0.02}^{+0.03} + i0.21_{-0.01}^{+0.02}$ | $-0.55_{-0.03}^{+0.02} - i0.41_{-0.04}^{+0.05}$ | $-0.28_{-0.02}^{+0.02} - i0.20_{-0.01}^{+0.02}$ |
| $f_+(0) = f_0(0)$ | $0.20_{-0.05}^{+0.03}$ constructive | $0.41_{-0.06}^{+0.04}$ | $0.61_{-0.07}^{+0.05}$ |

2π DAs $[\pi\pi]_S$

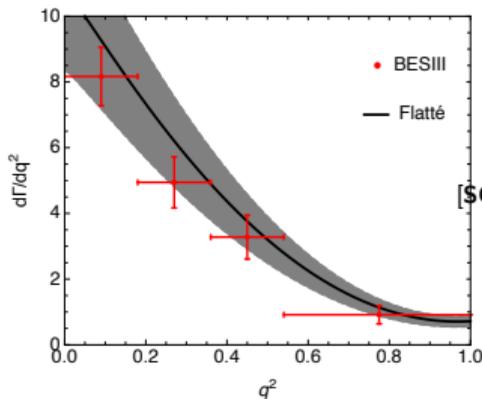
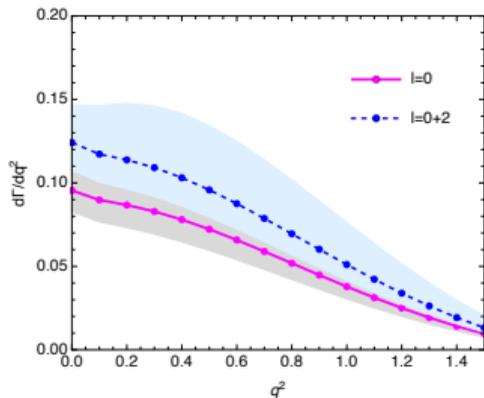


[Lü, et.al., PRD 75. 056001(2007)]
 [Han, et.al., EPJA (2013) 49: 78]



- asymmetric twist-3 2π DAs VS symmetric twist-3 f_0 LCDAs
- the asymmetry encoded by **PWE** that the single-meson picture fails to capture
- vanished asymptotic terms in twist-2 $[\pi\pi]_S / f_0$ LCDAs \Downarrow
- twist-3 (non-asymptotic twist-2) components dominates in charm (beauty) decays \Uparrow
- energy-dependent picture from the color transparency mechanism

$D_s \rightarrow [\pi\pi]_S e^+ \nu$ decay



[SC, Dai, Shen and Zhang, 2509.15659]

| $D_s \rightarrow [\pi\pi]_S e^+ \nu_e$ | $D_s \rightarrow [f_0 \rightarrow \pi\pi] e^+ \nu_e$ [23] | Data [25] |
|--|---|----------------|
| $0.81^{+0.34}_{-0.14}$ | $18.8^{+4.5}_{-3.8}$ | 17.2 ± 1.6 |

- Differential widths $d\Gamma/dq^2$ is two-order in magnitude smaller than the data
- $q\bar{q}$ Fock state is not the dominate component of $[\pi\pi]_S / f_0$ in charm decays, consists with the assessment of color transparency mechanism
- go further to multi-particle DiPion LCDAs in CHARM ($q\bar{q}g, q\bar{q}q\bar{q}$) $\Rightarrow |V_{cs}|$; different in $B \rightarrow f_0 / B \rightarrow \rho$ decays, (nonsymptotic) leading twist dominates

总结和展望

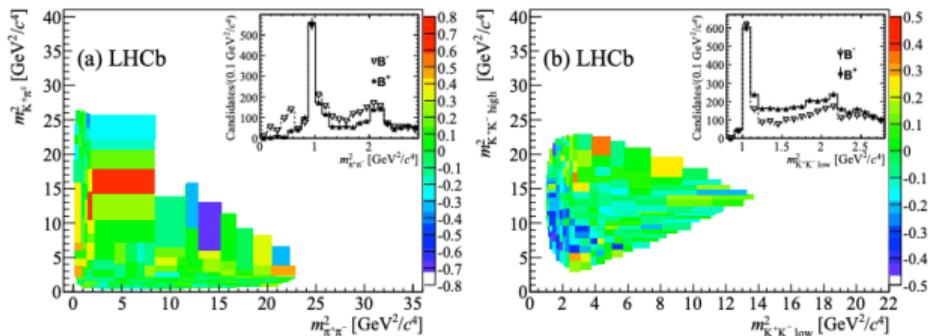
- $|V_{cb}|, |V_{ub}|$ 疑难和 $R_{D^{(*)}}, \rho'_5, \frac{d\Gamma}{dq^2}$ 等反常仍是半轻衰变研究的核心问题。上述问题的根本解决同时依赖于单举和遍举过程的独立研究，传统过程的精细研究和新过程的逐步探索是遍举研究的两条并行道路
- 四体半轻衰变 (H_{l4}) 提供了一个新方案 两介子系统光锥分布振幅
- 当前在 B_{l4} 过程抽取 $|V_{ub}|$ 的误差较大 (10%)，但是精度可以提高
Belle-II 积分亮度未来五年有望达到 $3ab^{-1}$ ，实验精度将提高 3 倍；LCSRs 和 LQCD 对形状因子的联合分析可以将理论误差减小一半
- 级联衰变方案在处理轻标量共振态过程时具有较大的不确定性，直接计算 D_{l4} 过程将是 $|V_{cs}|$ 测量和重味反常检验的一个重要候选 $D \rightarrow \pi l^+ l^-, \pi \pi l^+ l^-$
- H_{l4} 过程的研究正在进入高精度时代 $\gamma^* \rightarrow \pi \pi \gamma$ and $\gamma^* \gamma \rightarrow \pi \pi$

Thank you for your patience.

More opportunities/phenomena of 2π DAs

pion EMFF widely used in the three-body B decays studied from pQCD and QCDF is the asymptotic term of 2π DAs [Chai, SC and Ma, PRD 105. 033003 (2022)]

normalized to unit as $\Gamma_{M_1 M_2}^{J=1}(0) = 1$. When the invariant mass of dimeson system is small, the higher $\mathcal{O}(s)$ terms in the expansion of coefficient $B_{n1}(s, \mu)$ around the resonance pole can be safely neglected due to the large suppression $\mathcal{O}(s/m_b^2)$ in contrast to the energetic dimeson system in B decay, so the relation $B_{n1}(s, \mu) \rightarrow a_n(\mu)\Gamma_{M_1 M_2}^{J=1}(s)$ can be obtained in the lowest partial wave approximation. This argument induces the basic assumption in PQCD that the energetic dimeson DAs can be deduced from the DAs of resonant meson by replacing the decay constant by the timelike form factor.



$B \rightarrow K\pi\pi, KKK$

[PRL 111.101801(2013) LHCb]

Non-resonant contributions are small ($< 10\%$) in three-body D decays, large/dominate in the penguin dominated three-body B decays, the theoretically unresolved problem