



LHCb实验上CKM矩阵元和相角的测量

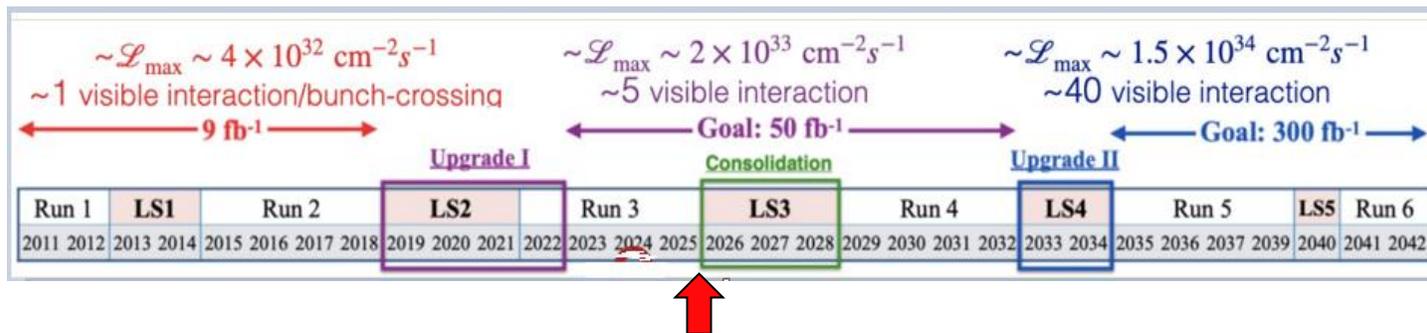
周晓康

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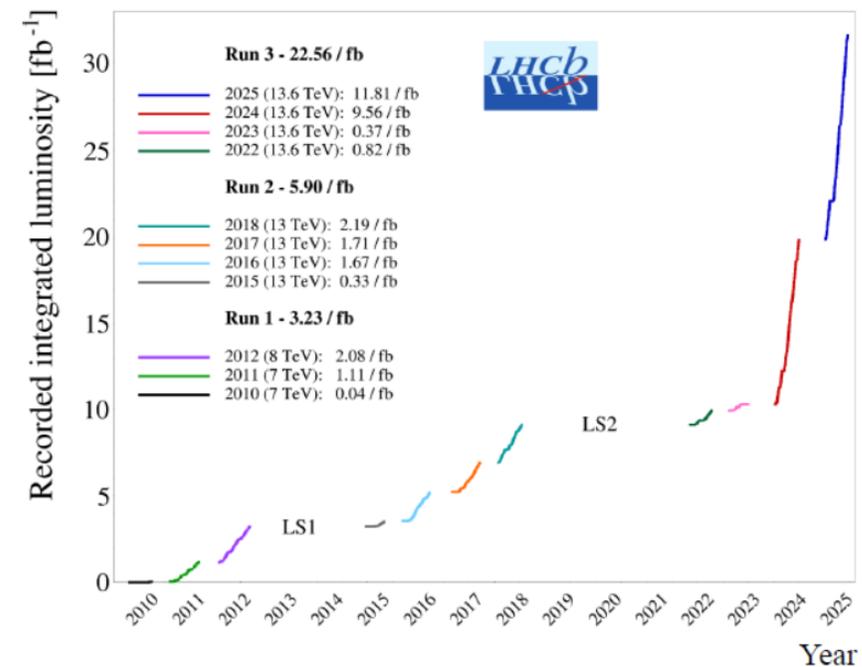
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味物理前沿研讨会暨味物理讲座100期特别活动, 2026@三亚

- ❖ Introduction
- ❖ Measure CKM elements in LHCb
- ❖ Measure CKM angle in LHCb
- ❖ Summary

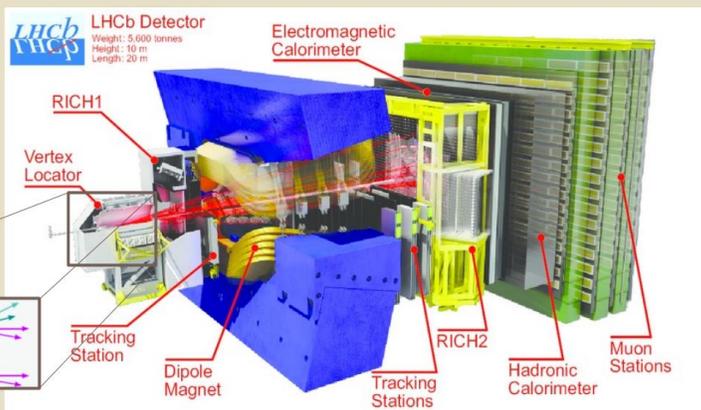


Total recorded luminosity – pp – 31.7 fb⁻¹

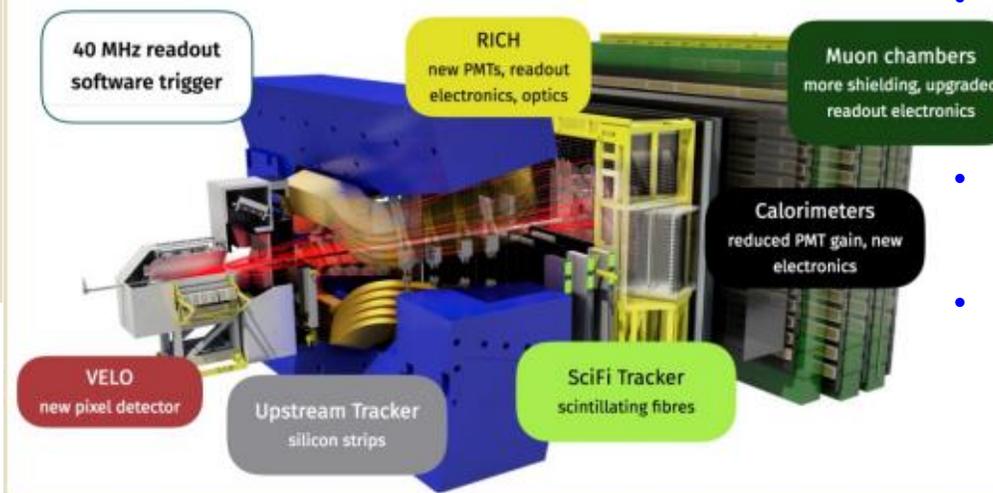


Run 1 & 2 (2011-2018)

- Single arm spectrometer designed for high precision flavour physics measurements
- Pseudorapidity range $\eta \in [2,5]$



New detector for LS2 (now)



- **Run 1:**
 - 2011 (7 TeV): 1 fb⁻¹
 - 2012 (8 TeV): 2 fb⁻¹
- **Run 2:**
 - 2015-2018 (13 TeV): 6 fb⁻¹
- **Run 3:**
 - 2024 alone (13.6 TeV): 9.56 fb⁻¹
 - 2025 (13.6 TeV): 11.81 fb⁻¹

- CKM matrix: 3×3 unitary matrix
- Unitarity test remains one of the main directions to probe NP beyond SM
- Precision essential to constrain NP scenarios

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix} .$$

V_{ud} : from $0^+ \rightarrow 0^+$ nuclear β decays,

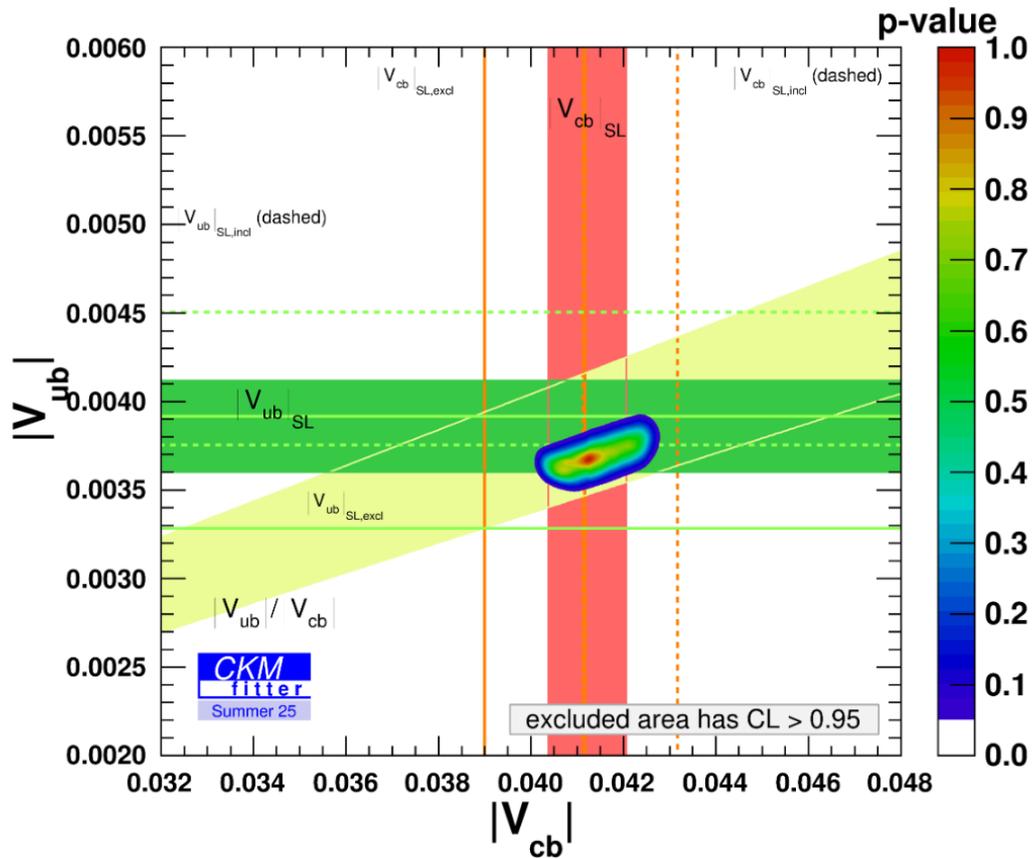
V_{us} : from kaon (semi-) leptonic decay, [LHCb-PAPER-2025-030](#)

V_{cd} and V_{cs} : from D/D_s (semi-) leptonic decay (see BaiCian's talk),

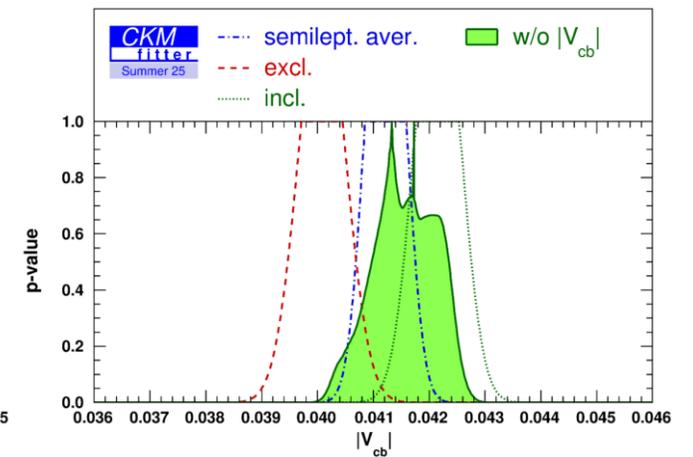
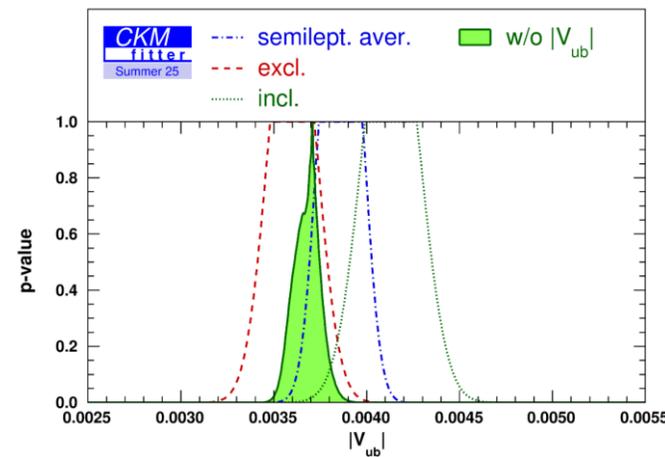
V_{cb} and V_{ub} : from $B \rightarrow X_{u/c} l \nu$

V_{td} and V_{ts} : determined from $B - \bar{B}$ mixing, or loop-mediated rare K and B decays,

V_{tb} : from top decays



- ❖ $|V_{ub}|$ and $|V_{cb}|$ remains one of the main limitations in CKM global fit
- ❖ Long standing tension: inclusive vs exclusive
- ❖ difficulties in reconstructing missing particles and more precise in measuring ratios in LHCb
- ❖ LHCb also plays a leading role

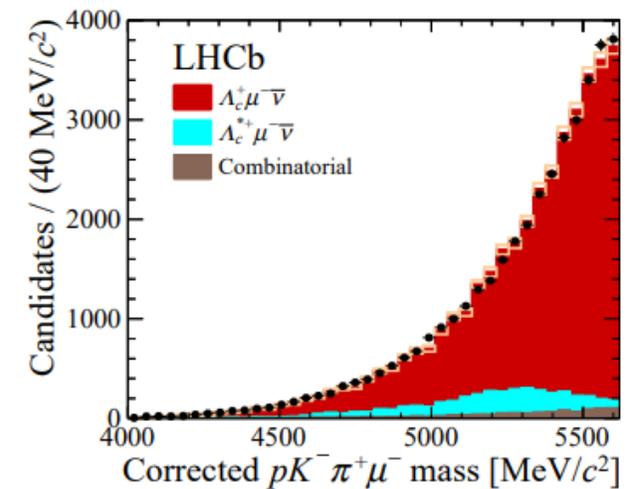
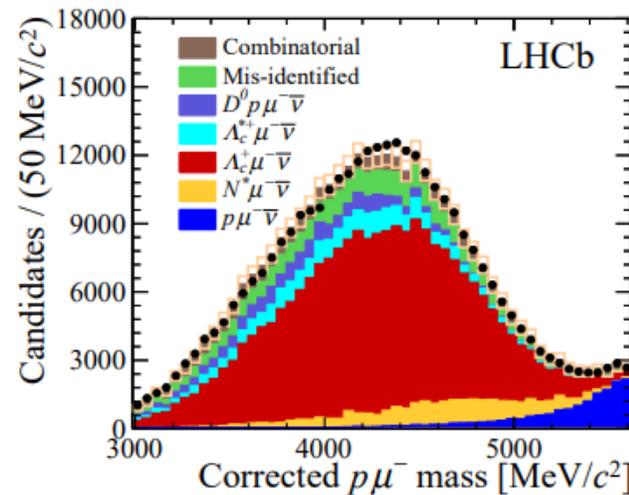
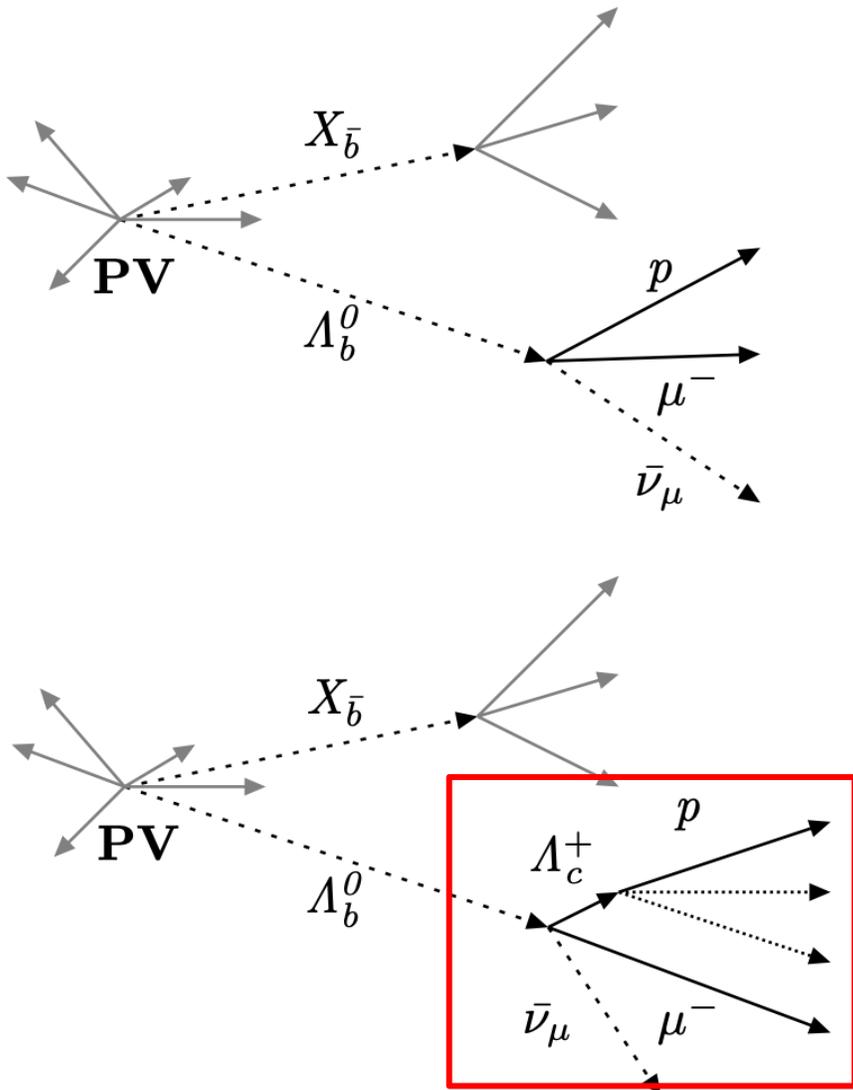


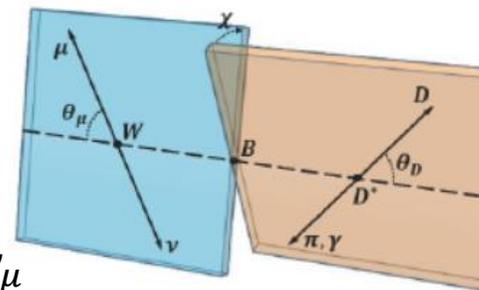
- ❖ Dataset: 2 fb^{-1} at $\sqrt{s} = 8 \text{ TeV}$ (2012)
- ❖ Ratio between $|V_{cb}|$ and $|V_{ub}|$ determined using $\Lambda_b^0 \rightarrow p\mu^-\bar{\nu}_\mu$ and $\Lambda_b^0 \rightarrow \Lambda_c^+\mu^-\bar{\nu}_\mu$

$$\frac{|V_{ub}|^2}{|V_{cb}|^2} = \frac{\mathcal{B}(\Lambda_b^0 \rightarrow p\mu^-\bar{\nu}_\mu)}{\mathcal{B}(\Lambda_b^0 \rightarrow \Lambda_c^+\mu^-\bar{\nu}_\mu)} R_{\text{FF}} \longrightarrow \text{Inputs from LQCD}$$

$$\frac{\mathcal{B}(\Lambda_b^0 \rightarrow p\mu^-\bar{\nu}_\mu)_{q^2 > 15 \text{ GeV}/c^2}}{\mathcal{B}(\Lambda_b^0 \rightarrow \Lambda_c^+\mu^-\bar{\nu}_\mu)_{q^2 > 7 \text{ GeV}/c^2}} = (1.00 \pm 0.04 \pm 0.08) \times 10^{-2}$$

Half systematic from $\text{Br}(\Lambda_c^+ \rightarrow pK^+\pi^-)$

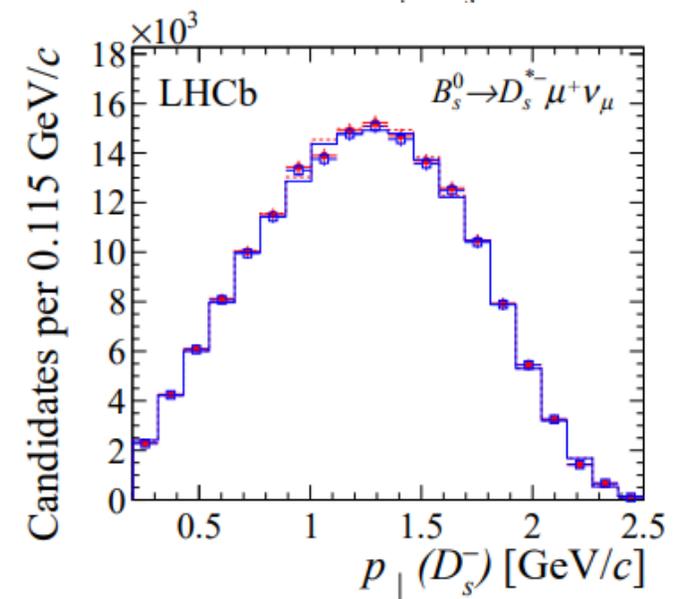
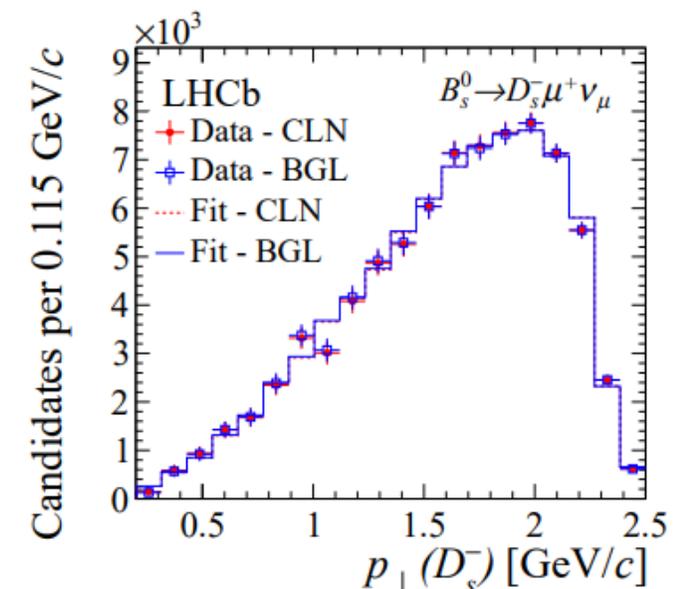




- ❖ Run1 data used (3 fb^{-1})
- ❖ Signal modes: $B_S^0 \rightarrow D_S^- ([KK]_\phi \pi) \mu^+ \nu_\mu$ and $B_S^0 \rightarrow D_S^{*-} \mu^+ \nu_\mu$
- ❖ Reference modes: $B^0 \rightarrow D^- ([KK]_\phi \pi) \mu^+ \nu_\mu$ and $B^0 \rightarrow D^{*-} \mu^+ \nu_\mu$
- ❖ either the CLN or the BGL parametrization used to model the form factors
- ❖ $p_\perp(D_s)$: component of the D_s momentum perpendicular to the B_s flight direction instead of q^2

- ❖ $|V_{cb}|_{\text{CLN}} = (41.4 \pm 0.6 \text{ (stat)} \pm 0.9 \text{ (syst)} \pm 1.2 \text{ (ext)}) \times 10^{-3}$,
- ❖ $|V_{cb}|_{\text{BGL}} = (42.3 \pm 0.8 \text{ (stat)} \pm 0.9 \text{ (syst)} \pm 1.2 \text{ (ext)}) \times 10^{-3}$,

- ❖ First determinations of $|V_{cb}|$ at a hadron-collider experiment
- ❖ First using B_s meson decays



- ❖ Dataset: 2 fb^{-1} at $\sqrt{s} = 8 \text{ TeV}$ (2012)
- ❖ First observation for $B_s^0 \rightarrow K^- \mu^+ \nu_\mu$
- ❖ Normalization: $B_s^0 \rightarrow D_s^- (KK\pi) \mu^+ \nu_\mu$
- ❖ $|V_{ub}|/|V_{cb}|$ extraction in two q^2 bins

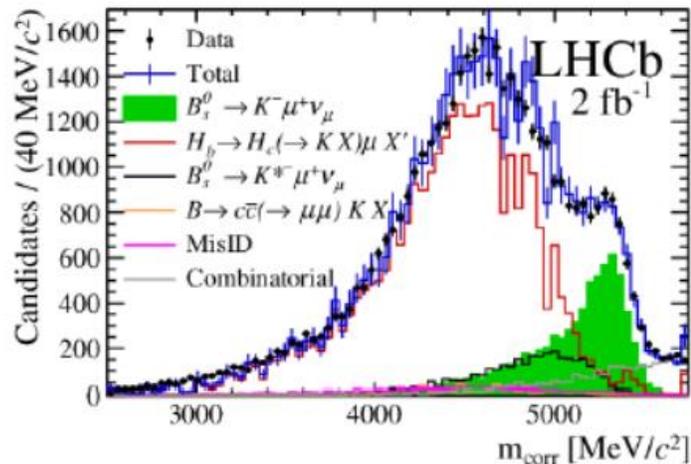
$$\frac{|V_{ub}|^2}{|V_{cb}|^2} \times \underbrace{\frac{\text{FF}_{B_s^0 \rightarrow K^-}}{\text{FF}_{B_s^0 \rightarrow D_s^-}}}_{\text{Theory Input}} = \underbrace{\frac{\mathcal{B}(B_s^0 \rightarrow K^- \mu^+ \nu_\mu)}{\mathcal{B}(B_s^0 \rightarrow D_s^- \mu^+ \nu_\mu)}}_{\text{Experiment}} = \underbrace{\frac{N_{B_s^0 \rightarrow K^- \mu^+ \nu_\mu}}{N_{B_s^0 \rightarrow D_s^- \mu^+ \nu_\mu}}}_{\text{Fit}} \times \underbrace{\frac{\epsilon_{B_s^0 \rightarrow K^- \mu^+ \nu_\mu}}{\epsilon_{B_s^0 \rightarrow D_s^- \mu^+ \nu_\mu}}}_{\text{Simulation}} \times \underbrace{\mathcal{B}(D_s^- \rightarrow K^- K^+ \pi^-)}_{\text{External input}}$$

$$|V_{ub}|/|V_{cb}| = 0.0607 \pm 0.0015(\text{stat}) \pm 0.0013(\text{syst}) \\ \pm 0.0008(D_s) \pm 0.0030(\text{FF})$$

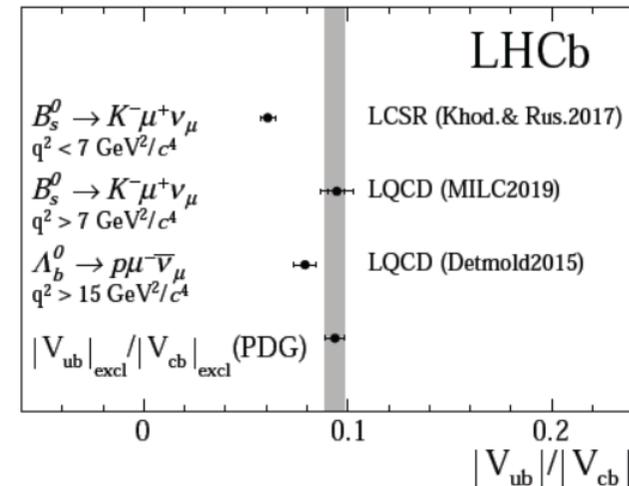
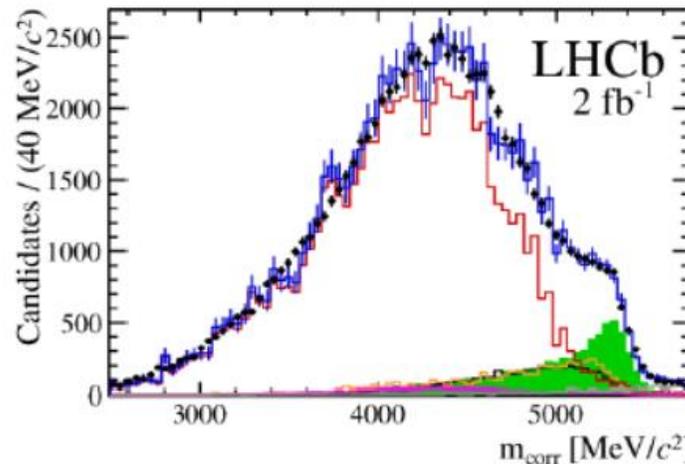
$$|V_{ub}|/|V_{cb}| = 0.0946 \pm 0.0030(\text{stat}) \begin{matrix} +0.0024 \\ -0.0025 \end{matrix}(\text{syst}) \\ \pm 0.0013(D_s) \pm 0.0068(\text{FF})$$

Tension is driven by difference in FF calculation

Signal fit at low q^2



Signal fit at high q^2



- ❖ **Extracting $|V_{ub}|$ from $B_s^0 \rightarrow K^- \mu^+ \nu_\mu$ and $B_s^0 \rightarrow K^-$ FF parameters with Run 2 data**
 - Measure shape of differential decay rate in **eight bins** of q^2

- ❖ **Extracting $|V_{ub}|$ from $B^+ \rightarrow \rho^0 \mu^+ \nu_\mu$ and $B^+ \rightarrow \rho^0$ FF parameters with Run 2 data**
 - Measure shape of differential decay rate in **10 bins** of q^2

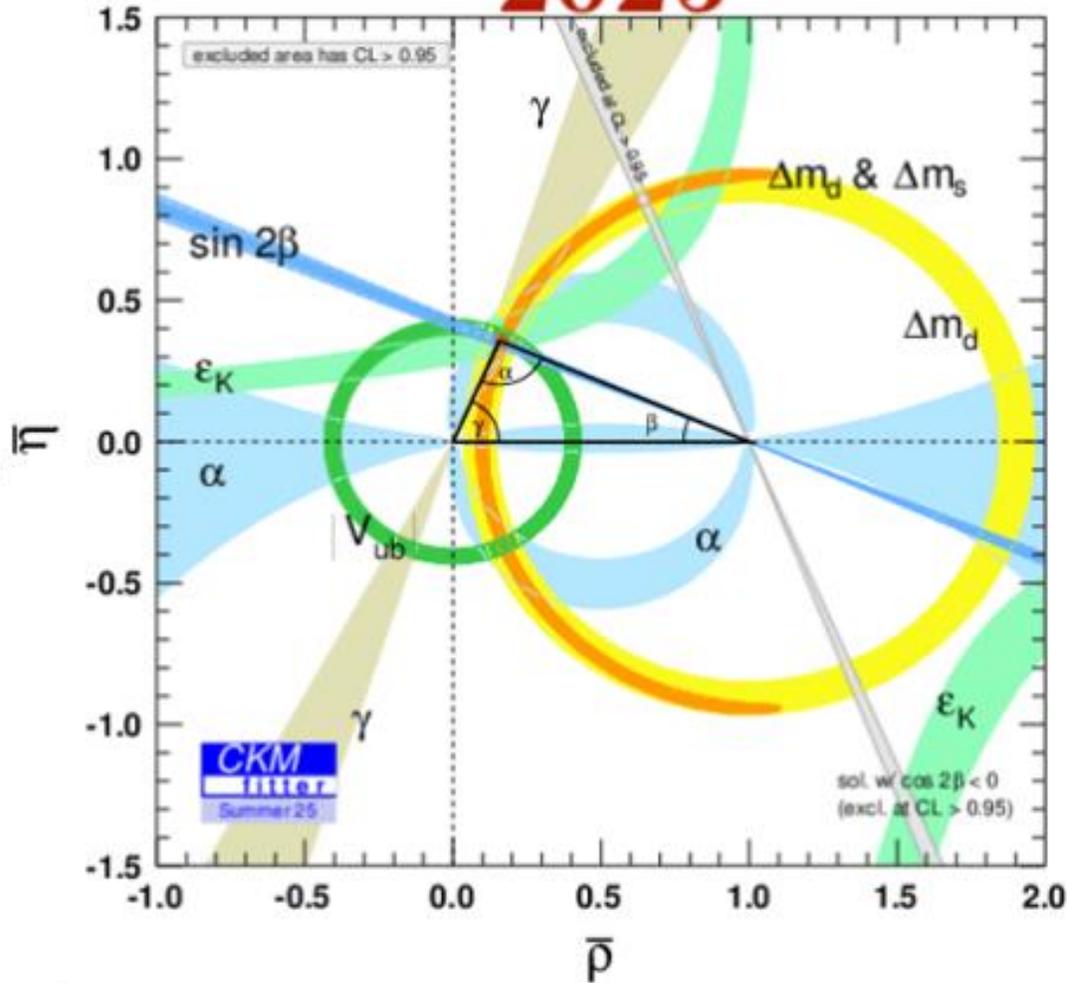
- ❖ **Extracting $|V_{ub}|/|V_{cb}|$ from $B_c^+ \rightarrow D^{(*)0} \mu^+ \nu_\mu$ with Run 2 data**
 - Normalize to $B_c^+ \rightarrow J/\psi \mu^+ \nu_\mu$
 - Profit from LQCD $B_c^+ \rightarrow D^0$ FF across full q^2 [Phys. Rev. D 105, 014503 \(2022\)](#)

- ❖ **Extracting $|V_{cb}|$ form $\Lambda_b^0 \rightarrow \Lambda_c^+ \mu^- \bar{\nu}_\mu$ with Run 2 data**
 - Measure shape of differential decay rate

- ❖ **Extracting $B^0 \rightarrow D^{*-}$ FF from $B^0 \rightarrow D^{*-} \mu^+ \nu_\mu$ using Run 2 data**

Many exciting Run 2 results upcoming
Expect more data in Run3

2025



CKM matrix **unitarity**: key test of the SM

$$V_{\text{CKM}} = \begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}|e^{-i\gamma} \\ -|V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}|e^{-i\beta} & -|V_{ts}|e^{i\beta_s} & |V_{tb}| \end{pmatrix} = \begin{pmatrix} \text{yellow} & \text{green} & \text{blue} \\ \text{green} & \text{yellow} & \text{red} \\ \text{blue} & \text{red} & \text{yellow} \end{pmatrix}$$

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0,$$

Latest results from HFLAV:

$$\alpha/\phi_2 = \arg\left(-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right) = (84.1^{+3.7}_{-3.0})^\circ$$

$$\beta/\phi_1 = \arg\left(-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right) = (22.63^{+0.45}_{-0.44})^\circ$$

$$\gamma/\phi_3 = \arg\left(-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right) = (66.4^{+2.7}_{-2.8})^\circ$$



- ❖ Time dependent CP -violation $\rightarrow \beta/\phi_1$ angle
- ❖ Golden decay: $B^0 \rightarrow J/\psi K^0$

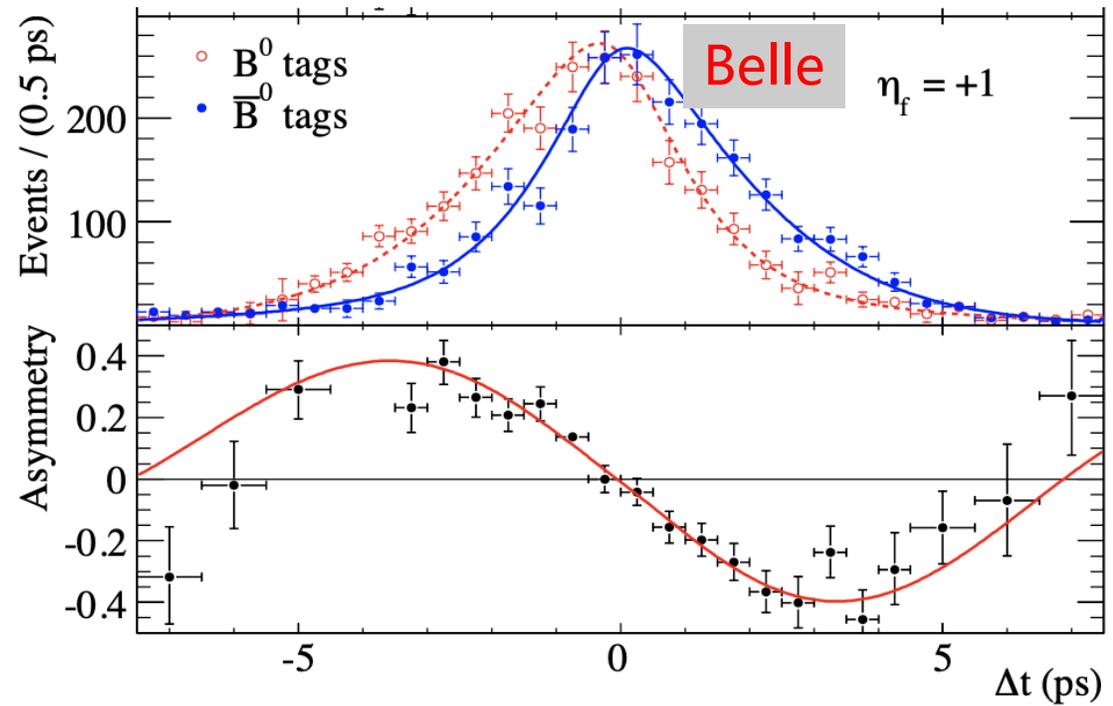
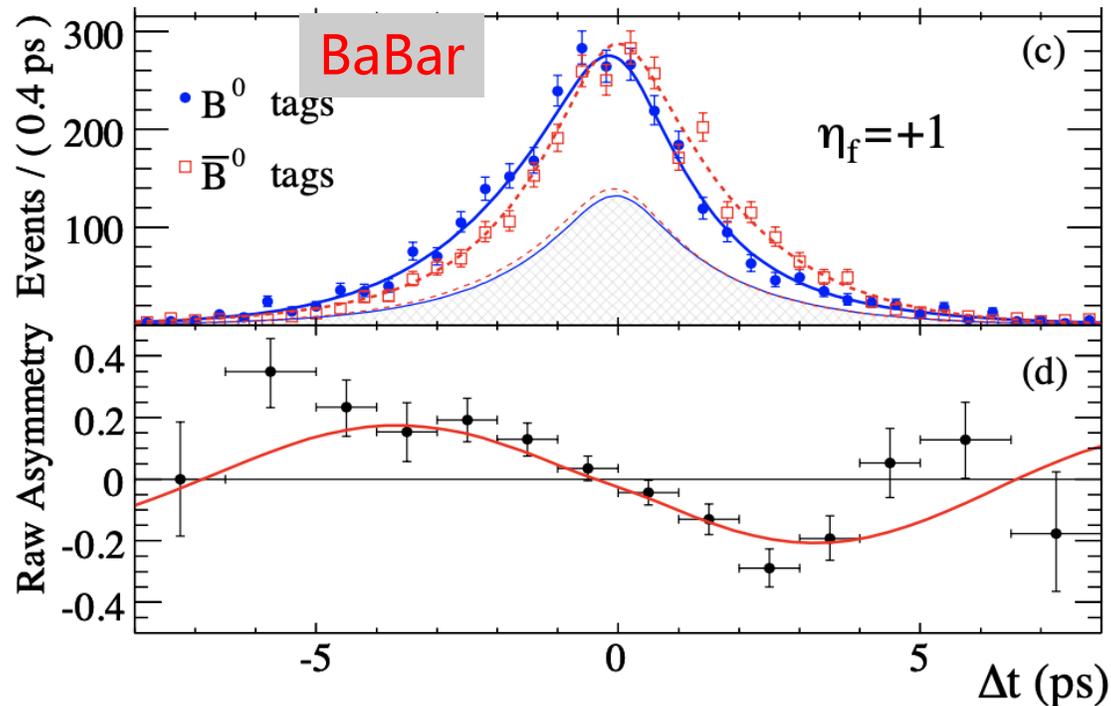
$$\sin 2\beta = 0.709 \pm 0.011$$

Dominated by LHCb run1&2
*PRL*132 (2024) 021801

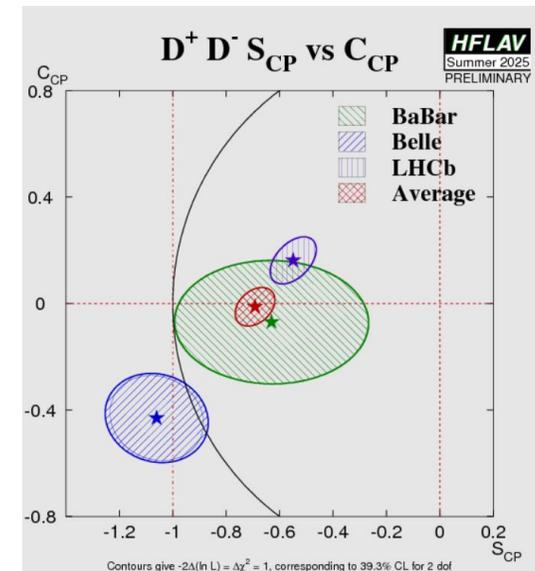
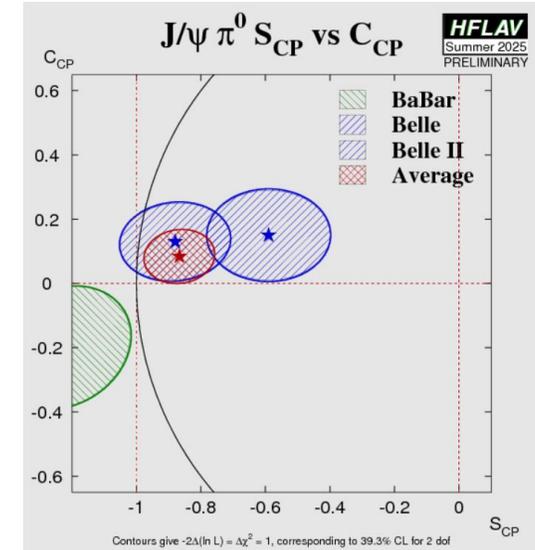
$$A_{f_{CP}}(t) = \frac{\Gamma(\bar{B}^0(t) \rightarrow f_{CP}) - \Gamma(B^0(t) \rightarrow f_{CP})}{\Gamma(\bar{B}^0(t) \rightarrow f_{CP}) + \Gamma(B^0(t) \rightarrow f_{CP})}$$

$$\propto S_f \sin(\Delta m_d t) - C_f \cos(\Delta m_d t)$$

$$\approx \eta_f \sin 2\beta \sin(\Delta m_d t) \quad \text{Penguin amplitude } \sim \lambda^2\text{-suppressed}$$



- ❖ In $b \rightarrow c\bar{c}d$: penguin amplitude is large ($\sim\lambda$)
- ❖ resulting in $S_f \neq -\eta_f$ and $C_f \neq 0$
- ❖ $B^0 \rightarrow J/\psi\pi^0, J/\psi\rho^0, D^{(*)+}D^{(*)-}$ modes studied
JHEP 01 (2025) 061
PLB 742 (2015) 38
- ❖ consistent with results from B^0 charmonium K^0 decays
- ❖ C_f 's are consistent with zero, the uncertainties are sizable



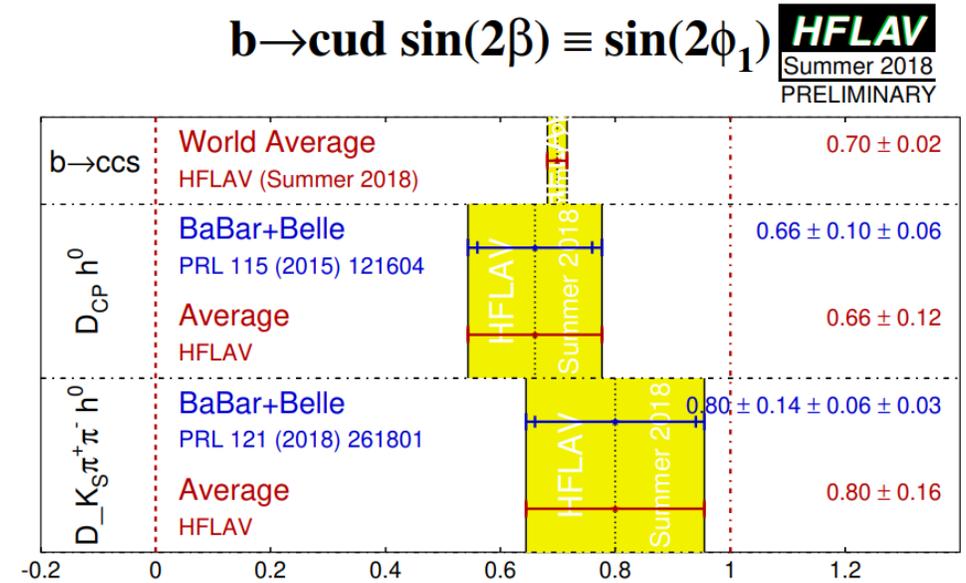
- ❖ $b \rightarrow c\bar{u}d$ transitions
 - Penguin free
 - Measure $\sin 2\beta$ and $\cos 2\beta$ at same time

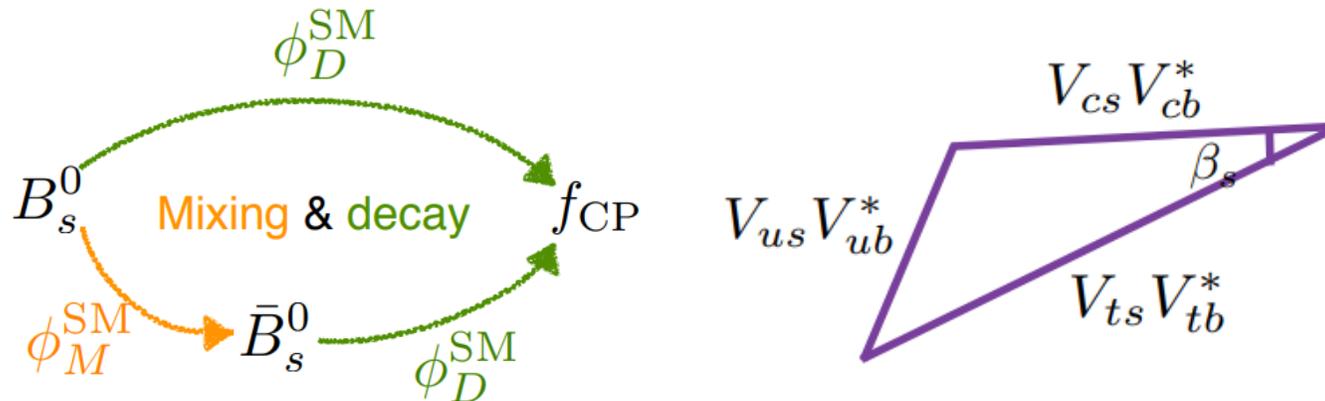
- ❖ Time-dependent analysis
 - $\bar{D} \rightarrow CP$
 - $\bar{D} \rightarrow K_S^0 \pi^+ \pi^-$ (Dalitz analysis)

- ❖ average of joint analyses of *BABAR* and Belle
- ❖ $\sin 2\beta = 0.71 \pm 0.09$

- ❖ LHCb run1&2 result come out soon

- ❖ Also $b \rightarrow q\bar{q}d$ penguin-dominated decays (many works in Belle & Babar), sensitive to effects from virtual new physics particles,

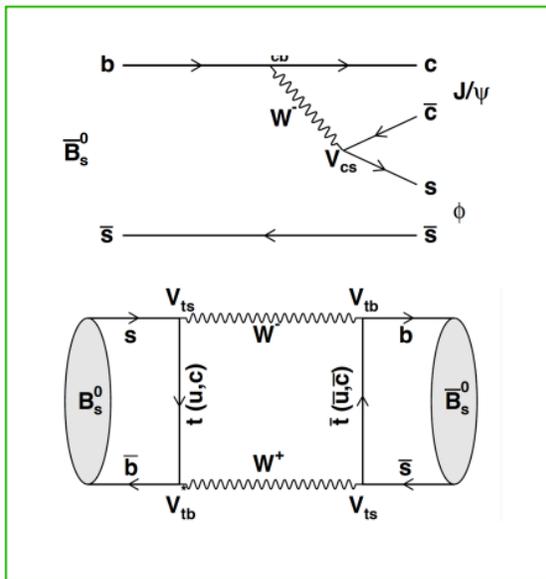




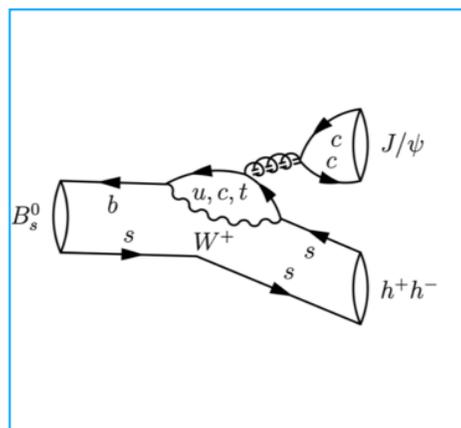
❖ ϕ_s mixing-induced CPV phase in B_s^0 decay through $b \rightarrow c\bar{c}s$ transitions

$$\phi_s^{meas.} = -2\beta_s + \delta\phi_s^{peng} + \delta\phi_s^{NP}$$

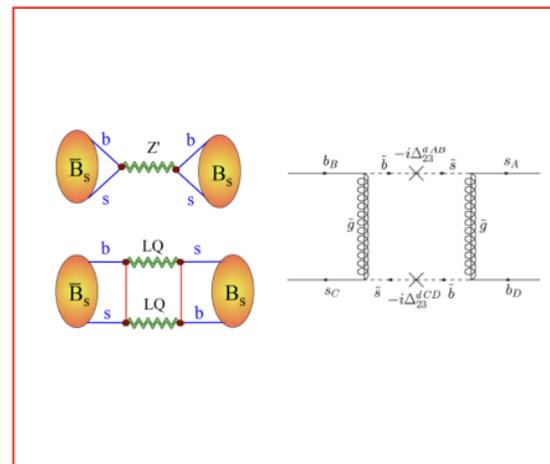
$$A_{CP}(t) = \frac{\Gamma_{\bar{B}_s^0 \rightarrow f}(t) - \Gamma_{B_s^0 \rightarrow f}(t)}{\Gamma_{\bar{B}_s^0 \rightarrow f}(t) + \Gamma_{B_s^0 \rightarrow f}(t)} \propto -\eta_f \sin 2\beta_s \sin(\Delta M_s t)$$



+



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$2\beta_s$:
 $36.8^{+0.9}_{-0.6}$ mrad [CKMfitter]
 (37 ± 1) mrad [UTFIT]

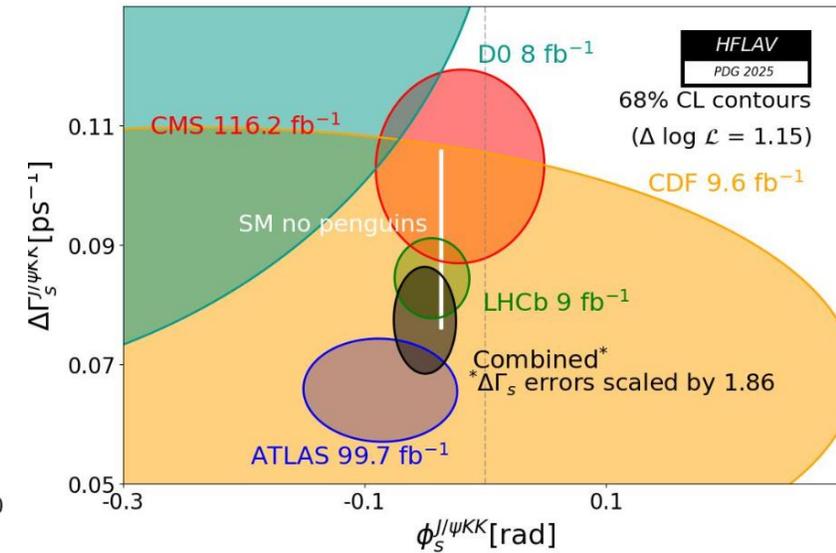
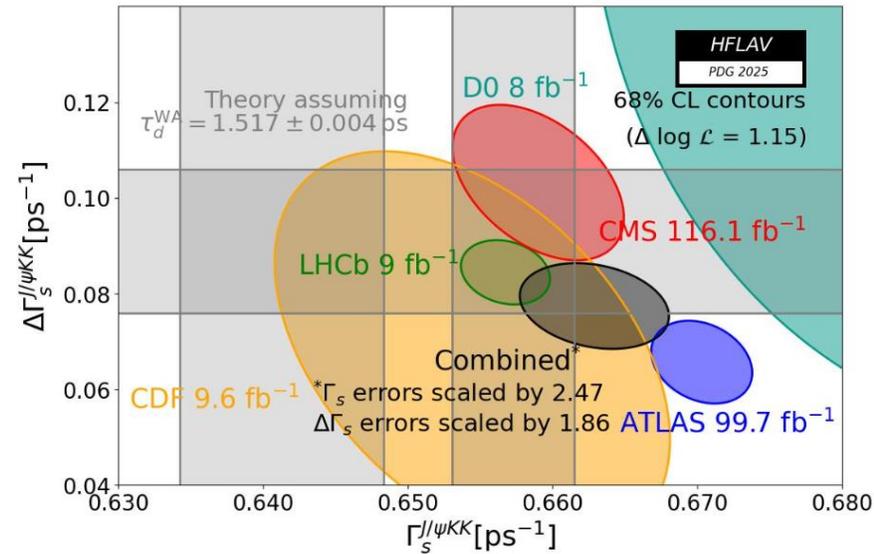
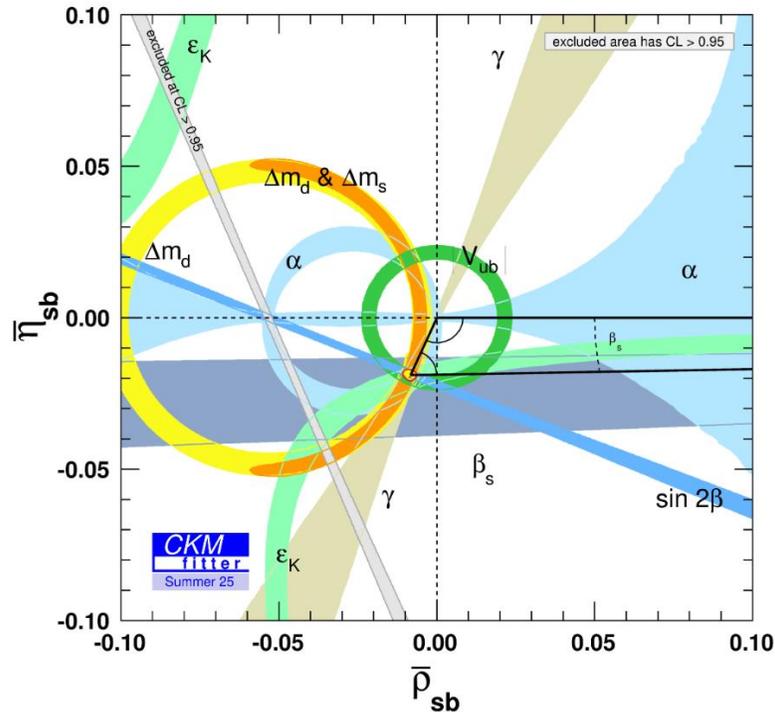
❖ $B_s \rightarrow J/\psi\phi$: Golden mode

■ Measure $\phi_s, \Delta\Gamma, \Gamma_s - \Gamma_d$

❖ Experimental Combination (CDF, D0, ATLAS, CMS, LHCb):

$$\phi_s^{c\bar{c}s} = (-50 \pm 17) \text{ mrad} \quad \Delta\Gamma_s = (0.077 \pm 0.006) \text{ ps}^{-1}$$

LHCb dominated
PRL 132 (2024) 051802



$$\phi_s^{meas.} = -2\beta_s + \delta\phi_s^{peng} + \delta\phi_s^{NP}$$

channel	Quark transition	Strength of tree	Strength of penguin
$B_S^0 \rightarrow J/\psi \phi$	$b \rightarrow ccs$	$\sim \lambda^2$	$\sim \lambda^4$
$B_S^0 \rightarrow J/\psi \bar{K}^{*0}$	$b \rightarrow ccd$	$\sim \lambda^2$	$\sim \lambda^4$

Angular Analysis

$$A(B_s^0 \rightarrow (J/\psi \bar{K}^{*0})_i) = -\lambda \mathcal{A}_i [1 - a_i e^{i\theta_i} e^{i\gamma}]$$

$$A(B_s^0 \rightarrow (J/\psi \phi)_i) = \left(1 - \frac{\lambda^2}{2}\right) \mathcal{A}'_i [1 + \epsilon a'_i e^{i\theta'_i} e^{i\gamma}]$$

Assuming $a'_i = a_i$, $\theta'_i = \theta_i$

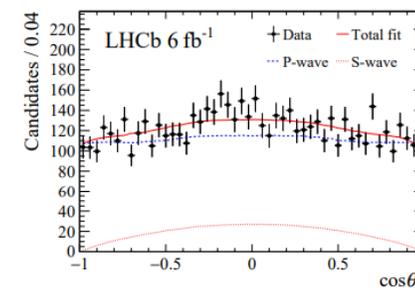
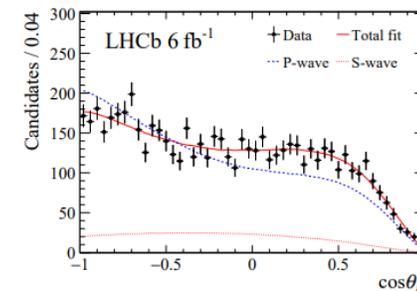
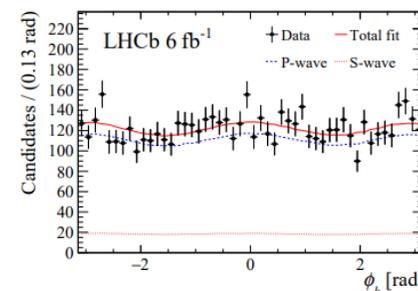
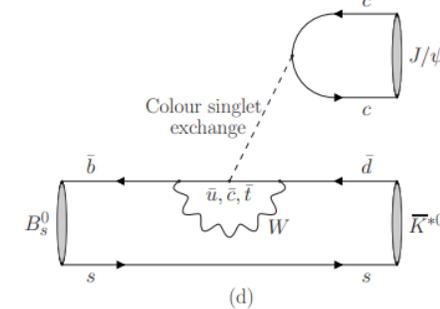
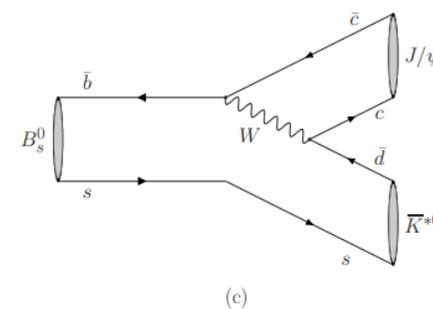
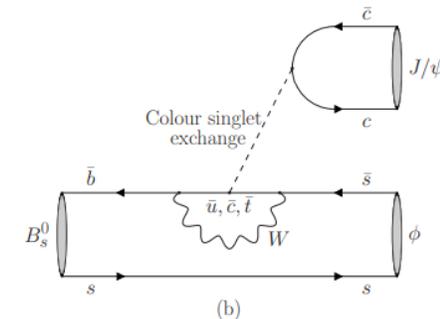
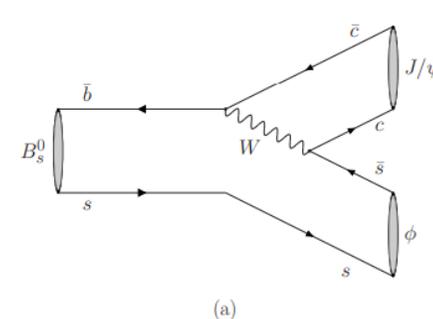
Run1 result (*JHEP 11 (2015) 082*)

$$\Delta\phi_{s,0}^{J/\psi\phi} = 0.000_{-0.011}^{+0.009} \text{ (stat)} \quad {}_{-0.009}^{+0.004} \text{ (syst) rad,}$$

$$\Delta\phi_{s,\parallel}^{J/\psi\phi} = 0.001_{-0.014}^{+0.010} \text{ (stat)} \pm 0.008 \text{ (syst) rad,}$$

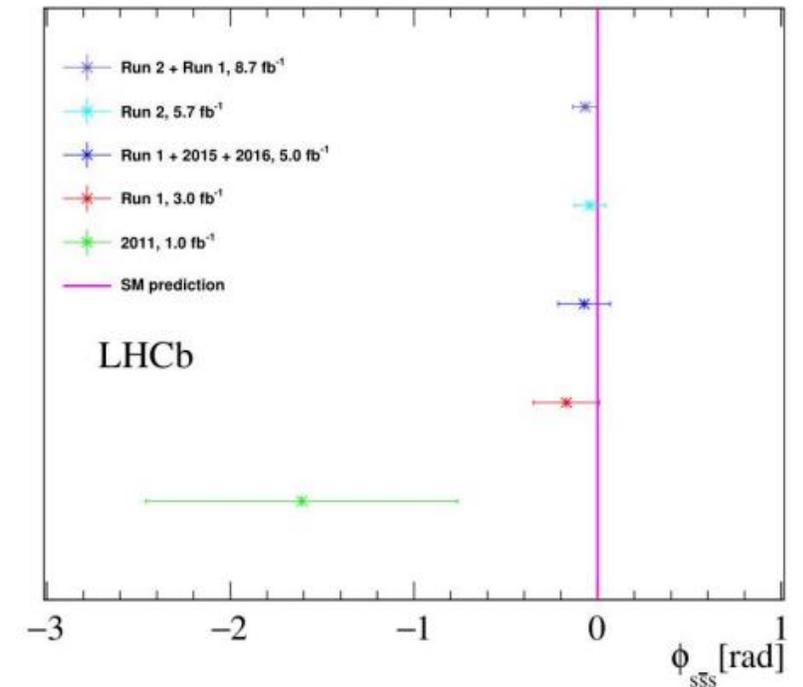
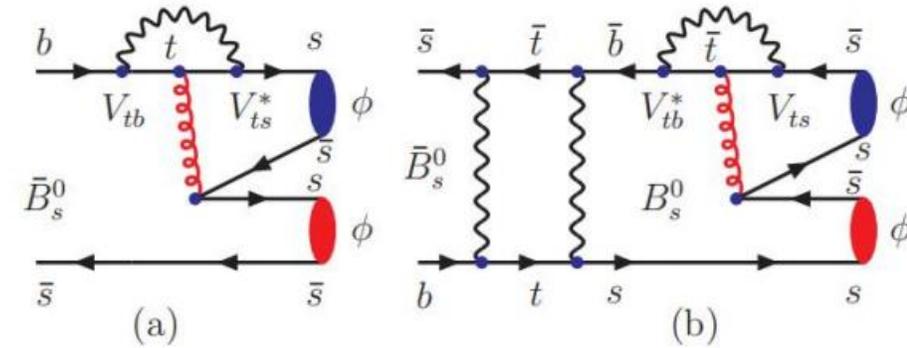
$$\Delta\phi_{s,\perp}^{J/\psi\phi} = 0.003_{-0.014}^{+0.010} \text{ (stat)} \pm 0.008 \text{ (syst) rad.}$$

Run2 analysis published *JHEP10(2025)173*



PRL 131 (2023) 171802

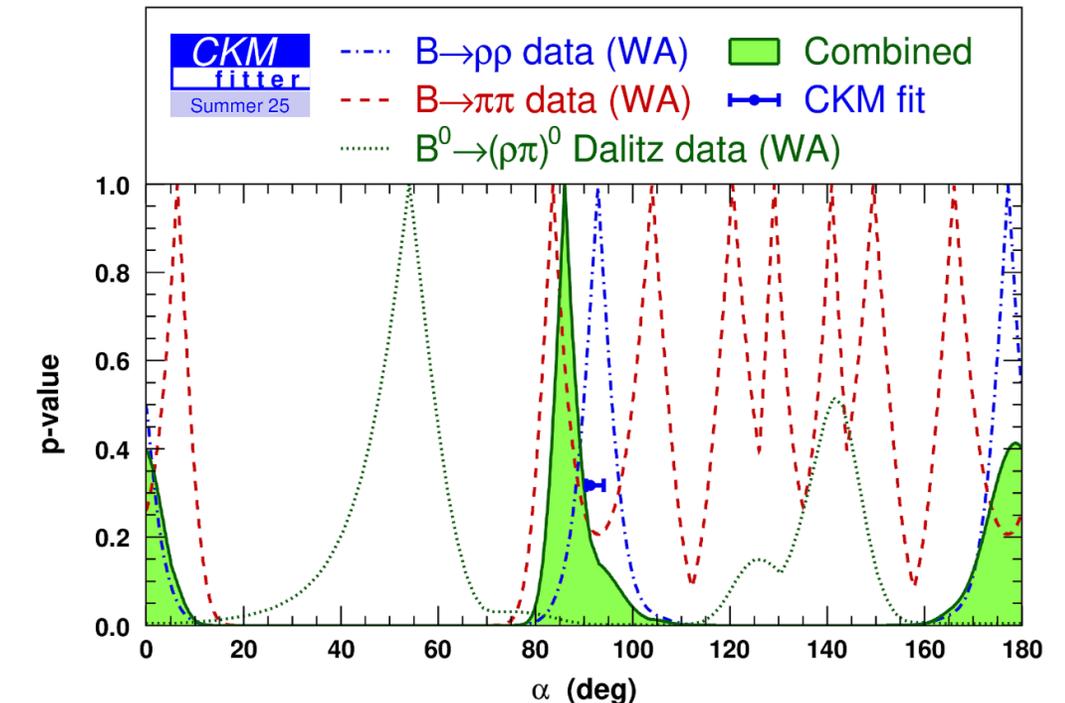
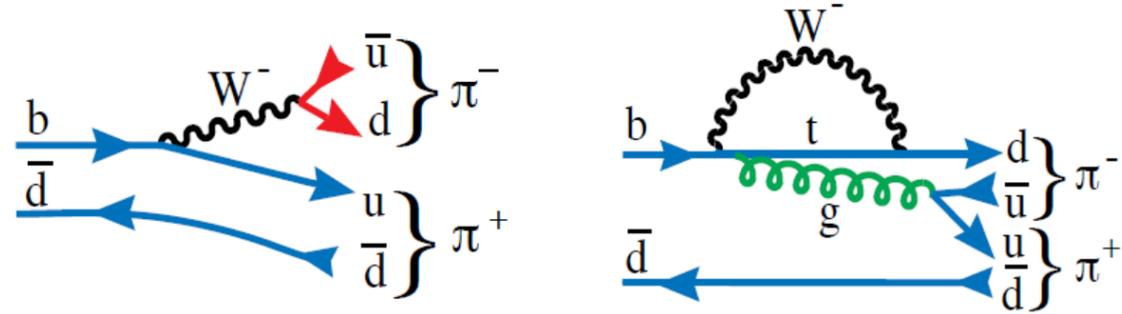
- ❖ $B_s \rightarrow \phi\phi$: **golden channel** to study CPV in $b \rightarrow s$ decays
 - Tiny CPV in SM
 - Sensitive to NP in B_s mixing and $b \rightarrow s$ decay
- ❖ CP violation observables from **flavor-tagged time-dependent angular analysis**
 - CP violating phase $\phi_{s,\bar{s}}^{S\bar{S}}$ and direct CP violation parameter $\lambda_i = \left| \frac{\bar{A}_i}{A_i} \right|$
 - SM predictions: $\phi_{s,\bar{s}}^{S\bar{S}} \approx 0$ and $\lambda_i \approx 1$ for $i = 0, \perp, \parallel$
 - Polarization-independent fit: common $\phi_s^{S\bar{S}}$ and λ for different Polarization
- ❖ Full Run 2 data, and make a combination with Run 1
 - Consistent with SM expectation
 - No sign of polarization-dependence observed
 - Most precise measurement of time-dependent CP violation in $b \rightarrow s$ decay



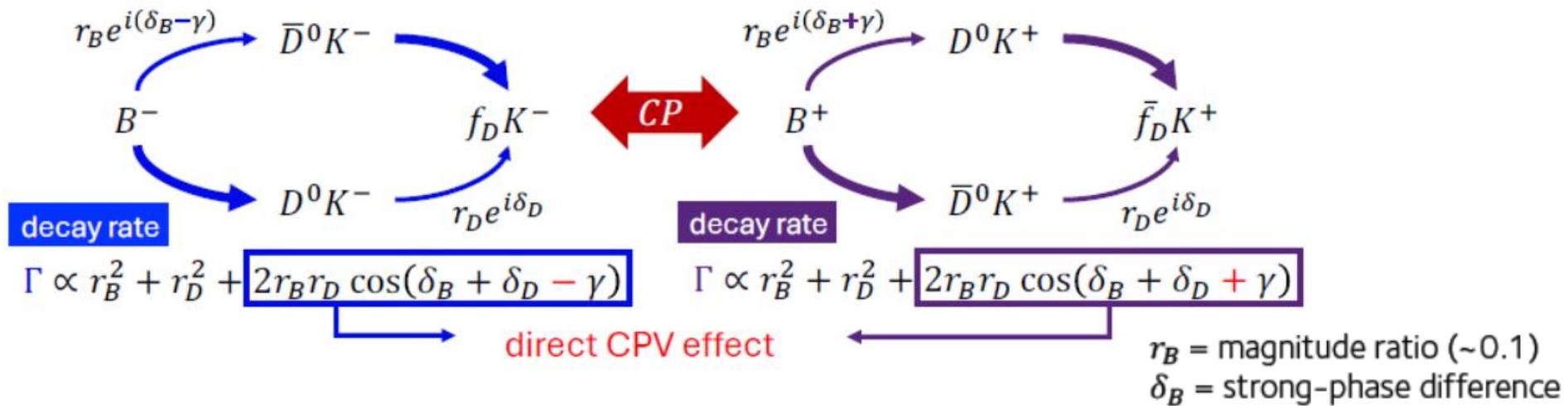
$$\phi_s^{S\bar{S}} = -0.074 \pm 0.069 \text{ rad}$$

$$|\lambda| = 1.009 \pm 0.030$$

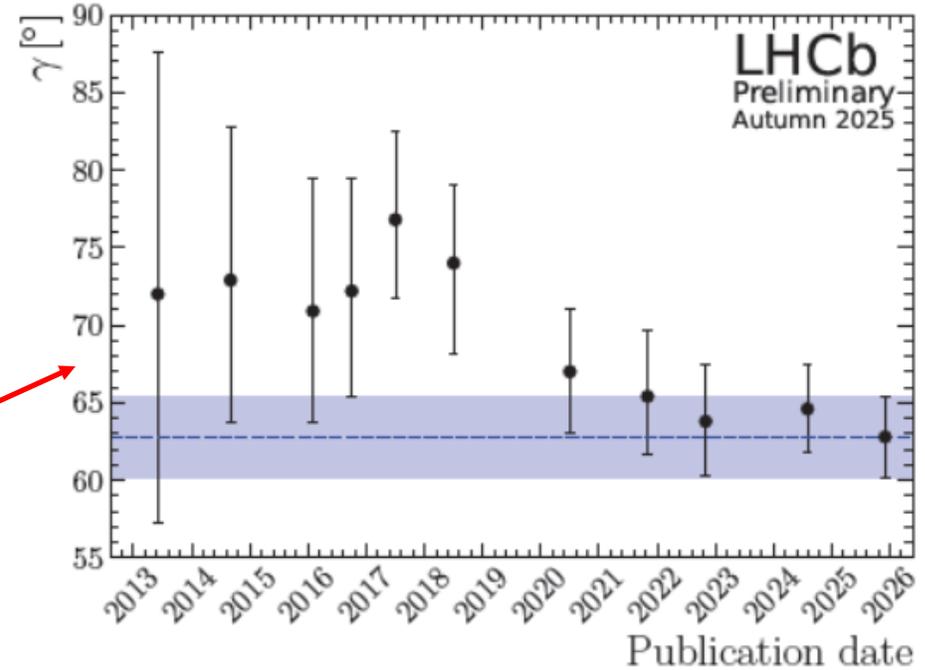
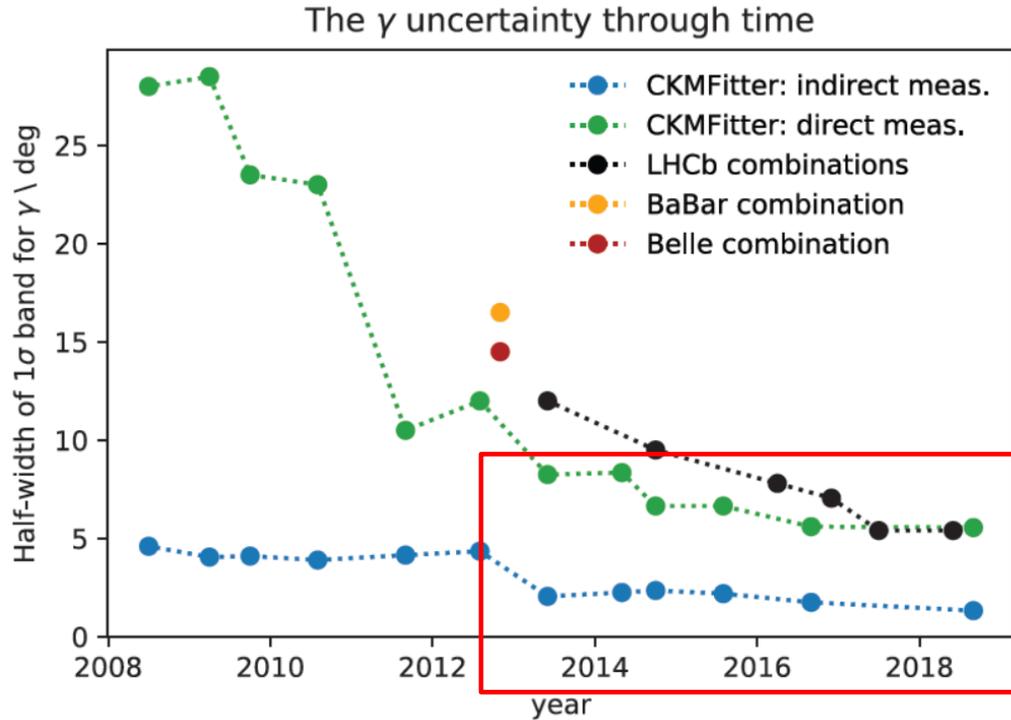
- ❖ Typical decay: $B^0 \rightarrow \pi^+\pi^-\pi^0, \pi\pi, \rho\rho \dots$
- ❖ If $B^0 \rightarrow \pi^+\pi^-$ only determined by $b \rightarrow u$ tree diagram, similar to β
- ❖ $B^0 \rightarrow \pi^+\pi^-$ penguin contribution same order to tree diagram
- ❖ M. Gronau & D. London: Isospin analysis of CP asymmetries in B decays, simultaneously measure $B_d^0 \rightarrow \pi^+\pi^-$, $B_d^0 \rightarrow \pi^0\pi^0$ and $B_u^+ \rightarrow \pi^+\pi^0$
PRL 65 (1990) 3381
- ❖ Other methods (e.g. $B \rightarrow \rho^{+0}\rho^{-0}$)
- ❖ Limited to the statistics, still much room for improvement in accuracy of α



- ❖ Interference between favoured $b \rightarrow c$ and suppressed $b \rightarrow u$ decay amplitude
- ❖ tree-level decays, theoretically clean ($\delta\gamma/\gamma < 10^{-7}$) *JHEP 1401(2014)051*
- ❖ Ideal decays: $B \rightarrow DK$ (clean background, large branching fraction)

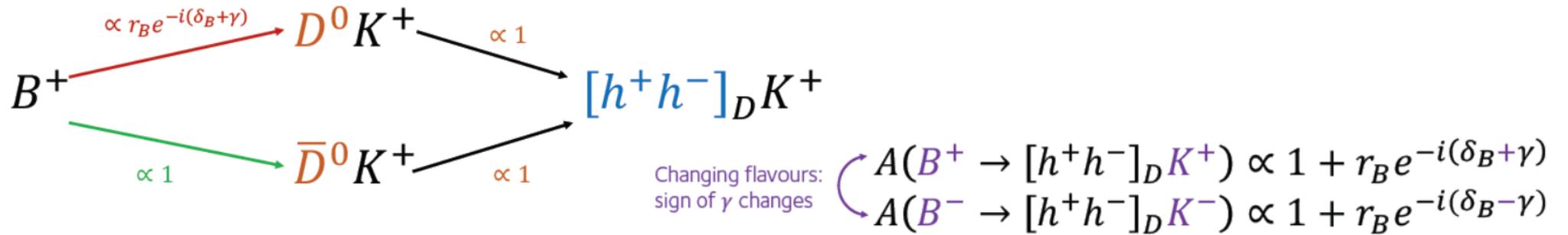


- ❖ Different method according to D decay modes
 - GLW method: CP modes such as $D \rightarrow K^+ K^-, \pi^+ \pi^-$
 - ADS method: Flavor modes such as $D \rightarrow K\pi, K\pi\pi^0$
 - BPGGSZ method: $D \rightarrow K_s \pi\pi / K_s KK$ (golden mode)



- ❖ The precision of γ improved significantly over the past two decades
- ❖ LHCb dominates current world averages of direct γ measurements
- ❖ Many more modes still need to add
- ❖ Need D strong parameters from charm factory as input (see BaiCian's talk)

- ❖ D (quasi-)CP-eigenstate states such as $D \rightarrow K^+K^-, \pi^+\pi^-, \pi^+\pi^-\pi^0 \dots$



- ❖ Use the yields of B+ and B- to construct observables related to γ

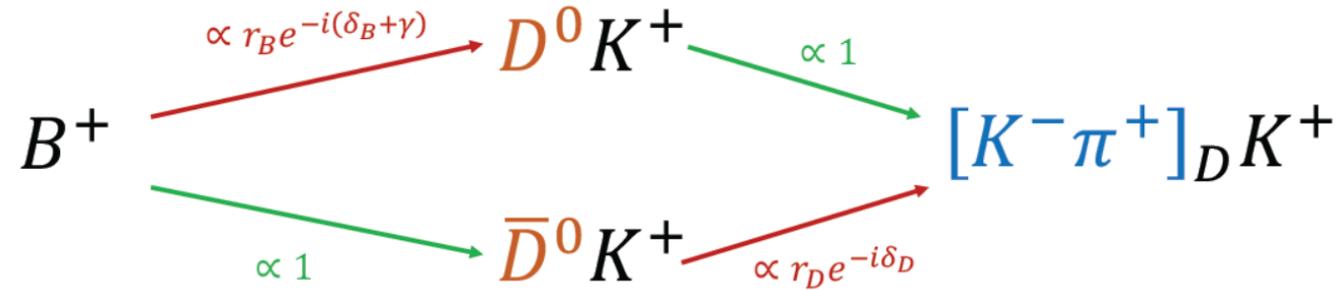
$$A^f = \frac{N(B^- \rightarrow f_D K^-) - N(B^+ \rightarrow f_D K^+)}{N(B^- \rightarrow f_D K^-) + N(B^+ \rightarrow f_D K^+)} = \frac{2\kappa r_B \sin\delta_B \sin\gamma}{R^f}$$

$$R^f = \frac{N(B^- \rightarrow f_D K^-) + N(B^+ \rightarrow f_D K^+)}{N(B^- \rightarrow [K\pi]_D K^-) + N(B^+ \rightarrow [K\pi]_D K^+)} = 1 + r_B^2 + 2\kappa r_B \cos\delta_B \cos\gamma$$

insert a factor of $(\kappa=2F_{+-}-1)$ before interference terms (F_{+-} =CP even content), need charm input

Notice r_B/δ_B need input

- ❖ Consider the Cabibbo-favored decay $D^0 \rightarrow K^- \pi^+$ and doubly-Cabibbo-suppressed decay $D^0 \rightarrow K^+ \pi^-$



- ❖ r_B/δ_B can be obtained directly, but external input r_D/δ_D

$$\Gamma(B^\pm \rightarrow f_D K^\pm) \propto r_B^2 + r_D^2 + 2R_f r_B r_D \cos(\delta_B + \delta_D \pm \gamma)$$

$$\Gamma(B^\pm \rightarrow \bar{f}_D K^\pm) \propto 1 + r_B^2 r_D^2 + 2R_f r_B r_D \cos(\delta_B - \delta_D \pm \gamma)$$

Need inputs from charm factory

$$R_{K3\pi} e^{-i\delta_{K3\pi}} = \frac{\int A_{K^-3\pi}(x) A_{K^+3\pi}(x) dx}{A_{K^-3\pi}(x) A_{K^+3\pi}(x)}$$

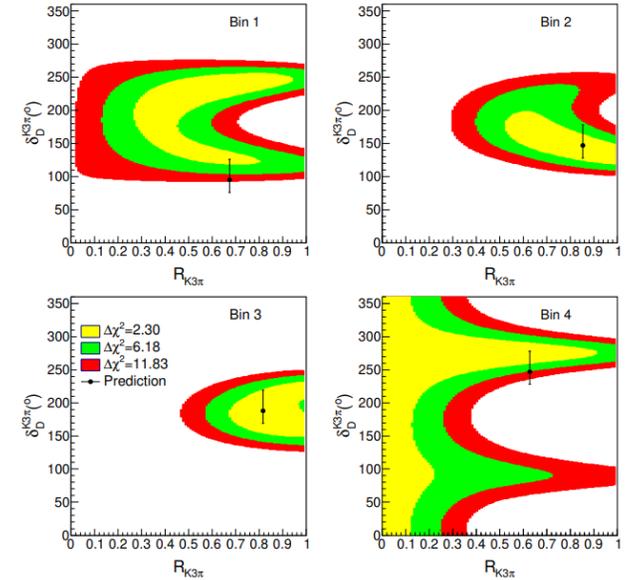
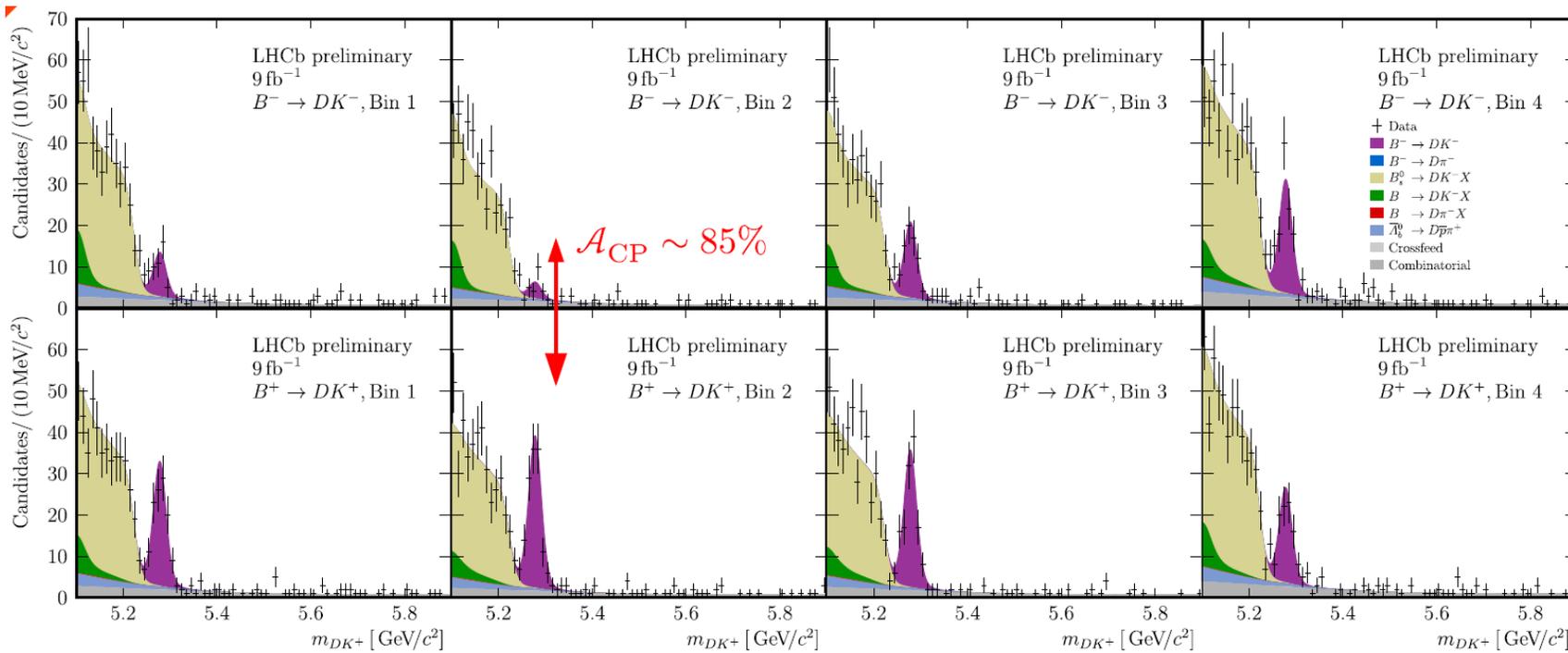
- For $K3\pi$ mode, coherence factor $R_{K3\pi}$ and $\delta_{K3\pi}$ averaged over phase space not good for whole space

[1] D. Atwood, I. Dunietz, and A. Soni, Phys. Rev. Lett. 78 (1997) 3257

[2] D. Atwood, I. Dunietz, and A. Soni, Phys. Rev. D63 (2001) 036005

❖ Large CPV observed in local bins!

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$$\gamma = \left(54.8 \begin{matrix} +6.0 \\ -5.8 \end{matrix} + 0.6 \begin{matrix} +6.7 \\ -4.3 \end{matrix} \right)^\circ$$

Comparable to golden mode!

Large expected improvement from incoming 20fb⁻¹ of BESIII $\psi(3770)$ data

- ❖ Golden mode: $D \rightarrow K_s \pi \pi / K_s K K$ (large statistic, large r_D)
 - Model-dependent method (not used now)
 - Model-independent binned method (BPGGSZ method^[1])
- ❖ Binned Dalitz plane according to δ_D , measure B^\pm yields in each bins
 - Sensitivity from **phase-space distribution**, not overall asymmetries \rightarrow not impacted by production/detection asymmetries
 - LHCb latest $K_s h h$ result: $\gamma = (68.7_{-5.1}^{+5.2})^\circ$ ([uncertainty \$\sim 1^\circ\$ from BESIII input](#))

$$r_B \exp[i(\delta_B \pm \gamma)] = x_\pm + iy_\pm$$

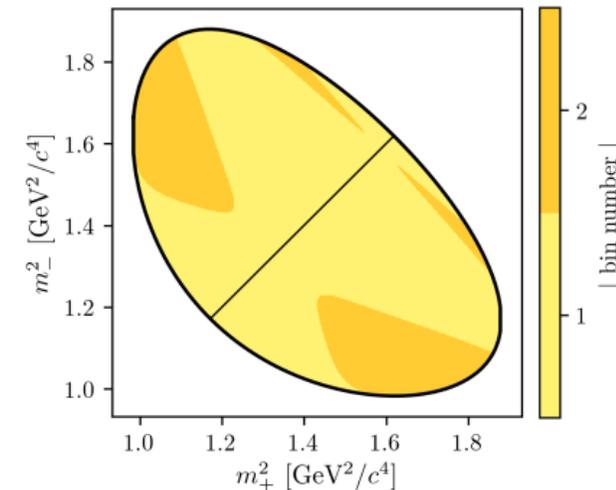
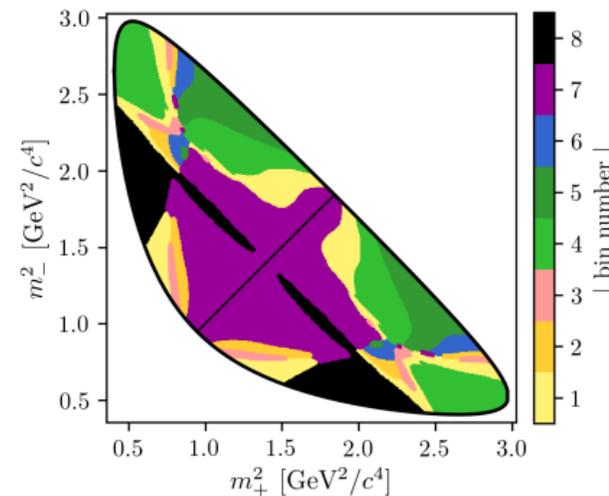
$$N_{\pm i}^- \propto F_{\pm i} + (x_\pm^2 + y_\pm^2) F_{\mp i} + 2\sqrt{F_i F_{-i}} (x_\pm c_{\pm i} \mp y_\pm s_{\pm i})$$

F_i : Fractional yield of flavour tagged D^0 into bin i

Measured in control channel:
 $\bar{B}^0 \rightarrow D^{*+} \mu^- \nu_\mu X$

c_i/s_i : Strong phase difference of $D^0 - \bar{D}^0$ decays

External input from BESIII and CLEO-c



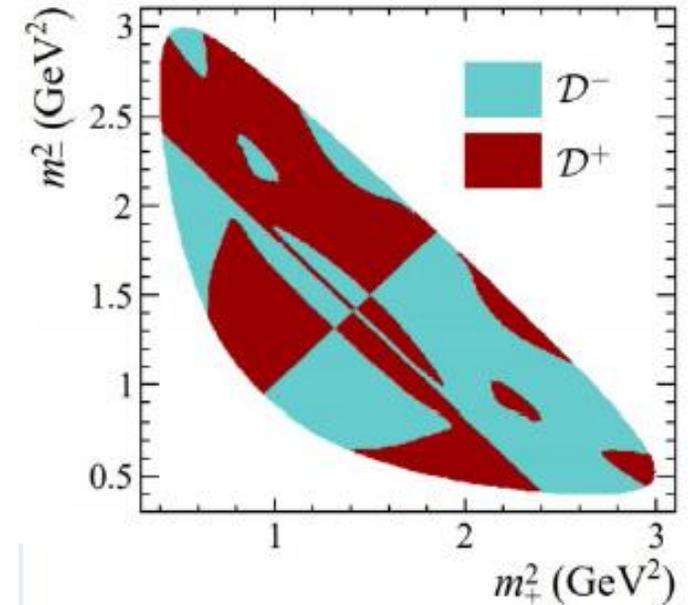
- ❖ Basic idea: Bins \rightarrow Events (*EPJC, 2018, 78(2)*)
 - Make most use of amplitude info in phase space
- ❖ Fourier expansion the amplitude by strong phase
 - Parameters definition similar to BPGGSZ method

$$\begin{aligned}
 \bullet \bar{a}_n^{B\pm} &= \bar{h}_B \{ a_n^{D\mp} + r_B^2 a_n^{D\pm} + 2[x_+ a_n^C \pm y_+ a_n^S] \} & \bullet x_{\pm} &= r_B \cos(\delta_B \pm \gamma) \\
 \bullet \bar{b}_n^{B\pm} &= \bar{h}_B \{ -b_n^{D\mp} + r_B^2 b_n^{D\pm} \pm 2[x_+ b_n^C - y_+ a_n^S] \} & \bullet y_{\pm} &= r_B \sin(\delta_B \pm \gamma) \\
 \bullet a_n^{B\pm} &= h_B \{ a_n^{D\pm} + r_B^2 a_n^{D\mp} + 2[x_- a_n^C \mp y_- a_n^S] \} \\
 \bullet b_n^{B\pm} &= h_B \{ b_n^{D\pm} - r_B^2 b_n^{D\mp} \pm 2[x_- b_n^C + y_- b_n^S] \}
 \end{aligned}$$

B sector

$$\begin{aligned}
 \bullet a_n^{D\pm} &= h_{Df} \{ a_n^{D\pm} + r_D^2 a_n^{D\mp} - 2R_D r_D [\cos(\delta_D) a_n^C \pm \sin(\delta_D) a_n^S] \} & \bullet \lambda_{CP} &= 2F_+ - 1 \\
 \bullet a_n^{CP\pm} &= h_{CP} [a_n^{D\pm} + a_n^{D\mp} - 2\lambda_{CP} a_n^C] \\
 \bullet a_{mn}^{DD\pm\pm} &= h_{DD} [a_m^{D\pm} a_n^{D\mp} + a_m^{D\mp} a_n^{D\pm} - 2(a_m^C a_n^C \pm a_n^S a_m^S)]
 \end{aligned}$$

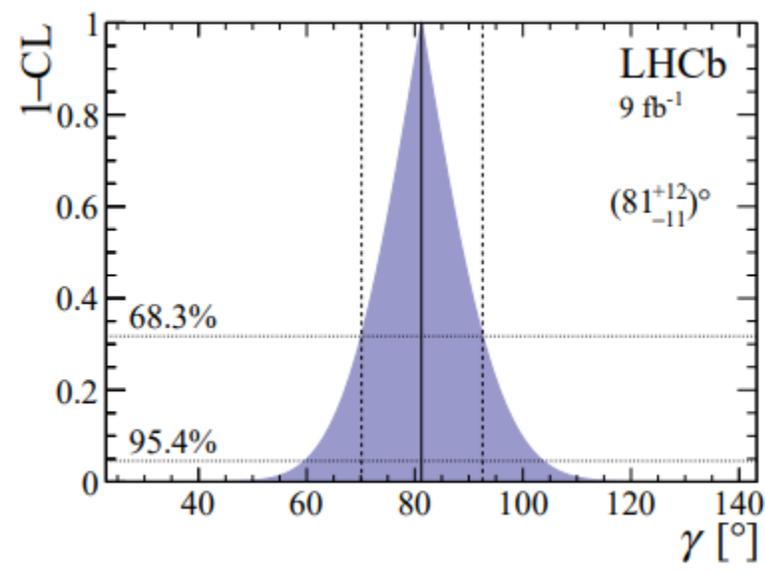
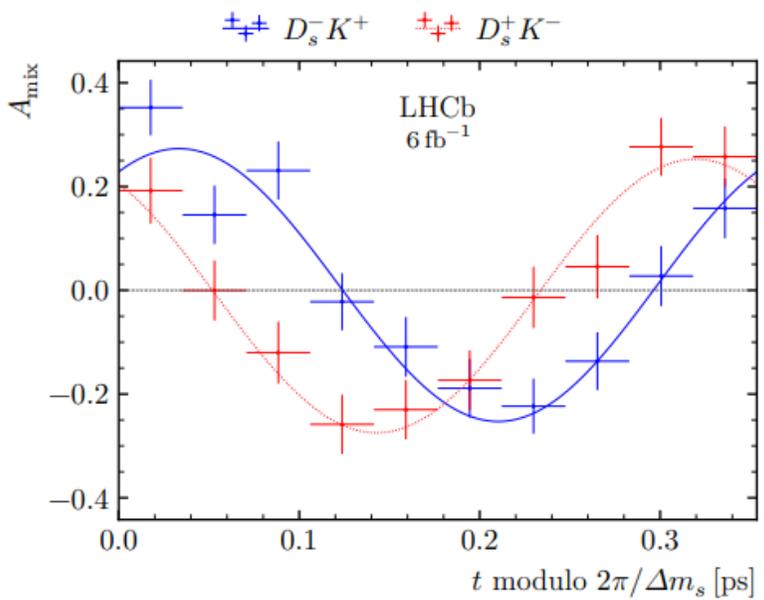
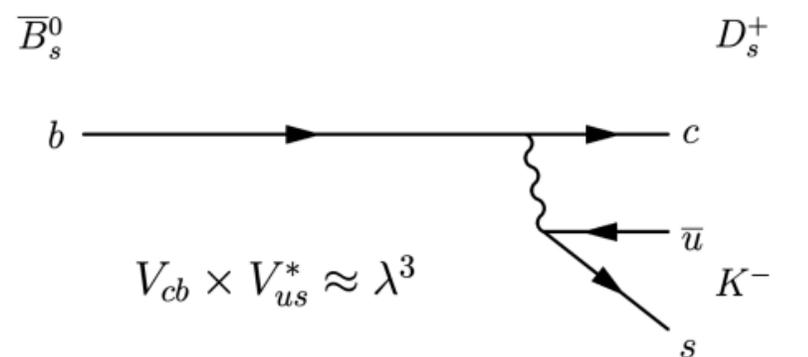
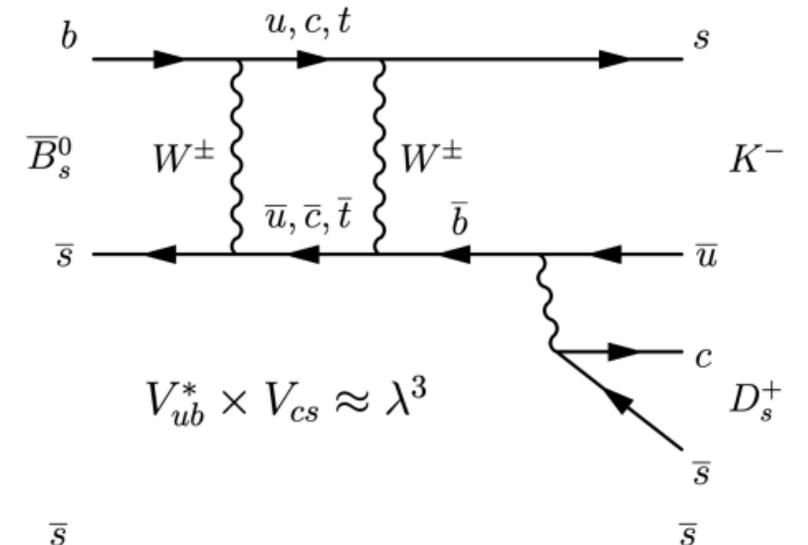
D sector



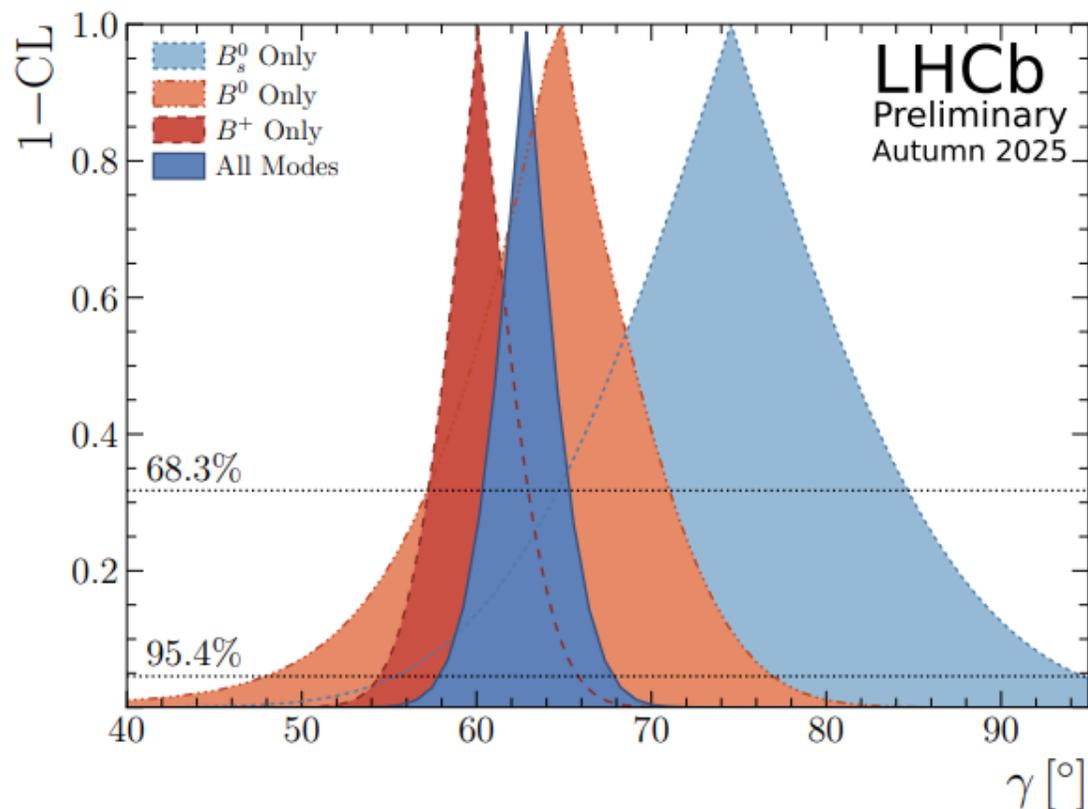
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❖ Golden decays: $B_s \rightarrow D_s K$

- larger interference: $r_B^{D_s K} \sim 0.4$ ($r_B^{D K^+} \sim 0.1$)
- Use flavor tagging to determine the initial flavor
- Interference between mixing and decay amplitudes gives sensitivity to $\gamma + (-)2\beta_{(s)}$



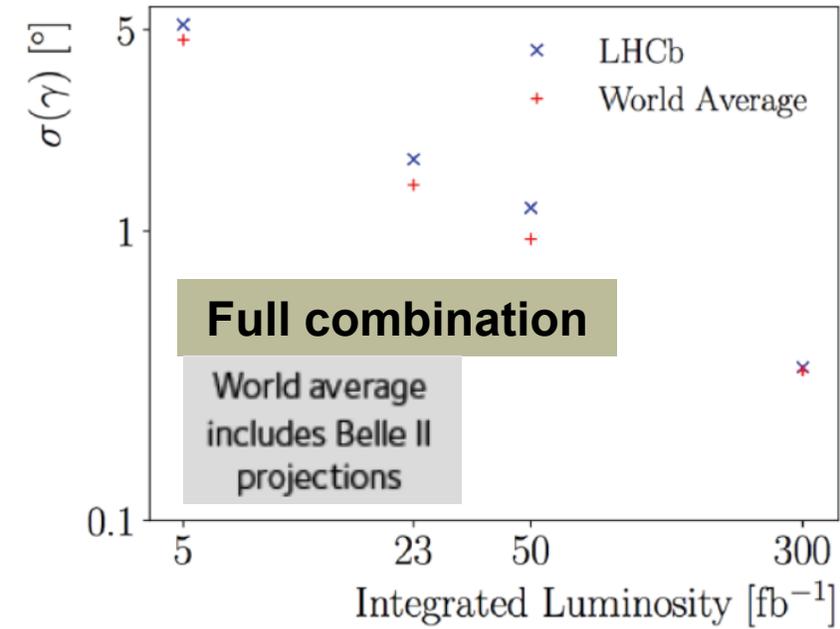
- ❖ Best knowledge of γ comes from combination of many measurements
- ❖ 20 LHCb B decay measurements + 12 D decay measurements + 14 inputs from LHCb, HFLAV, BESIII and CLEO-c = 29 physics parameters of interest + additional nuisance parameters



$$\gamma = (62.8 \pm 2.6)^\circ$$

surpass LHCb design: 4°

- Previous tension between B_s^0 and other modes smaller, B_s^0 modes still with largest uncertainty
- Sensitivity dominated by B^+ modes
- Charm inputs crucial for γ measurements

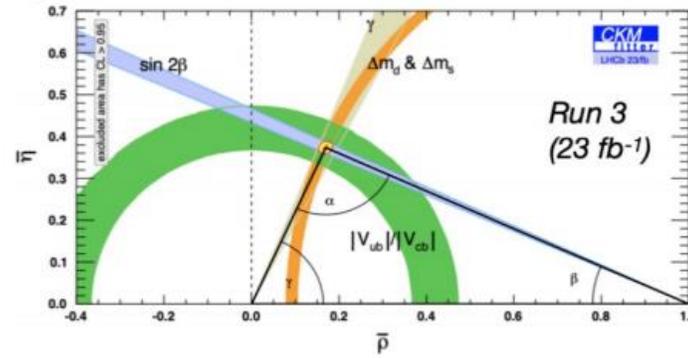
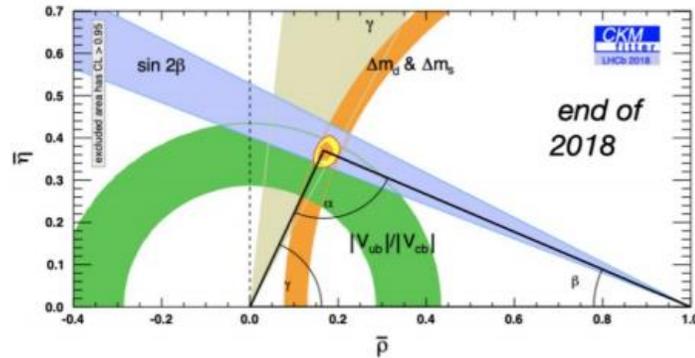


- ❖ Status now:
 - Error for γ is about 4°
 - BESIII contribute about 1°
- ❖ Around 2030
 - Less than 1° will be achieved
 - BESIII 20fb⁻¹ data → improve the error to 0.4°
- ❖ (>)2035
 - LHCb upgradell → sensitivity <0.4°
 - Need more charm factory data (STCF)

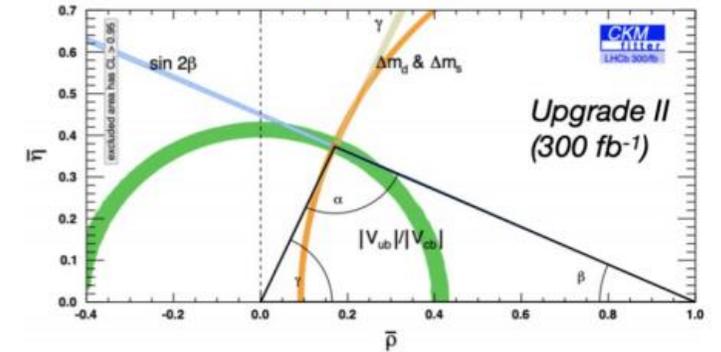
dataset	Int. Lum	Achieve year	γ precision
LHCb Run1 (7,8TeV)	3 fb ⁻¹	2012	8°
LHCb Run2 (13TeV)	6 fb ⁻¹	2018	4°
BelleII Run	50 ab ⁻¹	20xx	1-2°
LHCb upgrade (14TeV)	50 fb ⁻¹	2030	<1°
LHCb upgradeII (14TeV)	200 fb ⁻¹	(>)2035	<0.4°

- ❖ The precision of the CKM elements and angles have greatly improved

LHCb-only inputs for UT



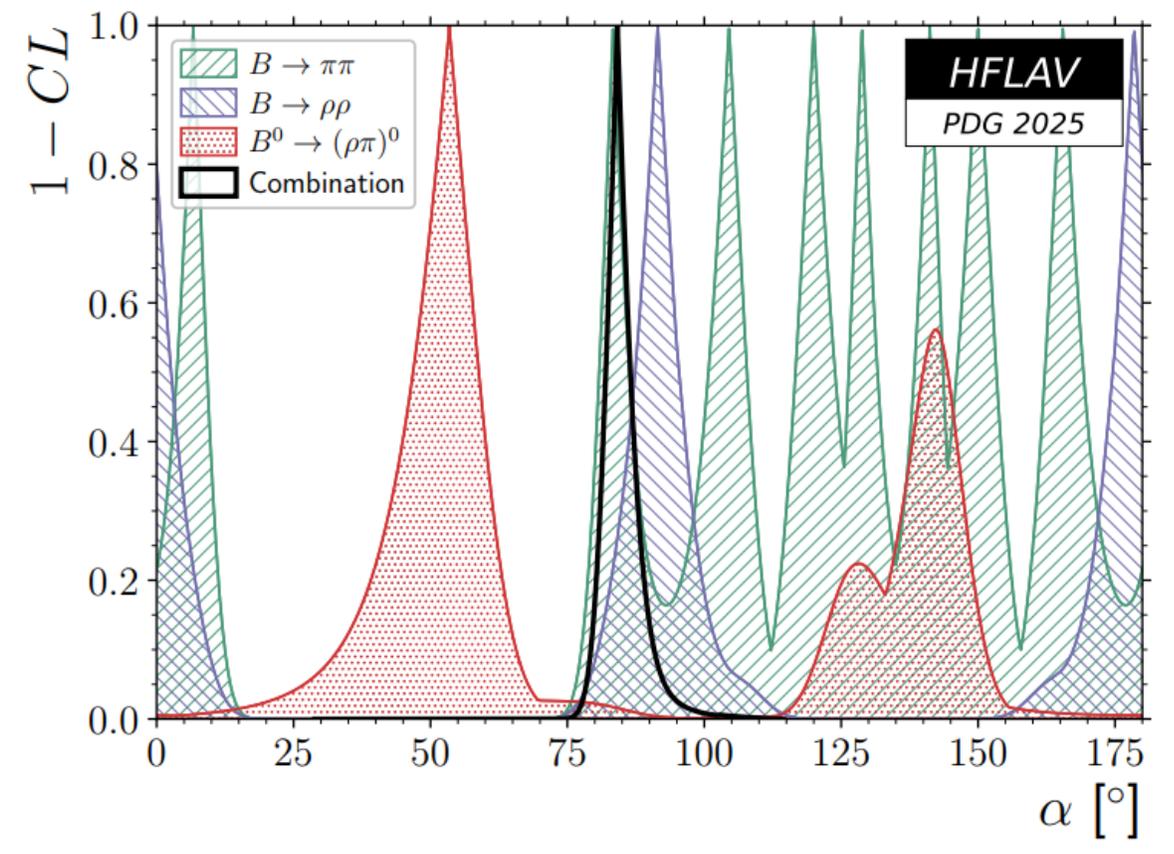
LHCb U2 + LQCD improvement



- ❖ LHCb play important role in measuring CKM elements and angle
- ❖ Already collected more data in Run 3 than in Run 1+2 combined, **Stay tuned !**
- ❖ Carry out an unprecedentedly rigorous test of the Standard Model in Upgrade II

Thank you!

- ❖ α is complex, isospin analysis by using:
 - $B_d^0 \rightarrow \rho^+ \rho^-, \rho^0 \rho^0, B_u^+ \rightarrow \rho^+ \rho^0$
- ❖ Time-dependent CP Asymmetries
- ❖ LHCb can contribute in many modes



$$\begin{aligned}\phi_s^{\text{SM}}(\text{B}_s^0 \rightarrow \phi\phi) &= -\arg(\eta_f \lambda_f) = -\arg\left(\eta_f \frac{q}{p} \frac{\bar{A}_f}{A_f}\right) = \Phi_M^{\text{SM}} - \Phi_D^{\text{SM}} \\ &\approx 2 \arg(V_{ts}^* V_{tb}) - \arg(V_{tb} V_{ts}^* / V_{tb}^* V_{ts}) = -2\chi + 2\chi = 0.\end{aligned}$$

$$\begin{aligned}\phi_s(\text{B}_s^0 \rightarrow \phi\phi) &\equiv \phi_s^{\text{SM}}(\text{B}_s^0 \rightarrow \phi\phi) + \Phi_M^{\text{NP}}(\text{B}_s^0) - \Phi_D^{\text{NP}}(\text{B}_s^0 \rightarrow \phi\phi) \\ &= \phi_s^{\text{SM}}(\text{B}_s^0 \rightarrow \phi\phi) + \phi_s^{\text{NP}}(\text{B}_s^0 \rightarrow \phi\phi) \approx \phi_s^{\text{NP}}(\text{B}_s^0 \rightarrow \phi\phi).\end{aligned}$$