

D-meson decays on the lattice

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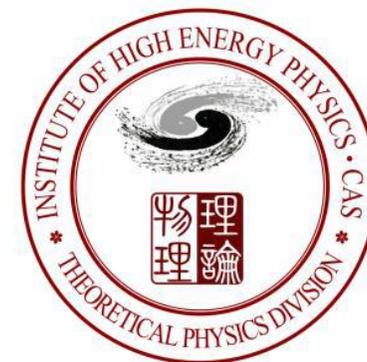
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味物理前沿研讨会

暨味物理讲座100期特别活动

2026.01.30-02.04



内容提纲

- 格点QCD
- 粲介子纯轻衰变
 - $D^{(*)}$ 、 $D_s^{(*)}$ 衰变常数
 - D_s^* 辐射衰变
- 粲介子半轻衰变
 - $D \rightarrow \pi/K l \nu$
 - $D_s \rightarrow \phi l \nu$

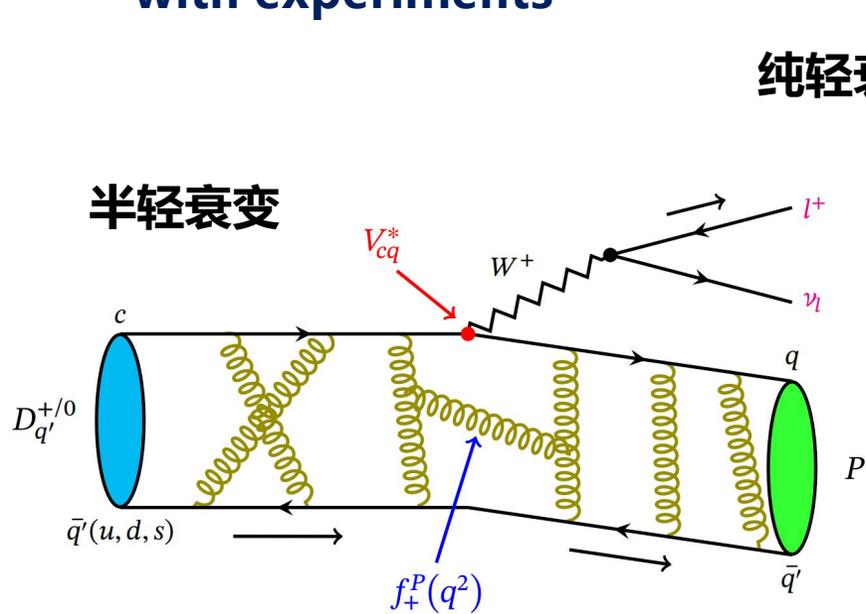
Y.-B. Yang, ..., ZL et al., PRD92.034517 (2015), arXiv:1410.3343
Y. Chen, ..., ZL et al., CPC45 (2021) 023109, arXiv:2008.05208
D. Li, ..., ZL et al., CPC48 (2024) 123105, arXiv:2407.03697
Y. Meng, ..., ZL et al., PRD109.074511, arXiv:2401.13475
G. Fan, Y. Meng, C. Liu, ZL et al., arXiv:2510.14478
FLAG review 2024, arXiv:2411.04268
PDG review 2025 update

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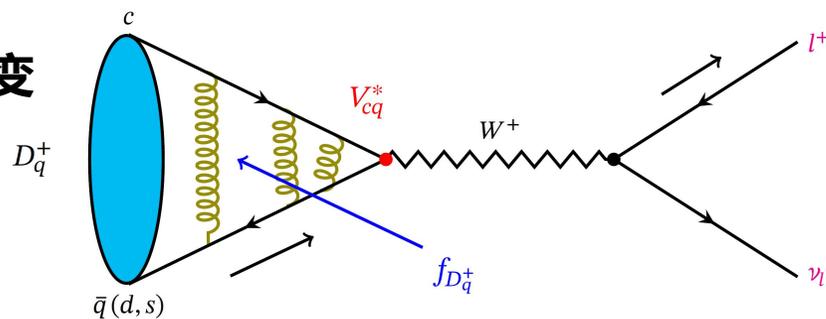
粲介子含轻衰变与LQCD

- LQCD can calculate form factors and meson decay constants appearing in weak decays of hadrons
- Combined with experiments, they can give us CKM matrix elements
- Test the SM (is the CKM matrix unitary?)
- Or use V_{ab} from elsewhere to compare QCD/SM results with experiments

V_{ud}	V_{us}	V_{ub}
$\pi \rightarrow \ell \nu$	$K \rightarrow \ell \nu$	$B \rightarrow \pi \ell \nu$
	$K \rightarrow \pi \ell \nu$	
V_{cd}	V_{cs}	V_{cb}
$D \rightarrow \ell \nu$	$D_s \rightarrow \ell \nu$	$B \rightarrow D \ell \nu$
$D \rightarrow \pi \ell \nu$	$D \rightarrow K \ell \nu$	$B \rightarrow D^* \ell \nu$
V_{td}	V_{ts}	V_{tb}
$B_d \leftrightarrow \bar{B}_d$	$B_s \leftrightarrow \bar{B}_s$	



纯轻衰变



CKM矩阵元

LQCD计算

例如:

$$\Gamma(P \rightarrow \ell \nu) = \frac{G_F^2 |V_{q_1 q_2}|^2 f_P^2 m_\ell^2 M_P \left(1 - \frac{m_\ell^2}{M_P^2}\right)^2}{8\pi}$$

实验测量

格点QCD (1973, Wilson; 1979, Creutz) 用数值模拟研究QCD的非微扰性质

- 4维闵氏时空 \rightarrow 4维欧氏空间 ($\tau = it$)

$$\langle O \rangle = \frac{\int DA_\mu D\bar{\psi} D\psi O[A, \bar{\psi}, \psi] e^{-\int \mathcal{L}_{QCD} d^4x}}{\int DA_\mu D\bar{\psi} D\psi e^{-\int \mathcal{L}_{QCD} d^4x}}, \quad \mathcal{L}_{QCD} = \bar{\psi} M[A] \psi + \mathcal{L}_G$$

$$M = \gamma \cdot D + m_q$$

$$\langle O \rangle = \frac{\int DU_\mu O[U, M^{-1}[U]] \text{Det}[M[U]] e^{-S_G}}{\int DU_\mu \text{Det}[M[U]] e^{-S_G}} \sim \frac{\int dx f(x) \rho(x)}{\int dx \rho(x)} \quad \rightarrow \quad \sim \frac{1}{N} \sum_{n=1}^N f(x_n)$$

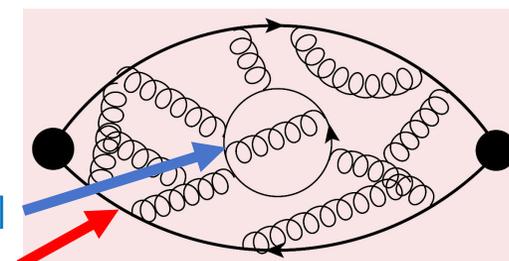
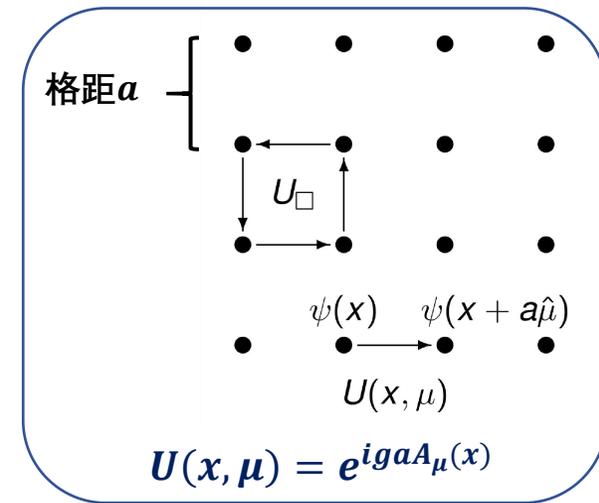
离散的 x_n 按 $\rho(x_n)$ 分布

上式为带权重 $\text{Det}[M[U]] e^{-S_G}$ 的平均，类似Boltzmann系综平均

- 在有限体积4维超立方格子上，自由度个数可数，路径积分具有良好定义
- 巨大高维积分，无法直接计算；用重点抽样按权重分布产生 U (组态)
- 路径积分变为对组态的统计平均： $\langle O \rangle = \frac{1}{N} \sum_{i=1}^N O_i$

N 有限，统计误差 $\sim 1/\sqrt{N}$

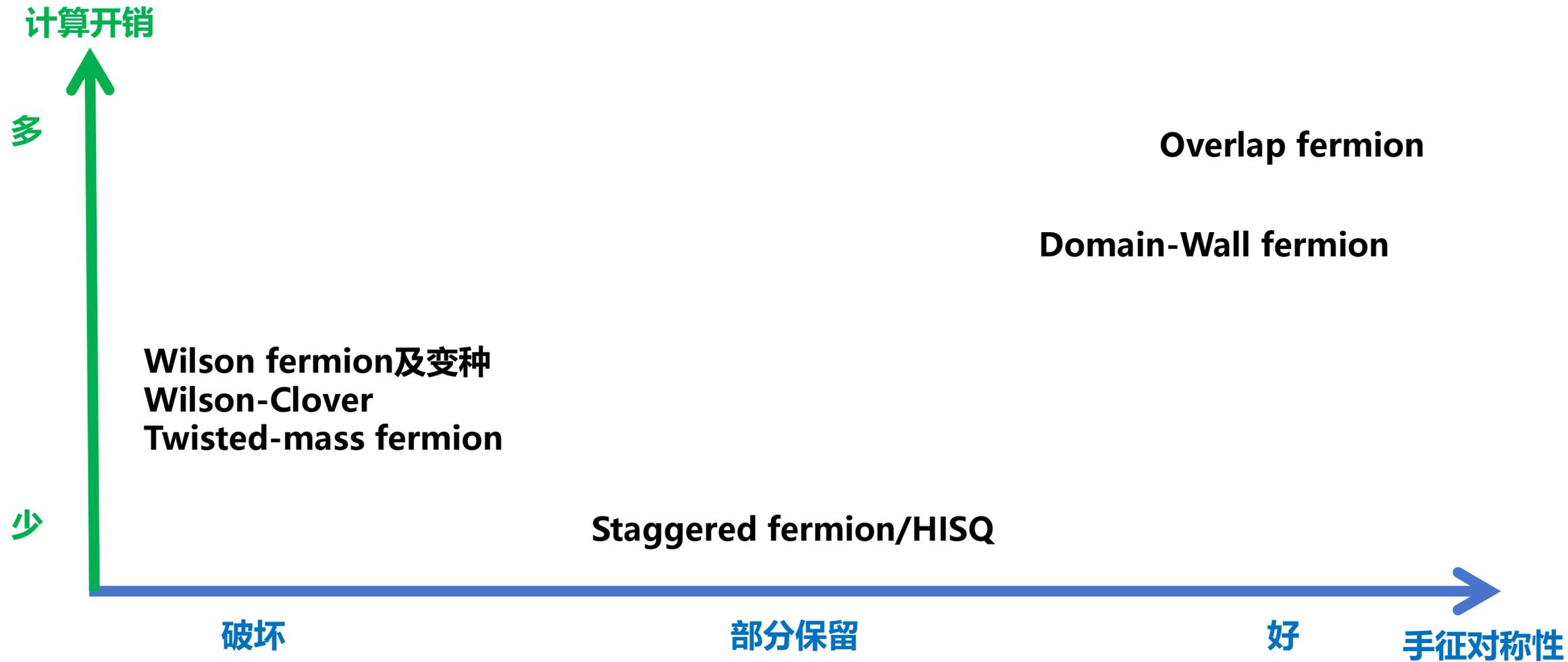
输入若干实验测量值，确定自由参数：
格距 a 和夸克质量 m_q 。计算其他结果



介子两点关联函数

格点费米子作用量

- 直接简单地离散化费米子作用量 (naive fermion) 存在加倍子问题



Flavor Lattice Averaging Group (FLAG) 味物理格点计算结果平均

<http://flag.unibe.ch/>

约三年综述一次格点结果

- 2010 *Eur. Phys. J. C* (2011) 71, 1695 (arXiv: 1011.4408)
- 2013 *Eur. Phys. J. C* (2014) 74, 2890 (arXiv: 1310.8555)
- 2016 *Eur. Phys. J. C* (2017) 77, 112 (arXiv: 1607.00299)
- 2019 *Eur. Phys. J. C* (2020) 80, 113 (arXiv: 1902.08191)
- 2021 *Eur. Phys. J. C* (2022) 82, 869 (arXiv: 2111.09849)
- 2024 arXiv:2411.04268, up to 2024.4.30

对正式发表的格点QCD结果的系统误差做评估

- 连续极限
- 手征外推
- 有限体积
- 重正化常数
- 重夸克作用量

- LECs
- α_s
- quark masses
- decay constants
- form factors
- nucleon matrix elements
-

Color-coding of systematic errors:

★ has been estimated in a satisfactory manner.

○ reasonable, could be improved.

■ no estimation, or unsatisfactory.

获得绿色评分的结果做平均

强子矩阵元的格点计算

$$C(t) = \sum_{\vec{x}} \langle \Omega | O(\vec{x}, t) O^\dagger(0) | \Omega \rangle \xrightarrow{t \rightarrow \infty} |\langle \Omega | O | P \rangle|^2 e^{-m_P t} \equiv A e^{-m_P t}$$

- 介子衰变常数从两点函数抽取, 例如 $O = \bar{q} \gamma_0 \gamma_5 c$

$$\langle 0 | \bar{q}(0) \gamma_\mu \gamma_5 c(0) | P(p) \rangle = i f_P p_\mu \quad q = d, s$$

- 结合两点和三点函数可抽取辐射/半轻过程强子矩阵元 (形状因子) :

$$C_3(\vec{p}, \vec{p}', T, t) = \sum_{\vec{z}} \sum_{\vec{y}} \langle 0 | O_P(\vec{z}, T) J(\vec{y}, t) O_D^\dagger(\vec{x}, 0) | 0 \rangle e^{-i\vec{p} \cdot \vec{z}} e^{i\vec{q} \cdot \vec{y}}$$

$$\xrightarrow[T \rightarrow \infty]{t \rightarrow \infty, (T-t) \rightarrow \infty} \langle 0 | O_P | P \rangle \langle P | J | D \rangle \langle D | O_D^\dagger | 0 \rangle e^{-m_D t} e^{-m_P (T-t)}$$

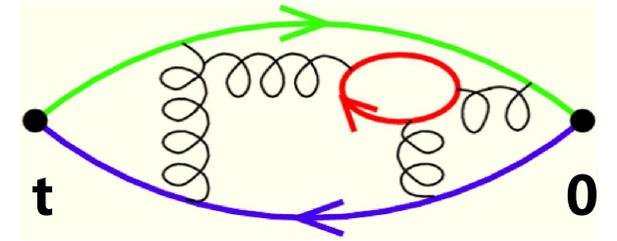
- 复合算符重正化常数

- 由于离散效应, 格子上的局域(轴)矢量流 $\bar{q} \gamma_\mu c$ ($\bar{q} \gamma_\mu \gamma_5 c$) 需要归一化常数 $Z_{V,A}$
- 标量及张量流算符随能标跑动, $Z_{S,T}$

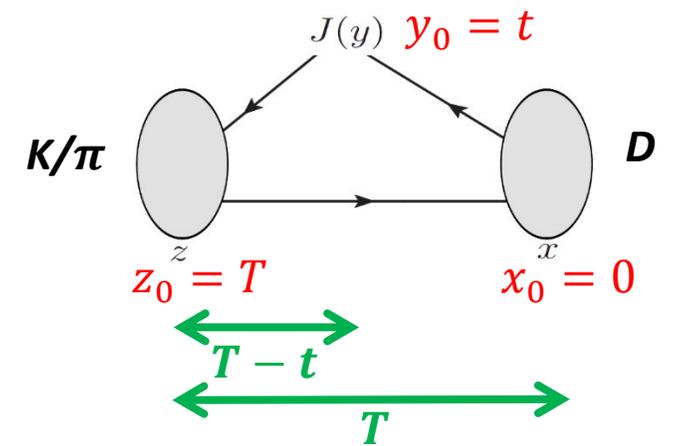
- 使用手征格点费米子有时可避免重正化常数的计算 ($Z_S Z_m = 1$)

- PCAC: $(m_q + m_c) \langle 0 | \bar{q}(0) \gamma_5 c(0) | P(p) \rangle = f_P m_{PS}^2$

- PCVC: $\langle K | S | D \rangle = f_0^{D \rightarrow K}(q^2) \frac{M_D^2 - M_K^2}{m_c - m_s}$



两点关联函数



三点关联函数

粲介子衰变常数

• $f_{D^{(*)}}$

$$\langle 0 | \bar{q}(0) \gamma_\mu \gamma_5 c(0) | P(p) \rangle = i f_P p_\mu \quad q = d, s$$

$$\langle 0 | \bar{q}(0) \gamma^\mu q'(0) | V(p, \lambda) \rangle = f_V m_V e_\lambda^\mu$$

• $f_V^T / f_V(\overline{\text{MS}}, 2 \text{ GeV})$ $\langle 0 | \left(\bar{q}(0) \sigma^{\mu\nu} q'(0) \right) (\mu) | V(p, \lambda) \rangle = i f_V^T(\mu) (e_\lambda^\mu p^\nu - e_\lambda^\nu p^\mu)$ (需要张量流重整化常数 $Z_T^{\overline{\text{MS}}}$)

$$\Gamma_{(D_s^* \rightarrow \ell \nu)} = \frac{G_F^2}{12\pi} |V_{cs}|^2 f_{D_s^*}^2 M_{D_s^*}^3 \left(1 - \frac{m_\ell^2}{M_{D_s^*}^2}\right)^2 \left(1 + \frac{m_\ell^2}{2M_{D_s^*}^2}\right)$$

• Determine CKM elements

• Not easy to measure f_V

$$\text{Br} = 4.26(52) \times 10^{-5} \quad [\text{D. Li, ..., ZL et al., CPC48 (2024) 123105}]$$

• Leptonic decay BRs are small; $D_s^* \rightarrow \ell \nu$ reported by BESIII [PRL131.141802(2023)]

• Test the accuracy of Heavy Quark Effective Theory: $f_V / f_P = 1 + \mathcal{O}(1/m_Q)$

• f_V^T / f_V for D^* and D_s^* are inputs for LCSR in calculations of $B \rightarrow V$ form factors at low q^2

• Input parameters for QCD factorization in studies of nonleptonic B decays, e.g., $B \rightarrow D^{(*)} M$

实验测量

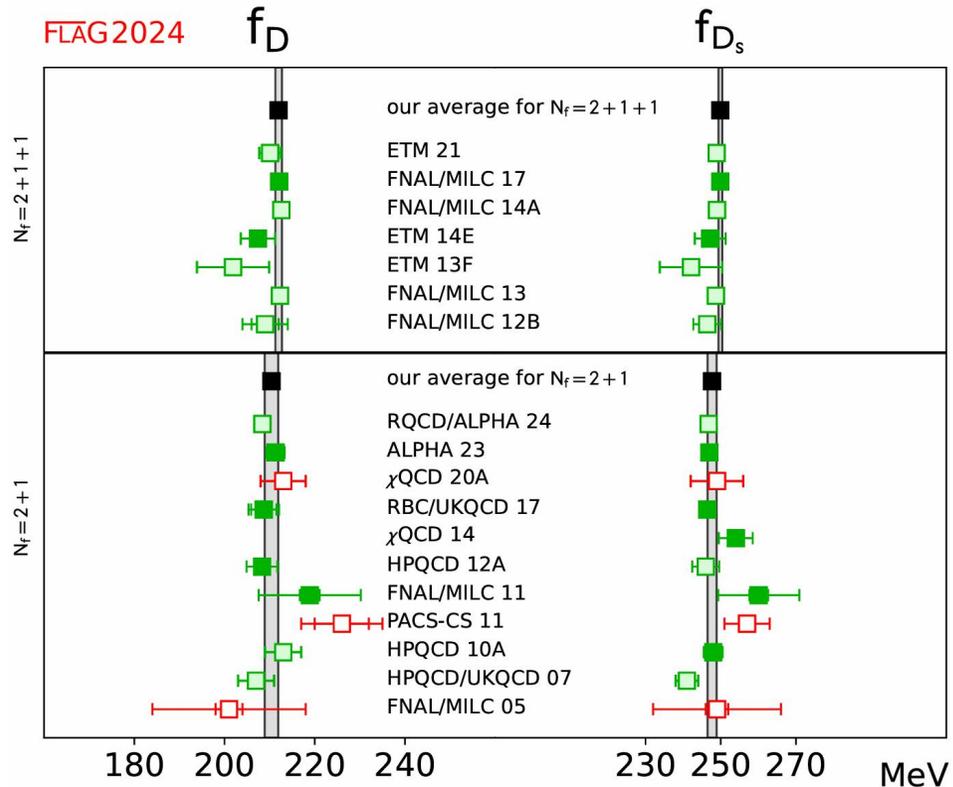
抽取CKM矩阵元

衰变常数可由LQCD给出

$$\Gamma(P \rightarrow \ell \nu) = \frac{G_F^2 |V_{q_1 q_2}|^2}{8\pi} f_P^2 m_\ell^2 M_P \left(1 - \frac{m_\ell^2}{M_P^2}\right)^2$$

粲介子衰变常数格点结果

- 赝标粲介子衰变常数的LQCD计算精度已达到 $\lesssim 1\%$
- 进一步提高精度需考虑同位旋破缺效应



FLAG Review 2024, arXiv:2411.04268
Results up to 2024.4.30

χ QCD, PRD92.034517 (2015), arXiv:1410.3343
 χ QCD, CPC45.023109 (2021), arXiv:2008.05208

2026-1-31 刘朝峰

- $N_f = 2 + 1$:
 $f_{D_s} = 247.7(1.2)$ MeV
 $f_D = 210.4(1.5)$ MeV
- $N_f = 2 + 1 + 1$:
 $f_{D_s} = 249.9(0.5)$ MeV
 $f_D = 212.0(0.7)$ MeV

• PDG2016 (CPC40):

$$f_{D_s}^{exp} = 257.8(4.1)$$
 MeV

格点结果与实验在 2σ 内一致

PDG2024 [PRD110.030001]:

$$f_D |V_{cd}| = 45.82(1.10)$$
 MeV

$$f_{D_s} |V_{cs}| = 243.5(2.7)$$
 MeV

BESIII:

$$f_{D_s} = 251.1(2.4)(3.0)$$
 MeV

$$[PRL127.171801.2021], 6.32 \text{ fb}^{-1}$$

$$f_{D_s^+} = 255.0(4.0)(3.2)(1.0)$$
 MeV

$$[PRD108.092014.2023], 7.33 \text{ fb}^{-1}$$

2021年之后的LQCD计算

A. Bussone et al. (Alpha Collab.),
2309.14154

2+1 味, twisted mass fermion on
improved Wilson fermion

$$f_D = 211.3(1.9)(0.6)$$
 MeV

$$f_{D_s} = 247.0(1.9)(0.7)$$
 MeV

$$f_{D_s}/f_D = 1.177(15)(5)$$

S. Kuberski et al. (RQCD & Alpha),
2405.04506

2+1 味, improved Wilson fermion

$$f_D = 208.4(1.5)$$
 MeV

$$f_{D_s} = 246.8(1.3)$$
 MeV

$$f_{D_s}/f_D = 1.1842(36)$$

CLQCD, arXiv:2408.03548

2+1味, Wilson clover fermion

$$f_D = 210.2(3.3)$$
 MeV

$$f_{D_s} = 248.7(2.8)$$
 MeV

同位旋破缺效应

$$\mathcal{O}((m_d - m_u)/\Lambda_{\text{QCD}}) \sim 1\%$$

$$\text{QED效应: } \mathcal{O}(\alpha_{\text{em}}) \sim 1\%$$

- 对于 f_{D_s} , 其 strong-isospin breaking effects 为海夸克效应, 目前可忽略 [PDG review, 2025 update (R. A. Briere et al.)]
- Strong-isospin breaking effects for f_{D^+}

$$f_{D^+} - f_D = +0.58(1)(7)(1) \text{ MeV} \quad 0.27\% \text{的修正}$$

[A. Bazavov et al. (Fermilab/MILC),
PRD98.074512 (2018)]

$$f_{D^+} = 212.6(7) \text{ MeV}, \quad f_{D_s} = 249.9(5) \text{ MeV}, \quad \frac{f_{D_s}}{f_{D^+}} = 1.175(2)$$

- Ongoing LQCD work (QED effects) on D mesons

虚修正 + 实修正: 红外发散相消

Desiderio et al., PRD103.014502 (2021)

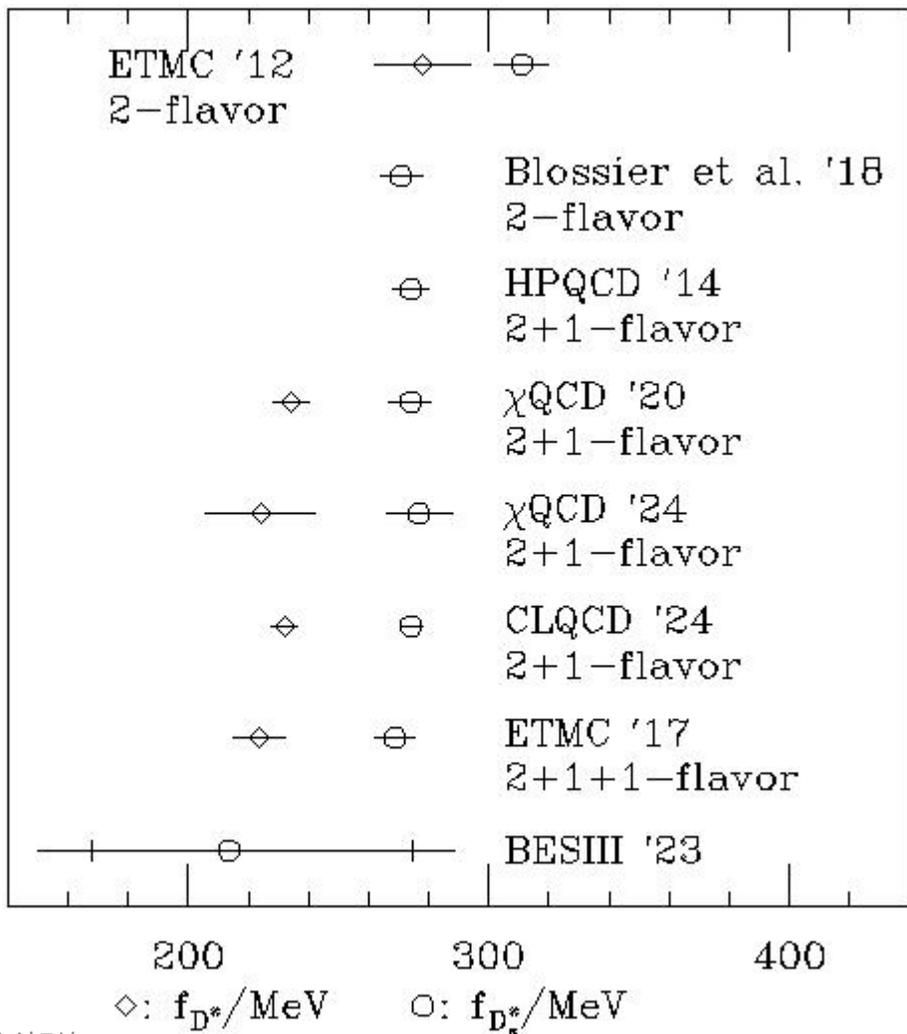
Radiative form factors, $D_s \rightarrow e\nu_e\gamma$, $\text{Br}(E_\gamma > 10 \text{ MeV}) = 4.4(3) \times 10^{-6}$ [Frezzotti et al., PRD108.074505 (2023)]

Giusti et al., PRD107.074507 (2023)

矢量粲介子衰变常数

实验给出分支比，需要总宽度才能得到分宽度

$$\Gamma(D_s^{*+} \rightarrow \ell \nu) = \frac{G_F^2}{12\pi} |V_{cs}|^2 f_{D_s^*}^2 M_{D_s^*}^3 \left(1 - \frac{m_\ell^2}{M_{D_s^*}^2}\right)^2 \left(1 + \frac{m_\ell^2}{2M_{D_s^*}^2}\right)$$



$D_s^{*\pm}$ WIDTH

<1.9 MeV CL=90%

D_s^{*+} DECAY MODES

pdgLive

D_s^{*-} modes are charge conjugates of the modes below.

Mode	Fraction (Γ_i / Γ)
Γ_1 $D_s^+ \gamma$	(93.6 ± 0.4) %
Γ_2 $D_s^+ \pi^0$	(5.77 ± 0.35) %
Γ_3 $D_s^+ e^+ e^-$	(6.7 ± 1.6) × 10 ⁻³
Γ_4 $e^+ \nu_e$	(2.1 ^{+1.2} _{-0.9}) × 10 ⁻⁵

Y. Chen, ..., ZL et al., CPC45 (2021) 023109

D. Li, ..., ZL et al., CPC48 (2024) 123105

CLQCD, PRD111 (2025) 054504

BESIII对 $D_s^{*+} \rightarrow e^+ \nu_e$ 的首个测量结果 [2304.12159, PRL131.141802(2023)]

D_s^* 总宽度及纯轻衰变分支比

HPQCD给出 D_s^* 总宽度: 0.070(28) keV

以及 $\text{Br}(D_s^* \rightarrow lv) = 3.4(1.4) \times 10^{-5}$

(PRL112, 212002 (2014))

$$\Gamma(D_s^* \rightarrow \gamma D_s) = 0.0549(54) \text{ keV}$$

D_s^* 总宽度: 0.0587(54) keV

$D_s^* \rightarrow D_s \gamma$ 分支比的实验值93.5(7)% PDG

Y. Meng, ..., ZL et al., arXiv:2401.13475 (PRD109.074511)

$D_s^* \rightarrow lv (l = e, \mu)$ 分支比实验测量, 结合总宽度, 可给出 $f_{D_s^*}|V_{cs}|$

BESIII对 $D_s^{*+} \rightarrow e^+ \nu_e$ 分支比的首个测量结果:

[2304.12159, PRL131.141802(2023)]

$$(2.1_{-0.9}^{+1.2} \pm 0.2_{\text{syst}}) \times 10^{-5}$$

实验理论对比

- BESIII + HPQCD

$$f_{D_s^*}|V_{cs}| = (207.9_{-44.6}^{+59.4} \pm 9.9_{\text{syst. exp}} \pm 41.5_{\text{syst. latt.}}) \text{ MeV}$$

- BESIII + arXiv:2401.13475

$$f_{D_s^*}|V_{cs}| = (190.5_{-41.7}^{+55.1} \pm 9.1_{\text{syst. exp}} \pm 8.7_{\text{syst. latt.}}) \text{ MeV}$$

等待实验提高精度

(D. Li et al., arXiv:2407.03697) + (Y. Meng, ..., ZL et al., arXiv:2401.13475)

$$f_{D_s^*} = 277(11) \text{ MeV} \longrightarrow \Gamma_{D_s^* \rightarrow lv} = 2.5(2) \times 10^{-6} \text{ keV} \longrightarrow \text{Br} = 4.26(52) \times 10^{-5}$$

[CLQCD, PRD111 (2025) 054504] $\text{Br} = 4.14(38)(29) \times 10^{-5}$ 12

粲介子半轻衰变

- $D \rightarrow \pi l \nu, D \rightarrow K l \nu$ 可用于确定 $|V_{cd}|$ 和 $|V_{cs}|$

$$\frac{d\Gamma(D \rightarrow K l \nu)}{dq^2} = (\text{known}) |\mathbf{p}_K|^3 |V_{cs}|^2 |f_+^{D \rightarrow K}(q^2)|^2$$

- 非微扰输入量: 形状因子 $f_{+/0}(q^2)$

$$\langle K | V^\mu | D \rangle = f_+(q^2) \left(p_D^\mu + p_K^\mu - \frac{m_D^2 - m_K^2}{q^2} q^\mu \right) + f_0(q^2) \frac{m_D^2 - m_K^2}{q^2} q^\mu$$

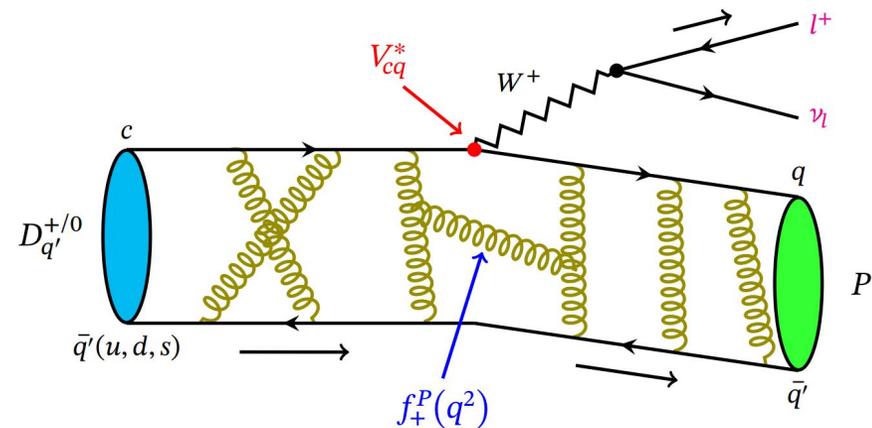
$f_+(0) = f_0(0)$

对于 $l = e, \mu$, 形状因子 f_0 对衰变宽度的贡献较小 (正比于 m_l^2)

- 标量流形状因子 $\langle K | S | D \rangle = f_0^{D \rightarrow K}(q^2) \frac{M_D^2 - M_K^2}{m_c - m_s}$

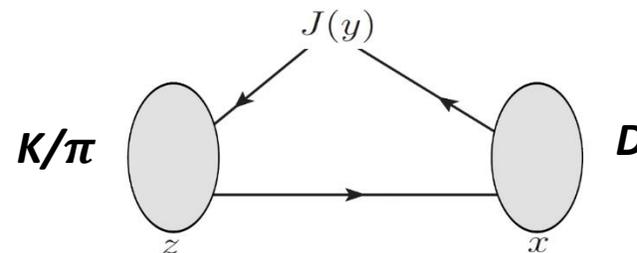
- 初末态强子四动量: p, p'

$q^2 = (p - p')^2$, 格点计算中3-动量取分立值



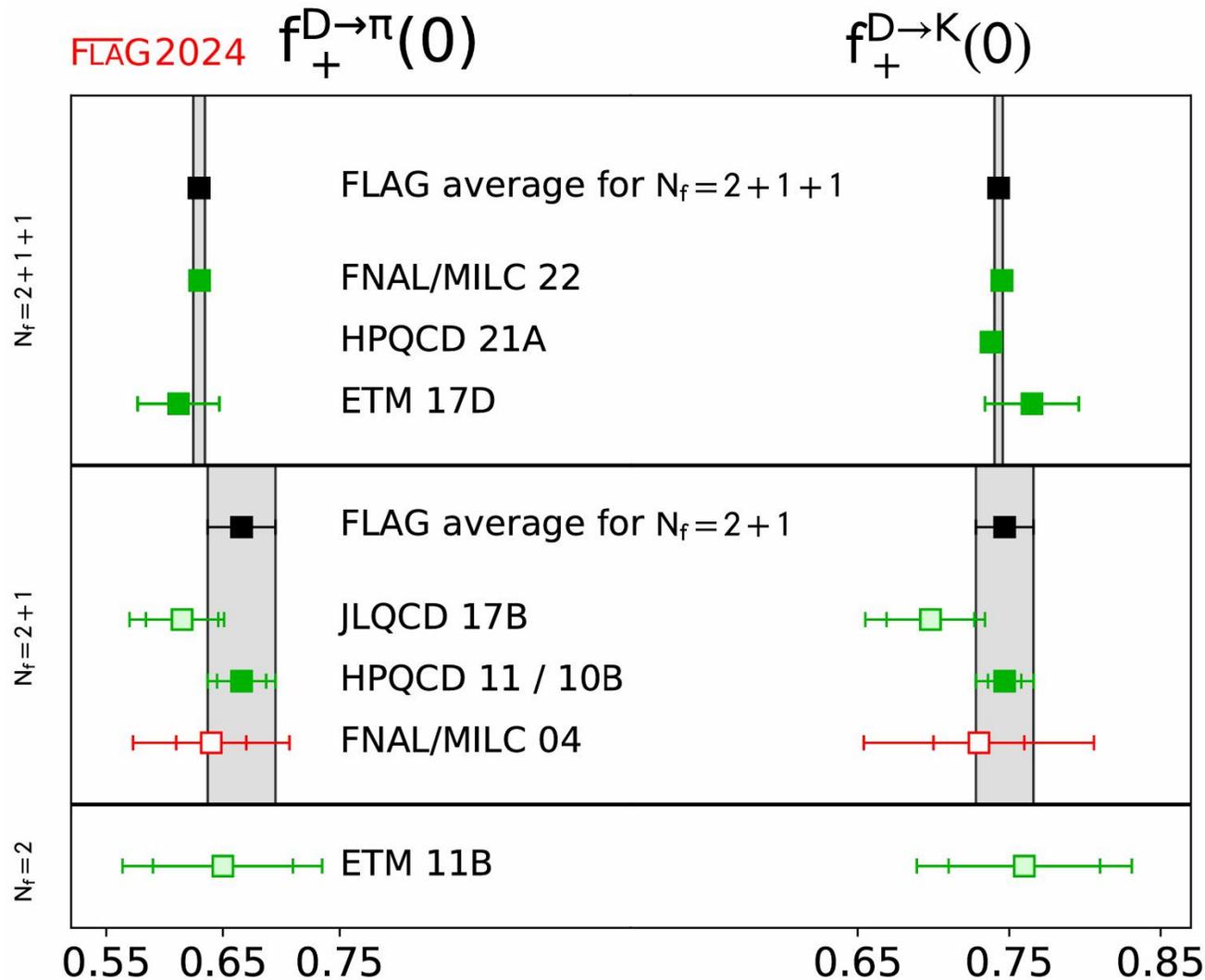
courtesy arXiv:2103.00908

(手征格点费米子有 $Z_S Z_m = 1$, 无需计算重整化常数)



三点关联函数

$f_+(q^2 = 0)$ for $D \rightarrow \pi/K$



FLAG Review 2024, arXiv:2411.04268

Results up to 2024.4.30

2+1+1味

- $f_+^{D\pi}(0) = 0.6296(50)$ [ETM 17D: PRD96 (2017) 054514, 1706.03017;
- [FNAL/MILC 22: 2212.12648, PRD107.094516]
- $f_+^{DK}(0) = 0.7430(27)$ [ETM 17D: PRD96 (2017) 054514, 1706.03017. HPQCD 21A: PRD104 (2021) 034505, 2104.09883. FNAL/MILC 22: 2212.12648, PRD107.094516]

2+1味

- JLQCD 17: LAT2017 [1711.11235]
- FNAL/MILC/HPQCD 04: 一个格距, m_π 大于 500 MeV
- $f_+^{D\pi}(0) = 0.666(29)$ [HPQCD 11, PRD84,114505, 1109.1501]
- $f_+^{DK}(0) = 0.747(19)$ [HPQCD 10B, PRD82,114506, 1008.4562]

$D \rightarrow \pi/K, D_s \rightarrow K$ 形状因子

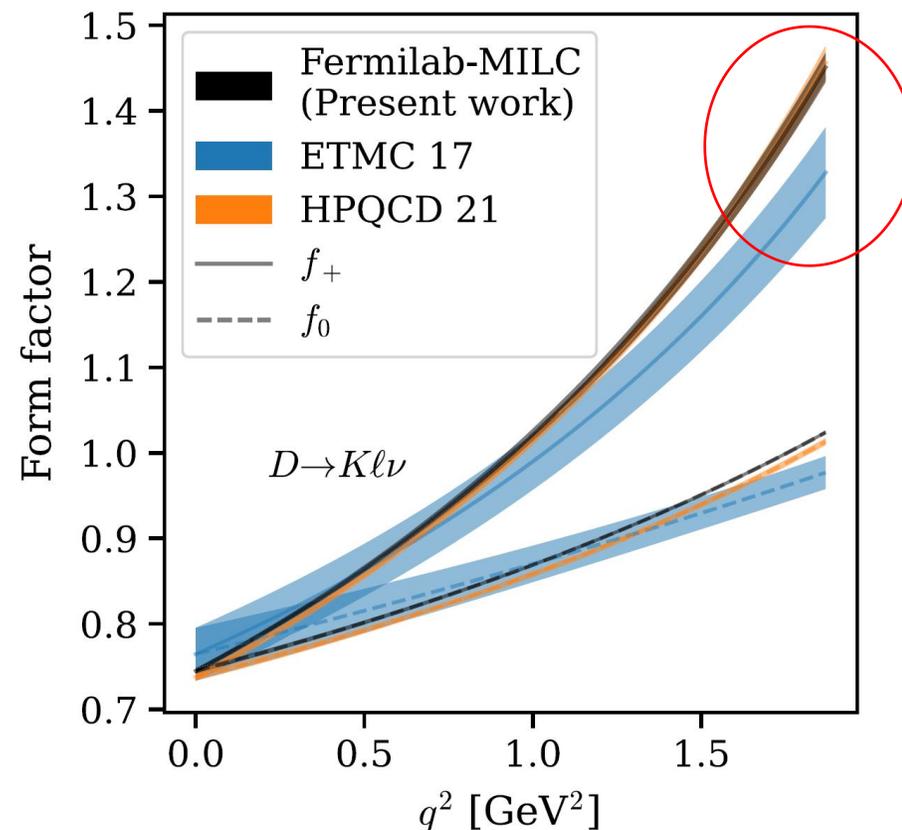
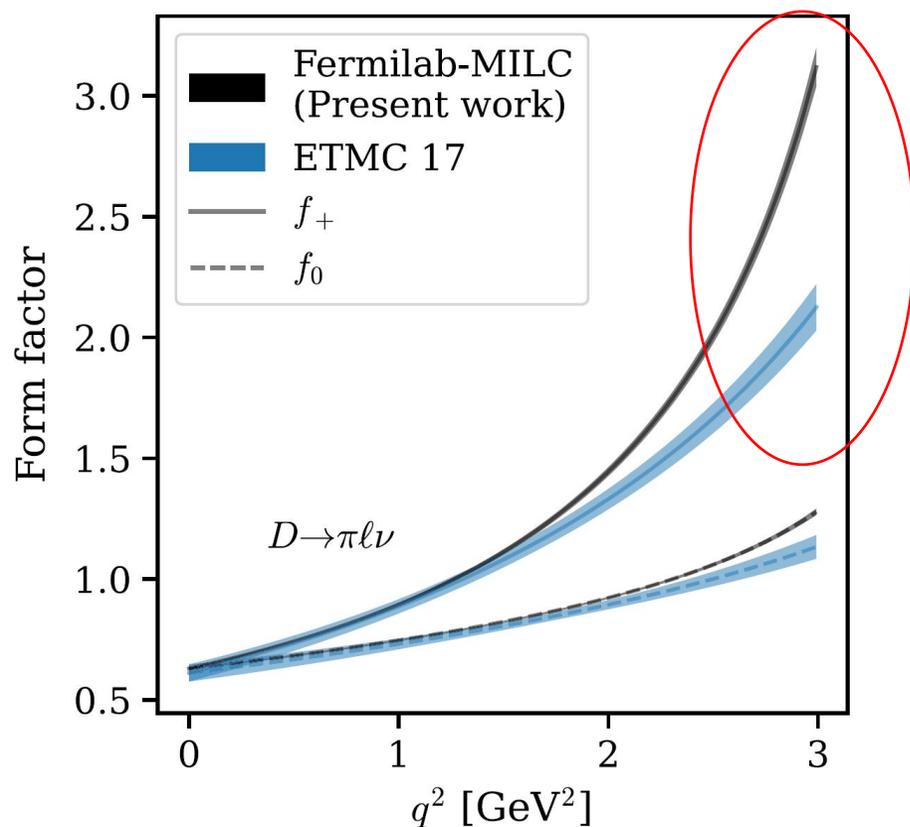
Fermilab/MILC, 2+1+1味HISQ

A. Bazavov et al., 2212.12648, PRD107.094516

- 四个格距: ~ 0.12 fm - 0.04 fm
- $\mathcal{O}(1000)$ 组态数, 多次测量/组态
- 两或三个体积@两个格距
- 物理轻夸克质量@三个格距

$$\begin{aligned} f_+^{D\pi}(0) &= 0.6300(51) \\ f_+^{DK}(0) &= 0.7452(31) \\ f_+^{D_s K}(0) &= 0.6307(20) \end{aligned}$$

Not included: systematic uncertainties associated with QED, isospin, and electroweak corrections

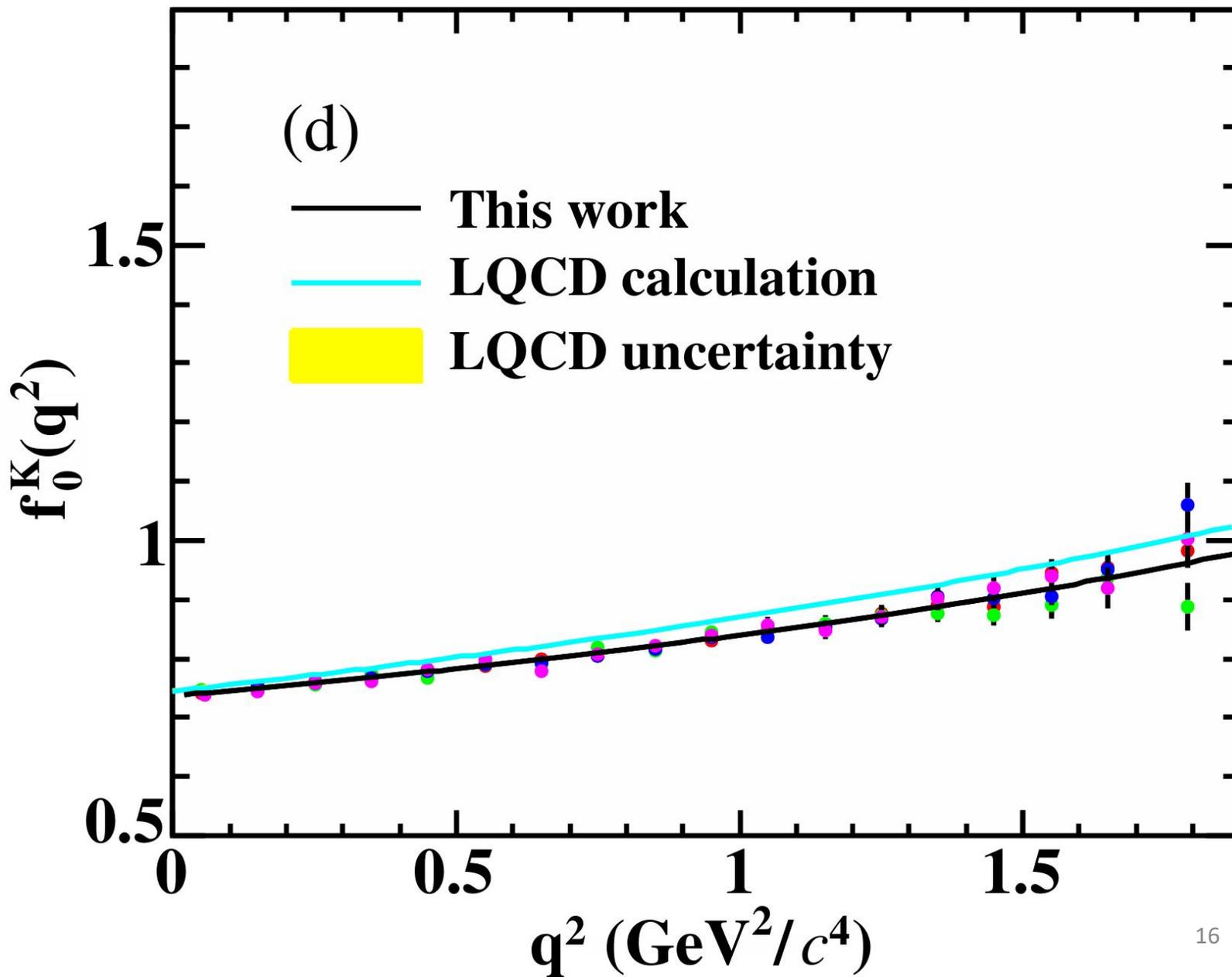


$D \rightarrow Kl\nu$ 形状因子 $f_0(q^2)$

BESIII, PRD110.112006 (2024)

BESIII VS LQCD

A. Bazavov et al., 2212.12648,
PRD107.094516



$|V_{cd}|$ 和 $|V_{cs}|$

$$f_D|V_{cd}| = 45.82(1.10) \text{ MeV}, \quad f_{D_s}|V_{cs}| = 243.5(2.7) \text{ MeV}$$

PDG2024, PRD110(2024).030001

$$f_+^{D\pi}(0)\eta_{EW}|V_{cd}| = 0.1426(18), \quad f_+^{DK}(0)\eta_{EW}|V_{cs}| = 0.7180(33)$$

HFLAV, EPJC81(2021) 226, 1909.12524

$$\eta_{EW}^2 = 1.018$$

- 纯轻过程，LQCD误差比实验误差小 (2+1, 2+1+1味)
- 介子半轻过程，LQCD误差比实验误差大
- Λ_c 半轻过程，格点误差比实验大

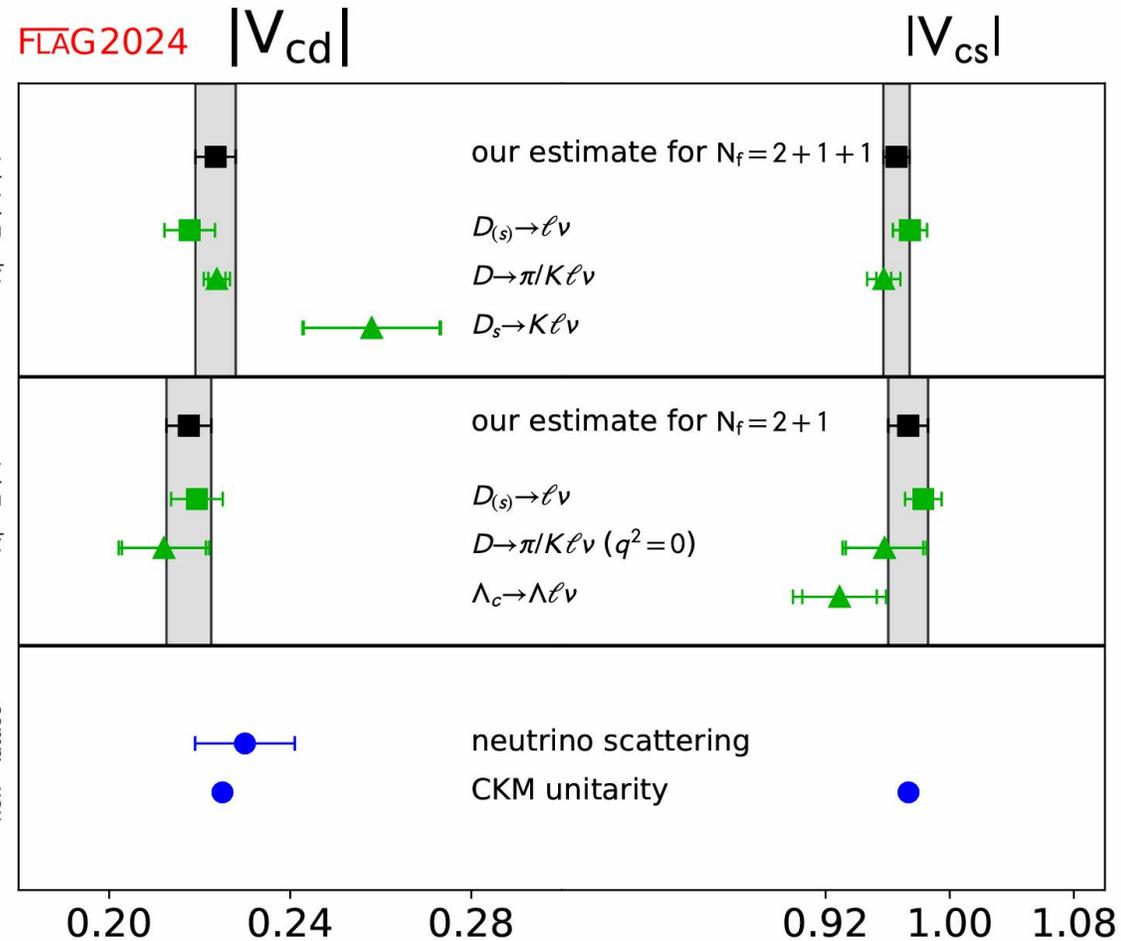
CKM幺正性 (第二行) 在 10^{-2} 精度上成立

$$N_f = 2 + 1: |V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 - 1 = 0.00(3)$$

$$N_f = 2 + 1 + 1: |V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 - 1 = -0.01(2)$$

FLAG Review 2024, arXiv:2411.04268

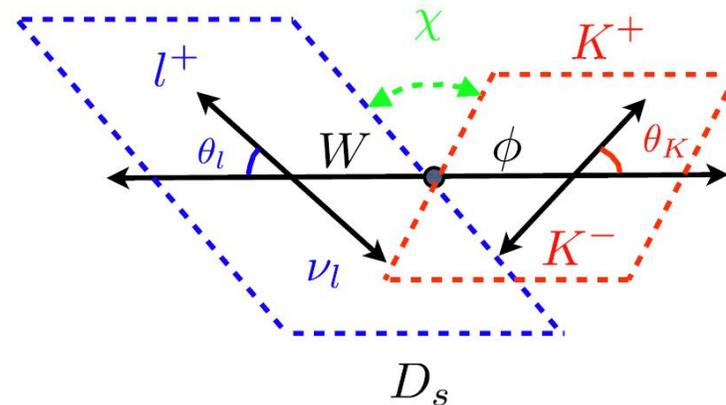
Results up to 2024.4.30



$D_s \rightarrow \phi l \nu$

- 除了在 q^2 -bin中比较截面，还有更多观测量， $\phi \rightarrow K\bar{K}$
- HPQCD之前的计算 [PRD90, 1311.6669]
 - 忽略非连通图贡献 (OZI压低)
 - 模拟中 ϕ 不衰变； ϕ 宽度较窄，预期阈效应小
 - 形状因子 V, A_2, A_1, A_0 ；z-expansion
 - 2+1味；两个格距；HISQ价夸克作用量
 - 和BABAR ($D_s \rightarrow \phi e^+ \nu_e$)，CLEO ($D \rightarrow K^*$) 的结果做比较 (假设spectator quark的影响很小)

$$\frac{d\Gamma(D_s^+ \rightarrow \phi l^+ \nu_\ell)}{dq^2 d\chi d \cos \theta_\ell d \cos \theta_K} = \frac{G_F^2 |V_{cs}|^2 |\vec{p}| q^2}{12(2\pi)^4 M^2} \left(1 - \frac{m_\ell^2}{q^2}\right)^2 W(\theta_K, \theta_\ell, \chi)$$



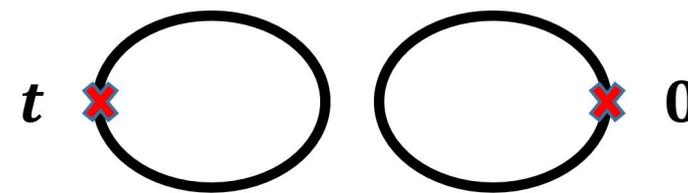
$$\langle \phi(p', \varepsilon) | V^\mu - A^\mu | D_s(p) \rangle$$

$$= \frac{2i\varepsilon^{\mu\nu\alpha\beta}}{M_{D_s} + M_\phi} \varepsilon_\nu^* p'_\alpha p_\beta V(q^2) - (M_{D_s} + M_\phi) \varepsilon^{*\mu} A_1(q^2)$$

$$+ \frac{\varepsilon^* \cdot q}{M_{D_s} + M_\phi} (p + p')^\mu A_2(q^2) + 2M_\phi \frac{\varepsilon^* \cdot q}{q^2} q^\mu A_3(q^2)$$

$$- 2M_\phi \frac{\varepsilon^* \cdot q}{q^2} q^\mu A_0(q^2).$$

$$A_3(q^2) = \frac{M_{D_s} + M_\phi}{2M_\phi} A_1(q^2) - \frac{M_{D_s} - M_\phi}{2M_\phi} A_2(q^2)$$



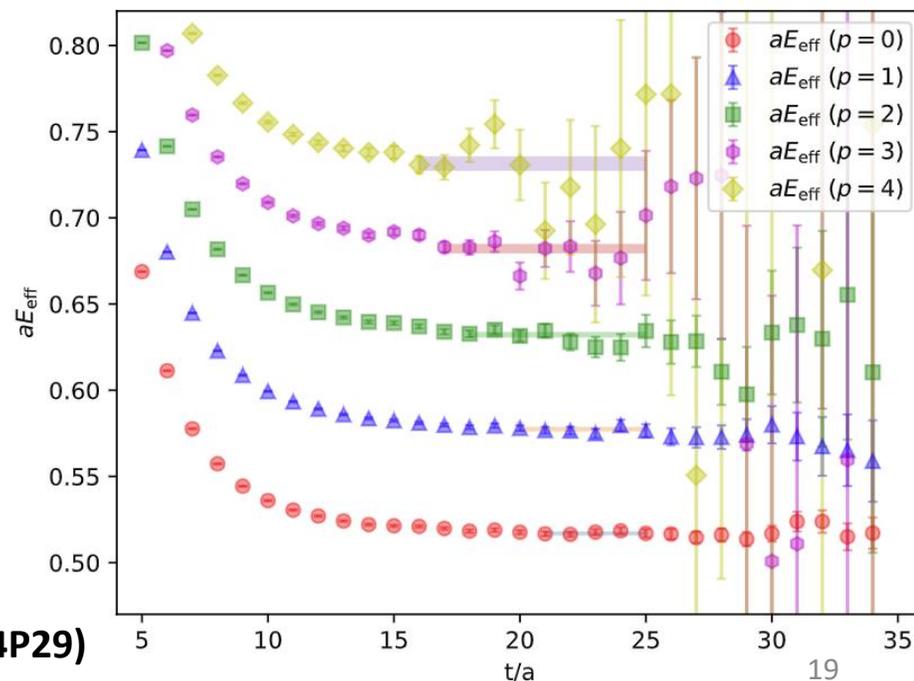
两点函数中的 (夸克线) 非连通图

$$A_0(0) = A_3(0)$$

	C24P29	C32P23	C32P29	F32P30	F48P21	G36P29	H48P32
a (fm)		0.10524(05)(62)		0.07753(03)(45)		0.06887(12)(41)	0.05199(08)(31)
am_l	-0.2770	-0.2790	-0.2770	-0.2295	-0.2320	-0.2150	-0.1850
am_s	-0.2400	-0.2400	-0.2400	-0.2050	-0.2050	-0.1926	-0.1700
am_s^V	-0.2356(1)	-0.2337(1)	-0.2358(1)	-0.2038(1)	-0.2025(1)	-0.1928(1)	-0.1701(1)
am_c^V	0.4159(07)	0.4190(07)	0.4150(06)	0.1974(05)	0.1997(04)	0.1433(12)	0.0551(07)
L (fm)	2.53	3.37	3.37	2.48	3.72	2.48	2.50
$L^3 \times T$	$24^3 \times 72$	$32^3 \times 64$	$32^3 \times 64$	$32^3 \times 96$	$48^3 \times 96$	$36^3 \times 108$	$48^3 \times 144$
N_{mea}	$450 \times 72 \times 2$	$333 \times 64 \times 3$	$397 \times 64 \times 2$	$360 \times 96 \times 2$	$241 \times 48 \times 4$	$300 \times 54 \times 2$	$300 \times 72 \times 2$
m_π (MeV)	292.3(1.0)	227.9(1.2)	293.1(0.8)	300.4(1.2)	207.5(1.1)	297.2(0.9)	316.6(1.0)

- 七套2+1味组态, Wilson-clover fermions [CLQCD]
- 四个格距, 多个pion 质量
- D_s 静止, 改变 ϕ 的三动量 $\vec{p} = \frac{2\pi}{La} (k_x, k_y, k_z)$
- $(k_x, k_y, k_z) = (100), (110), (111), (200), (210), (211)$
- (轴) 矢量流重正化常数 [CLQCD, arXiv:2408.03548]
- 组态 C32P29 用于检查有限体积效应

ϕ 两点函数能量平台



物理质量点及连续极限下的形状因子 $(D_s \rightarrow \phi lv)$

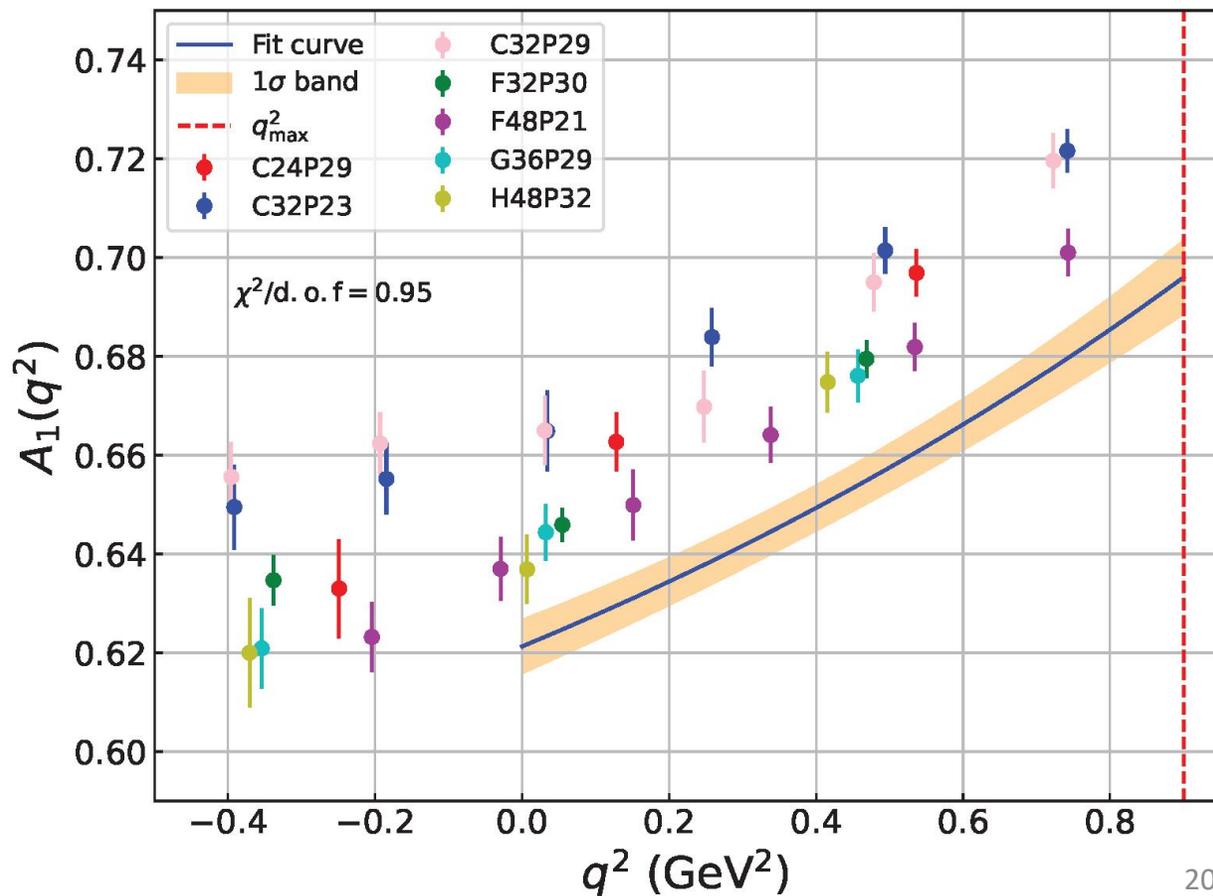
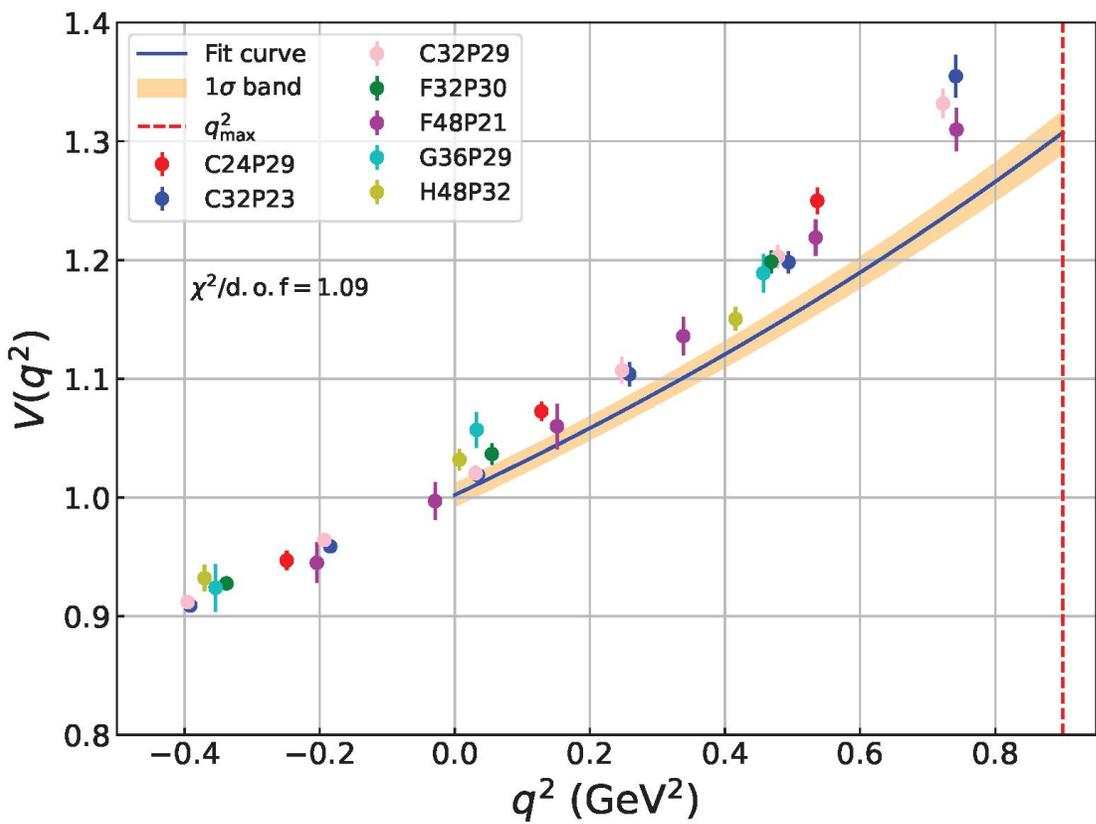
其它参数化形式给出误差内一致结果:

- Single pole form
- Modified pole form
- Phase moment form [Yao et al., PRD 101.034014 (2020)]

$$V(q^2, a, m_\pi) = \frac{1}{1 - q^2/m_{D_s^*}^2} \sum_{i=0}^2 (c_i + d_i a^2) \left[1 + f_i (m_\pi^2 - m_{\pi, \text{phys}}^2) + g_i (m_\pi^2 - m_{\pi, \text{phys}}^2)^2 \right] z^i$$

$$A_{0,1,2}(q^2, a, m_\pi) = \frac{1}{1 - q^2/m_{D_{s1}}^2} \sum_{i=0}^2 (c_i + d_i a^2) \left[1 + f_i (m_\pi^2 - m_{\pi, \text{phys}}^2) + g_i (m_\pi^2 - m_{\pi, \text{phys}}^2)^2 \right] z^i$$

$$m_{\pi, \text{phys}}^2 = 135.0 \text{ MeV}/c^2, m_{D_s^*}^2 = 2112.2 \text{ MeV}/c^2, m_{D_{s1}}^2 = 2459.5 \text{ MeV}/c^2$$

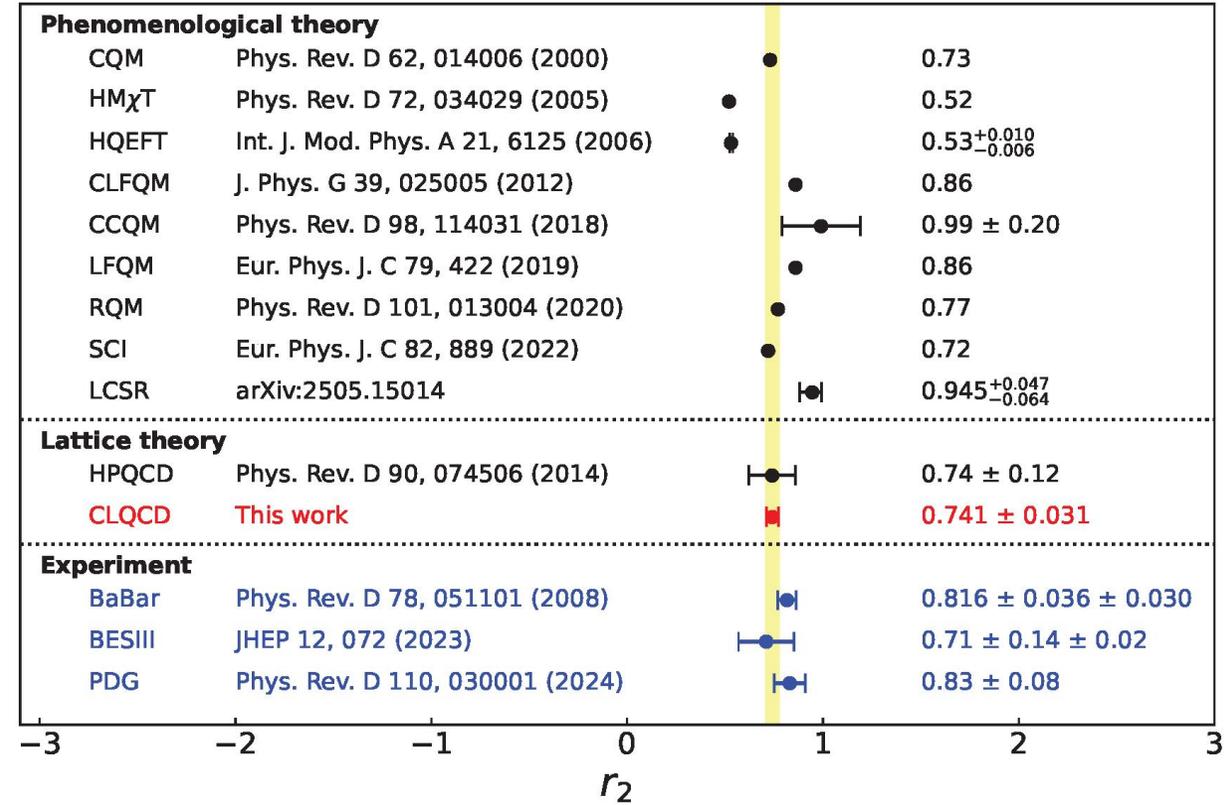
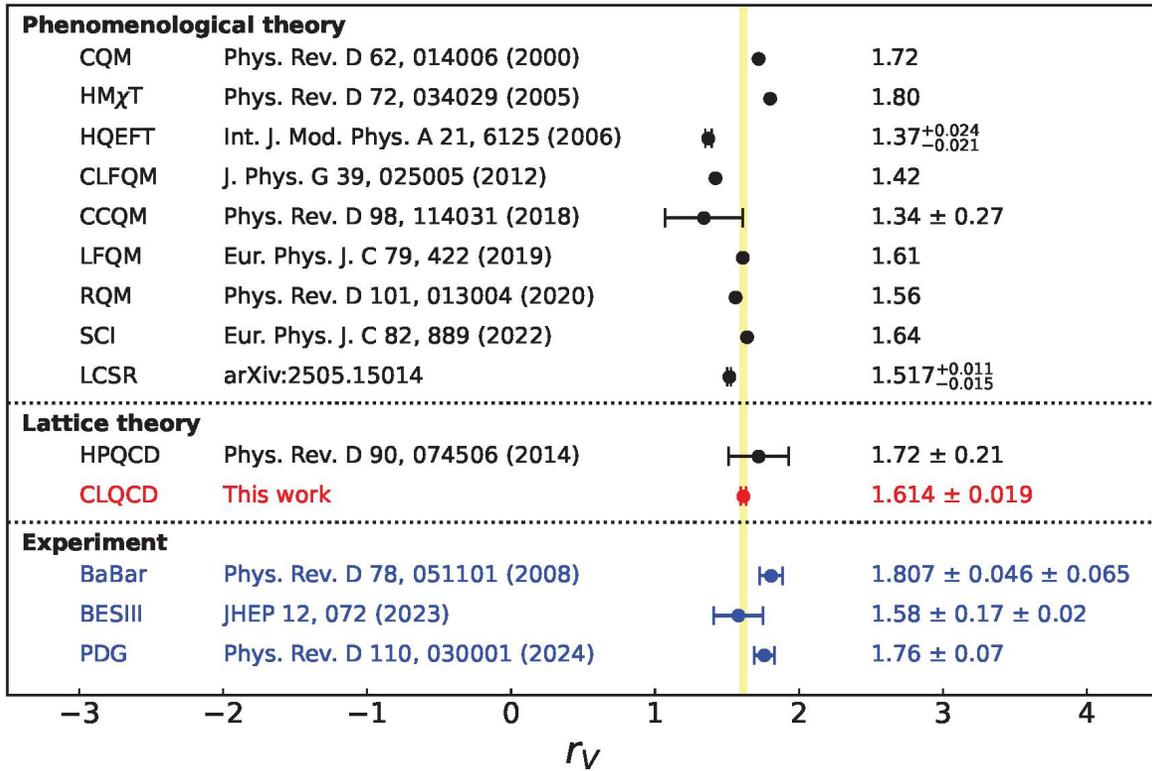


$A_0(0) - A_3(0) = -0.004(12)$
和零一致, 符合运动学限制

$D_s \rightarrow \phi l \nu$

$r_V = V(0)/A_1(0)$

$r_2 = A_2(0)/A_1(0)$



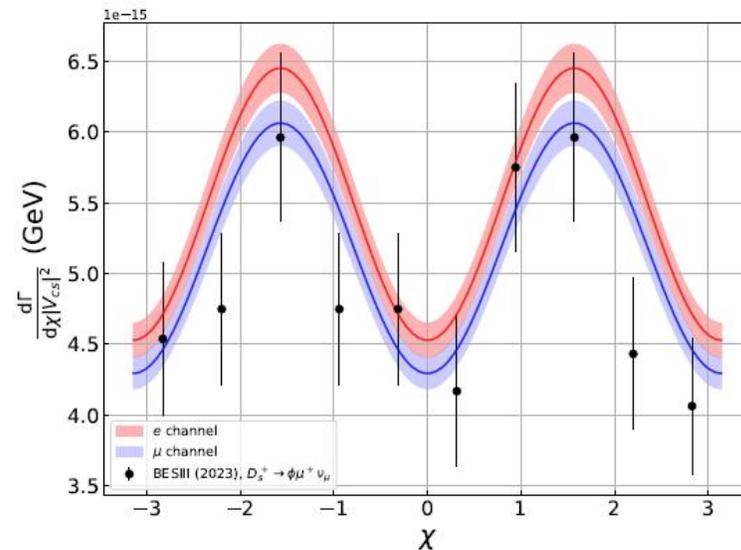
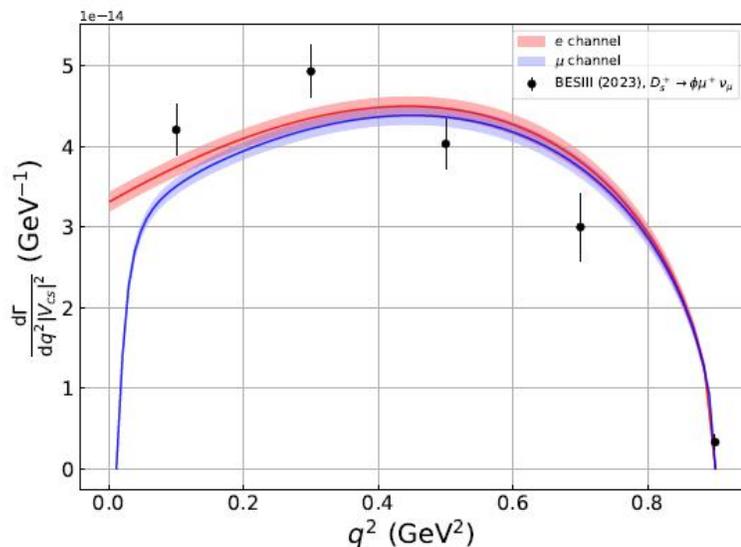
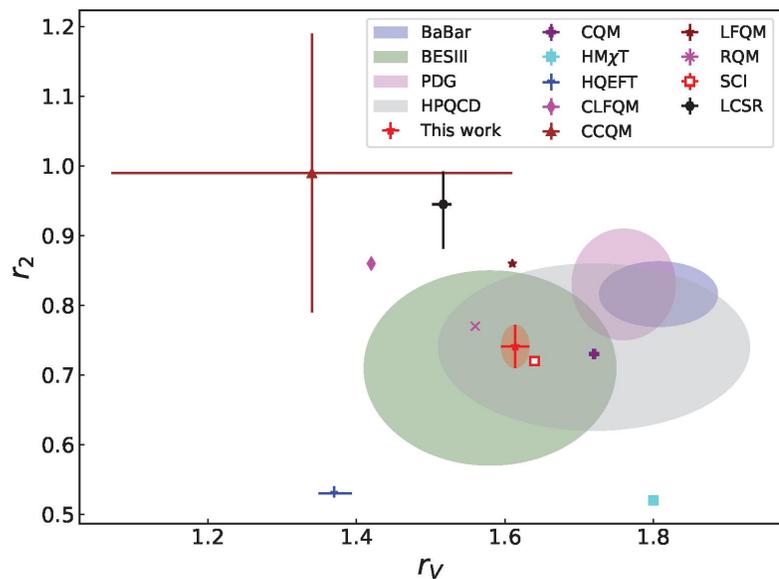
分支比及比值：
(精度比实验高)

	$\mathcal{B}(D_s \rightarrow \phi l \nu_e) \times 10^2$	μ channel	e channel	$\mathcal{R}_{\mu/e}$
This work	2.351(67)	2.493(73)	0.9432(13)	
BaBar [7]	—	2.61(17)	—	
CLEO [8]	—	2.14(19)	—	
BESIII (2018) [9]	1.94(54)	2.26(46)	0.86(29)	
BESIII (2023) [10]	2.25(11)	—	—	
PDG [39]	2.24(11)	2.34(12)	0.957(68)	

BABAR, PRD78.051101(R) (2008)
CLEO, PRD92.012009 (2015)
BESIII, PRD97.012006 (2018)
BESIII, JHEP12 (2023) 072
PDG, PRD110.030001 (2024)

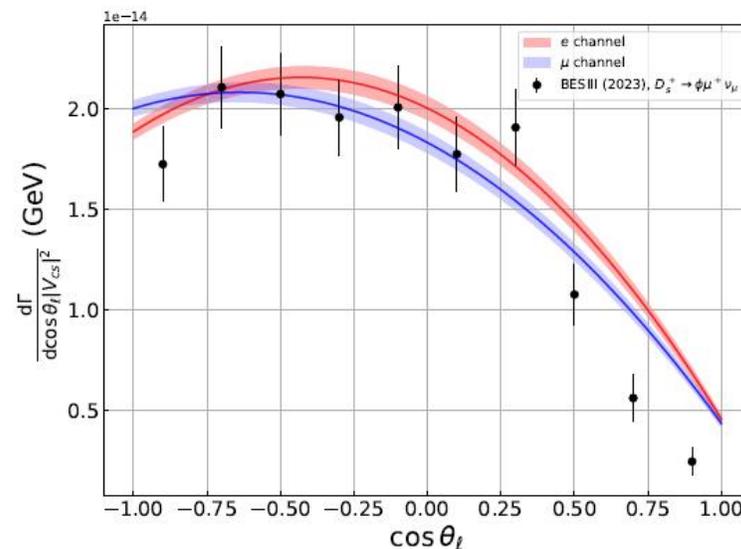
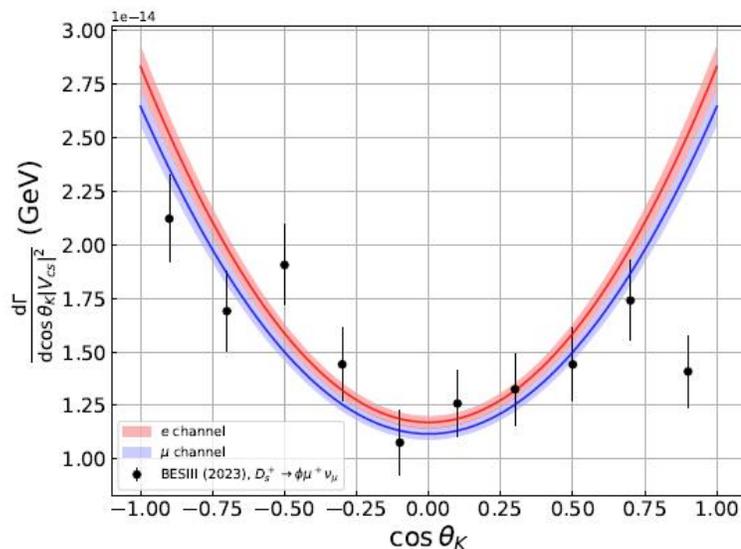
结果与实验及其它理论计算的比较

$$\frac{d\Gamma(D_s^+ \rightarrow \phi \ell^+ \nu_\ell)}{dq^2 d\chi d\cos\theta_\ell d\cos\theta_K} = \frac{G_F^2 |V_{cs}|^2 |\vec{p}| q^2}{12(2\pi)^4 M^2} \left(1 - \frac{m_\ell^2}{q^2}\right)^2 W(\theta_K, \theta_\ell, \chi)$$



Unitarity	●	0.97349 ± 0.00016	Phys. Rev. D 110, 030001 (2024)
HFLAV	●	0.9701 ± 0.0081	arXiv:2411.18639
PDG	●	0.975 ± 0.006	Phys. Rev. D 110, 030001 (2024)
BESIII	●	0.9623 ± 0.0046	Phys. Rev. D 110, 112006 (2024)
HPQCD	●	1.017 ± 0.064	Phys. Rev. D 90, 074506 (2014)
This work	●	0.945 ± 0.027	This work, e channel
This work	●	0.952 ± 0.026	This work, μ channel

$|V_{cs}|$



总结

- 粲介子纯轻、半轻衰变：精确检验标准模型、寻找新物理
- 格点QCD计算强子矩阵元（衰变常数、形状因子）
 - $D^{(*)}$ 、 $D_s^{(*)}$ 衰变常数
 - D_s^* 辐射衰变
 - $D \rightarrow \pi/Klv$ 、 $D_s \rightarrow \phi lv$
- 控制各种误差
 - 统计误差
 - 激发态污染
 - 离散误差
 - 同位旋破缺效应
 - 重整化常数误差
 - 格距不确定性
 -

谢谢!