味物理讲座

Flavour Physics Lecture

東南大學
SOUTHEAST UNIVERSITY

味物理前沿研讨会暨味物理讲座100期特别活动-三亚

From the Cornell model to the DeepQuark

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Feb. 3rd, 2026

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Based on: PRD108, 114016 (2023), PRD110, 034030 (2024), PRD109, 054034 (2024), arXiv:2506.20555 …

Phys.Lett. 8 (1964) 214-215

Volume 8, number 3 PHYSICS LETTERS 1 February 1964

A SCHEMATIC MODEL OF BARYONS AND MESONS *

M. GELL-MANN
California Institute of Technology, Pasadena, California

Received 4 January 1964

...

A simpler and more elegant scheme can be constructed if we allow non-integral values for the charges. We can dispense entirely with the basic baryon b if we assign to the triplet t the following properties: spin $\frac{1}{2}$, $z = -\frac{1}{3}$, and baryon number $\frac{1}{3}$. We then refer to the members $u^{\frac{2}{3}}$, $d^{-\frac{1}{3}}$, and $s^{-\frac{1}{3}}$ of the triplet as "quarks" 6) q and the members of the anti-triplet as anti-quarks $\bar{q}$. Baryons can now be constructed from quarks by using the combinations (qqq), $(qqqq\bar{q})$, etc., while mesons are made out of $(q\bar{q})$, $(qq\bar{q}\bar{q})$, etc. It is assuming that the lowest baryon configuration (qqq) gives just the represen-

8419/TH.412
21 February 1964

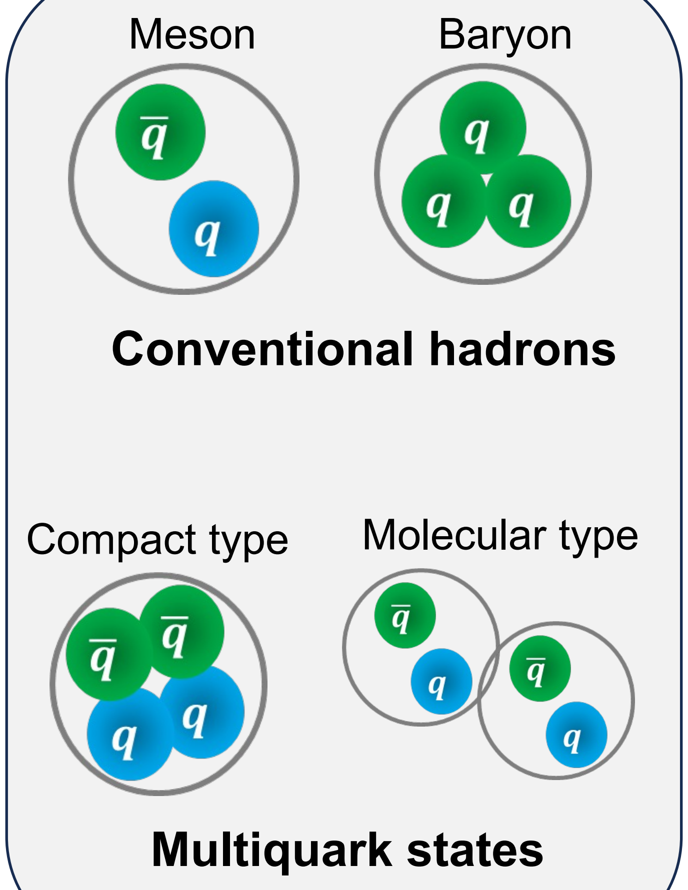
AN SU_3 MODEL FOR STRONG INTERACTION SYMMETRY AND ITS BREAKING
II *)

G. Zweig
CERN---Geneva

*) Version I is CERN preprint 8182/TH.401, Jan. 17, 1964.

...

6) In general, we would expect that baryons are built not only from the product of three aces, AAA, but also from $\overline{A}AAAA$, $\overline{A}\overline{A}AAAAA$, etc., where $\overline{A}$ denotes an anti-ace. Similarly, mesons could be formed from $\overline{A}A$, $\overline{A}\overline{A}AA$ etc. For the low mass mesons and baryons we will assume the simplest possibilities, $\overline{A}A$ and AAA, that is, "deuces and treys".



The multiquark states were predicted at the birth of quark model

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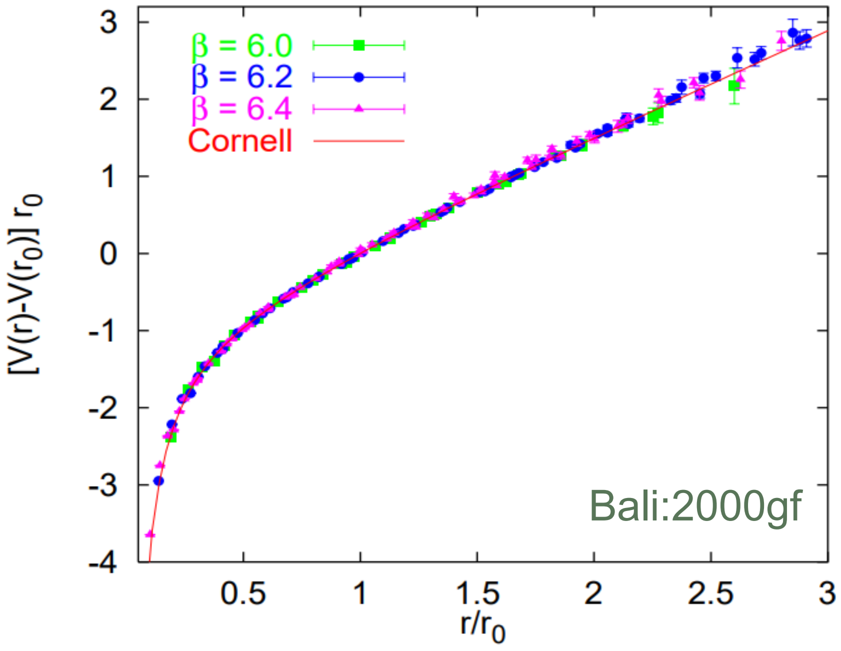
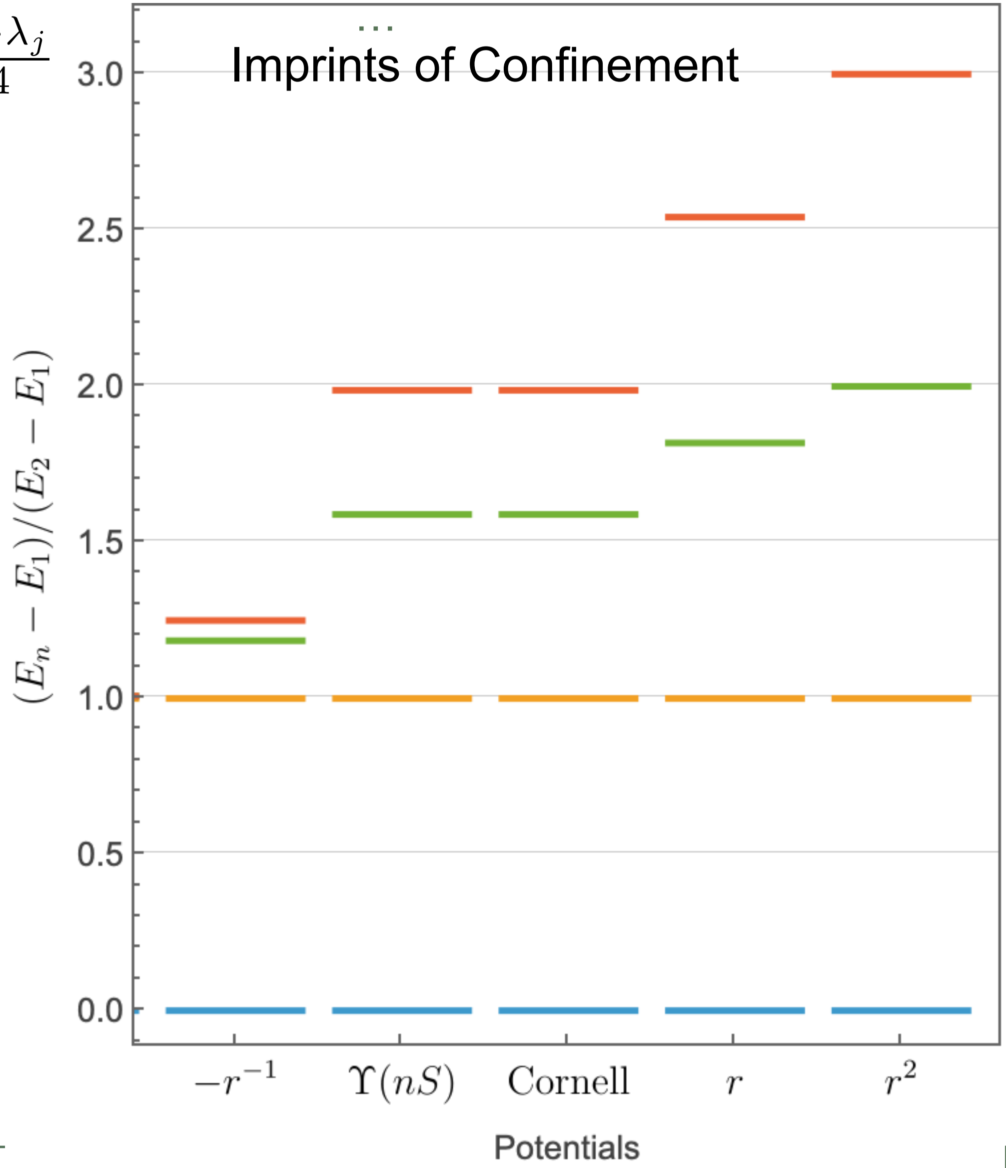
- A minimal model: one-gluon-exchange+Confinement

$$V_{ij}(r) = \left[\underbrace{\frac{\alpha_s}{r} - \frac{8\pi\alpha_s}{3m_i m_j} \frac{\tau^3}{\pi^{3/2}} e^{-\tau^2 r^2} \boldsymbol{s}_i \cdot \boldsymbol{s}_j}_{\text{OGE}} + \underbrace{\left(-\frac{3b}{4} r + V_c \right)}_{\text{Confinement}} \right] \frac{\lambda_i \cdot \lambda_j}{4}$$



Cornell model: Eichten:1978tg,
BGS model: Barnes:2005pb
…



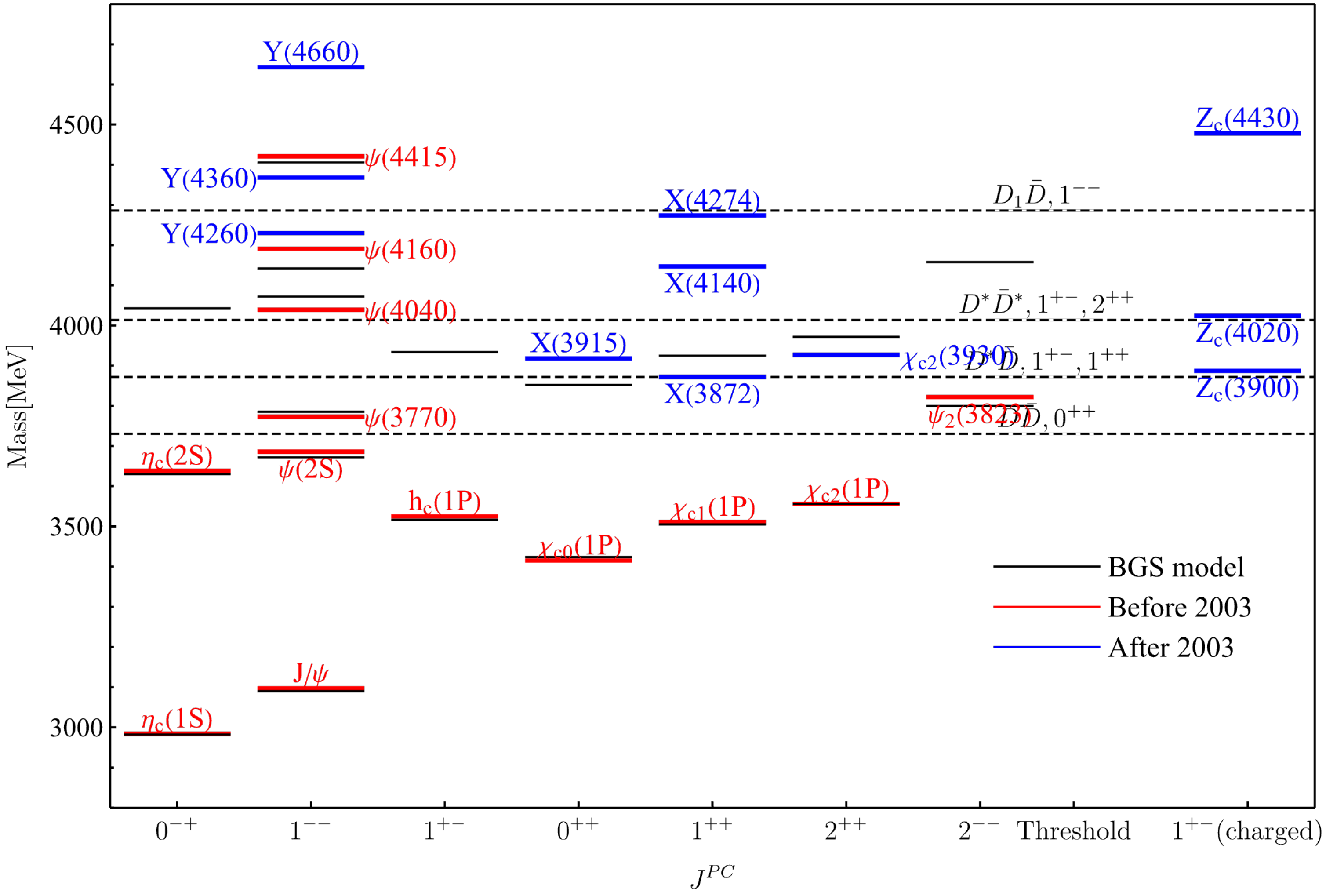
- Chiral quark models (π, K, η, exchange)

Manohar:1983md, Zhang:1997ny, Vijande:2004he, Gonzalez:2012gka…

- Relativized Godfrey-Isgur model Godfrey:1985xj…

Heavy quarkonium

BGS model
- Mass
- EM transitions
- Open flavor strong decays



- NRQM with only 4 paras.
- Work well below open flavor thresholds
- Work better for bottomonium

T. Barnes, S. Godfrey, and E. S. Swanson,, Phys. Rev. D 72, 054026 (2005).

Do we still need quark models?

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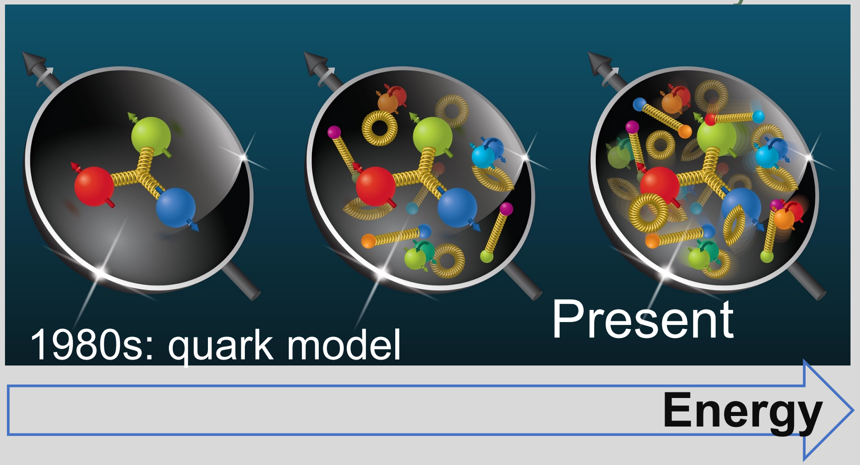
- Evolving view of the proton

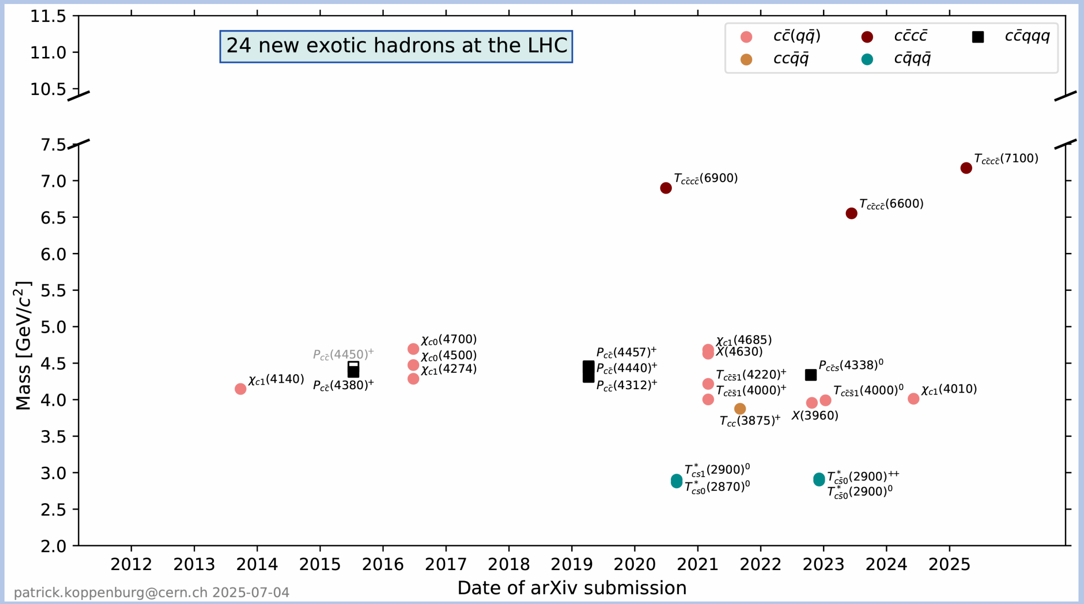
Courtesy of BNL



- Alternative methods:
  - ▶ Lattice QCD
  - ▶ Low energy EFTs
  - ▶ Dyson-Schwinger equations
  - ▶ …

Against

- We even do not know the **patterns**



- Quark models
  - ▶ A clear picture to uncover patterns
  - ▶ Lower computational costs
  - ▶ Prediction power
  - ▶ Also decay models
  - ▶ …

For

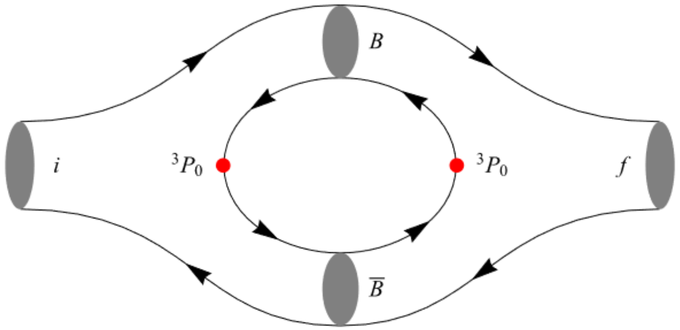
We still need quark model !

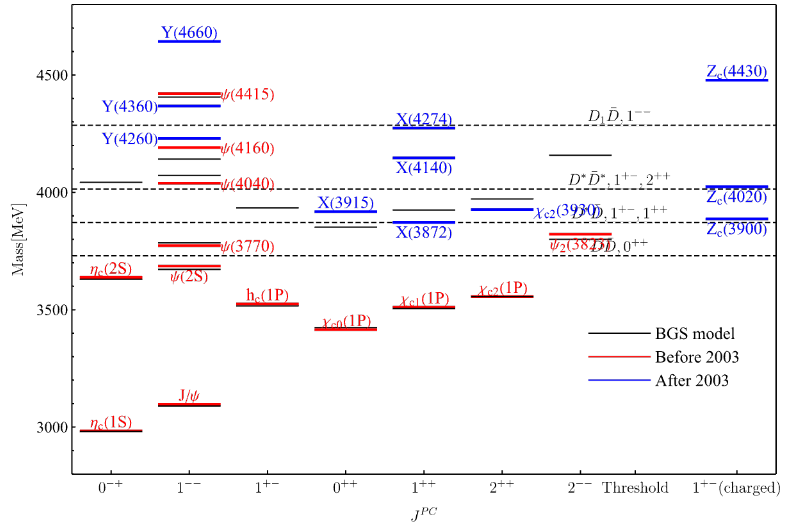
The right question: How to **adapt the quark models to meet the new demands**?

Quark models in new era

- Heavy-quarkonium-like states: unquenched quark model

B-S Zou's talks
https://indico.cern.ch/event/1457095/contributions/6563231/





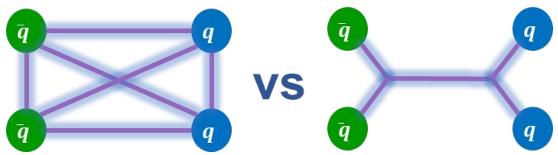
- **Multiquark systems**

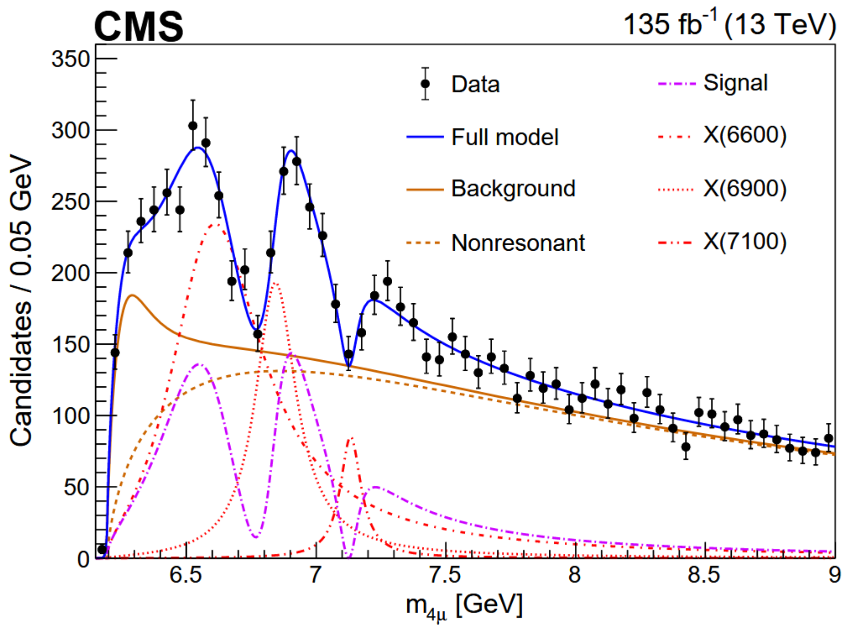
$[c\bar{c}qq]$ P_c	$[cc\bar{c}\bar{c}]$ $X(6900)$ $X(6600)$ $X(7100)$	$[cs\bar{u}\bar{d}]$ $T_{cs1}(2900)$ $T_{cs0}(2900)$	$[cs\bar{u}\bar{d}]$ $Z_{cs}(3985)$ $Z_{cs}(4000)$	$[cc\bar{u}\bar{d}]$ $T_{cc}(3875)^+$	$[c\bar{s}ud][c\bar{s}\bar{u}d]$ $T_{c\bar{s}0}(2900)^{++}$ $T_{c\bar{s}0}(2900)^0$	$[c\bar{c}qqq]$ $P_{cs}(4338)$ $P_{cs}(4459)$
	2006.16957 2306.07164 2304.08962 2506.07944	2009.00025 2009.00026	2011.07855 2103.01803	2109.01038 2109.01056	2212.02716 2212.02717 2411.19781	2210.10346 2012.10380 2502.09951

Particle Zoo 2.0

- **Opportunities:** all-charm tetraquark **family**
  - ▶ Great experimental advances: LHCb, CMS, ATLAS
  - ▶ Simple systems
    - **Constituent quark is almost the current quark**
    - **Small relativistic effect**
    - **Unlikely exchange light mesons between (anti)quarks**
  - ▶ Different confinements would leave imprints on the mass spectrum

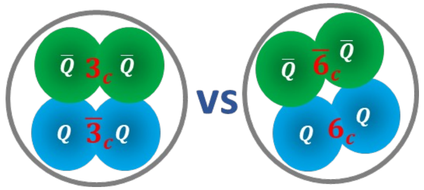
    Alexandrou:2004ak, Okiharu:2004ve, Bicudo:2017usw



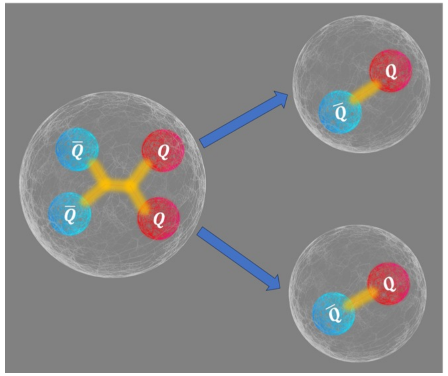


- **Challenges**
  - ▶ Color structures: e.g. $3-\bar{3}$ and $6-\bar{6}$ tetraquark
  - ▶ Matrix element of double Y-type potential
  - ▶ Four/five body problem
  - ▶ Resonance above the di-hadron thresholds
  - ▶ ….

Interactions?
few-body (resonance) problem?



Color structures



quark rearrangement

Contents

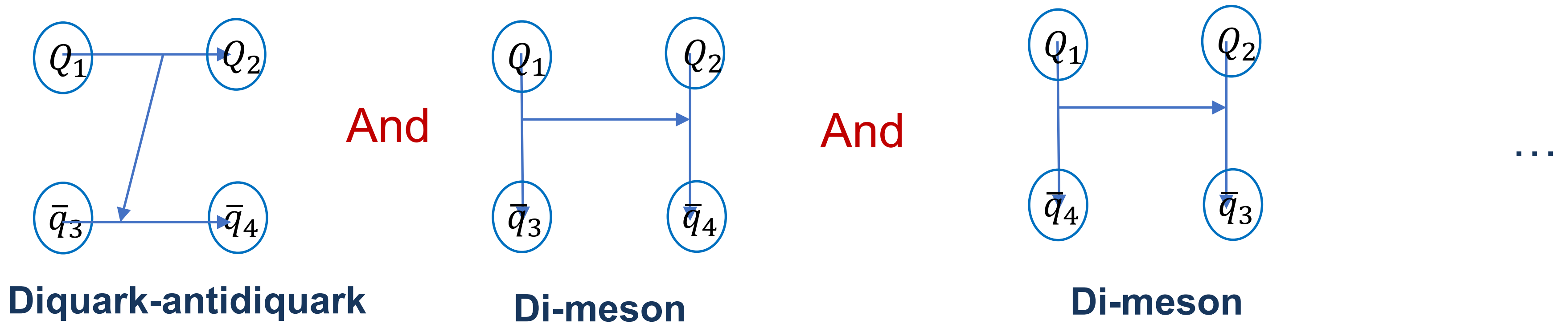
- Basis expansion + Complex scaling method

- **DNNs**: towards tetraquark **confinement**

Basis expansion + Complex scaling method

Gaussian Expansion Method

- Based on variational principle: $|\psi\rangle_{trial} = \sum_i c_i |\phi_i\rangle$
  - ▶ **upper limit** of the energy
  - ▶ Not necessary to be complete, but more general basis, more accurate results
- Spatial wave functions



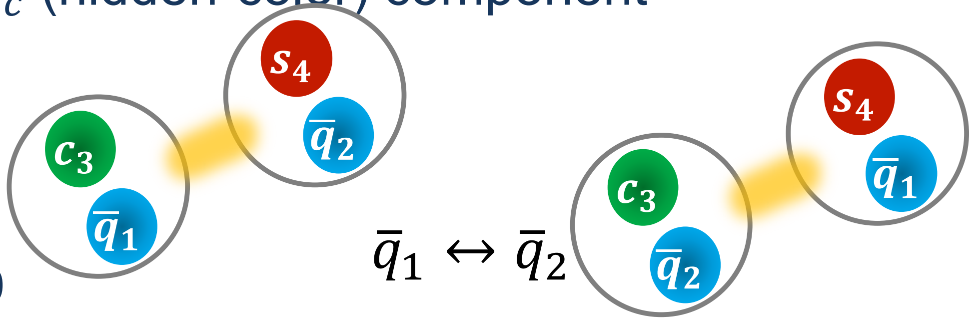
$$\phi_{nlm}(\boldsymbol{r}) = N_{lm} r^l e^{-\frac{r^2}{r_n^2}} Y_{lm}(\hat{r})$$

Hiyama:2003cu

- ▶ Geometric progression: $r_n = r_0 a^{n-1}$
- ▶ Embed both long- and short-range correlations

What can wave function tell us?

- Quark potential models: energies + **wave functions**
- Hadronic molecule could have large portion of the $8_c - 8_c$ (hidden-color) component



$$|\Psi\rangle = |1\rangle|\psi(31;42)\rangle - |1'\rangle|\psi(32;41)\rangle$$

$$|\psi(31;42)\rangle = |\psi_D(31)\rangle|\psi_{\bar{K}}(42)\rangle|\psi_{D\bar{K}}\rangle, \quad \langle\psi(31;42)|\psi(32;41)\rangle \approx 0$$

$$\langle\Psi|\Psi\rangle = 1 + 1 - 2\langle 1|1'\rangle\langle\psi(31;42)|\psi(32;41)\rangle \approx 2$$

$$\begin{cases} |(c_3\bar{q}_2)_{1_c}(s_4\bar{q}_1)_{1_c}\rangle = |1'\rangle \\ |(c_3\bar{q}_2)_{8_c}(s_4\bar{q}_1)_{8_c}\rangle = |8'\rangle \end{cases} \qquad \begin{cases} |(c_3\bar{q}_1)_{1_c}(s_4\bar{q}_2)_{1_c}\rangle = |1\rangle \\ |(c_3\bar{q}_1)_{8_c}(s_4\bar{q}_2)_{8_c}\rangle = |8\rangle \end{cases} \qquad \begin{bmatrix} \frac{1}{3} & \frac{2\sqrt{2}}{3} \\ \frac{2\sqrt{2}}{3} & -\frac{1}{3} \end{bmatrix} \begin{bmatrix} |1\rangle \\ |8\rangle \end{bmatrix} = \begin{bmatrix} |1'\rangle \\ |8'\rangle \end{bmatrix}$$

$$|\Psi\rangle = |1\rangle\left[|\psi(31;42)\rangle + \frac{1}{3}|\psi(32;41)\rangle - \right] + |8\rangle\frac{2\sqrt{2}}{3}|\psi(32;41)\rangle$$

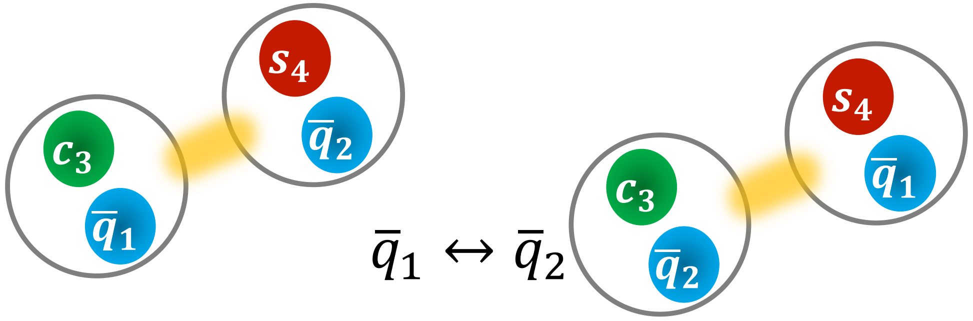
$$P_{8_c} = \frac{4}{9}, \qquad P_{1_c} = \frac{5}{9}$$

Compact state or Molecule?

- Wave functions: spatial distribution
- Proper quantities to discern

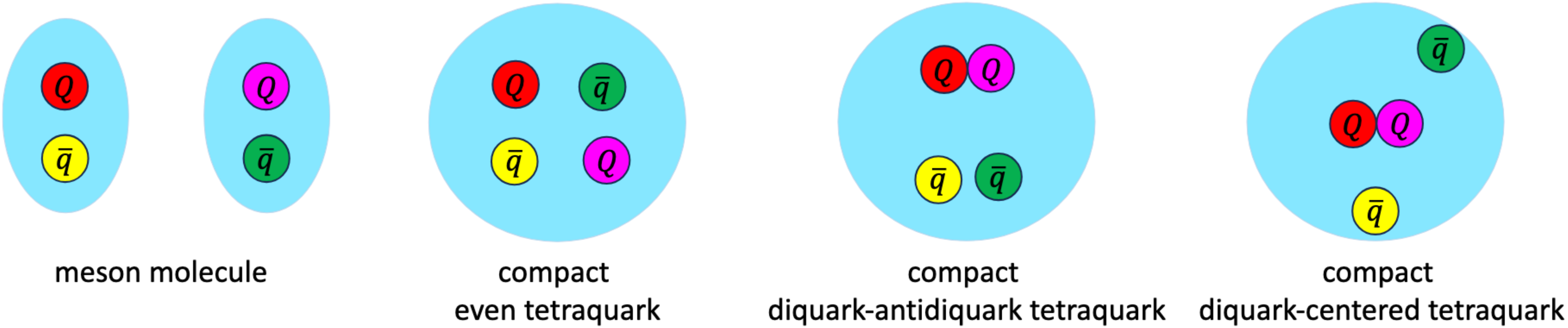
$$r_{13}^{rms} \equiv \langle \Psi | r_{13}^2 | \Psi \rangle$$

$$r_{c\bar{q}}^{rms} \equiv \langle \psi(31;43) | r_{13}^2 | \psi(31;42) \rangle \approx r_D^{rms}$$

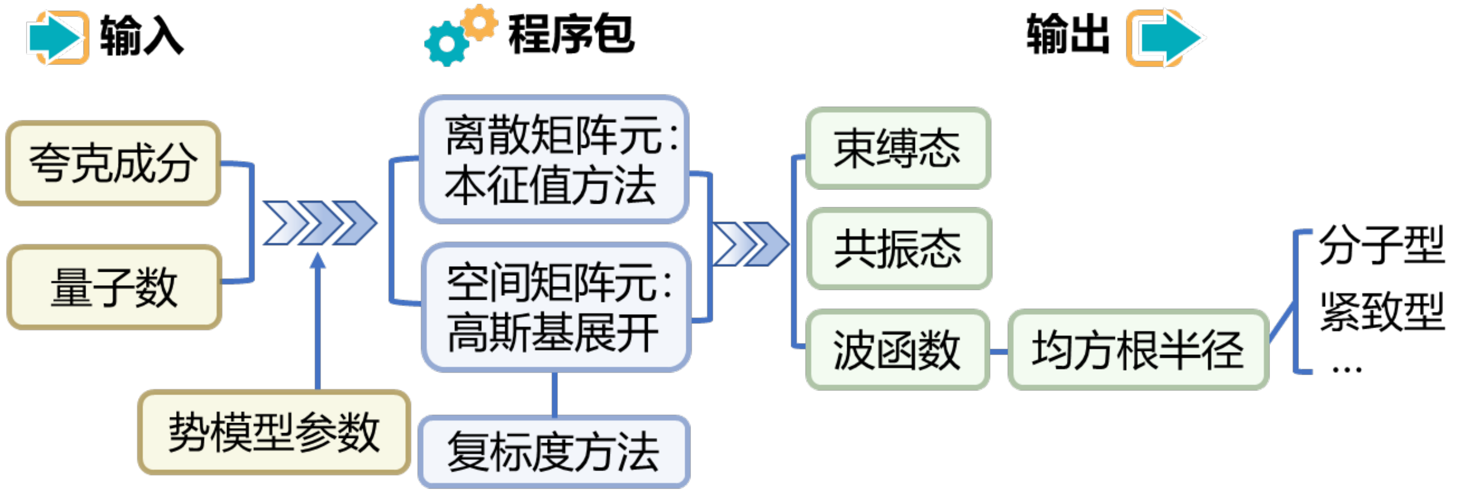


$$|\Psi\rangle = |1\rangle|\psi(31;42)\rangle - |1'\rangle|\psi(32;41)\rangle$$

- Categorization of spatial structures of tetraquark states.



Automatic and highly efficient package



- Fully heavy tetraquark states ($QQ\bar{Q}\bar{Q}$)
- Triply heavy tetraquark states ($QQ\bar{Q}\bar{q}$)
- Doubly heavy tetraquarks states ($QQ\bar{q}\bar{q}$)
- Single heavy strange states ($Qs\bar{q}\bar{q}$, $Q\bar{s}q\bar{q}$)

$q = u, d, s; \ \ Q = b, c$

$J^P = 0^+, 1^+, 2^+$, Only S-wave

Only bound states

Over 150 states

Total runtime:
2.5h on my laptop

$QQ\bar{Q}\bar{Q}$ tetraquark states

- Three models
  - ► BGS model — Barnes:2005pb
  - ► Semay-Silvestre-Brac Models — Semay:1994ht, Silvestre-Brac:1996myf

$$V_{ij}(r) = \left[-\frac{\kappa}{r} + \lambda r^p - \Lambda + \frac{2\pi}{3m_i m_j}\kappa' \frac{1}{\pi^{3/2} r_0^3} e^{(-r^2/r_0^2)} \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j \right] \lambda_i \cdot \lambda_j$$

AL1: $p = 1$, AP1: $p = 2/3$

- Our conclusions:

No bound states for any reasonable **pairwise** interaction

- Similar conclusions in literature

J. P. Ader, J. M. Richard, and P. Taxil, PR**D25**, 2370 (1982).
M.-S. Liu, Q.-F. Lü, X.-H. Zhong, and Q. Zhao, , PRD100, 016006 (2019).
X. Jin, Y. Xue, H. Huang, and J. Ping, EPJC**80**, 1083 (2020).
…

> tial models already used in the study of heavy mesons and baryons. We first consider the situation where the quarks have the same mass and interact through a two-body potential due to color-octet exchange. In this case, we show that for any reasonable confining potential there is no state below the threshold corresponding to the spontaneous dissociation into two mesons. We investigate in detail different possibilities of modifying this negative

Complex scaling method

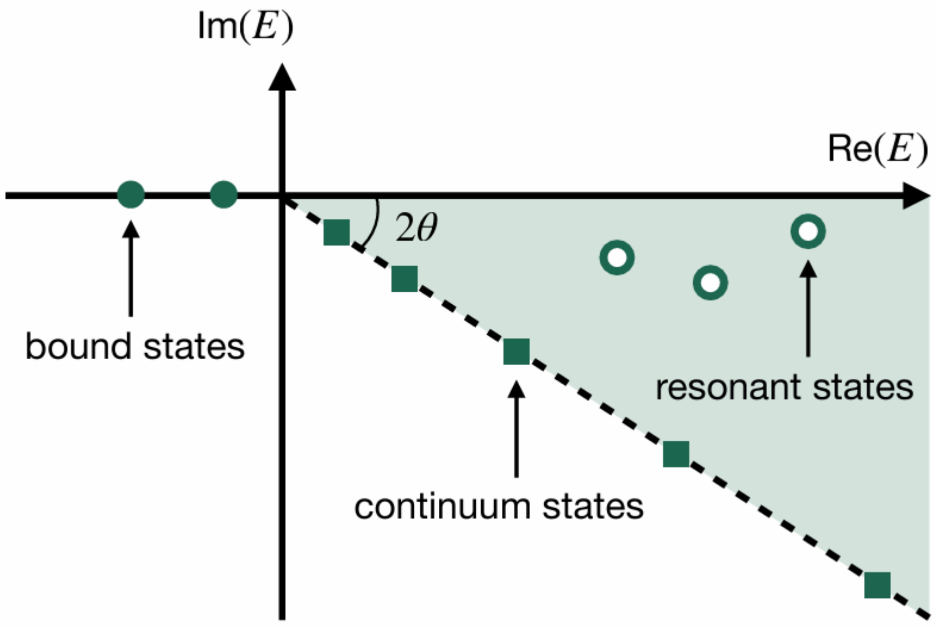
- Original Hamiltonian

$$H = \sum_{i}^{n} \left(m_i + \frac{\boldsymbol{p}_i^2}{2m_i} \right) + \sum_{i<j=1}^{n} V_{ij} \,,$$

- Complex scaling

$$U(\theta)\boldsymbol{r} = \boldsymbol{r}e^{i\theta}, \quad U(\theta)\boldsymbol{p} = \boldsymbol{p}e^{-i\theta}.$$

$$H(\theta) = \sum_{i=1}^{n} \left(m_i + \frac{p_i^2 e^{-2i\theta}}{2m_i} \right) + \sum_{i<j=1}^{n} V_{ij}\left(r_{ij}e^{i\theta}\right).$$

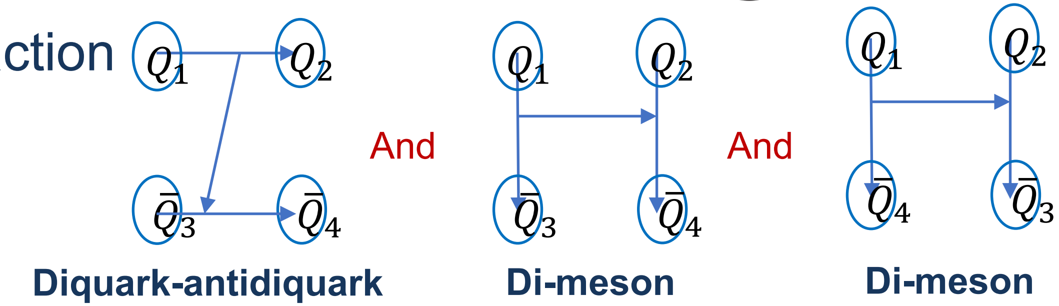


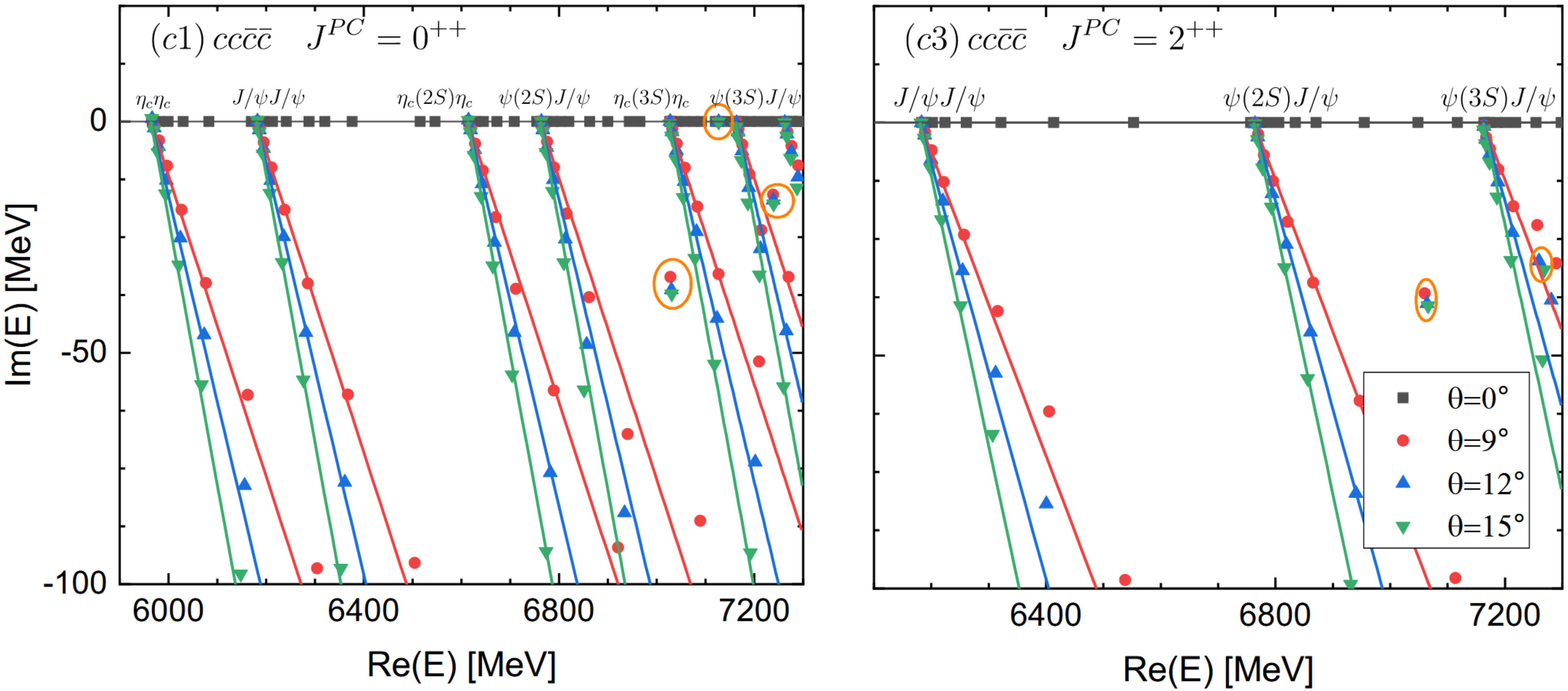
- Resonance appearing as the eigenvalue of $H(\theta)$
- Equivalence of the CSM and **contour deformation method** in 2-body systems

Aguilar:1971ve, Balslev:1971vb, Aoyama:2006hrz…

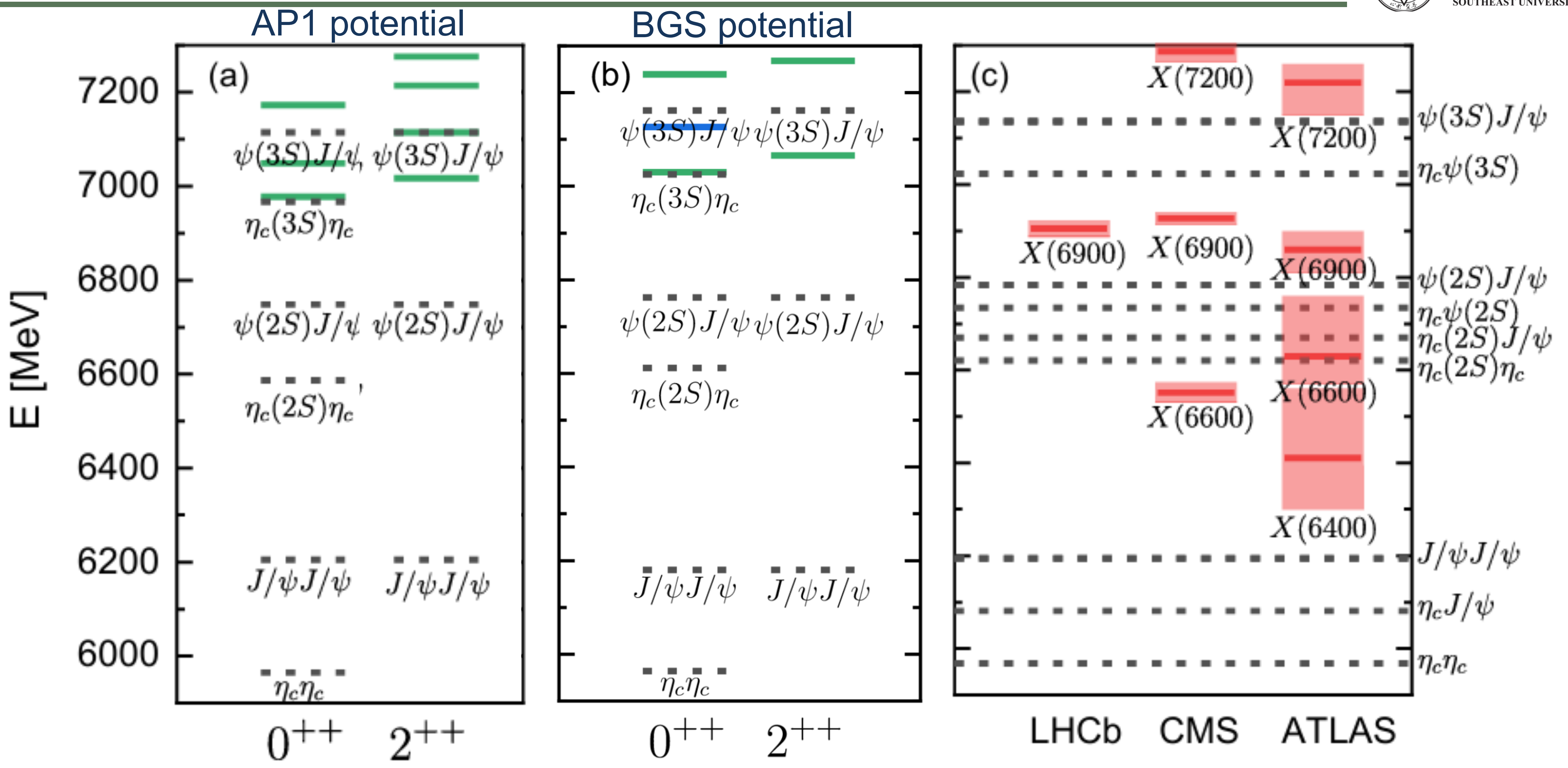
All-charm tetraquarks

- BGS, AL1, AP1 models: **pairwise** confinement interaction
- Configurations: diquark-antidiquark, di-meson×2
- Tetraquark as resonances





All-charm tetraquarks

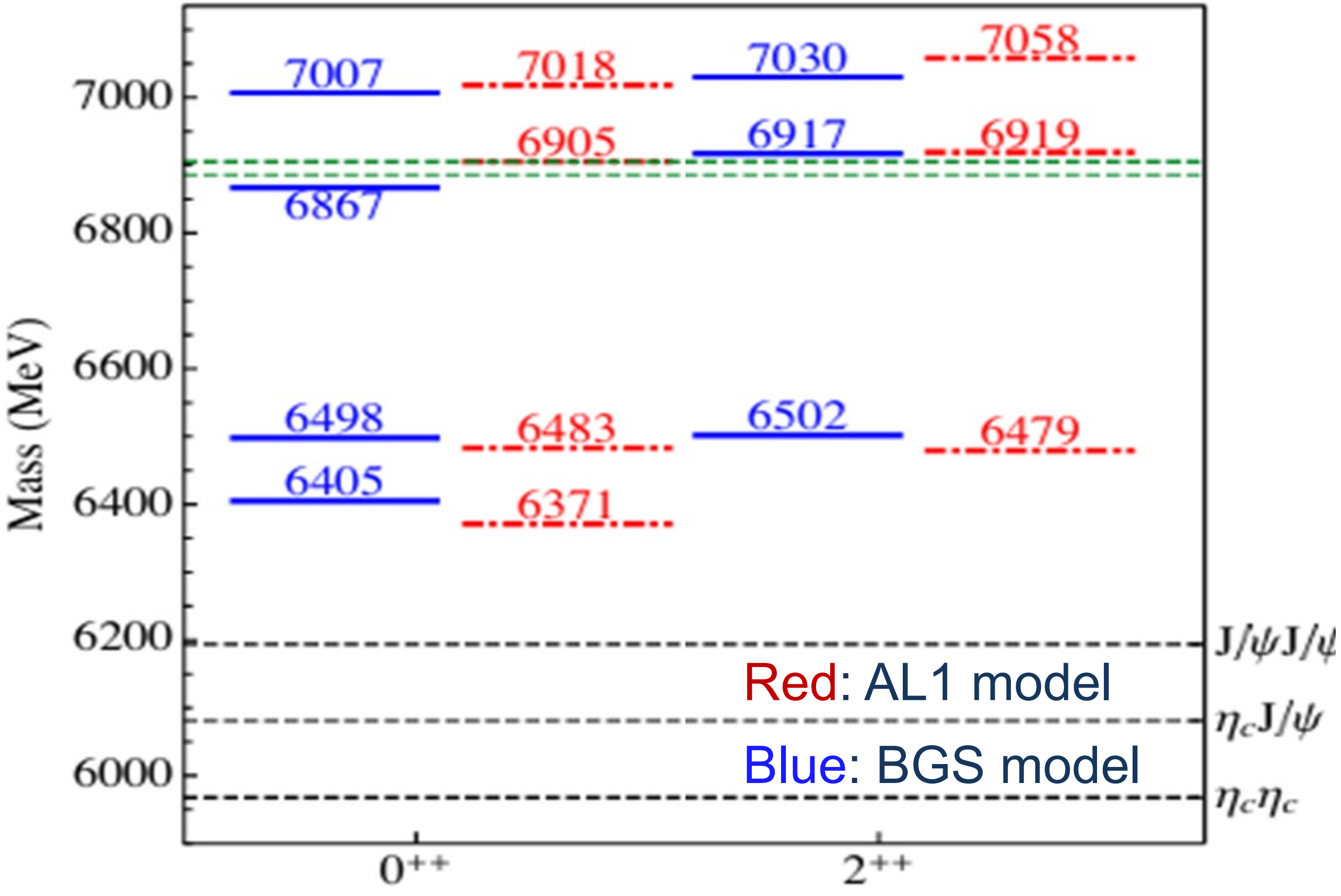


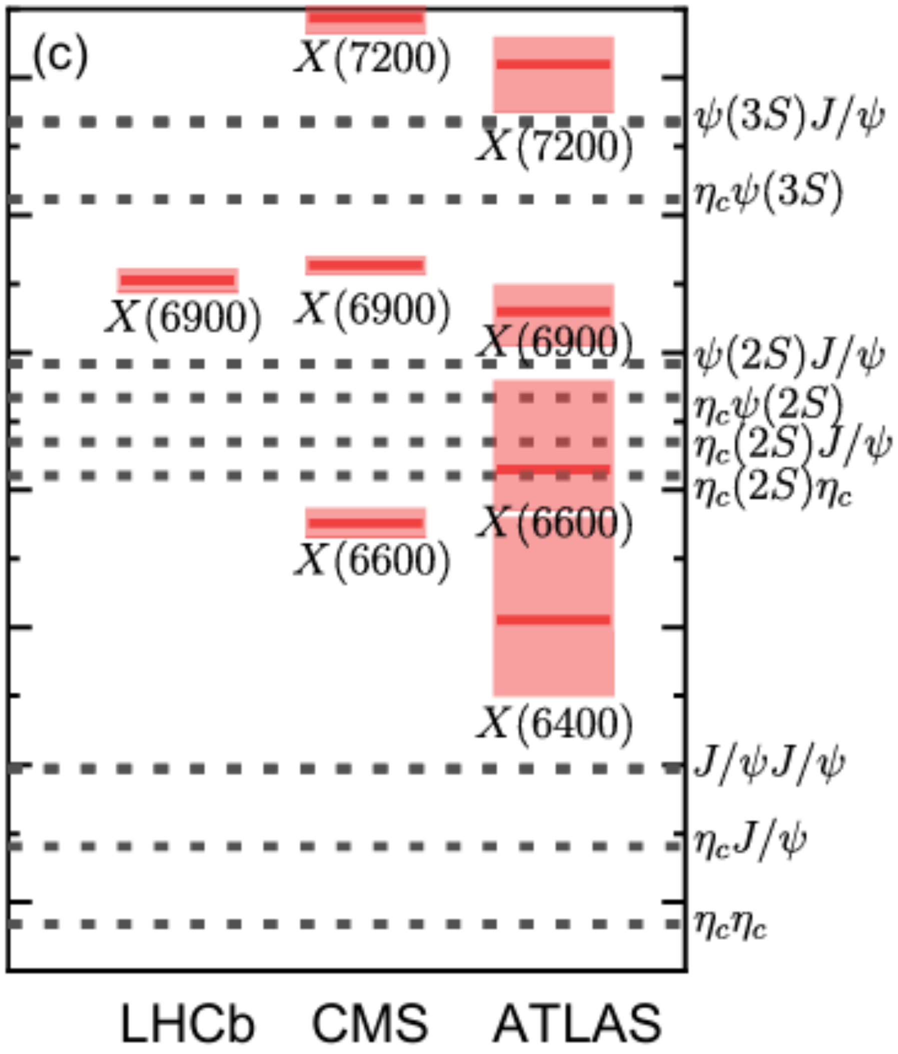
X(6600) is missing in pairwise confinement models

See also G.-J. Wang, Q. Meng, and M. Oka, PRD106, 096005 (2022).

All-charm tetraquarks

- Earlier calculation neglecting di-meson configurations

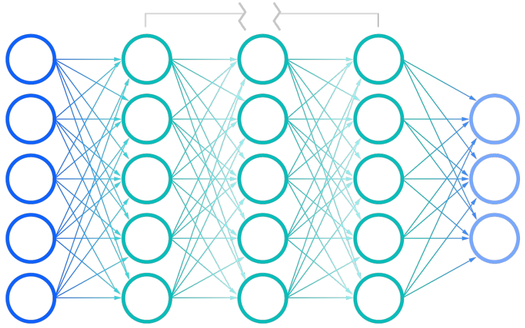


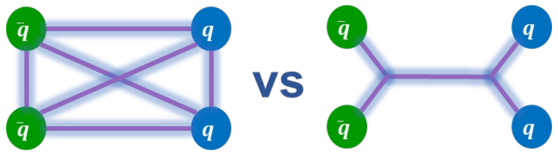


G-J Wang, LM, M Oka, S-L Zhu, PRD 104, 036016 (2021)

- Pairwise model: overestimate the transition between diquark-antidiquark and dimeson??

DNNs: towards tetraquark confinement



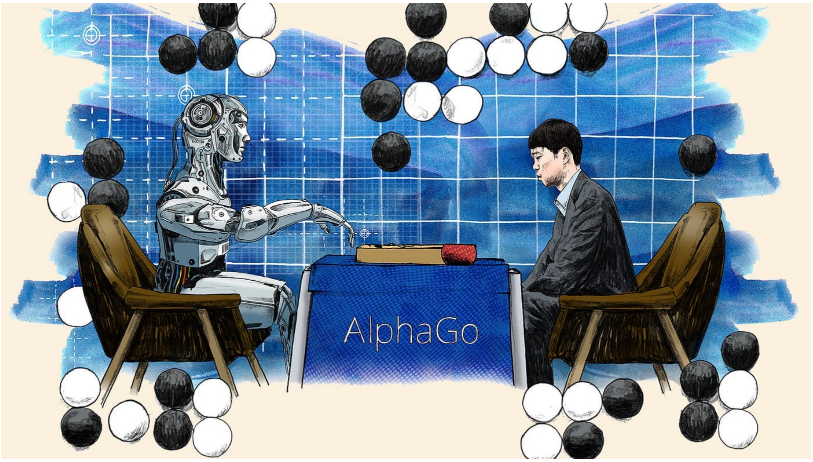


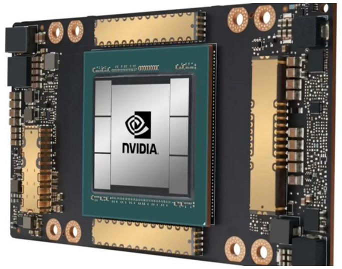
Turning point of AI

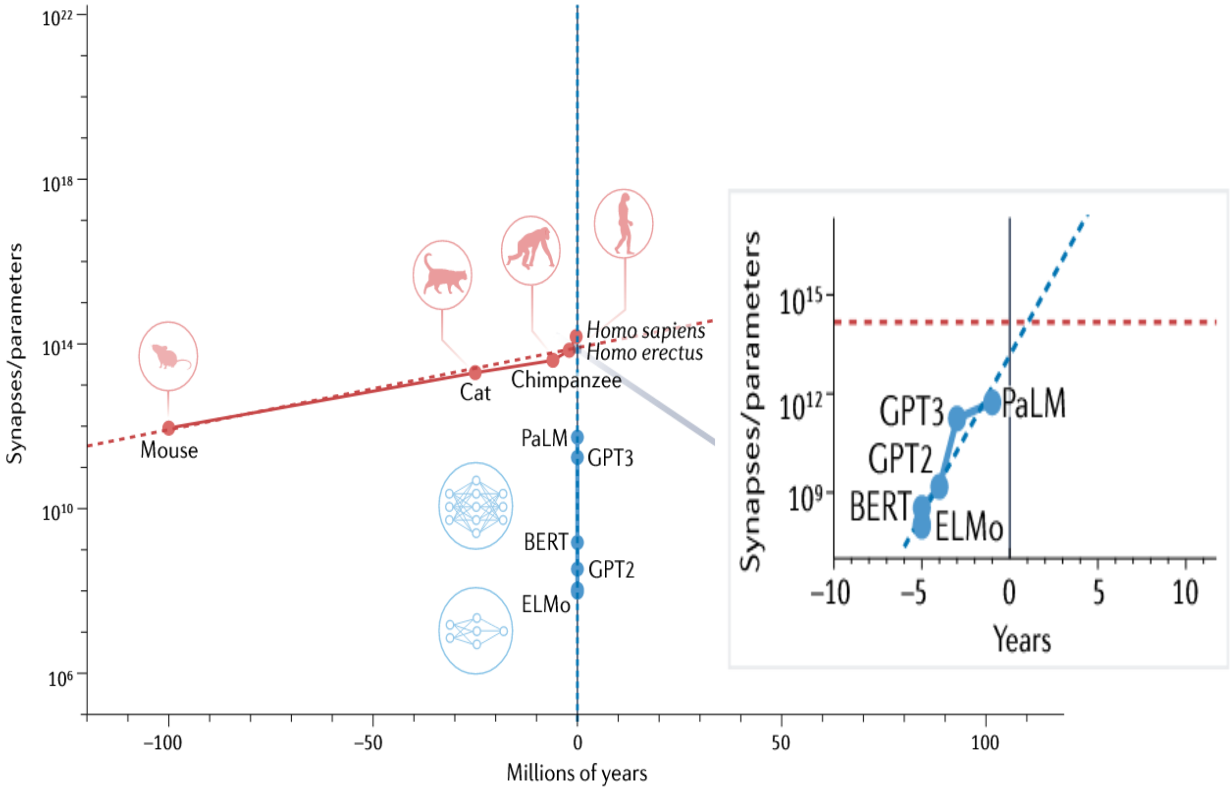
Year	Milestone	Who	Significance
2010	ImageNet	Fei-Fei Li	Large-scale dataset that enabled the deep learning revolution.
2012	AlexNet	Krizhevsky et al.	Proven power of deep CNNs and GPU acceleration for AI.
2014	GANs	Ian Goodfellow	Introduced framework for generative AI to create realistic data.
2015/16	TensorFlow/ Pytorch	Google/Meta	Critical open-source software that democratized AI research globally.
2016	AlphaGo	DeepMind	First AI to defeat a world champion in the game of Go.
2010s	AI Chips	NVIDIA, Google, Apple	Specialized GPUs/TPUs enabled training of massive AI models.
2017	Transformer	Google	Revolutionary architecture that made modern large language models possible.
2018	BERT	Google	Bidirectional language model that set new standards for NLP.
2020	AlphaFold 2	DeepMind	Solved the decades-old scientific challenge of protein folding.
2022	ChatGPT	OpenAI	Popularized large language models through accessible public interaction.

2025 AI agent

10 milestones of AI selected by DeepSeek

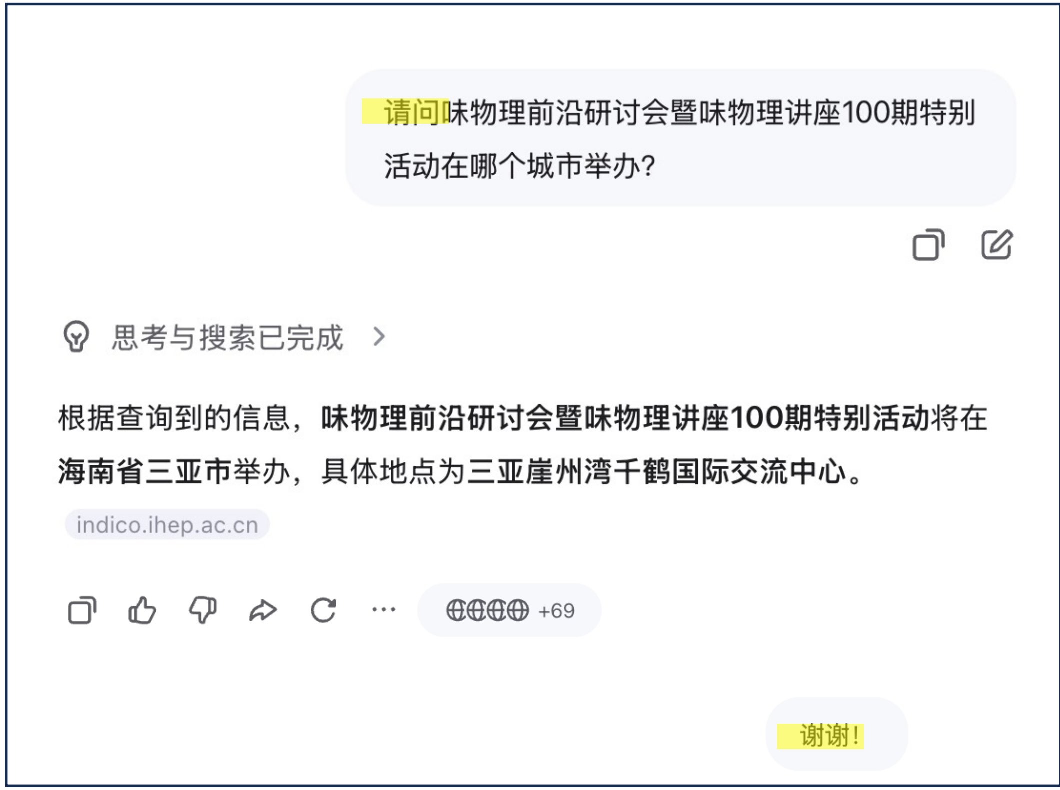






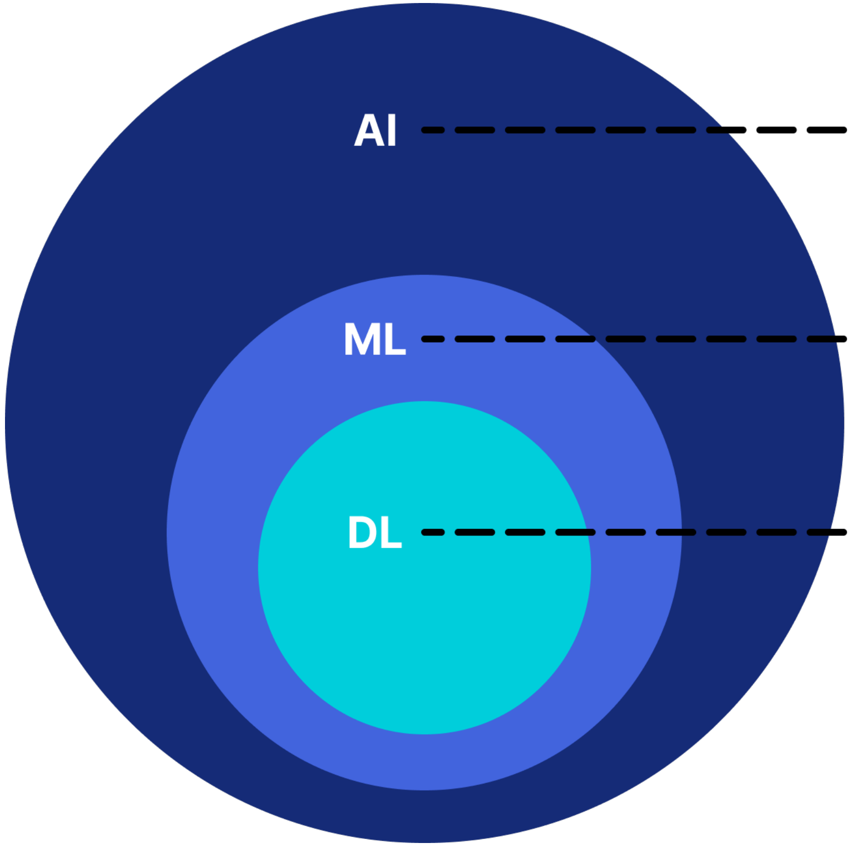
The evolution of biological and artificial intelligence

Schwartz, M. D. (2022). *Nature Reviews Physics*, *4*(12), 741–742.



Being more polite to chatbots may increase your chances of survival when robots win

AI vs ML vs DL



Artificial Intelligence
Engineering of making intelligent machines and programs.

Machine Learning
Ability to learn without being explicitly programmed.

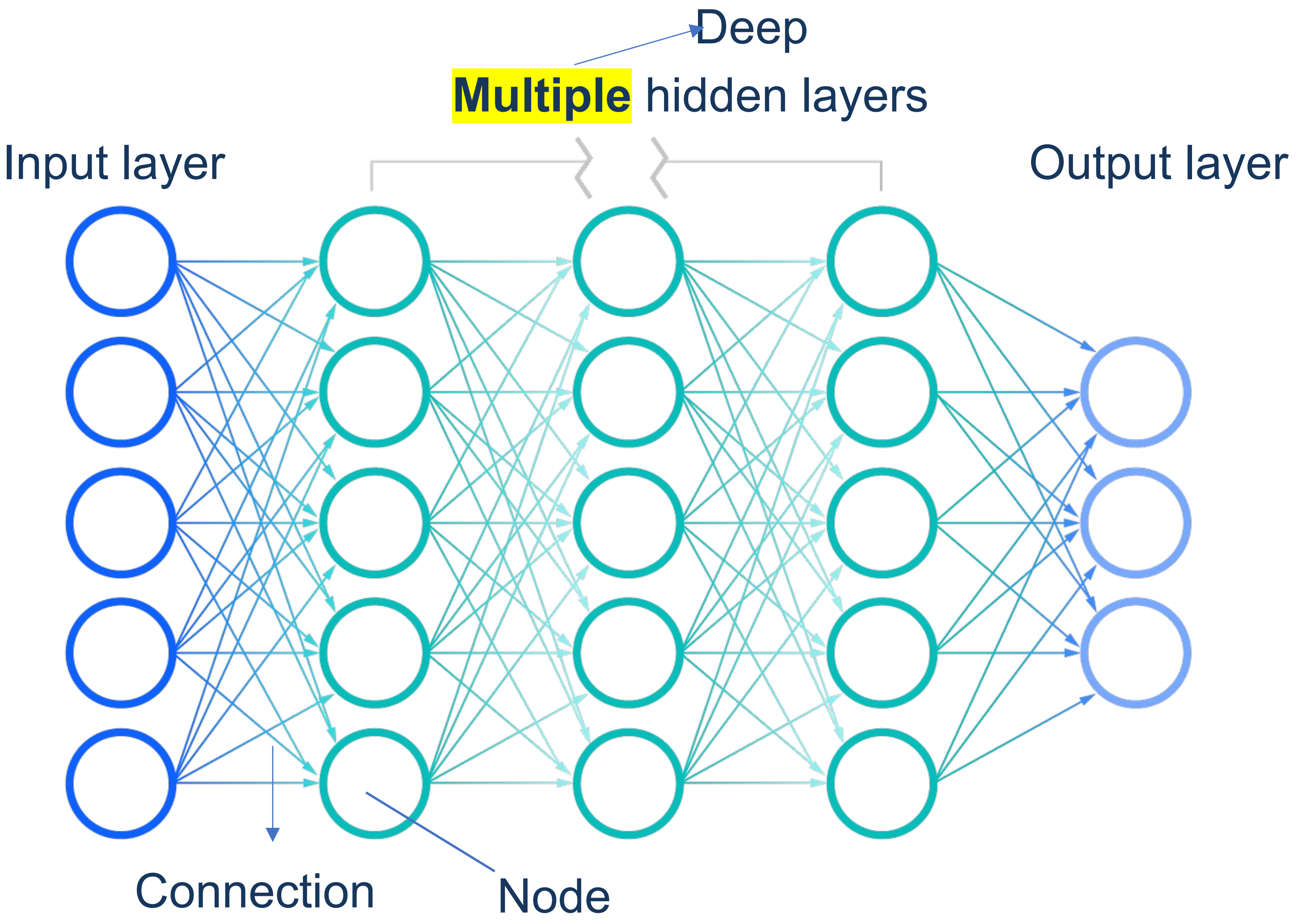
Deep Learning
Learning based on deep neural networks.

neurosnap.ai

https://neurosnap.ai/blog/post/64279cadfeb3e5ca5ba0904a

AI boom from 2010s was powered by **deep learning**

Deep neutral network



Neural network: simple units (neurons) connected to form complex network

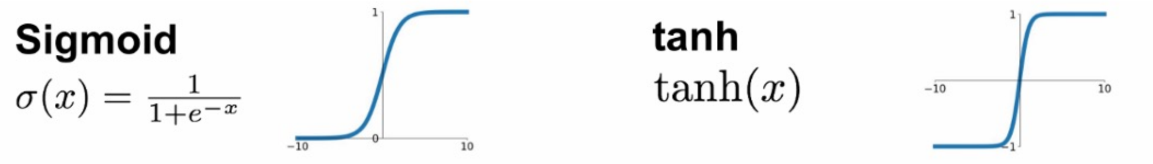
Deep： Enable learn complex relation in data

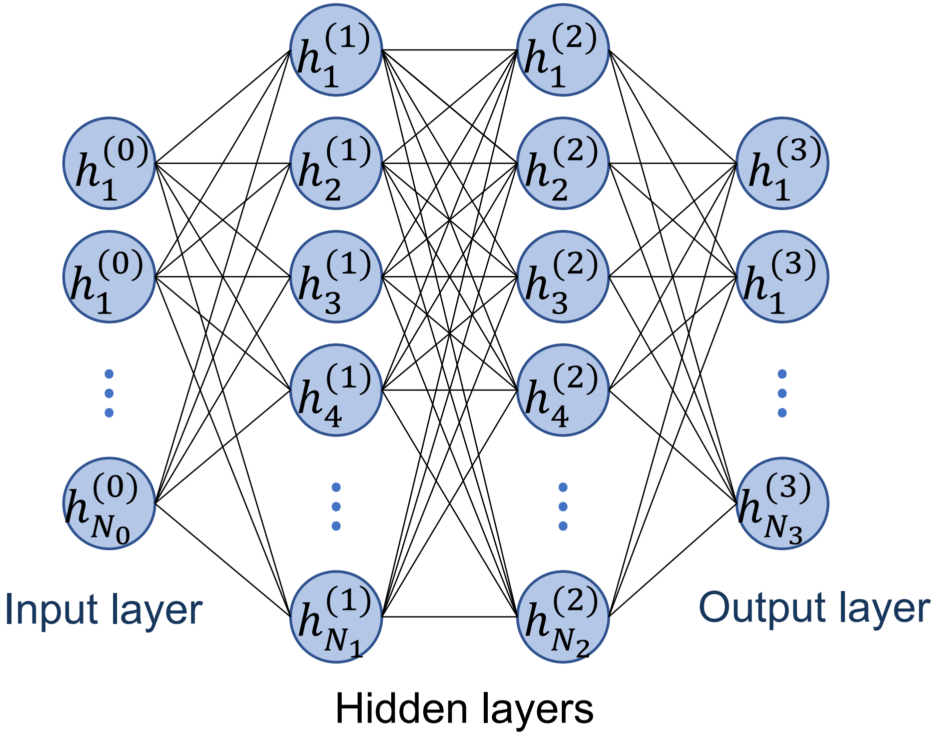
Connections between layers

$$\vec{h}^{(i+1)} = \sigma\left[W^{(i)}\vec{h}^{(i)} + \vec{b}^{(i)}\right]$$

Nonlinear Linear

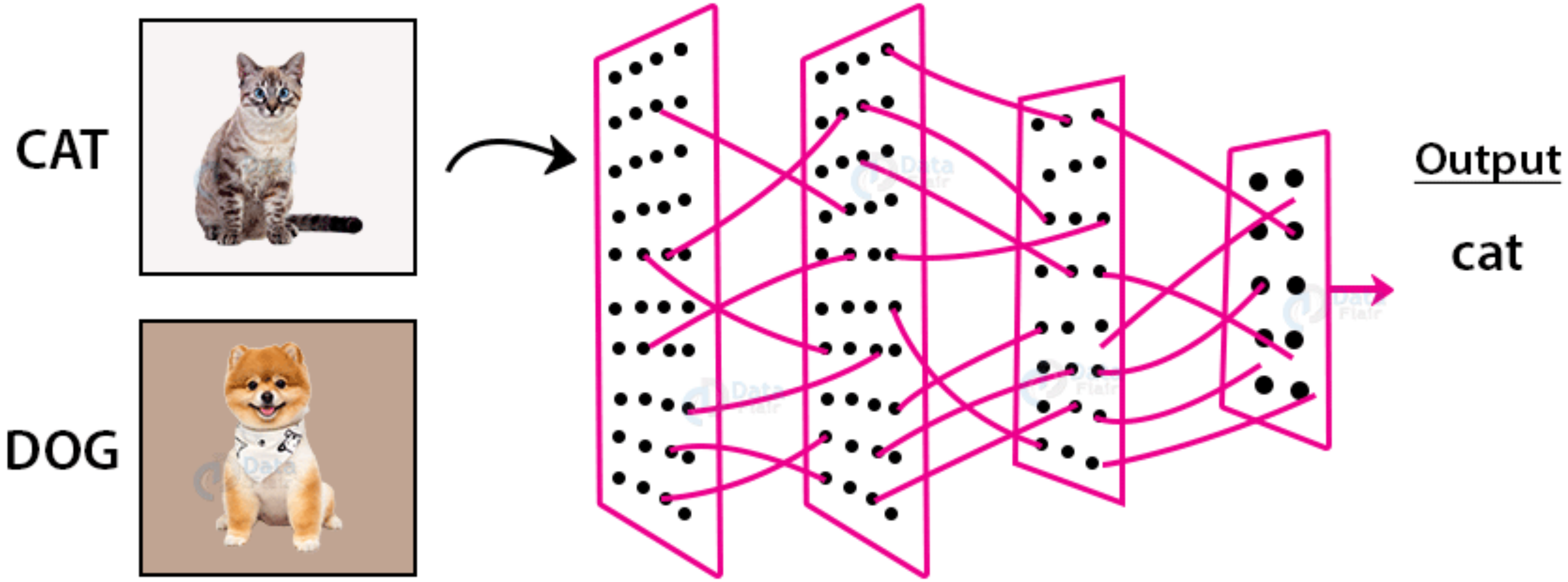
- Linear part: Weight matrices $W^{(i)}$ and biases $\vec{b}^{(i)}$
- Nonlinear part: Activation functions σ





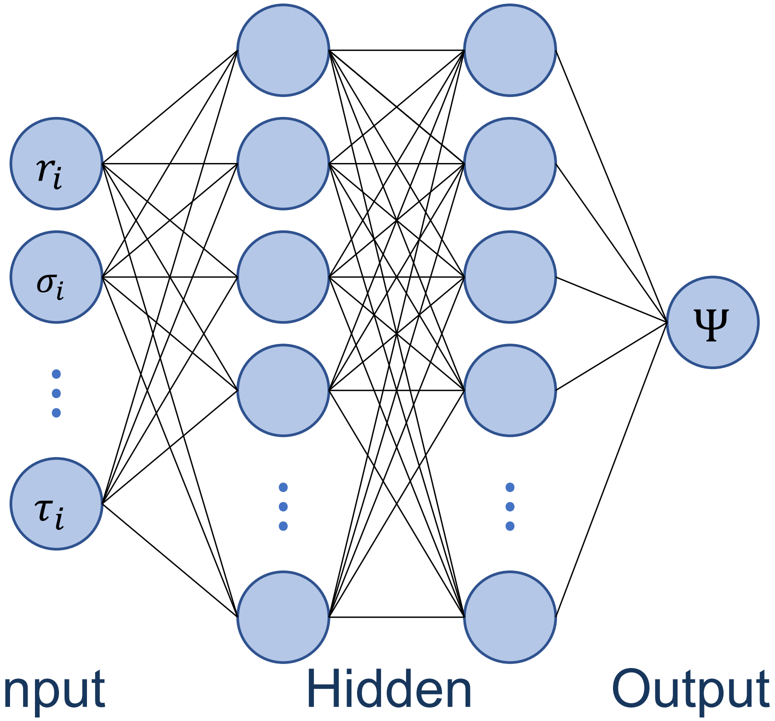
- Training: determine $W^{(i)}$ and biases $\vec{b}^{(i)}$ to minimize loss functions
  - ▶ Loss function： quantifies the difference between the expected and actual output.
- Highly efficient optimization
  - ▶ Backpropagation: automatic differentiation (AD)
  - ▶ GPUs
  - ▶ …

DNNs: Data-driven application



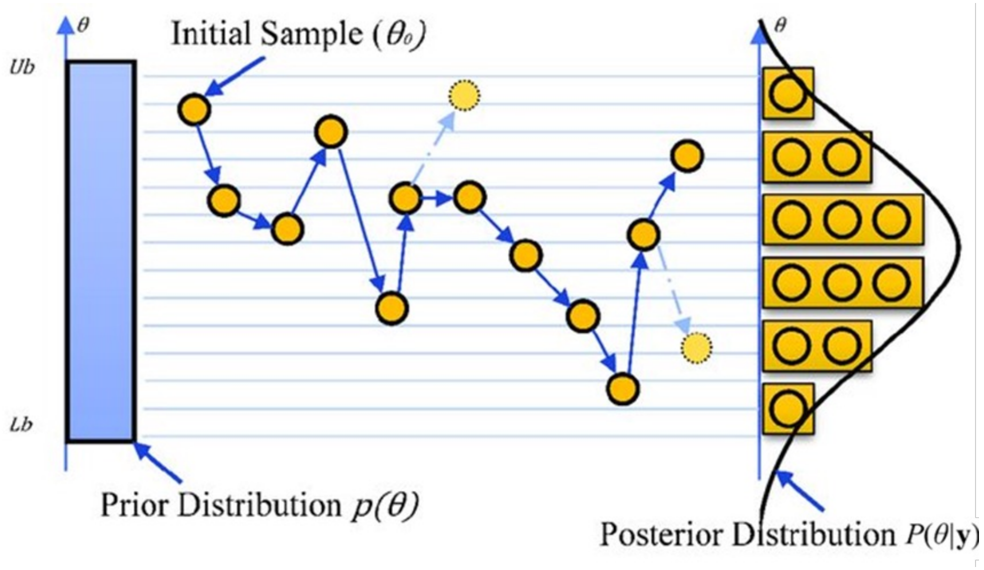
DNN-based VMC method

Neural Network Quantum States (NQSs):



Loss function: $E_{\boldsymbol{\theta}} = \frac{\langle \psi_{\boldsymbol{\theta}} | H | \psi_{\boldsymbol{\theta}} \rangle}{\langle \psi_{\boldsymbol{\theta}} | \psi_{\boldsymbol{\theta}} \rangle} \geq E_0$

Variational Monte Carlo (VMC)



Sample using Metropolis-Hasting algorithm

$$E_{\boldsymbol{\theta}} = \frac{\int |\psi_{\boldsymbol{\theta}}(r)|^2 \frac{H\psi(r)}{\psi(r)} dr}{\int |\psi_{\boldsymbol{\theta}}(r)|^2 dr}$$

Train the DNNs not by the data but by the physics equation: **physics-informed**

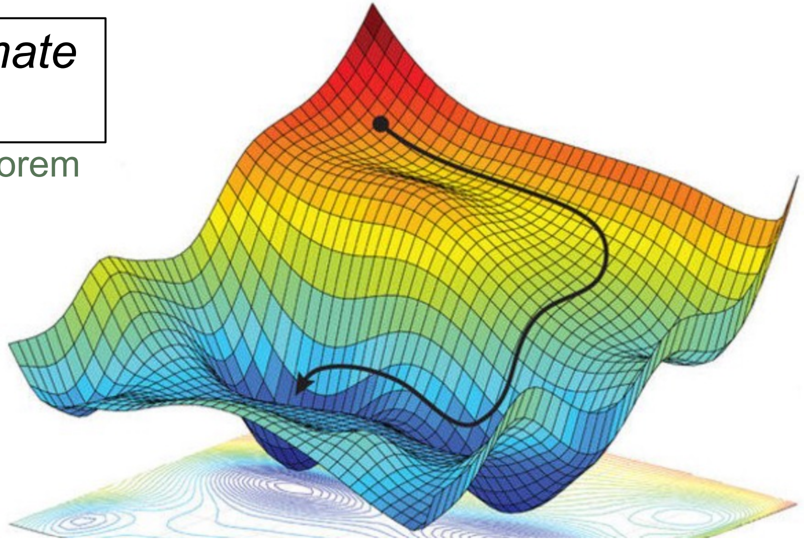
Advantages

- DNNs: approximate high-dimensional functions
  - ▶ **Universal approximation theorems**

sufficiently large or deep

> *Neural networks with a certain structure can, in principle, approximate* ***any continuous function*** *to* ***any desired accuracy***

https://en.wikipedia.org/wiki/Universal_approximation_theorem

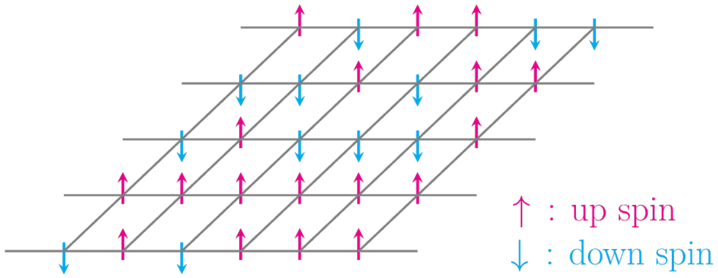


  - ▶ Variational principle: the more general the trial function, the more accurate the solution
  - ▶ Unbiasedly distinguish molecular and compact tetraquark states
- VMC: applicable for few-body potential
  - ▶ Flux-tube confinement potential
- Circumvents the sign problem in imaginary time evolution of DMC
- Fast optimization: AD+GPUs
- Easy-to-use open-source frameworks

TensorFlow PyTorch

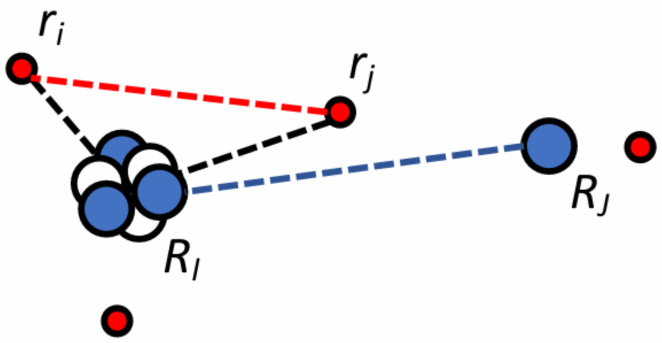
DNNs in many-body problems

Spin models



G. Carleo and M. Troyer, Science 355, (2017).

Electronic systems



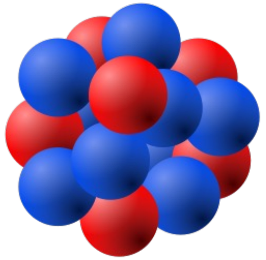
PauliNet: J. Hermann, Z. Schätzle, and F. Noé, Nat. Chem. 12, 891 (2020).

FermiNet: D. Pfau et al., Phys. Rev. Res. 2, 033429 (2020).

…

Review: J. Hermann et al., Nat. Rev. Chem.7, 692–709 (2023)

Nuclear structure



Deuteron: J. W. T. Keeble and A. Rios, Phys. Lett. B 809, 135743 (2020).

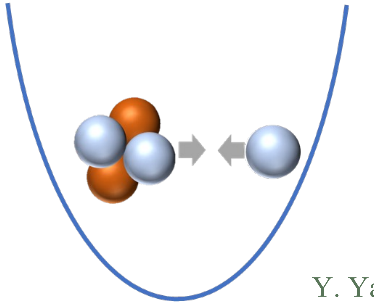
$A \leq 4$: C. Adams *et al.*, Phys. Rev. Lett. **127**, 022502 (2021).

FeynmanNet: Y. Yang and P. Zhao, Phys. Rev. C **107**, 034320 (2023).

Hypernuclei: Z.-X. Zhang *et al.*, arXiv:2508.03575

…

Neutron-α scattering



Chiral Nuclear force
Trapped five-body problems
(DNNs+VMC)

Y. Yang, E. Epelbaum, J. Meng, **LM**, and P. Zhao, Phys.Rev.Lett. 135 (2025) 17, 172502

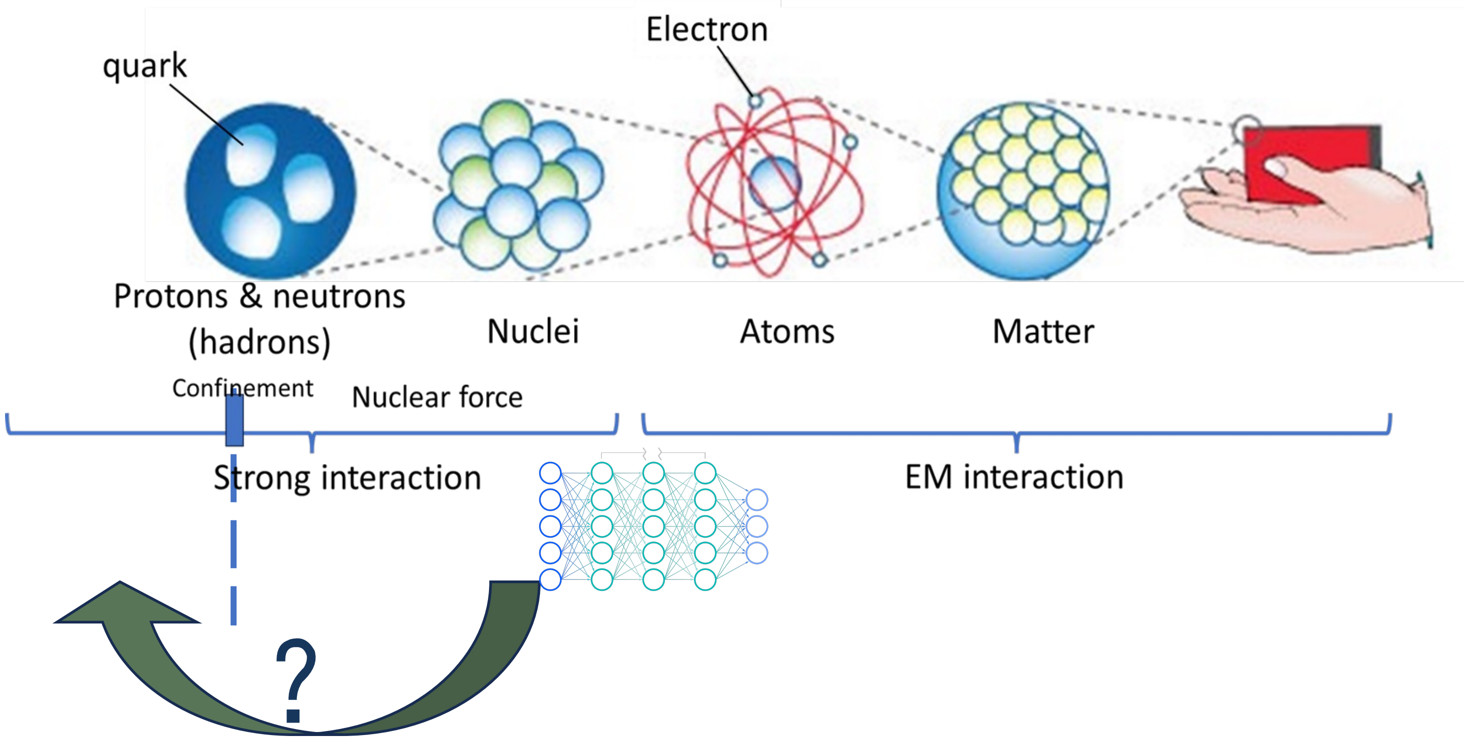
Quark potential models

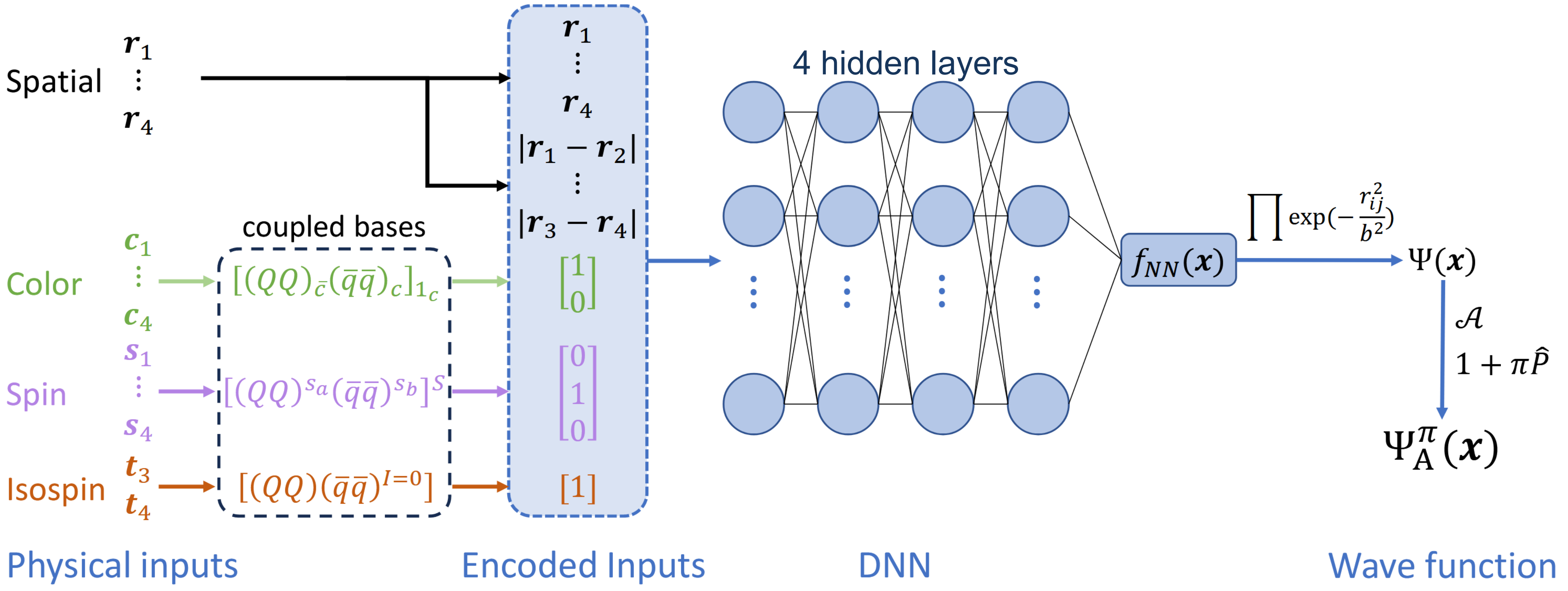
$$V_{ij}(r) = V_{ij}^{OGE} + V_{ij}^{conf.}$$
$$= \left[\frac{\alpha_s}{r} - \frac{8\pi\alpha_s}{3m_Q^2}\,\tilde{\delta}(\sigma;r)\boldsymbol{S_i}\cdot\boldsymbol{S_j} - \frac{3}{4}br\right]\frac{\lambda_i\cdot\lambda_j}{4}$$

Quark potential model

QCD-inspired model

- $SU(3)_c$: color and spin projections
- Confinement
- Strong correlation
- ….

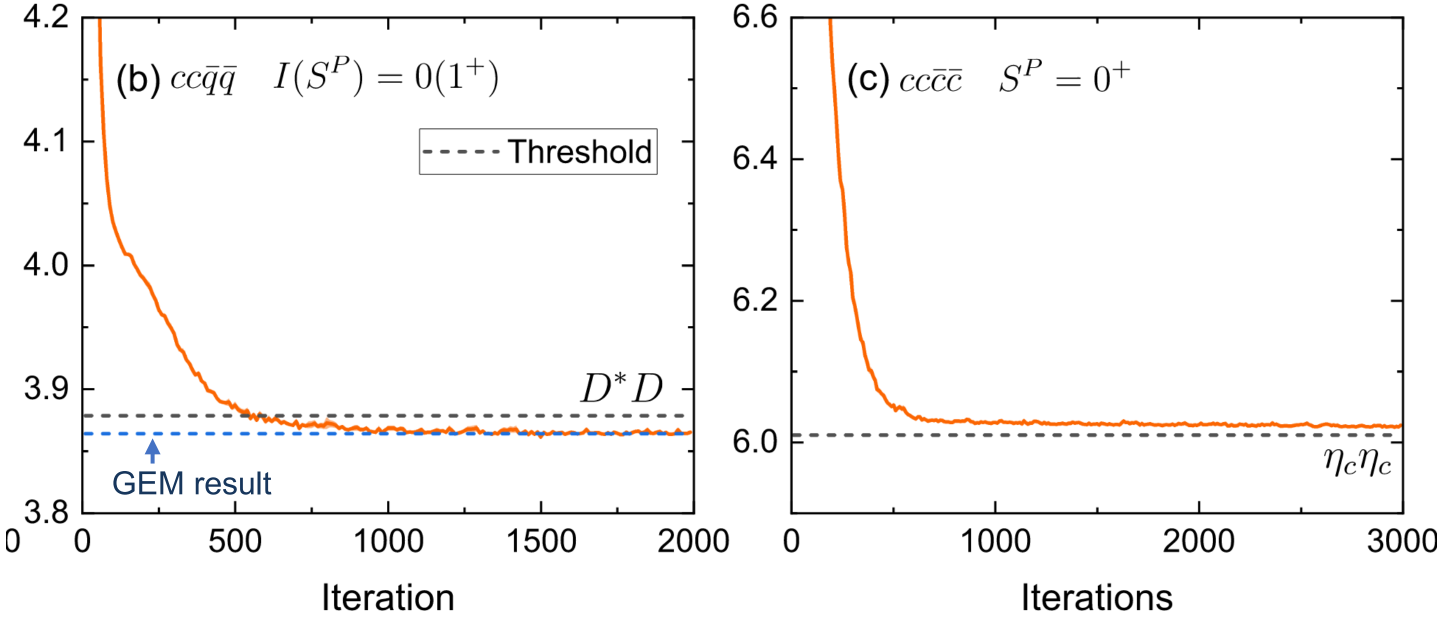




Powered by NetKet
https://github.com/netket

Tetraquark states

- Convergence after 1000 iterations
- Statistical errors less than 0.1 MeV
- Competitive performance with GEM
- $cc\bar{q}\bar{q}$: molecular state
- $bb\bar{q}\bar{q}$: compact heavy diquark
- $QQ\bar{Q}\bar{Q}$: no bound states



	$I(S^P)$	Thresh.	ΔE	$P_{\bar{3}_c\otimes 3_c}$	$P_{6\otimes 6_c}$	r_{QQ}	$r_{\bar{q}\bar{q}}$	$r_{Q\bar{q}}$
$cc\bar{q}\bar{q}$	$0(1^+)$	DD^*	-15	55%	45%	1.24	1.41	1.06
$bb\bar{q}\bar{q}$	$0(1^+)$	$\bar{B}\bar{B}^*$	-153	97%	3%	0.33	0.78	0.69
$QQ\bar{Q}\bar{Q}$	$0(0^+)$ $0(1^+)$ $0(2^+)$	$\eta_c\eta_c$ $\eta_c J/\psi$ $J/\psi J/\psi$	No bound					

ΔE in MeV, r in fm

Pentaquark states

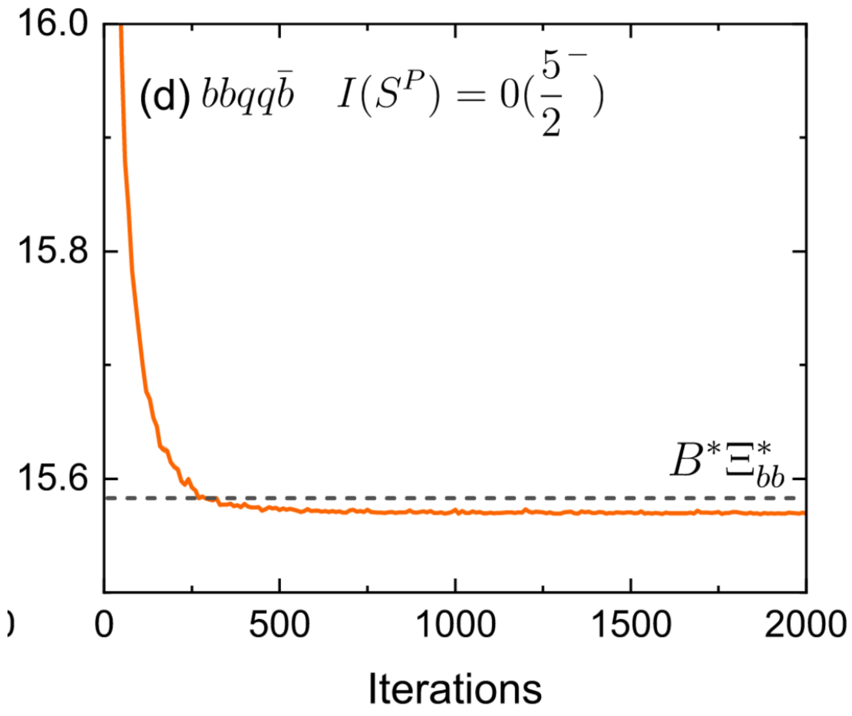
- Exact pentaquark calculations in are computationally prohibitive
  - ► Approximations in the spatial or color configurations
- Possible bound pentaquark systems
  - ► Heavy-diquark-antiquark-symmetry: $(QQ)_{\bar{3}_c} \to \bar{Q}$
  - ► $\bar{Q}\bar{Q}qq \to QQ\bar{Q}qq$
  - ► Given $M_{\bar{Q}\bar{Q}qq} < M_{\bar{Q}q} + M_{\bar{Q}q}$, $M_{QQ\bar{Q}qq} < M_{QQq} + M_{\bar{Q}q}$?
- Exact calculation via DeepQuark

$$\chi_{\bar{3}_c\otimes\bar{3}_c} = \left\{\left[(QQ)_{\bar{3}_c}(qq)_{\bar{3}_c}\right]_{3_c}\bar{Q}\right\}_{1_c},$$

$$\chi_{\bar{3}_c\otimes 6_c} = \left\{\left[(QQ)_{\bar{3}_c}(qq)_{6_c}\right]_{3_c}\bar{Q}\right\}_{1_c},$$

$$\chi_{6_c\otimes\bar{3}_c} = \left\{\left[(QQ)_{6_c}(qq)_{\bar{3}_c}\right]_{3_c}\bar{Q}\right\}_{1_c}.$$

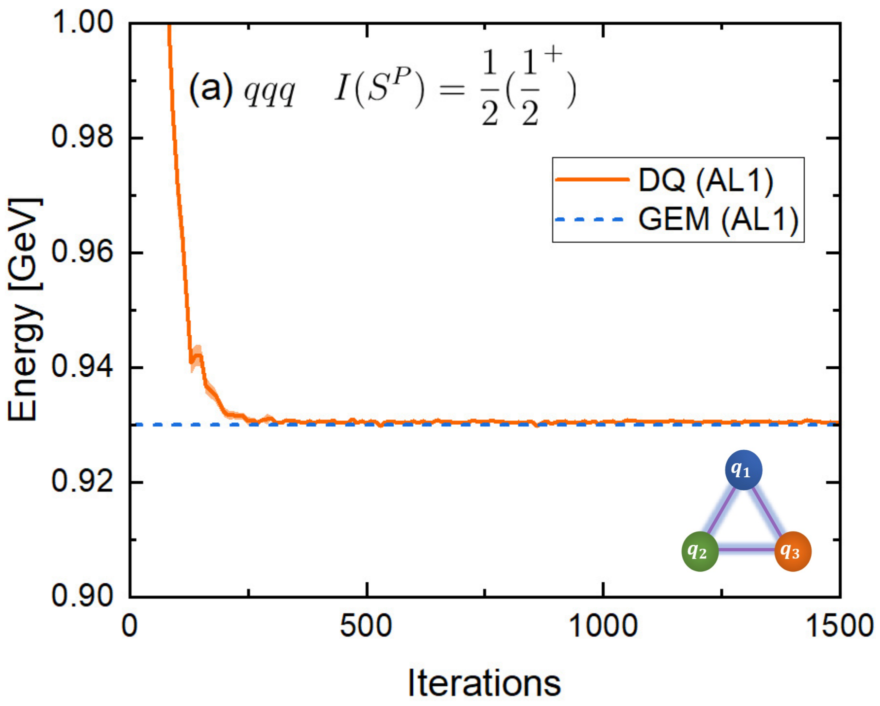
  - ► Moderate increase in comput. cost relative to the tetraquark

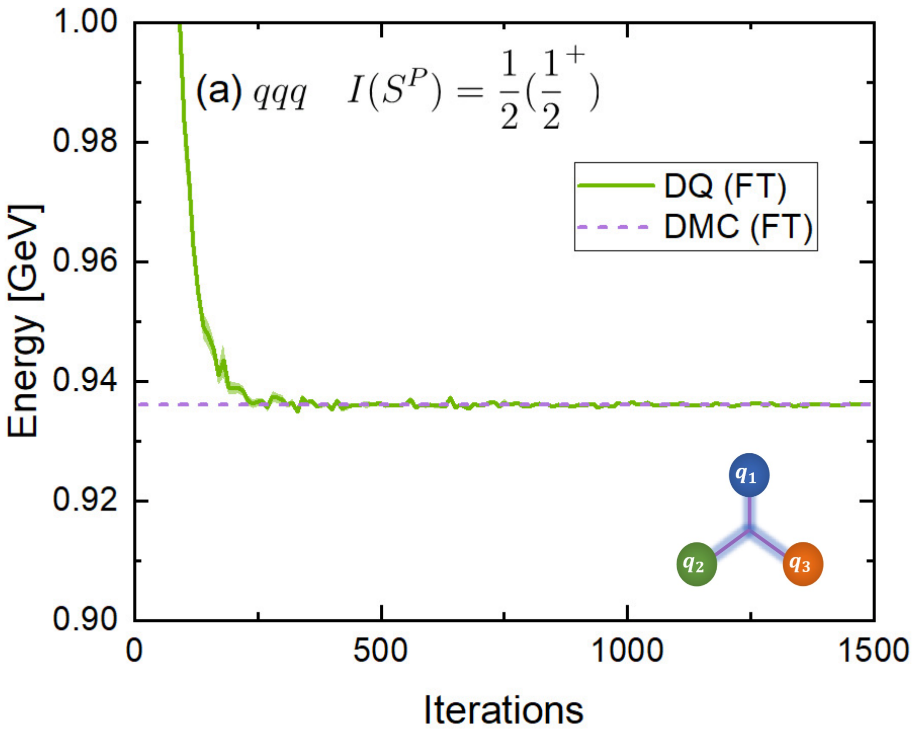


	S^P	Thresholds	ΔE	$\chi_{\bar{3}_c\otimes\bar{3}_c}$	$\chi_{\bar{3}_c\otimes 6_c}$	$\chi_{6_c\otimes\bar{3}_c}$	r_{QQ}	r_{Qq}	r_{qq}	$r_{Q\bar{Q}}$	$r_{q\bar{Q}}$
$ccqq\bar{c}$	$\frac{1}{2}^-, \frac{3}{2}^-$	$\eta_c\Lambda_c, J/\psi\Lambda_c$	NB	$\sim$35%	0%	$\sim$65%					
	$\frac{5}{2}^-$	$\bar{D}^*\Xi^*_{cc}$	-3	27%	73%	0%	0.50	1.39	1.90	1.73	1.38
$bbqq\bar{b}$	$\frac{1}{2}^-, \frac{3}{2}^-$	$\eta_b\Lambda_b, \Upsilon\Lambda_b$	NB	$\sim$35%	0%	$\sim$65%					
	$\frac{5}{2}^-$	$B^*\Xi^*_{bb}$	-14	19%	80%	1%	0.30	0.89	1.22	0.88	0.88

Flux-tube confinements

- Lattice QCD supports the flux-tube potential for qqq baryons and tetraquarks
- Baryons: Y-type interaction, junction position to minimize the total string length
  - ►Small impact on baryon spectrum, but complicate dramatically matrix element calculations
- DeepQuark: incorporate Y-type interaction without additional cost, thanks to the VMC
- Ready for tetraquark systems, where different confinement patterns lead to distinct signatures

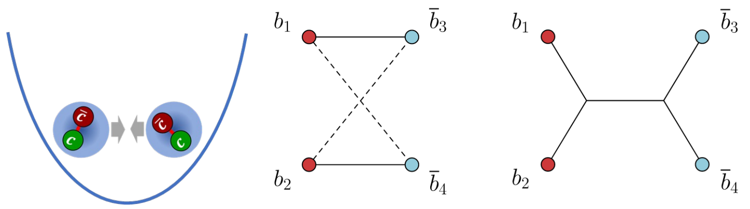




Summary and outlook

- We still need quark models to understander the **pattens** of multiquark states
  - ▶ **Difficulties:** interactions (confinement) and solving few-body (resonant) problem
  - ▶ **Chances:** All-charm tetraquark family ⇒ confinement pictures
- Gaussian expansion method + Complex scaling method
  - ▶ No bound $QQ\bar{Q}\bar{Q}$ for **pairwise** interaction
  - ▶ Exp. $cc\bar{c}\bar{c}$ results: disfavor the pairwise confinement interaction
  - ▶ Too strong coupling between dimeson and diquark-antidiquark (?)
- DeepQuark: DNN-based framework
  - ▶ Surpass traditional methods starting from pentaquark states
  - ▶ Ready for flux-tube confinement
- Outlook:
  - ▶ DNNs: excited states, scattering, resonances
  - ▶ All-charm tetraquark with flux-tube confinement
  - ▶ Convolutional transformer wave functions

Thanks for your attention!



Backup

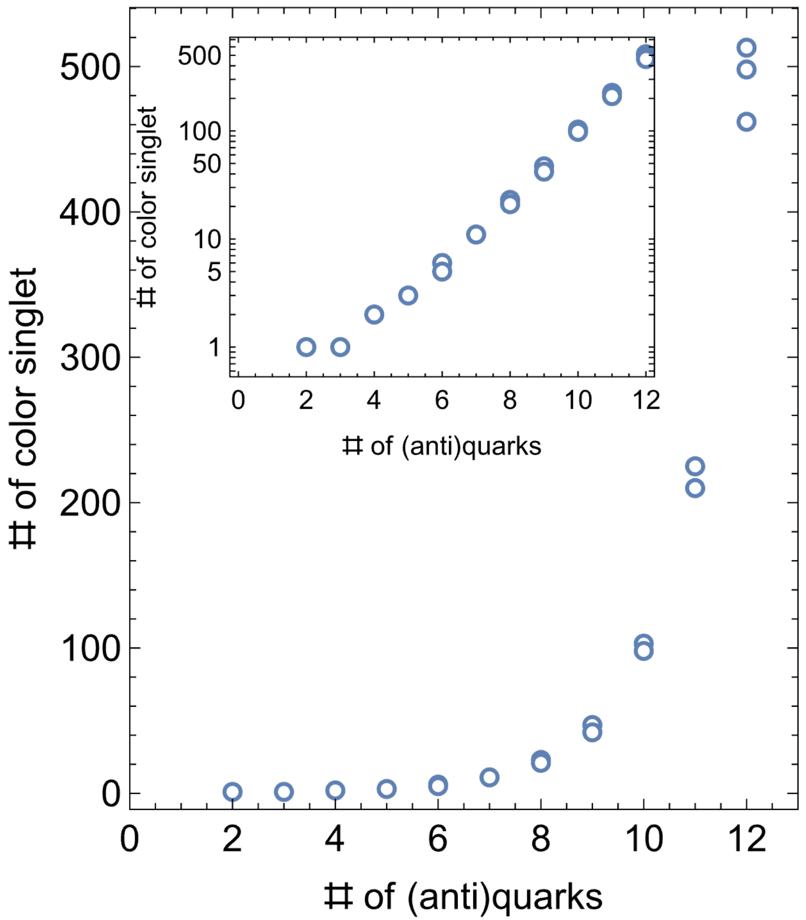
Challenge 1: Spin and color projection

- Conventional spin projection
  - ▶ Option 1： no projection, calculate the ground state
  - ▶ Option 2： penalty terms

$$f_{loss}(\theta) = \langle E\rangle_\theta + \langle S^2\rangle_\theta$$

However, for multiquark systems

- State with higher spin could more interesting
  - ▶ $[cc\bar{q}\,\bar{q}]_{S=0}$: DD threshold; $[cc\bar{q}\,\bar{q}]_{S=1}$: T_{cc} state
- Color projection is needed
  - ▶ color singlet
  - ▶ SU(3) symmetry

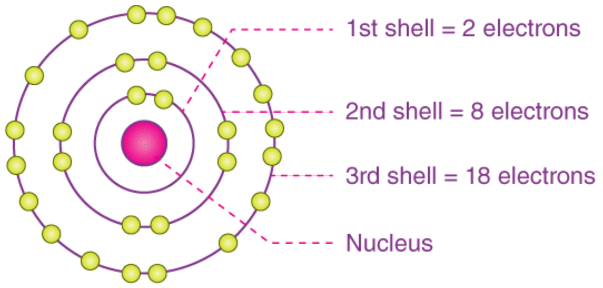


Challenge 2: Strong correlation

- Shell model: good guidance or initial point

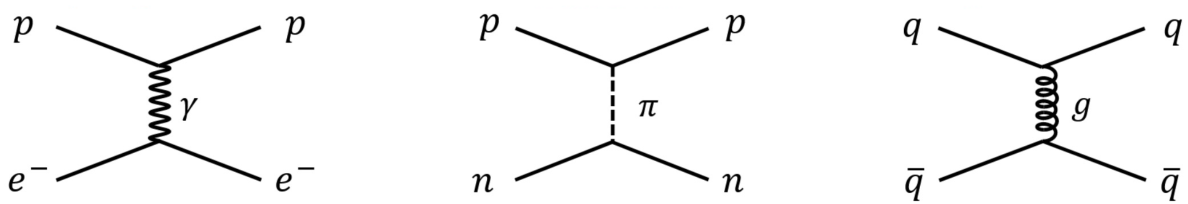
$$\Psi(\mathbf{x}_1,\mathbf{x}_2,\ldots,\mathbf{x}_N)=\frac{1}{\sqrt{N!}}\begin{vmatrix}\chi_1(\mathbf{x}_1) & \chi_2(\mathbf{x}_1) & \cdots & \chi_N(\mathbf{x}_1)\\ \chi_1(\mathbf{x}_2) & \chi_2(\mathbf{x}_2) & \cdots & \chi_N(\mathbf{x}_2)\\ \vdots & \vdots & \ddots & \vdots\\ \chi_1(\mathbf{x}_N) & \chi_2(\mathbf{x}_N) & \cdots & \chi_N(\mathbf{x}_N)\end{vmatrix}$$

For electron and nucleon systems



Electron Shell

- Much stronger interaction among quarks
  - No evidence of a multiquark shell structure
  - Few-body correlation could be important

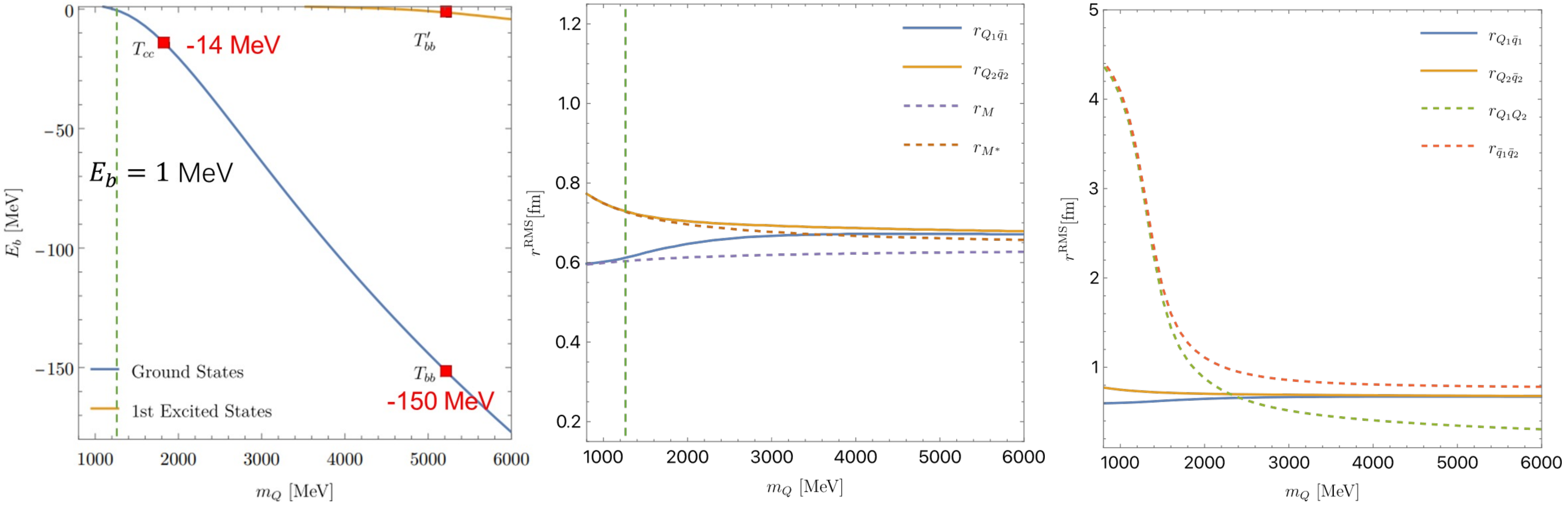


$$e^2=0.1$$
$$m_e\approx 0.5\ \mathrm{MeV}$$

$$\frac{m_\pi^2 g_A^2}{16\pi f_\pi^2}\sim 0.1$$
$$m_N=938\ \mathrm{MeV}$$

$$\frac{4}{3}\alpha_s(m_c)\sim 0.4$$
$$m_c\approx 1273\ \mathrm{MeV}$$

Molecular or compact ?



- Tuning the m_Q to m_b: (bb) compact diquark
- Tuning the m_Q to make $E_b < 1$ MeV: molecular states

Equal mass tetraquark

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- For color-singlet multiquark states $\{Q_1, Q_2, \dots, Q_n\}$, $Q_i = Q$ or $\bar{Q}$
  - ► 2-body interaction:

$$V_{Q_iQ_j} = V_8(r_{ij})\lambda_i \cdot \lambda_j; \qquad V_{\{Q_1,Q_2,\dots,Q_n\}} = \sum_{i<j} a_{ij} V_8(r_{ij}),$$

  - ► Corollary

$$\sum_{i<j} a_{ij} = -\frac{8}{3} n$$

Proof: $2\langle \sum_{i<j} \lambda_i \cdot \lambda_j \rangle = \langle \sum_i \lambda_i \rangle^2 - \sum_i \langle \lambda_i \rangle^2$

- A general question:

For fixed $\sum_{i<j} a_{ij} = C$, what distribution of $\{a_{ij}\}$ give the lowest mass?

- Answer:
  - ► The even case give the worse results: $M_n(a_{ij}) \le M_n^{(S)}$

Proof:

$$H = H^S + \Delta V = H^S + aV_8(r_{12}) - aV_8(r_{34}), \qquad \langle \psi^S | \Delta V | \psi^S \rangle = 0$$

$$\langle \psi^S | H^S | \psi^S \rangle = \langle \psi^S | H | \psi^S \rangle \ge \min_{\psi \in \mathcal{H}} \langle \psi | H | \psi \rangle$$

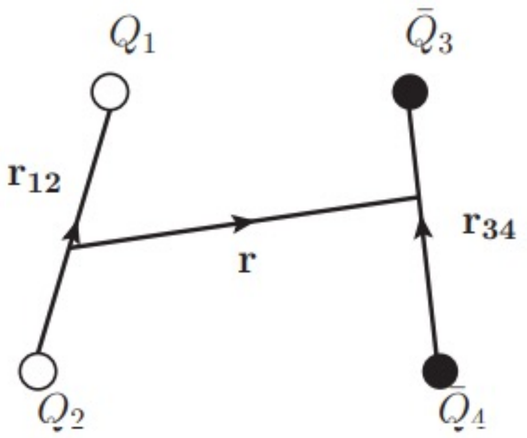
  - ► Intuitively, less even $\{a_{ij}\}$, more deeply bound ground states

J. P. Ader, J. M. Richard, and P. Taxil, Phys. Rev. D **25**, 2370 (1982).

Equal mass tetraquark

- $\{a_{ij}\}$ distribution for $QQ\bar{Q}\bar{Q}$

	$a_{12}=a_{34}$	$a_{13}=a_{24}$	$a_{14}=a_{24}$	Var[$\{a_{ij}\}$]
Dimeson	0	$-\frac{16}{3}$	0	$\frac{1024}{135}$
$\bar{3}_c-3_c$	$-\frac{8}{3}$	$-\frac{4}{3}$	$-\frac{4}{3}$	$\frac{64}{135}$
$\bar{6}_c-6_c$	$\frac{4}{3}$	$-\frac{10}{3}$	$-\frac{10}{3}$	$\frac{784}{135}$



  - ▶ Var[$\{a_{ij}\}$] as a measurement of evenness
  - ▶ $2M(Q\bar{Q}) < M(6_c-\bar{6}_c) < M(3_c-\bar{3}_c)$
  - ▶ Conclusion: Equal mass + color-electric + 2-body V ⇒ no state blow the dimeson threshold.
- Things become different, when
  - ▶ Unequal quark masses, e.g. $QQ\bar{q}\bar{q}$
  - ▶ multi body interaction, e.g. double-Y interaction
  - ▶ …

PRL118, 142001; PRL119, 202001; PRL119, 202002

Computational error in literature

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- Ref.[Ortega:2022efc]: Same SLM interactions
  - ▶ $[bb\bar{q}\bar{q}]^{I=1}$ bound state!
  - ▶ Get a $J^P(I) = 0^+(0)$ state dominated by S-wave BB states!

Our results: there is no isospin vector states

Violating Boson principle

Mass	E_B	$\mathcal{P}_{B^0 B^{*+}}$	$\mathcal{P}_{B^+ B^{*0}}$	$\mathcal{P}_{B^{*+} B^{*0}}$	$\mathcal{P}_{I=0}$	$\mathcal{P}_{I=1}$
10582.2	21.9	47.8	50.0	2.2	99.99	0.01
10593.5	10.5	51.0	48.6	0.4	0.02	99.98

J^P	I	Mass	Width	E_B	$\mathcal{P}_{BB}$	$\mathcal{P}_{B^*B^*}$	Γ_{BB}	$\Gamma_{B^*B^*}$
0^+	0	10553.0	0	6.0	92%	8%	0	0
		10640.7	2.8	8.7	76%	24%	2.8	0
	1	10545.9	0	13.1	93%	7%	0	0
		10672.6	72.0	−23.2	39%	61%	30.7	41.3
2^+	1	10642.3	0	7.1	-	100%	-	0

Computational error in literature

FERMILAB-PUB-23-660-T

Tetraquarks made of sufficiently heavy quarks are bound in QCD

Benoît Assi[1] and Michael L. Wagman[1]

[1] *Fermi National Accelerator Laboratory, Batavia, IL, 60510*

version 1

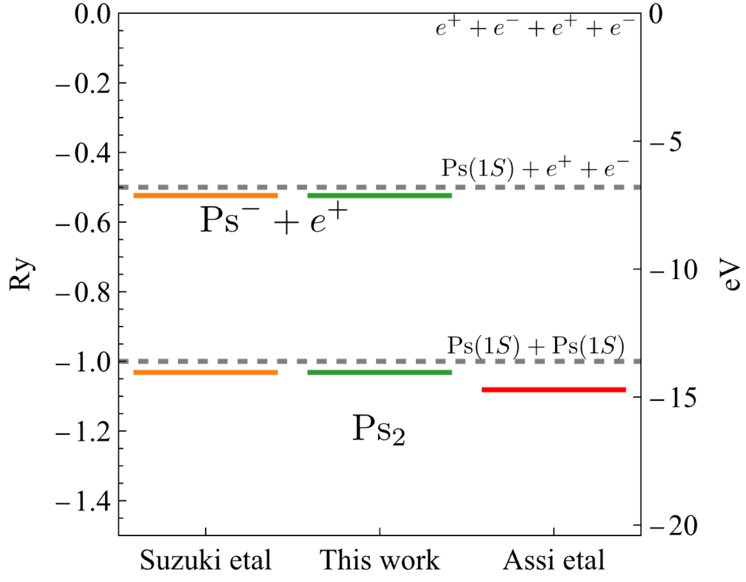
FERMILAB-PUB-23-660-T

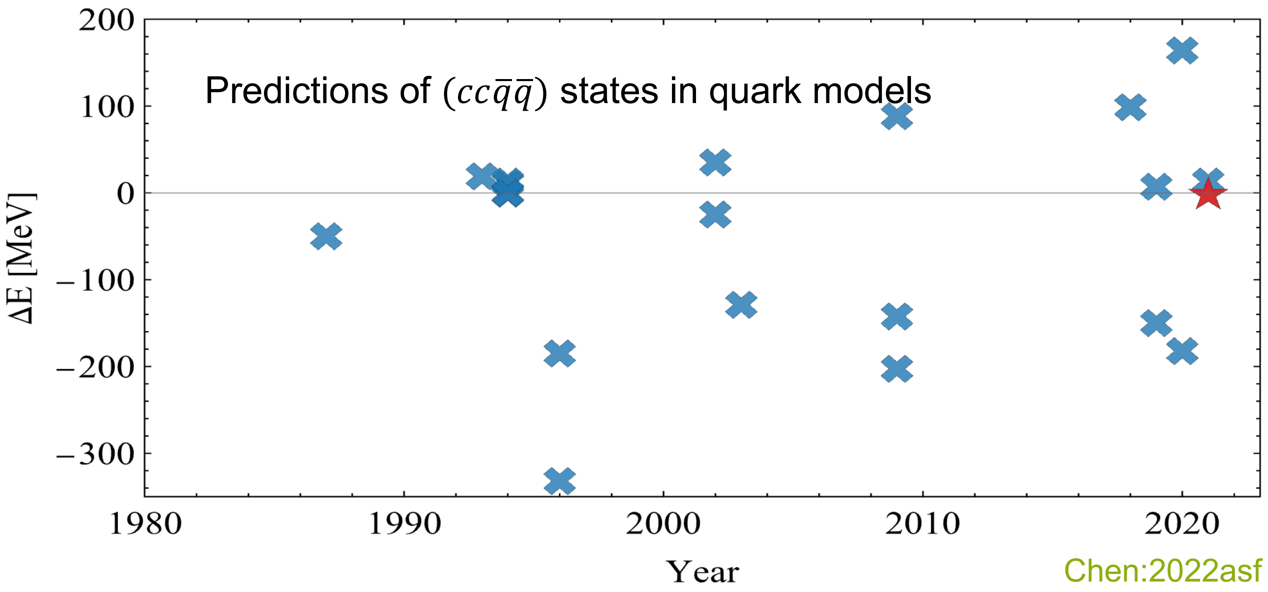
Tetraquarks made of sufficiently unequal-mass heavy quarks are bound in QCD

Benoît Assi[1] and Michael L. Wagman[1]

[1] *Fermi National Accelerator Laboratory, Batavia, IL, 60510*

version 2





Chen:2022asf

Baryons

$$V_{\rm conf}^{\Delta} = \sigma_\Delta \sum_{i<j} r_{ij}$$

$$V_{\rm conf}^{Y} = \sigma_Y L_{min}$$

$$0.5 \leq \frac{L_{min}}{L_\Delta} \leq 0.58$$

Other set

- Activation function: $\sigma(x) = \tanh(x)$
- Optimizer

$$\boldsymbol{\theta}^{i+1} = \boldsymbol{\theta}^{i} - \eta(S + \epsilon I)^{-1} \nabla_{\boldsymbol{\theta}^i} E_{\boldsymbol{\theta}^i}, \tag{8}$$

where i is the iteration step, η is the learning rate, $\epsilon = 10^{-3}$ is taken for numerical stability, and S is the Quantum Fisher information matrix,

$$S_{ab} = \frac{\langle \partial_{\theta_a} \psi_{\boldsymbol{\theta}} | \partial_{\theta_b} \psi_{\boldsymbol{\theta}} \rangle}{\langle \psi_{\boldsymbol{\theta}} | \psi_{\boldsymbol{\theta}} \rangle} - \frac{\langle \partial_{\theta_a} \psi_{\boldsymbol{\theta}} | \psi_{\boldsymbol{\theta}} \rangle}{\langle \psi_{\boldsymbol{\theta}} | \psi_{\boldsymbol{\theta}} \rangle} \frac{\langle \psi_{\boldsymbol{\theta}} | \partial_{\theta_b} \psi_{\boldsymbol{\theta}} \rangle}{\langle \psi_{\boldsymbol{\theta}} | \psi_{\boldsymbol{\theta}} \rangle}. \tag{9}$$

TABLE V. The number of nodes in each hidden layer and the total number of variational parameters in the DNN for different systems.

Systems	S^P	Nodes	Parameters
e^+e^-	0^+	(16, 16, 16, 16)	961
$e^+e^-e^-$	0^+	(16, 16, 16, 16)	1041
$e^+e^+e^-e^-$	0^+	(16, 16, 16, 16)	1137
qqq	$\frac{1}{2}^+$	(16, 16, 16, 16)	1105
$QQ\bar{q}\bar{q}$	1^+	(32, 16, 16, 16)	1889
$QQ\bar{Q}\bar{Q}$	0^+	(32, 16, 16, 16)	1825
$QQ\bar{Q}\bar{Q}$	1^+	(32, 16, 16, 16)	1857
$QQ\bar{Q}\bar{Q}$	2^+	(32, 16, 16, 16)	1793
$QQqq\bar{Q}$	$\frac{1}{2}^-$	(40, 20, 20, 20)	3081
$QQqq\bar{Q}$	$\frac{3}{2}^-$	(40, 20, 20, 20)	3041
$QQqq\bar{Q}$	$\frac{5}{2}^-$	(40, 20, 20, 20)	2921

Related work

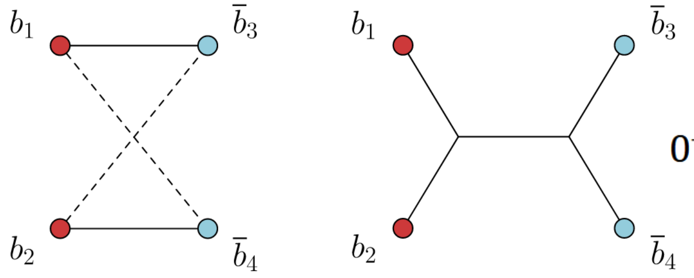


Fig. 1. Left panel: the flip-flop configuration of disconnected di-mesons. Right panel: the butterfly configuration with two connected diquarks. The two middle connecting points are chosen to minimize the total path.

$$V^{4Q} \equiv \min\left(V^{\text{flip-flop}}, V^{\text{butterfly}}\right)$$ + DMC

0^{++} state has a mass of 18.69 ± 0.03 GeV, which is around 100 MeV below twice the η_b mass.

Y. Bai, S. Lu, and J. Osborne, Beauty-full tetraquarks, Phys. Lett. B **798**, 134930 (2019).

$$\begin{aligned} V = {} & \Theta(V_{MM}-V_{YY})\Theta(V_{MM'}-V_{YY})(V_{YY}|\bar{\mathbf{3}}_{12}\mathbf{3}_{34}\rangle\langle\bar{\mathbf{3}}_{12}\mathbf{3}_{34}| + \min(V_{MM},V_{MM'})|\mathbf{6}_{12}\bar{\mathbf{6}}_{34}\rangle\langle\mathbf{6}_{12}\bar{\mathbf{6}}_{34}|) \\ & + \Theta(V_{YY}-V_{MM})\Theta(V_{MM'}-V_{MM})(V_{MM}|\mathbf{1}_{13}\mathbf{1}_{24}\rangle\langle\mathbf{1}_{13}\mathbf{1}_{24}| + \min(V_{YY},V_{MM'})|\mathbf{8}_{13}\mathbf{8}_{24}\rangle\langle\mathbf{8}_{13}\mathbf{8}_{24}|) \\ & + \Theta(V_{YY}-V_{MM'})\Theta(V_{MM}-V_{MM'})(V_{MM'}|\mathbf{1}_{14}\mathbf{1}_{23}\rangle\langle\mathbf{1}_{14}\mathbf{1}_{23}| + \min(V_{YY},V_{MM})|\mathbf{8}_{14}\mathbf{8}_{23}\rangle\langle\mathbf{8}_{14}\mathbf{8}_{23}|), \end{aligned}$$

+ multidimensional numerical integration

+ analytical continuation

$qq\bar{Q}\bar{Q}$ systems

P. Bicudo and M. Cardoso, Phys. Rev. D 94, 094032 (2016).