从氢分子、多轻子体系到双重味、全重味四夸克态
--散射态、束缚态与共振态的统一描述

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从量子力学角度看：多夸克态非常自然

Outline

- Motivation
- Theoretical framework

  -Scattering theory

  -Complex scattering method
- eeee, μμee, μμμμ, ppμμ
- ssss, Tcs(2900), X(6900), X(7100)
- Summary

隐粲五夸克分子态

CPC(HEP & NP), 2012, **36**(1): 6–13 Chinese Physics C Vol. 36, No. 1, Jan., 2012

arXiv: 1105.2901

Possible hidden-charm molecular baryons composed of an anti-charmed meson and a charmed baryon*

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LIU Xiang(刘翔)[2,4;2)] ZHU Shi-Lin(朱世琳)[1;3)]

Abstract: Using the one-boson-exchange model, we studied the possible existence of very loosely bound hidden-charm molecular baryons composed of an anti-charmed meson and a charmed baryon. Our numerical results indicate that the $\Sigma_c\bar{D}^*$ and $\Sigma_c\bar{D}$ states exist, but that the $\Lambda_c\bar{D}$ and $\Lambda_c\bar{D}^*$ molecular states do not.

PRL **105,** 232001 (2010) PHYSICAL REVIEW LETTERS week ending 3 DECEMBER 2010

Prediction of Narrow N^* and Λ^* Resonances with Hidden Charm above 4 GeV

Jia-Jun Wu,[1,2] R. Molina,[2,3] E. Oset,[2,3] and B. S. Zou[1,3]

The interaction between various charmed mesons and charmed baryons is studied within the framework of the coupled-channel unitary approach with the local hidden gauge formalism. Several meson-baryon dynamically generated narrow N^* and Λ^* resonances with hidden charm are predicted with mass above 4 GeV and width smaller than 100 MeV. The predicted new resonances definitely cannot be accommodated by quark models with three constituent quarks and can be looked for in the forthcoming PANDA/FAIR experiments.

PHYSICAL REVIEW D **88,** 114008 (2013)

Coupled-channel analysis of the possible $D^{(*)}D^{(*)}$, $\bar{B}^{(*)}\bar{B}^{(*)}$ and $D^{(*)}\bar{B}^{(*)}$ molecular states

Ning Li,[1,2,*] Zhi-Feng Sun,[3,4,†] Xiang Liu,[3,4,‡] and Shi-Lin Zhu[1,5,6,§]

TABLE IV. The numerical results for the $D^{(*)}D^{(*)}$ system. "$* * *$" means the corresponding state does not exist due to symmetry while "$\cdots$" means there does not exist binding energy with the cutoff parameter less than 3.0 GeV. The binding energies for the states $D^{(*)}D^{(*)}[I(J^P) = 0(1^+)]$ and $D^{(*)}D^{(*)}[I(J^P) = 1(1^+)]$ are relative to the threshold of DD^* while that of the state $D^{(*)}D^{(*)}[I(J^P) = 1(0^+)]$ is relative to the DD threshold.

			$D^{(*)}D^{(*)}$							
I	J^P		OPE				OBE			
	0^+		$* * *$				$* * *$			
		Λ (GeV)	1.05	1.10	1.15	1.20	0.95	1.00	1.05	1.10
		B.E. (MeV)	1.24	4.63	11.02	20.98	0.47	5.44	18.72	42.82
		M (MeV)	3874.61	3871.22	3864.83	3854.87	3875.38	3870.41	3857.13	3833.03
		$r_{\rm rms}$ (fm)	3.11	1.68	1.12	0.84	4.46	1.58	0.91	0.64
0	1^+	P_1 (%)	96.39	92.71	88.22	83.34	97.97	92.94	85.64	77.88
		P_2 (%)	0.73	0.72	0.57	0.42	0.58	0.55	0.32	0.15
		P_3 (%)	2.79	6.45	11.07	16.11	1.41	6.42	13.97	21.91
		P_4 (%)	0.08	0.13	0.14	0.13	0.04	0.09	0.08	0.05

We predicted a **very shallow DD* molecule**, **confirmed by LHCb in July 2021**

PHYSICAL REVIEW D **101**, 034018 (2020)

Spectrum of the strange hidden charm molecular pentaquarks in chiral effective field theory

BO WANG, LU MENG, and SHI-LIN ZHU PHYS. REV. D **101**, 034018 (2020)

TABLE III. The predicted binding energies ΔE and masses M for the $[\Xi_c'\bar{D}^{(*)}]_J$, $[\Xi_c^*\bar{D}^{(*)}]_J$, and $[\Xi_c\bar{D}^{(*)}]_J$ systems in $I=0$ channel, where the subscript "J" denotes the total spin of the system. We correspondingly use the thresholds of $\Xi_c'^+\bar{D}^{(*)0}$, $\Xi_c^{*+}\bar{D}^{(*)0}$, and $\Xi_c^+\bar{D}^{(*)0}$ as the benchmarks to calculate the values in this table (in units of MeV). The state that denoted by "♯" means which may be nonexistent at the upper limit.

System	$[\Xi_c'\bar{D}]_{\frac{1}{2}}$	$[\Xi_c'\bar{D}^*]_{\frac{1}{2}}$	$[\Xi_c'\bar{D}^*]_{\frac{3}{2}}$	$[\Xi_c^*\bar{D}]_{\frac{3}{2}}$	$[\Xi_c^*\bar{D}^*]_{\frac{1}{2}}$	$[\Xi_c^*\bar{D}^*]_{\frac{3}{2}}$	$[\Xi_c^*\bar{D}^*]_{\frac{5}{2}}^{\sharp}$	$[\Xi_c\bar{D}]_{\frac{1}{2}}$	$[\Xi_c\bar{D}^*]_{\frac{1}{2}}$	$[\Xi_c\bar{D}^*]_{\frac{3}{2}}$
ΔE	$-18.5^{+6.4}_{-6.8}$	$-15.6^{+6.4}_{-7.2}$	$-2.0^{+1.8}_{-3.3}$	$-7.5^{+4.2}_{-5.3}$	$-17.0^{+6.7}_{-7.5}$	$-8.0^{+4.5}_{-5.6}$	$-0.7^{+0.7}_{-2.2}$	$-13.3^{+2.8}_{-3.0}$	$-17.8^{+3.2}_{-3.3}$	$-11.8^{+2.8}_{-3.0}$
M	$4423.7^{+6.4}_{-6.8}$	$4568.7^{+6.4}_{-7.2}$	$4582.3^{+1.8}_{-3.3}$	$4502.9^{+4.2}_{-5.3}$	$4635.4^{+6.7}_{-7.5}$	$4644.4^{+4.5}_{-5.6}$	$4651.7^{+0.7}_{-2.2}$	$4319.4^{+2.8}_{-3.0}$	$4456.9^{+3.2}_{-3.3}$	$4463.0^{+2.8}_{-3.0}$

- **We predicted Pcs around 4457MeV with ChEFT**
- **Later, LHCb reported evidence of $P_{cs}(4459)$ in 2020, $P_{cs}(4338)$ in 2022**

PHYSICAL REVIEW D **104**, L051502 (2021)

Letter

Probing the long-range structure of the T_{cc}^{+} with the strong and electromagnetic decays

Lu Meng,[1] Guang-Juan Wang,[2] Bo Wang,[3,4,*] and Shi-Lin Zhu[5,†]

- LHCb first reported Tcc width to be (410±163) keV
- Within the molecular framework, we employed the couple-channel effective field theory and calculated the decay widths of Tcc
- In the isospin symmetry limit, we obtained its total decay width to be ($46.7^{+2.7}_{-2.9}$)keV
- One month later, the LHCb collaboration adopted the unitarized Breit-Wigner distribution and extracted the total width to be (47.8±1.9) keV, which further supports the molecular picture

New hadron states: where are we now?

- 实验发现了一大批新强子态,国内高能实验与理论团队作出了重要贡献
- 强子层次相互作用预言的多个近阈强子分子态如Pc、Tcc、Pcs相继被实验证实

- 郭奉坤、Hanhart、Meissner、王倩、赵强、邹冰松, Hadronic molecules, **Rev. Mod. Phys.** 90, 015004 (2018). **1501 citations**
- Brambilla, Eidelman, Hanhart, Nefediev, 沈振平, Thomas, Vairo,苑长征， The XYZ states: experimental and theoretical status and perspectives， **Phys. Rept. 873**, 1 (2020). **1021 citations**
- 陈华星、陈伟、刘翔、朱世琳, The hidden-charm pentaquark and tetraquark states, **Phys. Rept.** 639, 1 (2016). **1339 citations**
- 刘言锐、陈华星、陈伟、刘翔、朱世琳, Pentaquark and Tetraquark states, **Prog. Part. Nucl. Phys.** 107, 237 (2019). **816 citations** （紧致多夸克态）
- 孟璐、王波、王广娟、朱世琳, Chiral perturbation theory for heavy hadrons and chiral effective field theory for heavy hadronic molecules, **Phys. Rept.** 1019, 1 (2023). **273 citations** （有效场论处理分子态）
- 陈华星、陈伟、刘翔、刘言锐、朱世琳, An updated review of the new hadron states, **Rept. Prog. Phys.** 86, 026201 (2023). **650 citations**
- 陈华星、陈伟、刘翔、刘言锐、朱世琳,A review of the open charm and open bottom systems, **Rept. Prog. Phys**. 80, 076201 (2017). **452 citations**
- Liu, Pan, Liu, Wu, Lu, 耿立升, Three ways to decipher the nature of exotic hadrons: Multiplets, three-body hadronic molecules, and correlation functions,’Phys. Rept. 1108, 1 (2025). **144 citations**

2006年老问题

- 2006年双清论坛，我应邀作新强子态综述报告
- 邝老师和吴岳良追问："能否在同一个理论框架内统一描述这么多新强子态？"
- 我当时回答："这些新强子态内部的相互作用机制千差万别，由于目前色禁闭问题还没有解决，可能很难找到统一的描写方法"
- **一态一法，缺少统一框架**这问题一直困扰着我
- 课题组用过理论工具（QSR、QM、LS方程、AGS方程、OBE、RGM、EFT、ChPT、CHEFT）均有局限性

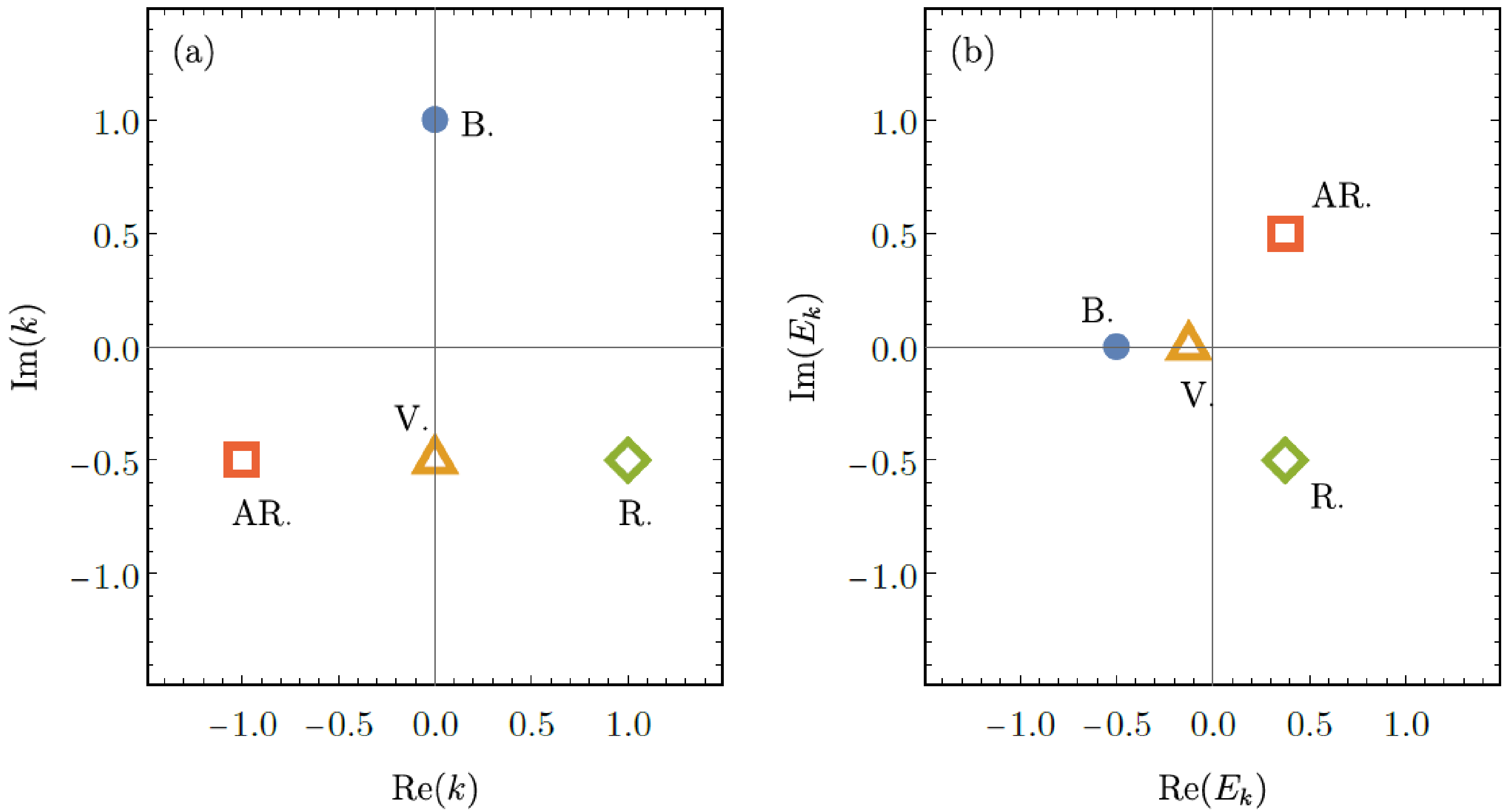
- 原子分子物理、核物理跨学科的启发：→找到了一大批新强子态的统一描写方法 ←：复标度变换+夸克模型

QED *vs* QCD

- QED: 包含库伦势的哈密顿量可以统一描述：
  - ➢ 自由电子、自由质子
  - ➢ 电子与质子散射
  - ➢ 电子与质子形成氢原子
  - ➢ 两个氢原子通过共价键形成氢分子…
- QCD：**应该能够统一描写：**
  - ✓ 正反粲夸克形成J/psi
  - ✓ J/psi J/psi散射
  - ✓ J/psi J/psi形成X(6900)共振态
- **夸克强子对偶性要求：夸克胶子层次上的统一描述能够再现强子层次上的各种奇特强子态**

束缚态、共振态和虚态

(根据极点在复平面上位置分类)



散射波函数在无穷远处渐近形式

$$\psi(r) \xrightarrow{r\to\infty} f_l^+(k)e^{-ikr} + f_l^-(k)e^{ikr}, \tag{2.26}$$

这是由于我们假定了无穷远处势能比$1/r^2$更快趋于0（所以对于库伦势，渐近形式需要改写）。其中，$f_l^+(k)$和$f_l^-(k)$称为Jost函数，其为散射动量k的解析函数，满足$f_l^+(k) = (f_l^-(k^*))^*$，关于其解析性质可参考文献[10]。Jost函数与分波S矩阵的关系为

$$s_(k) = \frac{f_l^-(k)}{f_l^+(k)}, \tag{2.27}$$

S矩阵的极点对应Jost函数$f_l^+(k)$的零点，第一项消失，对应的波函数在无穷远的渐近形式为$f_l^-(k)e^{ikr}$。对于束缚态，$k = i\kappa$，其中κ为正实数，所以波函数在无穷远处按$e^{-\kappa r}$收敛，因此平方可积。对于共振态，$k = \kappa_1 - i\kappa_2$，因此发散。

求解多夸克态传统方法的缺陷

- 利用高斯波函数（或其它束缚态波函数）构建多体波函数，通过变分法或把哈密顿量对角化求解本征值
- 多体哈密顿量是厄密算符，本征值为实数，本征波函数完备集为束缚态或散射态
- 多夸克态计算中，只能选有限个高斯基波函数(N～10)。有限个束缚态波函数的线性叠加仍然是束缚态波函数
- 传统方法求解得到的本征态都是离散的束缚态，更糟糕的是散射态被错误处理成束缚态，相应的能量连续谱被处理成离散谱
- 传统理论框架给出的解包含了大量非物理的态（散射态冒充束缚态）
- 大部分多夸克态是共振态，其能量本征值包含虚部，波函数不是平方可积的
- 需要寻找新的理论框架处理多夸克态体系

为何这问题以前没出现？

- ρ(770)是ππ的P波共振态，ψ(3770)是DDbar的P波共振态，Delta(1230)是Nπ P波共振态
- 为何之前夸克模型处理ρ, ψ(3770), Delta(1230)没碰到问题？
- ρ(770)和ψ (3770)实质上是qqbar态， Delta是qqq态，颜色结构唯一。色禁闭保证它们都是束缚态
- 对于多夸克态，颜色结构不唯一。对于X(6900)，有3*3, 6*6 (或1*1, 8*8)两种颜色组态
- J/ψ J/ψ → X(6900)散射过程、X(6900)→J/ψ J/ψ 衰变过程粒子数守恒（非相对论近似下）
- pipi→rho散射过程、 rho → pipi衰变过程涉及轻夸克对湮灭、产生，粒子数变化（非相对论夸克模型）

X(6900)包含J/ψ J/ψ 成分，不是颜色空间的禁闭态，是共振态

$$
\begin{aligned}
&|(Q_1Q_2)_{\bar{3}_c}(\bar{Q}_3\bar{Q}_4)_{3_c}\rangle \\
&= \sqrt{\frac{1}{3}}|(Q_1\bar{Q}_3)_{1_c}(Q_2\bar{Q}_4)_{1_c}\rangle - \sqrt{\frac{2}{3}}|(Q_1\bar{Q}_3)_{8_c}(Q_2\bar{Q}_4)_{8_c}\rangle \\
&= -\sqrt{\frac{1}{3}}|(Q_1\bar{Q}_4)_{1_c}(Q_2\bar{Q}_3)_{1_c}\rangle + \sqrt{\frac{2}{3}}|(Q_1\bar{Q}_4)_{8_c}(Q_2\bar{Q}_3)_{8_c}\rangle, \\
&|(Q_1Q_2)_{6_c}(\bar{Q}_3\bar{Q}_4)_{\bar{6}_c}\rangle \\
&= \sqrt{\frac{2}{3}}|(Q_1\bar{Q}_3)_{1_c}(Q_2\bar{Q}_4)_{1_c}\rangle + \sqrt{\frac{1}{3}}|(Q_1\bar{Q}_3)_{8_c}(Q_2\bar{Q}_4)_{8_c}\rangle \\
&= \sqrt{\frac{2}{3}}|(Q_1\bar{Q}_4)_{1_c}(Q_2\bar{Q}_3)_{1_c}\rangle + \sqrt{\frac{1}{3}}|(Q_1\bar{Q}_4)_{8_c}(Q_2\bar{Q}_3)_{8_c}\rangle.
\end{aligned}
$$

Complex scaling method (CSM)

A method to obtain energies and wave functions of bound states and resonances.

- In CSM, the coordinate r and its conjugate momentum p are transformed as

$$U(\theta)\boldsymbol{r} = \boldsymbol{r}e^{i\theta}, \quad U(\theta)\boldsymbol{p} = \boldsymbol{p}e^{-i\theta}$$

- The complex-scaled Hamiltonian

$$H(\theta) = \sum_{i=1}^{4}\left(m_i + \frac{p_i^2 e^{-2i\theta}}{2m_i}\right) + \sum_{i<j=1}^{4} V_{ij}\left(r_{ij}e^{i\theta}\right)$$

no longer hermitian, has complex eigenvalues

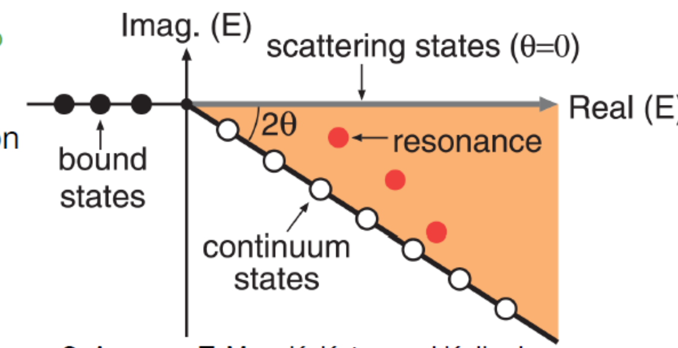
- The properties of solutions of the complex-scaled Schrödinger equation (the ABC theorem):

Aguilar:1971ve, Balslev:1971vb

Bound state: not change by scaling

Resonance $E_R = M_R - i\Gamma_R/2 \xrightarrow[2\theta > |\mathrm{Arg}(E_R)|]{\boldsymbol{r} \to \boldsymbol{r}e^{i\theta}}$ square-integrable function

Continuum spectra: start at the threshold, rotate clockwise by 2θ



S. Aoyama, T. Myo, K. Kat.o, and K. Ikeda, Progress of theoretical physics 116, 1 (2006)

- CSM was advocated to derive resonances in many-body systems.

B. Simon, Communications in Mathematical Physics, 27(1): 1–9 (1972)

为何CSM方法能够求解共振态？

$$\tilde{\psi}(r) \xrightarrow{r\to\infty} f_l^+(k)e^{-ikre^{i\theta}} + f_l^-(k)e^{ikre^{i\theta}}. \tag{2.28}$$

波函数在无穷远的渐近形式为$e^{i(\kappa_1\cos\theta+\kappa_2\sin\theta)r+(\kappa_2\cos\theta-\kappa_1\sin\theta)r}$，在$\kappa_2\cos\theta-\kappa_1\sin\theta<0$即$\theta>\mathrm{Arg}(k)$时，波函数平方可积。

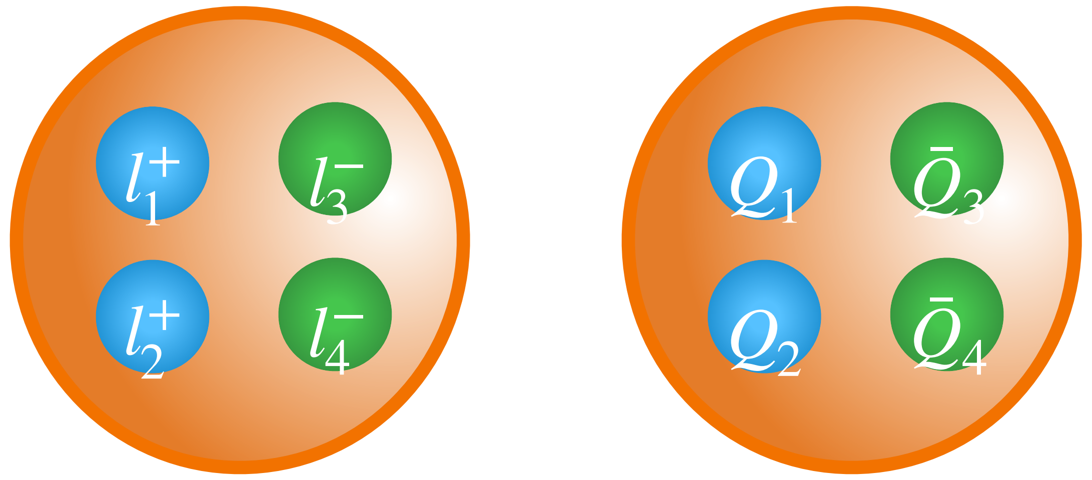
对于不对应极点的连续态，方程（2.28）中两项均不消失，同时不发散要求$\theta=\mathrm{Arg}(k)$，因此能解出的连续态分布在$\mathrm{Arg}(k)=\theta$或$\mathrm{Arg}(E)=2\theta$的直线上。

- 束缚态、共振态不随转角变化
- 束缚态在负实轴上，共振态在下半平面
- 连续态随转角变化，分布在Arg(k)=theta直线上
- 非常方便辨认区分束缚态、共振态与散射态
- 同一框架同时描写束缚态、共振态与散射态

为何之前高能同事们不采用CSM？2003年之前，高能实验发现强子包括大量共振态，在夸克胶子层次上都是束缚态，完全用不着利用CSM处理。这些共振态通过真空产生轻夸克对来强衰变（非相对论夸克模型，粒子数目变化）

Introduction

Charged leptonic systems



- primarily governed by QED, simplicity
- Share similarities with tetraquark states——QED counterpart of tetraquark state
  - 4-lepton and tetraquark systems: 2 particles, 2 antiparticles
  - Heavy tetraquark systems: dominated by color-electric Coulomb interaction
- Experiment
  - Di-positronium (Ps_2) has been observed by experiment
  - 3- and 4-lepton states with muon may be detectable by future experiments
- Previous works
  - Mainly focus on bound states
  - Resonant states studied: $e^+e^+e^-$, $e^+e^+e^-e^-$, $e^+e^+\mu^-$ Ho:1979zz, ho1989resonant, liverts2013three
  - To be explored: $\mu^+e^+e^-$, $\mu^+\mu^+e^-$, $\mu^+e^+\mu^-$, $\mu^+\mu^+e^-e^-$, $\mu^+e^+\mu^-e^-$,……

Hamiltonian

Hamiltonian of an n-body system

$$H = \sum_i^n \left(m_i + \frac{\boldsymbol{p}_i^2}{2m_i} \right) + \sum_{i<j=1}^n V_{ij}$$

QED Coulomb potential

$$V_{ij}(r) = \frac{Q_i Q_j}{r_{ij}}$$

Without including spin-dependent interaction

⇒ No coupling between spin channels

⇒ Spin and C-parity degenerate

⇒ Threshold degenerate

TABLE I. The exact binding energies $\Delta E_{\mathrm{exact}}$, calculated binding energies ΔE_{calc} and rms radii $r^{\mathrm{rms}}_{\mathrm{calc}}$ of the $l^+ l^{(\prime)-}$ systems.

J^{PC}	System	$\Delta E_{\mathrm{exact}}$	ΔE_{calc}	$r^{\mathrm{rms}}_{\mathrm{calc}}$
$0^{-+}/1^{--}$	Ps(1S)	-6.80 eV	-6.80 eV	0.18 nm
	Ps(2S)	-1.70 eV	-1.70 eV	0.69 nm
	Ps(3S)	-0.76 eV	-0.76 eV	1.52 nm
	$\mu^+ e^- (1S)$	-13.5 eV	-13.5 eV	0.09 nm
	$\mu^+ e^- (2S)$	-3.4 eV	-3.4 eV	0.34 nm
	$\mu^+ e^- (3S)$	-1.5 eV	-1.5 eV	0.77 nm
	$\mu^+ \mu^- (1S)$	-1.41 keV	-1.41 keV	0.9 pm
	$\mu^+ \mu^- (2S)$	-0.35 keV	-0.35 keV	3.3 pm
	$\mu^+ \mu^- (3S)$	-0.16 keV	-0.16 keV	7.4 pm

Wave function construction

$$\psi = \mathcal{A}\left(\phi \otimes \chi_s\right)$$

- Spatial wave function:

$$\phi_{nlm}(\boldsymbol{r}) = \sqrt{\frac{2^{l+5/2}}{\Gamma\left(l+\frac{3}{2}\right) r_n^3}} \left(\frac{r}{r_n}\right)^l e^{-\frac{r^2}{r_n^2}} Y_{lm}(\hat{r})$$

Only S-wave is considered.

- Spin wave function:

  - $3-\text{body}:$

$$S=\frac{1}{2}: \begin{cases} \left[\left(l_1^+ l_2^+\right)_0 l_3^-\right]_{\frac{1}{2}} \\ \left[\left(l_1^+ l_2^+\right)_1 l_3^-\right]_{\frac{1}{2}} \end{cases},$$

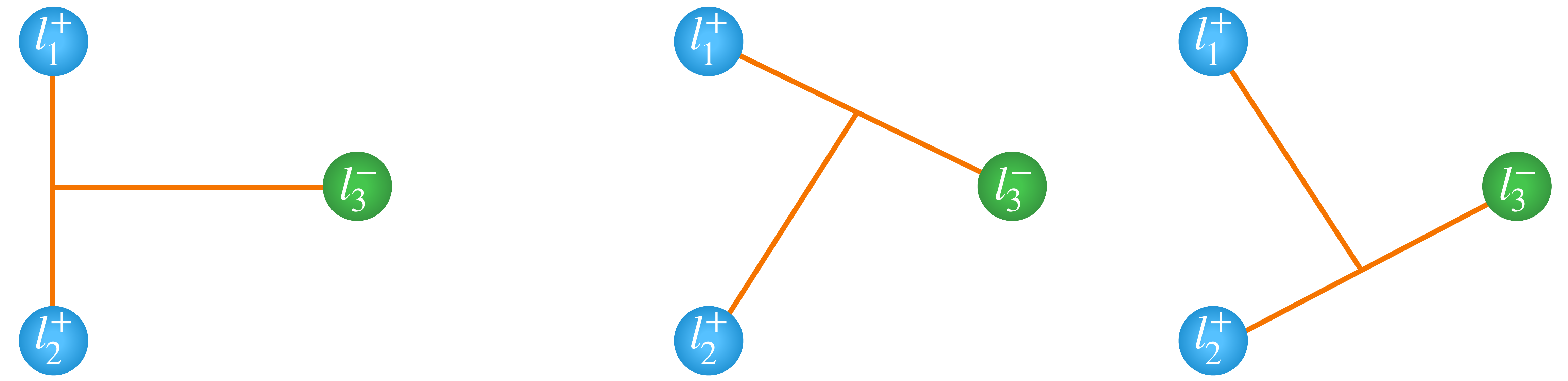
$$S=\frac{3}{2}: \left[\left(l_1^+ l_2^+\right)_1 l_3^-\right]_{\frac{3}{2}}.$$

  - $4-\text{body}:$

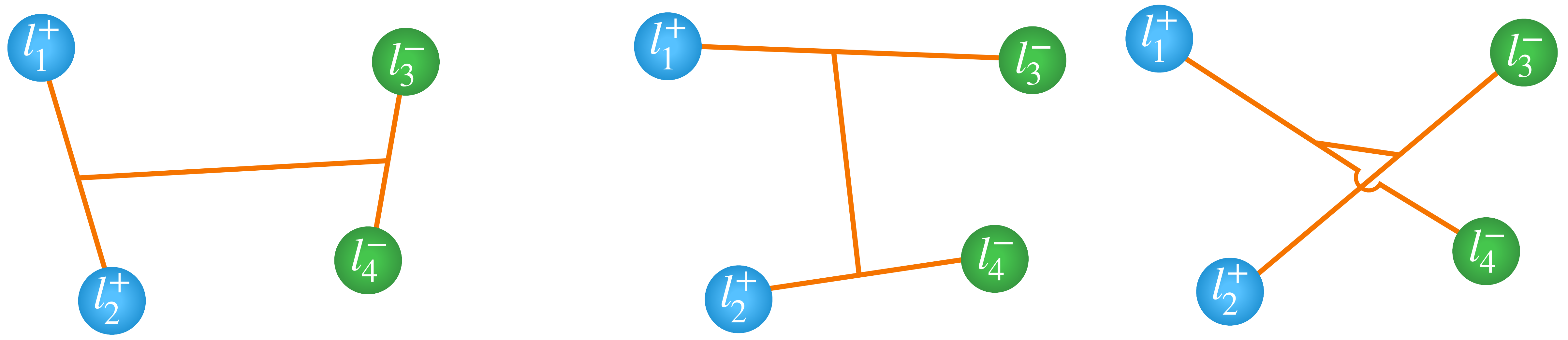
$$S=0: \begin{cases} \left[\left(l_1^+ l_2^+\right)_0 \left(l_3^- l_4^-\right)_0\right]_0 \\ \left[\left(l_1^+ l_2^+\right)_1 \left(l_3^- l_4^-\right)_1\right]_0 \end{cases},$$

$$S=1: \begin{cases} \left[\left(l_1^+ l_2^+\right)_0 \left(l_3^- l_4^-\right)_1\right]_1 \\ \left[\left(l_1^+ l_2^+\right)_1 \left(l_3^- l_4^-\right)_0\right]_1 \\ \left[\left(l_1^+ l_2^+\right)_1 \left(l_3^- l_4^-\right)_1\right]_1 \end{cases},$$

$$S=2: \left[\left(l_1^+ l_2^+\right)_1 \left(l_3^- l_4^-\right)_1\right]_2.$$



(a) (b)



(a) (b)

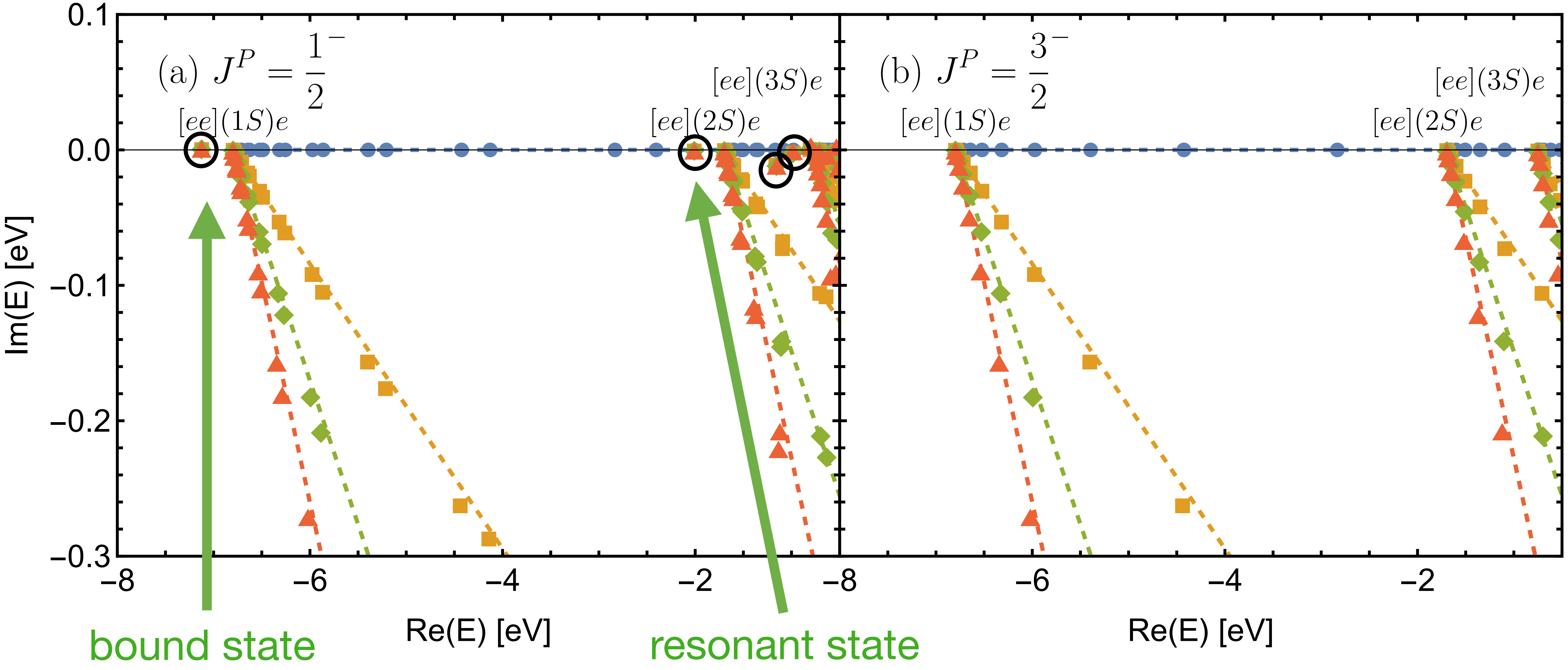
- $\mu^+\mu^+e^-e^-$:

Gaussian bases parameters e.g.

$$\begin{cases} \mu^+-\mu^+: & r_n \in [0.0001, 0.01]\ \text{nm},\ n=12 \\ e^--e^-: & r_n \in [0.01, 1.06]\ \text{nm},\ n=12 \\ (\mu^+\mu^+)-(e^-e^-): & r_n \in [0.025, 0.7]\ \text{nm},\ n=12 \\ \mu^+-e^-: & r_n \in [0.01, 1.06]\ \text{nm},\ n=12 \\ (\mu^+e^-)-(\mu^+e^-): & r_n \in [0.025, 0.7]\ \text{nm},\ n=30 \end{cases}$$

3-lepton system

$e^+e^+e^-$



- 1 bound state
- 3 resonant states
- Only S=1/2 system have bound and resonant states.

system	$\Delta E - i\Gamma/2$	type	component	$r_{l^+l^+}$	$r_{l^+l^{(\prime)-}}$
$e^+e^+e^-$	-7.12	B	$[0, \frac{1}{2}]_{\frac{1}{2}}$	0.51	0.37
	-2.01	R	$[0, \frac{1}{2}]_{\frac{1}{2}}$	1.32	0.83
	$-1.16 - 0.01i$	R	$[0, \frac{1}{2}]_{\frac{1}{2}}$	1.85	1.31
	-0.98	R	$[0, \frac{1}{2}]_{\frac{1}{2}}$	2.62	1.61

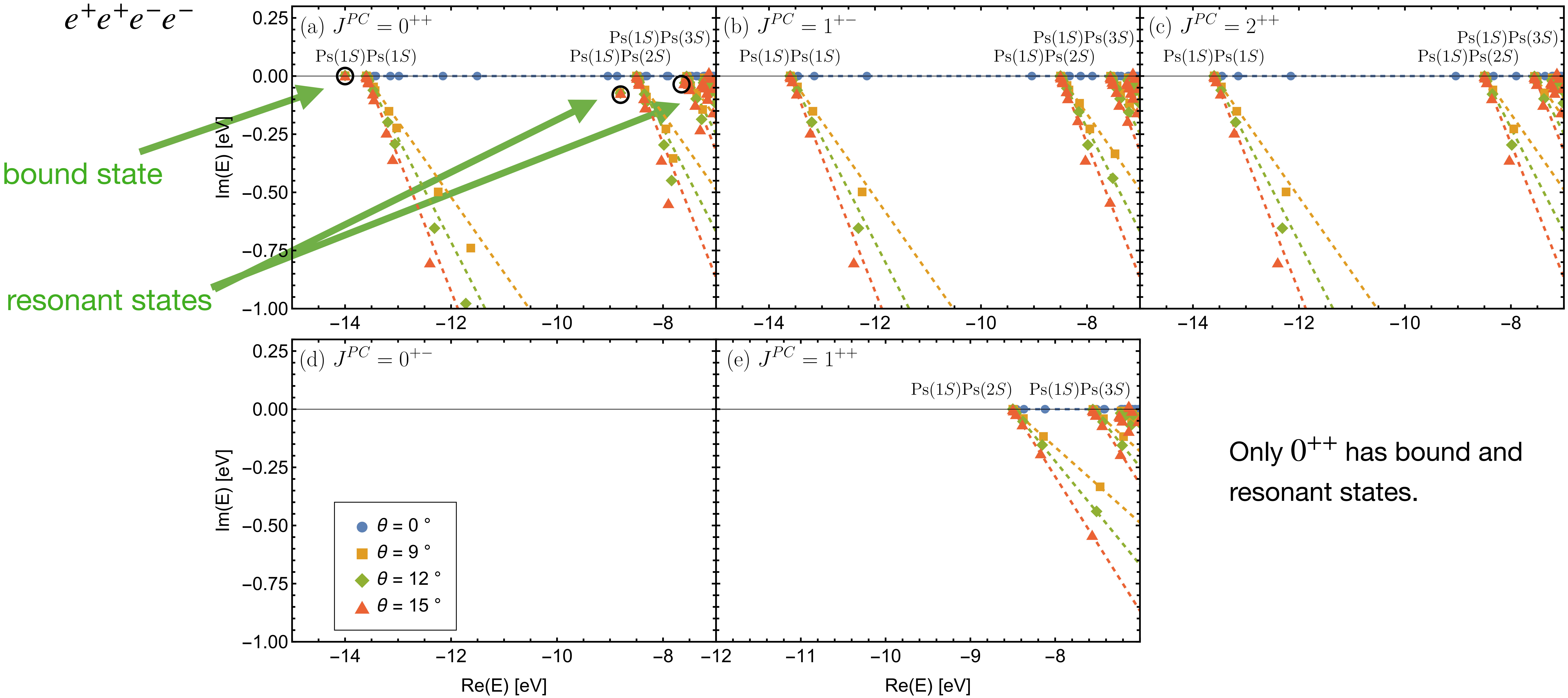
covalent one-electron bond structure

$[s_{12}, s_3]_S = \left[0, \frac{1}{2}\right]_{\frac{1}{2}}$: spin anti-aligned

4-lepton system

- 4-lepton systems with bound state or resonant state solutions

$e^+e^+e^-e^-$



Only 0^{++} has bound and resonant states.

4-lepton system

- 4-lepton systems with bound state or resonant state solutions

$e^+e^+e^-e^-$

J^{PC}	$\Delta E - i\Gamma/2$	type	component	$r_{e^+e^+} = r_{e^-e^-}$	$r_{e^+e^-}$	$\Delta E'$
0^{++}	-14.00	B	$[0,0]_0$	0.37	0.29	-0.40
	$-8.80 - 0.07i$	R	$[0,0]_0$	0.63	0.56	...
	$-7.59 - 0.03i$	R	$[0,0]_0$	1.30	1.16	...

$[(e^+e^+)_0(e^-e^-)_0]_{S=0}$ spin configuration

$\Rightarrow$ S-wave in spatial wave function

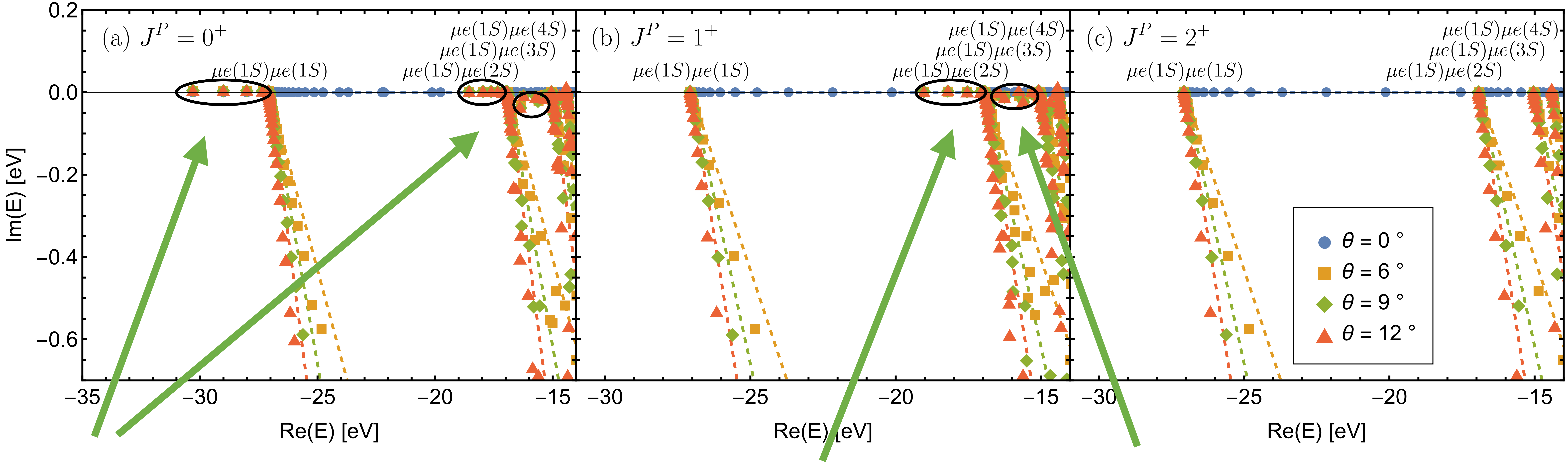
$\Rightarrow$ lower energy

even distribution

4-lepton system

- 4-lepton systems with bound state or resonant state solutions

$\mu^+\mu^+e^-e^-$



$[(\mu^+\mu^+)_0(e^-e^-)_0]_{S=0}$

$[(\mu^+\mu^+)_0(e^-e^-)_1]_{S=1}$
or $[(\mu^+\mu^+)_1(e^-e^-)_0]_{S=1}$

$[(\mu^+\mu^+)_0(e^-e^-)_1]_{S=1}$

- S=2 system: pure $[(\mu^+\mu^+)_1(e^-e^-)_1]_{S=2}$ component, higher wave, higher energy, more difficult to form bound states and resonant states.

4-lepton system

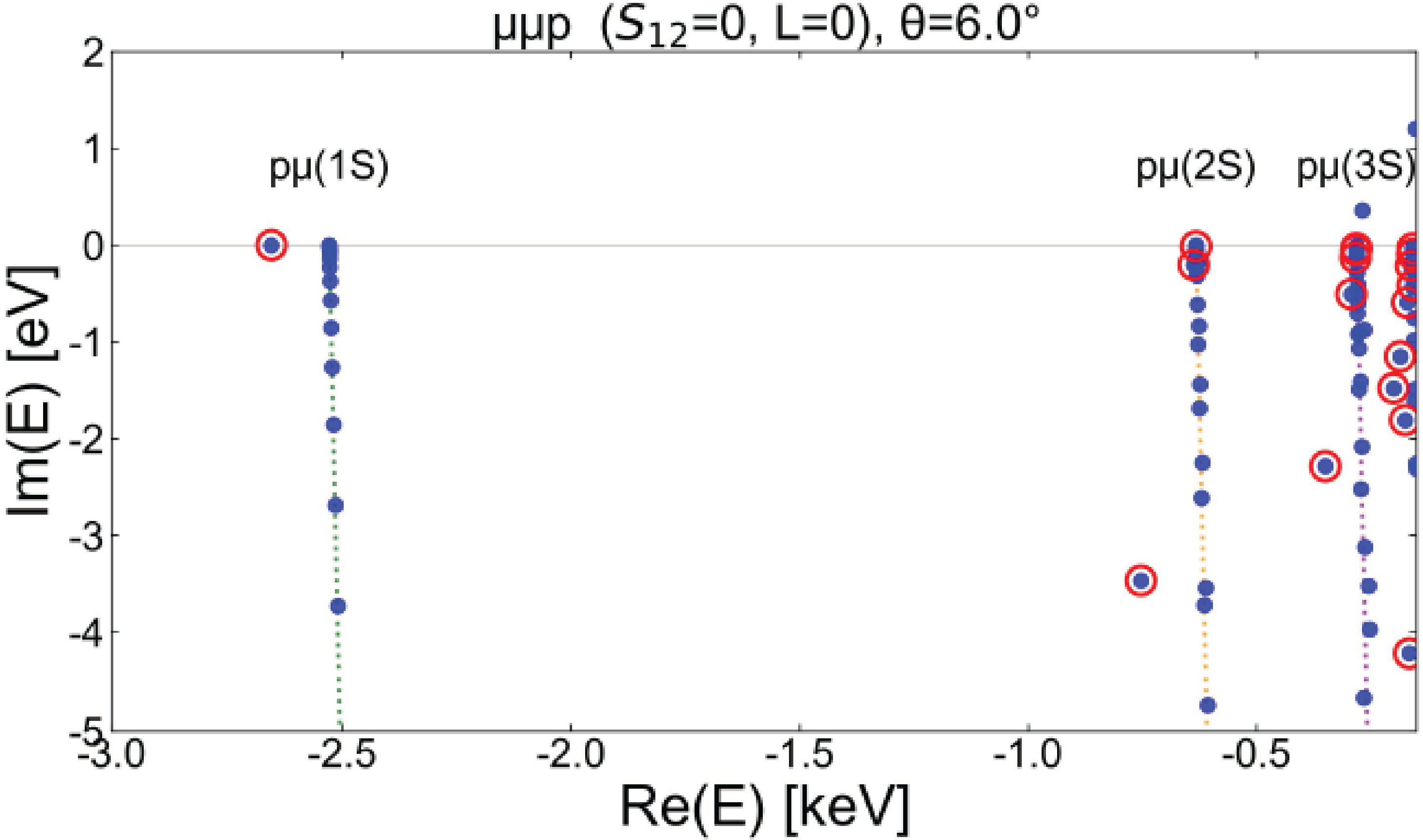
$\mu^+\mu^+e^-e^-$

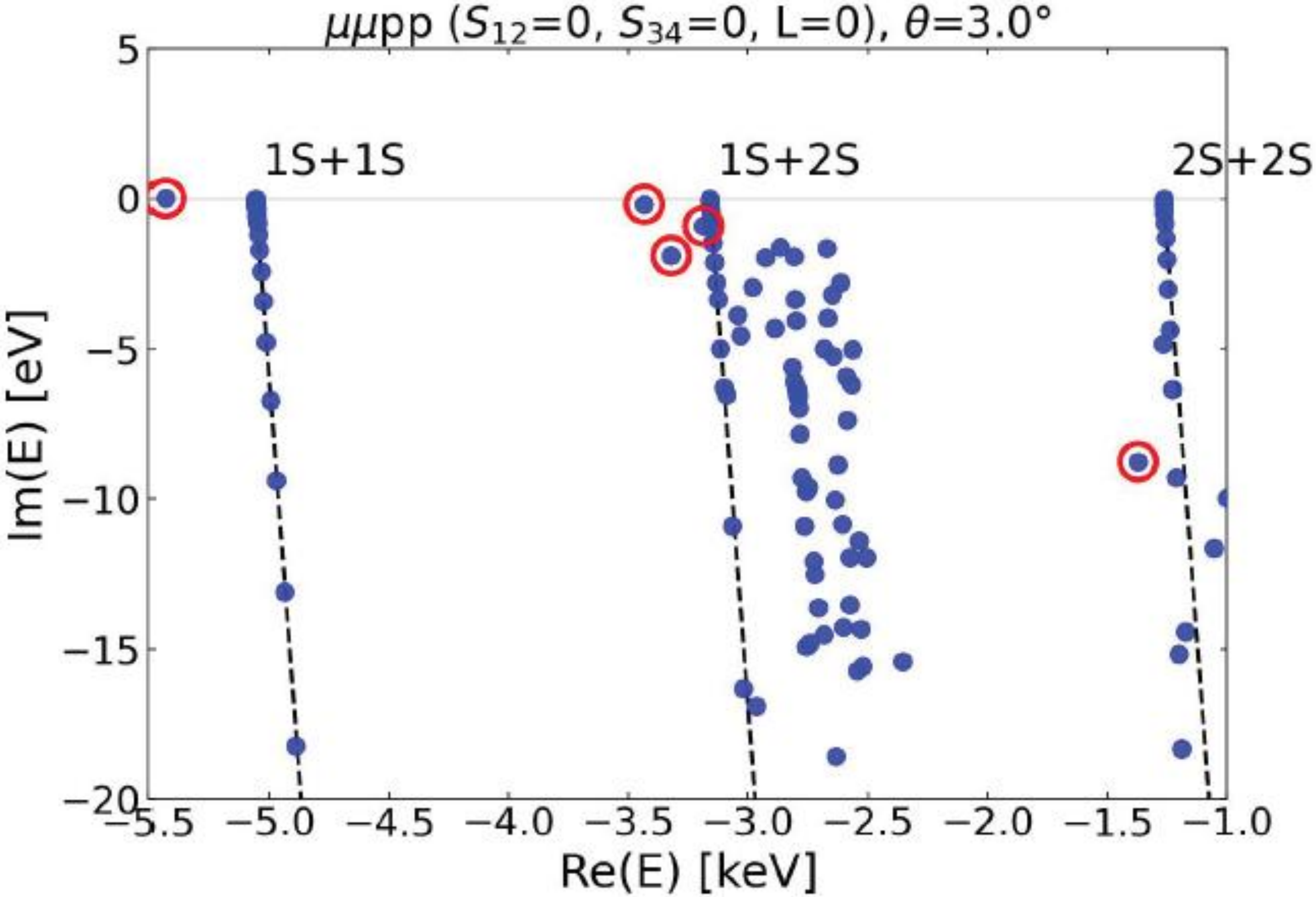
J^P	$\Delta E - i\Gamma/2$	type	component	$r_{\mu^+\mu^+}$	$r_{e^-e^-}$	$r_{\mu^+e^-}$
0^+	-30.30	B	$[0,0]_0$	0.08	0.14	0.10
	-29.01	B	$[0,0]_0$	0.11	0.16	0.11
	-28.01	B	$[0,0]_0$	0.13	0.18	0.13
	-27.34	B	$[0,0]_0$	0.18	0.22	0.16
	-18.55	R	$[0,0]_0$	0.12	0.41	0.29
	-17.96	R	$[0,0]_0$	0.16	0.41	0.29
	-17.61	R	$[0,0]_0$	0.21	0.41	0.30
	-17.34	R	$[0,0]_0$	0.26	0.44	0.31
	-17.12	R	$[0,0]_0$	0.32	0.48	0.34
	-16.98	R	$[0,0]_0$	0.41	0.56	0.40
	$-16.33-0.03i$	R	$[0,0]_0$	0.12	1.27	0.90
	$-16.22-0.01i$	R	$[0,0]_0$	0.15	0.91	0.65
	$-15.72-0.01i$	R	$[0,0]_0$	0.18	0.86	0.61
	$-15.60-0.02i$	R	$[0,0]_0$	0.16	1.30	0.92
	$-15.33-0.01i$	R	$[0,0]_0$	0.24	0.85	0.60
1^+	-18.20	R	$[0,1]_1$	0.13	0.39	0.27
	-17.53	R	$[0,1]_1$	0.16	0.41	0.29
	-17.07	R	$[1,0]_1$	0.35	0.48	0.34
	-17.04	R	$[0,1]_1$	0.16	0.66	0.47
	-17.03	R	$[0,1]_1$	0.17	0.64	0.45
	-16.95	R	$[1,0]_1$	0.46	0.58	0.41
	$-16.42-0.01i$	R	$[0,1]_1$	0.12	1.22	0.87
	$-16.95-0.01i$	R	$[0,1]_1$	0.14	0.95	0.68
	$-15.75-0.01i$	R	$[0,1]_1$	0.17	1.00	0.72
	$-15.58-0.02i$	R	$[0,1]_1$	0.17	1.18	0.84
	-15.30	R	$[0,1]_1$	0.23	0.88	0.63

Structure: covalent bond like hydrogen molecule, electron pairs shared by two muons

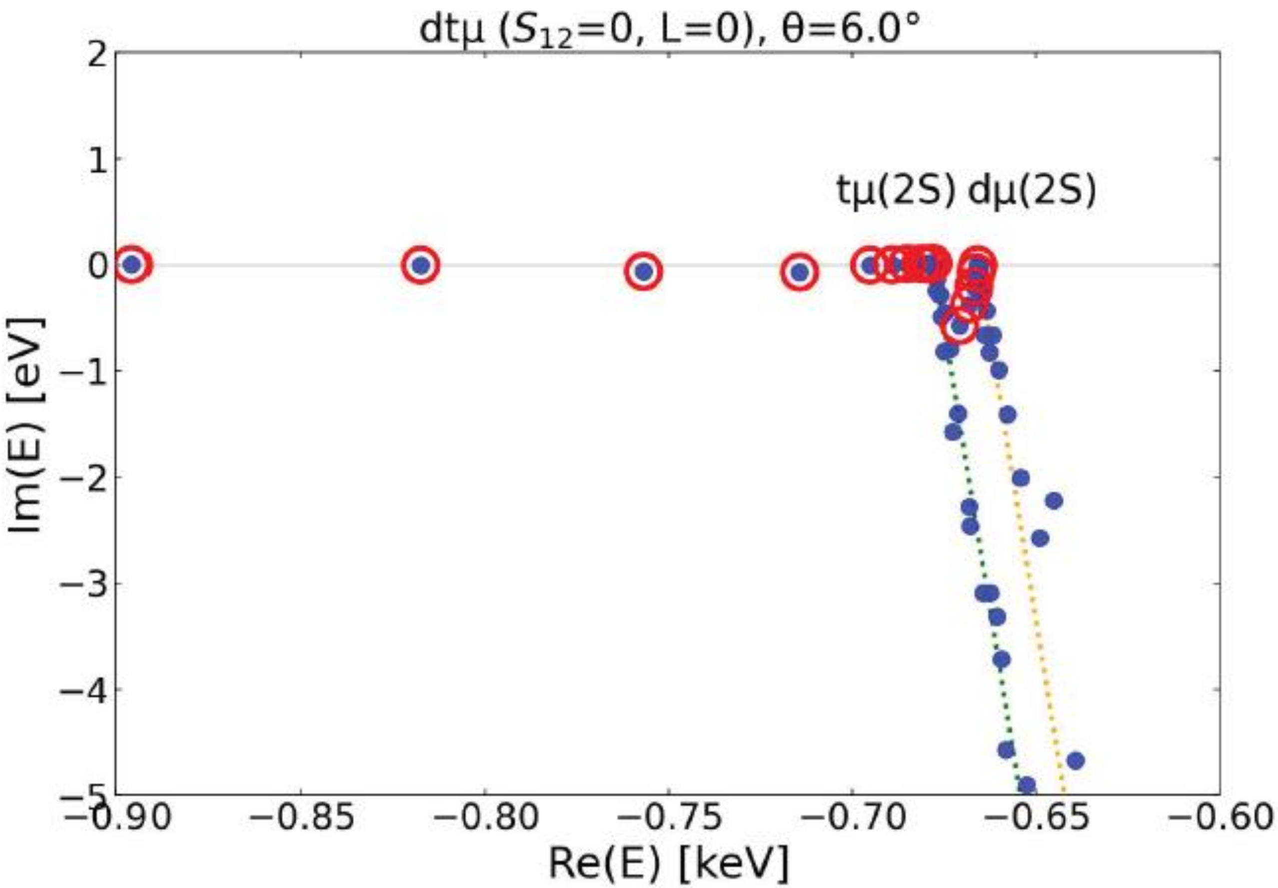
$[s_{12}, s_{34}]_S = [0,0]_0$: spin anti-aligned

$[s_{12}, s_{34}]_S = [0,1]_1$ or $[s_{12}, s_{34}]_S = [1,0]_1$: at least one spin anti-aligned pair


μμp (S12=0, L=0), θ=6.0°
pμ(1S)
pμ(2S)
pμ(3S)
Im(E) [eV]
Re(E) [keV]
2
1
0
-1
-2
-3
-4
-5
-3.0
-2.5
-2.0
-1.5
-1.0
-0.5



μμpp (S_{12}=0, S_{34}=0, L=0), θ=3.0°
1S+1S
1S+2S
2S+2S
Im(E) [eV]
Re(E) [keV]
5
0
−5
−10
−15
−20
−5.5
−5.0
−4.5
−4.0
−3.5
−3.0
−2.5
−2.0
−1.5
−1.0



dtμ (S_{12}=0, L=0), θ=6.0°
tμ(2S) dμ(2S)
Im(E) [eV]
Re(E) [keV]
2
1
0
−1
−2
−3
−4
−5
−0.90
−0.85
−0.80
−0.75
−0.70
−0.65
−0.60

TABLE IV. The binding energies and rms radii of $dt\mu$ are presented below. Benchmark results from previous calculations are listed in the last column for comparison. Newly identified states are marked with a dagger (†).

L	E [eV]	r_{dt}^{rms} [pm]	$r_{d\mu}^{rms}$ [pm]	$r_{t\mu}^{rms}$ [pm]	E_{bench} [eV]
0	-3030.38	0.74	0.62	0.59	-3030.38 [23]
0	-2746.08	1.42	1.21	0.88	-2746.08 [23]
1	-2943.71	0.41	0.49	0.47	-2943.70 [24]
1	-2711.90	1.48	1.50	0.55	-2711.90 [24]
$\mathbf{t\mu(n=1)}$	-2711.24				
$\mathbf{d\mu(n=1)}$	-2663.20				
0	-895.70	2.6	1.8	1.7	
0	-817.54	3.2	2.1	2.1	
0	-756.92	3.9	2.6	2.4	
0	-714.41	5.1	3.7	2.7	
0	-695.26	6.9	5.8	2.7	
0	-689.23	7.1	4.9	4.5	
0	-685.04	10.2	9.2	2.9	
0	-681.38	14.6	13.8	2.5	
0	-679.54	21.0	20.2	2.4	
0†	-678.64	30.6	29.8	2.3	
0†	-678.20	48.3	47.6	2.0	
1	-890.35	1.3	1.3	1.3	
1	-813.19	1.6	1.5	1.5	
1	-753.52	2.0	1.8	1.6	
1	-711.96	2.6	2.6	1.3	
1	-696.97	3.2	2.3	2.0	
1	-694.11	3.5	3.3	1.8	
1	-688.30	3.9	3.4	2.3	
1	-684.23	5.6	5.4	1.6	
1	-680.97	8.0	7.7	1.7	
1	-679.32	11.6	11.4	1.8	
1†	-678.52	17.2	16.9	1.8	
1†	-678.14	23.6	23.3	1.6	
$\mathbf{t\mu(n=2)}$	-677.81				
0	-670.73-0.6i	37.5	35.7	11.5	
0	-668.18 - 0.4i	7.1*	7.3*	18.5	
0	-666.87 - 0.2i	29.1	15.3	26.4	
0†	-666.30 - 0.1i	41.0	10.6	39.1	
0†	-666.03	60.8	6.9	59.7	
1†	-670.62 - 0.7i	23.7	23.1	5.9	
1†	-668.03 - 0.4i	7.9	3.4*	9.1	
1†	-666.82 - 0.2i	13.0	0.3*	13.5	
1†	-666.25-0.1i	21.8	1.0	21.7	
1†	-665.97	35.6	2.4	35.3	
$\mathbf{d\mu(n=2)}$	-665.80				

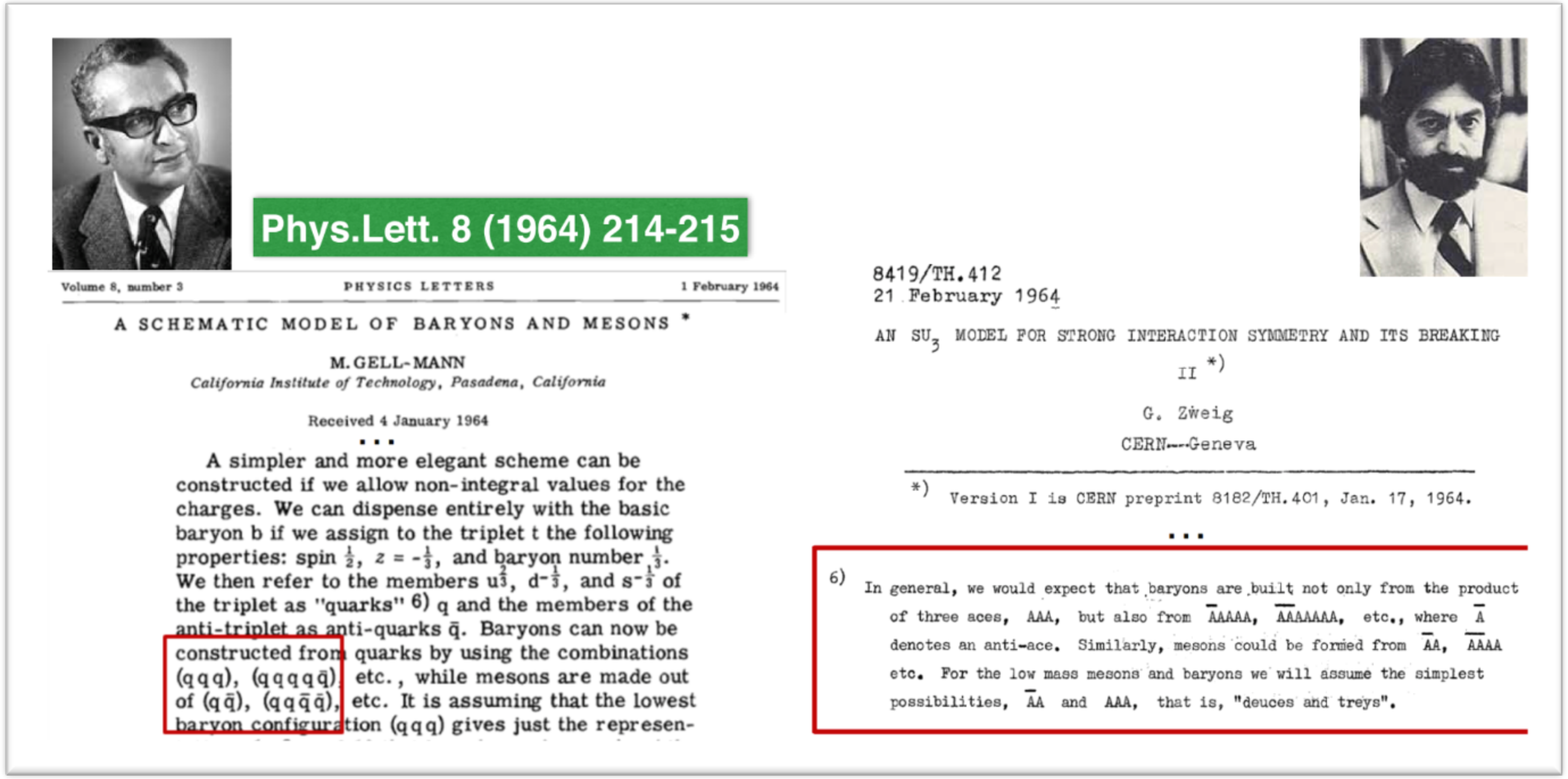
TABLE V. The binding energies and rms radii of $dd\mu$ and tritium are presented below. Newly identified states are marked with a dagger (†).

$[S_{dd}, L]$	E [eV]	r_{dd}^{rms}[pm]	$r_{d\mu}^{rms}$[pm]
[0(2),0]	-2988.27	0.76	0.62
[0(2),0]	-2699.04	1.57	1.16
[1,1]	-2889.88	0.43	0.49
[1,1]	-2665.18	1.37	1.06
$\mathbf{d\mu(n=1)}$	-2663.20		
[0(2),0]	-883.91	2.7	1.8
[0(2),0]	-801.08	3.4	2.2
[0(2),0]	-738.76	4.2	2.7
[0(2),0]	-697.59	5.7	3.7
[0(2),0]	-678.32	8.4	5.5
[0(2),0]	-671.06	12.6	8.4
[0(2),0]	-668.05	19.0	13.0
[0(2),0]	-666.76	28.8	19.9
[0(2),0]	-666.21	44.3	30.8
[0(2),0]†	-665.96	69.8	48.8
[1,0]	-686.96	6.6	4.3
[1,0]	-675.21	9.5	6.3
[1,0]	-669.88	14.2	9.6
[1,0]	-667.56	21.3	14.6
[1,0]	-666.56	32.4	22.4
[1,0]	-666.12	50.1	34.9
[1,0]†	-665.92	76.3	53.4
[0(2),1]	-688.44	3.3	2.3
[0(2),1]	-685.92	3.3	2.5
[0(2),1]	-674.60	4.8	3.5
[0(2),1]	-669.55	7.3	5.1
[0(2),1]	-667.39	11.0	7.7
[0(2),1]	-666.47	17.2	12.0
[0(2),1]	-666.04	29.3	20.6
[1,1]	-877.70	1.4	1.3
[1,1]	-795.98	1.7	1.5
[1,1]	-734.51	2.2	1.8
[1,1]	-694.78	2.9	2.2
[1,1]	-677.07	4.3	3.1
[1,1]	-670.47	6.6	4.6
[1,1]	-667.76	10.0	7.0
[1,1]	-666.62	15.6	10.9
[1,1]	-666.09	25.1	17.7
[1,1]†	-665.90	32.4	22.9
$\mathbf{d\mu(n=2)}$	-665.80		

TABLE X. Complex eigenenergies E of the $\mu\mu pp$ system. Benchmark results from previous calculations are listed in the last column for comparison. The column "Type" indicates bound states ("B") and resonant states ("R") in the pure Coulomb system.

$[S_{\mu\mu}, S_{pp}, L]$	E [eV]	Type	$r_{p\mu}^{rms}$[pm]	r_{pp}^{rms}[pm]	$r_{\mu\mu}^{rms}$[pm]	E_{bench} [eV]
[0,0,0]	-5431.91	B	0.62	0.64	0.81	-5431.5 [28]
[0,1,1]	-5147.57	B	0.56	0.44	0.77	-5147.3 [28]
$\mathbf{p}\mu(\mathbf{n}=\mathbf{1})+\mathbf{p}\mu(\mathbf{n}=\mathbf{1})$	-5056.99					
[0,0,0]	-3433.18-0.4i	R	1.4	1.5	1.5	-3370-0.16i [22]
[0,0,0]	-3321.23-2.5i	R	1.6	1.2	2.1	-3259-4.3i [22]
[0,0,0]	-3186.55-0.9i	R	2.0	2.6	2.1	
[1,0,0]	-3430.15	B	1.4	0.8	1.9	-3406 [22]
[0,1,0]	-3554.44	B	1.1	1.2	1.2	-3449 [22]
[0,1,0]	-3296.14	B	1.5	1.9	1.4	-3194 [22]
[0,0,1]	-3523.22	B	0.8	0.7	1.0	
[0,0,1]	-3317.24	B	1.0	0.9	1.1	
[0,0,1]	-3184.78	B	1.3	1.4	1.7	
[1,0,1]	-3345.64-26.9i	R	1.0	1.1	1.2	
[0,1,1]	-3385.60-1.3i	R	1.0	0.8	1.3	
[0,1,1]	-3191.26-1.7i	R	1.4	1.0	1.9	
[1,1,1]	-3254.64	B	1.4	0.5	1.9	
$\mathbf{p}\mu(\mathbf{n}=\mathbf{1})+\mathbf{p}\mu(\mathbf{n}=\mathbf{2})$	-3160.62					
[0,0,0]	-1374.56-10.3i	R	4.0	5.1	5.2	
[0,1,1]	-1347.06-13.2i	R	2.1	2.0	3.0	
$\mathbf{p}\mu(\mathbf{n}=\mathbf{2})+\mathbf{p}\mu(\mathbf{n}=\mathbf{2})$	-1264.25					

Introduction


Phys.Lett. 8 (1964) 214-215

Volume 8, number 3 PHYSICS LETTERS 1 February 1964

A SCHEMATIC MODEL OF BARYONS AND MESONS *

M. GELL-MANN
California Institute of Technology, Pasadena, California

Received 4 January 1964

. . .

A simpler and more elegant scheme can be constructed if we allow non-integral values for the charges. We can dispense entirely with the basic baryon b if we assign to the triplet t the following properties: spin $\frac{1}{2}$, $z = -\frac{1}{3}$, and baryon number $\frac{1}{3}$. We then refer to the members $u^{\frac{2}{3}}$, $d^{-\frac{1}{3}}$, and $s^{-\frac{1}{3}}$ of the triplet as "quarks" 6) q and the members of the anti-triplet as anti-quarks $\bar{q}$. Baryons can now be constructed from quarks by using the combinations (qqq), $(qqqq\bar{q})$, etc., while mesons are made out of $(q\bar{q})$, $(qq\bar{q}\bar{q})$, etc. It is assuming that the lowest baryon configuration (qqq) gives just the represen-

8419/TH.412
21 February 1964

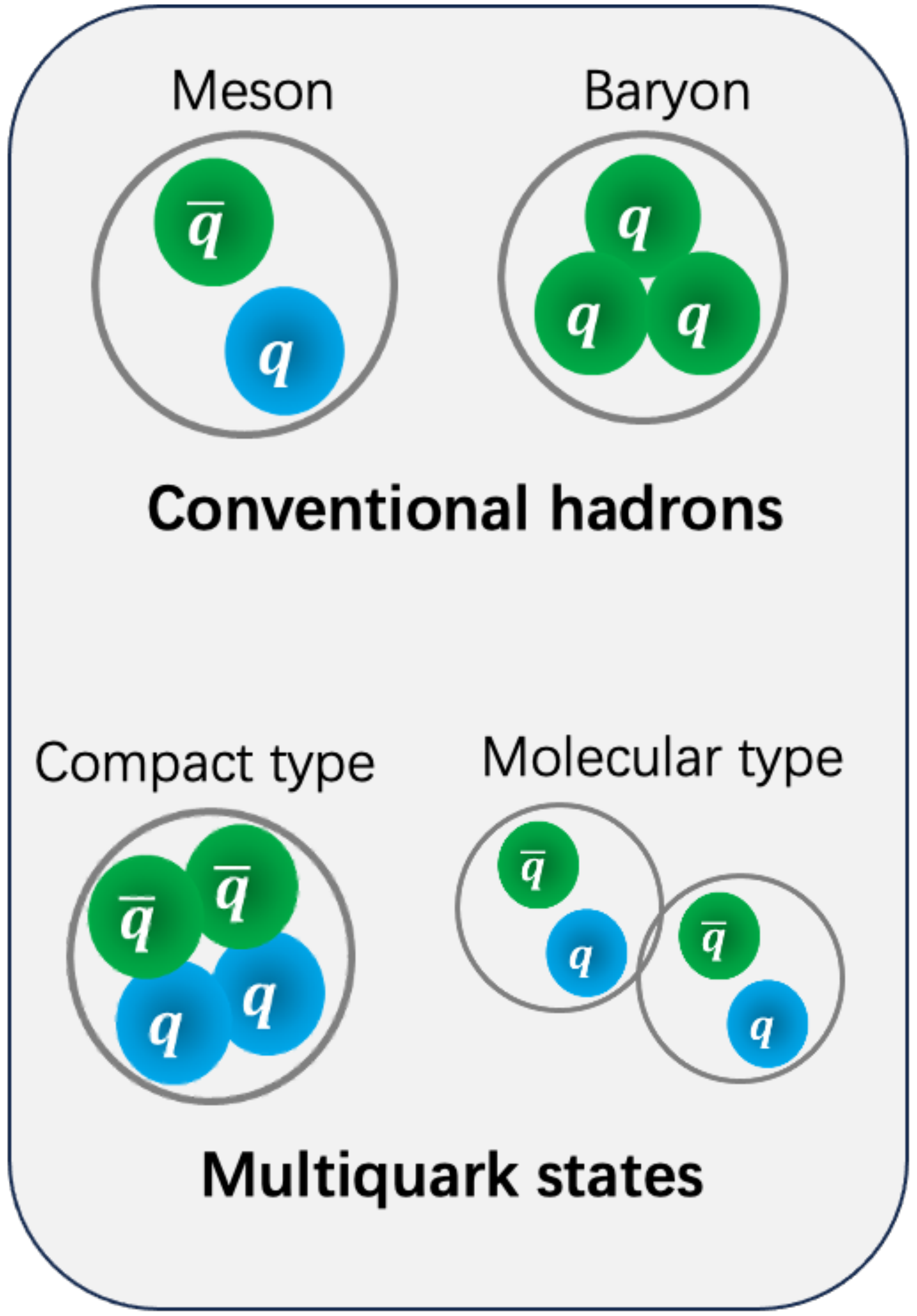
AN SU_3 MODEL FOR STRONG INTERACTION SYMMETRY AND ITS BREAKING II *)

G. Zweig
CERN---Geneva

*) Version I is CERN preprint 8182/TH.401, Jan. 17, 1964.

. . .

6) In general, we would expect that baryons are built not only from the product of three aces, AAA, but also from $\bar{A}AAAA$, $\bar{A}\bar{A}AAAAA$, etc., where $\bar{A}$ denotes an anti-ace. Similarly, mesons could be formed from $\bar{A}A$, $\bar{A}\bar{A}AA$ etc. For the low mass mesons and baryons we will assume the simplest possibilities, $\bar{A}A$ and AAA, that is, "deuces and treys".



- Multiquark states were predicted at the birth of quark model
- Quark potential model——a useful theoretical tool to describe the interaction between quarks

Motivation

The BaBar Collaboration first reported the observation of strangenium-like state $Y(2175)$ in $e^+e^- \to \phi(1020)f_0(980)$ in 2006. BaBar:2006gsq

Later it was confirmed by Belle in the $e^+e^- \to \phi\pi^+\pi^-$ and $e^+e^- \to \phi f_0(980)$ processes. It was also observed by BESII/BESIII in the $J/\psi \to \eta\phi f_0(980)$ process. Belle:2008kuo, BES:2007sqy, BESIII:2014ybv, BESIII:2017qkh

$X(2060)$, $X(2239)$, $X(2500)$, $f_0(2200)$, $f_2(2340)$,…… BESIII:2016qzq, BESIII:2018ldc, BESIII:2018zbm

Recently, the LHCb Collaboration discovered a fully charmed tetraquark candidate $X(6900)$, and confirmed by CMS and ATLAS. LHCb:2020bwg, CMS:2023owd, ATLAS:2023bft

As an analogy of the $T_{cc\bar{c}\bar{c}}$ system, there might exist $T_{ss\bar{s}\bar{s}}$ states.

Quark potential model

- AL1 model

Semay:1994ht, Silvestre-Brac:1996myf

$$V_{ij}(r) = \left[-\frac{\kappa}{r} + \lambda r^p - \Lambda + \frac{2\pi}{3m_i m_j}\kappa' \frac{1}{\pi^{3/2} r_0^3} e^{(-r^2/r_0^2)} \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j \right] \lambda_i \cdot \lambda_j$$

$$p = 1$$

J^{PC}	Meson	Exp. [MeV]	AL1 [MeV]	r^{rms} [fm]
0^{-+}	η'[a]	-	713.5	0.54
	$\eta'(2S)$	-	1565.2	1.17
	$\eta'(3S)$	-	2140.9	1.65
1^{--}	ϕ	1019.5	1021.0	0.70
	$\phi(2S)$	1680	1695.1	1.25
	$\phi(3S)$	2188	2231.6	1.70

[a]For simplicity, we assume there is no mixing effects between $\eta(n\bar{n})$ with $I = 0$ and $\eta'(s\bar{s})$.

Tetraquark wave function construction

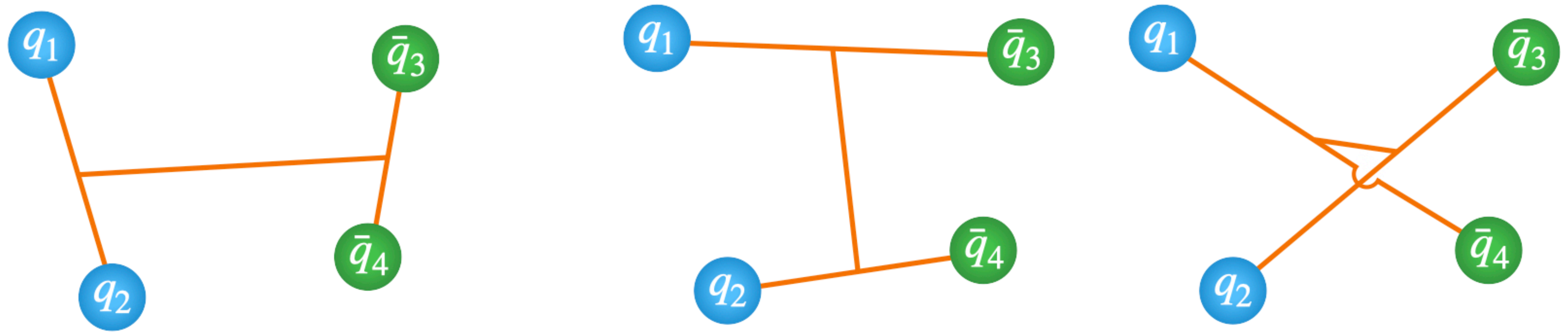
$$\Psi = \mathscr{A}\left(\chi_f \otimes \chi_c \otimes \chi_s \otimes \psi\right)$$

$\mathscr{A}$: antisymmetrization

Spatial wave function:

$$\phi_{nlm}(\boldsymbol{r}) = \sqrt{\frac{2^{l+5/2}}{\Gamma\left(l+\frac{3}{2}\right) r_n^3}} \left(\frac{r}{r_n}\right)^l e^{-\frac{r^2}{r_n^2}} Y_{lm}(\hat{r})$$

Only S-wave is considered.



(a) (b)

$$\begin{cases} r_0 = 0.4\text{fm}, r_{\max} = 2\text{fm} & q-q \text{ or } \bar{q}-\bar{q} \\ r_0 = 0.4\text{fm}, r_{\max} = 2\text{fm} & (qq)-(\bar{q}\bar{q}) \\ r_0 = 0.4\text{fm}, r_{\max} = 1.3\text{fm} & q-\bar{q} \\ r_0 = 0.4\text{fm}, r_{\max} = 5\text{fm} & (q\bar{q})-(q\bar{q}) \end{cases}$$

Color wave function:

$$\begin{cases} [(q_1q_2)_{\bar{3}}(\bar{q}_3\bar{q}_4)_3]_1 \\ [(q_1q_2)_6(\bar{q}_3\bar{q}_4)_{\bar{6}}]_1 \end{cases}$$

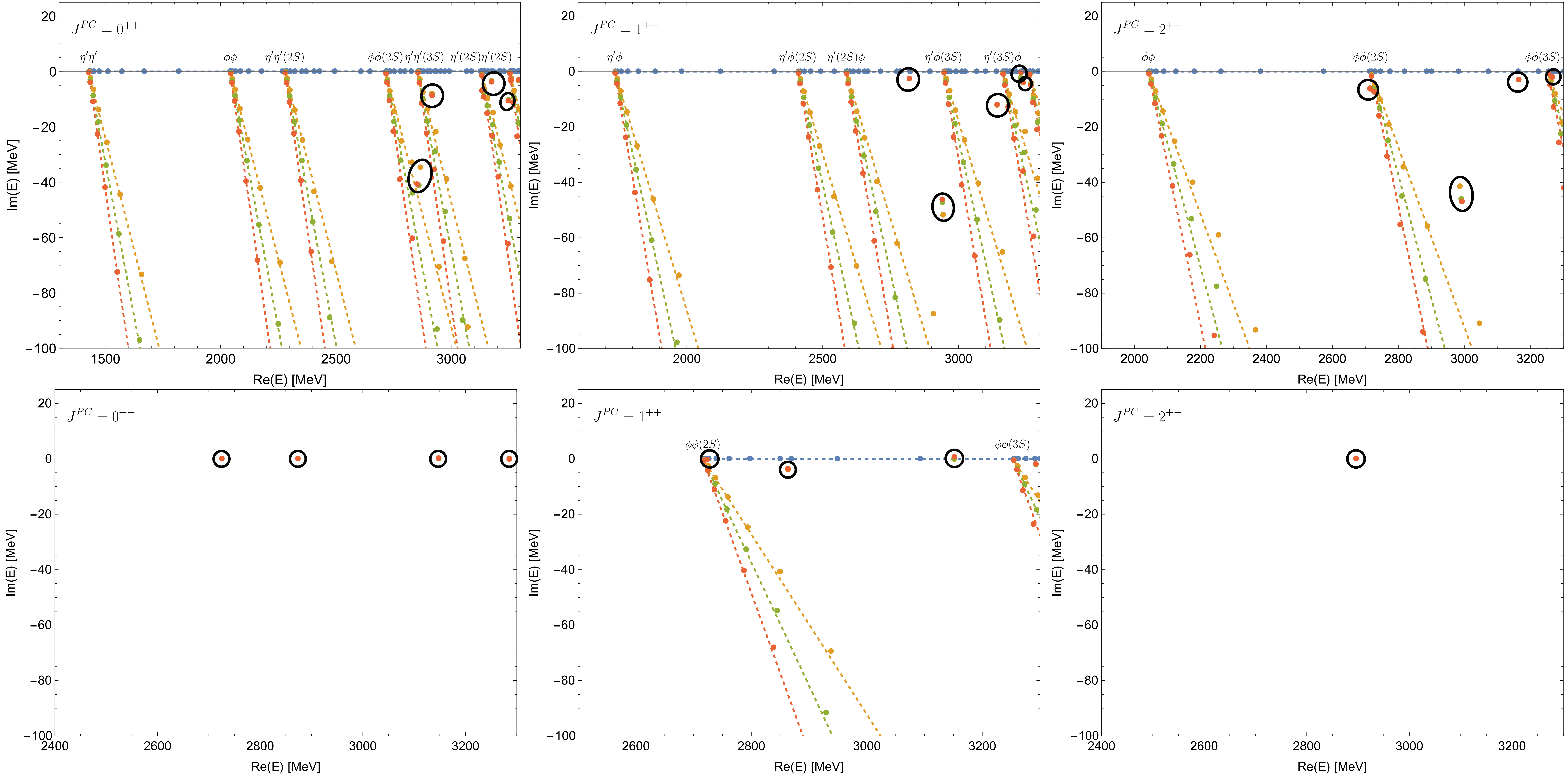
Spin wave function:

$$S=0: \begin{cases} [(q_1q_2)_0(\bar{q}_3\bar{q}_4)_0]_0 \\ [(q_1q_2)_1(\bar{q}_3\bar{q}_4)_1]_0 \end{cases}$$

$$S=1: \begin{cases} [(q_1q_2)_0(\bar{q}_3\bar{q}_4)_1]_1 \\ [(q_1q_2)_1(\bar{q}_3\bar{q}_4)_0]_1 \\ [(q_1q_2)_1(\bar{q}_3\bar{q}_4)_1]_1 \end{cases}$$

$$S=2: \left\{ [(q_1q_2)_1(\bar{q}_3\bar{q}_4)_1]_2 \right.$$

Numerical results



Numerical results

- "normal" C-parity

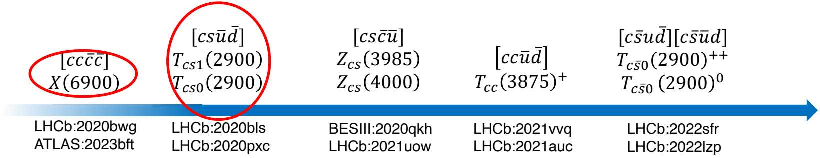
J^{PC}	$M - i\Gamma/2$ [MeV]	$\chi_{\bar{3}_c\otimes 3_c}$	$\chi_{6_c\otimes\bar{6}_c}$	$r^{\rm rms}_{ss}$ [fm]	$r^{\rm rms}_{s\bar{s}}$ [fm]
0^{++}	$2859 - 41i$	60%	40%	3.01	2.28
	$2917 - 8i$	41%	59%	1.51	1.36
	$3175 - 4i$	46%	54%	1.41	1.32
	$3248 - 10i$	34%	66%	1.37	1.36
1^{+-}	$2819 - 3i$	63%	37%	1.01	1.11
	$2940 - 47i$	89%	11%	0.88	1.05
	$3142 - 12i$	78%	22%	1.11	1.42
	$3229 - 2i$	66%	34%	1.21	1.37
	$3237 - 4i$	64%	36%	1.16	1.30
2^{++}	$2714 - 6i$	74%	26%	1.28	1.22
	$2990 - 46i$	84%	16%	1.23	1.16
	$3164 - 3i$	92%	8%	0.94	1.47
	$3266 - 2i$	66%	34%	1.28	1.35

- "exotic" C-parity

J^{PC}	$M - i\Gamma/2$ [MeV]	$\chi_{\bar{3}_c\otimes 3_c}$	$\chi_{6_c\otimes\bar{6}_c}$	$r^{\rm rms}_{ss}$ [fm]	$r^{\rm rms}_{s\bar{s}}$ [fm]
0^{+-}	2725	33%	67%	1.13	0.96
	2873	65%	35%	1.17	1.03
	3148	21%	79%	1.44	1.20
	3285	78%	22%	1.30	1.28
1^{++}	$2723 - 0.4i$	58%	42%	1.57	1.27
	$2863 - 4i$	99%	1%	1.05	1.00
	$3151 - 0.1i$	66%	34%	1.17	1.31
2^{+-}	2896	100%	0%	1.09	1.02

The "exotic" C-parity systems refer to the ones that cannot be composed of two S-wave ground mesons.

Recently, more and more hadrons composed of at least four quarks were observed



Theoretical interpretations: molecular VS compact state

- Unified description
- Dynamic calculations that treat the molecular and compact state equally
- Constituent quark model + 4-body Schrödinger equation

In this report, we focus on $T_{cs0}(2900)$ state in $B^+ \to (D^+K^-)D^+$ [LHCb:2020pxc]:

$$M = 2866 \pm 7 \pm 2\text{MeV}, \quad \Gamma = 57 \pm 12 \pm 4\text{MeV}$$

Model

AL1 quark potential model [Silvestre-Brac:1996myf]:

$$H = \sum_{j=1}^{4} \frac{p_j^2}{2m_j} - T_{\mathrm{c.m.}} + \sum_{i<j=1}^{4} V_{ij} + \sum_j m_j$$

$$V_{ij} = -\frac{3}{16} \boldsymbol{\lambda}_i^c \cdot \boldsymbol{\lambda}_j^c \left(-\frac{\kappa}{r_{ij}} + \lambda r_{ij} - \Lambda + \frac{2\pi\kappa'}{3m_i m_j} \frac{\exp\left(-r_{ij}^2/r_0^2\right)}{\pi^{3/2} r_0^3} \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j \right)$$

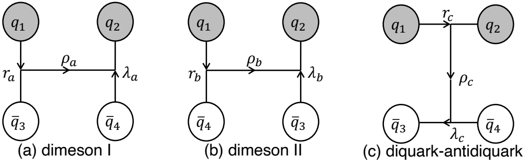
- One-gluon-exchange + Linear confinement
- Parameters were fitted by the meson spectrum (We do not introduce any additional free parameters)

Mesons $\overset{?}{\Rightarrow}$ tetraquark states, not trivial

- richer color-structure:
  $\mathbf{3} \otimes \bar{\mathbf{3}} = \mathbf{1} + \mathbf{8}, \qquad \mathbf{3} \otimes \mathbf{3} \otimes \bar{\mathbf{3}} \otimes \bar{\mathbf{3}} = 2(\mathbf{1}) + 4(\mathbf{8}) + \mathbf{10} + \overline{\mathbf{10}} + \mathbf{27}$
- interactions and color confinement mechanism are not well understood

Solving 4-body Schrödinger equation

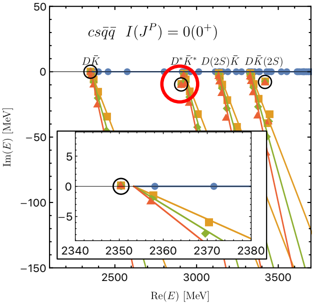
Gaussian Expansion Method (GEM) [Hiyama:2003cu]



$$\psi = \sum_{\alpha} \sum_{\beta=1}^{3} \sum_{n_1=1}^{n_{\max}} \sum_{n_1=2}^{n_{\max}} \sum_{n_3=1}^{n_{\max}} C_{\alpha,\beta,n_\beta} \chi_{sc}^{(\alpha)} \exp[-\nu_{n_1,\beta} r_\beta^2 - \nu_{n_2,\beta} \lambda_\beta^2 - \nu_{n_3,\beta} \rho_\beta^2]$$

- Include both meson-meson and diquark-antidiquark correlations
- Embed both long-range and short-range correlations
- Treat the molecular and compact state
- $10^4 \times 10^4$ non-Hermitian matrices
- By now, we only focus on S-wave states, $J^P = (0,1,2)^+$

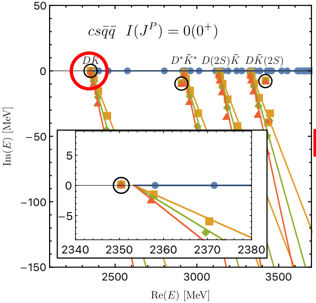
$(cs\bar{n}\bar{n})$ with $I(J^P) = 0(0^+)$



States	$M - i\Gamma/2$	ΔM	$r_{c\bar{q}}^{\rm rms}$	$r_{s\bar{q}}^{\rm rms}$	$r_{cs}^{\rm rms}$	$r_{\bar{q}\bar{q}}^{\rm rms}$	Type
$D\bar{K}$	2353		0.61	0.59			S.
$D^*\bar{K}^*$	2920		0.70	0.81			S.
$0(0^+)$	2350	-3	0.61	0.59	2.45	2.52	M.
	$2906 - 10i$	-14	0.74	0.86	1.12	1.26	M.
	$3419 - 7i$		0.91	1.09	0.87	1.22	C.

- mass and width: $M_T = 2906$ MeV, $\Gamma_T = 20$ MeV
- type: $D^*\bar{K}^*$ molecular quasi-bound(-14 MeV) state (Feshbach resonance)
- good candidate for the experimental $T_{cs0}(2900)$ in $B^+ \to (D^+K^-)D^+$
- cross-verification channel [Chen:2020eyu]: $B^+ \to (D^+K^-)\pi^+$

$(cs\bar{n}\bar{n})$ with $I(J^P) = 0(0^+)$



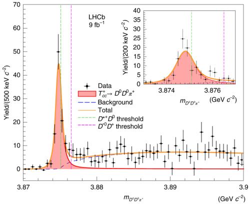
States	$M - i\Gamma/2$	ΔM	$r^{\rm rms}_{c\bar{q}}$	$r^{\rm rms}_{s\bar{q}}$	$r^{\rm rms}_{cs}$	$r^{\rm rms}_{\bar{q}\bar{q}}$	Type
$D\bar{K}$	2353		0.61	0.59			S.
$D^*\bar{K}^*$	2920		0.70	0.81			S.
$0(0^+)$	2350	−3	0.61	0.59	2.45	2.52	M.
	$2906 - 10i$	−14	0.74	0.86	1.12	1.26	M.
	$3419 - 7i$		0.91	1.09	0.87	1.22	C.

- mass: $M_T = 2350$ MeV.
- type: $D\bar{K}$ molecular bound(-3 MeV) state
- below the $D\bar{K}$ threshold and can only decay weakly
- possible channel [Yu:2017pmn]: $B^+ \to T_{cs} D^+$ with $T_{cs} \to K^- K^- \pi^+ \pi^+$

Introduction

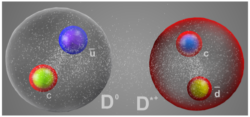
Experimentally,

- The first doubly charmed tetraquark $T_{cc}(3875)^+$ discovered in the $D^0 D^0 \pi^+$ channel [LHCb:2021vvq]
  - $m_{T_{cc}^+} - (m_{D^{*+}} + m_{D^0}) \sim -300$ keV
  - $\Gamma \sim 400$ keV
- $X(3872) \to$ XYZ states
  $T_{cc}(3875)^+ \to$ doubly heavy exotic states?



Theoretically,

- $T_{cc}^+(3875)$: natural interpretation as the $D^{*+}D^0$ molecular state
- Other doubly heavy tetraquarks: compact tetraquark / hadronic molecule



Quark Model + Effective Few-Body Methods:

A possible framework for unified descriptions

Bound states

波恩-奥本海默近似不适用

System	$I(J^P)$	M	ΔE	$\chi_{\bar{3}_c\otimes 3_c}$	$\chi_{6_c\otimes\bar{6}_c}$	$r^{\rm rms}_{Q_1\bar{q}_1}$	$r^{\rm rms}_{Q_2\bar{q}_2}$	$r^{\rm rms}_{Q_1\bar{q}_2}$	$r^{\rm rms}_{Q_2\bar{q}_1}$	$r^{\rm rms}_{Q_1Q_2}$	$r^{\rm rms}_{\bar{q}_1\bar{q}_2}$	Configuration
$cc\bar{q}\bar{q}$	$0(1^+)$	3864	-14	58%	42%	0.71	0.64	1.13	1.16	1.02	1.22	M. (D^*D)
$bb\bar{q}\bar{q}$	$0(1^+)$	10642	-1	33%	67%	0.66	0.63	2.06	2.07	1.98	2.15	M. $(\bar{B}^*\bar{B})$
$bc\bar{q}\bar{q}$	$0(2^+)$	7363	-3	27%	73%	0.66	0.70	1.95	1.97	1.86	2.05	M. $(\bar{B}^*D^*)$
$bc\bar{q}\bar{q}$	$0(0^+)$	7129	-26	48%	52%	0.64	0.64	0.91	0.95	0.76	1.03	C.E.
	$0(1^+)$	7185	-27	60%	40%	0.67	0.66	0.88	0.93	0.71	1.00	C.E.
$bb\bar{q}\bar{q}$	$0(1^+)$	10491	-153	97%	3%	0.68	0.67	0.70	0.71	0.33	0.78	C.DC.
$bb\bar{s}\bar{q}$	$\frac{1}{2}(1^+)$	10647	-64	91%	9%	0.56	0.67	0.71	0.61	0.36	0.76	C.DC.

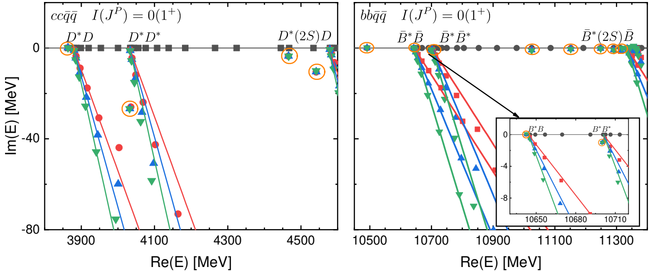
- D^*D molecule with $\Delta E = -14$ MeV as candidate for $T_{cc}(3875)^+$
- Bound states with various configurations
  - Molecular (M.) shallow bound state
  - Compact even tetraquark (C.E.)
  - Compact diquark-centered tetraquark (C.DC.) deeply bound state
- No isovector bound state $\to$ importance of "good" antidiquark $(\bar{q}\bar{q})^{S=0,I=0}_{3_c}$

松散分子态

类氦原子态、全新结构

紧致态，全新结构

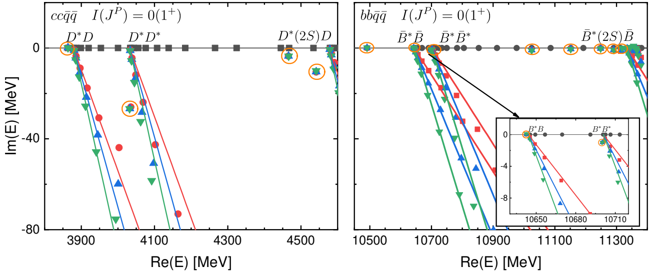
Resonant states: isoscalar $QQ\bar{q}\bar{q}$



- Lowest resonant states near the D^*D^* and $\bar{B}^*\bar{B}^*$ thresholds
- First T_{bb} resonance as $\bar{B}^*\bar{B}^*$ molecule

$M - i\Gamma/2$	$\chi_{\bar{3}_c\otimes 3_c}$	$\chi_{6_c\otimes\bar{6}_c}$	$r^{\rm rms}_{Q_1\bar{q}_1}$	$r^{\rm rms}_{Q_2\bar{q}_2}$	$r^{\rm rms}_{Q_1\bar{q}_2}$	$r^{\rm rms}_{Q_2\bar{q}_1}$	$r^{\rm rms}_{Q_1Q_2}$	$r^{\rm rms}_{\bar{q}_1\bar{q}_2}$	Config.
10491	97%	3%	0.68	0.67	0.70	0.71	0.33	0.78	C.DC.
10642	33%	67%	0.66	0.63	2.06	2.07	1.98	2.15	M. ($\bar{B}^*\bar{B}$)
10700 − 1i	44%	56%	0.67	0.67	1.96	1.96	1.88	2.02	M. ($\bar{B}^*\bar{B}^*$)
11025 − 1i	98%	2%	1.08	1.07	1.08	1.08	0.33	0.83	C.DC.

Resonant states: isoscalar $QQ\bar{q}\bar{q}$



- Second T_{bb} resonance as radial excitation of the deeply bound T_{bb} state
- More resonant states in higher energy region and $bc\bar{q}\bar{q}$ system

$M - i\Gamma/2$	$\chi_{\bar{3}_c \otimes 3_c}$	$\chi_{6_c \otimes \bar{6}_c}$	$r^{\rm rms}_{Q_1\bar{q}_1}$	$r^{\rm rms}_{Q_2\bar{q}_2}$	$r^{\rm rms}_{Q_1\bar{q}_2}$	$r^{\rm rms}_{Q_2\bar{q}_1}$	$r^{\rm rms}_{Q_1 Q_2}$	$r^{\rm rms}_{\bar{q}_1\bar{q}_2}$	Config.
10491	97%	3%	0.68	0.67	0.70	0.71	0.33	0.78	C.DC.
10642	33%	67%	0.66	0.63	2.06	2.07	1.98	2.15	M. $(\bar{B}^*\bar{B})$
$10700 - 1i$	44%	56%	0.67	0.67	1.96	1.96	1.88	2.02	M. $(\bar{B}^*\bar{B}^*)$
$11025 - 1i$	98%	2%	1.08	1.07	1.08	1.08	0.33	0.83	C.DC.

Background for $QQ\bar{Q}\bar{Q}$

Experimentally,

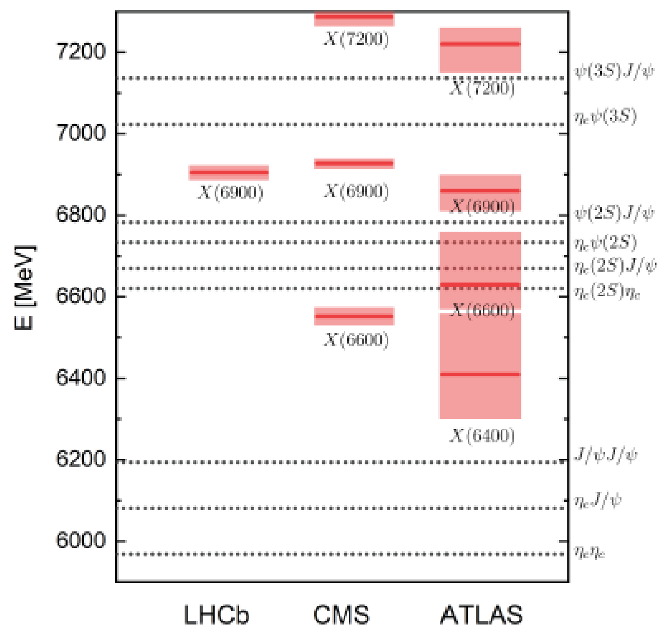
- Observation / Evidence of a series of $cc\bar{c}\bar{c}$ states [LHCb:2020bwg, ATLAS:2023bft, CMS:2023owd]

$$X(6900) \qquad X(7200)$$
$$X(6400) \qquad X(6600)$$

- No signals for $bb\bar{b}\bar{b}$ states are seen

Theoretically,

- Identify genuine resonant states from dynamical calculations
- Distinguish between compact and molecular configurations



Quark Potential Model

- Success in conventional hadrons → extension to multiquark
- Do not priorly assume structures of multiquark states
- Interaction: one-gluon-exchange + confinement

$$V_{ij} = -\frac{3}{16}\boldsymbol{\lambda}_i \cdot \boldsymbol{\lambda}_j \left(-\frac{\kappa}{r_{ij}} + \lambda r_{ij}^p - \Lambda + \frac{8\pi\kappa'}{3m_i m_j} \frac{\exp\left(-r_{ij}^2/r_0^2\right)}{\pi^{3/2} r_0^3} \boldsymbol{S}_i \cdot \boldsymbol{S}_j \right)$$

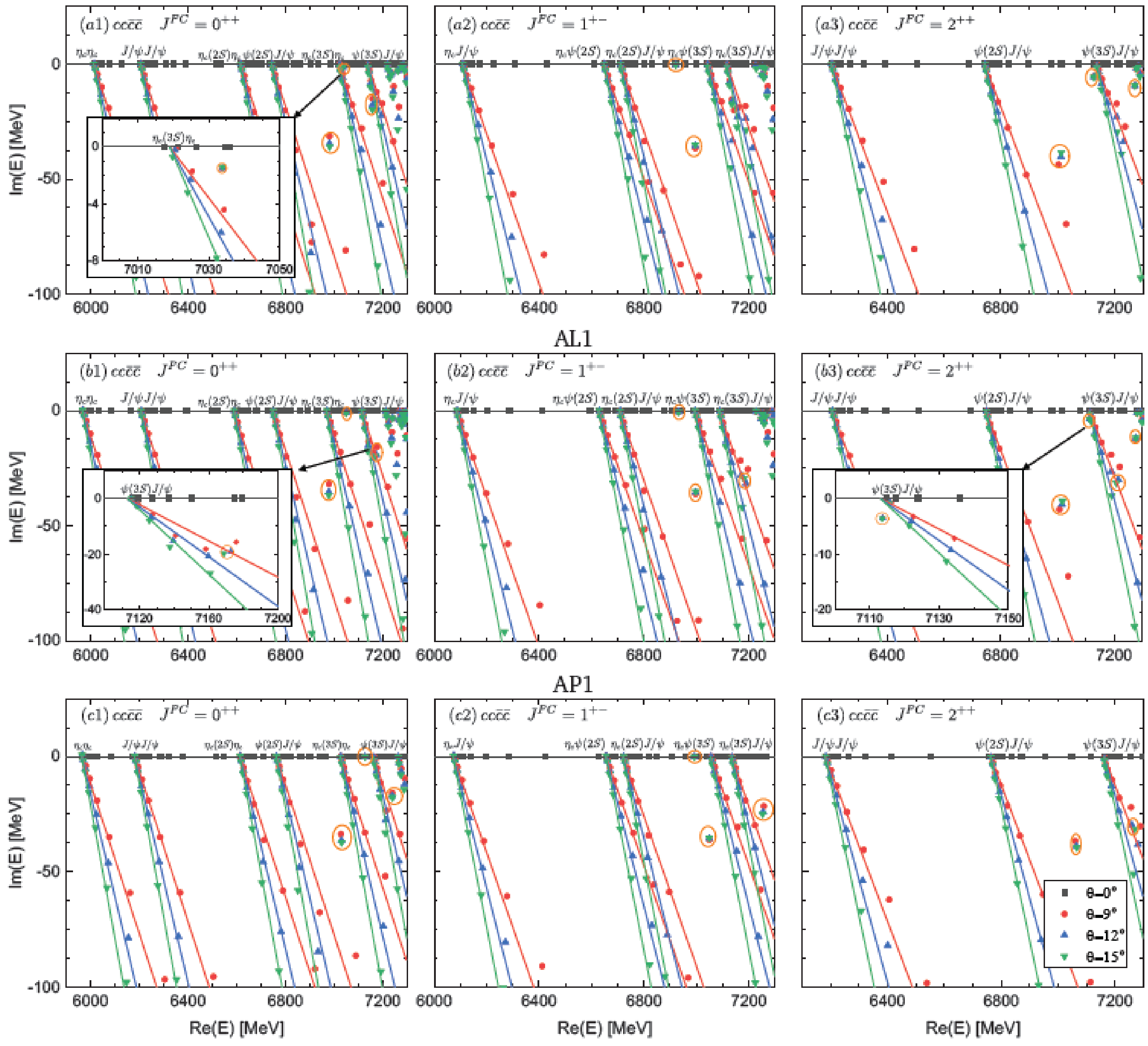
We use 3 models with different sets of parameters:

AL1, AP1 [Silvestre-Brac:1996myf] and BGS [Barnes:2005pb]

Mesons	$m_{\mathrm{Exp.}}$	m_{AL1}	m_{AP1}	m_{BGS}	$r^{\mathrm{rms}}_{\mathrm{AP1}}$
η_c	2984	3006	2982	2982	0.35
$\eta_c(2S)$	3638	3608	3605	3630	0.78
$\eta_c(3S)$	$\cdots$	4014	3986	4043	1.15
J/ψ	3097	3102	3102	3090	0.40
$\psi(2S)$	3686	3641	3645	3672	0.81
$\psi(3S)$	4039	4036	4011	4072	1.17
η_b	9399	9424	9401	$\cdots$	0.20
$\eta_b(2S)$	9999	10003	10000	$\cdots$	0.48
$\eta_b(3S)$	$\cdots$	10329	10326	$\cdots$	0.73
Υ	9460	9462	9461	$\cdots$	0.21
$\Upsilon(2S)$	10023	10012	10014	$\cdots$	0.49
$\Upsilon(3S)$	10355	10335	10335	$\cdots$	0.74

Fully charmed tetraquark

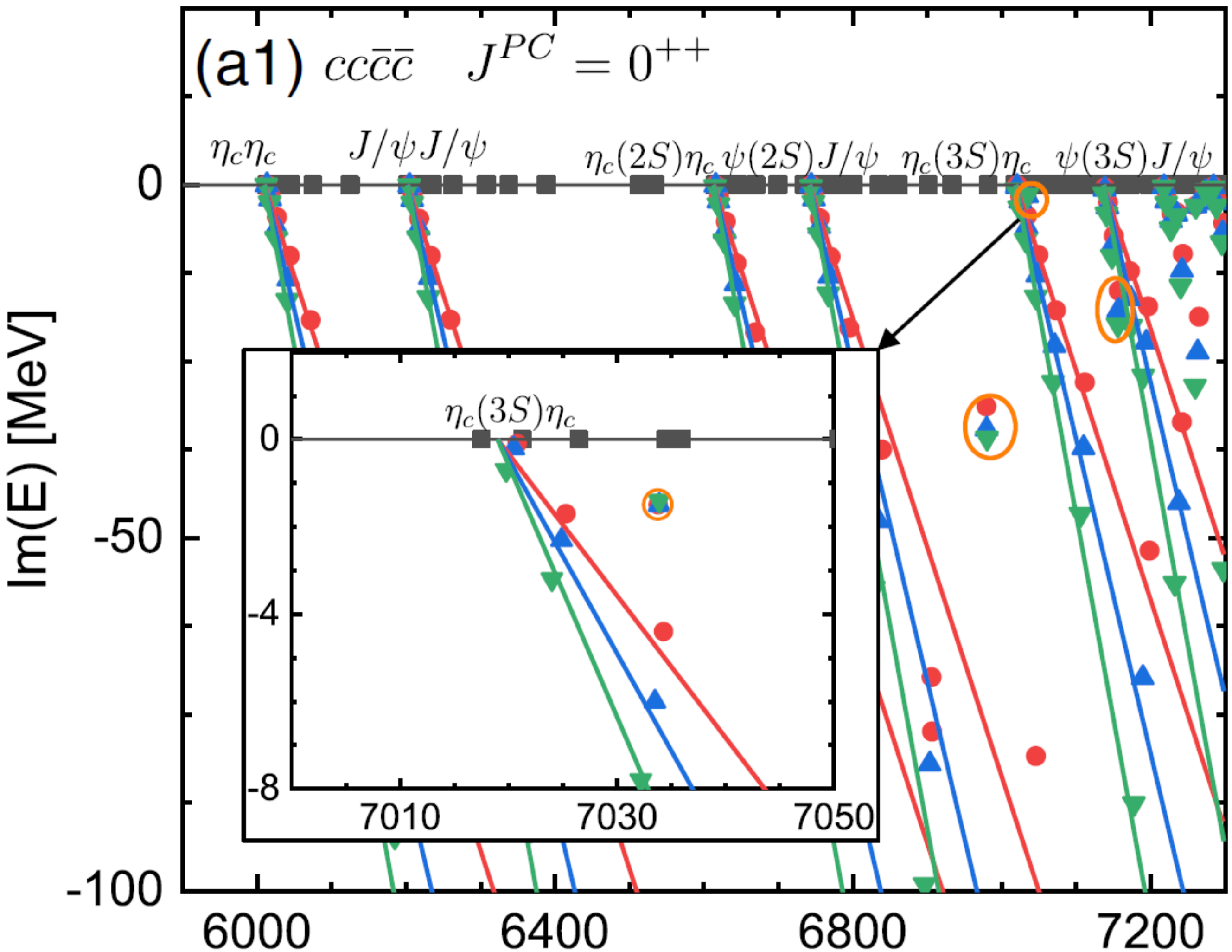
- Different models give qualitatively consistent results
- Similar pattern in different J^{PC} systems
- Candidates for $X(6900), X(7200)$ are found in $J^{PC} = 0^{++}, 2^{++}$ systems



三种常用夸克模型结果大体一致

J^{PC}	AL1	AP1	BGS	BGS, Wang *et al.*
0^{++}	$6980-35i$	$6978-36i$	$7030-36i$	$7035-39i$
	$7034-1i$	$7049-1i$	$7127-0.1i$	—
	$7156-20i$	$7173-20i$	$7239-17i$	$7202-30i$
1^{+-}	$6921-0.5i$	$6932-0.5i$	$6991-0.1i$	—
	$6995-35i$	$6998-35i$	$7048-35i$	$7050-35i$
	?	$7191-32i$	$7254-24i$	$7273-25i$
2^{++}	$7013-38i$	$7017-39i$	$7066-39i$	$7068-42i$
	$7127-6i$	$7114-4i$	—	—
	?	$7214-30i$	$7268-32i$	$7281-46i$
	$7272-9i$	$7276-12i$	$7337-8i$	—

黑点对应转角为零、被误认为离散态的散射态

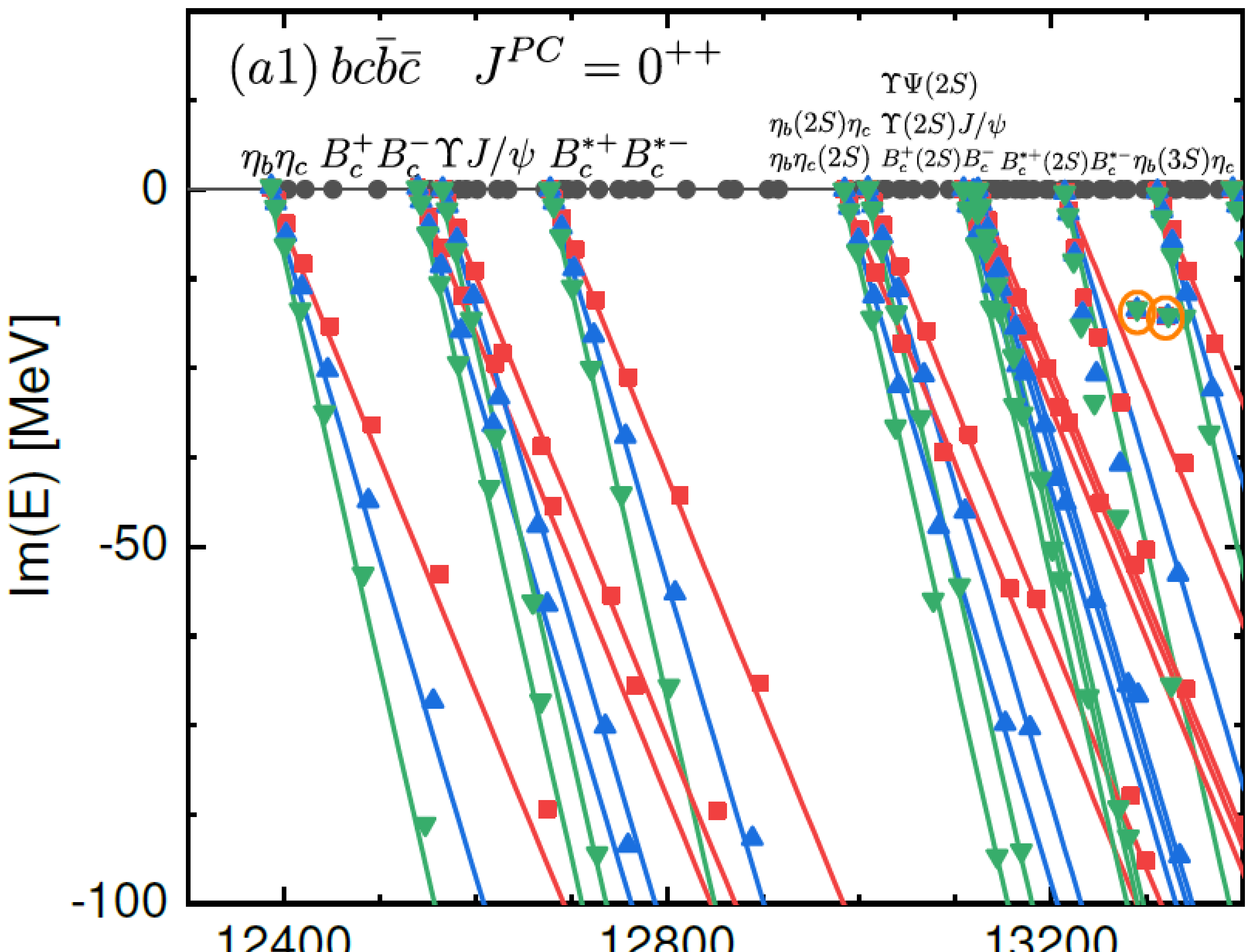


All fully charmed tetraquark states are compact

J^{PC}	$M - i\Gamma/2$	$\chi_{\bar{3}_c \otimes 3_c}$	$\chi_{6_c \otimes \bar{6}_c}$	$r^{\rm rms}_{c_1\bar{c}_3}$	$r^{\rm rms}_{c_2\bar{c}_4}$	$r^{\rm rms}_{c_1\bar{c}_4} = r^{\rm rms}_{c_2\bar{c}_3}$	$r^{\rm rms}_{c_1c_2} = r^{\rm rms}_{\bar{c}_3\bar{c}_4}$
0^{++}	$6978 - 36i$	86%	14%	0.81	0.81	0.86	0.66
	$7049 - 1i$	37%	63%	0.70	0.70	0.82	0.75
1^{+-}	$6932 - 0.5i$	65%	35%	0.66	0.66	0.73	0.63
	$6998 - 35i$	88%	12%	0.79	0.80	0.77	0.59
2^{++}	$7017 - 39i$	90%	10%	0.79	0.79	0.71	0.56
	$7114 - 4i$	69%	31%	0.92	0.92	0.65	0.55

Mesons	$r^{\rm rms}_{\rm Theo.}$
η_c	0.35
$\eta_c(2S)$	0.78
$\eta_c(3S)$	1.15
J/ψ	0.40
$\psi(2S)$	0.81
$\psi(3S)$	1.17

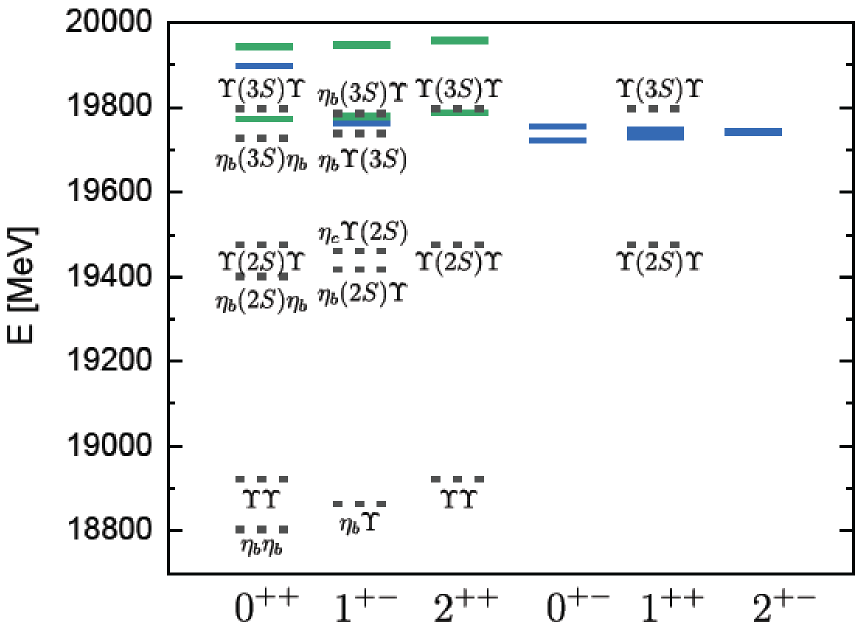
没有看到6600/6400共振态
6800 MeV没有看到共振态



$(a1)\, bc\bar{b}\bar{c} \quad J^{PC} = 0^{++}$
$\Upsilon\Psi(2S)$
$\eta_b(2S)\eta_c$ $\Upsilon(2S)J/\psi$
$\eta_b\eta_c$ $B_c^+ B_c^-$ $\Upsilon J/\psi$ $B_c^{*+} B_c^{*-}$
$\eta_b\eta_c(2S)$ $B_c^+(2S)B_c^-$ $B_c^{*+}(2S)B_c^{*-}$ $\eta_b(3S)\eta_c$
Im(E) [MeV]
0
-50
-100
12400
12800
13200

Fully bottomed tetraquark

- AP1 model is used for $bb\bar{b}\bar{b}$ systems
- Resonant states obtained in the region (19.7,20.0) GeV
- All states have compact configuration



Green (Blue) lines: states with widths larger (smaller) than 1 MeV

还有未来吗？

A theorist’s wish list（谦卑愿望）：

- 澄清X(6600/6400)是否存在？
- X(6900)与X(7100)的J^{PC}测量，共振态参数与质量谱的精细结构→有助于研究禁闭机制
- J/ψ ϒ末态寻找13.3GeV 共振态
- ϒϒ 末态寻找19.8GeV 共振态
- ϕϕ 末态寻找2.7GeV共振态
- 两个Zcs(4000)各种衰变模式→是否同一个态
- 多个Zc(4200/4430/4475)态精细测量→分子态、紧致态？
- 确认4GeV 的1^{++} χ_{c1}→ 对粲偶素谱定标
- 确定Pc态量子数，寻找多重态伙伴
- 寻找并确立Pcs多重态
- 寻找重味混杂态
- ...

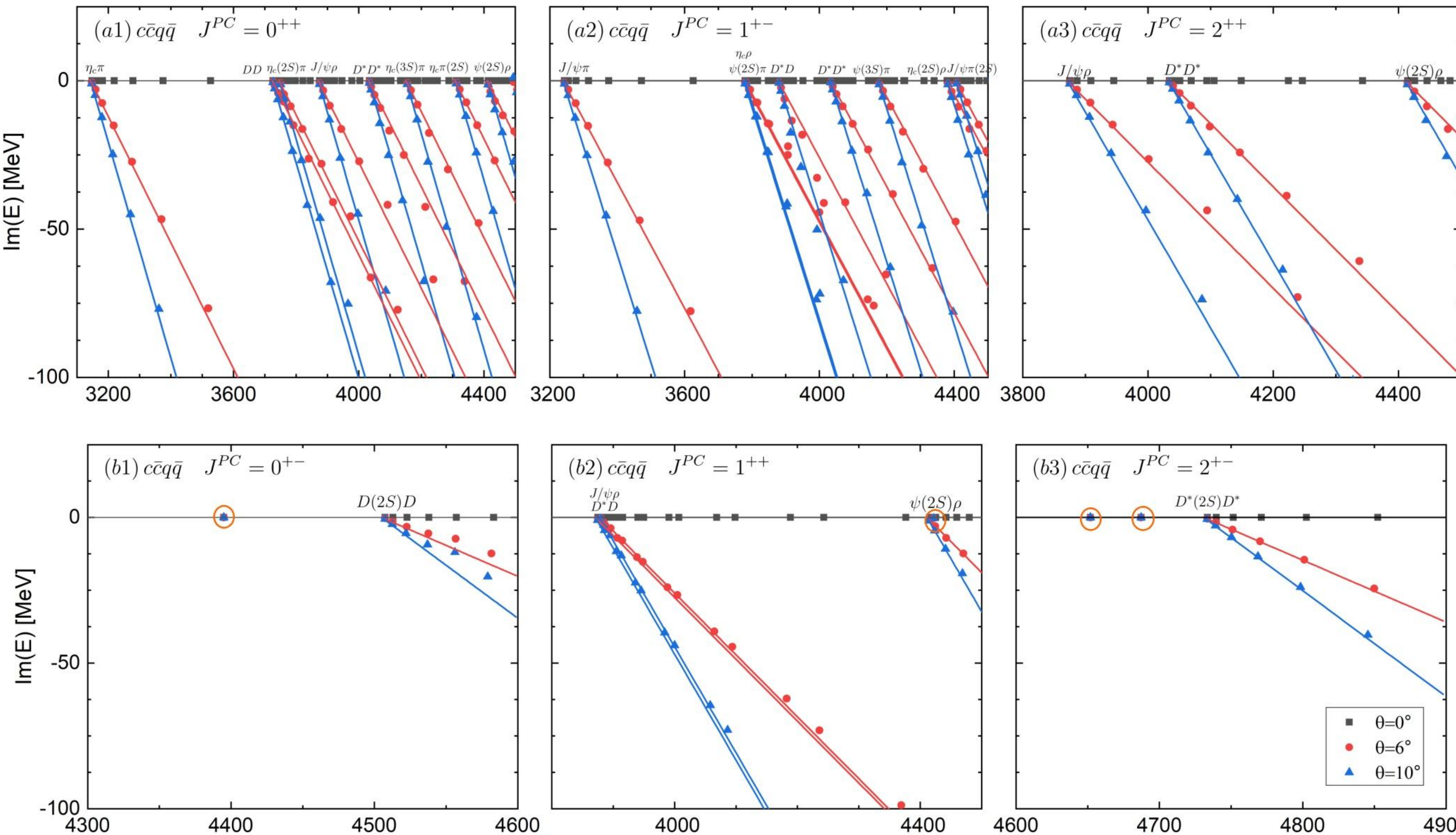
理论家能鼓捣点啥？
A theorist’s ambition/hope

把目前框架推广到

- 宇称为负奇特四夸克态体系
- 粒子数N=5、6多夸克体系，回答是否存在紧致隐粲五夸克态…
- **需要克服10^6*10^6非厄密稀疏矩阵求本征值问题，北大超算中心存储资源不够**
- CSM+耦合道分析→涉及粒子数目变化体系如X3872，LHCb电磁衰变分枝比的测量与4GeV 1^{++}态
- 正常量子数的四夸克态体系
- 多夸克体系强、电磁衰变
- …

Endless Frontier:
New horizon and landscape

- ➢ 紧致重味四夸克态体系提供了深入研究色禁闭机制的独特机会
- 强子分子态大多松散，由强子层次上的相互作用主导（各种介子交换相互作用）
- 核子中的禁闭势（胶子场、色流管）就存在争议：格点QCD、（最近QGP分析？）倾向于Y型三体力，可惜两体和Y型禁闭势导致的普通重子谱的数值差异不大
- 激发态粲偶素谱对禁闭势具体形式敏感，问题是4GeV以上的粲偶素非常混乱（实验、理论）
- X(6900)似乎是唯一确立的紧致四夸克态，内部颜色结构复杂，禁闭相互作用是两体力、Y型三体力、更复杂的K型结构？实验测量结合可靠理论计算，筛选理论模型，确定禁闭势形式？
- ➢ 紧致重味四夸克态体系提供了研究低能三胶子规范相互作用导致的三体力的平台
- 传统夸克模型中各种精细、超精细相互作用（自旋轨道耦合、自旋自旋相互作用）起源于单胶子交换等两体相互作用
- QCD颜色SU(3)规范相互作用，包含三胶子、四胶子顶角，它们对普通介子和重子没有贡献
- X(6900)中任意三个粲夸克处于颜色三维表示，三胶子顶角导致的三体力不为零。
- ➢ …


(a1) cc̄qq̄ J^PC = 0^++
η_cπ
DD
η_c(2S)π
J/ψρ
D*D*
η_c(3S)π
η_cπ(2S)
ψ(2S)ρ
(a2) cc̄qq̄ J^PC = 1^+-
J/ψπ
η_cρ
ψ(2S)π
D*D
D*D*
ψ(3S)π
η_c(2S)ρ
J/ψπ(2S)
(a3) cc̄qq̄ J^PC = 2^++
J/ψρ
D*D*
ψ(2S)ρ
(b1) cc̄qq̄ J^PC = 0^+-
D(2S)D
(b2) cc̄qq̄ J^PC = 1^++
J/ψρ
D*D
ψ(2S)ρ
(b3) cc̄qq̄ J^PC = 2^+-
D*(2S)D*
Im(E) [MeV]
Im(E) [MeV]
θ=0°
θ=6°
θ=10°

Zc (1^+1^{+-})在哪？