GUANGXI NORMAL UNIVERSITY

Suppression of the jet quenching parameter near the critical temperature

Yun Guo

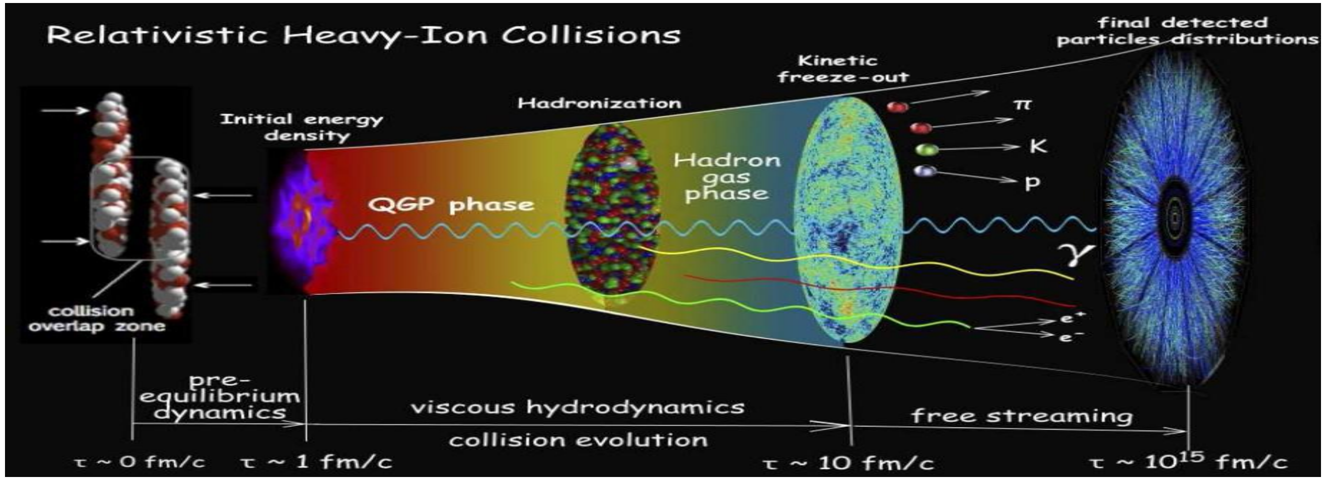
Based on: ArXiv: 2601.11230

Physics Department, Guangxi Normal University

- **Introduction**
- **Theoretical approaches to computing the jet quenching parameter**
- **The jet quenching parameter in the effective theory**
- **Results and discussions**
- **Summary**

The 1st Workshop on Jet and Heavy Quark Physics, Wuhan, 24 Jan. 2026

Introduction



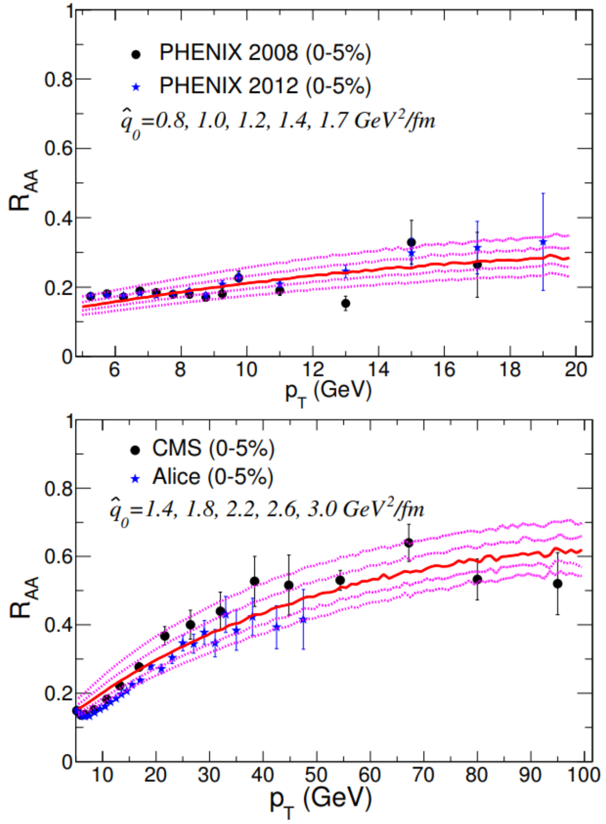
Jets are ideal Probes that can used to study the QGP:

- the suppression of high-transverse-momentum hadrons
- the modification of di-hadron momentum balance

Governed by the jet quenching parameter $\hat{q}$

$$\hat{q}(\Lambda_\perp) = \int_{q_\perp < \Lambda_\perp} d^2\mathbf{q}_\perp \frac{d\Gamma_{el}}{d^2\mathbf{q}_\perp} q_\perp^2 \,,$$

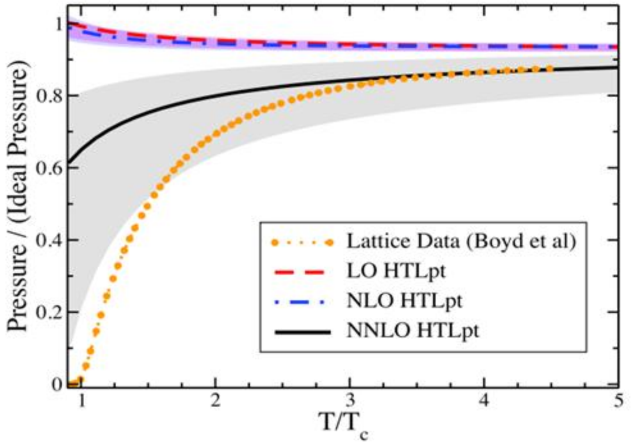
measures the average squared transverse momentum transfer per unit path length to a fast-moving parton due to elastic collisions



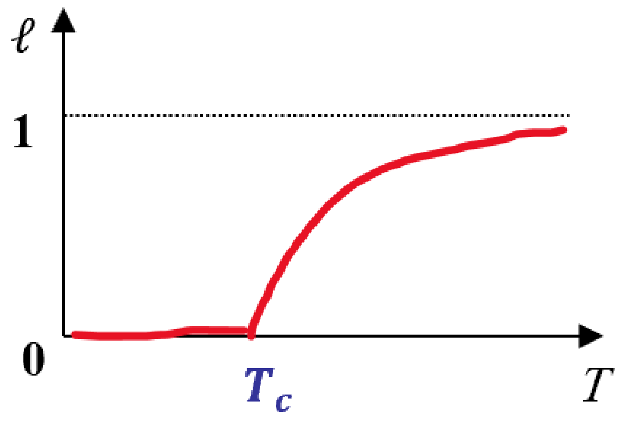
The JET Collaboration, PRC 2014

Introduction

Thermodynamics: perturbative calculation vs. Lattice results



Polyakov loop: the order parameter of the deconfining phase transition



phase	temperature	ℓ from LQCD	method
hadronic	$T < T_c$	$\ell = 0$	eff. theory (HRG)
QGP	$T >\sim 3T_c$	$\ell \approx 1$	(HTL) pert. theory
"semi"-QGP	$T_c < T <\sim 3T_c$	$0 < \ell < 1$	BF eff. theory

an effective theory in the "semi"-QGP: that can describe the thermodynamics and non-trivial *T*-dependence of the Polyakov loop, which reflects the influence of the deconfining phase transition on the physical quantities in consideration.

Theoretical approaches to computing the jet quenching parameter

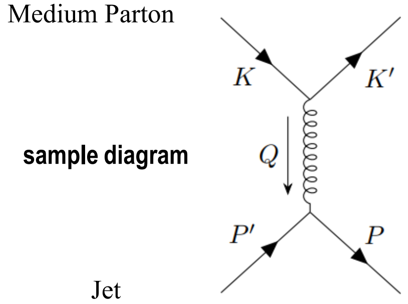
- **The differential interaction rate for the elastic scatterings**

Aurenche, JHEP 2002 Arnold, PRD 2008

$$\frac{d\Gamma_{\rm el}}{d^2q_\perp} \simeq \frac{1}{3\pi^2} \times \begin{cases} \dfrac{g^2 T m_D^2}{q_\perp^2(q_\perp^2+m_D^2)}, & q_\perp \ll T\,, \\ \dfrac{6I(q_\perp/T)}{q_\perp^4}\dfrac{g^4T^3}{\pi^2}, & q_\perp \gtrsim T\,, \end{cases}$$

soft process: HTL resumed propagator + sum rule

hard process: bare propagator



sample diagram

an **artificial cutoff** needs to be introduced to separate the hard and soft scale

- **The jet quenching parameter**

$$\hat{q}_{\rm soft} = \frac{g^2 C_R T m_D^2}{4\pi} \ln\left(1+\frac{\lambda_\perp^2}{m_D^2}\right) .$$

$$\hat{q}_{\rm hard1} = \frac{4g^4T^3}{\pi^3}\zeta(3)\ln\left(\frac{\Lambda_\perp}{\lambda_\perp}\right) ,$$

$$\hat{q}_{\rm hard2} = \frac{4g^4T^3}{\pi^3}\left\{\frac{\zeta(2)-\zeta(3)}{2}\left[1-2\gamma_E+2\ln 2-2\ln\left(\frac{\lambda_\perp}{T}\right)\right]-\sigma\right\} .$$

the dependence on the cutoff $\lambda_\perp$ is **cancelled** between the soft and hard contributions when $m_D \ll \lambda_\perp \ll T$

Theoretical approaches to computing the jet quenching parameter

- **Treating the hard and soft processes in an unified framework with the HTL resumed propagator**

$$\hat{q}_{\mathrm{I}} = \frac{g^4}{(2\pi)^3}\int_0^\infty dk \int_{-k}^\infty d\omega \int_{|\omega|}^{\min(2k+\omega,\sqrt{\omega^2+\Lambda_\perp^2})} dq\, q^2(1-\hat{w}^2)n(k)\left(1+n(k+w)\right) \times \left[|\Delta_T(Q)|^2(1-\hat{w}^2)^2\left((w+2k)^2+3q^2\right)+2|\Delta_L(Q)|^2\left((w+2k)^2-2q^2\right)\right].$$

✓ gauge independent & unphysical gluon polarizations eliminated by the ghost field   Cai & YG, 2512.12959

✓ For high-energy jet, *t*-channel gluon exchange is dominated

✓ For isotropic medium, no cross term ~ $\Delta_T\, \Delta_L^*$ or $\Delta_L \Delta_T^*$ appears

✓ As compared to a simplified version for computing the squared matrix element   Arnold, JHEP 2003   Boguslavsk, PRD 2024

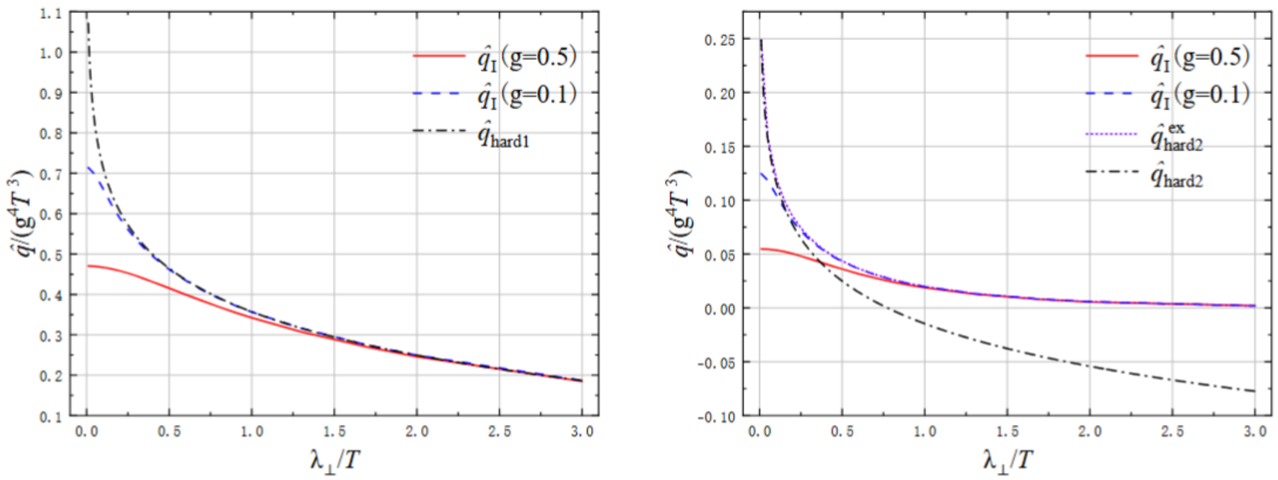
$|\mathcal{M}|^2 \sim |D^{\mu\nu}(Q)(P+P')_\mu(K+K')_\nu|^2$, the difference is given by $4q^2|\Delta_L(Q)|^2 - 4q^2(1-\hat{\omega}^2)^2|\Delta_T(Q)|^2$

✓ **Reproduce the previous known results in the hard and soft limit**

- **Numerical comparison among different theoretical approaches**

hard contribution

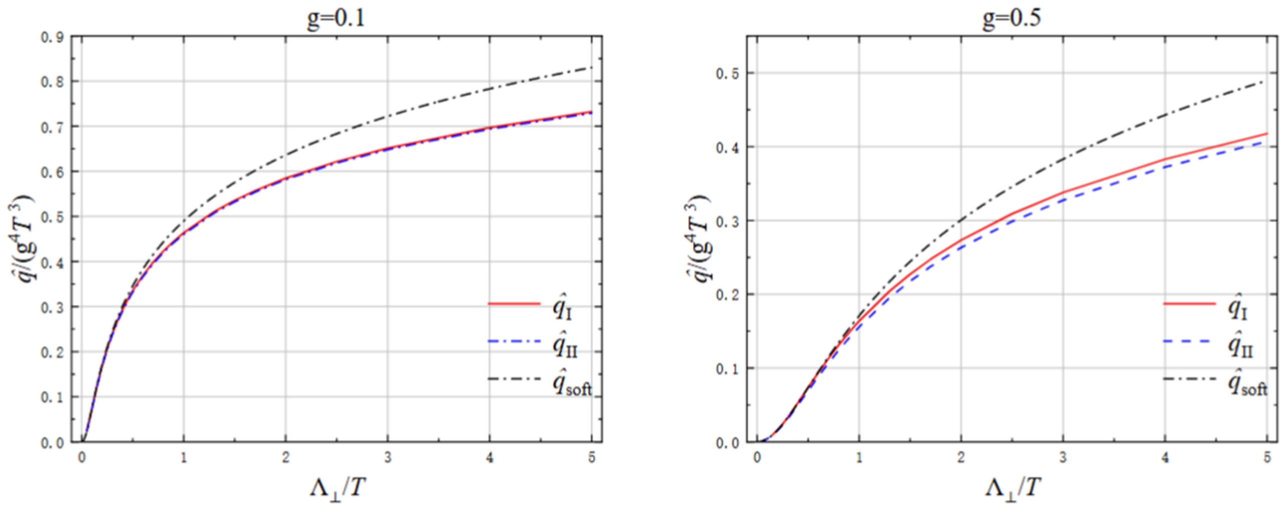
$\Lambda_\perp/T = 10$



Left: contributions linear in the distribution function

Right: contributions quadratic in the distribution function

soft contribution



using soft contribution to approximate the full jet quenching parameter turns to be reasonable in the small-coupling limit

The jet quenching parameter in the effective theory

- ***how to describe the semi-QGP?***

  ✓ The semi-QGP is characterized by a T-dependent Polyakov loop which is non-zero but less than 1 in the phase transition region.

$$\mathbf{L} = \mathcal{P}\exp\left(ig\int_0^{1/T} A_0^{cl}\, d\tau\right) \qquad\qquad \ell = \frac{1}{N}\mathrm{Tr}\mathbf{L}\,.$$

✓ to generate a T-dependent Polyakov loop in the "semi"-QGP region, introduce a classical background field for the gauge field.

$$A_\mu = A_\mu^{\mathrm{cl}} + gB_\mu \qquad \textit{with} \qquad (A_\mu^{\mathrm{cl}})_{ab} = \mathcal{Q}^a \delta_{ab}\delta_{\mu 0}$$

✓ one-loop result for the pressure with/without the BF

$$P_{\mathrm{BF}=0} = \sum_{ab} \frac{1}{3\pi^2}\int_0^\infty q^3 n_B(q) dq (1-\delta_{ab}/N)$$

$$P_{\mathrm{BF}\neq 0} = \sum_{ab} \frac{1}{6\pi^2}\int_0^\infty q^3 (n_B^{+ab}(q) + n_B^{-ab}(q)) dq (1-\delta_{ab}/N)$$

BF modified distribution function

$$n_B(q) \Longrightarrow \frac{n_B^{+ab}(q) + n_B^{-ab}(q)}{2} \qquad \textit{with} \qquad n_B^{\pm ab}(q) = \frac{1}{e^{\beta q \mp i\beta \mathcal{Q}^{ab}} - 1} \qquad \mathcal{Q}^{ab} \equiv \mathcal{Q}^a - \mathcal{Q}^b$$

The jet quenching parameter in the effective theory

- ***however,*** **the perturbation theory with BF**

  predicts a system which always favors the complete QGP phase where BF=0 & $\ell = 1$, no phase transition !

- **to drive the system to the confined phase, one needs to consider effective theory with the background field.**

matrix model for deconfinement

$$\mathcal{V}(\mathrm{q}) = \sum_{a,b=1}^{N} \mathcal{P}^{ab}_{ab} \left(\frac{2\pi^2 T^4}{3} B_4(|\mathrm{q}^{ab}|) + \frac{CT^2}{2} B_2(|\mathrm{q}^{ab}|) \right)$$

with $\mathrm{q}^a \equiv \mathcal{Q}/(2\pi T)$ $\quad \mathcal{P}^{ab}_{cd} \equiv \delta_{ac}\delta_{bd} - \frac{1}{N}\delta_{ab}\delta_{cd}$

$B_n(x)$: Bernoulli Polynomials

C: parameter with dimension of energy square.

- **equation of motion for the BF**

$$\sum_{b=1}^{N} \mathrm{sign}(\mathrm{q}^{ab}) \left(\frac{8\pi^2 T^2}{3} B_3(|q^{ab}|) + C B_1(|q^{ab}|) \right) = 0$$

non-pert. contribution is necessary to generate a non-zero BF from the eom

for SU(3) $\quad \mathcal{Q} = (\mathrm{q}, 0, -\mathrm{q})$ $\quad C = 40\pi^2 T_d^2/81$

$$\begin{cases} \mathrm{q}_{\mathrm{decon}} = \frac{1}{36}\left(9 - \sqrt{81 - 80(T/T_d)^2}\right) \\ \mathrm{q}_{\mathrm{con}} = \frac{1}{3} \end{cases}$$

$$\ell = \frac{1}{3}[1 + 2\cos(2\pi \mathrm{q})]$$

qualitatively agree with the T-dependence of the Polyakov loop as found in LQCD !!!

The jet quenching parameter in the effective theory

- **two ways to get the non-pert. term**

✓ **adding a mass term M^2 in the bare gluon pro., then expand the eff. potential in high T limit**

Meisinger, Miller and Ogilvieeldon, PRD 2002

✓ **adding a contribution from two dimensional ghost field to the eff. potential**

Hidaka and Pisarski, PRD 2021

$$\mathcal{S}_{\rm 2D} = \int_0^{1/T} d\tau \int \frac{d\Omega_{\hat{n}}}{4\pi} \int_{-\infty}^{\infty} d\hat{x} \int_{|x_\perp^2|>1/C} d^2x_\perp \, \mathrm{tr}\left((\hat{D}\bar{\phi})(\hat{D}\phi) + (D_\perp\bar{\phi})(D_\perp\phi) \right)$$

$$C \sim \int_{|k_\perp|<\sqrt{C}} \frac{d^2k_\perp}{(2\pi)^2}$$

$$\mathcal{V}_{\rm ghost} = (-)\, T \int_{-\infty}^{\infty} \frac{d\hat{k}}{2\pi} \int_{|k_\perp|<\sqrt{C}} \frac{d^2k_\perp}{(2\pi)^2} \sum_{a,b=1}^{N} \mathcal{P}_{ab}^{ab} \left(\log\left(1 - \mathrm{e}^{-\sqrt{\hat{k}^2+k_\perp^2}/T - 2\pi i q^{ab}}\right) + \mathrm{c.c.}\right)$$

- **the gluon self-energy from the effective theory**

$$\Pi^{ab,cd}_{\mathrm{eff};\mu\nu}(Q) = \left((\mathcal{M}^2)^{ab,cd} + g^2 C \frac{N}{4\pi^2} \mathcal{P}^{ab,cd} \right) \left(\Pi_T(Q) A_{\mu\nu} + \Pi_L(Q) B_{\mu\nu} \right)$$

$$(\mathcal{M}^2)^{ab,cd} = g^2 T^2 \left[\delta^{ad}\delta^{bc} \sum_{e=1}^{N} \left(B_2(q^{ae}) + B_2(q^{eb}) \right) - 2\delta^{ab}\delta^{cd} B_2(q^{ac}) \right]$$

The jet quenching parameter in the effective theory

- **the HTL resumed gluon propagator in the BF effective theory**

YG and Kuang, PRD 2021

off-diagonal components

$$i\tilde{D}^{ab,cd}_{\mu\nu}(Q,\vec{q}) \overset{a\neq b}{=\!=\!=} \delta^{ad}\delta^{bc}\left[\tilde{\Delta}^{(ab)}_T(Q,\vec{q})A_{\mu\nu} + \tilde{\Delta}^{(ab)}_L(Q,\vec{q})B_{\mu\nu}\right]$$

$$\begin{cases} \tilde{\Delta}^{(ab)}_T(Q,\vec{q}) = [Q^2 - \tilde{\Pi}^{(ab)}_T(\hat{\omega},\vec{q})]^{-1} \\ \tilde{\Delta}^{(ab)}_L(Q,\vec{q}) = [q^2 - \tilde{\Pi}^{(ab)}_L(\hat{\omega},\vec{q})]^{-1} \end{cases}$$

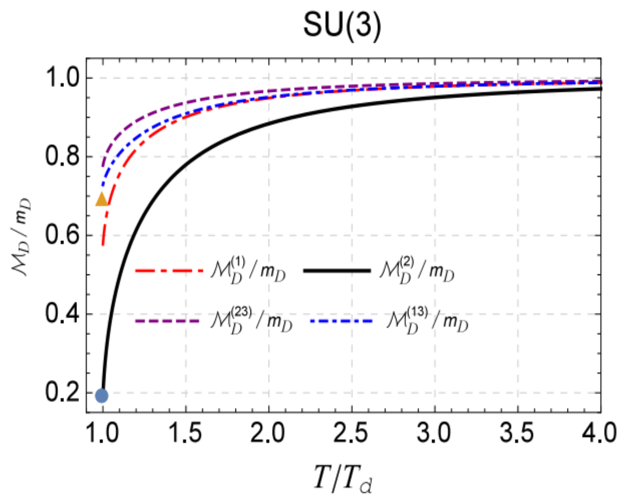
$$\mathcal{M}^{(23)}_D = \mathcal{M}^{(32)}_D = \mathcal{M}^{(12)}_D = \mathcal{M}^{(21)}_D = m_D\sqrt{1+\beta+7\mathrm{q}^2-5\mathrm{q}}\,, \qquad \mathcal{M}^{(13)}_D = \mathcal{M}^{(31)}_D = m_D\sqrt{1+\beta+10\mathrm{q}^2-6\mathrm{q}}\,,$$

diagonal components

$$i\sum_{ab}\mathcal{P}^{aa,bb}\tilde{D}^{aa,bb}_{\mu\nu}(Q,\vec{q}) = \sum_{i=1}^{2}\left[\tilde{\Delta}^{[i]}_T(Q,\vec{q})A_{\mu\nu} + \tilde{\Delta}^{[i]}_L(Q,\vec{q})B_{\mu\nu}\right]$$

$$\begin{cases} \tilde{\Delta}^{[i]}_T(Q,\vec{q}) = [Q^2 - \tilde{\Pi}^{[i]}_T(\hat{\omega},\vec{q})]^{-1} \\ \tilde{\Delta}^{[i]}_L(Q,\vec{q}) = [q^2 - \tilde{\Pi}^{[i]}_L(\hat{\omega},\vec{q})]^{-1} \end{cases}$$

$$\mathcal{M}^{[1]}_D = m_D\sqrt{1+\beta+6\mathrm{q}^2-6\mathrm{q}}\,, \qquad \mathcal{M}^{[2]}_D = m_D\sqrt{1+\beta+18\mathrm{q}^2-10\mathrm{q}}\,.$$



- **The screening strength is weakened by the BF**
- **The same Lorentz structure as vanishing BF**

The jet quenching parameter in the effective theory

- **Contributions from off-diagonal gluons to the jet quenching parameter**

➢ color structure

$$
\begin{aligned}
|\mathcal{M}|^2_{\rm off} &\sim \sum_{\rm colors}^{e\neq f, g\neq h} {\rm Tr}[t^{ef} t^{gh}] f^{ab,cd,ef} \tilde{D}^{fe,ef} f^{ba,dc,gh} (\tilde{D}^*)^{hg,gh} n^{ab}(k)(1+n^{cd}(k+w)) \\
&= \frac{1}{4} \sum_{a}^{e\neq f} \left[n^{ae}(k)(1+n^{fa}(k+w)) + n^{ea}(k)(1+n^{af}(k+w)) \right] \tilde{D}^{fe,ef} (\tilde{D}^*)^{ef,fe}
\end{aligned}
$$

each off-diagonal gluon, labelled by a pair of color indices *ef*, contributes similarly as its counterpart at vanishing background field

➢ result for SU(3)

$$
\begin{aligned}
\hat{q}_{\rm off} &= \frac{g^4}{3(4\pi)^3} \sum_{i=3}^{4} \int_0^\infty dk \int_{-k}^\infty d\omega \int_{|\omega|}^{\min(2k+\omega, \sqrt{\omega^2+\Lambda_\perp^2})} dq\, q^2 (1-\hat{w}^2) \mathcal{F}^{[i]} \\
&\times \left[|\tilde{\Delta}_T^{[i]}(Q, \vec{\mathrm{q}})|^2 (1-\hat{w}^2)^2 \left((w+2k)^2 + 3q^2 \right) + 2 |\tilde{\Delta}_L^{[i]}(Q, \vec{\mathrm{q}})|^2 \left((w+2k)^2 - 2q^2 \right) \right]
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{F}^{[3]} &= \left(2n(k) + n^{13}(k)\right)\left(1+n^{21}(k+w)\right) + \left(2n(k)+n^{31}(k)\right)\left(1+n^{12}(k+w)\right) \\
&+ n^{12}(k)\left(3+2n(k+w)+n^{31}(k+w)\right) + n^{21}(k)\left(3+2n(k+w)+n^{13}(k+w)\right),
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{F}^{[4]} &= n^{12}(k)\left(1+n^{12}(k+w)\right) + n^{21}(k)\left(1+n^{21}(k+w)\right) + \left(n^{13}(k)+n^{31}(k)\right) \\
&\times \left(1+n(k+w)\right) + n(k)\left(2+n^{13}(k+w)+n^{31}(k+w)\right).
\end{aligned}
$$

The jet quenching parameter in the effective theory

- **Contributions from diagonal gluons to the jet quenching parameter**

  - color structure

$$i\mathcal{M} \sim \sum_{e,f} f^{ab,cd,ee}\tilde{\mathcal{D}}^{ee,ff}t^{ff} = \sum_{e\neq f} f^{ab,cd,ee}\tilde{\mathcal{D}}^{ee,ff}t^{ff}$$

$$\tilde{\mathcal{D}}^{aa,bb} \equiv \tilde{\mathcal{D}}^{aa,bb} - \tilde{\mathcal{D}}^{bb,bb}$$

No ambiguity in determining the scattering amplitude

$$|\mathcal{M}|^2_{\rm dia} \sim \sum_{\rm colors}^{e\neq f, g\neq h} {\rm Tr}[t^{ff}t^{hh}] f^{ab,cd,ee}\tilde{\mathcal{D}}^{ee,ff} f^{ba,dc,gg}(\tilde{\mathcal{D}}^*)^{gg,hh} n^{ab}(k)(1+n^{cd}(k+w))$$
$$= \frac{1}{2}\sum_{i=1}^{2}\left[|\tilde{\Delta}_T^{[i]}(Q,\vec{\mathrm{q}})|^2 A^{\rho\sigma}A^{\rho'\sigma'} + |\tilde{\Delta}_L^{[i]}(Q,\vec{\mathrm{q}})|^2 B^{\rho\sigma}B^{\rho'\sigma'}\right]\mathcal{F}^{[i]}\,.$$

$$\mathcal{F}^{[1]} = \frac{3}{2}[n^{21}(k)(1+n^{12}(k+w)) + n^{12}(k)(1+n^{21}(k+w))]$$

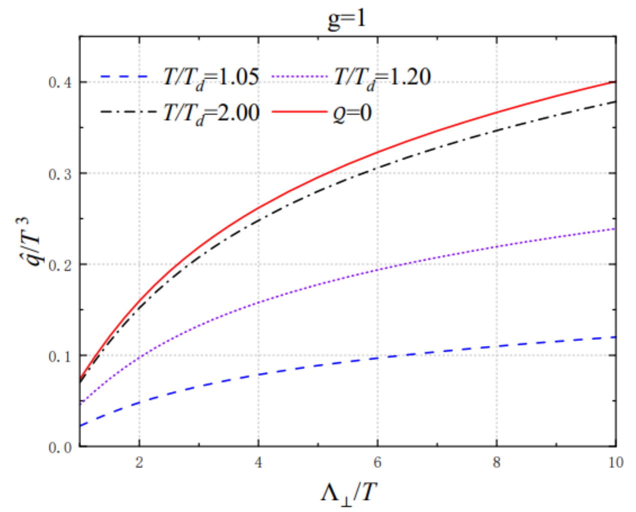
$$\mathcal{F}^{[2]} = n^{31}(k)(1+n^{13}(k+w)) + n^{13}(k)(1+n^{31}(k+w)) + \frac{1}{2}[n^{12}(k)(1+n^{21}(k+w)) + n^{21}(k)(1+n^{12}(k+w))]$$

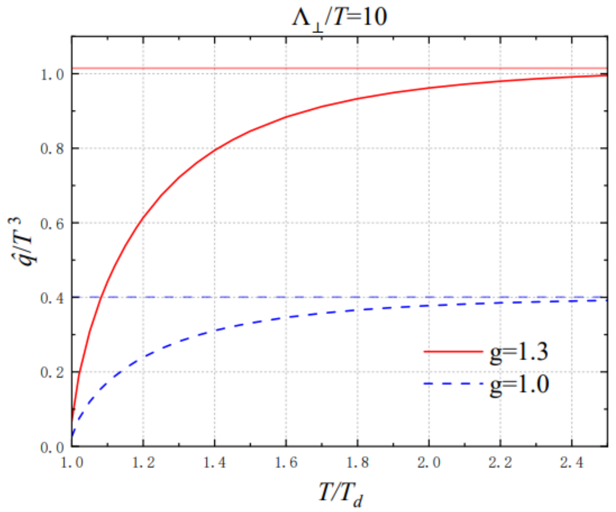
  - total result (diagonal+off-diagonal) for SU(3)

$$\hat{q} = \frac{g^4}{3(4\pi)^3}\sum_{i=1}^{4}\int_0^\infty dk\int_{-k}^{\infty} d\omega\int_{|\omega|}^{\min(2k+\omega,\sqrt{\omega^2+\Lambda_\perp^2})} dq\, q^2(1-\hat{w}^2)\mathcal{F}^{[i]}$$
$$\times\left[|\tilde{\Delta}_T^{[i]}(Q,\vec{\mathrm{q}})|^2(1-\hat{w}^2)^2\left((w+2k)^2+3q^2\right) + 2|\tilde{\Delta}_L^{[i]}(Q,\vec{\mathrm{q}})|^2\left((w+2k)^2-2q^2\right)\right]$$

Results and discussions

- **at fixed gauge coupling**

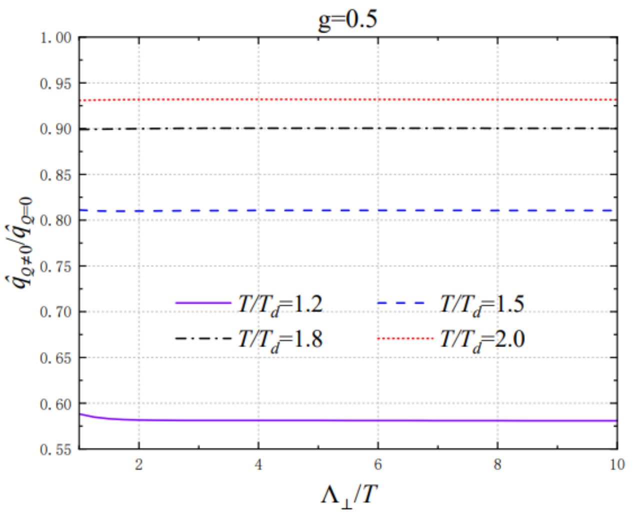


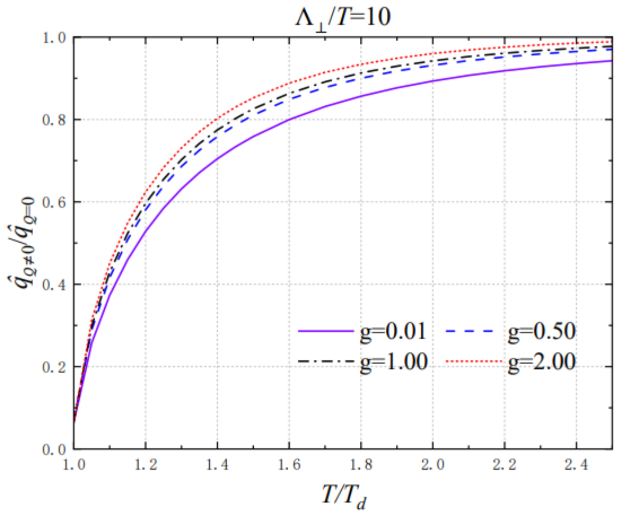


Reduced $\hat{q}/T^3$

As a function of the cutoff, growth rate of $\hat{q}/T^3$ declines, especially near T_d

BF effects only significant in semi-QGP



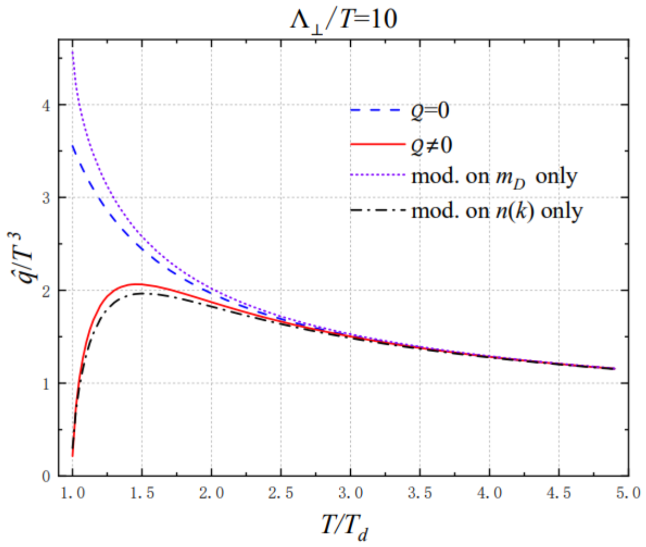


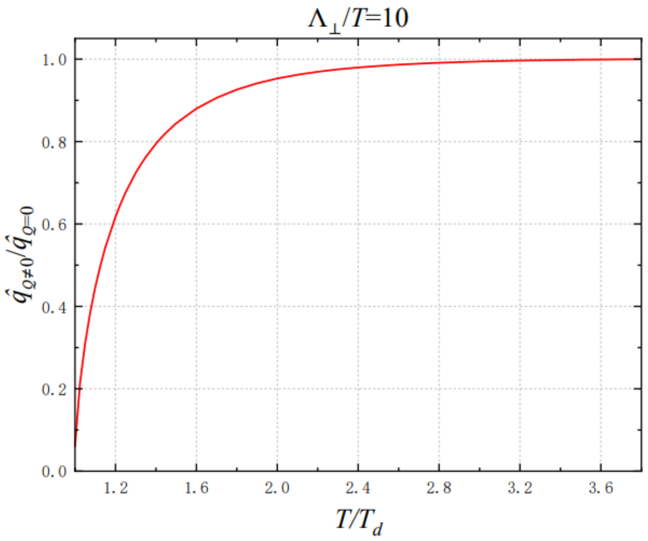
$\hat{q}$-ratio is insensitive to the cutoff

$\hat{q}$-ratio has moderate dependence on the gauge coupling

$\hat{q}$-ratio can be considered as a function of T only

- with 2-loop running coupling





non-monotonic *T*-dependence
- **high *T*-region, running effect dominant**
- **low *T*-region, BF effect dominant**

agree with Lattice result

Kumar, Majumder and Weber, PRD 2022

in perturbation theory with BF=0, increased $\hat{q}/T^3$ with decreasing T due to running coupling effect

BF-modification
- **on the screening mass** ⟹ **enhanced $\hat{q}/T^3$**
- **on the dis. functions** ⟹ **reduced $\hat{q}/T^3$ (dominant)**

$$\hat{q}_{\rm soft} = \frac{g^2 C_R T m_D^2}{4\pi} \ln\left(1 + \frac{\Lambda_\perp^2}{m_D^2}\right) .$$

$\hat{q}$-ratio encodes all the BF effects, for phenomenological applications

$$1 - a\,\mathsf{q}^2 + b\,\mathsf{q}^3$$ **with** $a \approx 43.89$ and $b \approx 113.74$

Summary

- We studied the jet-quenching parameter with an effective theory in semi-QGP, where a background field is self-consistently introduced to describe the non-trivial *T*-dependence of the Polyakov loop.

- The background field modifies both the thermal distribution function and the resumed propagator, the net effect is a reduction of the dimensionless $\hat{q}/T^3$.

- With the running coupling effect, $\hat{q}/T^3$ shows a non-monotonic *T*-dependence. A strong suppression in semi-QGP is found which agrees with the Lattice simulations.

- The $\hat{q}$-ratio encodes all the background field effects. Its *T*-dependence can be reproduced by a simple parametrization that could be useful for phenomenological applications.

Thank You for Your Attention