Overview of parton showers and jet properties in proton-proton collisions

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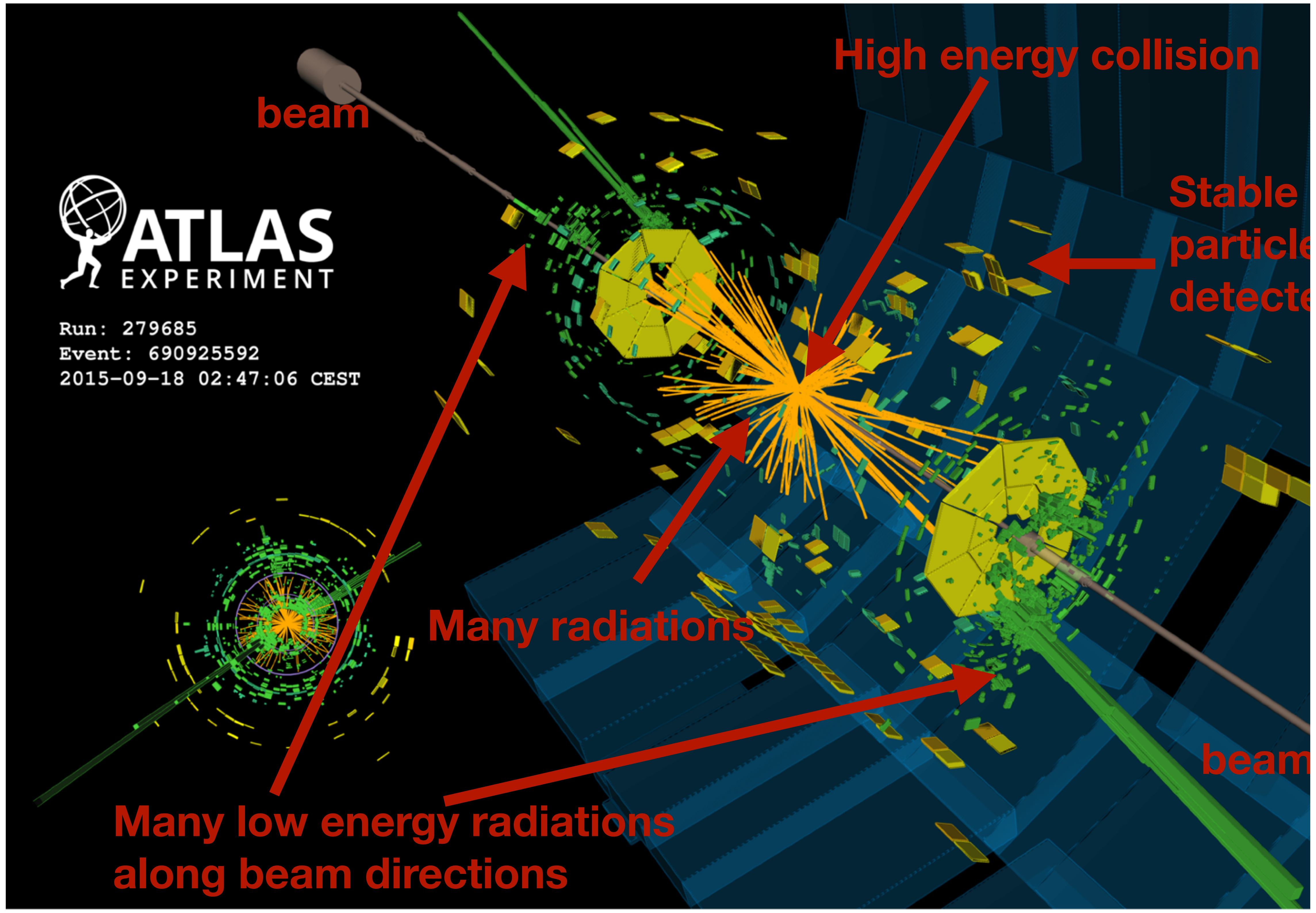
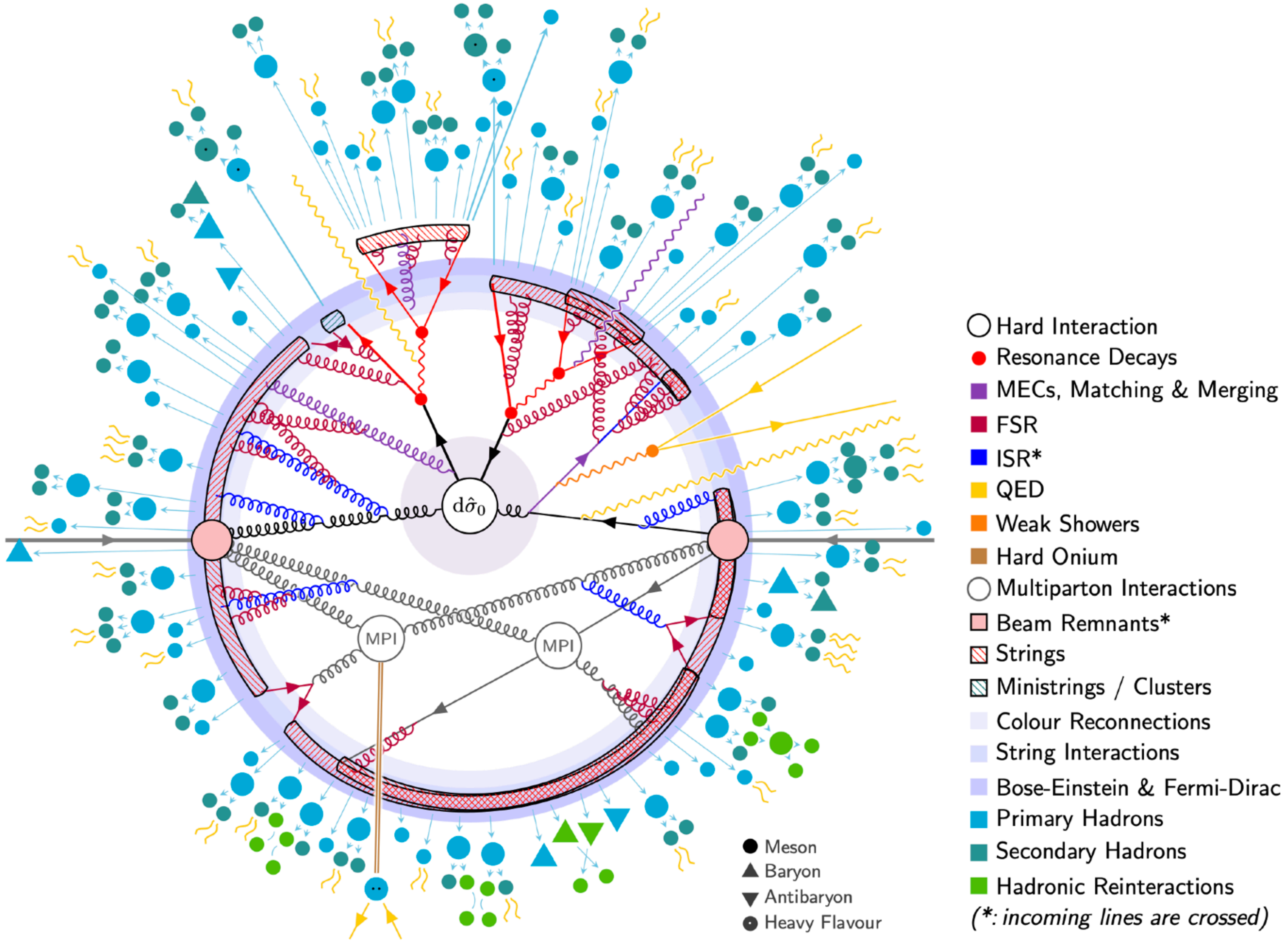
华中师范大学粒子物理研究所华中核理论中心 (C3NT)

Outline

1. Introduction
2. Parton Showers
3. Matching and Merging
4. Showers and Jet
5. Summary

1.Introduction





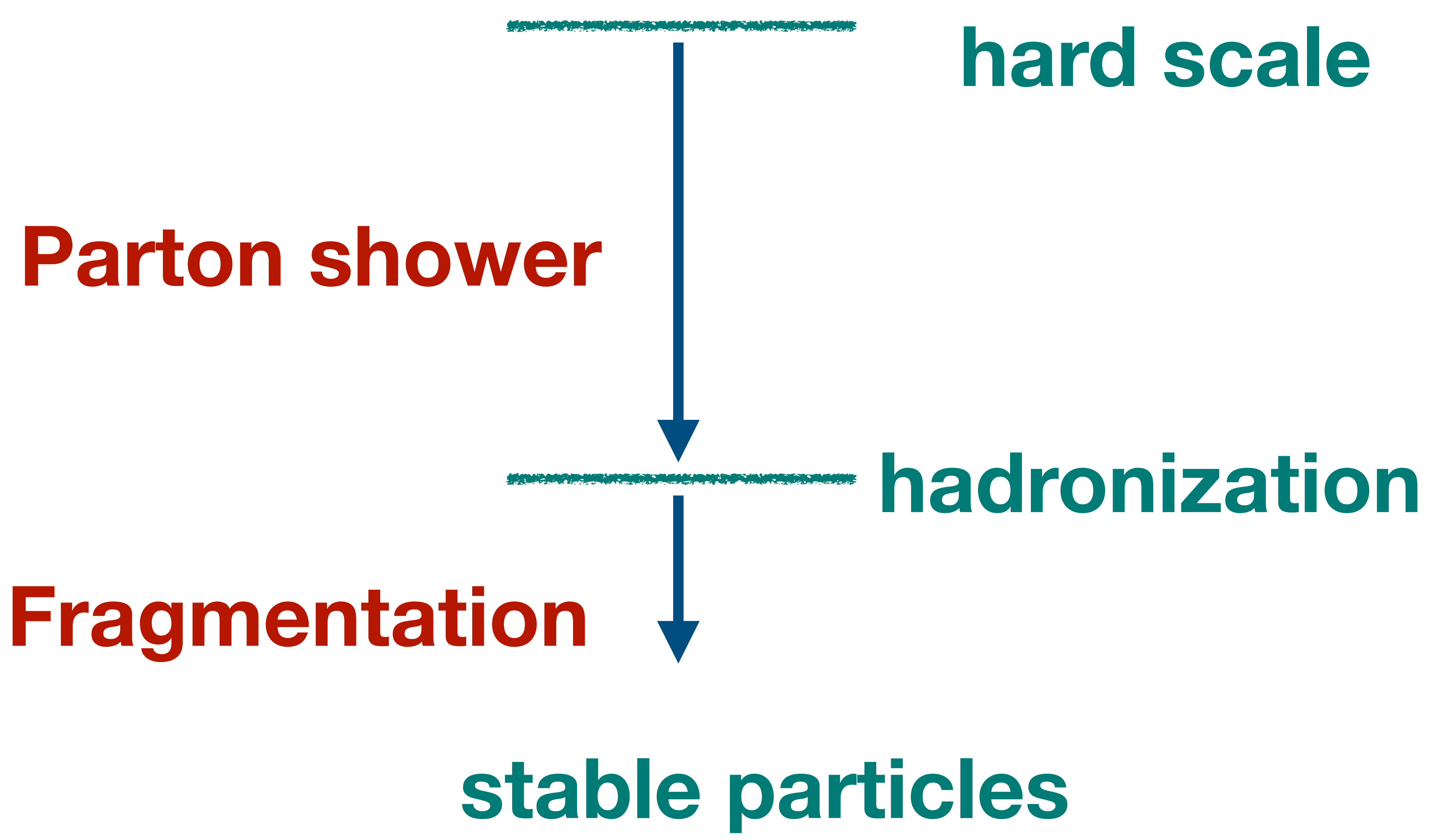
From PYTHIA 8.3

1.Introduction

The purpose of Monte Carlo event generators is to generate events in as much details as nature (generate average and fluctuation right)

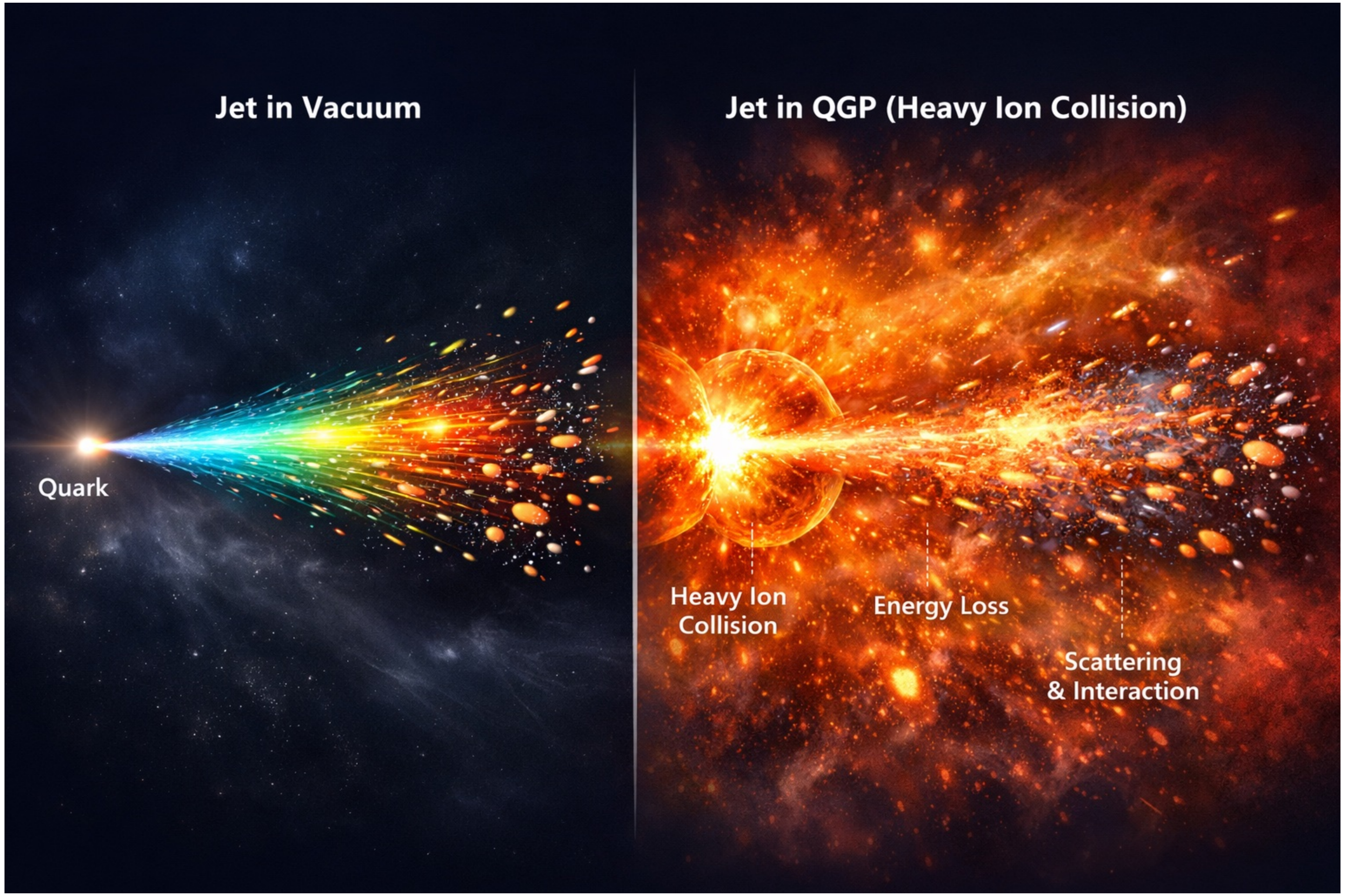
$$\mathscr{P}_{\text{event}} = \mathscr{P}_{\text{Hard}} \otimes \mathscr{P}_{\text{Decay}} \otimes \mathscr{P}_{\text{ISR}} \otimes \mathscr{P}_{\text{FSR}} \otimes \mathscr{P}_{\text{MPI}} \otimes \mathscr{P}_{\text{Had}} \cdots$$

- ❑ **Hard process in high energy**
- ❑ **Transition from high energy to low energy**
  —parton shower
- ❑ **Low energy soft regime**
  —fragmentation



Parton shower: a model for the evolution from high scale to hadronization scale

1.Introduction



$\mathscr{P}_{\text{event in p+p}} \otimes \mathscr{P}_{\text{FSR in QGP}}$

In heavy-ion collisions

- initial state: PDFs to nPDFs
- branching probabilities are modified by the presence of the QGP
- alters the Sudakov form factor and introduces medium-induced emissions

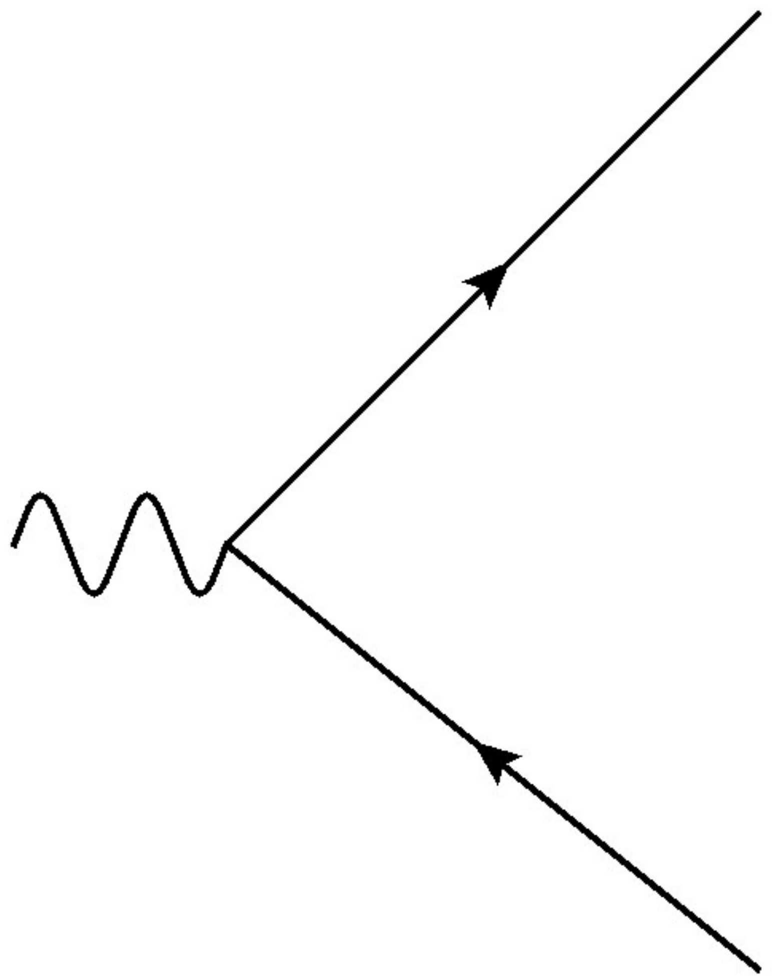
Physical consequences

- Partons lose energy as they pass through the QGP
- Jet substructures, such as jet mass, fragmentation, and angular distributions, are modified

2. Parton Showers

Starting points

Parton showers approximate higher-order real-emission corrections to the hard scattering process



In the collinear or soft limit, the matrix element can be factorized as

$$|M(\cdots, p_i, p_j, \cdots)|^2 \xrightarrow{i||j} g_s^2 \mathscr{C} \frac{P(z)}{s_{ij}} |M(\cdots, p_i + p_j, \cdots)|^2$$

$$|M(\cdots, p_i, q, p_j, \cdots)|^2 \xrightarrow{q \to 0} g_s^2 \mathscr{C} \frac{p_i \cdot p_j}{p_i \cdot q \; p_j \cdot q} |M(\cdots, p_i, p_j, \cdots)|^2$$

n+1 external legs | **n external legs**

Together with phase space integration, the cross section is

$$d\sigma_{n+1} = \frac{1}{2s} \int d\phi_{n+1} |M_{n+1}|^2 = d\sigma_n \otimes d\phi_{n \to n+1} \times \frac{|M_{n+1}|^2}{|M_n|^2}$$

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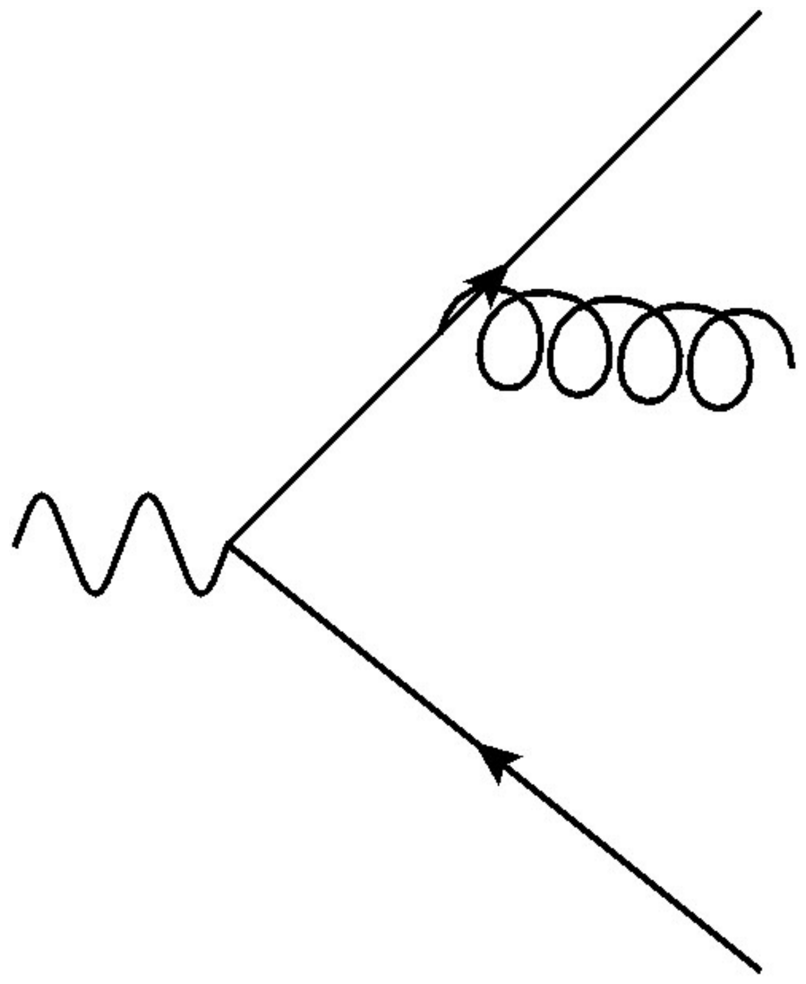

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n+1 external legs (left side) — **n external legs** (right side)

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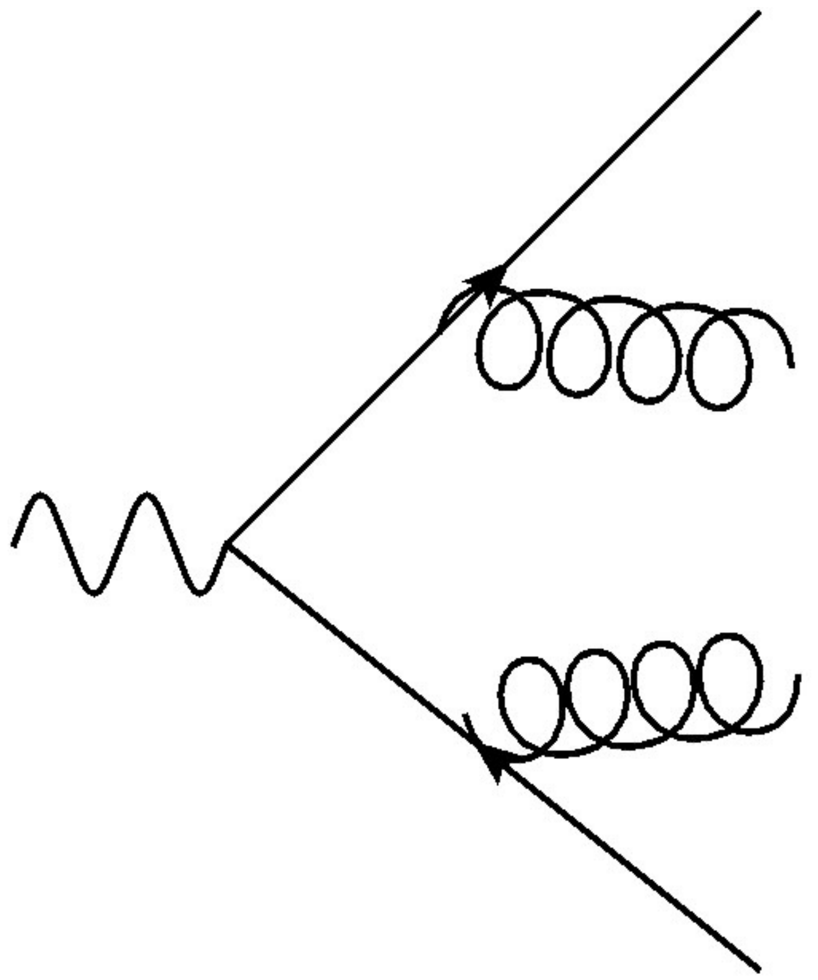

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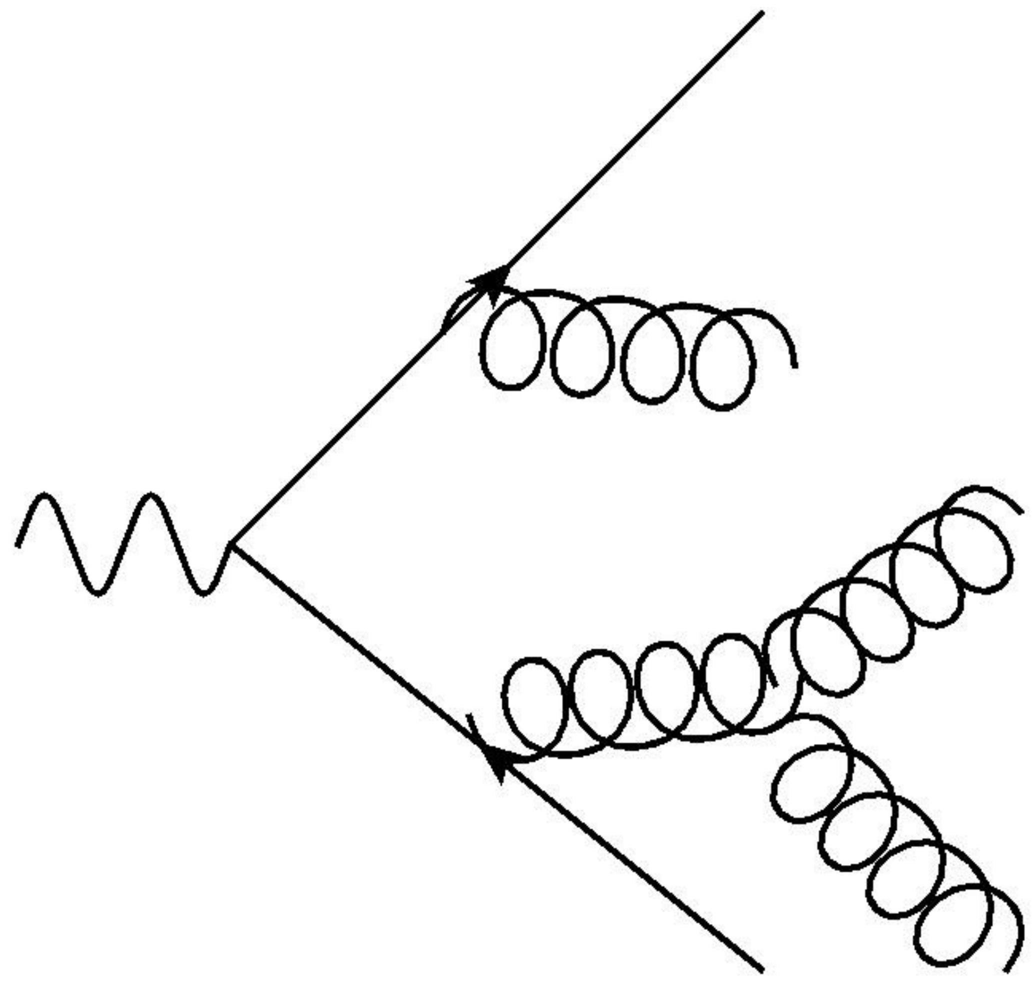

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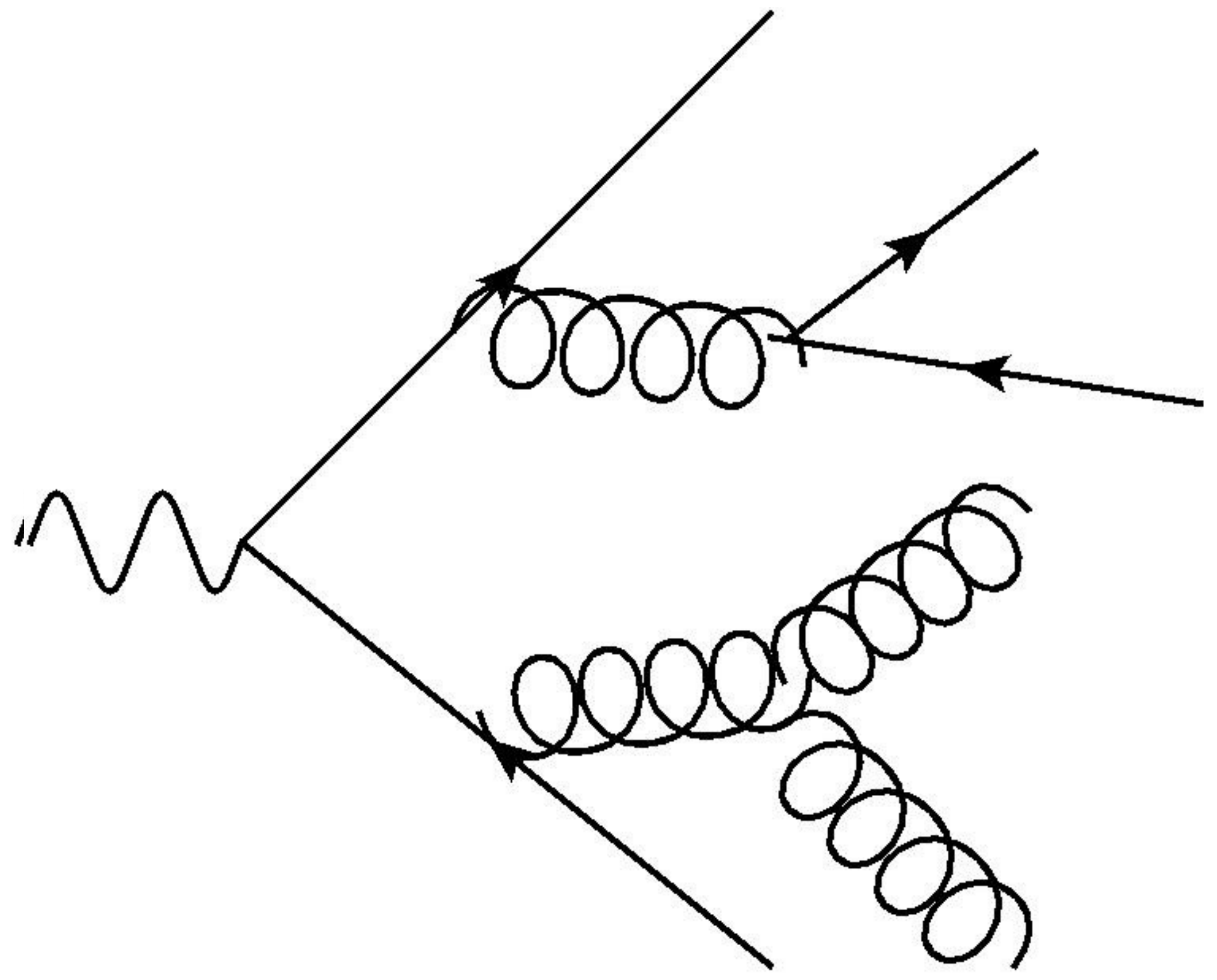

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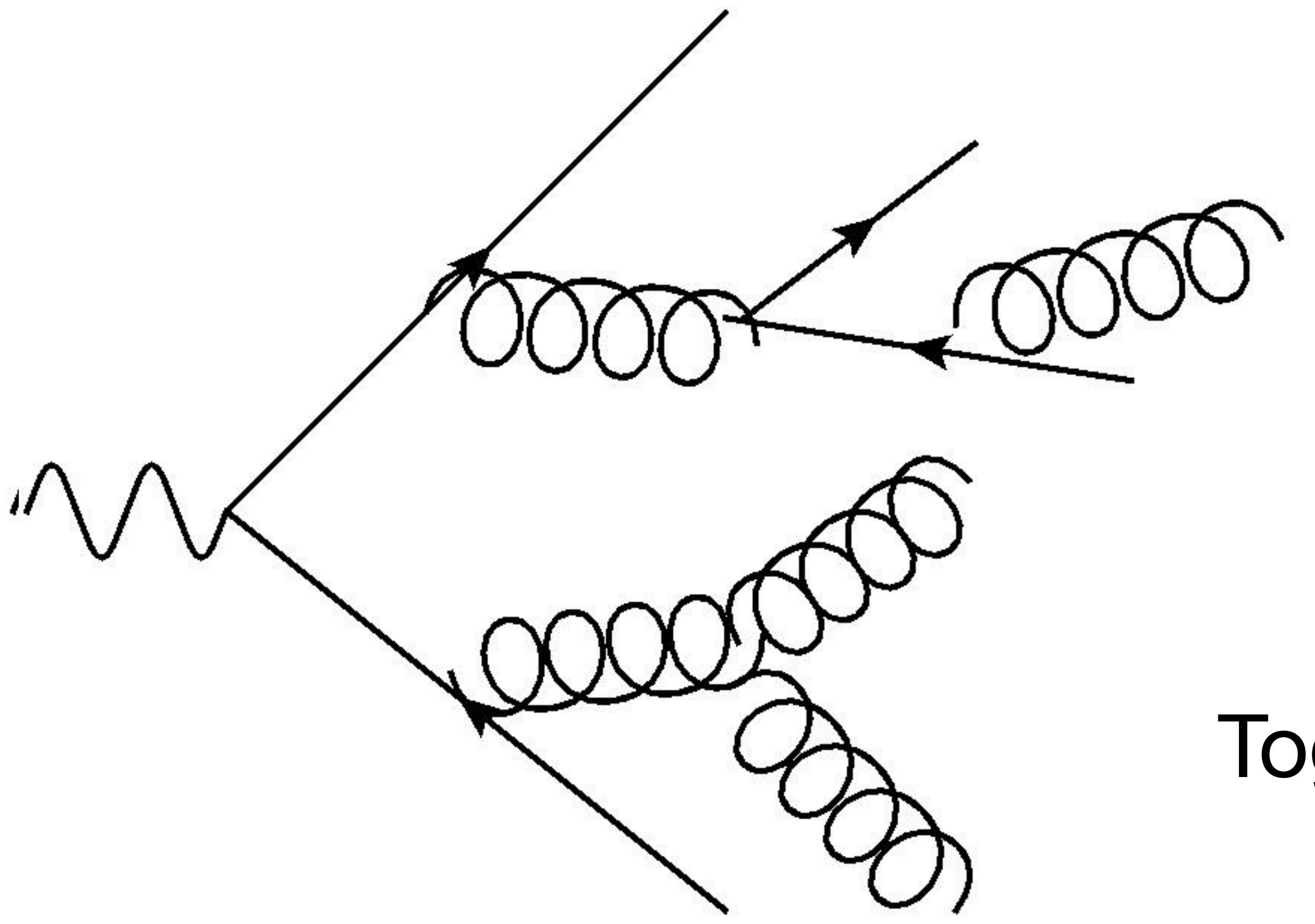

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2. Parton Showers

Starting points

Sudakov form factor: Non-branching probability $\exp\left[\int d\phi_{n\to n+1}\frac{|M_{n+1}|^2}{|M_n|^2}\right]$

choose kinematic variable as the evolution scale

Probability that there is no branching from Q to q is $\Delta_i\left(Q^2, q^2\right)$

$$\Delta\left(Q^2, q^2\right) = \exp\left\{\int_{Q^2}^{q^2} d\phi_{n\to n+1}\frac{|M_{n+1}|^2}{|M_n|^2}\right\}$$

Probability for one observed branching $1-\Delta\left(Q^2, q^2\right)$

Probability one branching between the scale q^2 to q^2+dq^2

$$\frac{d}{dq^2}\Delta\left(Q^2, q^2\right) = \Delta\left(Q^2, q^2\right)\times d\phi_{n\to n+1}\frac{|M_{n+1}|^2}{|M_n|^2}$$

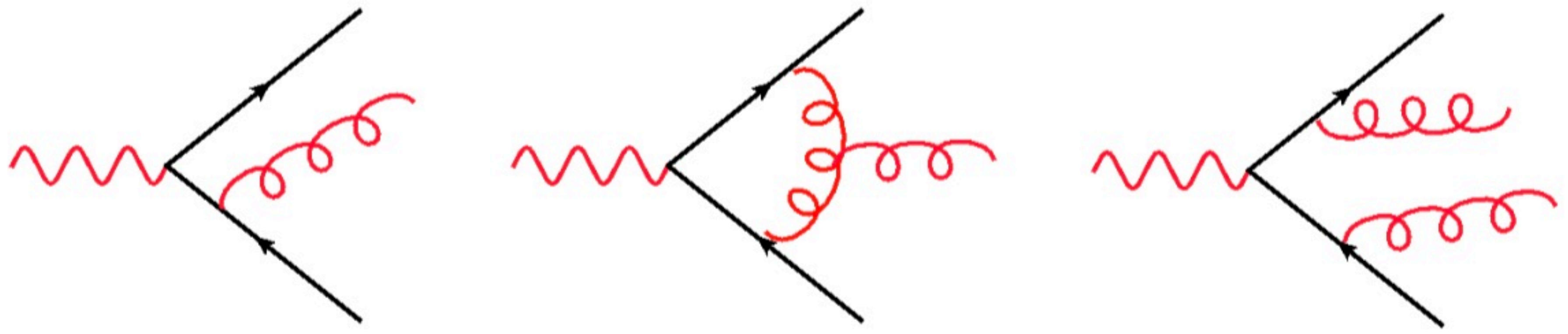
Additional radiations can be added according to the function $\Delta\left(Q^2, q^2\right)$

2. Parton Showers

Improve the splitting kernels

NLO corrections to resummation kernel

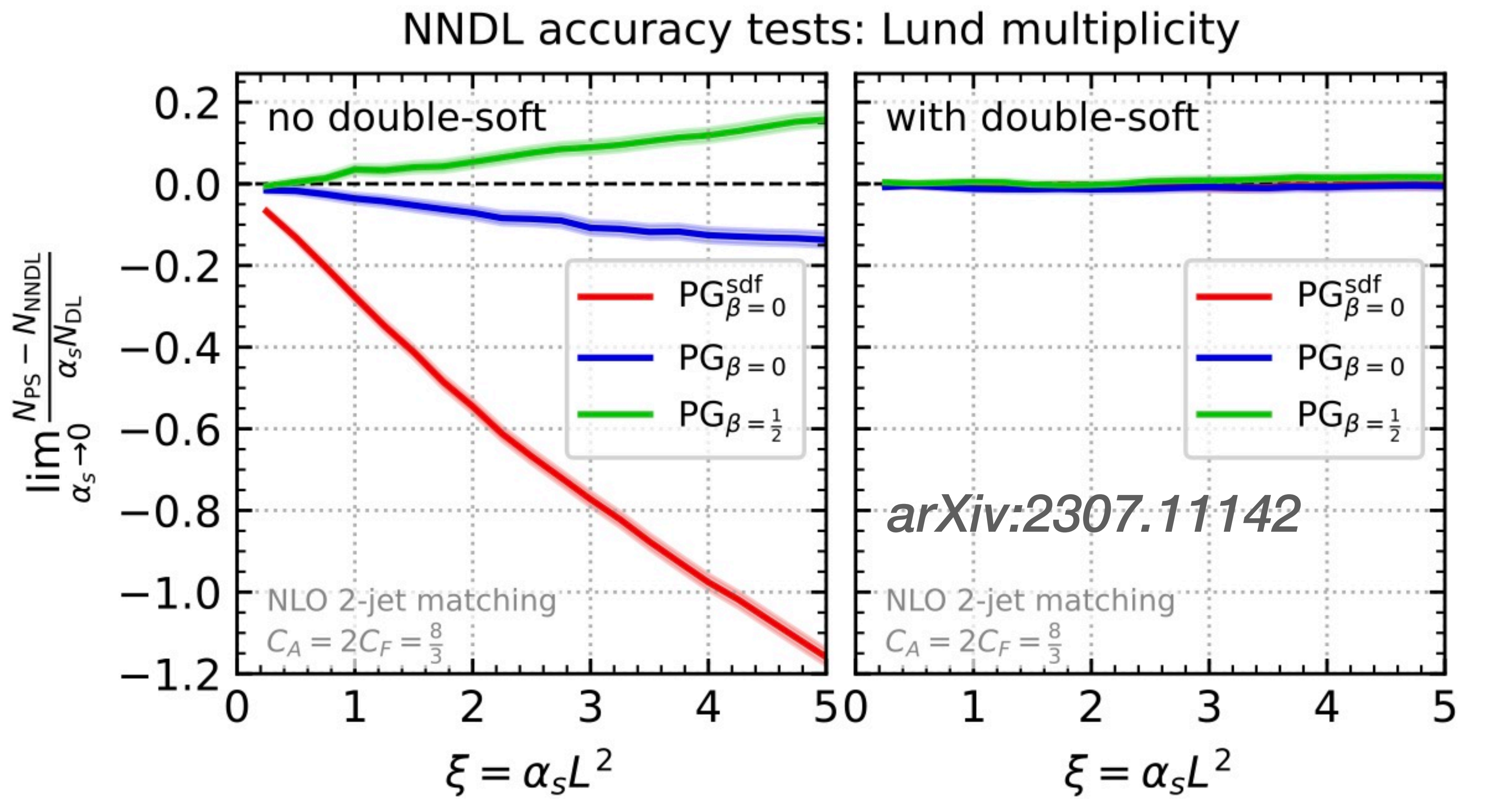
What we expect for NLO showers

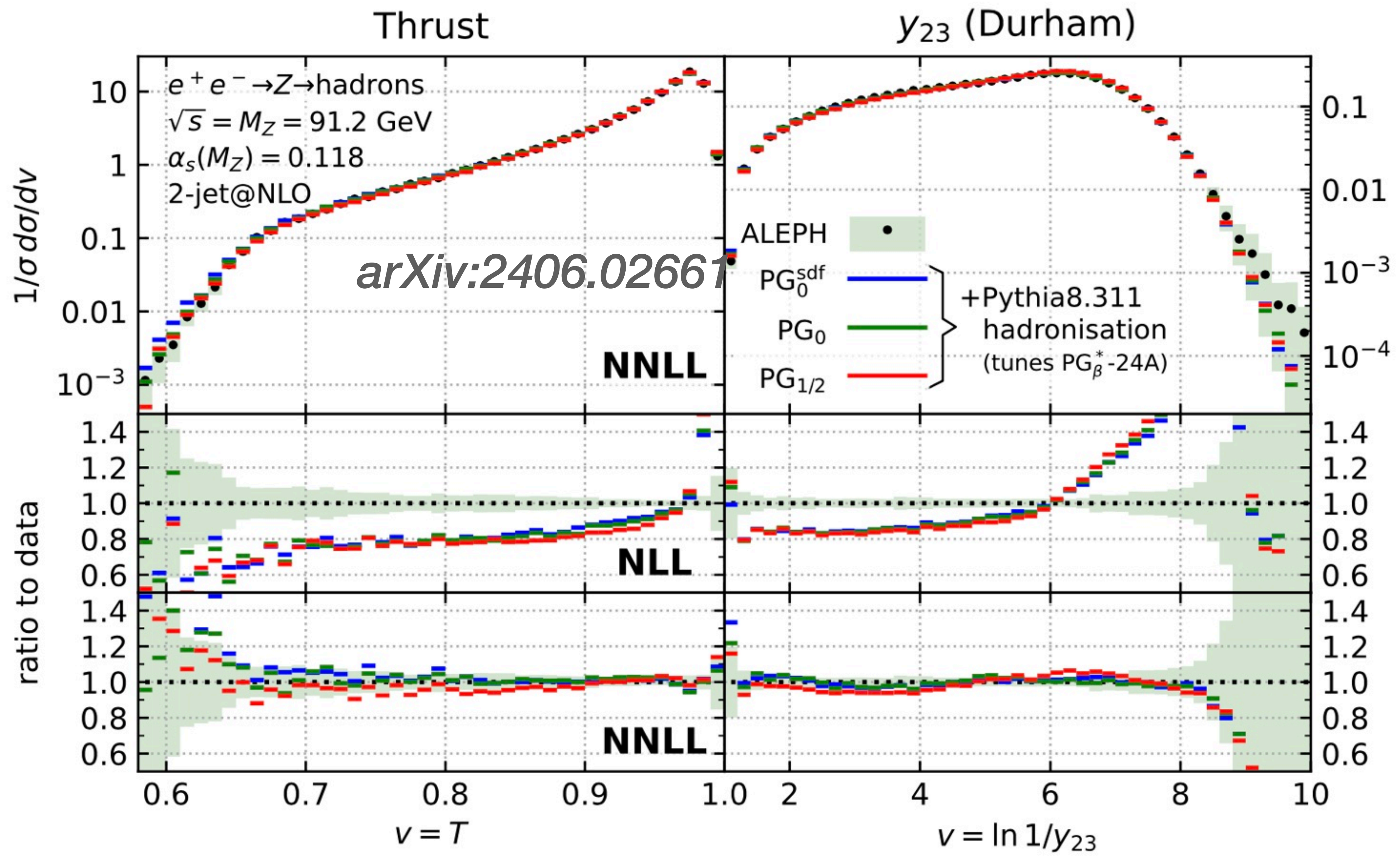


NLO parton shower

$$\frac{d}{dQ^2}\underbrace{\left(1-\Delta\left(Q_0^2,Q^2\right)\right)}_{\text{branching probability}} = -\underbrace{\int \frac{d\Phi_3}{d\Phi_2}\delta\left(Q^2-Q^2\left(\Phi_3\right)\right)\left(a_3^0+a_3^1\right)\Delta\left(Q_0^2,Q^2\right)}_{\text{born and virtual correction}} - \underbrace{\int \frac{d\Phi_4}{d\Phi_2}\delta\left(Q^2-Q^2\left(\Phi_4\right)\right)a_4^0\,\Delta\left(Q_0^2,Q^2\right)}_{\text{real correction}}$$

HTL, Skands, arXiv:1611.00013

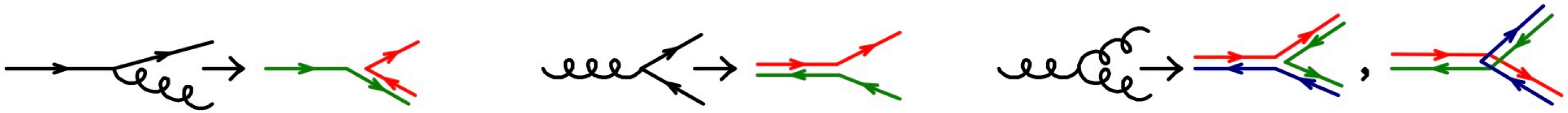




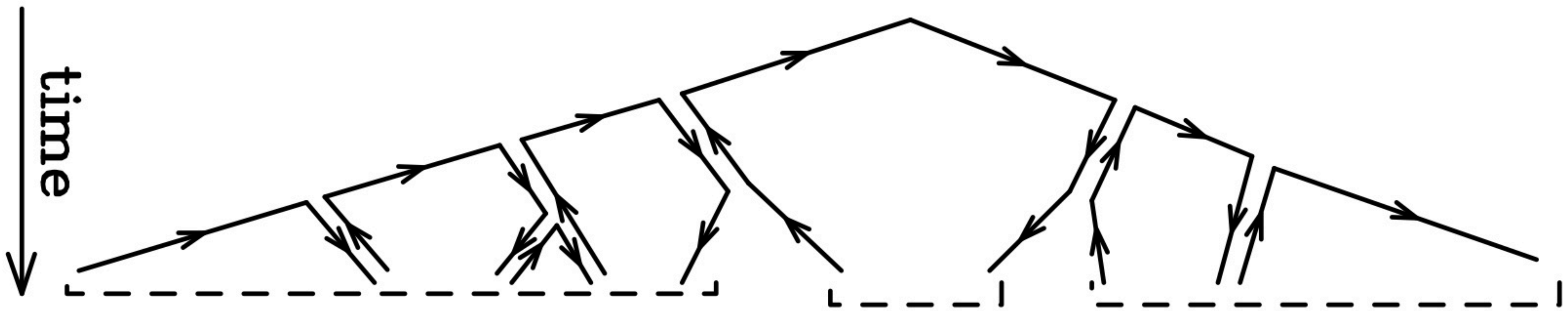
2. Parton Showers

Improve the Colors

Leading Color Approximation: Dipole Shower

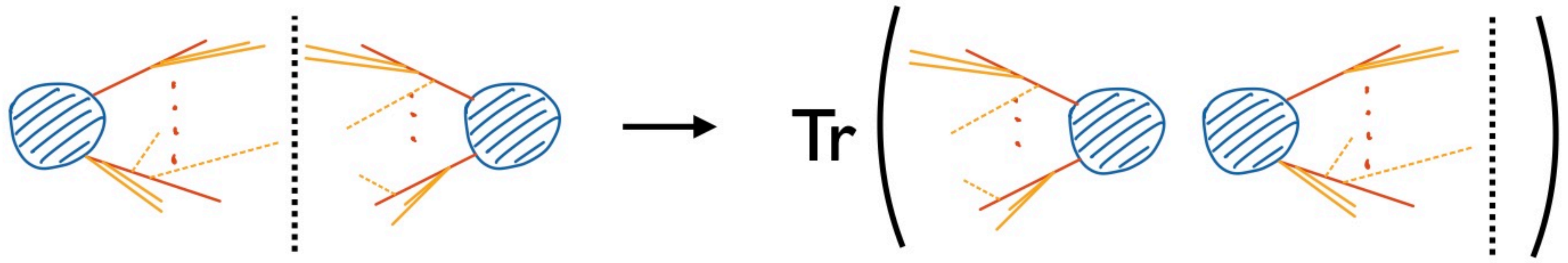


QCD radiation in this approximation is always simulated as the radiation from a single color dipole, rather than a coherent sum from a color multipole.



a color density operator   *Deductor, arXiv:1902.02105*   *CVolver, arXiv:2502.12133*

simulates parton showers at the amplitude level with full color information



$$\mathbf{A}_n(E) = \mathbf{V}_{E,E_n}\mathbf{D}_n^{\mu}\mathbf{A}_{n-1}(E_n)\mathbf{D}^{\dagger}_{n\mu}\mathbf{V}^{\dagger}_{E,E_n}\Theta(E \le E_n),$$

2. Parton Showers

Fixed order

Sunshine by @vector_corp on freepik.es

Sunshine

Sudakov Nesting of Hard Integrals

Using generalized parton shower to generate fixed order corrections

Fixed order should look like $$\frac{\mathrm{d}\mathcal{P}}{\mathrm{d}\Phi_n} = |M_0|^2 \prod_{i=0}^{n-1} \mathrm{ant}_{i\mapsto i+1}$$

matrix element ratio $(0 \to 1) \times (1 \to 2) \times \cdots \times (n-1 \to n)$

Usually showers will give $(0 \to n)$

$$\frac{\mathrm{d}\mathcal{P}_{00\ldots0}}{\mathrm{d}\Phi_n} = |M_0|^2 \prod_{i=0}^{n-1} \mathrm{ant}_{i\mapsto i+1} \Delta_i(t_i, t_{i+1})$$

Sudakov factor from showers $\Delta_0 \times \Delta_1 \times \cdots \times \Delta_{(n-1)}$

Altmann, HTL, Scyboz, Skands, arXiv:2507.00111

2. Parton Showers

Fixed order

Sunshine by @vector_corp on freepik.es

Sunshine

Sudakov Nesting of Hard Integrals

Using generalized parton shower to generate fixed order corrections

keep the parent events after branching, and ask the event branches m times at stage $0 \to 1$, then shower them afterwards

$$\frac{\mathrm{d}\mathcal{P}_{m0\ldots0}}{\mathrm{d}\Phi_n} = \frac{\mathrm{d}\mathcal{P}_{00\ldots0}}{\mathrm{d}\Phi_n} \prod_{j=1}^{m} \int_{t_1}^{\tilde{t}_{j-1}} \mathrm{ant}_{0\mapsto1}(\tilde{t}_j)\,\mathrm{d}\tilde{t}_j$$

keep all the intermediate states and shower them m_k times from $k-1$ partons to k partons

$$\frac{\mathrm{d}\mathcal{P}_{m_1m_2\ldots m_n}}{\mathrm{d}\Phi_n} = \frac{\mathrm{d}\mathcal{P}_{00\ldots0}}{\mathrm{d}\Phi_n} \prod_{k=1}^{n} \prod_{j=1}^{m_k} \int_{t_k}^{\tilde{t}_{k_j-1}} \mathrm{ant}_{k-1\mapsto k}(\tilde{t}_{k_j})\,\mathrm{d}\tilde{t}_{k_j}$$

sum m_k to infinity

$$\sum_{m_k\geq0} \frac{\mathrm{d}\mathcal{P}_{m_1m_2\ldots m_n}}{\mathrm{d}\Phi_n} = \frac{\mathrm{d}\mathcal{P}_{00\ldots0}}{\mathrm{d}\Phi_n} \prod_{k=1}^{n} \frac{1}{\Delta_k(t_{k-1},t_k)}$$

$$\textsc{Sunshine}: \quad \sum_{m_k\geq0} \frac{\mathrm{d}\mathcal{P}_{m_1m_2\ldots m_n}}{\mathrm{d}\Phi_n} = |M_0|^2 \prod_{i=0}^{n-1} \mathrm{ant}_{i\mapsto i+1}\,.$$

Altmann, HTL, Scyboz, Skands, arXiv:2507.00111

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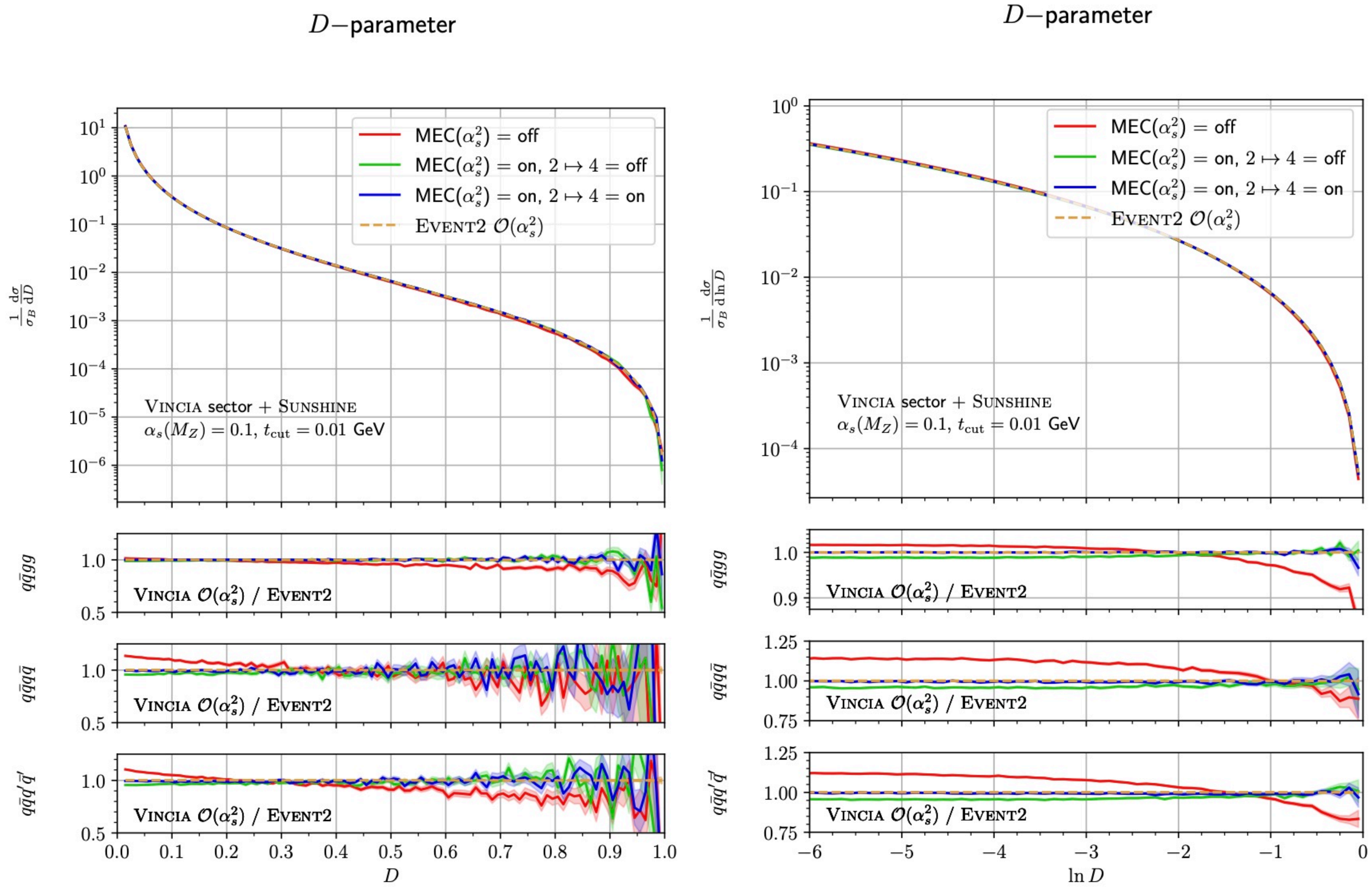
Fixed order

Sunshine by @vector_corp on freepik.es

Sunshine

Sudakov Nesting of Hard Integrals

Using generalized parton shower to generate fixed order corrections



Altmann, HTL, Scyboz, Skands, arXiv:2507.00111

2. Parton Showers

Resummation and accuracy

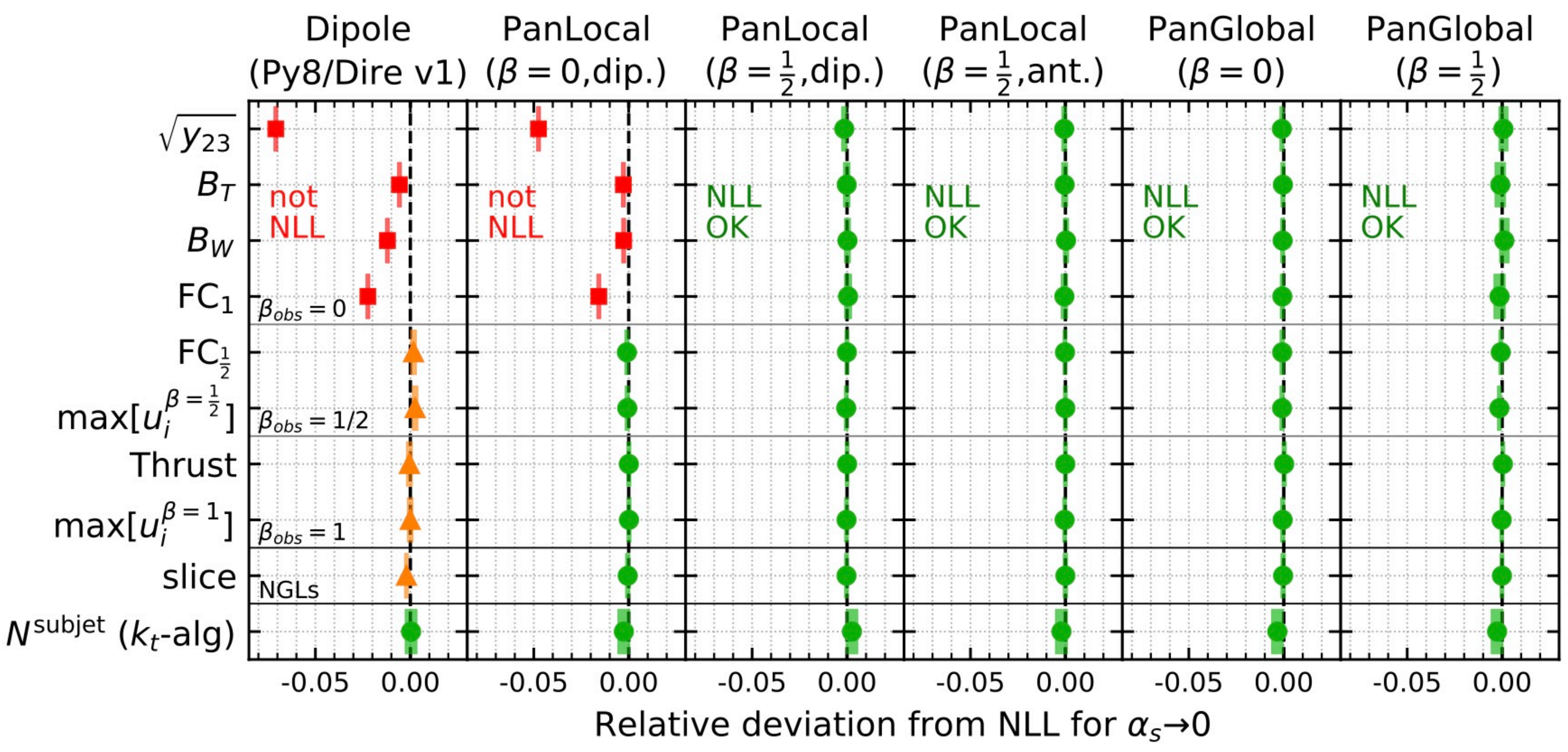
固定阶计算

	NLO	NNLO	NNNLO		重求和
$z\frac{1}{\sigma_0}\frac{d\sigma}{dz} \propto z +$	$\frac{\alpha_s}{4\pi} a_{1,2} \ln z +$	$\left[\frac{\alpha_s}{4\pi}\right]^2 a_{2,4} \ln^3 z +$	$\left[\frac{\alpha_s}{4\pi}\right]^3 a_{3,6} \ln^5 z$	$+ \cdots$	LL
$+$	$\frac{\alpha_s}{4\pi} a_{1,1} +$	$\left[\frac{\alpha_s}{4\pi}\right]^2 a_{2,3} \ln^2 z +$	$\left[\frac{\alpha_s}{4\pi}\right]^3 a_{3,5} \ln^4 z$	$+ \cdots$	NLL
$+$	$\frac{\alpha_s}{4\pi} a_{1,0} z +$	$\left[\frac{\alpha_s}{4\pi}\right]^2 a_{2,2} \ln z +$	$\left[\frac{\alpha_s}{4\pi}\right]^3 a_{3,4} \ln^3 z$	$+ \cdots$	NNLL
	$+$	$\left[\frac{\alpha_s}{4\pi}\right]^2 a_{2,1} +$	$\left[\frac{\alpha_s}{4\pi}\right]^3 a_{3,3} \ln^2 z$	$+ \cdots$	NNNLL
	$+$	$\left[\frac{\alpha_s}{4\pi}\right]^2 a_{2,0} z +$	$\left[\frac{\alpha_s}{4\pi}\right]^3 a_{3,2} \ln z$	$+ \cdots$	NNNNLL
		$+$	$\left[\frac{\alpha_s}{4\pi}\right]^3 a_{3,2}$	$+ \cdots$	
		$+$	$\left[\frac{\alpha_s}{4\pi}\right]^3 a_{3,2} z$	$+ \cdots$	

NLL: PanScales, Alaric, Herwig et al

with higher order effects: Vincia, DIRE et al

PanScales:



For observables that involve scale hierarchies resummation is required

arXiv:2002.11114

2. Parton Showers

Resummation and accuracy

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$+$	$\frac{\alpha_s}{4\pi}a_{1,1}$ $+$	$\left[\frac{\alpha_s}{4\pi}\right]^2 a_{2,3}\ln^2 z +$	$\left[\frac{\alpha_s}{4\pi}\right]^3 a_{3,5}\ln^4 z$	$+\cdots$	NLL
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	$+$	$\left[\frac{\alpha_s}{4\pi}\right]^2 a_{2,1}$ $+$	$\left[\frac{\alpha_s}{4\pi}\right]^3 a_{3,3}\ln^2 z$	$+\cdots$	NNNLL
	$+$	$\left[\frac{\alpha_s}{4\pi}\right]^2 a_{2,0}z$ $+$	$\left[\frac{\alpha_s}{4\pi}\right]^3 a_{3,2}\ln z$	$+\cdots$	NNNNLL
		$+$	$\left[\frac{\alpha_s}{4\pi}\right]^3 a_{3,2}$	$+\cdots$	
		$+$	$\left[\frac{\alpha_s}{4\pi}\right]^3 a_{3,2}z$	$+\cdots$	

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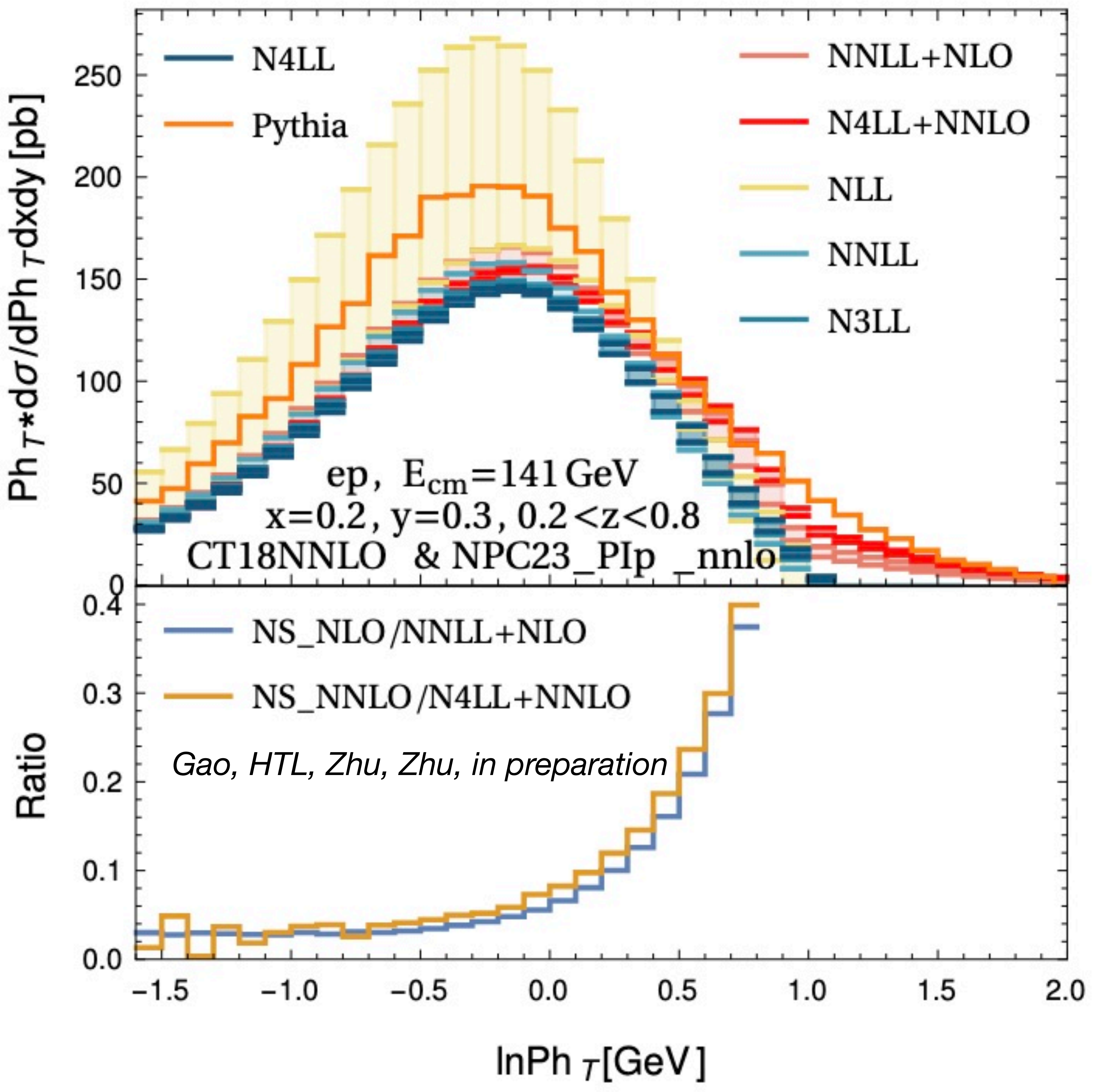

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2. Parton Showers

Resummation and accuracy

NLL accuracy is the becoming the new standard

PanScales

Logarithmic accuracy of parton showers: a fixed-order study
Dasgupta, Dreyer, Hamilton, Monni, Salam [1805.09327]

Parton showers beyond leading logarithmic accuracy
Dasgupta, Dreyer, Hamilton, Monni, Salam, Soyez [2002.11114]

Colour and logarithmic accuracy in final-state parton showers
Hamilton, Medves, Salam, Scyboz, Soyez [2011.10054]

Spin correlations in final-state parton showers and jet observables
Karlberg, Salam, Scyboz, Verheyen [2103.16526]

Soft spin correlations in final-sta parton showers
Hamilton, Karlberg, Salam, Scyboz, Verhe [2111.01161]

PanScales parton showers for hadron collisions: formulation and fixed-order studies
van Beekveld, Ferrario Ravasio, Salam, Soto Ontoso, Soyez, Verheyen [2205.02237]

PanScales parton showers for hadron collisions: all-order validation
van Beekveld, Ferrario Ravasio, Hamilton, Salam, Soto Ontoso, Soyez, Verheyen [2207.09467]

Next-to-leading-logarithmic PanScales showers for deep inelastic scattering and vector boson fusion
van Beekveld, Ferrario Ravasio [2305.08645]

Introduction to the PanScales framework, version 0.1
van Beekveld, Dasgupta, El-Menoufi, Ferrario Ravasio, Hamilton, Helliwell, Karlberg, Medves, Monnim Salam, Scyboz, Soto Ontoso, Soyez, Verheyen [2312.13275]

Building a consistent parton shower
Forshaw, Holguin, Plätzer [2003.06400]

Improvements on dipole shower colour
Forshaw, Holguin, Plätzer [2011.15087]

Alaric

A new approach to color-coherent parton evolution
Herren, Höche, Krauss, Reichelt, Schönherr [2208.06057]

New approach to QCD final-state evolution i processes with massive partons
Assi, Höche [2307.00728]

The Alaric parton shower for hadron colliders
Höche, Krauss, Reichelt [2404.14360]

Apollo

A partitioned dipole-antenna shower with improved transverse recoil
Preuss [2403.19452]

Deductor

Summation of large logarithms by parton showers
Nagy, Soper [2011.04773]

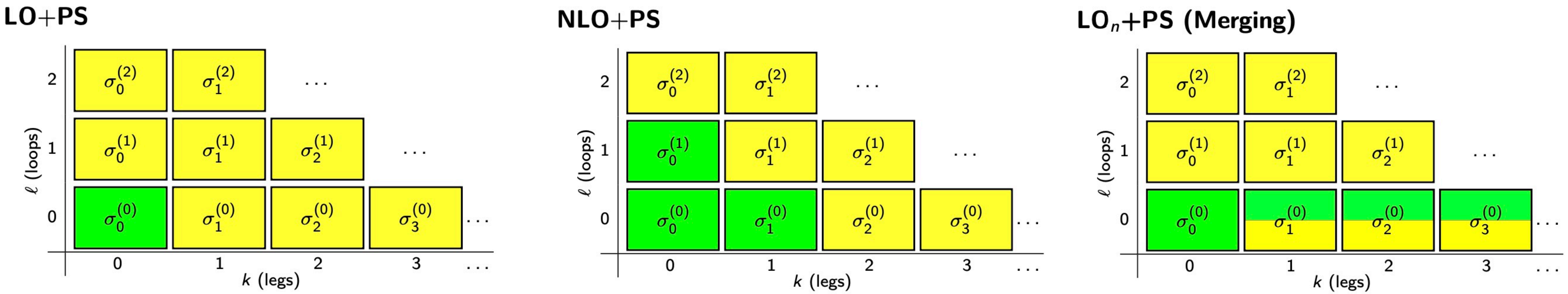
Summation by parton showers of large logarithms in electron-positron annihilation
Nagy, Soper [2011.04777]

Herwig

Logarithmic accuracy of angular-ordered parton showers
Bewick, Ferrario Ravasio, Richar Seymour [1904.11866]

Initial state radiation in the Herwig 7 angular-ordered parton shower
Bewick, Ferrario Ravasio, Richardson, Seymour [2107.04051]

3. Matching and Merging



Matching:

- combine a fixed-order (typically NLO) calculation with a parton shower, avoiding double-counting in overlap regions

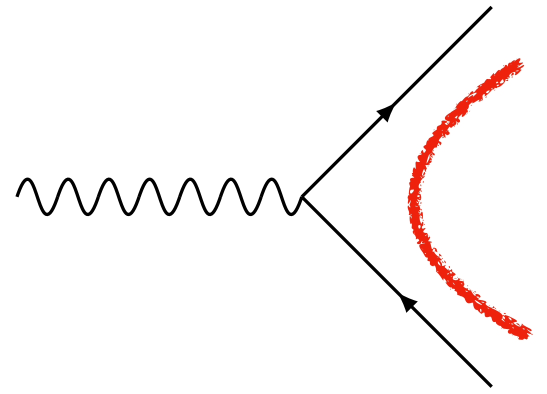
Merging:

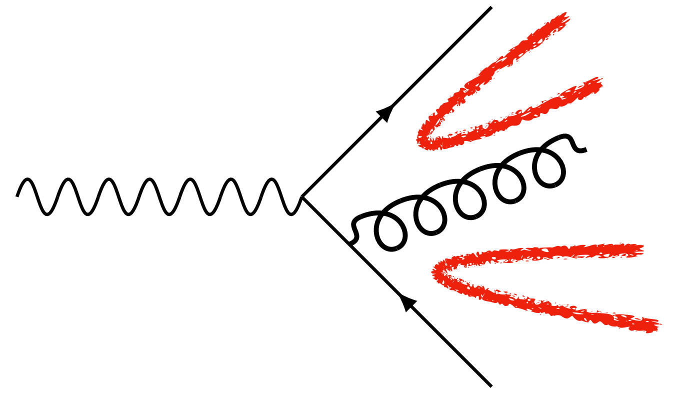
- combine multiple inclusive (N)LO event samples into a single inclusive one with additional shower radiation, accounting for Sudakov suppression and avoiding double-counting in overlap regions (typically via phase-space slicing)

3. Matching and Merging

From Showers

From parton shower





$1 - \Delta_i(Q^2, Q_0^2)$

$$\sigma_{\rm NLO}^{\rm PS} = \sigma_0 \Pi_i \left(\Delta_i(Q^2, Q_0^2) + \int_{Q_0^2}^{Q^2} \frac{dq^2}{q^2} \Delta_i(Q^2, q^2) \int dz P_{ji}(z) \right)$$

0-radiation **1-radiation (Sudakov suppressed)**

From the definition of Sudakov factor, we have

$$\mathscr{P}(\text{unresolved}) + \mathscr{P}(\text{resolved}) = 1$$

probability conservation from the definition of Δ

From NLO calculations

$$\sigma_{\rm NLO} = \sigma_0 + \left(\int d\Phi_n V + \int d\Phi_{n+1} S \right) \mathcal{O}_n + \int d\Phi_{n+1} (R\mathcal{O}_{n+1} - S\mathcal{O}_n)$$

virtual **integrated subtraction** **subtracted real**

$$\sigma_{\rm NLO} = \sigma_0^n + \int_0^{t_n} d\sigma_{(1)}^n + \int_{t_n} d\sigma_{(1)}^{n+1}$$

t_n as the resolution scale for 1-radiation

LO parton showers reproduce the NLO singular behavior of the underlying hard process with unitarity assumption $V + \int R = 0.$

Resummation from Showers + Hard emissions From fixed orders

3. Matching and Merging

Matching

Additive (MC@NLO-like)

- Using Parton Shower evolution kernel as infrared subtraction terms
- Multiply LO event weighted by Born-local K factor including the loop corrections and integrated subtraction terms
- Add hard remainder function consisting of subtracted real corrections

born   loop   Integrated subtraction

$$d\sigma^{\mathrm{MC@NLO}} = d\Phi_0 \left[B\left(\Phi_0\right) + \alpha_s V_1\left(\Phi_0\right) + \alpha_s B\left(\Phi_0\right) \otimes \int d\Phi_{1|0} P\left(\Phi_{1|0}\right)\right]$$
$$\times \left[\Delta\left(Q^2, Q_0^2\right) + \int_{Q_0^2} \frac{dq_1^2}{q_1^2} \int dz_1 \frac{\alpha_s}{2\pi} P\left(z_1\right) \Delta\left(Q^2, q_1^2\right)\right]$$
$$+ d\Phi_1 \alpha_s \left[R_1\left(\Phi_1\right) - B\left(\Phi_0\right) \otimes P\left(\Phi_{1|0}\right)\right]$$

subtracted real   generated by shower

Preserves logarithmic accuracy of PS

Parametrically $\mathcal{O}(\alpha_s)$ correct

3. Matching and Merging

Matching

Multiplicative (POWHEG-like)

- Use matrix-element corrections to replace parton-shower splitting kernel in first shower branching
- Multiply LO event weight by Born-local NLO K-factor
- Eliminate negative weights.
- In order to cover full phase space for real-emission correction.
- Enhance the large p_T contribution

Modify the Sudakov factor for first emission using full MEC

$$\bar{\Delta}\left(Q^2, q^2\right) = \exp\left[-\int \mathrm{d}\Phi_{1|0(>q^2)} \alpha_s \frac{R_1^s\left(\Phi_1\right)}{B\left(\Phi_0\right)}\right]$$

NLO-local k factor

$$\bar{B}\left(\Phi_0\right) = B\left(\Phi_0\right) + \alpha_s V_1\left(\Phi_0\right) + \alpha_s \int \mathrm{d}\Phi_{1|0} S_1\left(\Phi_1\right) + \alpha_s \int \mathrm{d}\Phi_{1|0}\left[R_1\left(\Phi_1\right) - S_1\left(\Phi_1\right)\right]$$

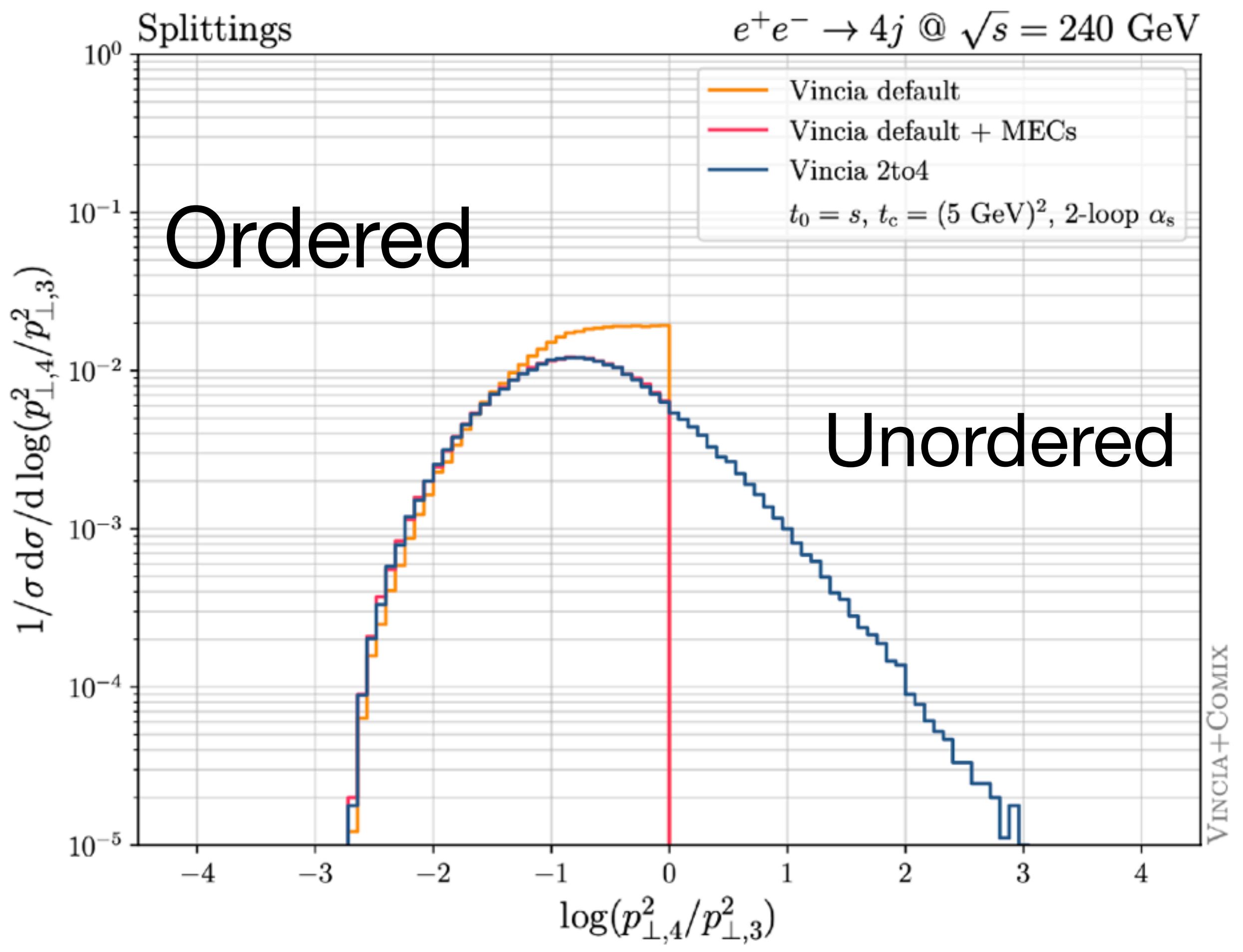
NLO corrections

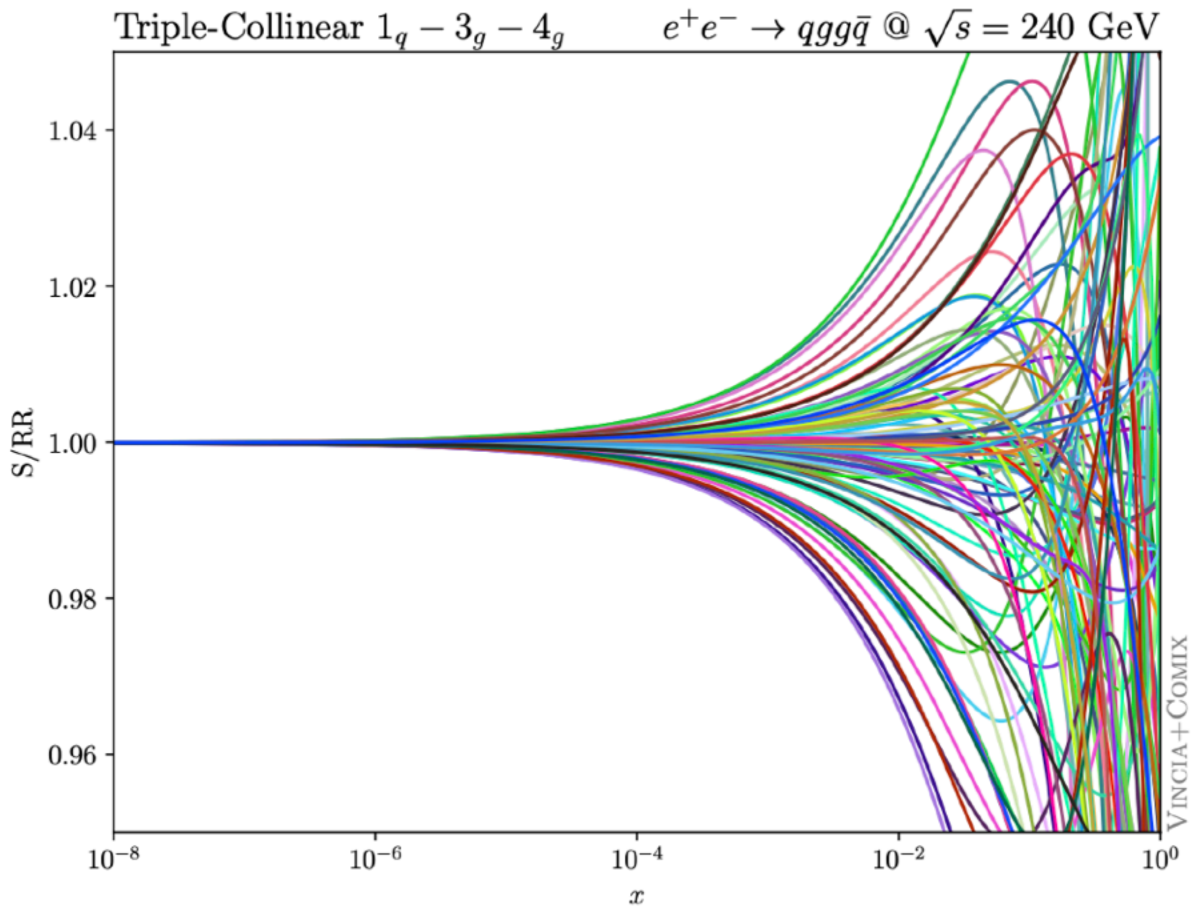
$$\mathrm{d}\sigma^{\mathrm{POWHEG}} = \mathrm{d}\Phi_0 \bar{B}\left(\Phi_0\right)\left[\bar{\Delta}\left(Q^2, Q_0^2\right) + \int \mathrm{d}\Phi_{1|0(>Q_0^2)} \alpha_s \frac{R_1\left(\Phi_1\right)}{B\left(\Phi_0\right)} \bar{\Delta}\left(Q^2, q_1^2\right)\right]$$

enhanced hard radiation

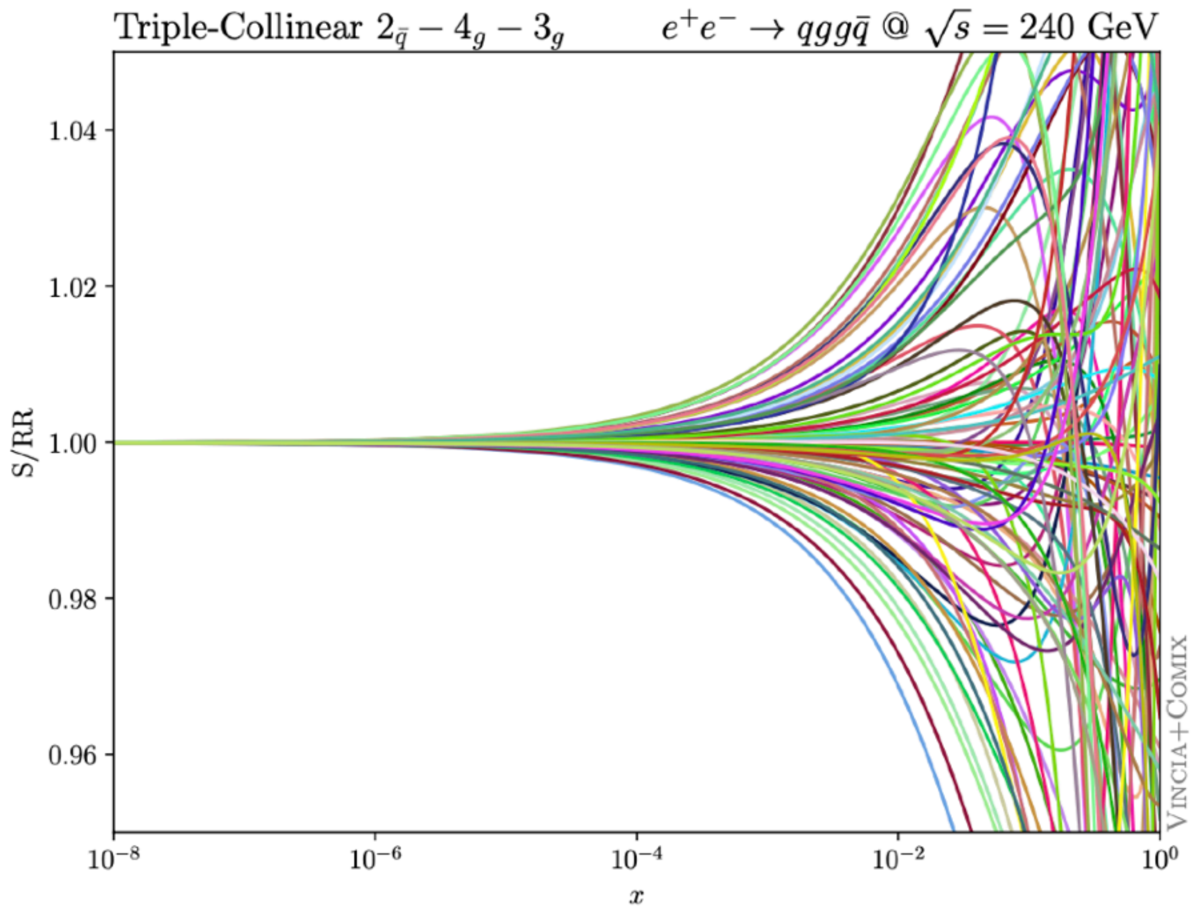
3. Matching and Merging

Matching@NNLO





$x = s_{134}/s_{1234}$



$x = s_{234}/s_{1234}$

- ❑ Sharp cut-off for Strongly ordered sector shower
- ❑ Ordered phases space with Matrix element corrections
- ❑ With 2to4 showers the full phase space is populated

Subtraction terms from Shower kernel V.S. Double Real

- ❑ Convergence in triple-collinear limit
- ❑ With full spin correlations upto 2 radiations

A first study by Campbell, Hoech, HTL, Pruss, Skands, arXiv:2108.07133

4. Showers and Jet

Phase space is always exact

$$p_b = (1-k_b)p_b^{(0)} + k_c p_c^{(0)} ,$$
$$p_c = (1-k_c)p_c^{(0)} + k_b p_b^{(0)} ,$$
$$k_{b,c} = \frac{m_a^2 - \sqrt{(m_a^2 - m_b^2 - m_c^2)^2 - 4m_b^2 m_c^2} \pm (m_c^2 - m_b^2)}{2m_a^2} .$$

$$p_{\perp\text{evol}}^2 = z(1-z)Q^2 = z(1-z)(m_{a'}^2 - m_a^2)$$

Splitting kernel is approximated

Leading power From SCET

$$\left(\frac{dN^{\text{vac}}}{dzd^2\mathbf{k}_\perp}\right)_{Q\to Qg} = \frac{\alpha_s}{2\pi^2}\frac{C_F}{\mathbf{k}_\perp^2 + z^2m^2} \times \left(\frac{1+(1-z)^2}{z} - \frac{2z(1-z)m^2}{\mathbf{k}_\perp^2 + z^2m^2}\right)$$

$$\left(\frac{dN^{\text{vac}}}{dzd^2\mathbf{k}_\perp}\right)_{g\to Q\bar{Q}} = \frac{\alpha_s}{2\pi^2}\frac{T_R}{\mathbf{k}_\perp^2 + m^2} \times \left(z^2 + (1-z)^2 + \frac{2z(1-z)m^2}{\mathbf{k}_\perp^2 + m^2}\right)$$

actual splitting function in Pythia

$$\mathrm{d}P_{\mathrm{g}\to\mathrm{Q}\overline{\mathrm{Q}}} = \frac{\alpha_\mathrm{s}}{2\pi}\frac{\mathrm{d}m^2}{m^2}\frac{\beta_\mathrm{Q}}{2}\left(z^2 + (1-z)^2 + 8r_\mathrm{Q}z(1-z)\right)\mathrm{d}z ,$$

Norrbin, Sjostrand arXiv:hep-ph/0010012

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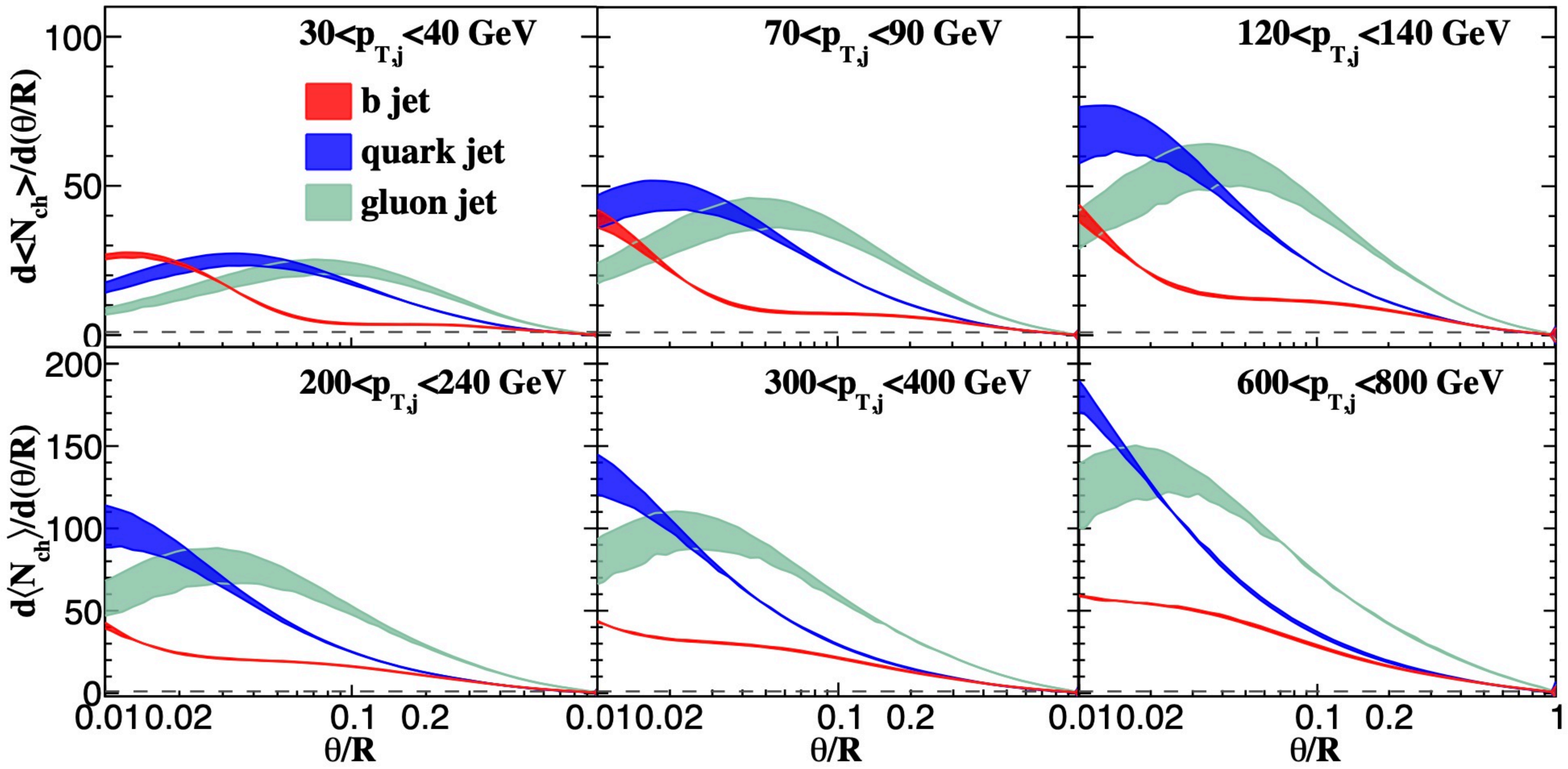

Jiang, HTL, Li, Si, arXiv:2401.09033

Norrbin, Sjostrand arXiv:hep-ph/0010012

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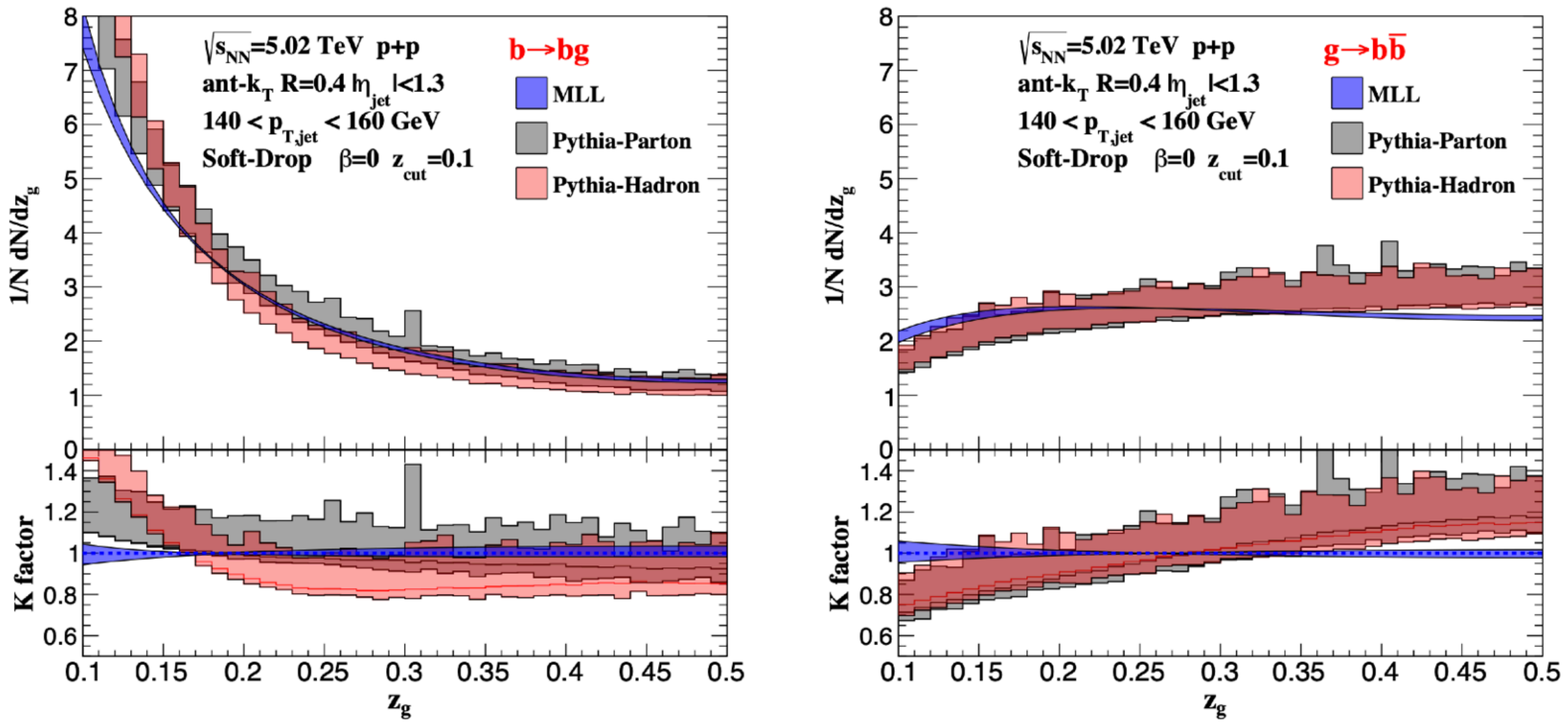
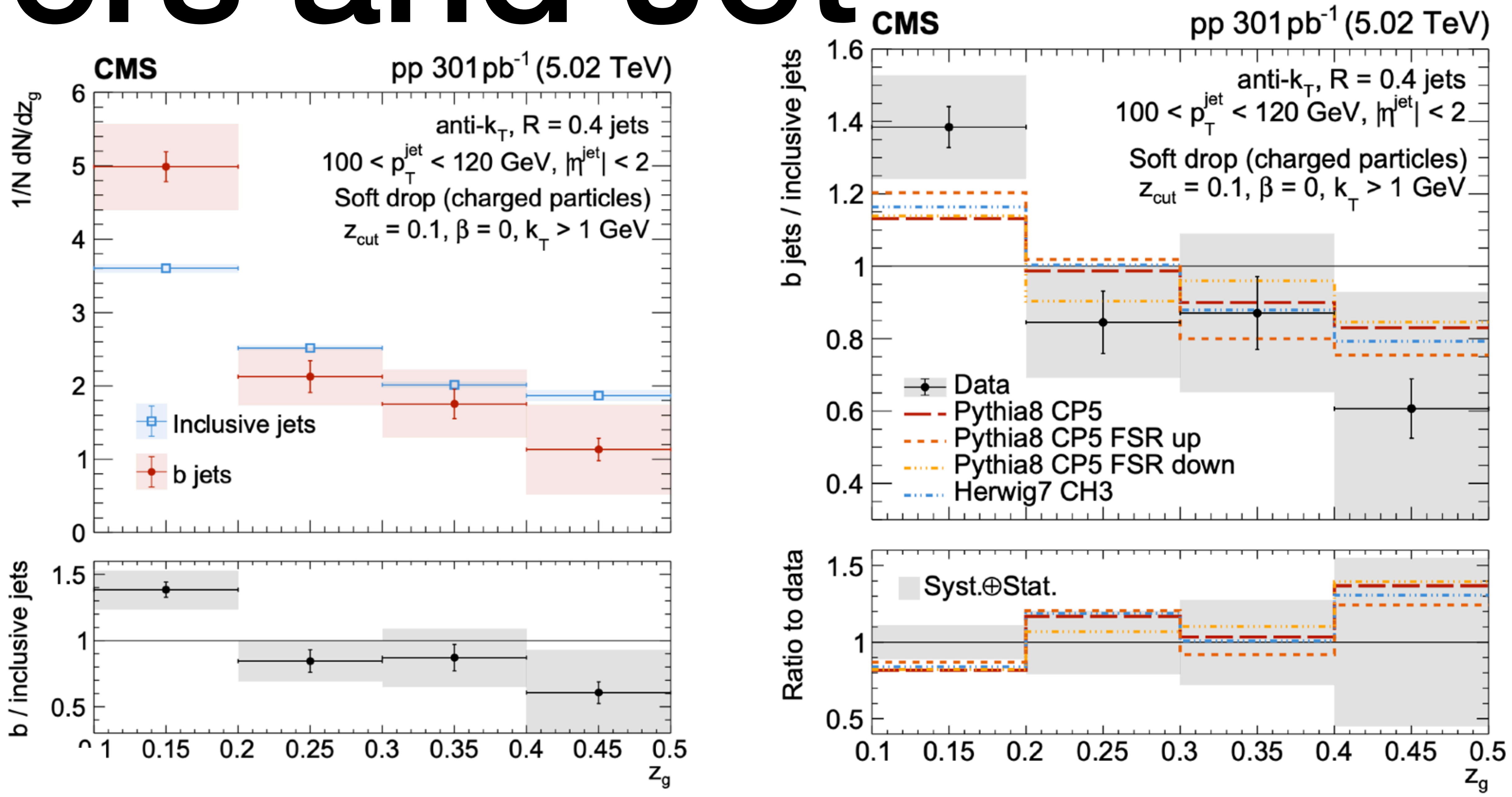
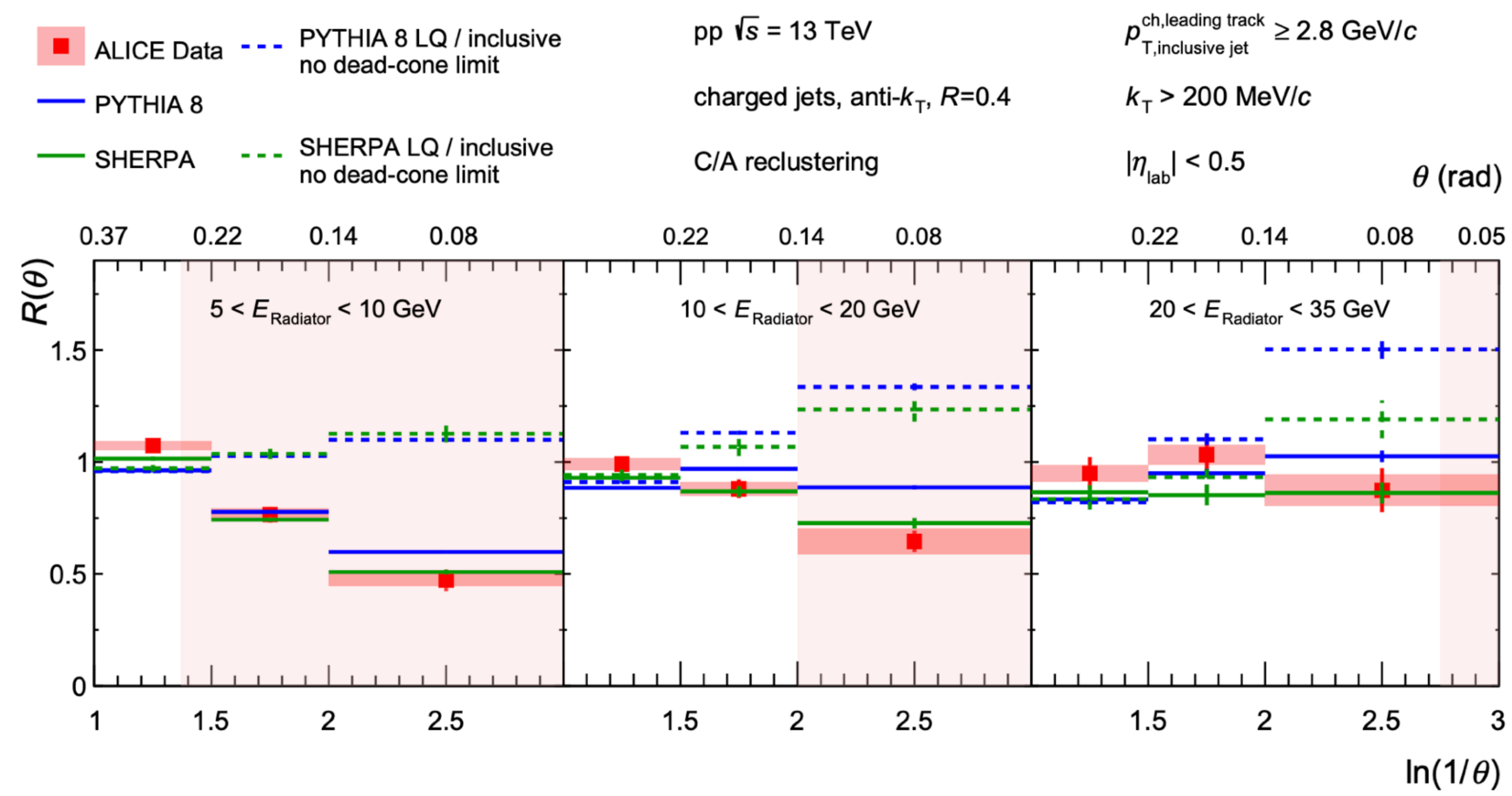
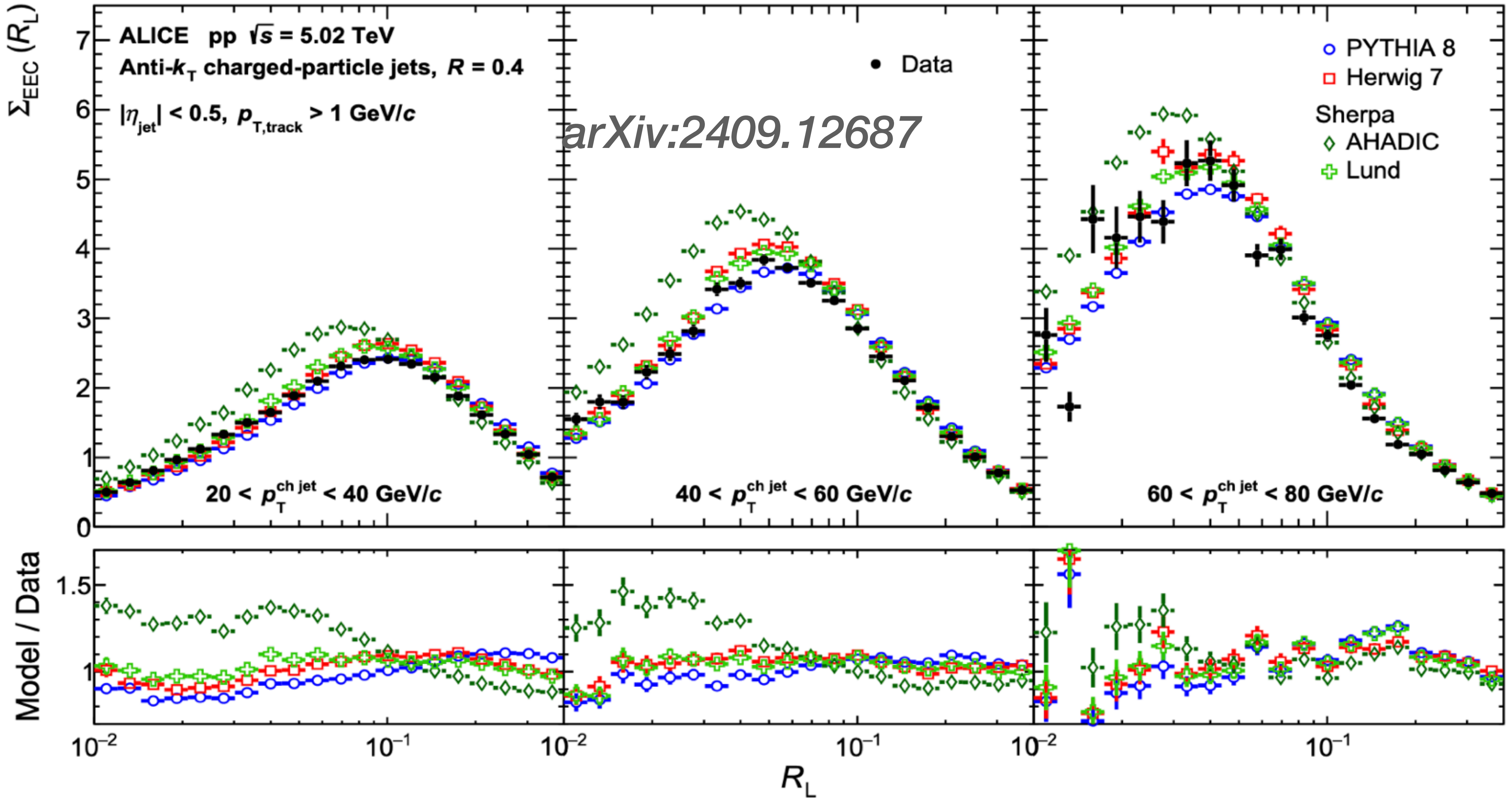


HTL, Vitev, arXiv:1801.00008







arXiv:2409.12687

5.Summary

- Parton showers are built on soft and collinear approximations to the full cross sections
  - conserve flavor and four momentum, and
  - constructed with the assumption unitarity,
- Matching and merging
- Briefly discussed antenna shower and its NNLO expansions
- Present several examples of jet distributions

Indispensable tools for phenomenology studies in p+p and A+A colliders

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Thank you!