Overview of LQCD calculations for heavy flavors
(in extreme conditions)

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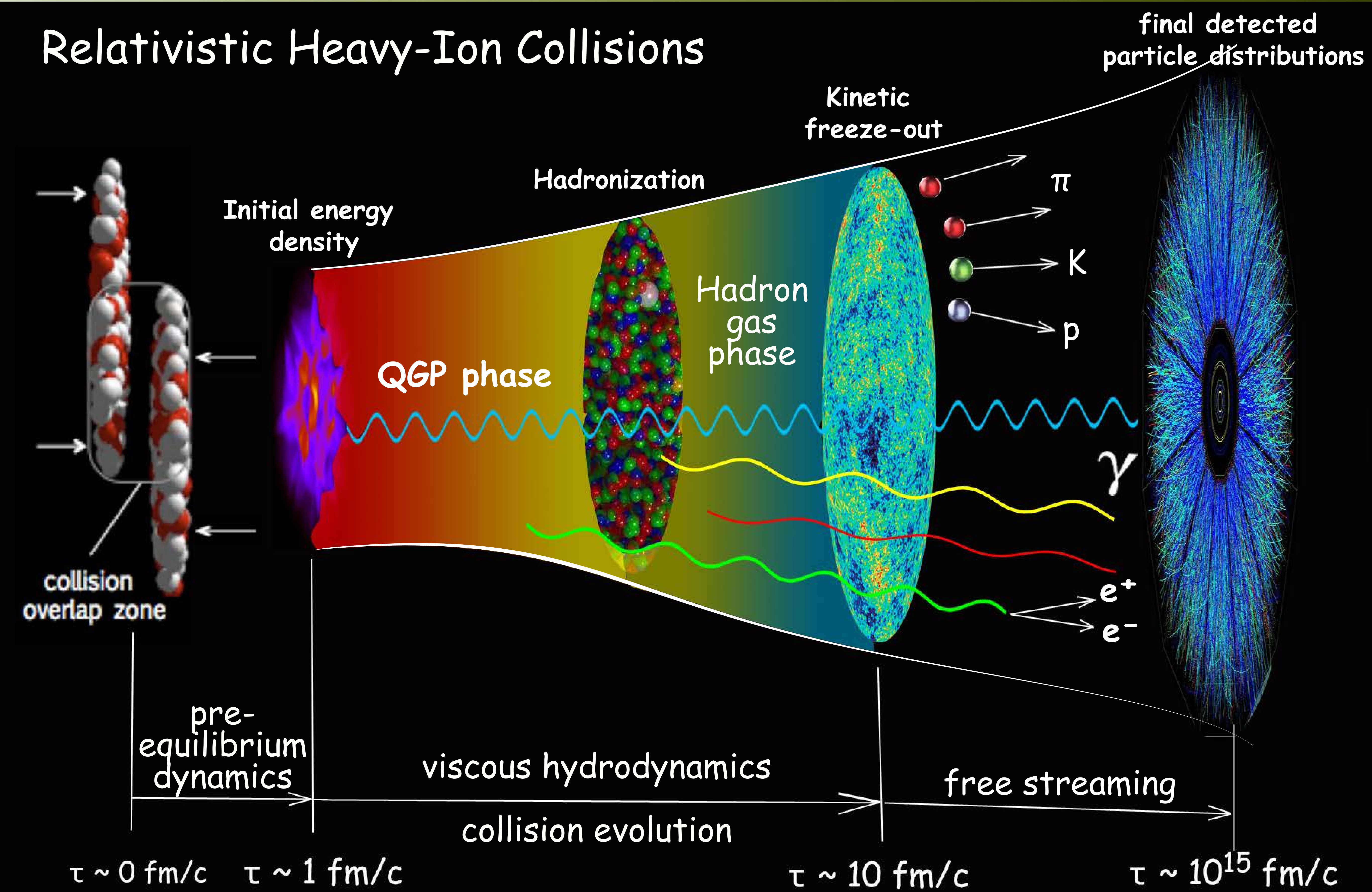
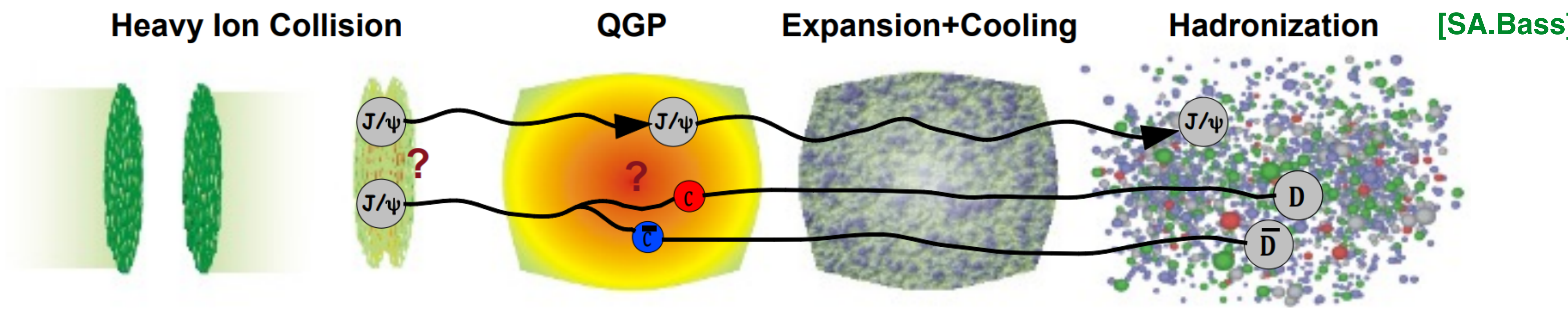
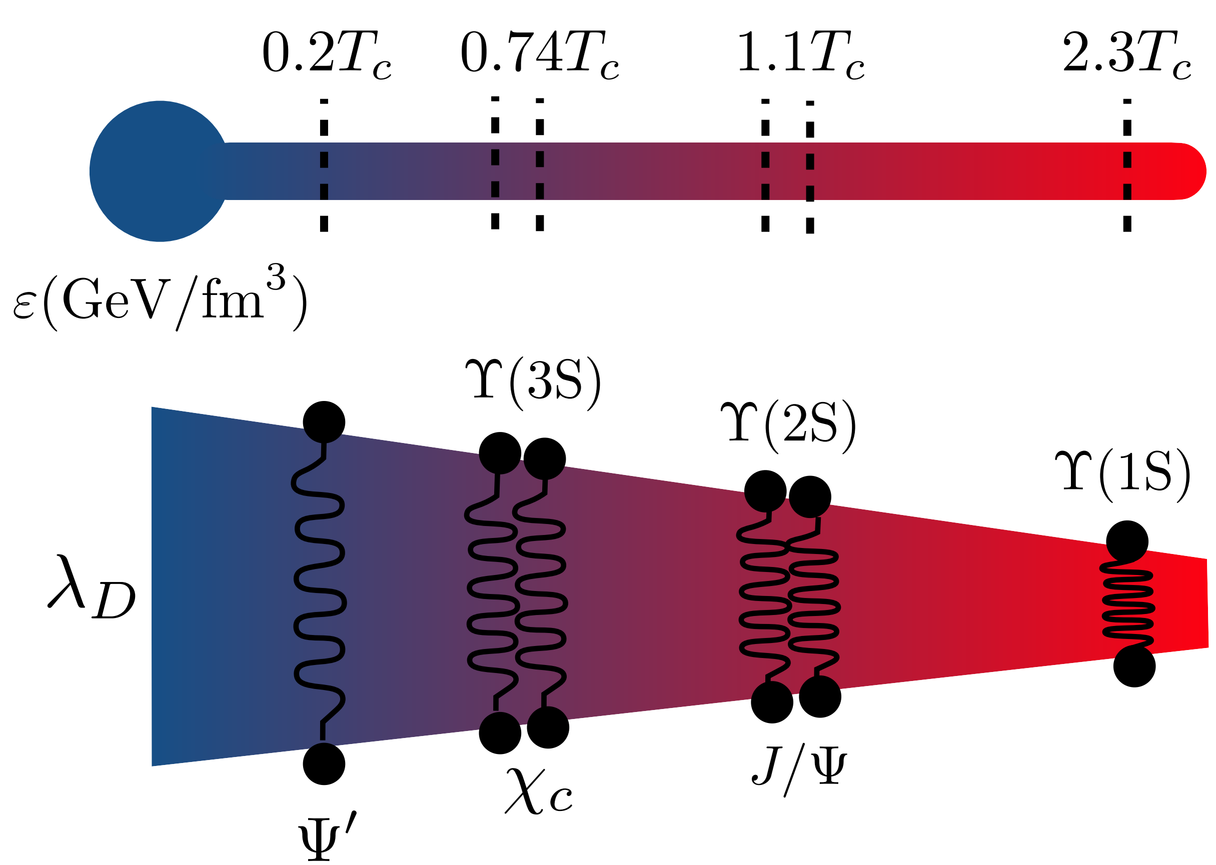
Heavy-ion collisions at LHC and RHIC



Heavy flavor probes



Probe the hot medium with different length scales



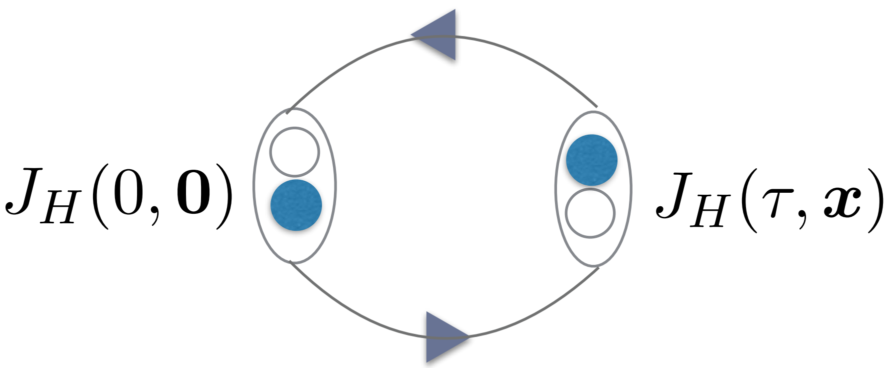
2015 Long Run Plan Nuclear Science

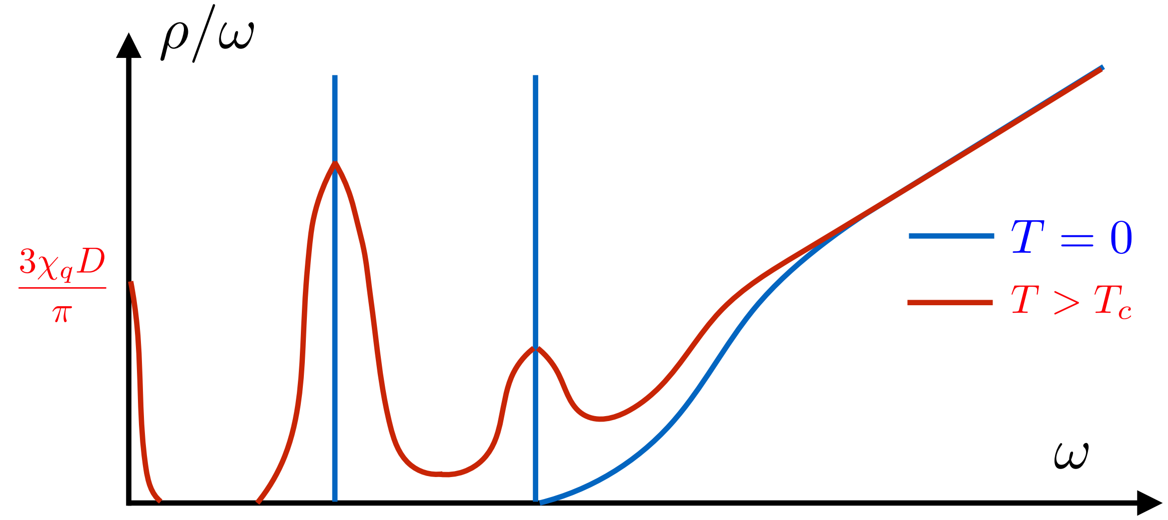
Where Lattice QCD can help …

- In-medium quarkonium properties: masses, widths, melting T
- Complex quark-antiquark potential: $\mathrm{Re}[V]$, $\mathrm{Im}[V]$
- Charm degrees of freedom
- Heavy quark diffusion: D_s

From Imag.-time lattice to real-time physics

An example: meson spectral function





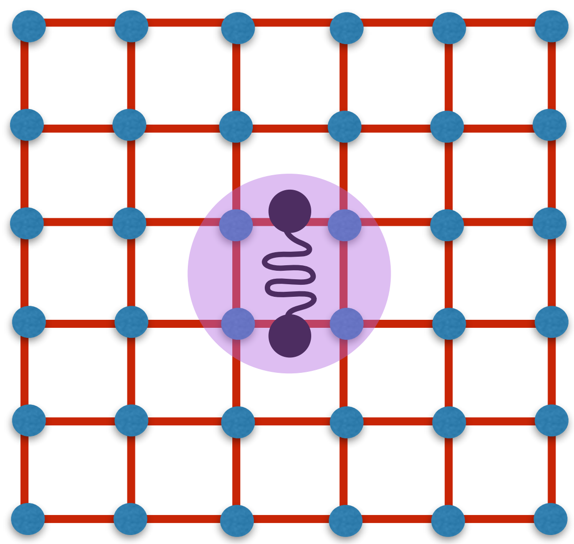
$$G_H(\tau,\vec{p}) = \sum_{x,y,z} exp(-i\,\vec{p}\cdot\vec{x})\langle J_H(0,\vec{0})J_H^\dagger(\tau,\vec{x}) = \int \frac{d\omega}{2\pi}\frac{\cosh(\omega(\tau T-1/2)/T)}{\sinh(\omega/2T)}\rho_H(\omega,\vec{p},T)$$

$K(\omega,\tau,T)$

- Meson spectral function tells melting temperature and heavy quark diffusion

Relativistic heavy quarks on the lattice

- Need high temperature to study thermal effects on heavy flavor $T = 1/(aN_\tau)$, but N_τ can not be too small, so a must be small



Compton wavelength of heavy quarks: $\lambda_Q \sim \frac{1}{m_Q}$

Small λ_Q requires fine a to resolve the dynamics

N_f	β_0	a [fm]	T[MeV]	$N_\sigma^3 \times N_\tau$	confs
$2+1$	8.249	0.028	110	$64^3 \times 64$	1010
			220 $1.2T_c$	$96^3 \times 32$	1750
			251 $1.4T_c$	$96^3 \times 28$	612
			293 $1.6T_c$	$96^3 \times 24$	856

Large & fine ensembles generated by HotQCD

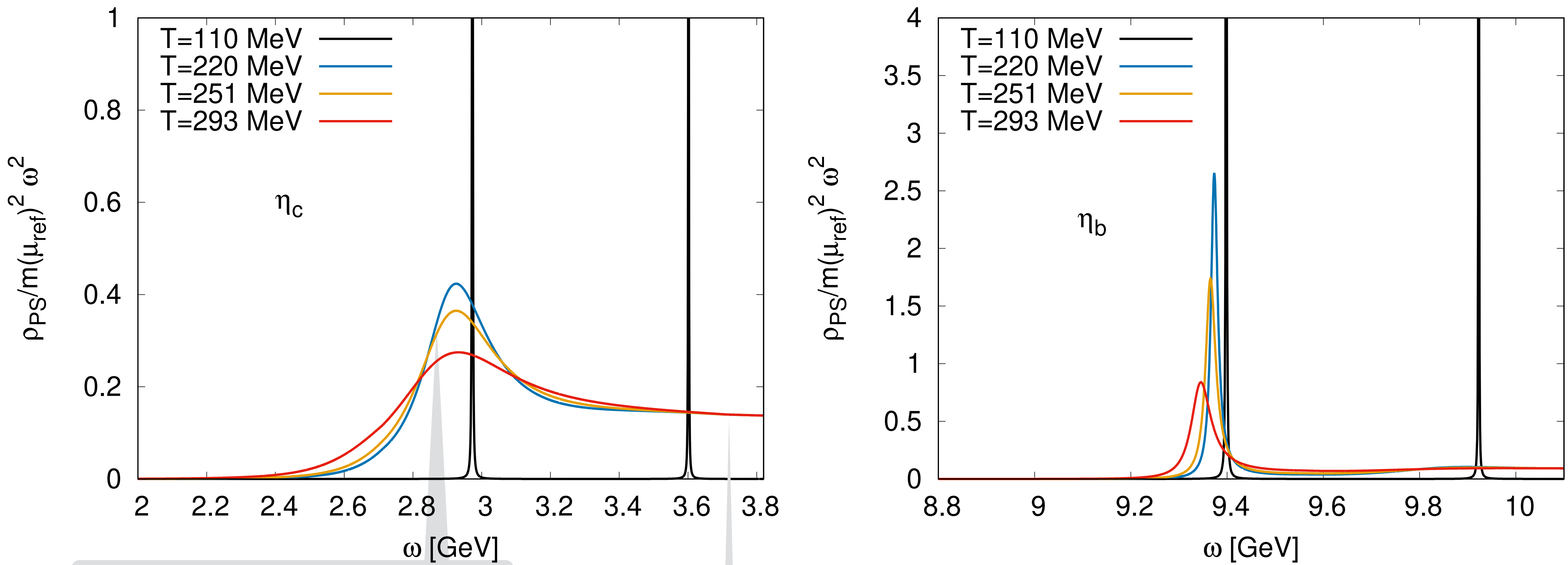
$$aM_c = 0.028 \text{ fm} \times 1.273 \text{ GeV} \times 5.068 = 0.18 \ll 1$$
$$aM_b = 0.028 \text{ fm} \times 4.183 \text{ GeV} \times 5.068 = 0.59 < 1$$

[HotQCD, PRD 112 (2025) 5, 054510]

First full QCD calculation with relativistic heavy quarks

Quarkonium spectral function in PS channel

[HotQCD, PRD 112 (2025) 5, 054510]



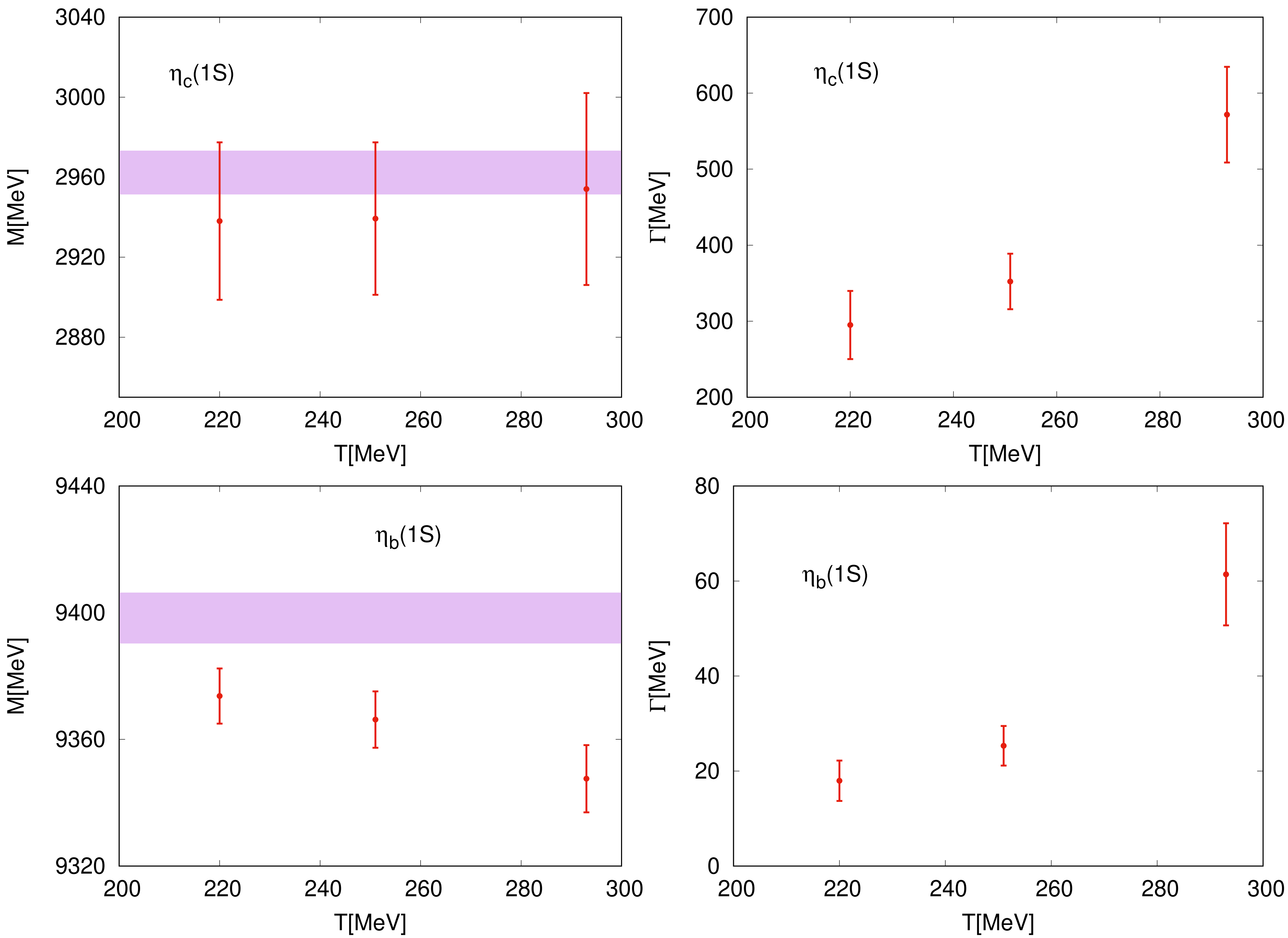
$$\left(2M_q - \frac{\nabla^2}{M_q} + V(r)\right) C^{PS}_{>}(r,t) = i\frac{\partial C^{PS}_{>}(r,t)}{\partial t}$$

Asymptotic vacuum perturbative SPF

- Thermal broadening for both charm and bottom
- More broadening for charm than bottom
- Excited state peaks melt at the temperatures above T_c

Thermal mass and thermal width of quarkonium

[HotQCD, PRD 112 (2025) 5, 054510]



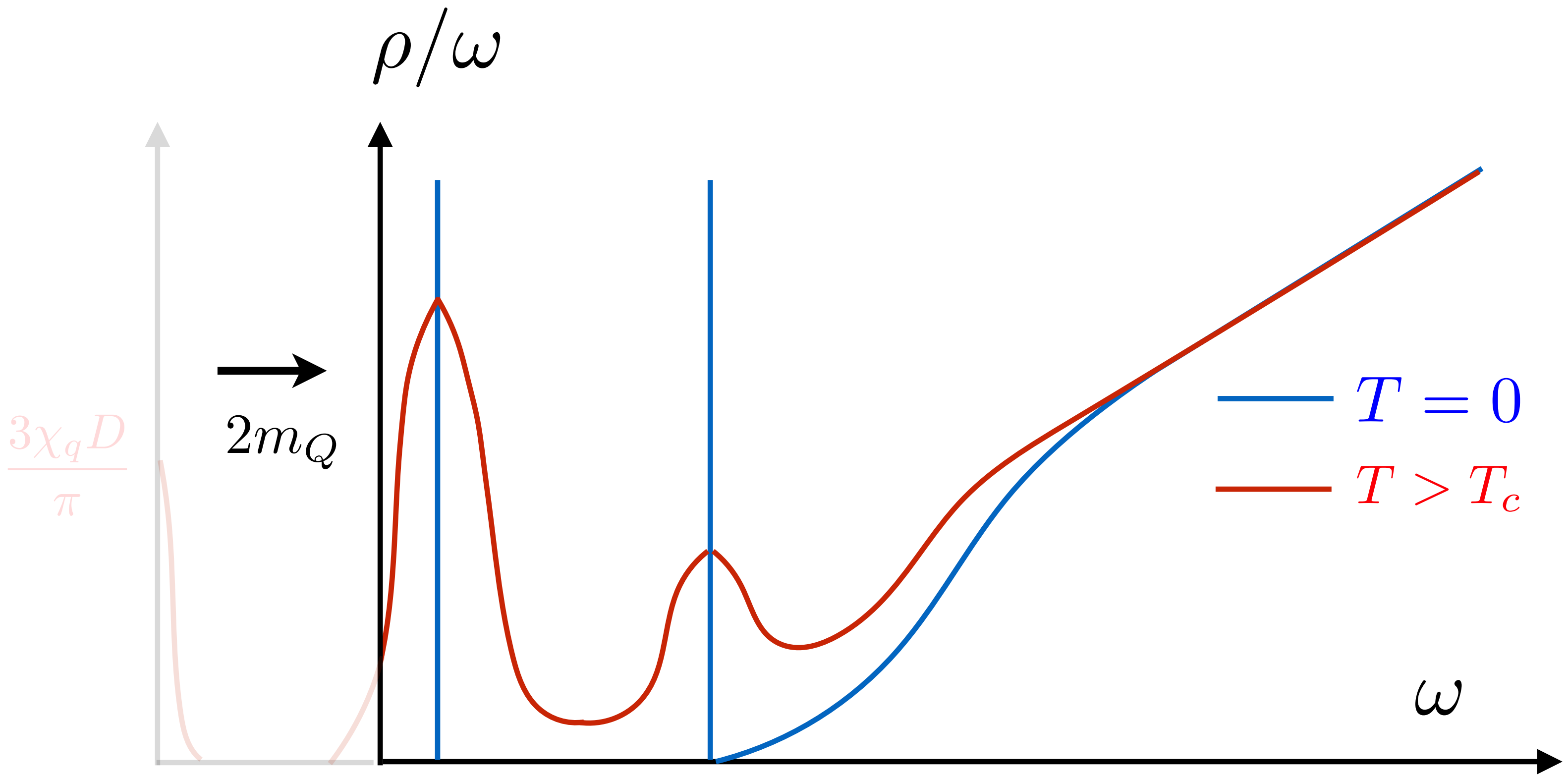
- Negligible thermal mass for $\eta_c(1S)$ at the studied temperatures
- Visible thermal mass for $\eta_b(1S)$, but small compared to its mass
- Substantial thermal width in the $\eta_c(1S)$ state —> $\eta_c(1S)$ is nearing dissolution
- $\eta_b(1S)$ exhibits little change and remains well-defined

Non-relativistic heavy quark

NRQCD becomes possible due to scale separation: $m_Q \gg m_Q v \gg m_Q v^2$

~~m_Q : hard scale, quark creation and annihilation~~
$m_Q v$: soft scale, momentum exchange between $Q\bar{Q}$
$m_Q v^2$: ultrasoft scale, binding/melting of $Q\bar{Q}$

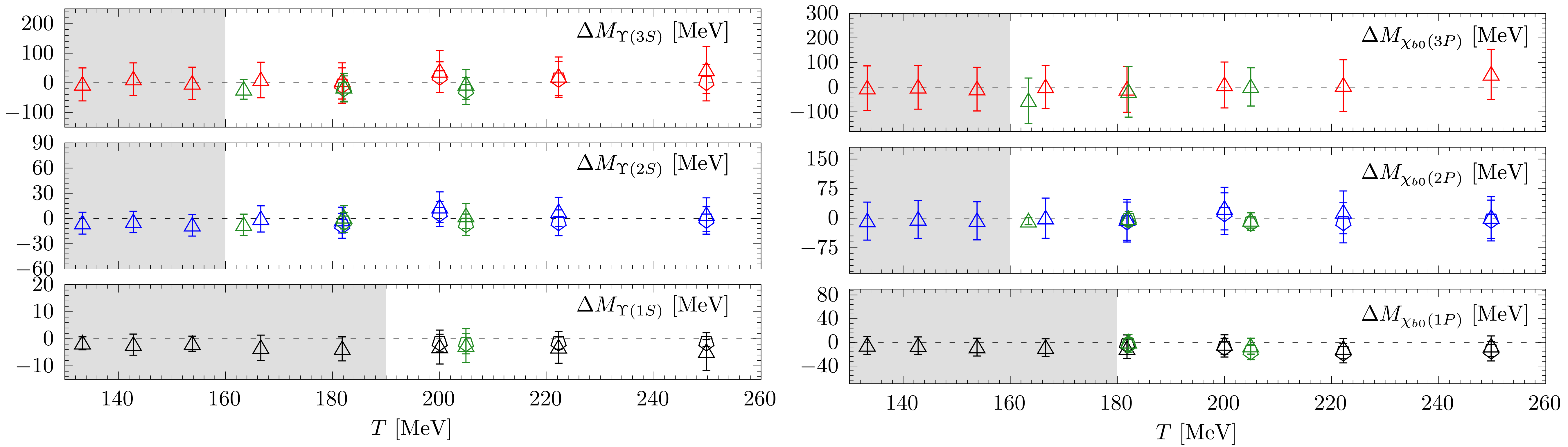
[G. Aarts, et al., JHEP 07 (2014) 097]
[S. Kim, et al., JHEP 11 (2018) 088]
[R. Larsen, et al., PRD 100 (2019) 7, 074506]
[R. Larsen, et al., PLB 800 (2020) 135119]



Quarkonium correlators can be computed with better resolution!

Mass shift of bottomonium from NRQCD

Model spectra: $\rho_{\text{med}}(\omega,T) = A_{\text{low}}(T)\delta\left(\omega-\omega_{\text{low}}(T)\right) + A_{\text{med}}(T)\dfrac{\Gamma(\omega,T)}{\left(\omega - M_{\text{med}}(T)\right)^2+\Gamma^2(\omega,T)}$

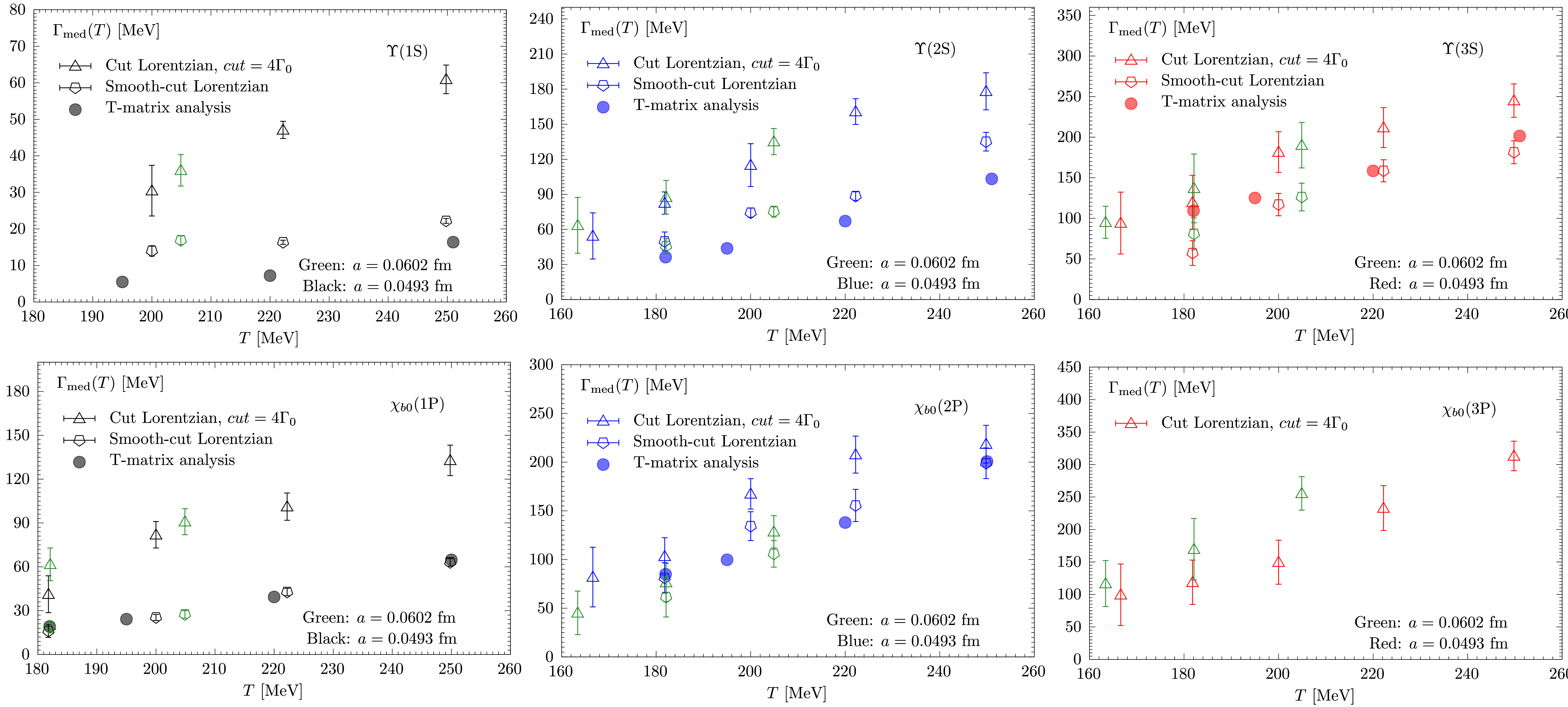


[丁亨通，黄玮平, et al.，JHEP 05 (2025) 149]

a [fm]	0.0493	0.0493	0.0493	0.0493	0.0493
T [MeV]	151	173	199	251	341

- No mass shift for all states

Thermal width of bottomonium from NRQCD



[丁亨通，黄玮平, et al.，JHEP 05 (2025) 149]

- Increasing thermal width with temperature for all states
- Visible model dependence for thermal width
- Thermal broadening follows the hierarchical increasing pattern when using the same model

Static quark potential

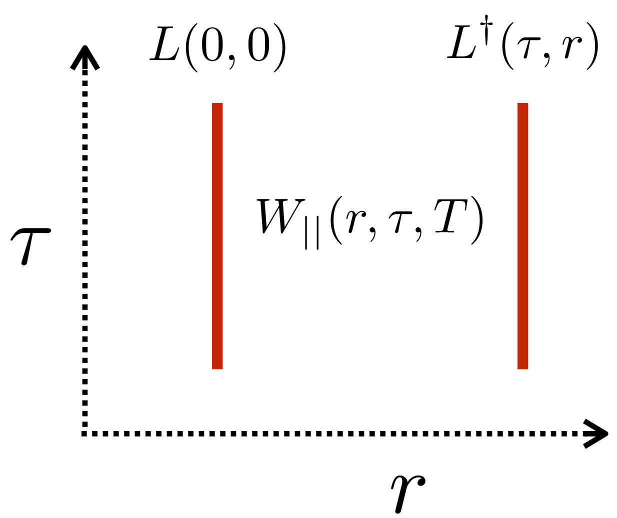
- Hard Thermal Loop resummed perturbation theory **[M. Laine, JHEP0703,054 (2007)]**

$$\lim_{t\to\infty} V_{>}^{(2)}(t,r) = -\frac{g^2 C_F}{4\pi}\left[m_{\rm D} + \frac{\exp(-m_{\rm D}r)}{r}\right] - \frac{ig^2 T C_F}{4\pi}\phi(m_{\rm D}r)$$

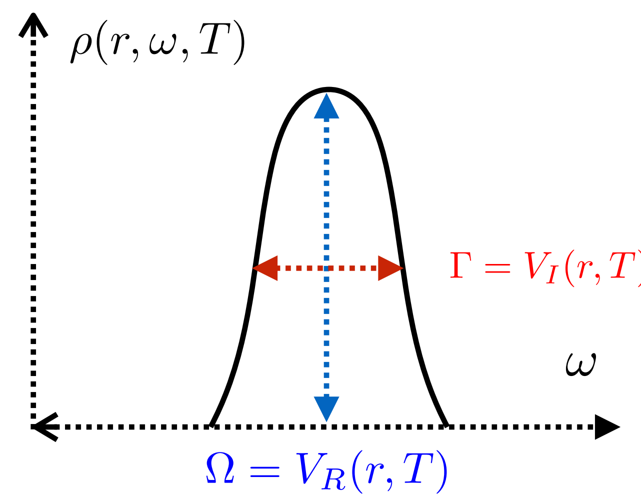
Imaginary part becomes important for physical bottomonium at T>250 MeV!

- Non-perturbative determination matters at around and above Tc

Wilson loop/thermal Wilson line correlators in Coulomb gauge **[A. Rothkopf et al., PRL. 108 (2012) 162001]**



$$W_{||}(r,\tau,T) = \int d\omega \rho(r,\omega,T) e^{-\omega\tau}$$



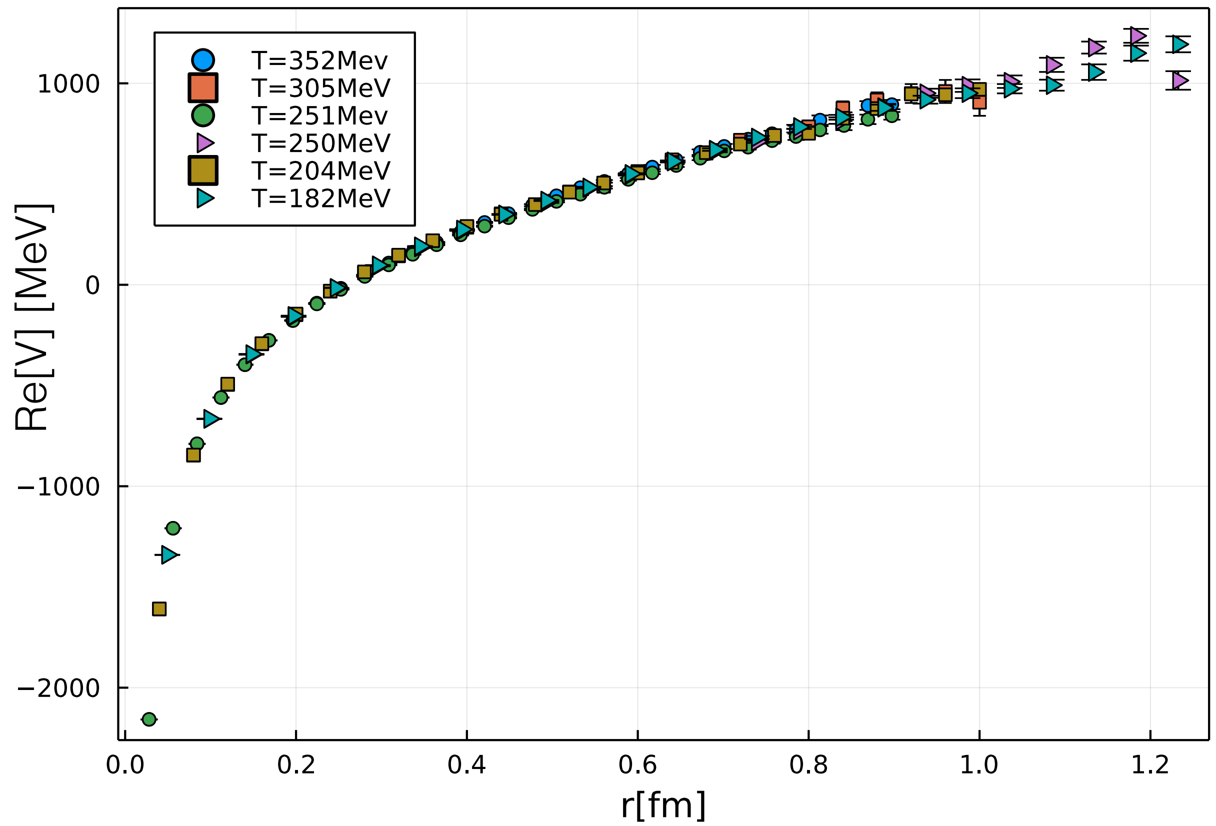
- Subtract continuum contribution from zero temperature correlators
- Model the potential as arguments of spectral function

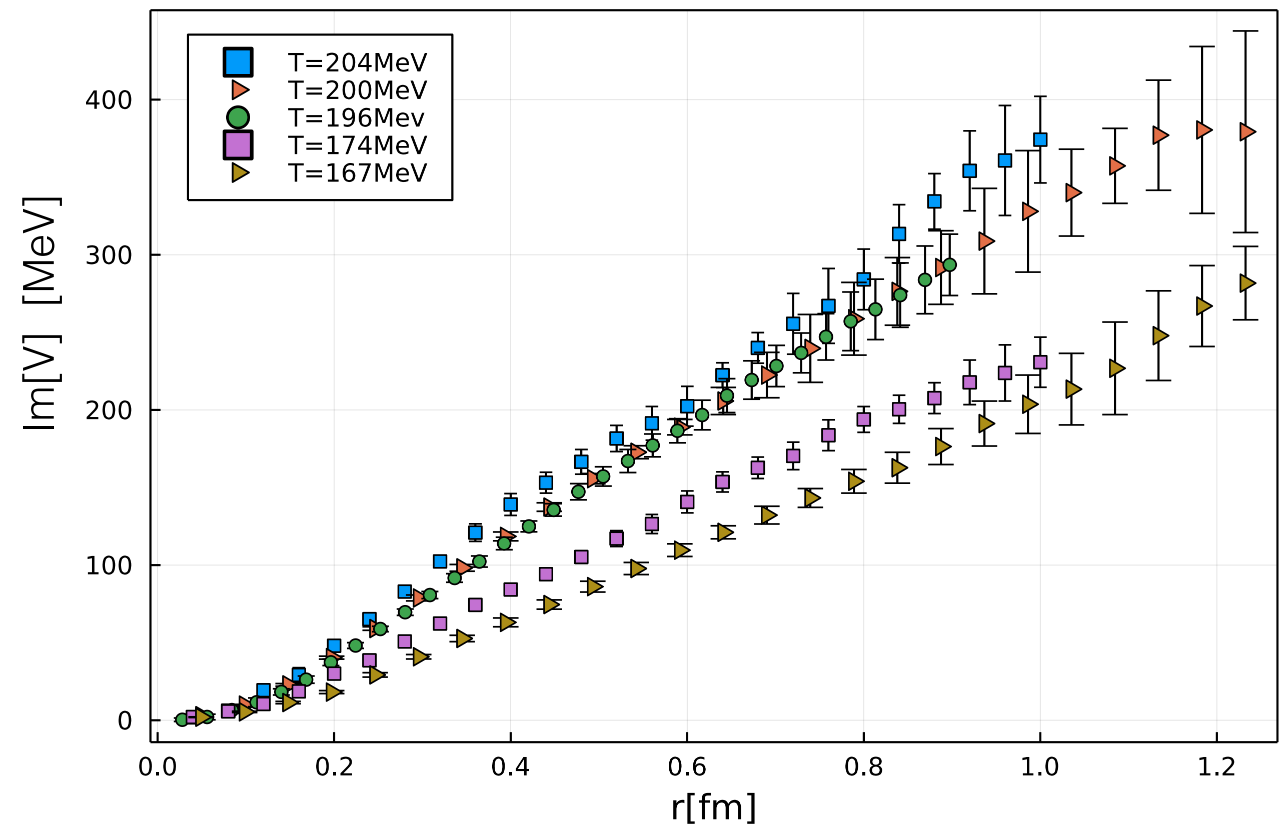
Static quark potential in hot QCD

$$N_f = 2+1, m_\pi = 160 \text{ MeV}, a \to 0$$

Real part: temperature insensitive

Imag. part: increasing with T & r





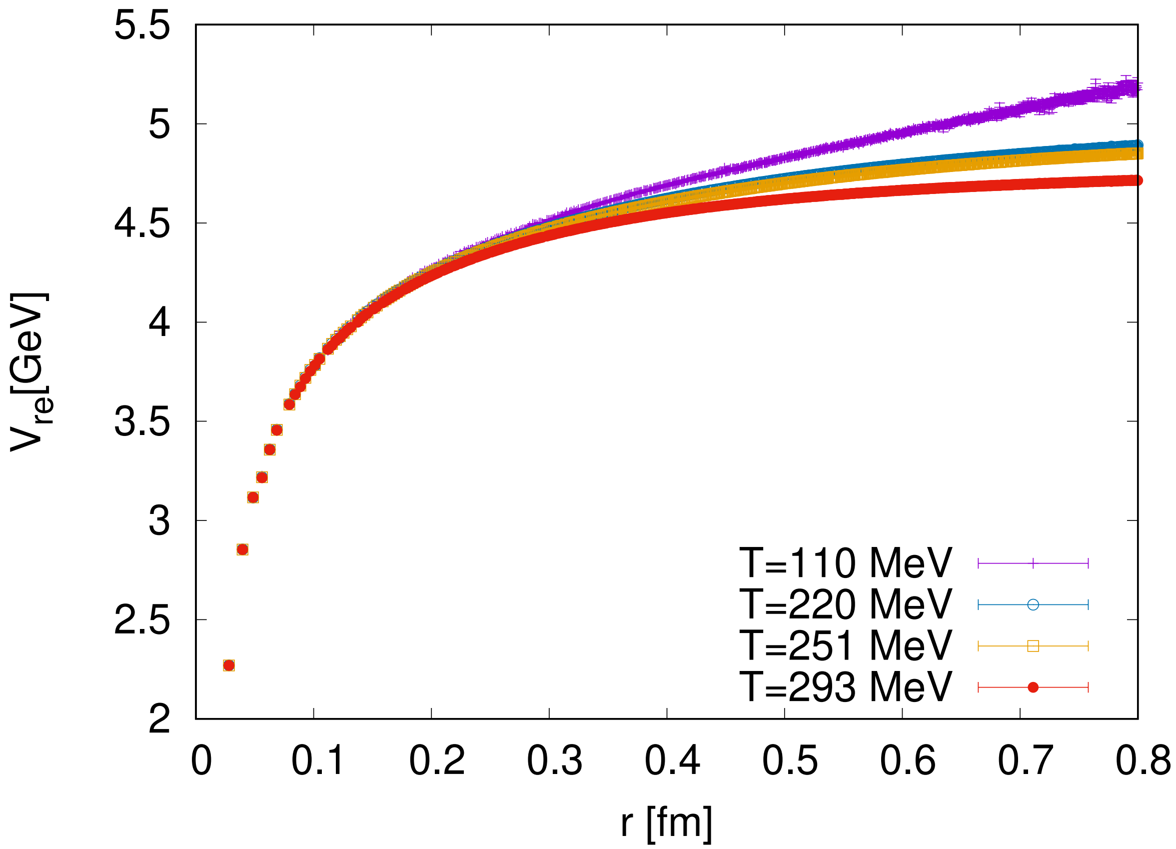
[HotQCD, PRD 109 (2024) 7, 074504]

$$\rho(r,\omega) = \frac{1}{\pi}\mathrm{Im}\frac{A(T)}{\omega - \mathrm{Re}V(r,T) - i\Gamma(\omega,r,T)}$$

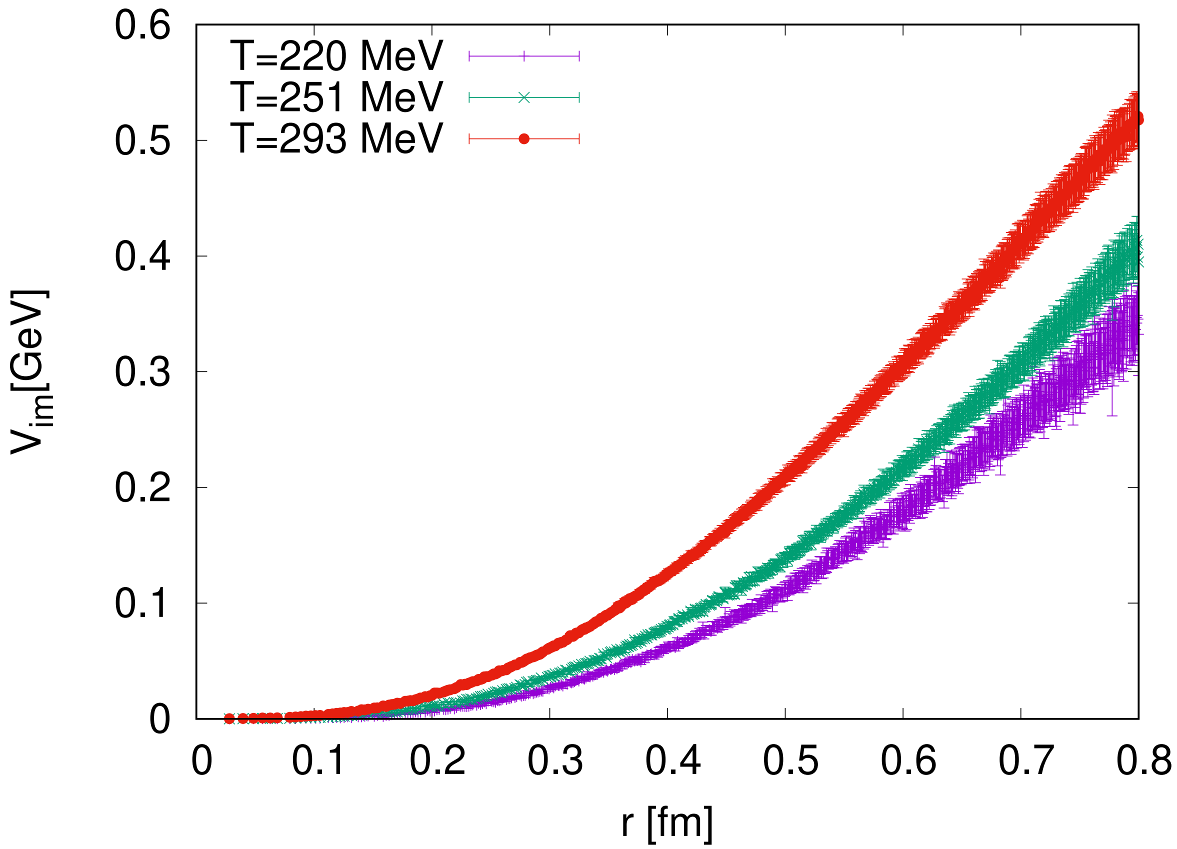
- No color screening from cut-Lorentzian model SPF (no mass shift for bottomonium)

Static quark potential in hot QCD

Real part: temperature sensitive



Imag. part: increasing with T & r



[HotQCD, PRD 112 (2025) 5, 054510]

$$\rho(r,\omega) = n_b(\omega)\left(\frac{\beta V_{im}(r)}{2\pi\,\omega} + \sum_{l=0}^{\infty} c_{2l+1}\,\omega^{2l+1}\right)$$

- Color screening evident from HTL-inspired model SPF
- Model dependence demands further investigation

Charm degrees of freedom in hot matter

- Strong interaction matter undergoes a chiral crossover at T_{pc} ~156.5 MeV
- In heavy-ion collisions, relevant degrees of freedom change from partonic to hadronic in going from high temperature phase to temperatures below T_{pc}
- What are the relevant charmed d.o.f.s. after the onset of hadron melting?
- Do charmed hadrons start melting at T_{pc}?
- Can we get a signal for the appearance of new degrees of freedom at T_{pc}?
- Find missing open-charm states by comparing LQCD and HRG

Charm degrees of freedom in hot matter

- Compare pressure (including contributions from meson&baryon&quark) computed using LQCD and HRG models

$$P = \frac{T}{V} \ln Z(T, V, \hat{\mu}_B, \hat{\mu}_Q, \hat{\mu}_S, \hat{\mu}_C)$$

$$P_C(T, \vec{\mu}) = P_M^C(T, \vec{\mu}) + P_B^C(T, \vec{\mu}) + P_q^C(T, \vec{\mu})$$

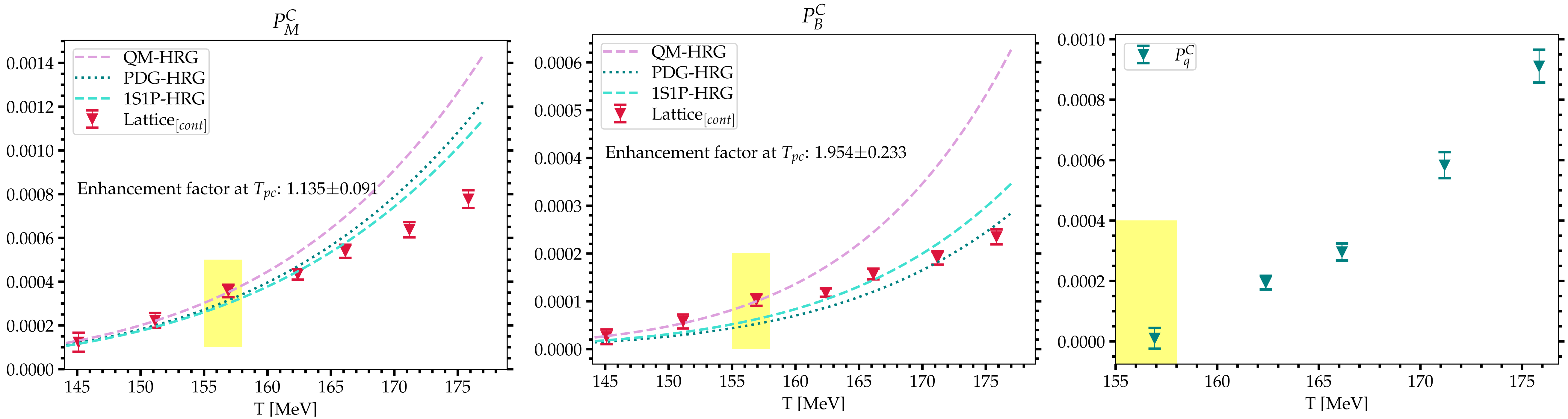
Lattice proxy:

$$P_q^C = 9(\chi_{13}^{BC} - \chi_{22}^{BC})/2$$
$$P_B^C = (3\chi_{22}^{BC} - \chi_{13}^{BC})/2$$
$$P_M^C = \chi_4^C + 3\chi_{22}^{BC} - 4\chi_{13}^{BC}$$

HRG:

$$P_{B/M}^C(T, \vec{\mu}) = \frac{1}{2\pi^2} \sum_{i \in \text{C-B/M}} g_i \left(\frac{m_i}{T}\right)^2 K_2(m_i/T) \cosh(B_i\hat{\mu}_B + Q_i\hat{\mu}_Q + S_i\hat{\mu}_S + C_i\hat{\mu}_C)$$

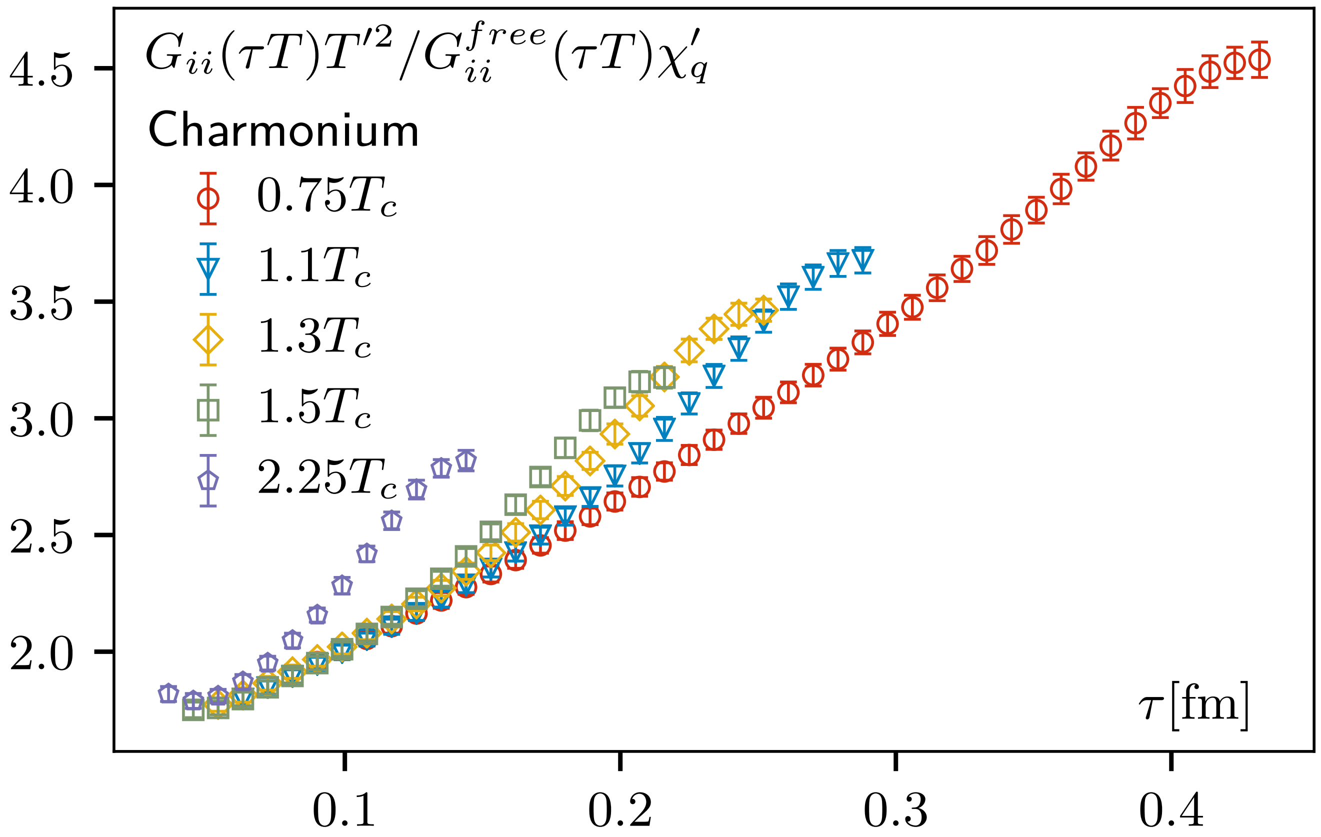
$$P_q^C(T, \vec{\mu}) = \frac{6}{\pi^2} \left(\frac{m_q^C}{T}\right)^2 K_2(m_q^C/T) \cdot \cosh\left(\frac{2}{3}\hat{\mu}_Q + \frac{1}{3}\hat{\mu}_B + \hat{\mu}_C\right)$$

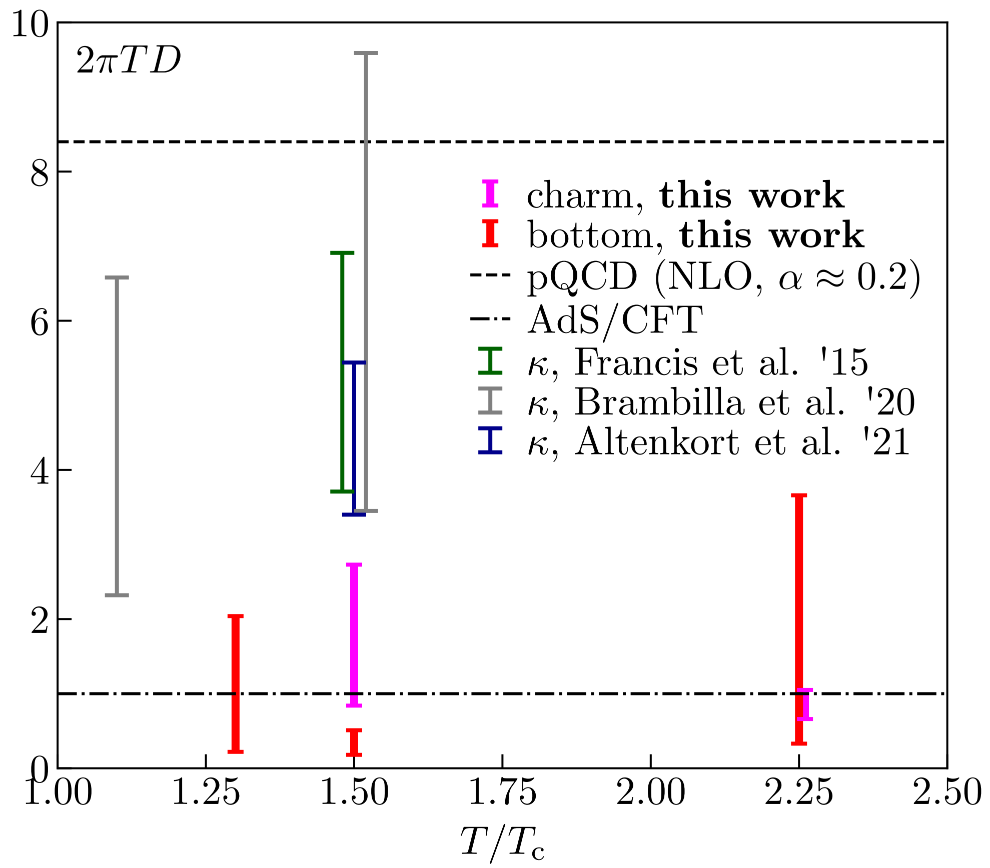


[HotQCD, PRD 112 (2025) 3, 034509]

- Below T_{pc} : QM-HRG describes the susceptibilities while PDG-HRG not: missing charmed hadrons in PDG (isospin partners)
- Above T_{pc} : QM-HRG overpredicts the lattice results —> charmed hadrons melt
- Above T_{pc} : non-zero contributions to pressure from charm quark

Heavy quark diffusion from meson correlators





[丁亨通, OK, AL, HO, HS, 舒海涛, PRD104(2021) 11, 114508]

- Still limited to the quenched approximation (but in the continuum limit)
- Extension to full QCD on the way　(pseudo-scalar -> vector)

Heavy quark diffusion via HQEFT

- Langevin equations of heavy quark motion

$$\partial_t p_i = -\eta_D p_i + \xi_i(t) \qquad \langle \xi_i(t)\xi_j(t')\rangle = \kappa \delta_{ij}\delta(t-t')$$

- Mass dependent **momentum** diffusion coefficient

$$\kappa^{(M)} \equiv \frac{M^2\omega^2}{3T\chi_q}\sum_i \frac{2T\rho_V^{ii}(\omega)}{\omega}\Big|^{\eta\ll|\omega|\lesssim\omega_{\rm UV}}$$

- Large quark mass limit in HQ effective field theory

$$\kappa \equiv \frac{\beta}{3}\sum_{i=1}^{3}\lim_{\omega\to 0}\left[\lim_{M\to\infty}\frac{M^2}{\chi_q}\int_{-\infty}^{\infty}{\rm d}t\; e^{i\omega(t-t')}\int {\rm d}^3\vec{x}\,\left\langle \frac{1}{2}\{\mathcal{F}^i(t,\vec{x}),\mathcal{F}^i(0,\vec{0})\}\right\rangle\right]$$

[J. Casalderrey-Solana and D. Teaney, PRD 74, 085012]

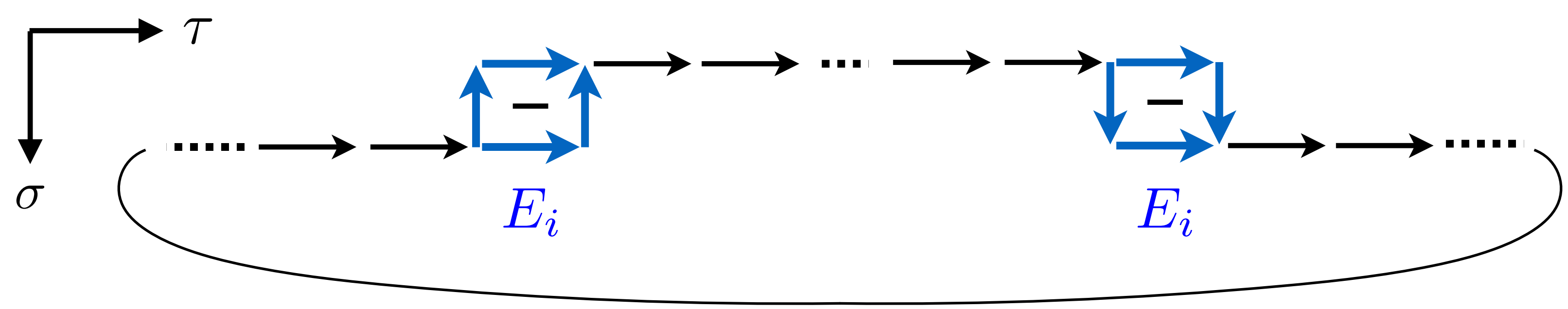
[S. Caron-Huot et al., JHEP 0904 (2009) 053]

[A. Bouttefeux, M. Laine, JHEP 12 (2020) 150]

$$\partial_t \mathbf{p} = q(\mathbf{E} + \mathbf{v}\times\mathbf{B}) = \mathbf{F}$$

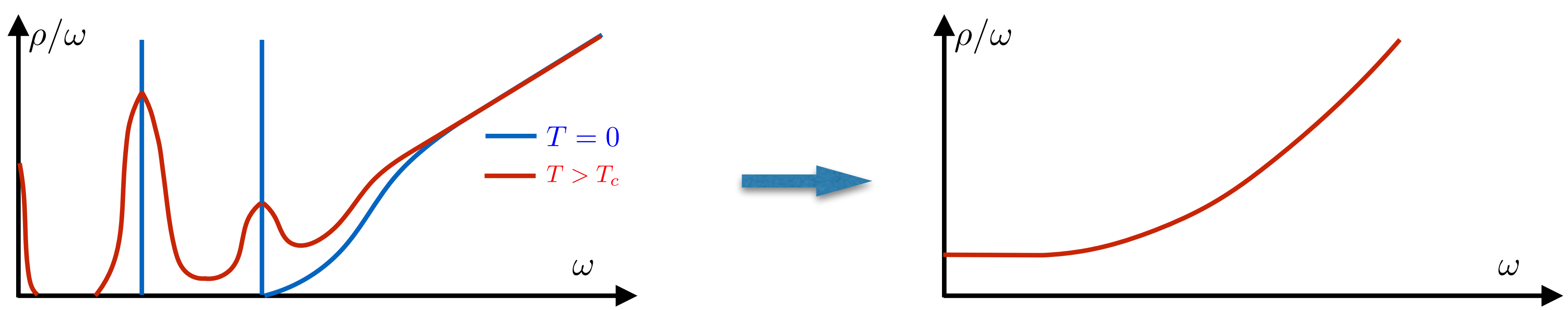
Momentum diffusion on the lattice

$$\kappa = \kappa_E + \frac{2}{3}\langle \mathbf{v^2} \rangle \kappa_{\mathbf{B}} \qquad \langle \mathbf{v}^2 \rangle = \frac{3T}{M} \qquad \frac{1}{2\pi T D} = \frac{\kappa}{4\pi T^3} = \frac{1}{2\pi T^2} \lim_{\omega \to 0} \frac{\rho(\omega)}{\omega}$$



Color-electric field correlation function

$$G(\tau, T) = \int \frac{d\omega}{\pi} K(\omega, \tau, T) \rho(\omega, T)$$

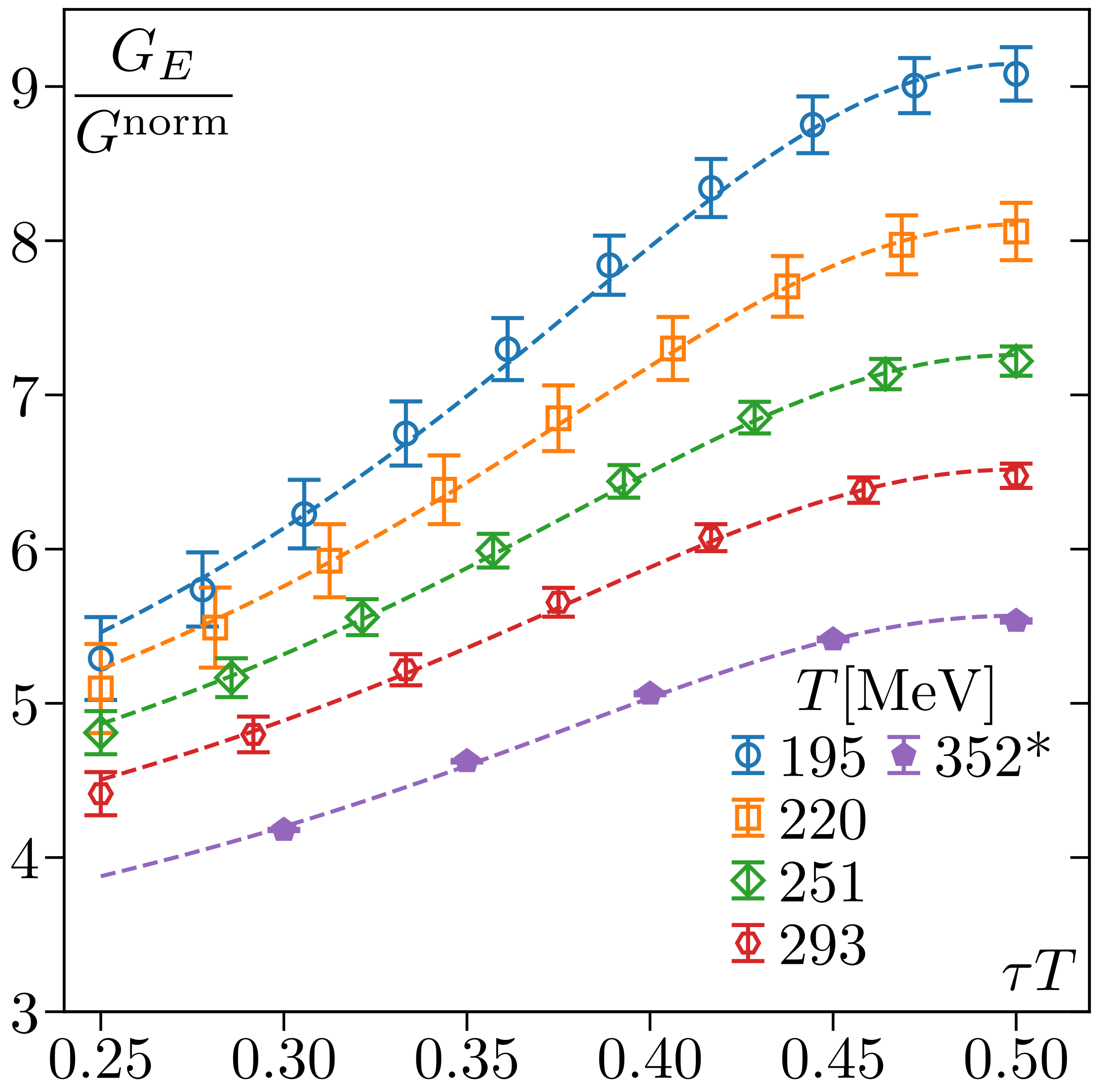


- Cheaper to measure on the lattice
- No peak structures in spectral functions
- Absence of transport peak

First full QCD calculation of EE correlators

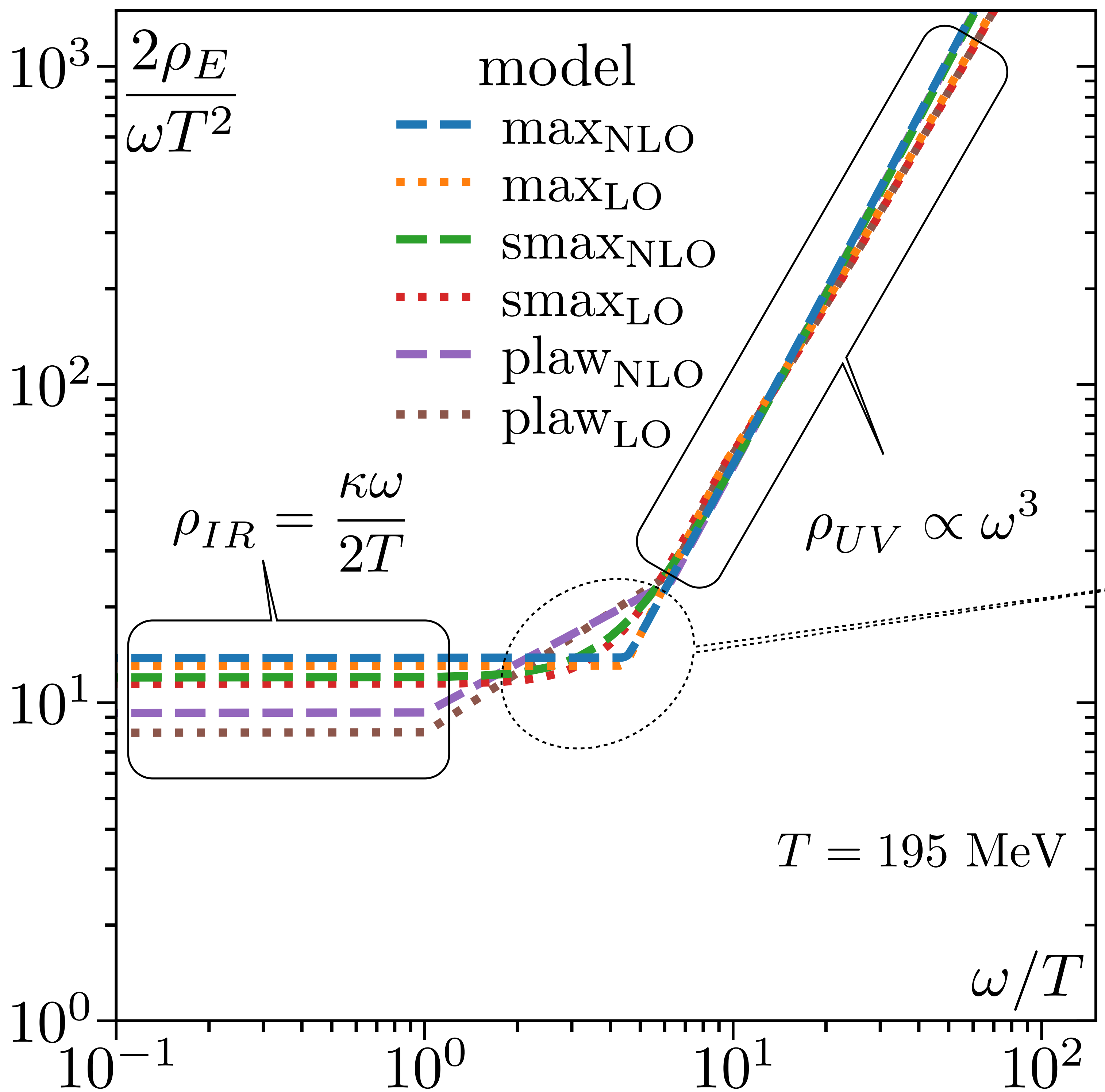
First full QCD calculation of κ, only possible via Gradient Flow!

[Luscher & Weisz, JHEP1102(2011)051]
[Narayanan & Neuberger, JHEP0603(2006)064]



[LA, OK, RL, SM, PP, 舒海涛, SS, PRL 130 (2023) 23, 231902]

- 195 MeV <= T <= 352 MeV
- 2+1 flavor in the sea
- pion mass 320 MeV



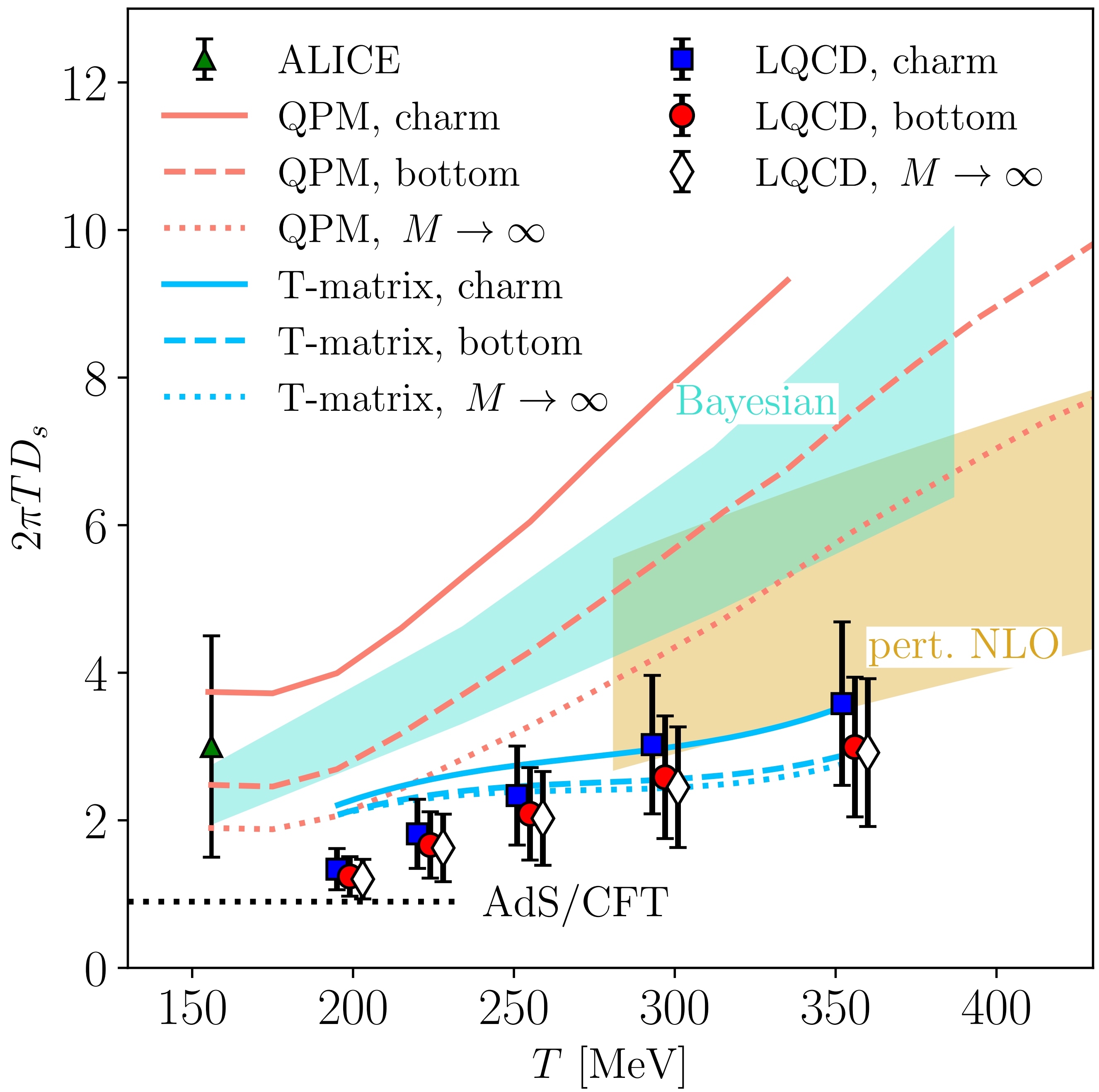
$$G(\tau, T) = \int \frac{d\omega}{\pi} K(\omega, \tau, T)\rho(\omega, T)$$

$$\begin{cases} \rho_{\text{max}} \equiv \max(\phi_{\text{IR}}, \phi_{\text{UV}}) \\ \rho_{\text{smax}} \equiv \sqrt{\phi_{\text{IR}}^2 + \phi_{\text{UV}}^2} \\ \rho_{\text{plaw}} \equiv \begin{cases} \phi_{\text{IR}} & \omega \leq \omega_{\text{IR}}, \\ a\omega^b \quad \text{for} & \omega_{\text{IR}} < \omega < \omega_{\text{UV}}, \\ \phi_{\text{UV}} & \omega \geq \omega_{\text{UV}}, \end{cases} \end{cases}$$

[LA, OK, RL, SM, PP, 舒海涛, SS, PRL 130 (2023) 23, 231902]

Charm and Bottom quark diffusion

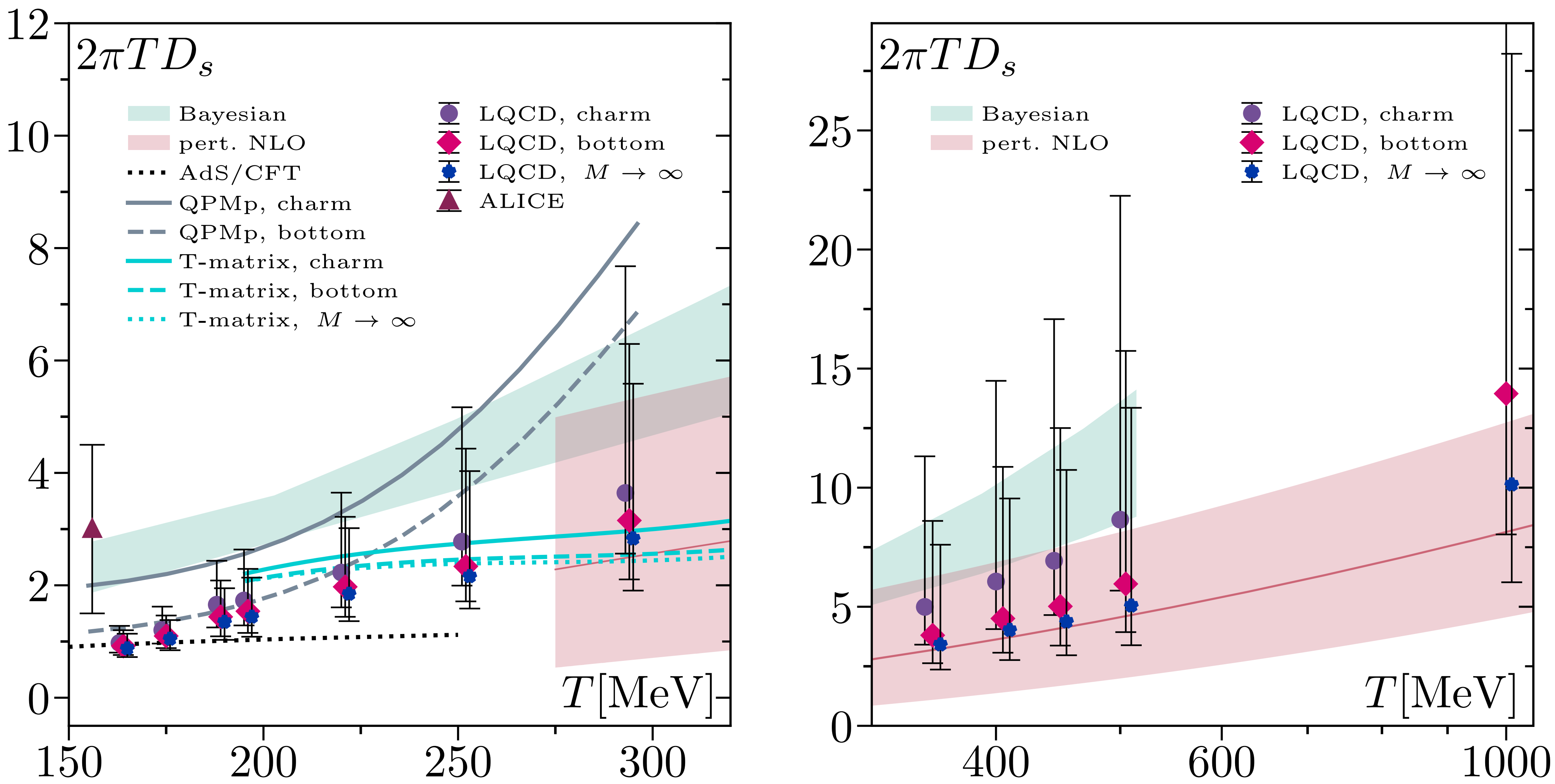
First full QCD determination of charm&bottom quark diffusion!



- HQ mass dependence of HQ diffusion: mild
- Universal change pattern with quark mass
- Weak quark mass dependence in LQCD & T-matrix
- Weaker than quasi-particle model (QPM) calculations

[LA, DC, OK, RL, GDM, SM, PP, 舒海涛, SS, PRL 132 (2024) 5, 051902]

HQ diffusion at the physical point



[JDG, SM, PP, 舒海涛, JHW, et al., JHEP 09 (2025) 180]

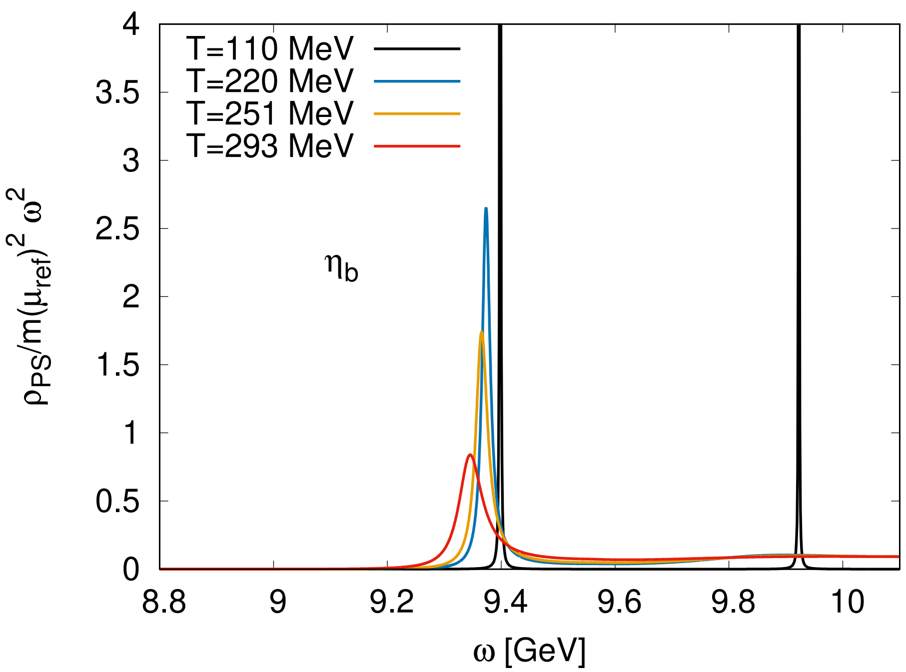
$m_\pi \sim 320\text{MeV}$ → $m_\pi \sim 160\text{MeV}$

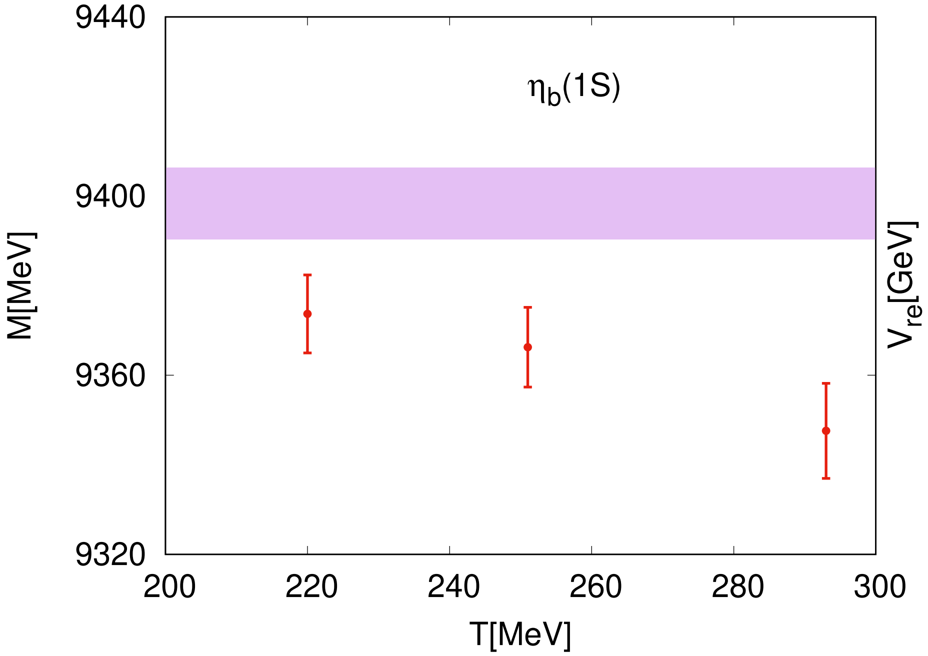
$195 \text{ MeV} \leq T \leq 352 \text{ MeV}$ → $163 \text{ MeV} \leq T \leq 10000 \text{ MeV}$

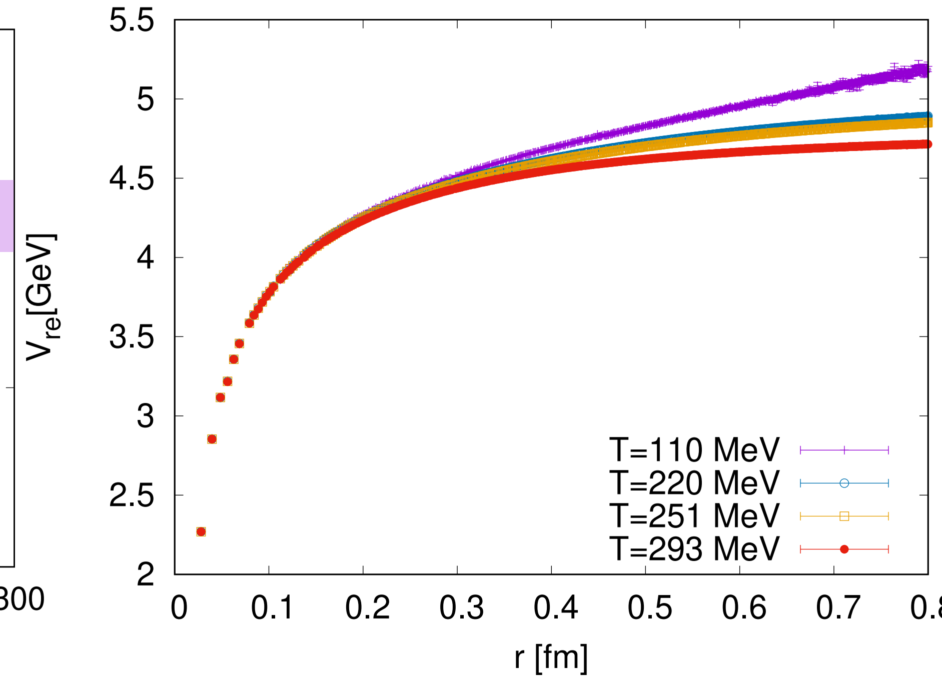
- Similar magnitudes as previous studies
- Approaching AdS/CFT limit near T_c
- Consistent with NLO at high temperature

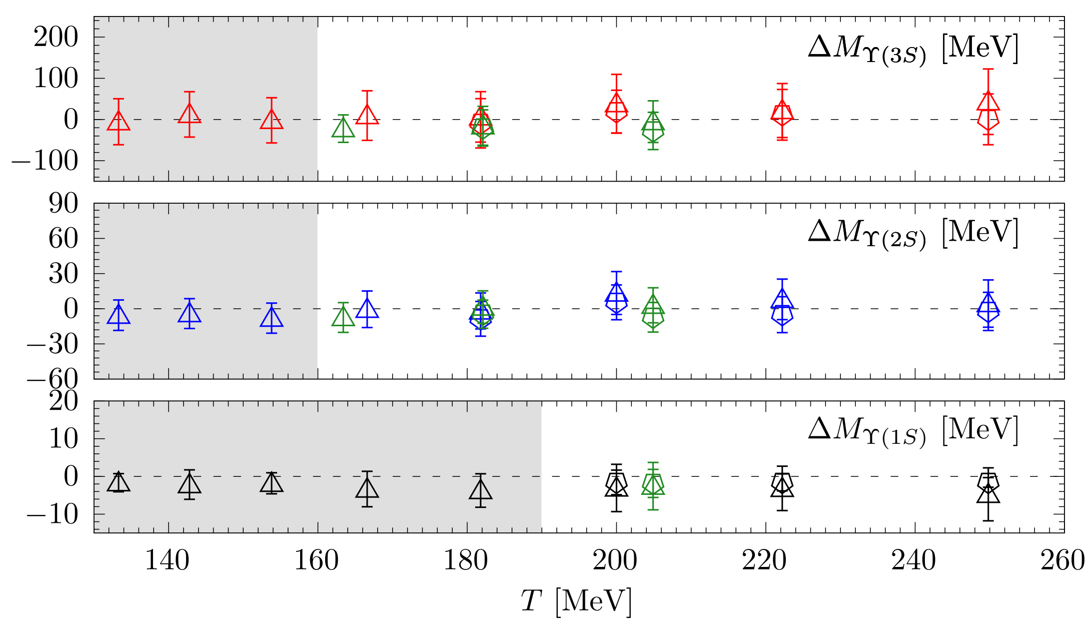
Heavy-flavor diffusion is insensitive to changes in the light degrees of freedom across the phase transition

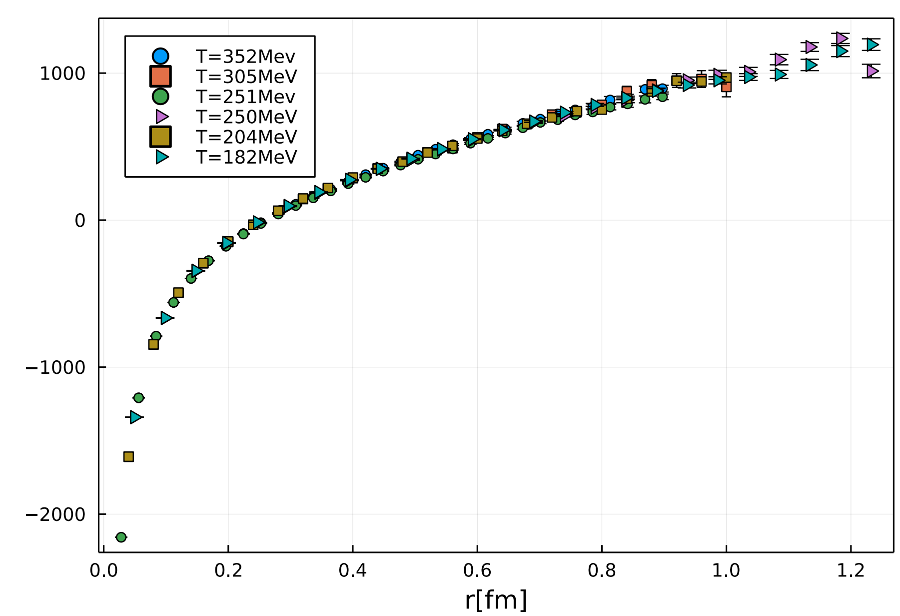
Summary of LQCD on heavy flavors





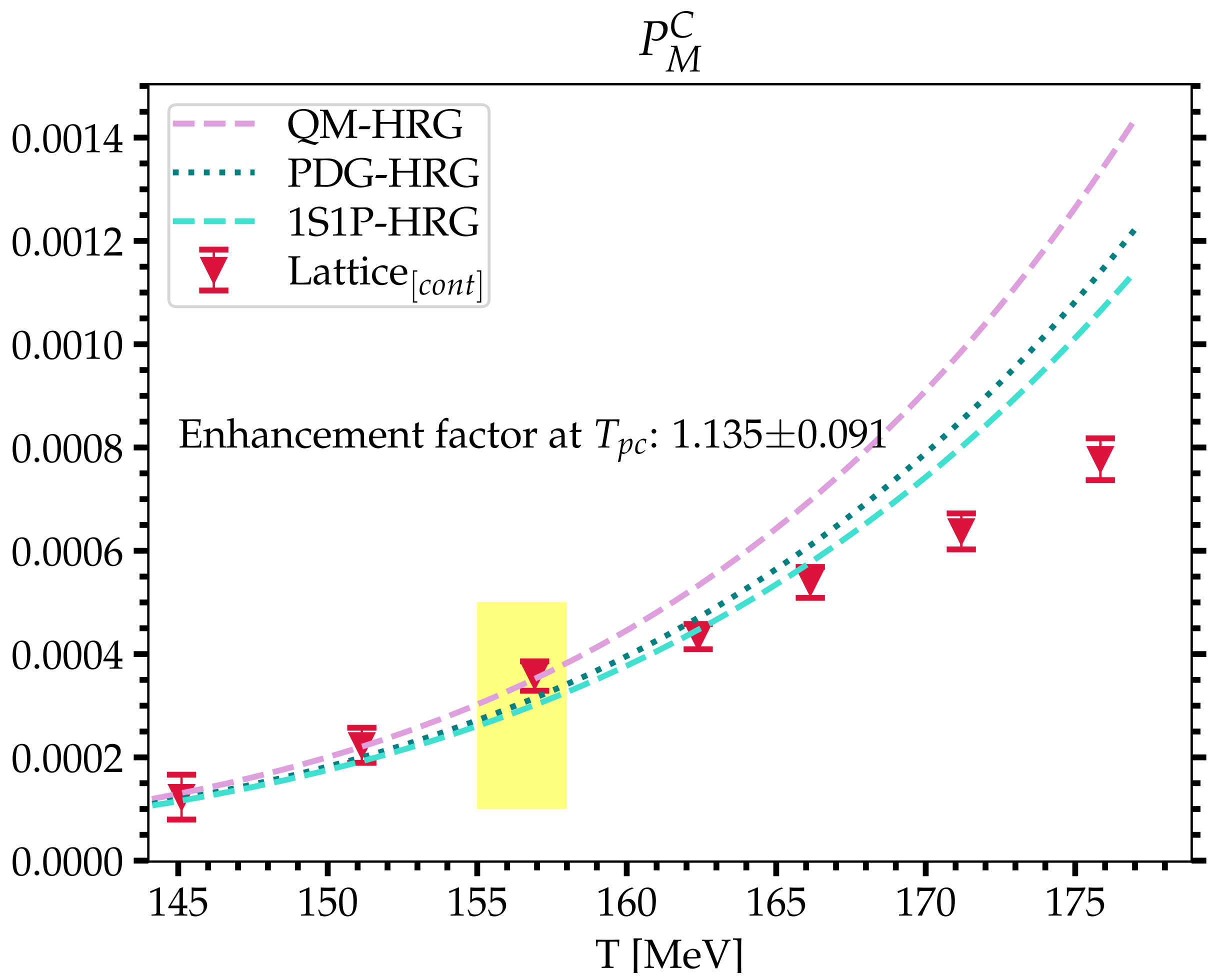


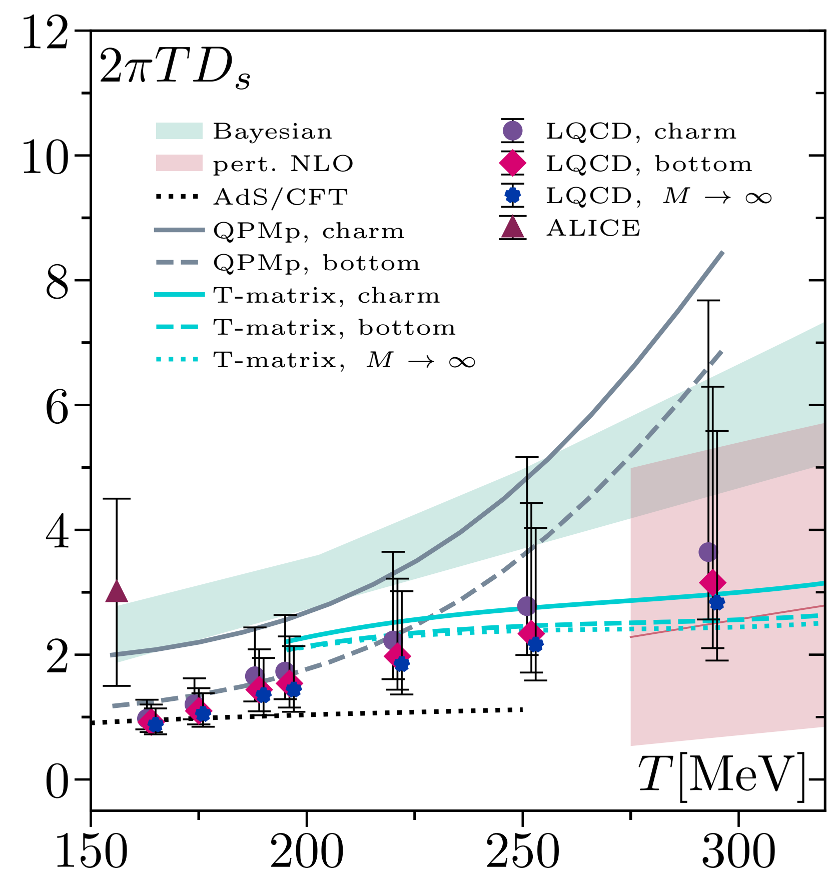




- First full QCD study of quarkonium using relativistic heavy quarks
- Screening or not requires more theory guidance
- increasing $\mathrm{Im}[V]$ with $T\ \&\ r$

Summary of LQCD on heavy flavors





- Comparing LQCD and HRG reveals missing PDG charmed hadrons and provides information on the dissociation of charm hadrons above T_{pc}
- First full QCD calculation of HQ diffusion coefficient at the physical point in wide temperature range

Lattice techniques provide crucial insights into heavy-flavor physics