Multiplicity distributions in QCD jets and Large-radius jet suppression with substructure dependence at the LHC

Xiang-Pan Duan (段香盼)
Fudan University (复旦大学)

Based on: arXiv:2503.24200, arXiv:2509.06158

In collaboration with
Lin Chen (陈林), Guo-Liang Ma (马国亮), Carlos A. Salgado, and Bin Wu (吴斌)

第一届喷注与重夸克物理研讨会

Outline

- KNO scaling in QCD jets within the DLA framework
  - Multiplicity probability distributions and KNO scaling in quark and gluon jets
  - X.-P. Duan, L. Chen, G.-L. Ma, C. A. Salgado, and B. Wu, PRD **112**, 094022 (2025)
- Multiplicity distributions in QCD jets within the MDLA framework
  - Modified DLA (MDLA) = DLA + energy conservation
  - X.-P. Duan, L. Chen, G.-L. Ma, C. A. Salgado, and B. Wu, arXiv:2509.06158
- Large-radius jet suppression with substructure dependence at the LHC
  - LO + DLA + BDMPS-Z
  - X.-P. Duan, L. Chen, G.-L. Ma, C. A. Salgado, and B. Wu, work in progress

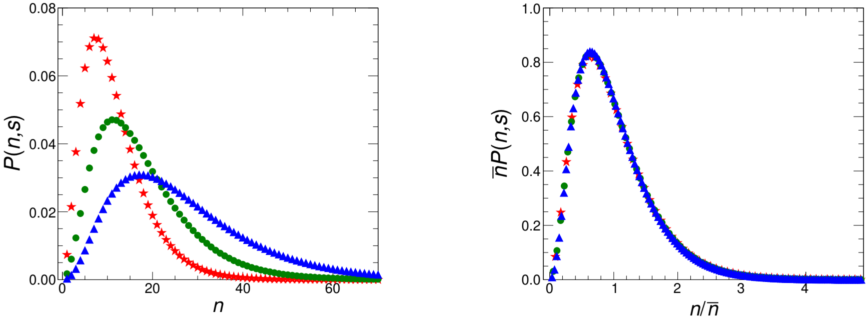
Koba-Nielsen-Olesen (KNO) scaling

- **KNO scaling** describes the universal behavior of multiplicity distributions:

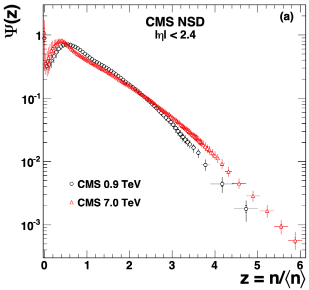
$$\bar{n}P(n,s) = \Psi(n/\bar{n})$$

Koba, Nielsen, Olesen, NPB **40**, 317 (1972)

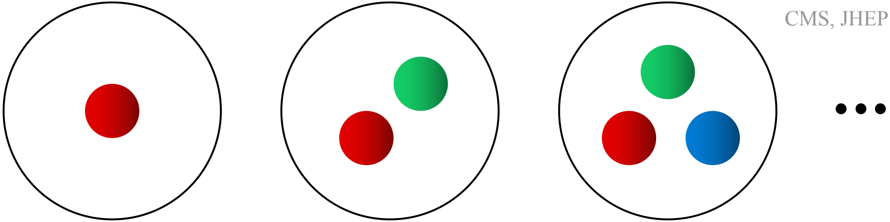
- $\bar{n}$: mean multiplicity
- $P(n,s)$: probability to produce n final-state particles in a collision at $\sqrt{s}$
- $\Psi(x)$: universal function with $\int \mathrm{d}x\, \Psi(x) = 1$ and $x = n/\bar{n}$



- **KNO scaling support:**
  - e^+e^- collisions, Deep Inelastic Scattering (DIS)
- **KNO scaling violation:**
  - pp collisions
- **KNO scaling in QCD jets**
- **QCD jets with p_T and R**



CMS, JHEP **01**, 079 (2011)



P(1) P(2) P(3)

Generating functions and evolution equation in QCD jets within DLA

Ellis, Stirling, and Webber, 2 (2011)
Dremin and Gary, Phys. Rept. **349**, 301 (2001)

- GF for a jet initiated by a parton a at scale Q is defined as

$$Z_a(u,Q) \equiv \sum_{n=0}^{\infty} u^n P_a(n,Q), \quad \text{with } Z_a(1,Q) = 1$$

- In DLA, the splitting function suffices:

$$\boldsymbol{\hat{P}_{a\to ga}(z) \approx 2C_a/z}$$

- Evolution equation in DLA:

$$\frac{\partial}{\partial \ln Q} Z_a(u,Q) = Z_a(u,Q) c_a \int \frac{dz}{z} \gamma_0^2 \left[Z_g(u,zQ) - 1\right]$$

with the anomalous dimension $\gamma_0 = \sqrt{\frac{2N_c\alpha_s}{\pi}}$

- The initial jet scale $\boldsymbol{Q = p_T R}$

Multiplicity probability distributions in quark and gluon jets within DLA

- Restricting to DLA phase space in $zQ = k^0\theta > k_\perp > Q_0$, GF is given by:

$$Z_a(u,y) = u\exp\left\{c_a\int_0^y \mathrm{d}\bar{y}\,(y-\bar{y})\gamma_0^2[Z_g(u,\bar{y})-1]\right\}$$

with $y \equiv \ln(Q/Q_0)$ and $\bar{y} \equiv \ln(k_\perp/Q_0)$

Dokshitzer, Khoze, Mueller, and Troian, (1991)

- By solving this equation:

$$P_a(n,Q) = \frac{1}{n!}\frac{\partial^n}{\partial u^n}Z_a(u,Q)\bigg|_{u=0}$$

- **Multiplicity probability distributions in DLA**:

$$P_a(1,Q) = \exp\left\{-c_a\int_0^y \mathrm{d}\bar{y}\,(y-\bar{y})\gamma_0^2\right\},$$

$$P_a(n+1,Q) = \sum_{k=1}^{n}\frac{k}{n}P_a(n+1-k,Q)\times c_a\int_0^y \mathrm{d}\bar{y}\,(y-\bar{y})\gamma_0^2 P_g(k,\bar{y})$$

- The normalization condition: $\sum_{n=0}^{\infty} P_a(n,Q) = 1$

Mean multiplicity in quark and gluon jets within DLA

- Mean multiplicity from the first derivative of the GF at $u = 1$:

$$\bar{n}_a(Q) = 1 + c_a \int_0^y \mathrm{d}\bar{y}\,(y - \bar{y})\gamma_0^2 \bar{n}_g(\bar{y})$$

- Relation between quark and gluon jets in DLA:

$$\bar{n}_q(Q) - 1 = c_q[\bar{n}_g(Q) - 1]$$

- Mean multiplicity for gluon jets (from the second-order ODE):

$$\bar{n}_g = \begin{cases} \cosh(\gamma_0 y) & \text{for fixed coupling} \\ z_1\,\{I_1[z_1]K_0[z_2] + K_1[z_1]I_0[z_2]\} & \text{for running coupling} \end{cases} \quad (1)$$

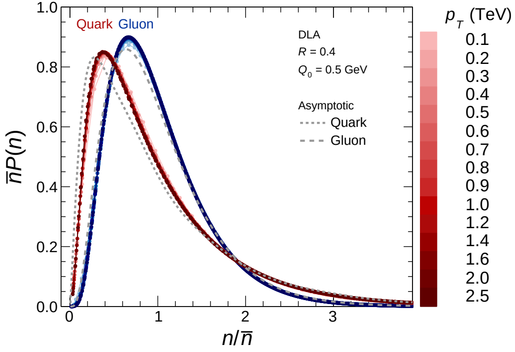
- From multiplicity probability distributions:

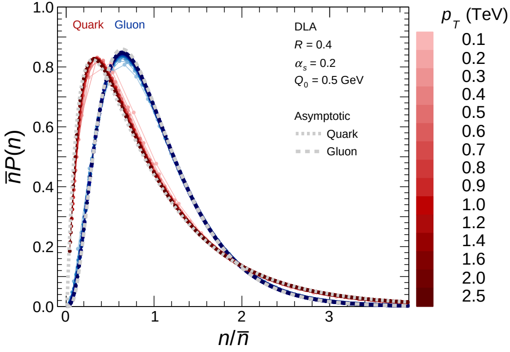
$$\bar{n}_a(Q) = \sum_{n=1}^{\infty} nP_a(n, Q) \quad (2)$$

Here, the agreement of these two approaches has been verified in our calculations

KNO scaling in quark and gluon jets within DLA

- $n = 1000$, p_T range: $0.1 - 2.5$ TeV, $R = 0.4$, $Q_0 = 0.5$ GeV
- Running coupling α_s and fixed coupling $\alpha_s = 0.2$
- Our DLA results are consistent with the asymptotic DLA ($Q \to \infty$)
- **The universal KNO scaling in quark and gluon jets**





Multiplicity distributions in inclusive jets

- Multiplicity distributions in inclusive jets expressed as:

$$P(n) = r_q P_q(n) + r_g P_g(n)$$
$$\bar{n} = r_q \bar{n}_q + r_g \bar{n}_g$$
$$r_a \equiv \frac{\mathrm{d}\sigma_a/\mathrm{d}p_T}{\mathrm{d}\sigma_q/\mathrm{d}p_T + \mathrm{d}\sigma_g/\mathrm{d}p_T}$$

- Leading-order (LO) differential cross section in pQCD:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}p_T} = 2p_T \sum_{a,b,c,d} \int \mathrm{d}y_c \mathrm{d}y_d x_a f_{a/p}(x_a, \mu^2) x_b f_{b/p}(x_b, \mu^2) \frac{\mathrm{d}\hat{\sigma}_{ab\to cd}}{\mathrm{d}\hat{t}}$$

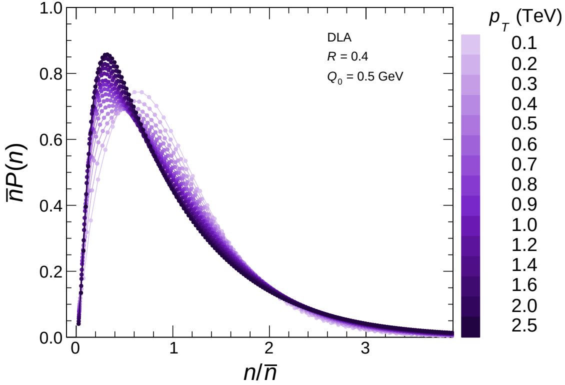
- KNO scaling for inclusive jets:

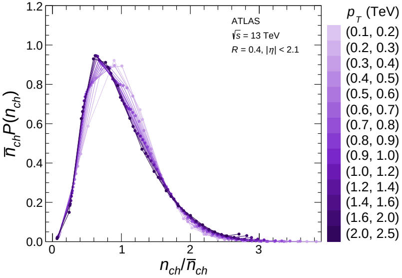
$$\bar{n}P(n) = \bar{n}[r_q P_q(n) + r_g P_g(n)] = r_q \frac{\bar{n}}{\bar{n}_q} \Psi_q(x\bar{n}/\bar{n}_q) + r_g \frac{\bar{n}}{\bar{n}_g} \Psi_g(x\bar{n}/\bar{n}_g)$$

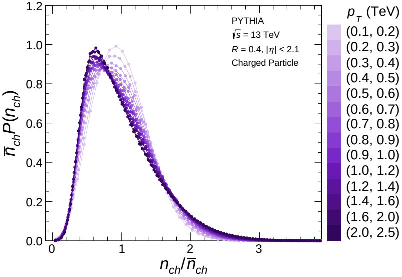
with the relation $n = x\bar{n} = x_q \bar{n}_q = x_g \bar{n}_g$

KNO functions in inclusive jets: ATLAS vs. PYTHIA vs. DLA

- No universal KNO scaling in inclusive jets across a wide p_T range
- **Low p_T: gluon jets** and **High p_T: quark jets**







Evolution equations in QCD jets within MDLA

- **Modified DLA (MDLA) = DLA + energy conservation:** Dokshitzer, PLB **305**, 295 (1993)

$$\frac{\partial}{\partial \ln Q} Z_a(u,Q) = c_a \int \frac{dz}{z} \gamma_0^2 \left[Z_g(u,zQ) Z_a(u,(1-z)Q) - Z_a(u,Q) \right]$$

- By equaling the following two expansions

$$\Phi_a(\beta) \equiv \int_0^\infty \mathrm{d}x \; \Psi_a(x) e^{-\beta x} = \sum_{k=0}^{\infty} \frac{(-\beta)^k}{k!} \int_0^\infty \mathrm{d}x \; x^k \Psi_a(x) = \sum_{k=0}^{\infty} \frac{(-\beta)^k}{k!} f_a^{(k)}$$

$$\Phi_a(\beta) = \lim_{Q\to\infty} \sum_{k=0}^{\infty} \frac{(e^{-\frac{\beta}{\bar{n}_a(Q)}} - 1)^k}{k!} n_a^{(k)}(Q) = \sum_{k=0}^{\infty} \frac{(-\beta)^k}{k!} \lim_{Q\to\infty} \frac{n_a^{(k)}(Q)}{[\bar{n}_a(Q)]^k}$$

where the second expression corresponds to the expansion of Z_a at $u = 1$:

$$Z_a(u,Q) = \sum_{k=0}^{\infty} \frac{(u-1)^k}{k!} n_a^{(k)}(Q)$$

KNO functions in QCD jets within MDLA

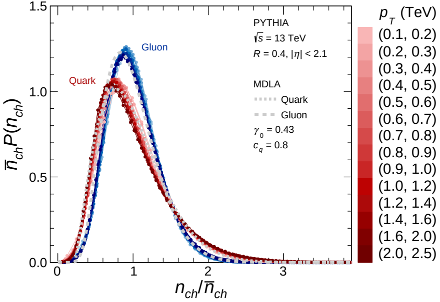
- The evolution equations at $u = 1$:

$$f_g^{(m)} = \frac{\gamma_0 m}{m^2 - 1} \sum_{k=1}^{m-1} \frac{m!}{k!(m-k)!} \frac{\Gamma(\gamma_0 k)\Gamma(\gamma_0(m-k)+1)}{\Gamma(\gamma_0 m + 1)} f_g^{(k)} f_g^{(m-k)}$$

$$f_q^{(m)} = \frac{c_q^{1-m}}{m^2} f_g^{(m)} + \gamma_0 \sum_{k=1}^{m-1} \frac{(m-1)!}{k!(m-k)!} \frac{\Gamma(\gamma_0 k)\Gamma(\gamma_0(m-k)+1)}{\Gamma(\gamma_0 m + 1)} c_q^{1-k} f_g^{(k)} f_q^{(m-k)}$$

with $f_a^{(0)} = f_a^{(1)} = 1$ for $m \geq 2$

- **PYTHIA** samples are generated with A14 tune
- **MDLA** uses two parameters:
  - $\gamma_0 = 0.43$
  - $c_q = 0.8$

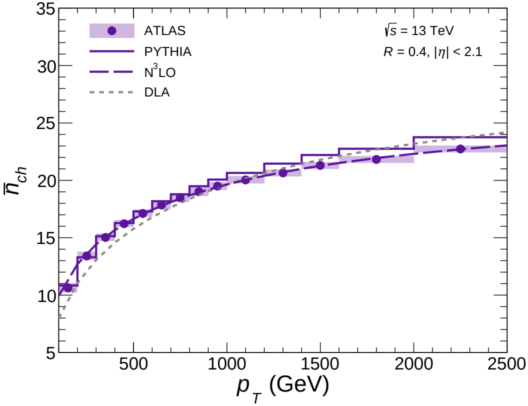


Mean charged-particle multiplicities: ATLAS vs. DLA vs. N^3LO

- **ATLAS:** two leading jets (anti-k_t, $R = 0.4$), $|\eta| < 2.1$, and $p_T^{\rm lead}/p_T^{\rm sublead} < 1.5$
- **DLA:** $\bar{n}_{\rm ch} = K_{\rm LPHD} \times \bar{n}_{\rm parton}$, with $K_{\rm LPHD} = 0.8$ (fit for running α_s)
- **N^3LO:** normalization constant $K = 0.0353$

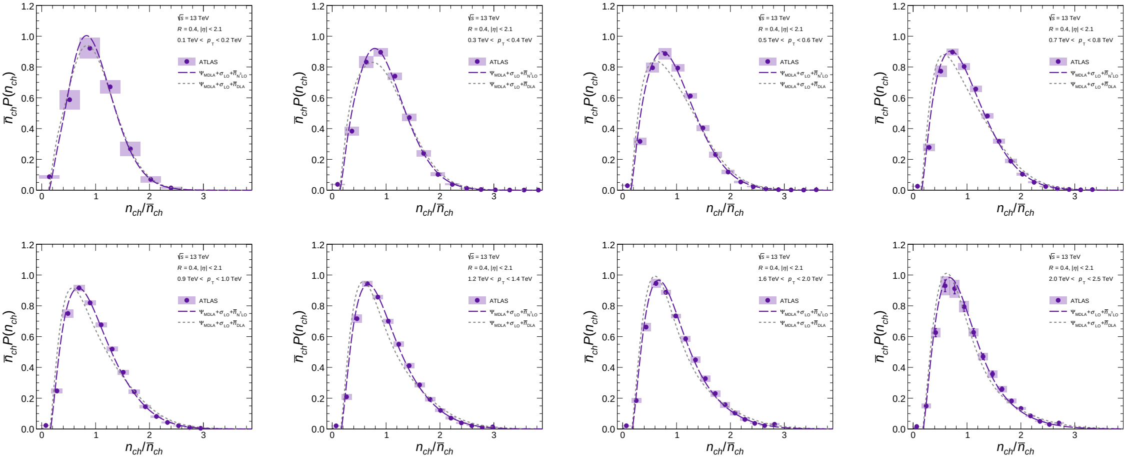
Dremin and Gary, PLB **459**, 341 (1999)

Capella *et al.*, PRD **61**, 074009 (2000)



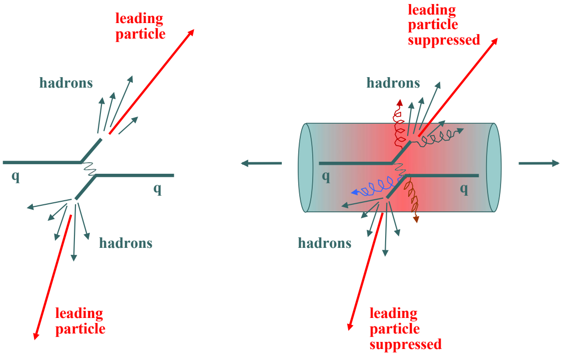
KNO functions in inclusive jets: ATLAS vs. MDLA

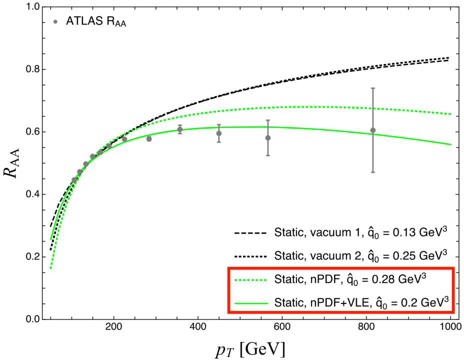
- **MDLA + LO + N^{3}LO**
- **Our KNO functions show excellent agreement with ATLAS data**



Jet quenching

- **Jet quenching**: energy loss and transverse-momentum broadening
- **Energy loss**: elastic scattering and inelastic processes via medium-induced radiation
- **Vacuum-like emissions** and **Medium-induced radiation**





https://indico.cern.ch/event/751767/contributions/3846600/ Adhya, Salgado, Spousta, and Tywoniuk, EPJC, **82**, 20 (2022)

Nuclear modification factor R_{AA}

- The differential cross section in AA collisions:

$$\frac{\mathrm{d}\sigma^{AA}}{\mathrm{d}p_T} = \int \mathrm{d}^2\vec{r}\, T_A(\vec{r}+\vec{b}/2) T_B(\vec{r}-\vec{b}/2) \frac{\mathrm{d}\phi}{2\pi} \times \sum_{i=q,g} \int \mathrm{d}\epsilon D_i(\epsilon) \left.\frac{\mathrm{d}\sigma_i^{pp}}{\mathrm{d}p'_T}\right|_{p'_T = p_T+\epsilon}$$

- The (2+1)-D viscous hydrodynamic model
- In **BDMPS-Z** formalism, the probability distribution for total medium-induced energy loss ϵ:

$$D(\epsilon) = \sum_{n=0}^{\infty} \frac{1}{n!} \left[\prod_{k=1}^{n} \int \mathrm{d}\omega_k \frac{\mathrm{d}I(\omega_k)}{\mathrm{d}\omega}\right] \delta\left(\epsilon - \sum_{k=1}^{n}\omega_k\right) \exp\left[-\int_0^{\infty} \mathrm{d}\omega \frac{\mathrm{d}I}{\mathrm{d}\omega}\right]$$

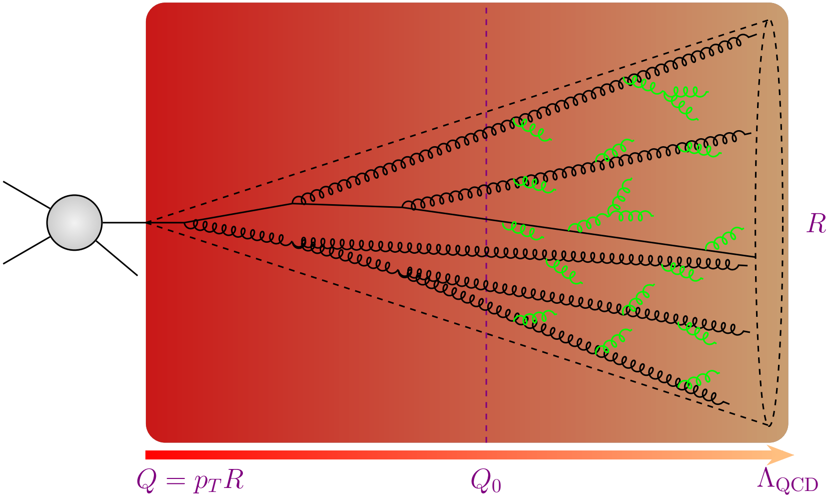
- Nuclear modification factor:

$$R_{AA} = \frac{1}{\langle T_{AA}\rangle} \frac{\mathrm{d}\sigma^{AA}/\mathrm{d}p_T}{\mathrm{d}\sigma^{pp}/\mathrm{d}p_T}$$

- Jet energy loss is treated as **color coherence**

A new framework

- **Vacuum-like emissions** and **Medium-induced radiation**



A new framework: LO + DLA + BDMPS-Z

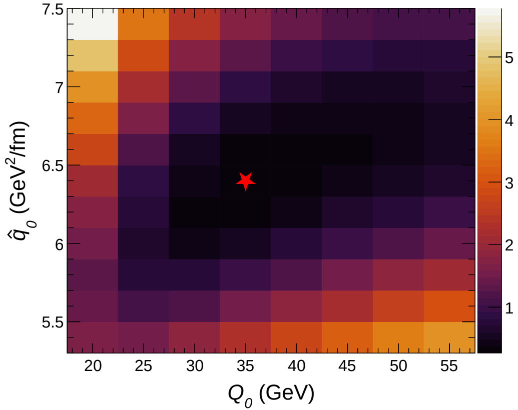
- **A new framework in C++: LO + DLA + BDMPS-Z**

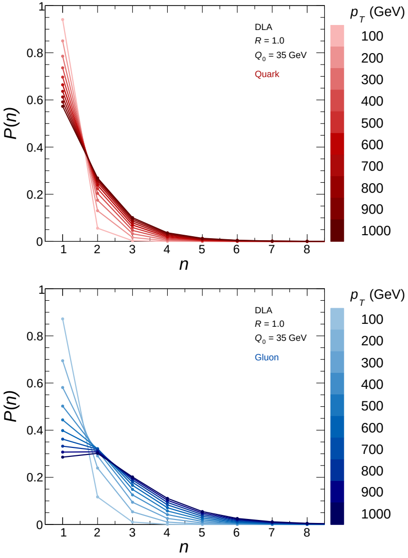
$$\frac{\mathrm{d}\sigma^{AA}}{\mathrm{d}p_T} = \int \mathrm{d}^2\vec{r}\, T_A(\vec{r}+\vec{b}/2) T_B(\vec{r}-\vec{b}/2) \frac{\mathrm{d}\phi}{2\pi}$$
$$\times \left[\sum_{i=q,g} P_i(1, p_T' R) \int \mathrm{d}\epsilon_1 D_i(\epsilon_1) \left. \frac{\mathrm{d}\sigma_i^{pp}}{\mathrm{d}p_T'} \right|_{p_T'=p_T+\epsilon_1} \right.$$
$$\left. + \sum_{n=2}^{N} \sum_{i=q,g} P_i(n, p_T' R) \int \mathrm{d}\epsilon_1 D_i(\epsilon_1) \left(\prod_{m=2}^{n} \int \mathrm{d}\epsilon_m D_g(\epsilon_m) \right) \left. \frac{\mathrm{d}\sigma_i^{pp}}{\mathrm{d}p_T'} \right|_{p_T'=p_T+\sum_{k=1}^{n}\epsilon_k} \right]$$

- The first term corresponds to the energy loss of a single parton
- The second term corresponds to the energy loss of multiple partons
- Jet energy loss is treated as **color decoherence**

Large-radius jet suppression

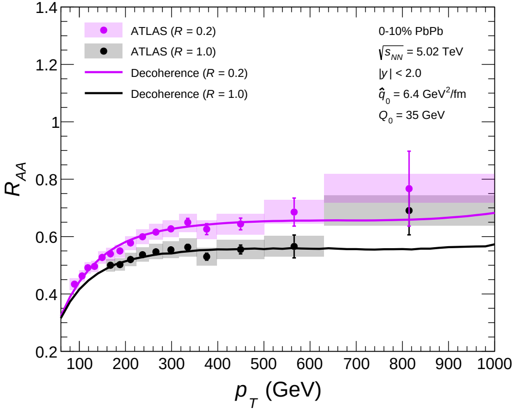
- Large-radius jet for $R = 1.0$ and also for $R = 0.2$
- The two-dimensional $\chi^2/\text{d.o.f.}$ distribution
- $\boldsymbol{Q_0 = 35}$ **GeV and** $\boldsymbol{\hat{q}_0 = 6.4}$ **GeV^2/fm**

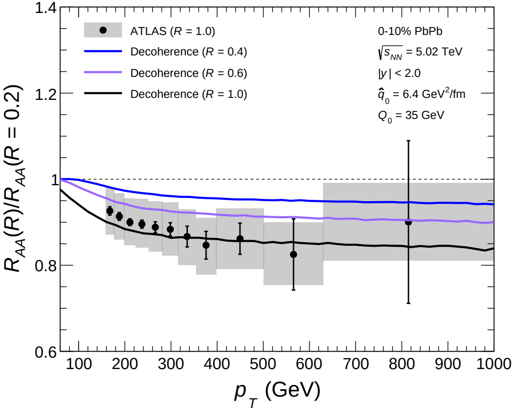




Jet suppression with cone-size dependence

- Our theoretical results show good agreement with the ATLAS data
- The R_{AA} distributions exhibit an approximately **flat behavior** at high transverse momentum
- **Large-radius jets are more strongly suppressed than narrow jets**





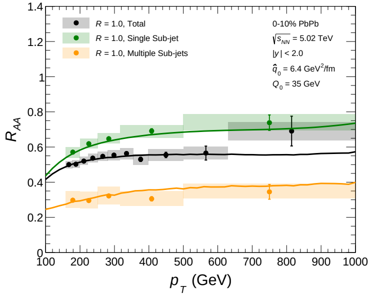
Large-radius jet suppression with substructure dependence

- Single subjet:

$$R_{AA,\mathrm{S}} = \frac{1}{\langle T_{AA}\rangle} \frac{\sum_{i=q,g} P_i(1, p'_T R) \left.\frac{\mathrm{d}\sigma_i^{AA}}{\mathrm{d}p'_T}\right|_{p'_T = p_T + \epsilon_1}}{\sum_{i=q,g} P_i(1, p_T R) \frac{\mathrm{d}\sigma_i^{pp}}{\mathrm{d}p_T}}$$

- Multiple subjets:

$$R_{AA,\mathrm{M}} = \frac{1}{\langle T_{AA}\rangle} \frac{\sum_{n=2}^{N} \sum_{i=q,g} P_i(n, p'_T R) \left.\frac{\mathrm{d}\sigma_i^{AA}}{\mathrm{d}p'_T}\right|_{p'_T = p_T + \sum_{k=1}^{n} \epsilon_k}}{\sum_{n=2}^{N} \sum_{i=q,g} P_i(n, p_T R) \frac{\mathrm{d}\sigma_i^{pp}}{\mathrm{d}p_T}}$$



- Our theoretical predictions agree well with the ATLAS data
- **Single subjet exhibits the weakest suppression**
- **Multiple subjets show significantly stronger suppression**

Summary

- **KNO scaling in QCD jets within the DLA framework**
  - Multiplicity probability distributions in quark and gluon jets
  - KNO scaling in quark and gluon jets
  - No universal KNO scaling in inclusive jets
- **Multiplicity distributions in QCD jets within the MDLA framework**
  - MDLA = DLA + energy conservation
  - Mean charged-particle multiplicities: N^3LO
  - MDLA + LO + N^3LO
- **Large-radius jet suppression with substructure dependence at the LHC**
  - A new framework: LO + DLA + BDMPS-Z
  - Large-radius jet suppression
  - Jet substructure: single subjet and multiple subjets

Thank You!