T-matrix approach for QGP and heavy-quark physics

Shuai Liu
Hunan University

The 1st Workshop on Jet and Heavy Quark Physics
, Jan 23-26, 2026 CCNU, Wuhan

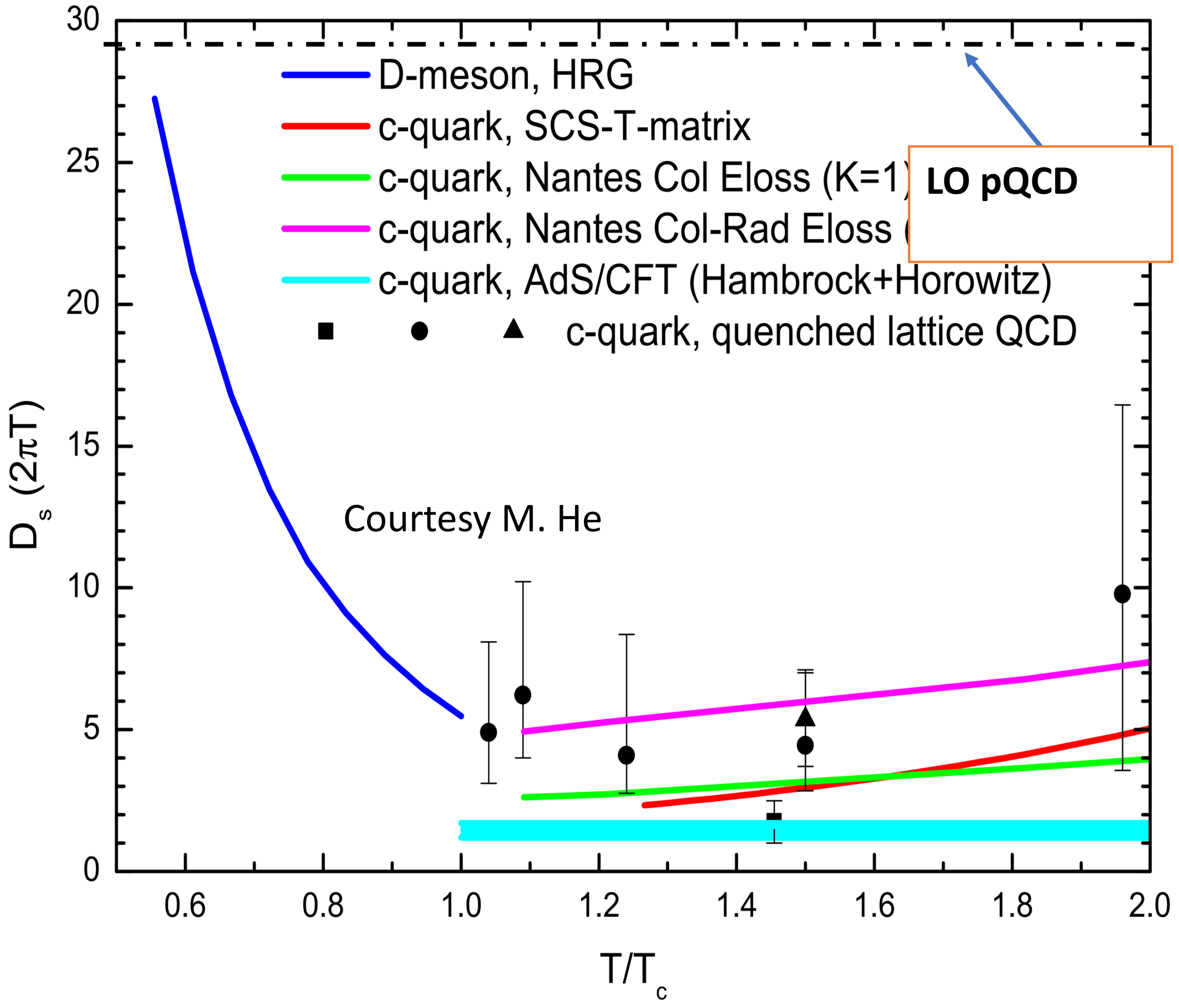
**Based on work: Liu and Rapp, PRC 97 (2018) 3, 034918, PRC 106 (2022) 5, 055201;
Li and Liu, arXiv:2206.11890**

Outline

1. Background and motivation
2. T-matrix approach
   - for Lattice QCD data
   - for transport properties
   - for spectral properties
3. Implication for spin alignment
4. Summary

A Strongly Coupled Picture of QGP

- $(2\pi T)D_s$, close to strongly coupled limit, an order of magnitude smaller than LO pQCD.
- QGP is strongly coupled
- Microscopically?

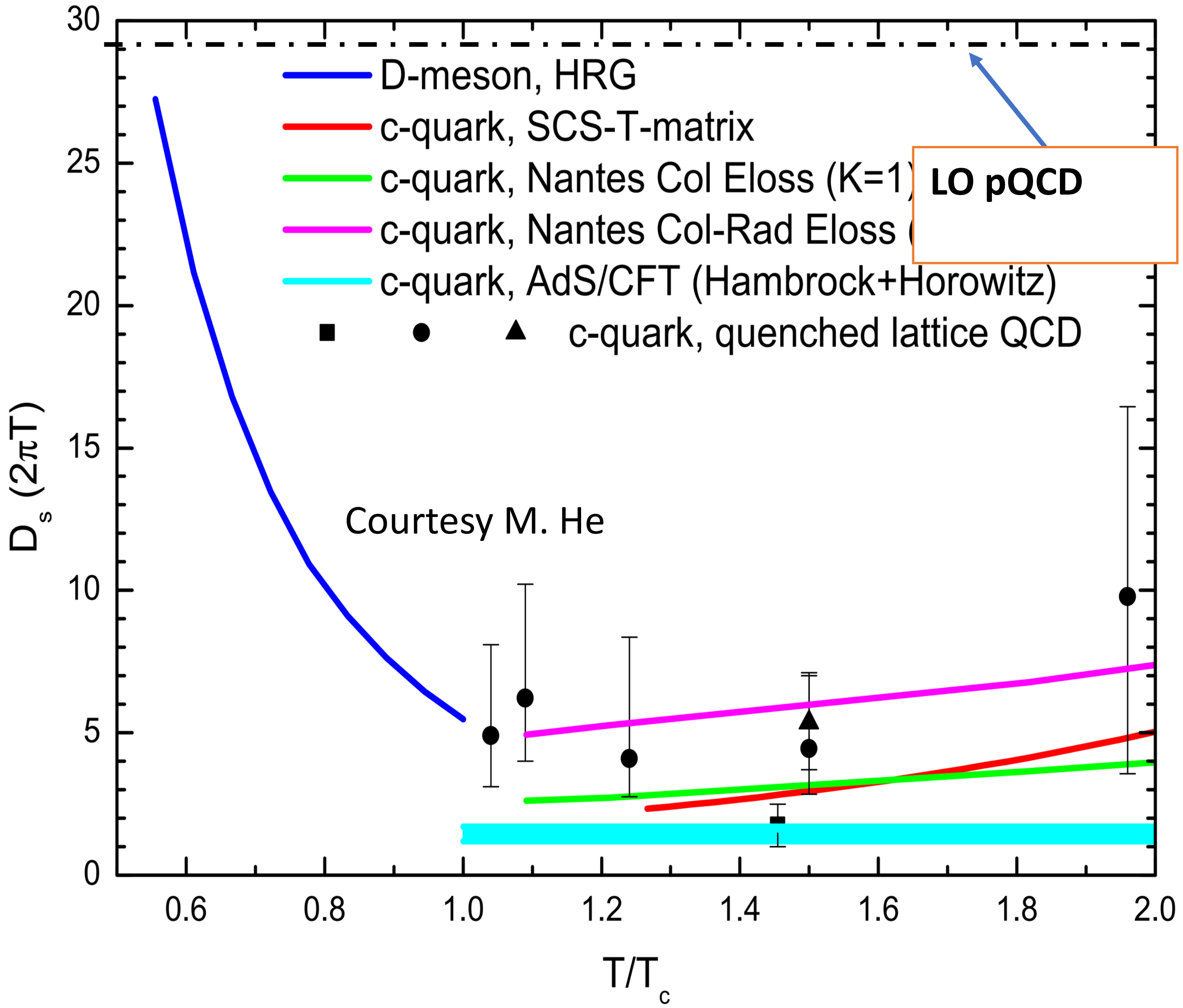


A Strongly Coupled Picture of QGP

- ❖ $(2\pi T)D_s$, close to strongly coupled limit, an order of magnitude smaller than LO pQCD.
- ❖ QGP is strongly coupled
- ❖ Microscopically:

T-matrix Approach:

Liu&Rapp, PRC 2018



Outline

1. Background and motivation
2. T-matrix approach
   - for Lattice QCD data
   - for transport properties
   - for spectral properties
3. Implication for spin alignment
4. Summary

Relativistic Constituent Quark & Gluon and Relativistic Potential

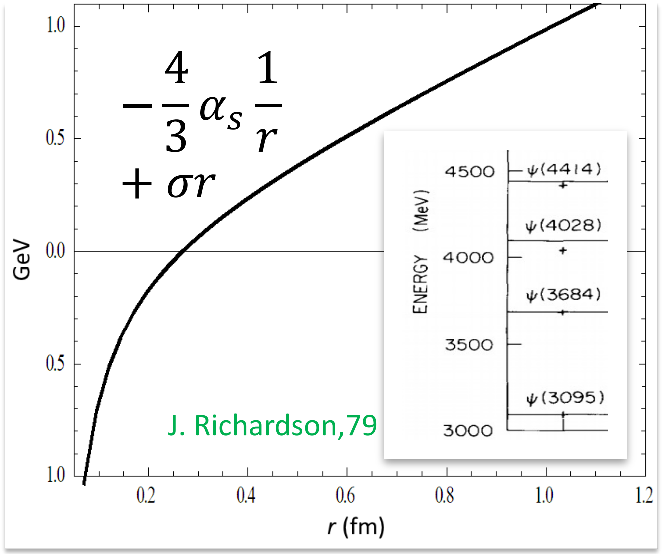
- An In-medium Unified Hamiltonian for Heavy/Light Quarks and Gluons

$$H = \sum \varepsilon_i(\mathbf{p})\psi_i^\dagger(\mathbf{p})\psi_i(\mathbf{p}) + \psi_i^\dagger(\frac{\mathbf{P}}{2}-\mathbf{p})\psi_j^\dagger(\frac{\mathbf{P}}{2}+\mathbf{p})V_{ij}^a\psi_j(\frac{\mathbf{P}}{2}+\mathbf{p}')\psi_i(\frac{\mathbf{P}}{2}-\mathbf{p}')$$

- Degrees of freedoms: relativistic constituent quark and gluons
  - With relativistic dispersion relation: $\varepsilon_i(\mathbf{p}) = \sqrt{M_i^2 + \mathbf{p}^2}$
  - With effective non-perturbative masses: M_i
- Interaction: relativistic non-perturbative color potential

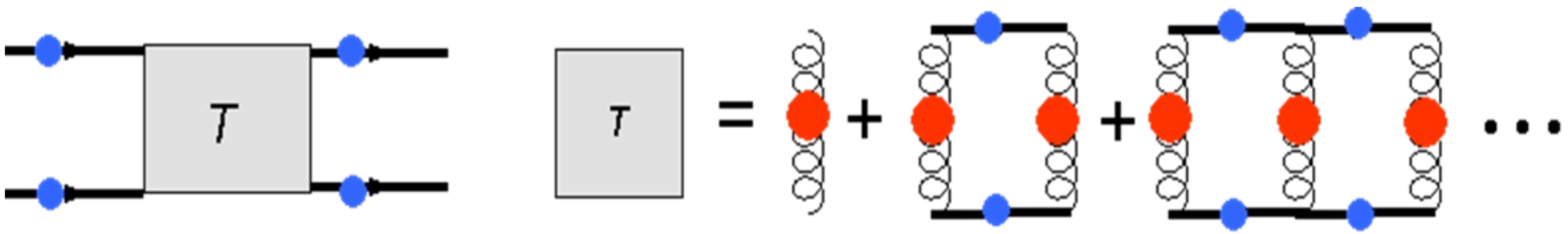
$$V_{ij}^a(\mathbf{p},\mathbf{p}') = \mathcal{R}_{ij}^{\mathcal{C}}\mathcal{F}_a^{\mathcal{C}}V_{\mathcal{C}}(\mathbf{p}-\mathbf{p}') + \mathcal{R}_{ij}^{\mathcal{S}}\mathcal{F}_a^{\mathcal{S}}V_{\mathcal{S}}(\mathbf{p}-\mathbf{p}')$$

  - With relativistic factors $\mathcal{R}_{ij}^{\mathcal{C}}$
  - With color Casimir factors $\mathcal{F}_{ij}^{\mathcal{C}}$ **[Liu+RR 16&18]**
  - Includes both color Coulomb and non-perturbative "confining" force
- **Serve as An Effective Theory for QCD**



- Recover QCD at HQ limit
- Including the confining interaction effectively **(nonperturbative effect)**

T-matrix Approach to the Strongly Coupled Plasma



- Dynamical bound states generation
- Scattering states in equal footing

- T-matrix equation: $T(E,\mathbf{p},\mathbf{p}') = V(\mathbf{p},\mathbf{p}') + \int \frac{d^3p}{(2\pi)^3} V(\mathbf{p},\mathbf{k}) G_{(2)}(E,\mathbf{k}) T(z,\mathbf{p},\mathbf{p}')$

$$G_{(2)} = G\,G \qquad G = \frac{1}{z-\varepsilon_{\mathbf{p}}-\Sigma} \qquad \Sigma = \sum_{s,c,f} \int \widetilde{d^4k}\, T(\mathrm{G})\, G$$

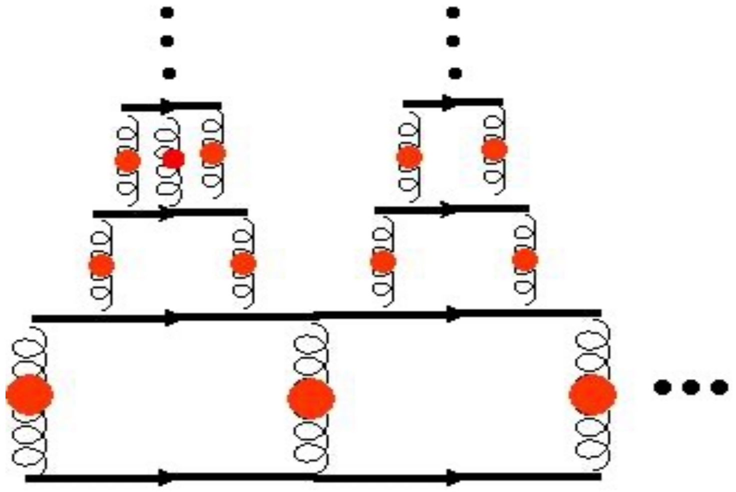
Self consistent self energy and many-body effects

- Luttinger–Wald functional (LWF):

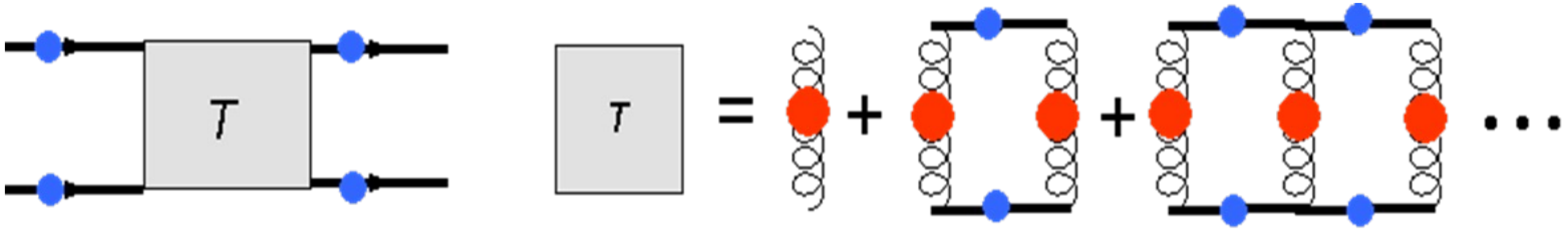
$$\Phi = \sum_{n,\nu} \mathrm{Tr}\left\{\frac{1}{2\nu}\Sigma_\nu\, G\right\} = \frac{1}{2}\sum_n \mathrm{Tr}\{-\ln(1-GGV)\}$$

- Grand potential in Luttinger–Wald Formalism:

$$\Omega = \mp\frac{-1}{\beta}\sum_n \mathrm{Tr}\{\ln(-G^{-1}) + (G_0^{-1}-G^{-1})G\} \pm \Phi$$



T-matrix Approach to the Strongly Coupled Plasma



- Dynamical bound states generation
- Scattering states in equal footing

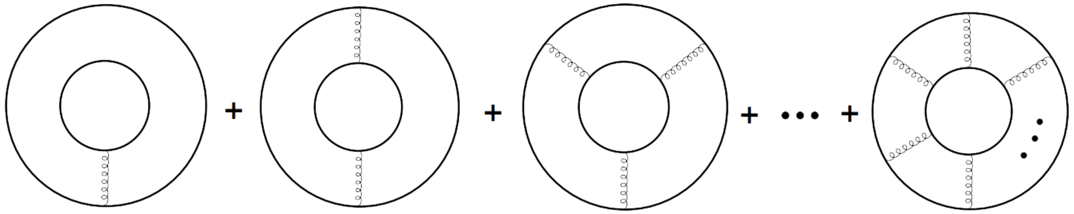
❖T-matrix equation: $T(E,\mathbf{p},\mathbf{p}') = V(\mathbf{p},\mathbf{p}') + \int \frac{d^3p}{(2\pi)^3} V(\mathbf{p},\mathbf{k}) G_{(2)}(E,\mathbf{k}) T(z,\mathbf{p},\mathbf{p}')$

$$G_{(2)} = G\ G \qquad G = \frac{1}{z - \varepsilon_{\mathbf{p}} - \Sigma} \qquad \Sigma = \sum_{s,c,f} \int \widetilde{d^4k}\, T(\mathrm{G})\, G$$

❖Luttinger–Wald functional (LWF):

$$\Phi = \sum_{n,\nu} \mathrm{Tr}\left\{\frac{1}{2\nu} \Sigma_\nu\, G\right\} = \frac{1}{2} \sum_n \mathrm{Tr}\{-\ln(1 - GGV)\}$$

Resonance/Interacting contribution to pressure



❖Grand potential in Luttinger–Wald Formalism:

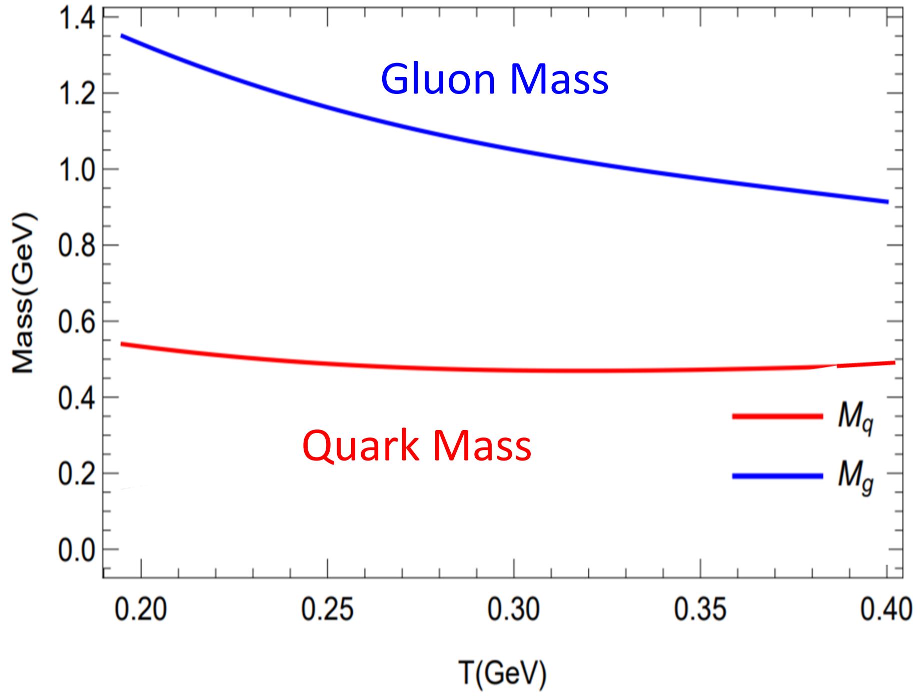
$$\Omega = \mp \frac{-1}{\beta} \sum_n \mathrm{Tr}\{\ln(-G^{-1}) + (G_0^{-1} - G^{-1})G\} \pm \Phi$$

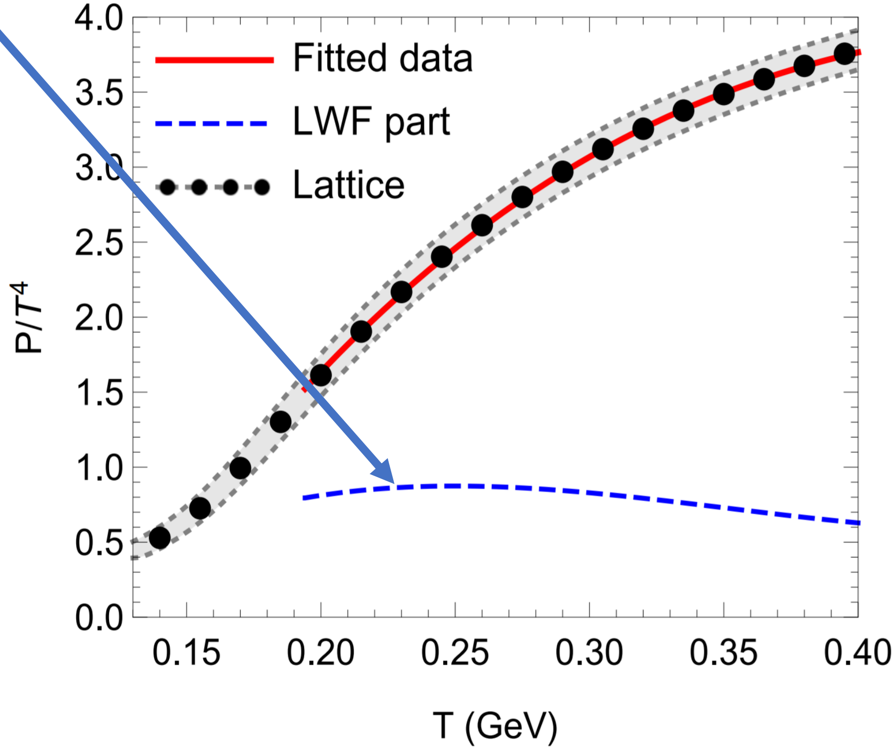
Outline

1. Background and motivation
2. T-matrix approach
   - for Lattice QCD data
   - for transport properties
   - for spectral properties
3. Implication for spin alignment
4. Summary

Lattice Data: Equation of States and Effective Masses

- Effective mass to reproduce the lattice EoS data
  - Large effective parton masses (thermal, gluonic condensates, ….)
- Dynamical transition from patrons to hadronic resonances

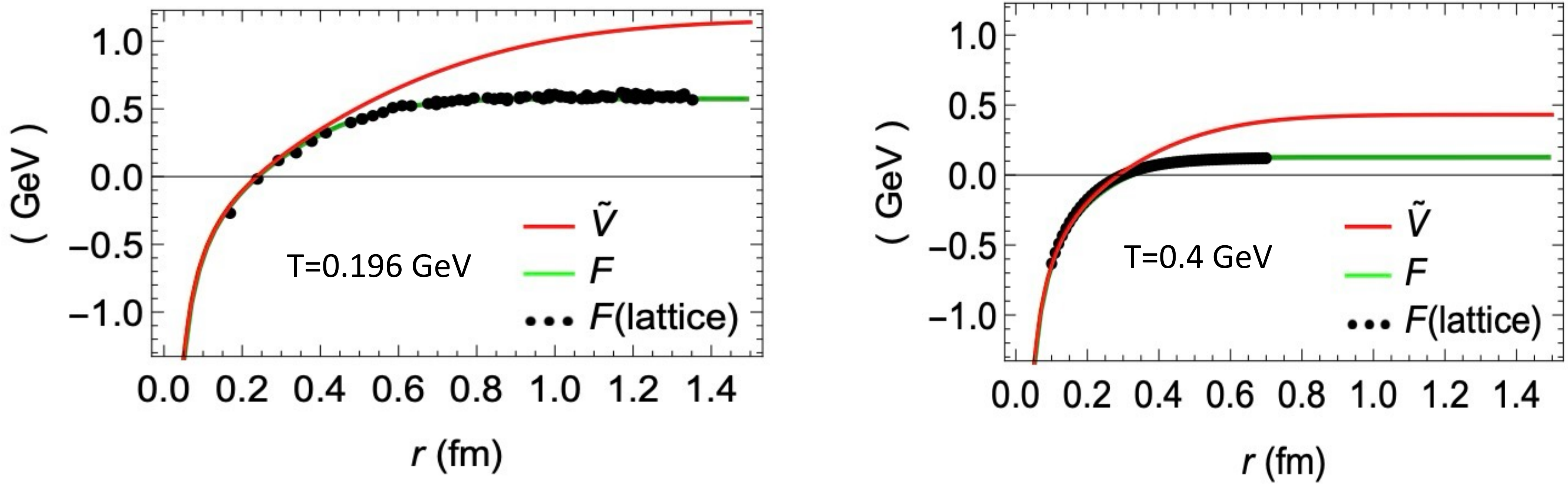




Liu &Rapp, PRC 2018

Lattice Data: Free Energy and Potential

- The potential and the HQ free energy data
  - Large remnant of the confining term
  - Especially strong around phase boundary
  - Much smaller at higher temperature

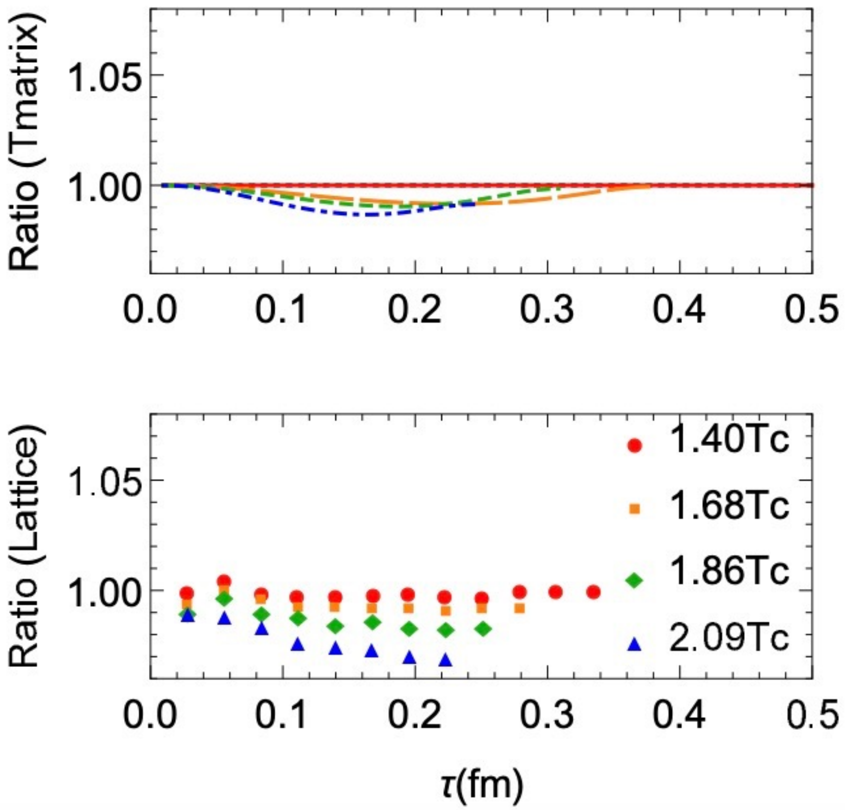


Liu &Rapp, PRC 2018

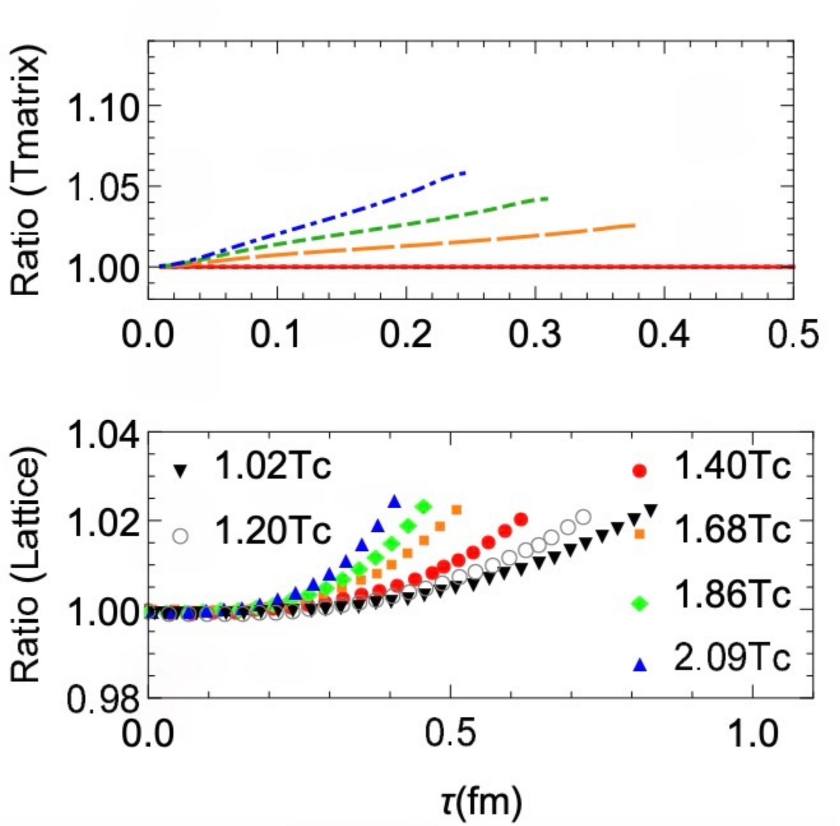
Lattice Data: Correlator Ratio of Quarkonium

- Correlator ratio: similar to lattice QCD data

Charmonium slightly below to 1



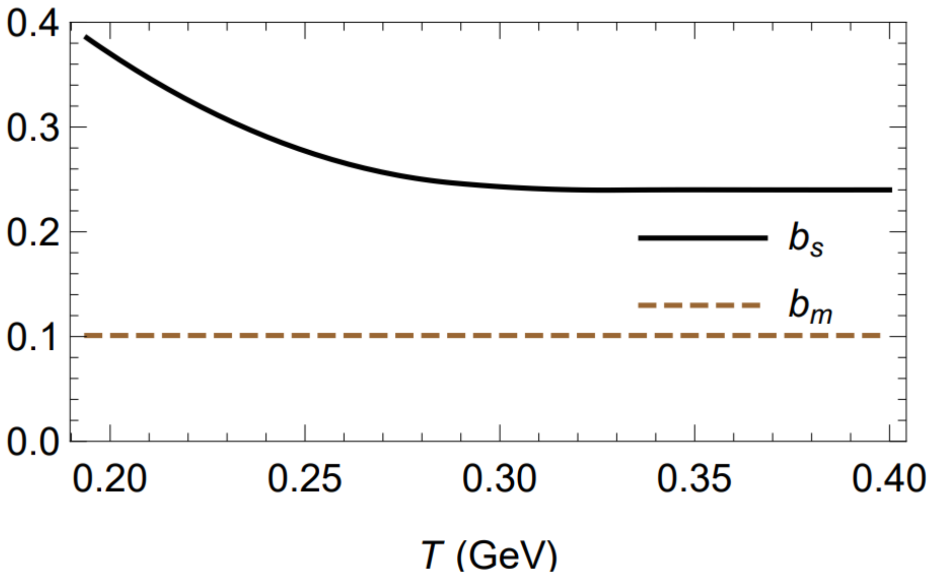
Bottomonium, larger than 1

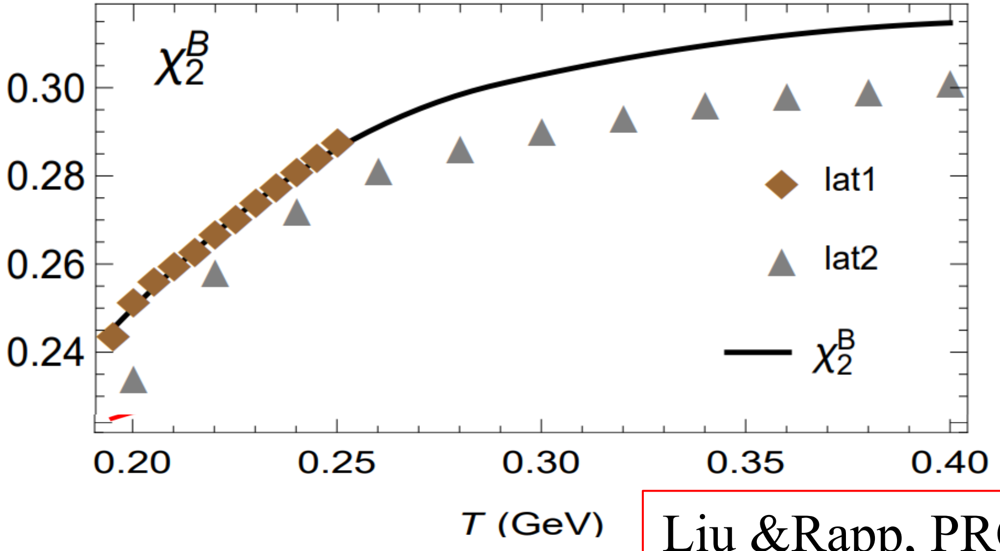


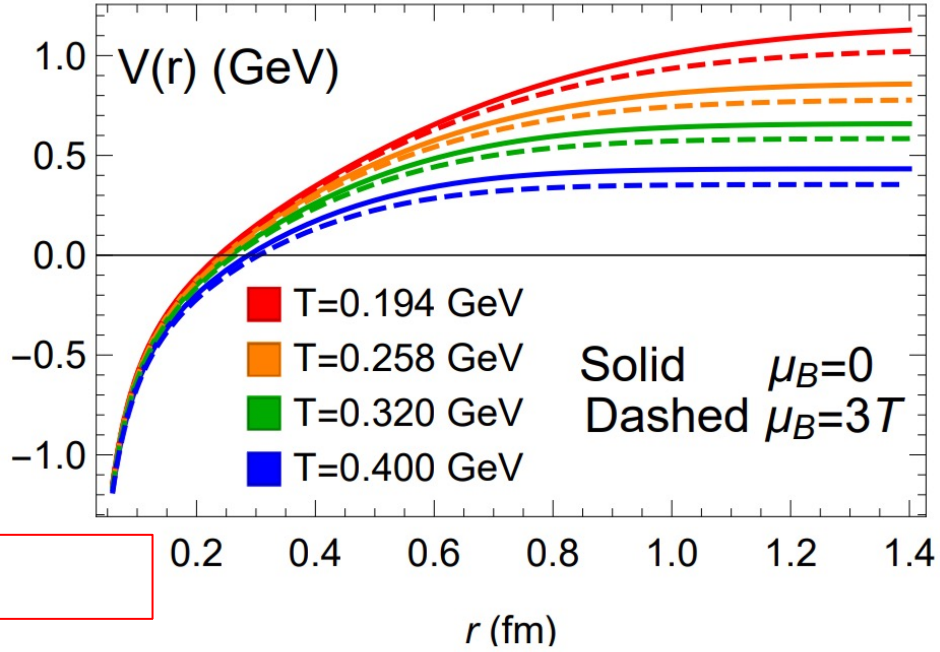
Liu &Rapp, PRC 2018

Lattice Data: Susceptibilities at Finite μ_B

- Propagator: $G_i^0(z,\boldsymbol{p}) = \dfrac{1}{z-\varepsilon_{\boldsymbol{p}} \pm \mu_i}$, $\varepsilon_{\boldsymbol{p}} = \sqrt{M_i^2 + p^2}$
- Masses: $M_i = M_i^0 \sqrt{1 + b_m \left(\dfrac{\mu_q}{T}\right)^2}$
- Screening masses: $m_d = m_d^0 \sqrt{1 + b_s \left(\dfrac{\mu_q}{T}\right)^2}$
- The parameters are fitted to the χ_2^B data







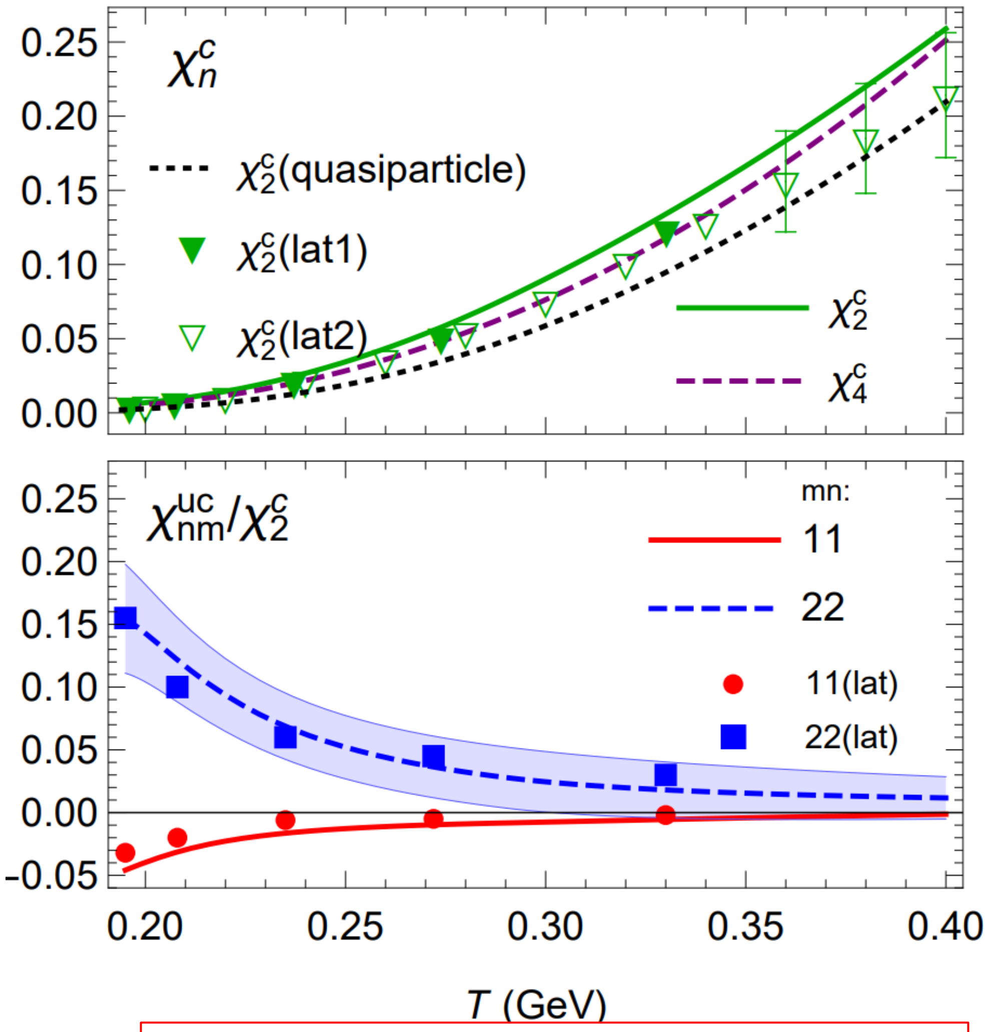
Liu &Rapp, PRC 2022

Lattice Data: Charm & Heavy-Light Susceptibilities

- Parameters are all fixed in previous steps
- χ_n^c NOT depend on the two new parameters b_m, b_s, since only need $\mu_B = 0$ information
- $\chi_2^c \approx \chi_4^c$

Mukherjee, Petreczky, Sharma, PRD 2016

- x_{22}^{uc} similar to HTL results in Ref
- $x_{11}^{uc} = 0$ for HTL in Ref
- $x_{11}^{uc} \neq 0$ for T-matrix approach



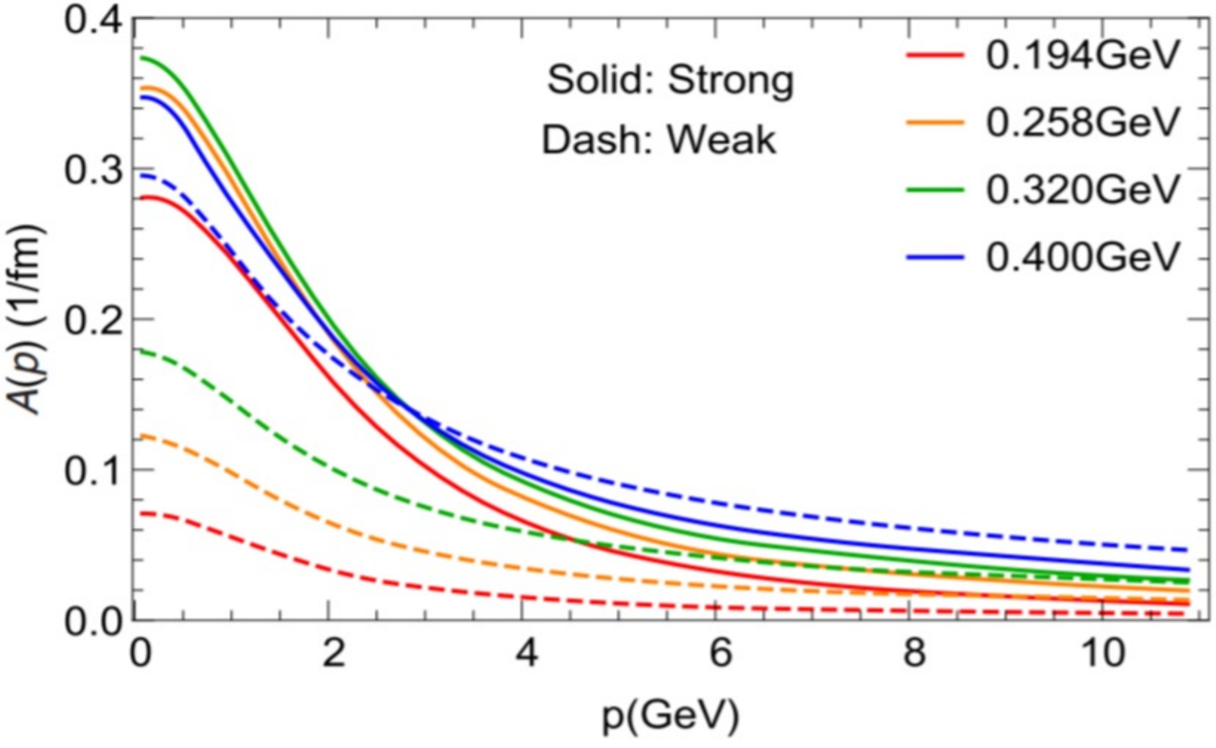
Liu &Rapp, PRC 2022

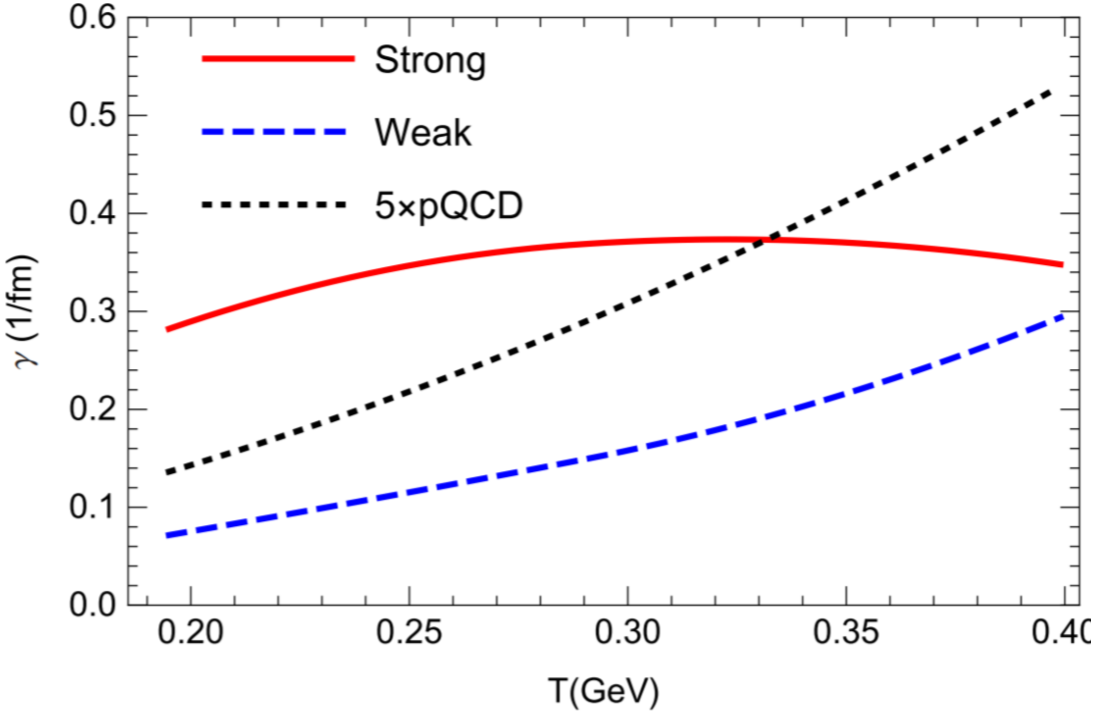
Outline

1. Background and motivation
2. T-matrix approach
   - for Lattice QCD data
   - for transport properties
   - for spectral properties
3. Implication for spin alignment
4. Summary

Transport: Drag for Heavy-Quark

- Overall large drag, flat *T* dependence

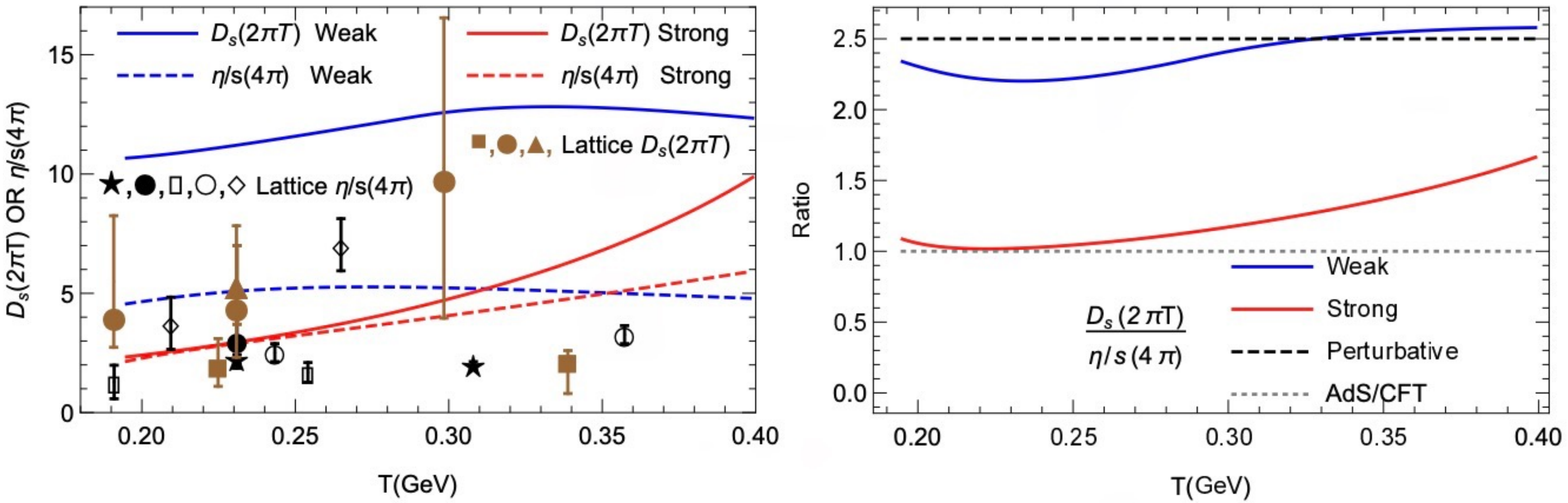




Liu, He, Rapp PRC 2019

Viscosity and Heavy-Quark Diffusion

- Spatial diffusion coefficient& Viscosity



- Strongly coupled: $(2\pi T)D_s \sim (4\pi)\ \eta/s$
- Perturbative: $(2\pi T)D_s \sim 5/2(4\pi)\ \eta/s$
- Transition as *T* increases

Liu, Rapp EPJA 2020

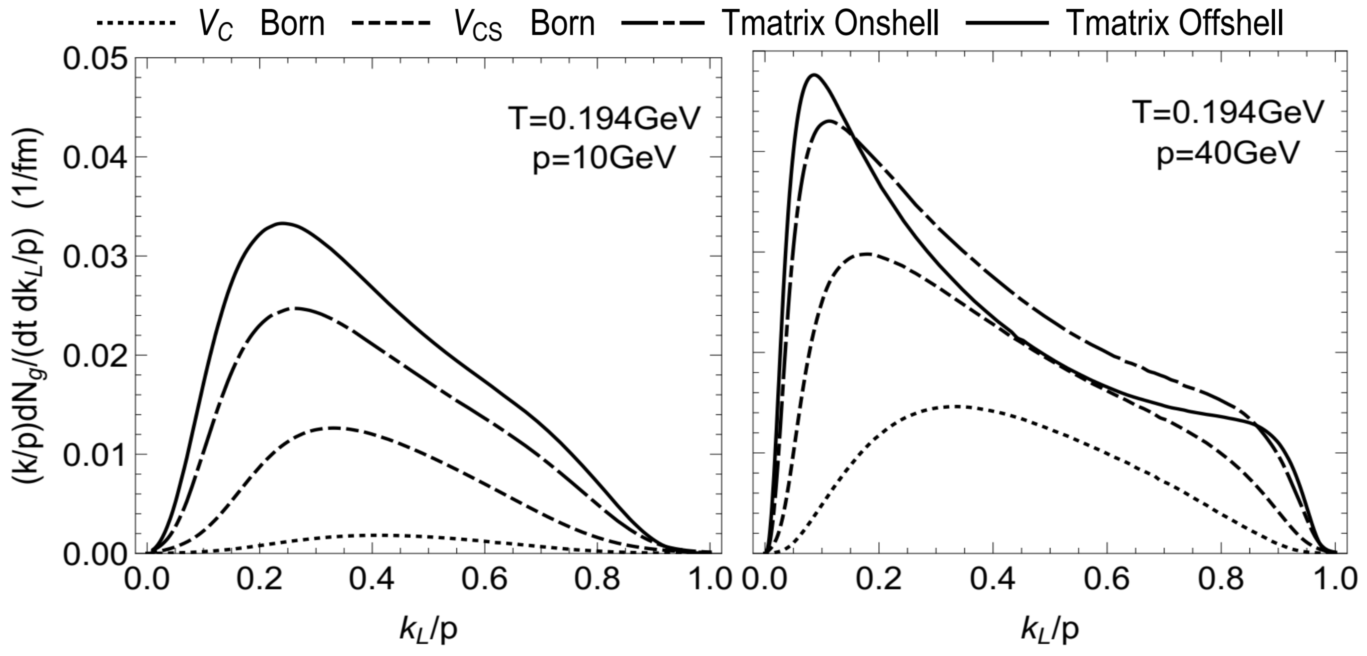
Strongly coupled solution preferred by experimental observables

Transport: Radiative Energy Loss and $\hat{q}$ by T-matrix

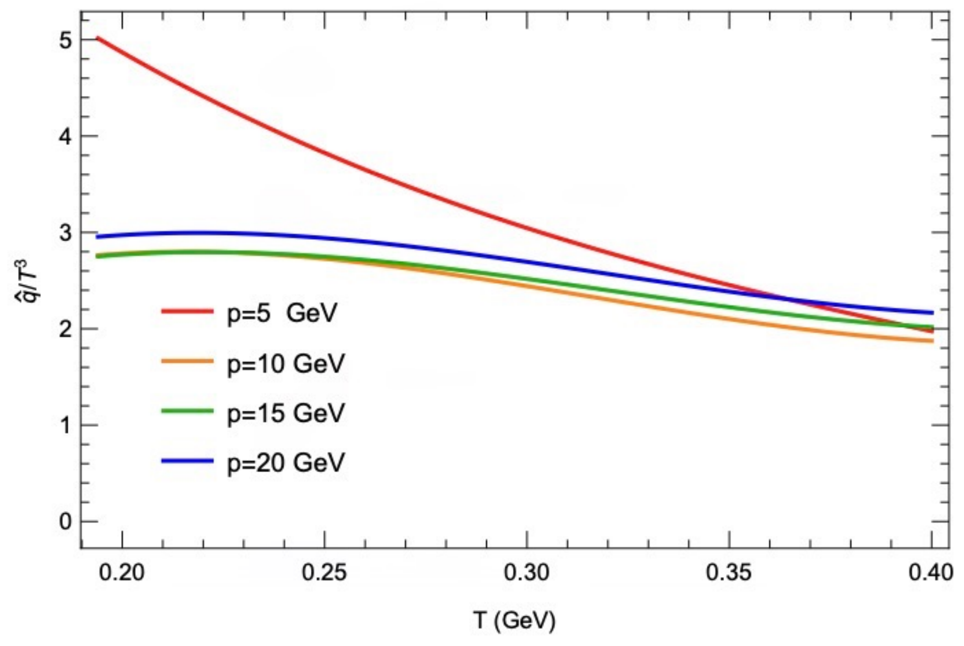
- Large interaction from confining force
- Low number density due to the fit of EoS
- Confining interaction is important at low T and low momentum region

Liu, Rapp JHEP 2020

Power spectrum $\frac{(k/p)dN_g}{dt\, d(k_L/p)} \approx \frac{x dN_g}{dt\, dx}$ at different T,p



$\hat{q}$ at different momentum



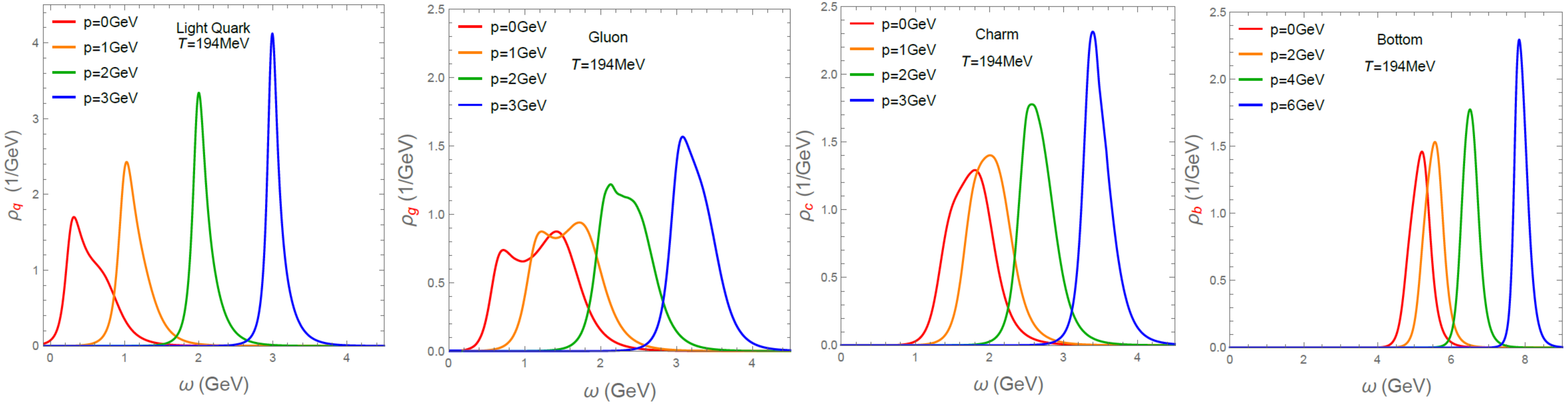
Outline

1. Background and motivation
2. T-matrix approach
   - for Lattice QCD data
   - for transport properties
   - for spectral properties
3. Implication for spin alignment
4. Summary

Spectral Properties: Quarks and Gluons

- QGP structure changes with resolution scale
- Distorted non-quasi-particle spectral function at low momentum
- Quasi-particle re-emerge at higher momentum
- Broad but still quasi-particle like heavy quark

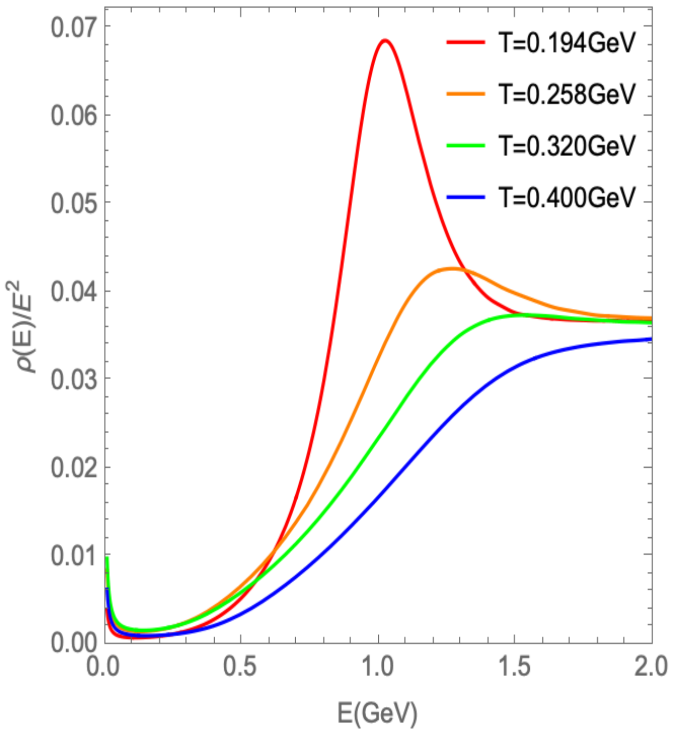
Liu, Rapp PRC 2018



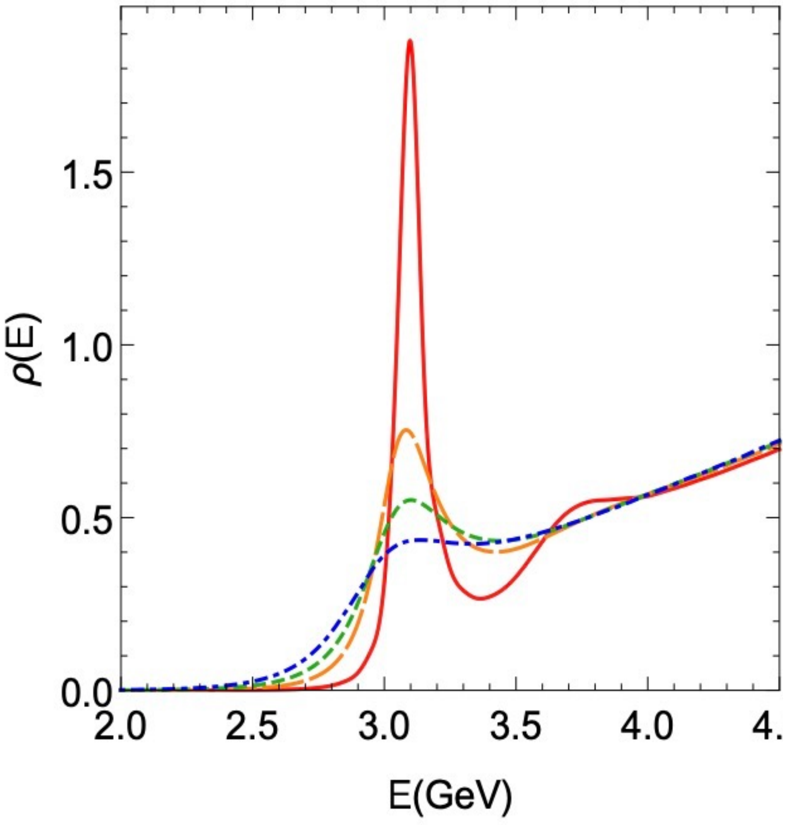
Spectral properties: hadronic states

Liu, Rapp PRC 2018

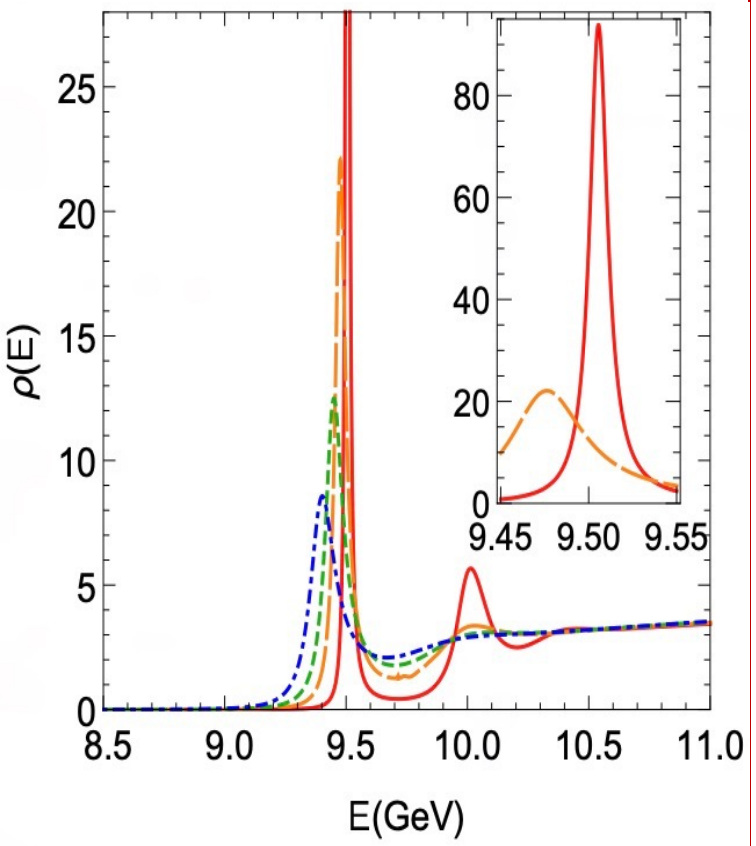
Preliminary ρ,ϕ like resonances spectral function



Charmonium Spectral function



Bottomonium Spectral Function

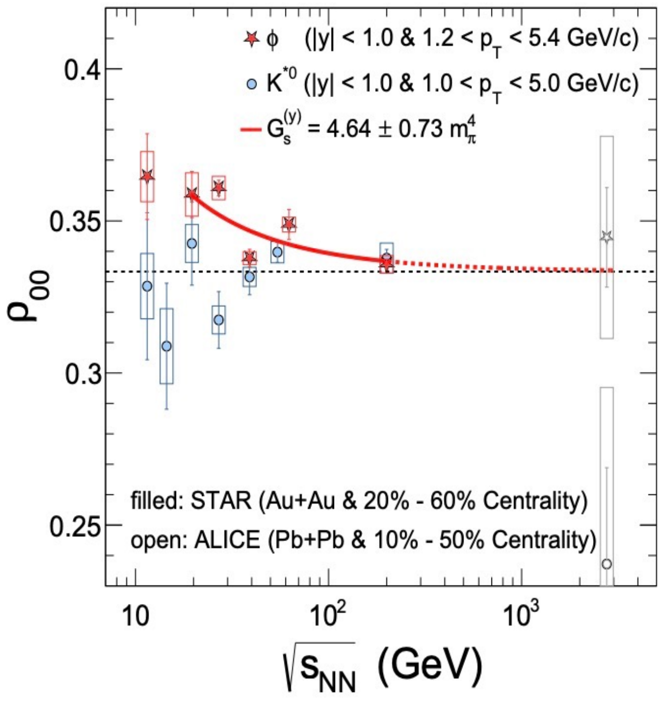


Outline

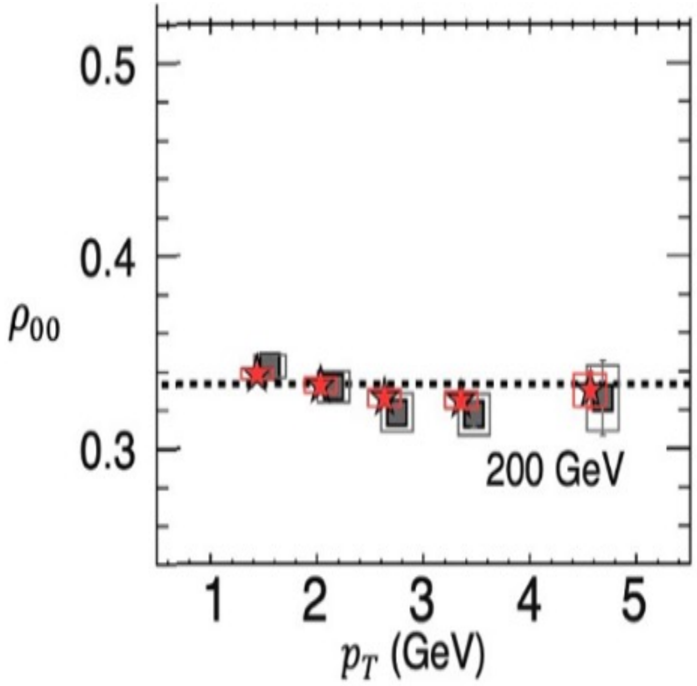
1. Background and motivation
2. T-matrix approach
   - for Lattice QCD data
   - for transport properties
   - for spectral properties
3. Implication for spin alignment
4. Summary

Puzzling Behavior of Spin Alignment

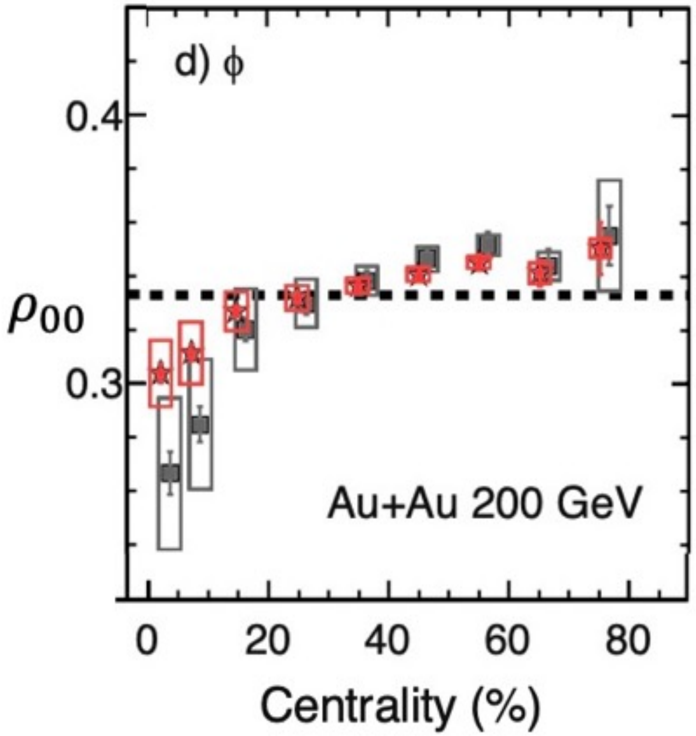
Beam energy dependence



p_T dependence



Centrality dependence



Large magnitudes, sign flipping behaviors …

Linear Response Theory

- The tensor polarization and spin alignment of vector meson （ϕ, ρ ）

  Li & Liu, arxiv: 2206.11890 , Li& Liu arxiv: : 2501.17861

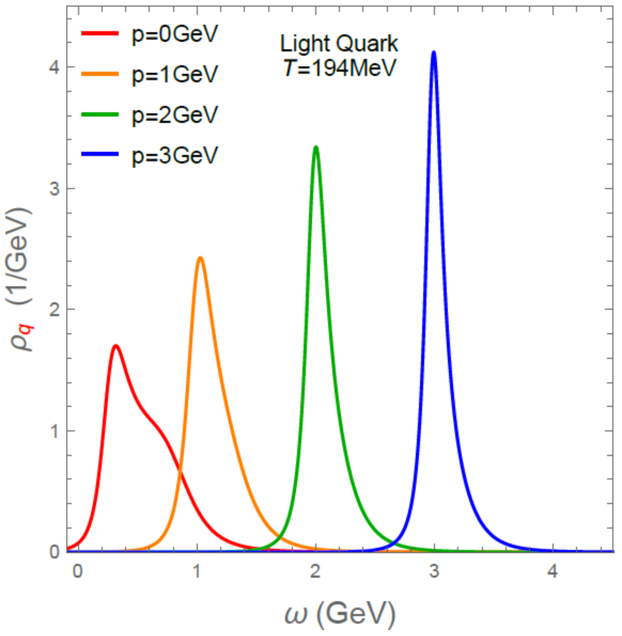
$$\delta\rho_{00}(\hat{n}_{\rm pr}, \boldsymbol{p}) = \frac{\int d\Sigma^\lambda p_\lambda \mathcal{T}^{\mu\nu}(x, \boldsymbol{p}) \hat{n}_\mu(\boldsymbol{p}) \hat{n}_\nu(\boldsymbol{p})}{d\Sigma^\lambda p_\lambda 3 n(\varepsilon_u)}$$

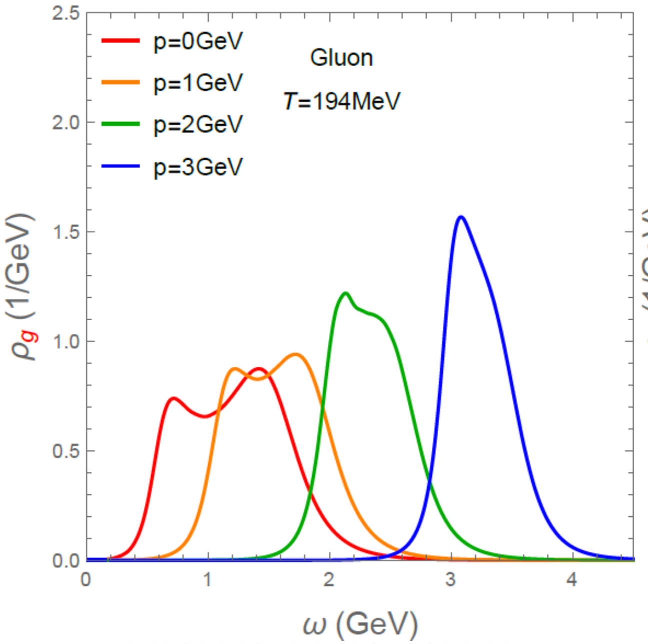
$$\mathcal{T}^{\mu\nu} = \tilde{\Delta}^{\langle\mu}_{\lambda} \tilde{\Delta}^{\nu\rangle}_{\gamma} \beta n_0 \left[\kappa_0^u u^\lambda u^\gamma + \kappa_1^u u^\lambda u^\gamma + \boxed{\alpha_{\rm sh}\sigma^{\lambda\gamma}} + \cdots\right]$$

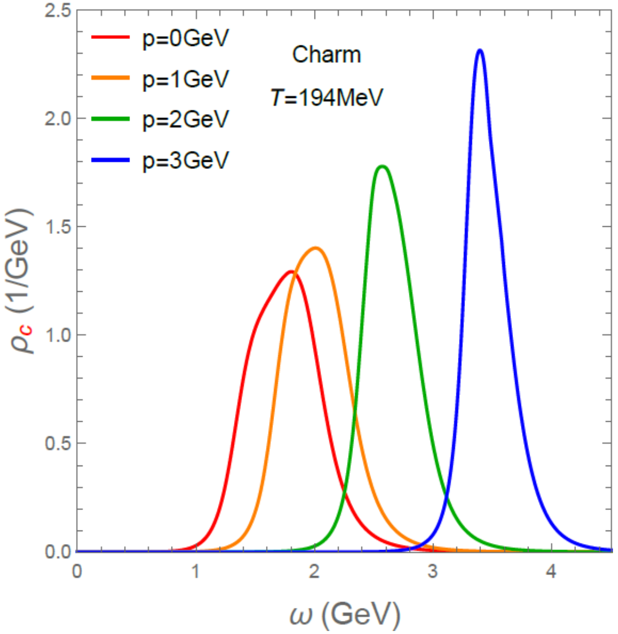
- Coefficients related to spectral properties of the vector meson

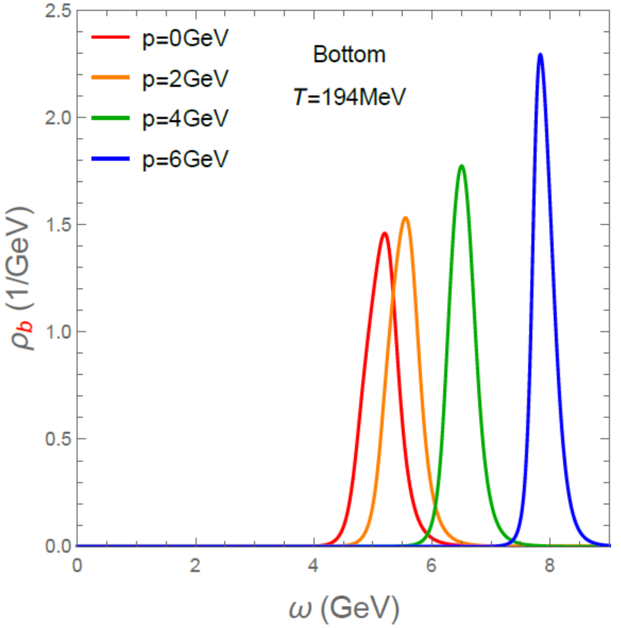
$$\alpha_{\rm sh} = \frac{4\varepsilon_{\boldsymbol{p}}\pi}{\beta n(\varepsilon_{\boldsymbol{p}})} \int_0^\infty d\omega \frac{\partial n(\omega)}{\partial \omega} (\omega^2 - \varepsilon_{\boldsymbol{p}}^2) A^2(\omega, \boldsymbol{p}) \approx -\frac{2\Delta\varepsilon_{\boldsymbol{p}}}{\Gamma_{\boldsymbol{p}}} + 2\frac{\Delta\varepsilon_{\boldsymbol{p}}}{\Gamma_{\boldsymbol{p}}}\frac{\Delta\varepsilon_{\boldsymbol{p}}}{T} + \frac{\Gamma_{\boldsymbol{p}}}{2T},$$

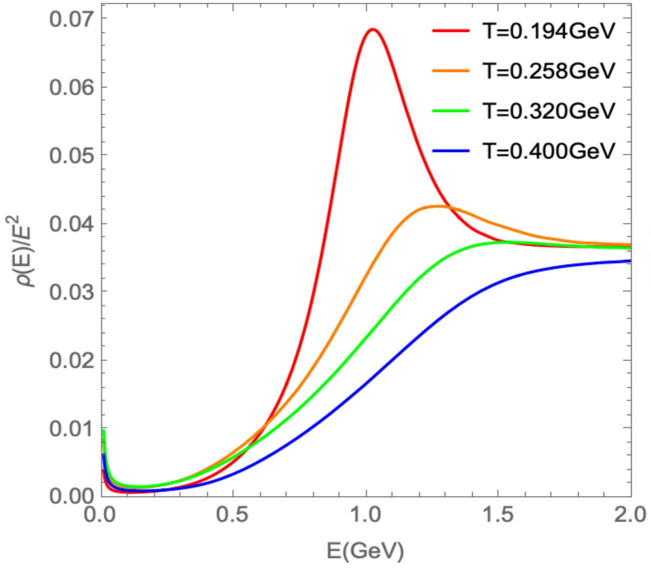
Reminder of Nontrivial In-medium Spectral Function

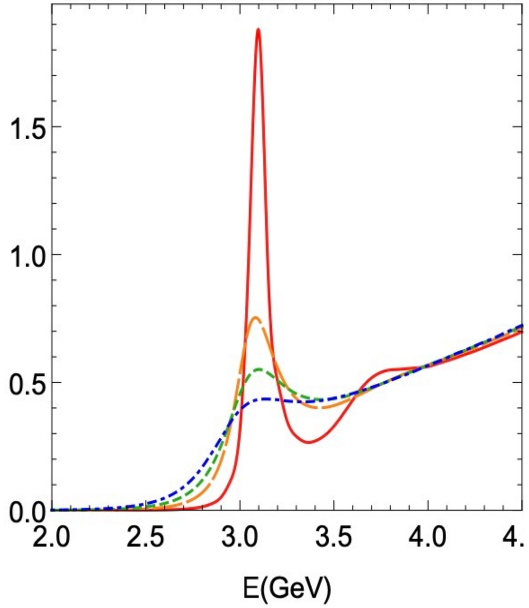


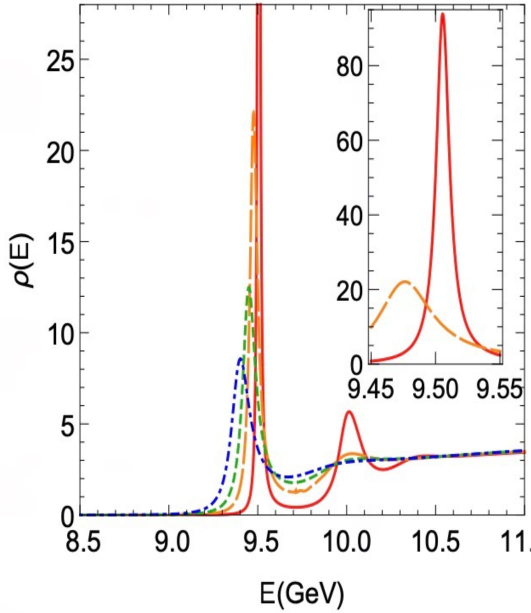










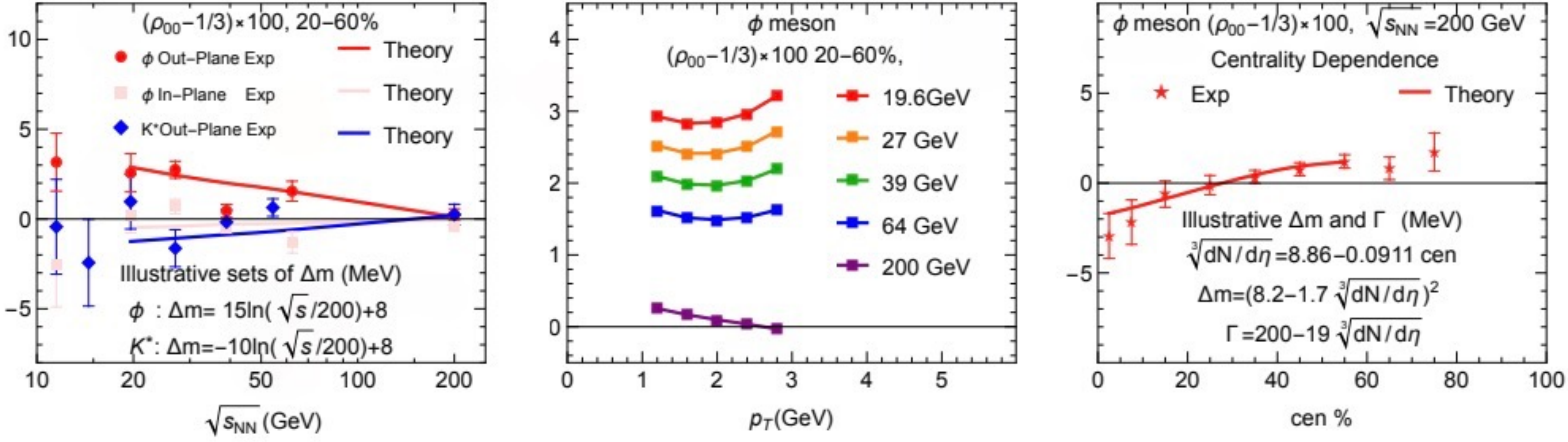


- Width can be large
- Nontrivial dependence on momentum
- Mass shift can be positive or negative

A Tune to Reproduce the Data

❖Tune the coefficients $\alpha_{sh}(\Gamma, \Delta m)$ with mass shift and width

❖Reproduce the beam energy (ϕ, K^*), p_T, centrality dependence

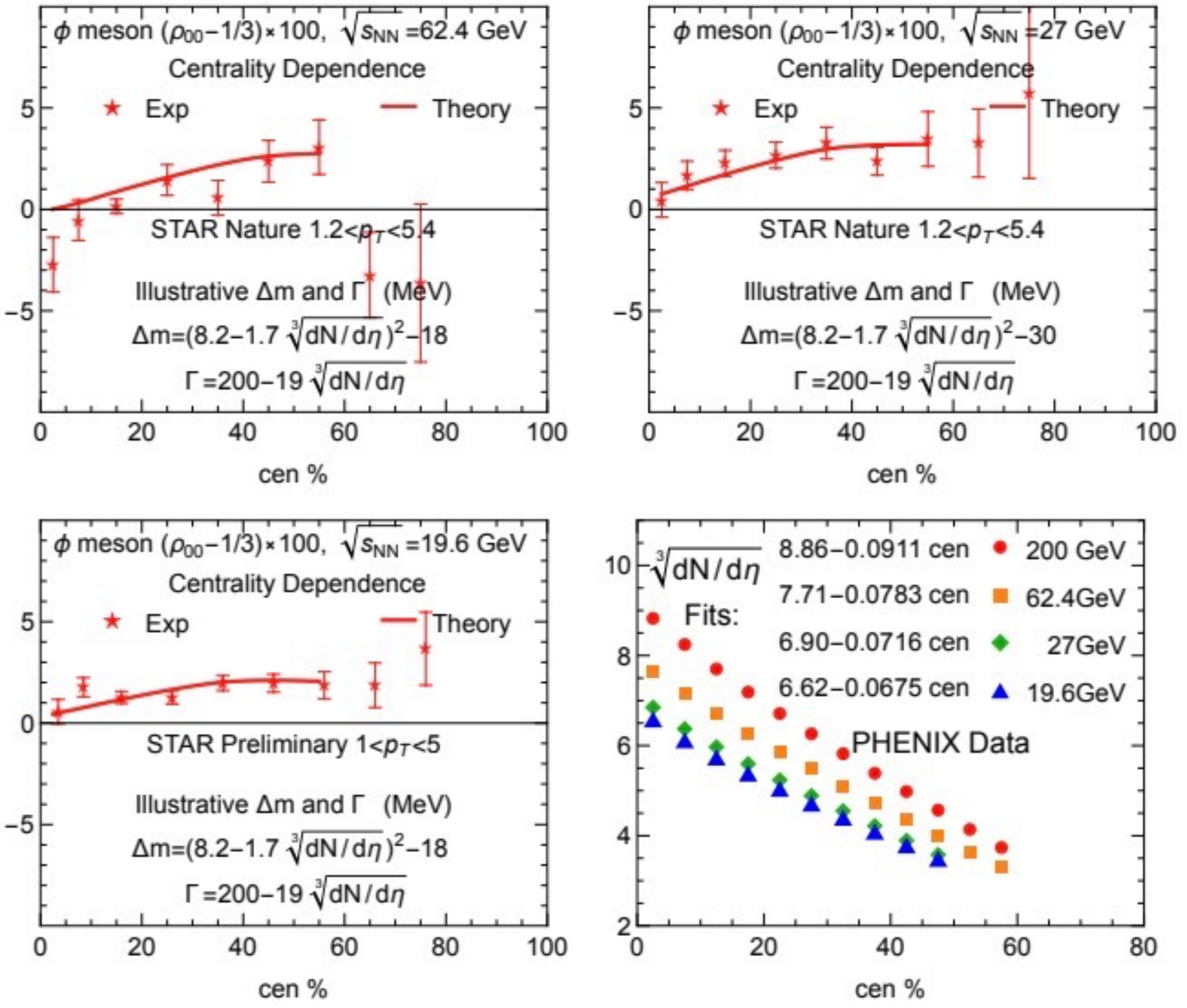


Li and Liu, arXiv:2206.11890

More for Centrality Dependence

Centrality Freezeout Correlation

- More central, longer hadronic phase
- Longer hadronic phase, ϕ will re-equilibrate in hadronic phase and get freezeout there
- Short hadronic phase, ϕ do not have time to re-equilibrate so it have memory of spin alignment in QGP or mix phase



Li and Liu, arXiv:2206.11890

Summary

- T-matrix approach: a versatile approach for a wide range of physics
  - Lattice data: EoS, HQ free energy, correlator ratios, various susceptibilities, …
  - Transport phenomena: heavy quark transport, viscosity, radiative energy loss, …
  - Spectral properties: light parton's spectral functions, heavy flavor spectral functions, hadron's spectral functions, …

- Motivated by those spectral properties, we find a tune that can reproduce the puzzling spin alignment data (beam energy, p_T, particle species, centrality dependence)

Backup, rapidity dependence