Disentangling Sub-Eikonal Broadening Effects

JAQ 2026
Jo Bahder
CCNU
25th Jan., 2026

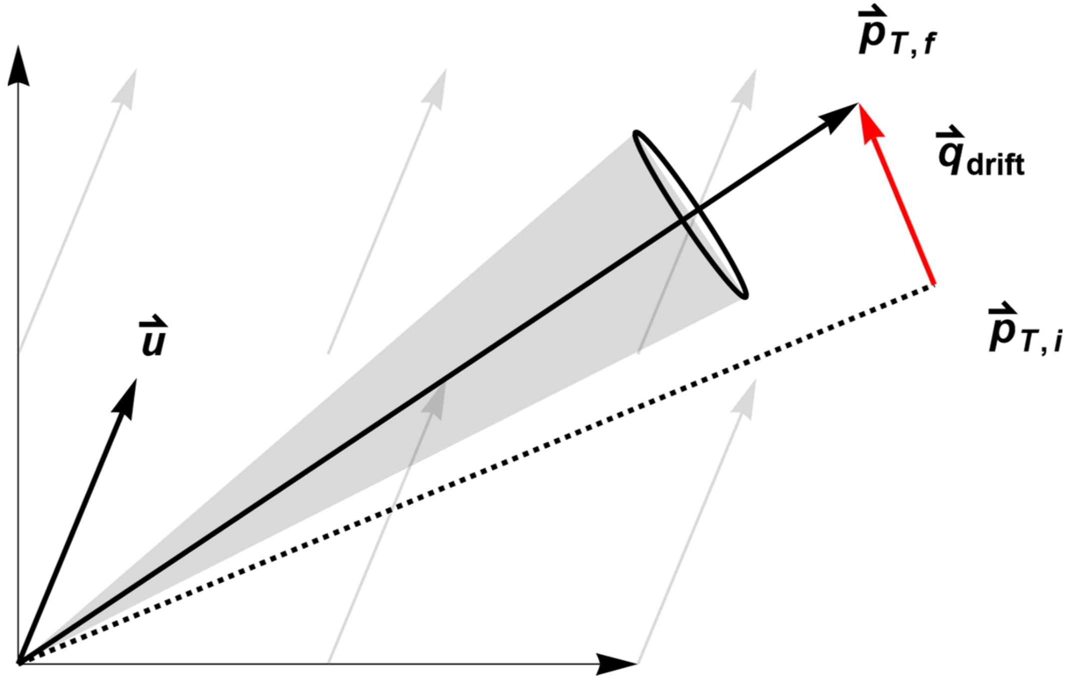
Outline

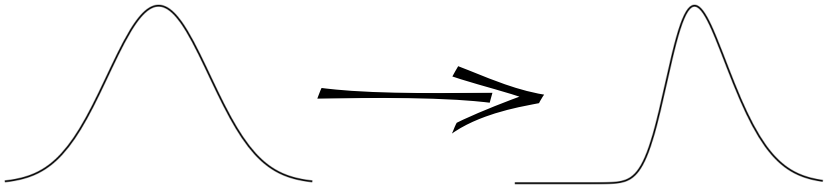
I. Intro to Flow Effects

II. Theoretical Ingredients – Boost vs Potential

III. Gradient Expansion & Implementation

Flow-Induced Broadening ("Drift")



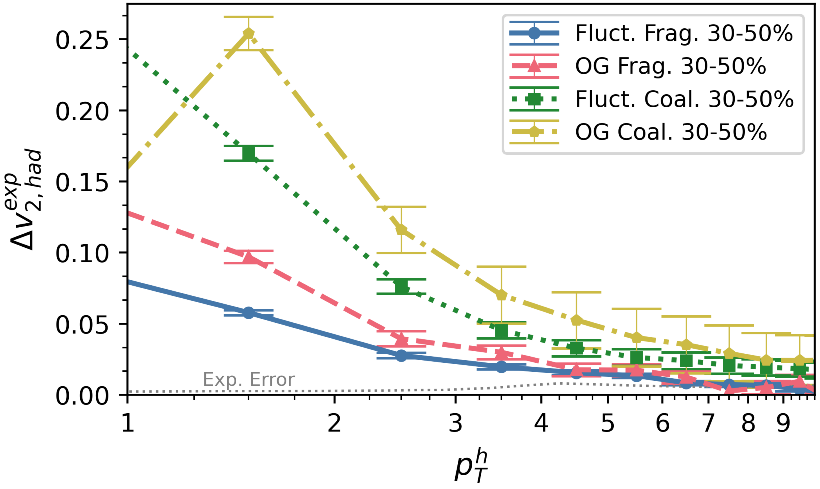


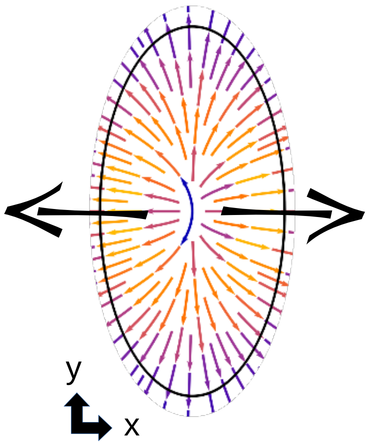
- Moving scattering centers skew broadening distribution
- On average, jets deflect in direction of medium flow
- Traditionally accounted for with global boosts
- Recent advancement: velocity of scattering centers enters at the level of the potential

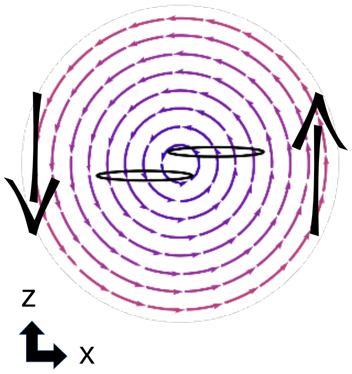
Importance for Phenomenology

- Drift couples to preferred directions in the medium
  - Event plane (azimuthal)
  - Vorticity (rapidity)
- Modulates distribution of hard particles, mainly at low & intermediate p_T
  - e.g. v_2 enhancement

See Tan Luo's Talk!







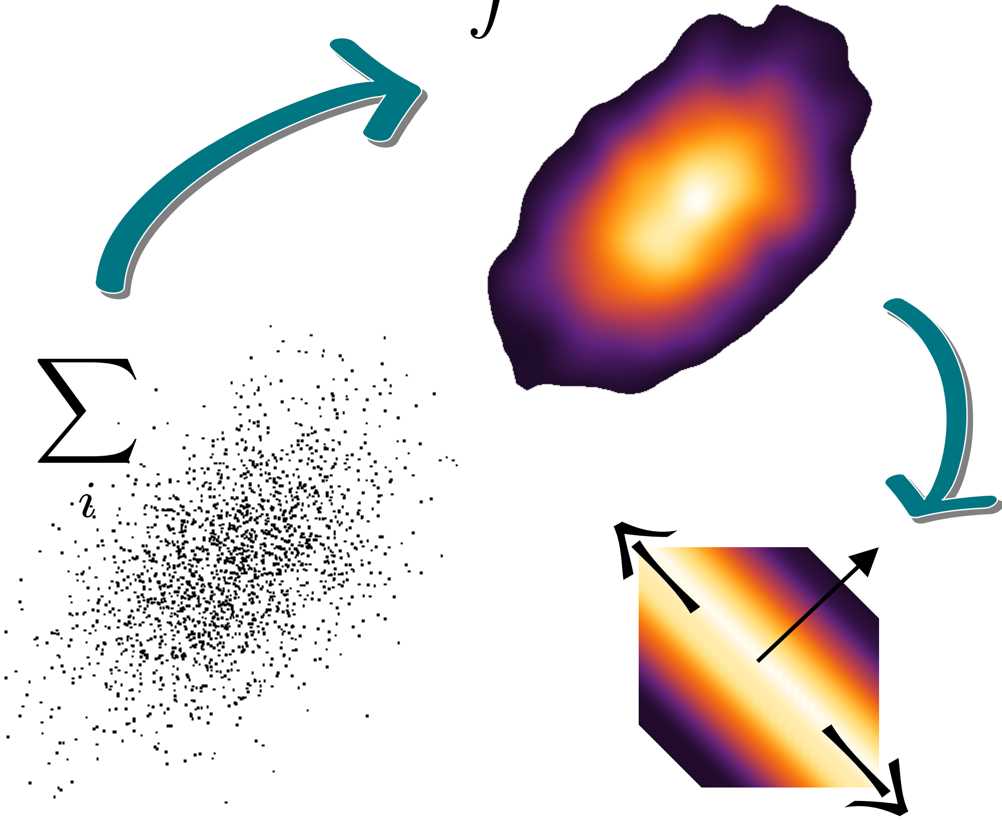
Initial Drift Pheno.
L. Antiporda, J. Bahder, H. Rahman, & M. D. Sievert
Phys.Rev.D 105 (2022)
(arXiv:2110.03590)

APE & Drift Pheno.
J. Bahder, H. Rahman, M. D. Sievert, & I. Vitev
Physical Review Research (2026)
(arXiv:2412.05474)

Treatment of Scattering Centers in Opacity Expansion

$$\int d^3x \, \rho(x)$$

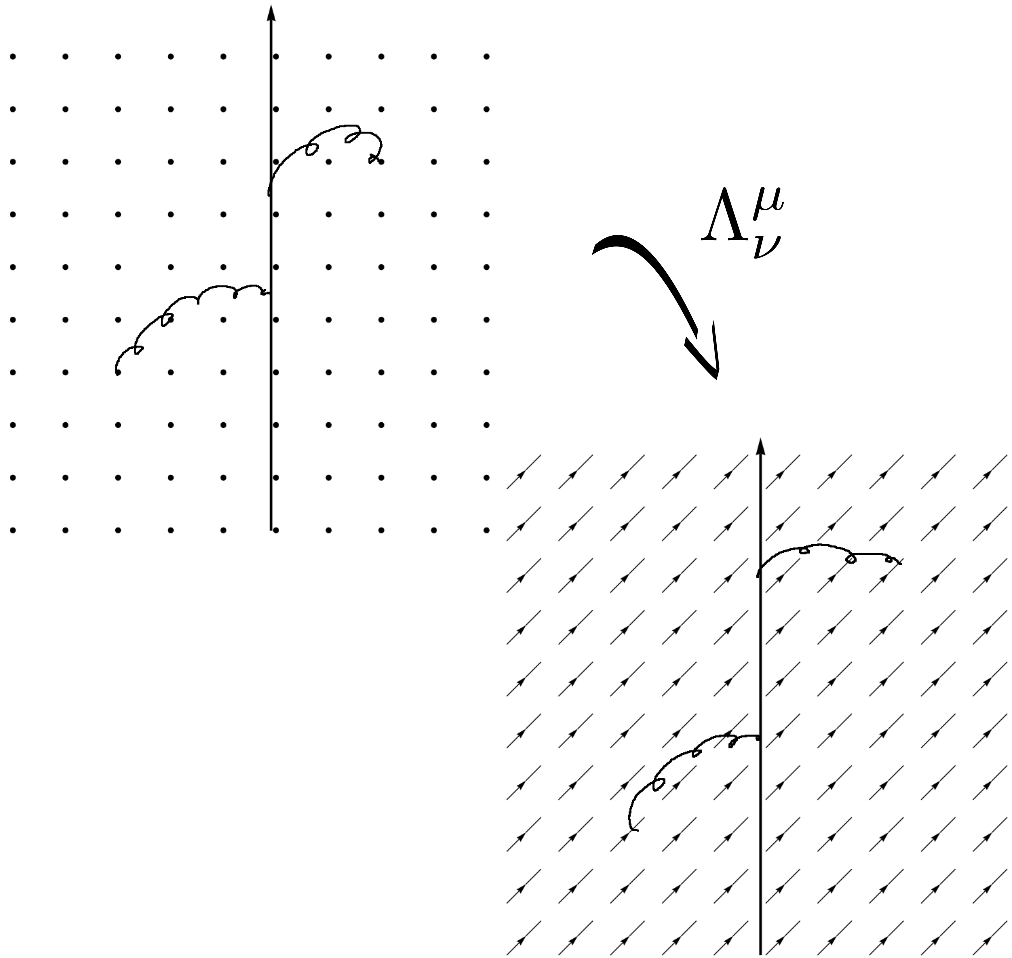
- Summation over scattering centers to spatial integral over density
- ***All*** scattering centers contribute to amplitude!
- Traditionally, transverse variation is neglected to simplify integration
- Longitudinal integration alone is analytically (simple models) or computationally (e.g. hydro) tractable
- **Imagine we perform the spatial integral exactly…**



$$\sum_i$$

$$e^{-i\alpha \vec{x}_\perp - i\beta\tau}$$

Boost Treatment (Most MCs)

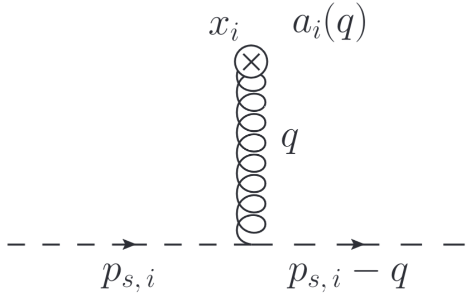


- Standard result neglects transverse variation of medium
- Calculate broadening in fluid rest frame, boost to moving frame
- Spatially *constant* velocity field

Drift Treatment

$$v_i(q) \equiv v_i\left(\boldsymbol{q}^2 - (\boldsymbol{u}_i \cdot \boldsymbol{q})^2\right) \equiv \frac{-g^2}{\boldsymbol{q}^2 + \mu_i^2 - (\boldsymbol{u}_i \cdot \boldsymbol{q})^2 - i\epsilon}$$

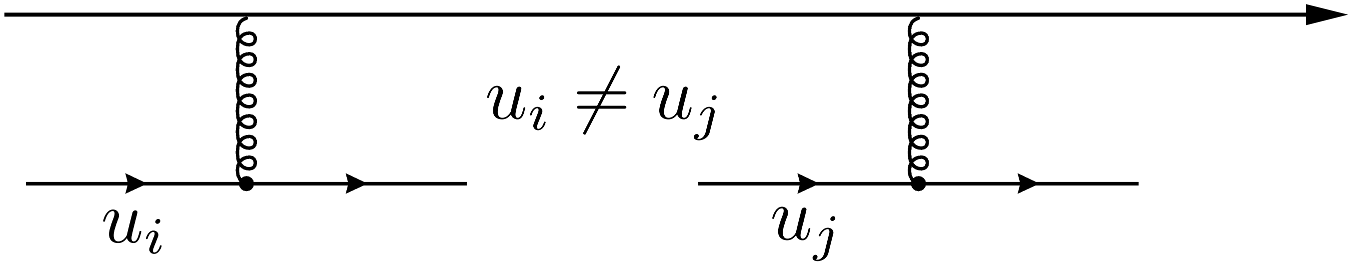
Directional scattering centers



$$g\, a_i^{\mu a}(q) = t_i^a\, u_i^\mu\, v_i(q)\, (2\pi)\, \delta\left(q^0 - \boldsymbol{u}_i \cdot \boldsymbol{q}\right)$$

A. V. Sadofyev,
I. Vitev, & M. D. Sievert
Phys.Rev.D 104 (2021)
(arXiv:2104.09513)

Individual scattering centers have spatially varying velocity profile!

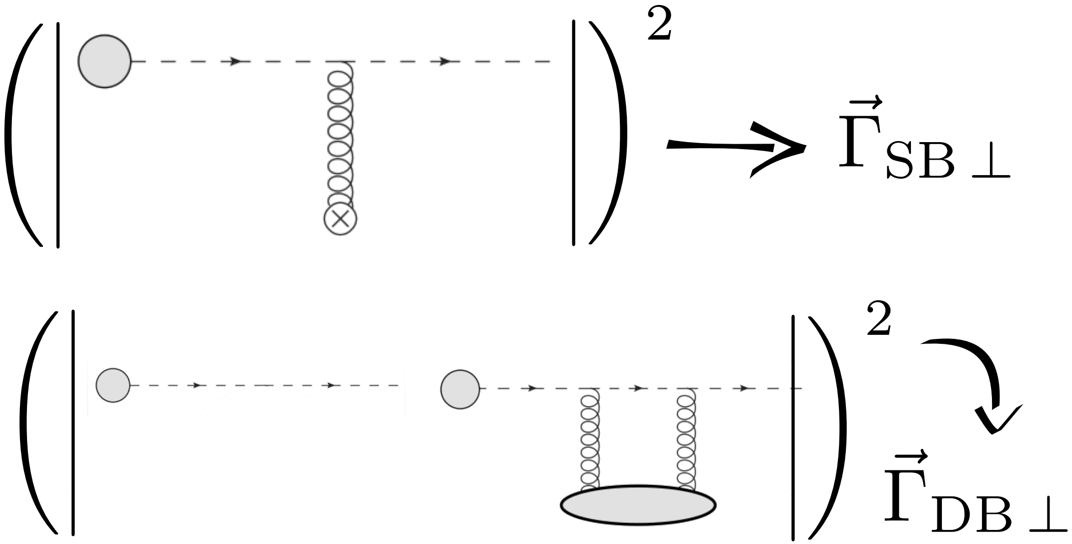


Interference of Cut Diagrams

Different effect on SB & DB diagram

- Flow effect cannot be reduced to a single boost!
- DB comes w/ $\delta^2(p_\perp)$: explicit unitarity

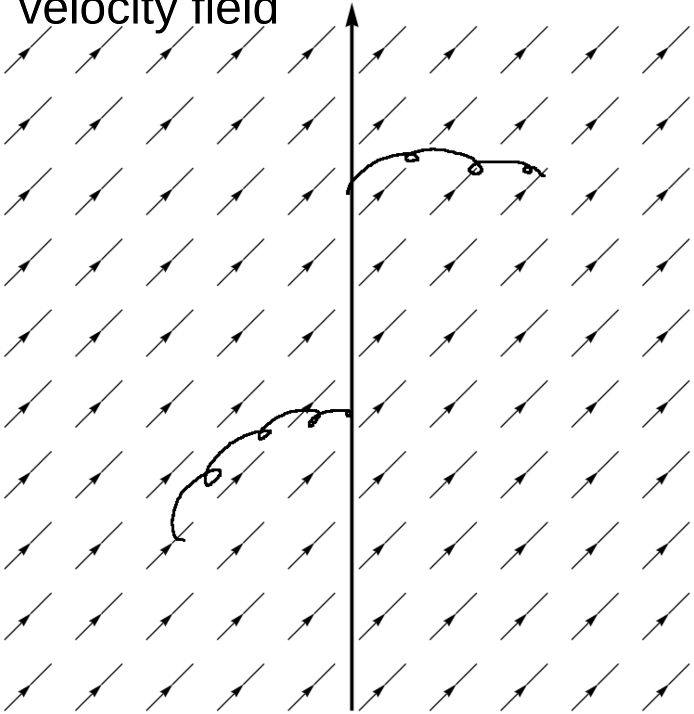
- **SB / DB effects not captured with boost – different final state distribution**

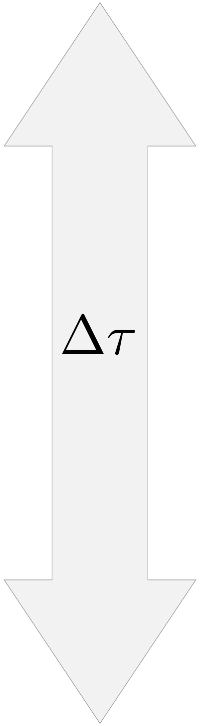


$$E\frac{dN^{(1)}}{d^3p} = \int d\tau\, d^2Q_\perp\, \rho_0\, \bar{\sigma}(Q) \times \left[E\frac{dN^{(0)}}{dE\, d^2(\vec{p}_\perp - Q_\perp)} \left(1 + \vec{u}_\perp \cdot \vec{\Gamma}_{\mathrm{SB}\perp}(Q_\perp)\right) - E\frac{dN^{(0)}}{dE\, d^2(\vec{p}_\perp)} \left(1 + \vec{u}_\perp \cdot \vec{\Gamma}_{\mathrm{DB}\perp}(Q_\perp)\right) \right]$$

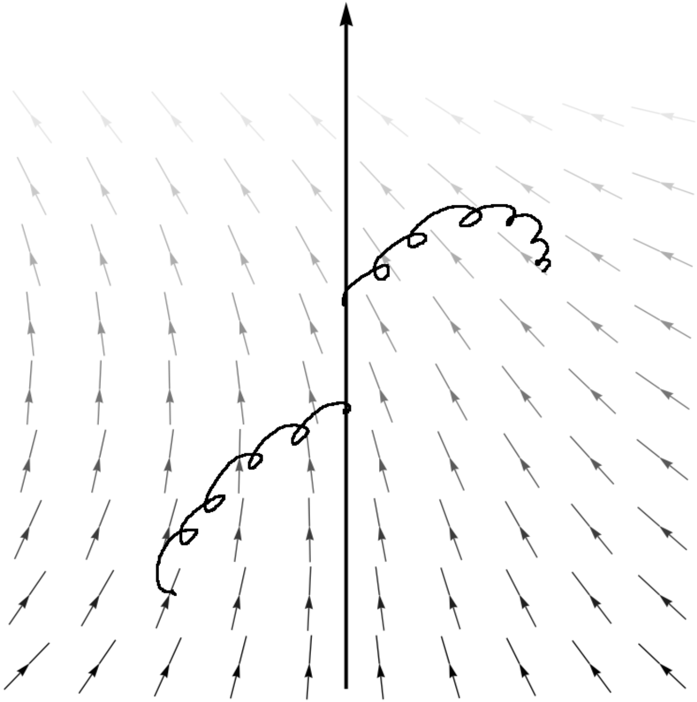
Velocity Fields $\Delta\tau$

Boost:
Longitudinally uniform, transversely uniform velocity field





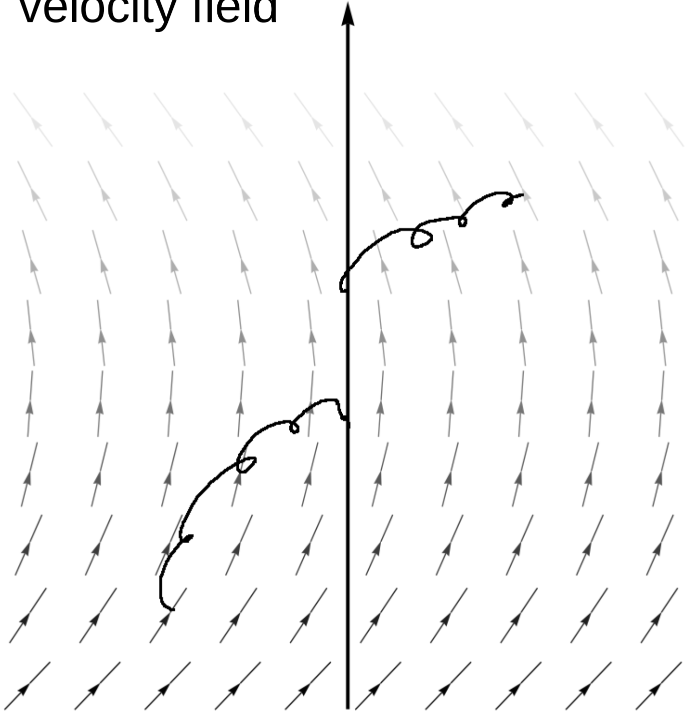
Drift:
Dynamic velocity field

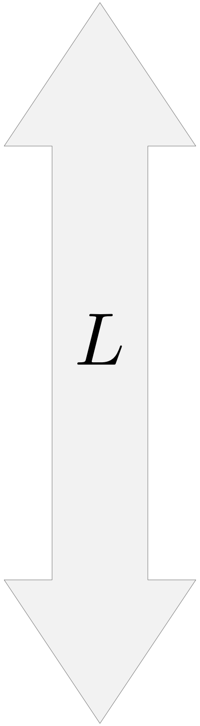


Velocity Fields

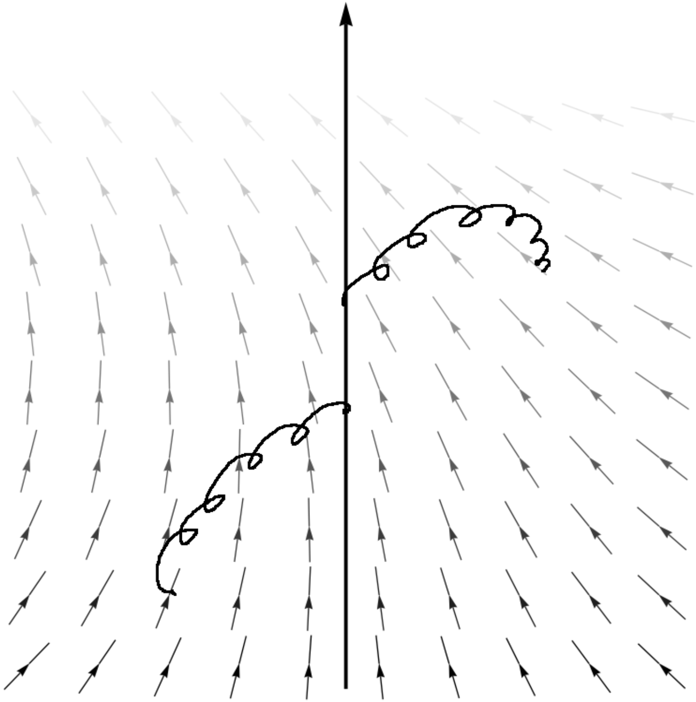
$$L = \int d\tau$$

<u>Boost:</u>
Longitudinally dynamic,
transversely uniform
velocity field





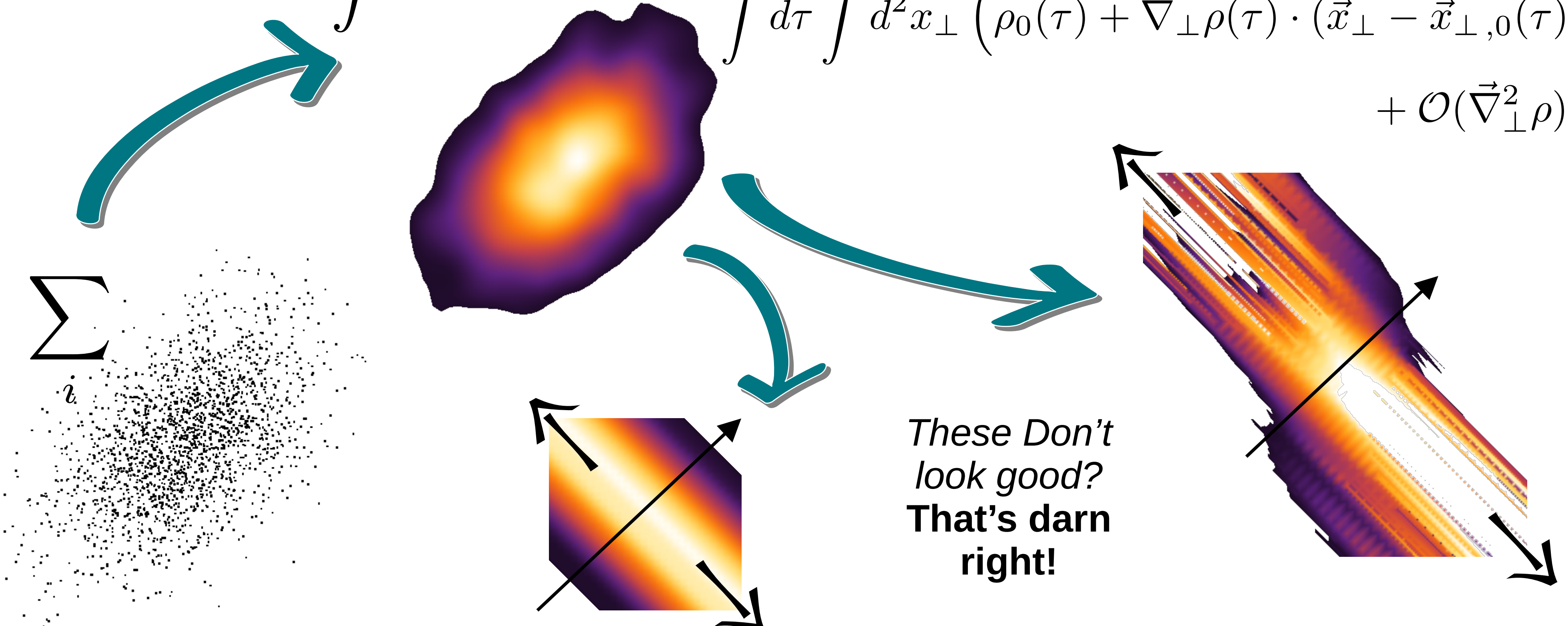
<u>Drift:</u>
Dynamic velocity field



Gradient Expansion to Implement

$$\int d^3x\,\rho(x)$$

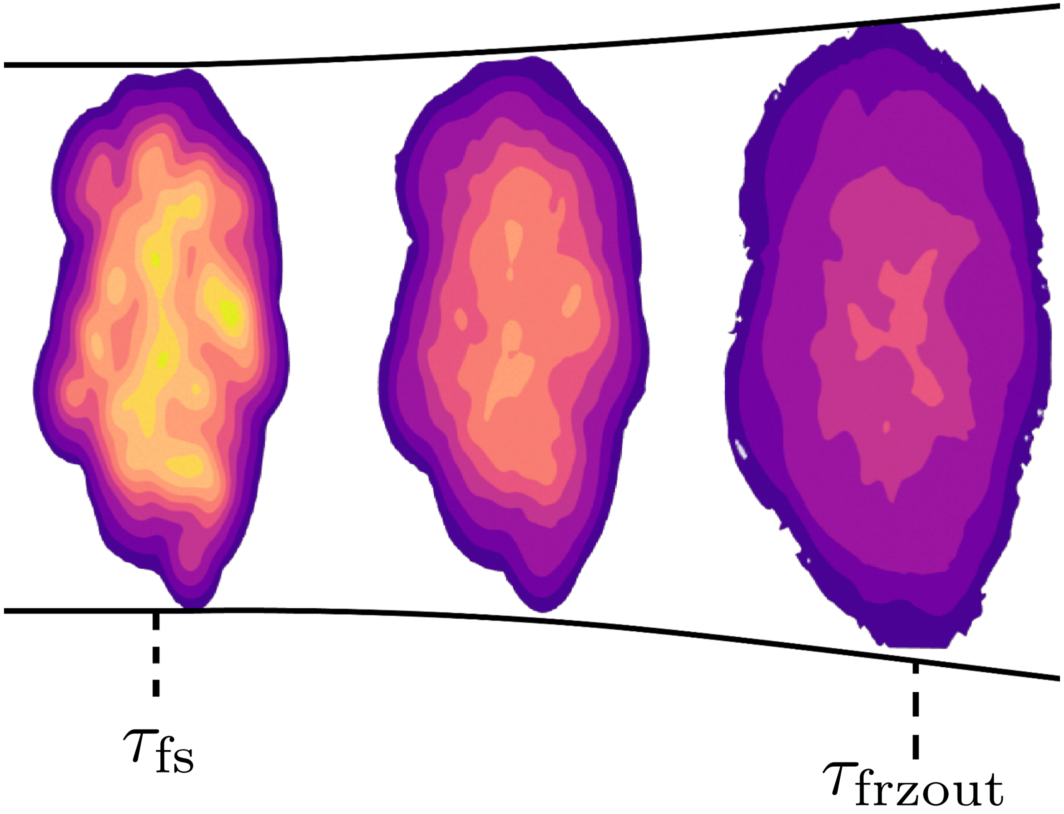
$$\int d\tau \int d^2x_\perp \left(\rho_0(\tau) + \vec{\nabla}_\perp \rho(\tau)\cdot(\vec{x}_\perp - \vec{x}_{\perp,0}(\tau))\right) + \mathcal{O}(\vec{\nabla}^2_\perp \rho)$$



These Don't look good?

That's darn right!

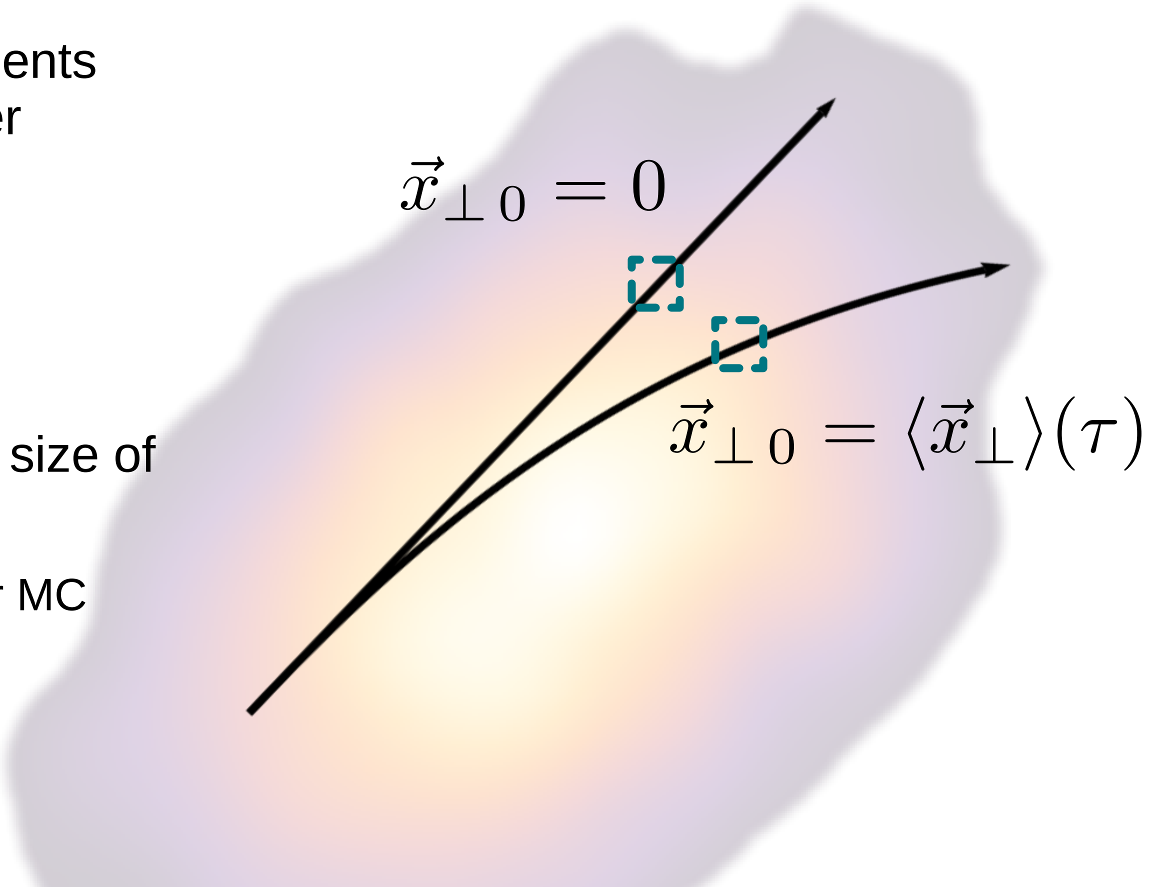
Are Gradients Small?



- Short answer:
  Not always
- Gradients are small…
  - at late times in hydro evolution
  - away from hot spots & edges
- Implementation in MC needs either:
  - careful event selection
  - higher order gradients
    (quickly becomes algebraically miserable)
- For the moment, let's put a pin in this problem...

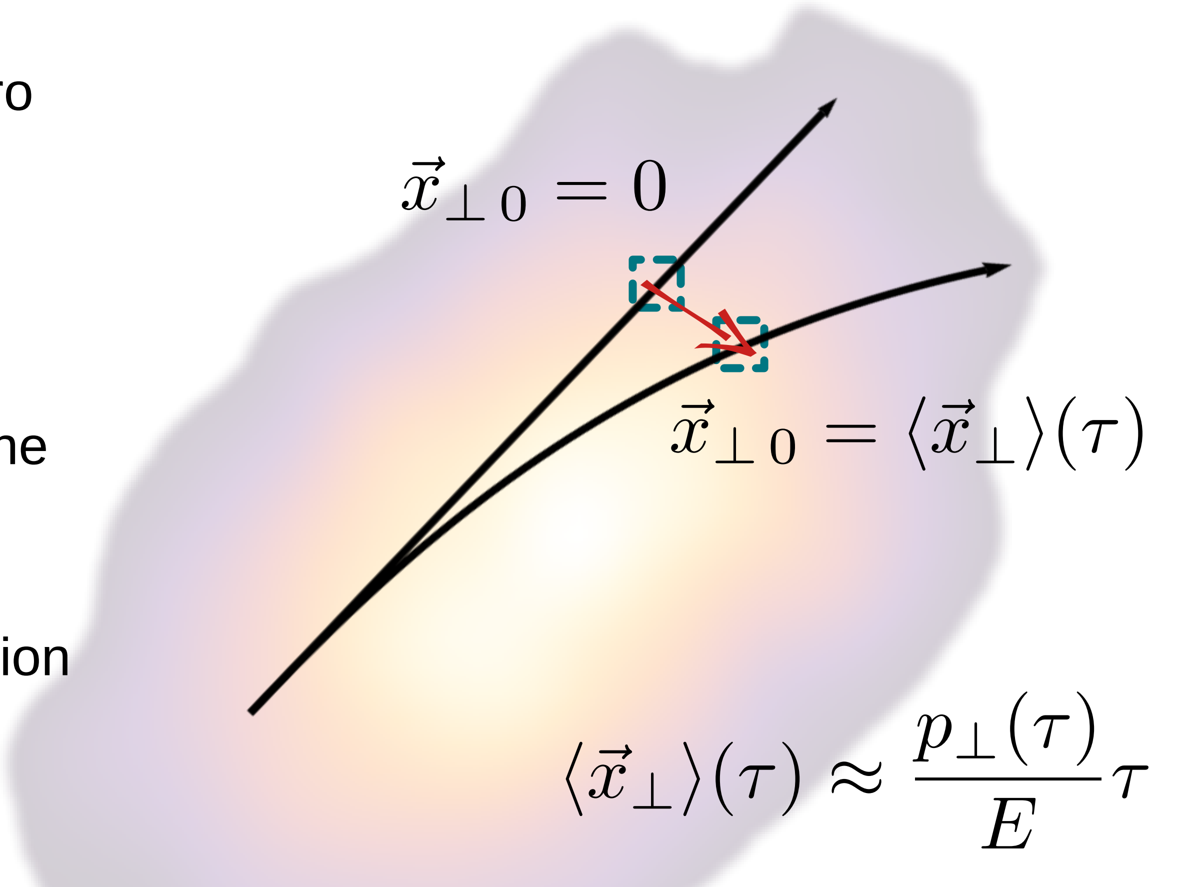
Gradients of… What and Where?

- Independent terms enter for gradients of… *separate terms* at linear order
  - Density
  - Debye Mass
  - Flow
- Choice of expansion point affects size of effects
  - Propose new expansion point for MC implementation:
    - Old – zero, straight traj.
    - New – Avg. transverse pos.



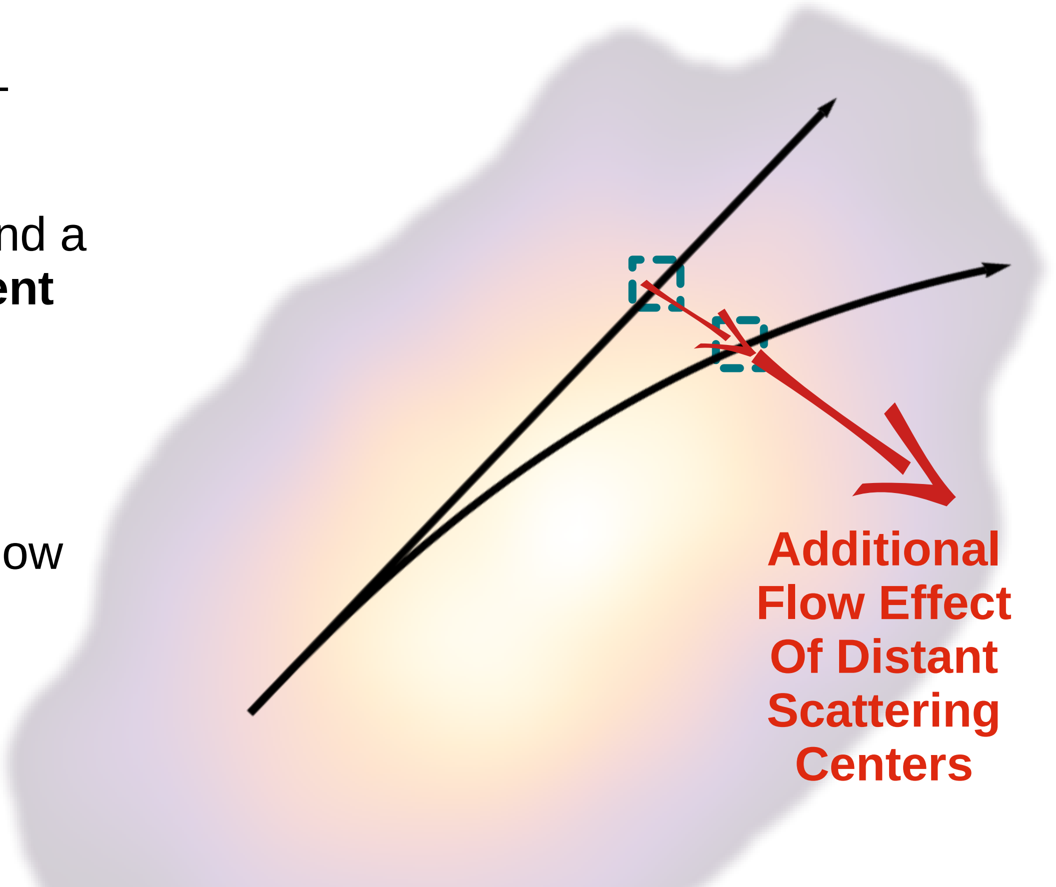
"Pure" Gradients: Pure Coordinate Transformation

- Linear order pure gradients (zero flow) are a coordinate transformation
  - Exact cancellation at first sub-eikonal order
- Centers the "waveform" about the mean $x_\perp$ position
  - In MCs, this is the dynamic updating of transverse position between $\Delta\tau$ steps



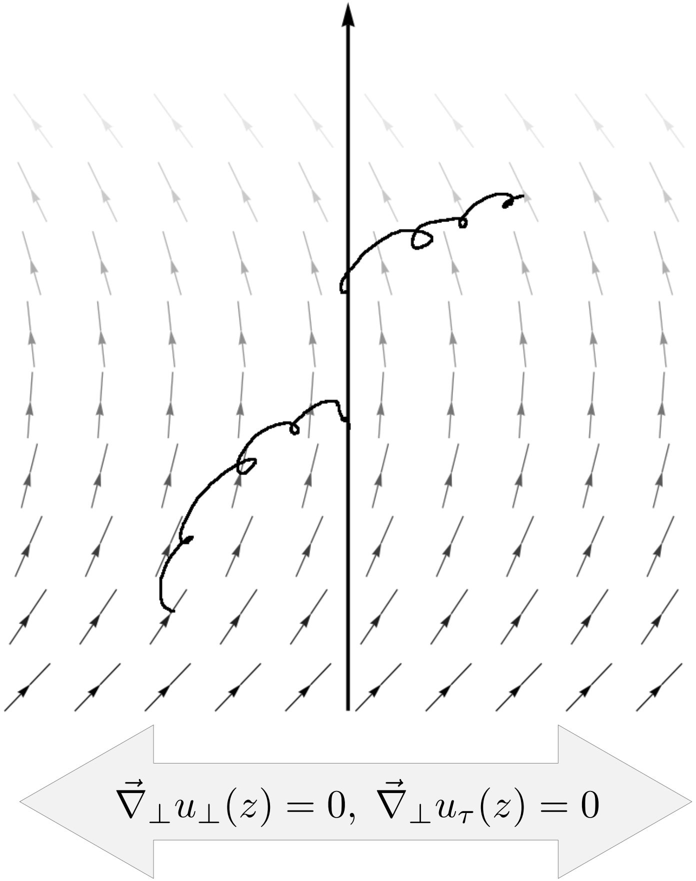
Flow x Gradients – Partial Coordinates, Partial Intrinsic

- Contains coordinate transformation + intrinsic correction
- Each can be zeroed expanding around a particular **energy and flow dependent** point
  - Not a necessary feature: special to gradient effects
- **Cannot** zero (flow x grad) + (flow x flow grad)
  - Unique interaction with flow of distant scattering centers



Punchlines

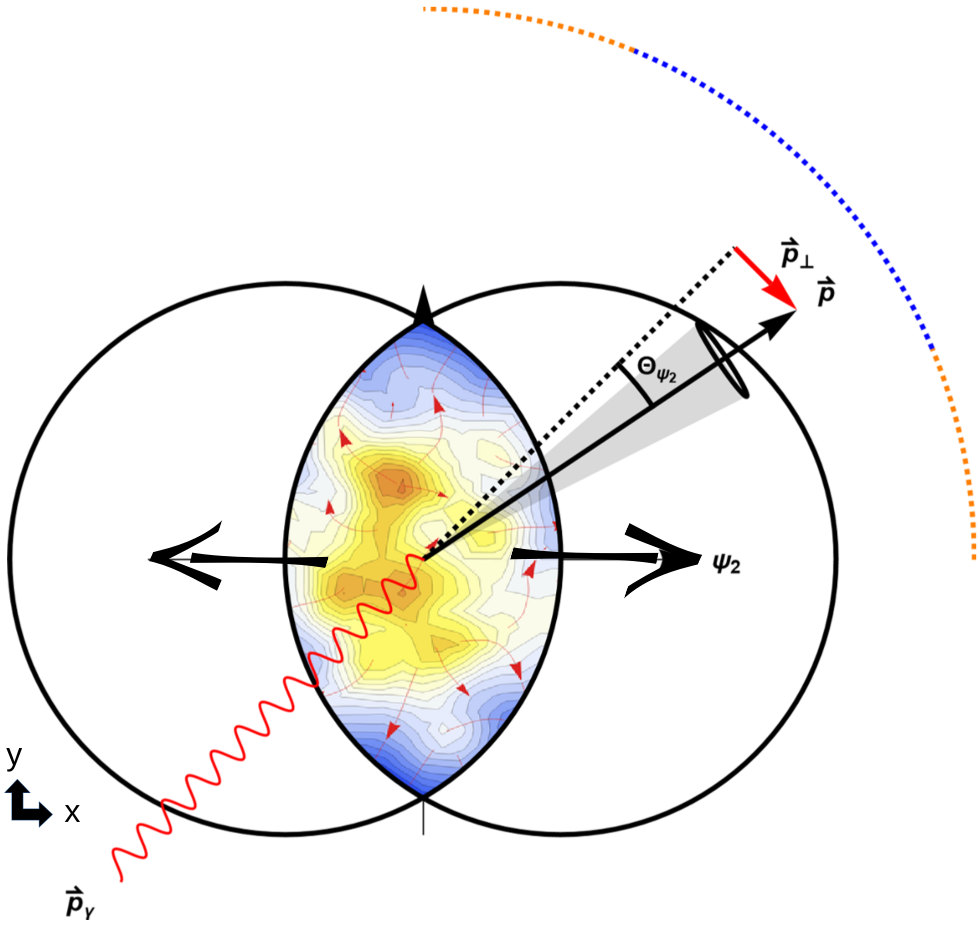
- Boost driven MCs:
  - Pure gradient effects are generally included to linear order via dynamic transverse position
- Jet Drift:
  - Different contributions from SB & DB
  - Locally-varying flow of scattering centers
- **Boost includes no analogue of “flow x flow gradient” or SB / DB interference effects**

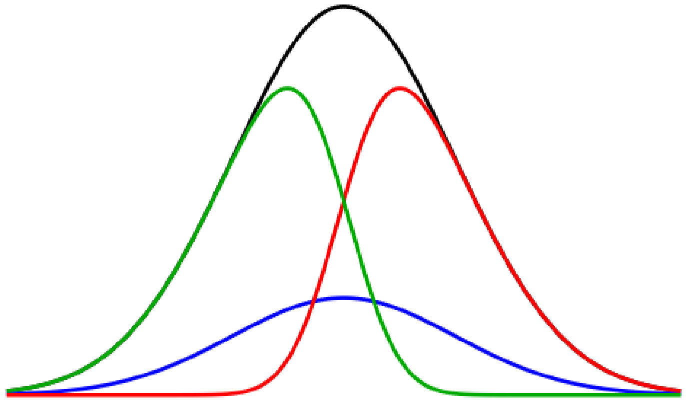


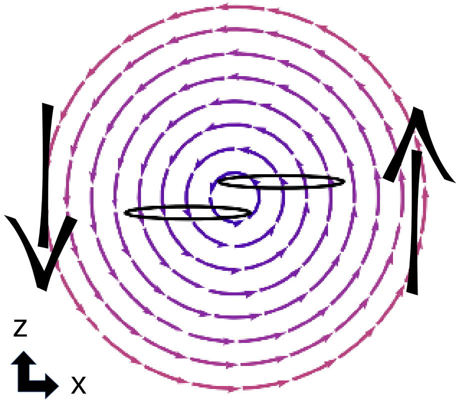
Thanks a bunch!

See APE Phenomenology:
(arXiv:2412.05474)

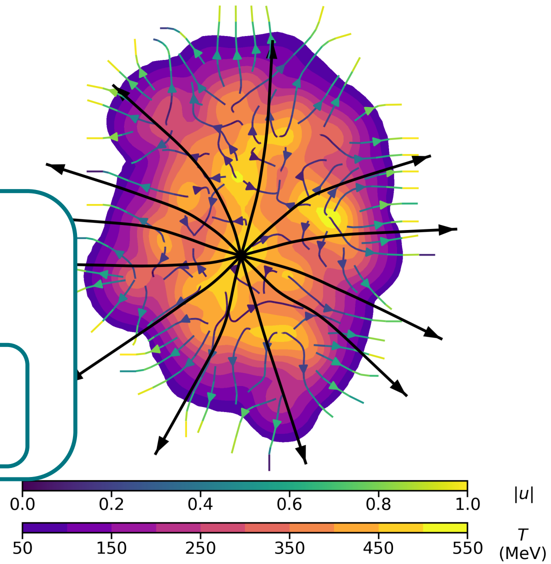
On Observables...







Drift in APE



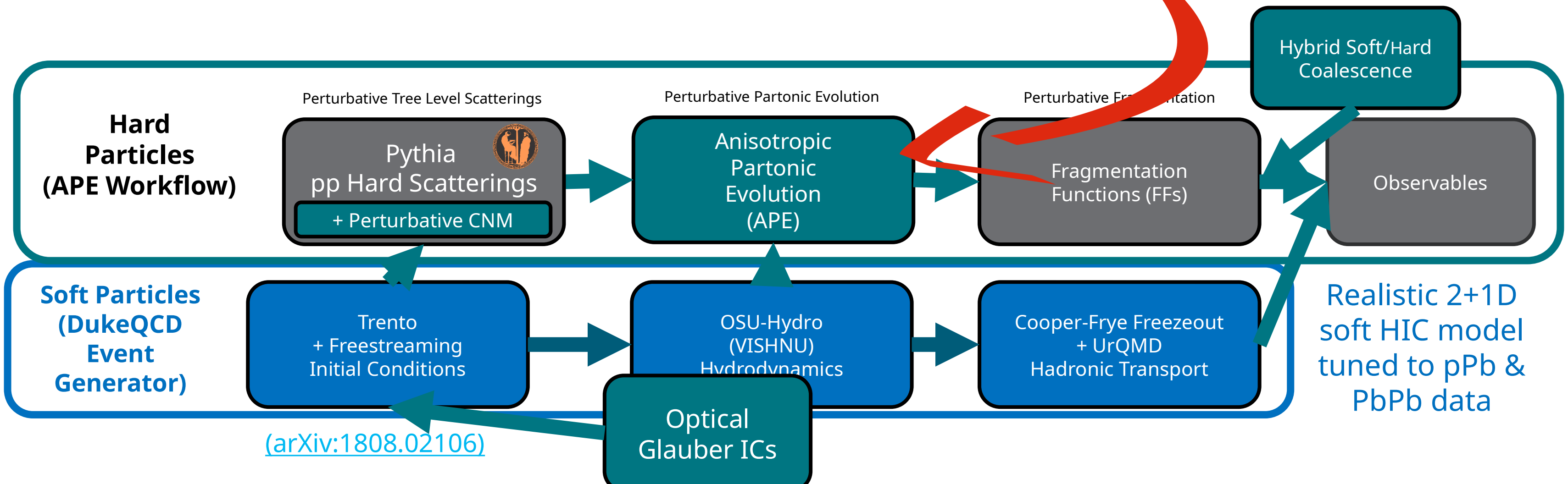
Average effect as transport equation

Neglects longitudinal variation over $\Delta\tau$

Neglects transverse variation

APE & Drift Pheno.
J. Bahder, H. Rahman, M. D. Sievert, & I. Vitev
Physical Review Research (2025/2026)
(arXiv:2412.05474)

Just a fancy boost?



(arXiv:1808.02106)

Realistic 2+1D soft HIC model tuned to pPb & PbPb data

Not exactly…
Sensitive to different features of flow!

Different phenomenology!