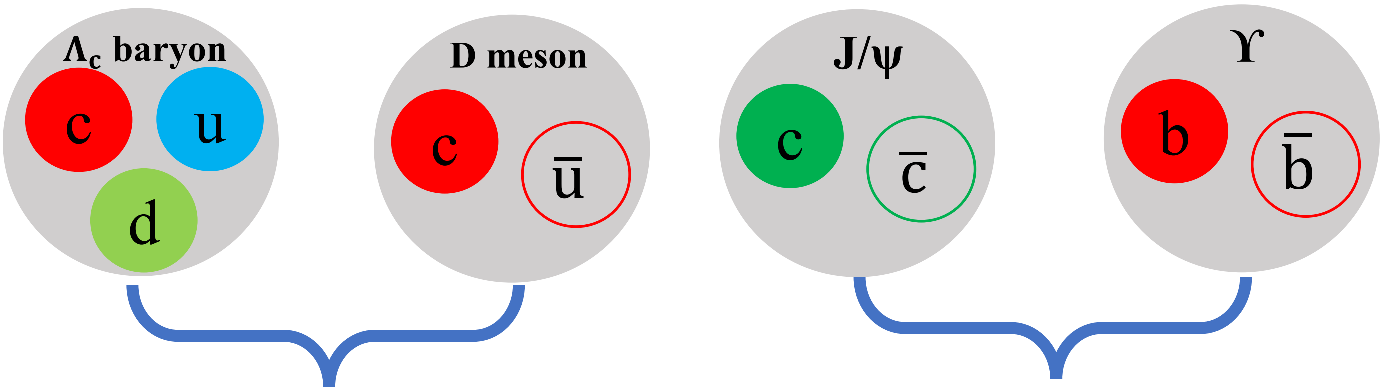
The 1st workshop on Jet and Heavy Quark Physics

# Heavy quarkonium dissociation, regeneration & equilibration in the Quark-Gluon Plasma

Based on S. Zhao & M. He, PRD 110, 074040 (2024)
S. Zhao & M. He, PRD 112, 094018 (2025)

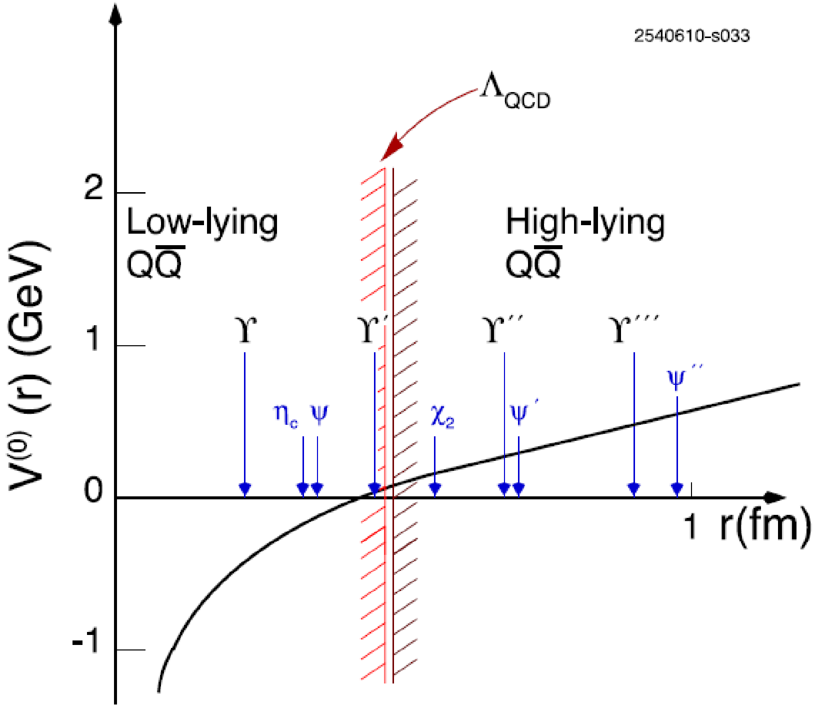
Shouxing Zhao (赵守杏) NJUST, Jan. 25, Wuhan

# Heavy quarkonia in vacuum




Open heavy flavors

- Hidden heavy flavors → quarkonia
  Non-relativistic bound states of $Q$-$\bar{Q}$




- Schrödinger eq. $$\left[2m_Q - \frac{\nabla^2}{m_Q} + V(r)\right]\psi_i(r) = M_i\psi_i(r)$$

→ ψ below $D\bar{D}$ threshold and Υ below $B\bar{B}$ threshold

| state | $J/\psi$ | $\chi_c$ | $\psi'$ | $\Upsilon$ | $\chi_b$ | $\Upsilon'$ | $\chi_b'$ | $\Upsilon''$ |
|---|---|---|---|---|---|---|---|---|
| mass [GeV] | 3.10 | 3.53 | 3.68 | 9.46 | 9.99 | 10.02 | 10.26 | 10.36 |
| $\Delta E$ [GeV] | 0.64 | 0.20 | 0.05 | 1.10 | 0.67 | 0.54 | 0.31 | 0.20 |

Kluberg & Satz, 0901.3831

- Cornell potential =
  short distance→ Coulomb part
  long distance → confining part

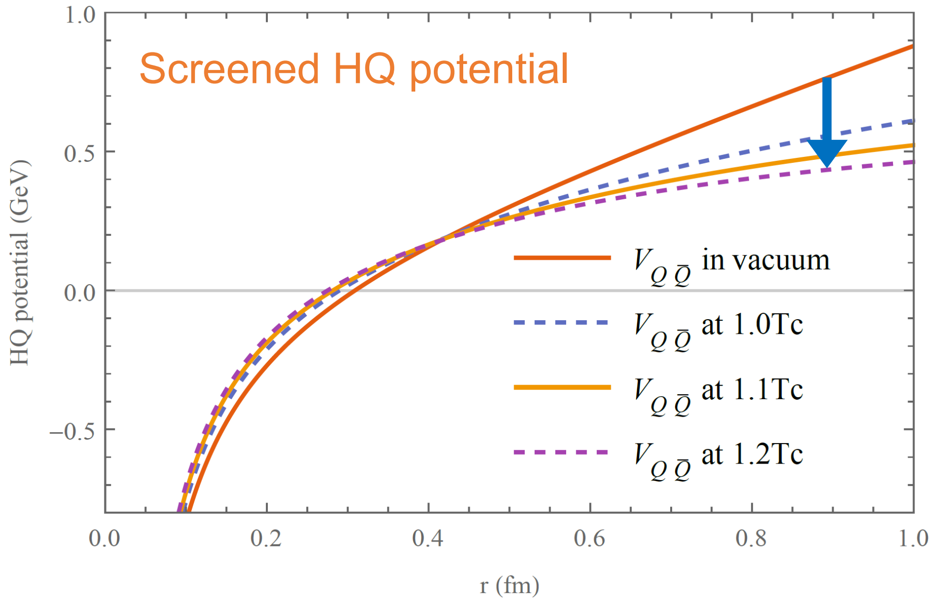
$$V_{Q\bar{Q}}(r) = -\frac{4}{3}\frac{\alpha_s}{r} + \sigma r$$

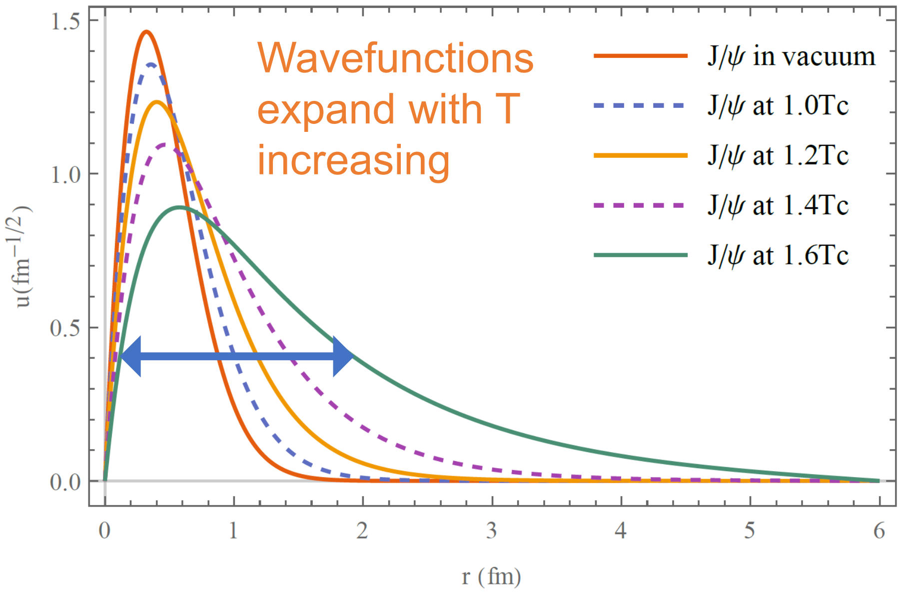
# Quarkonium static deconfinement in QGP

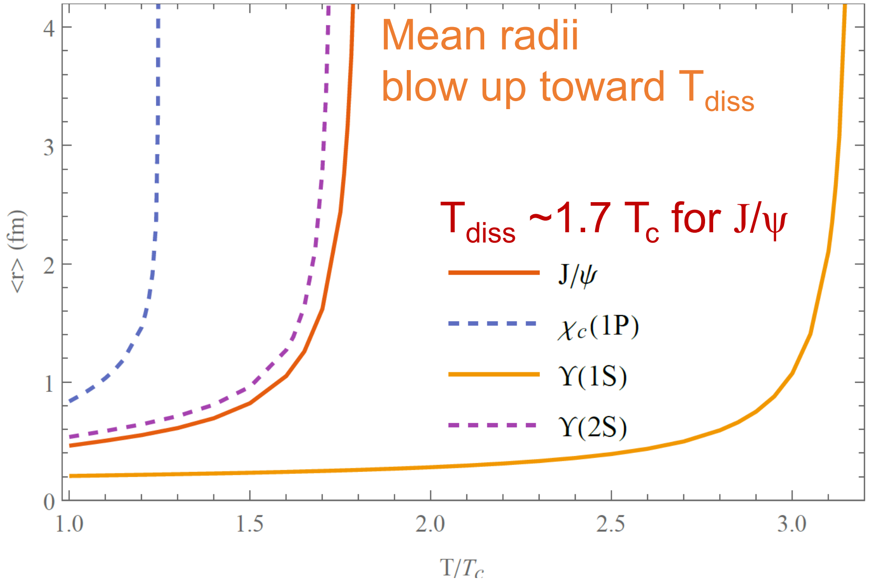
- Color screened Cornell potential

$$V_{Q\bar{Q}}(r,T) = -\frac{4}{3}\frac{\alpha_s}{r}e^{-m_D r} + \frac{\sigma}{m_D}(1 - e^{-m_D r})$$

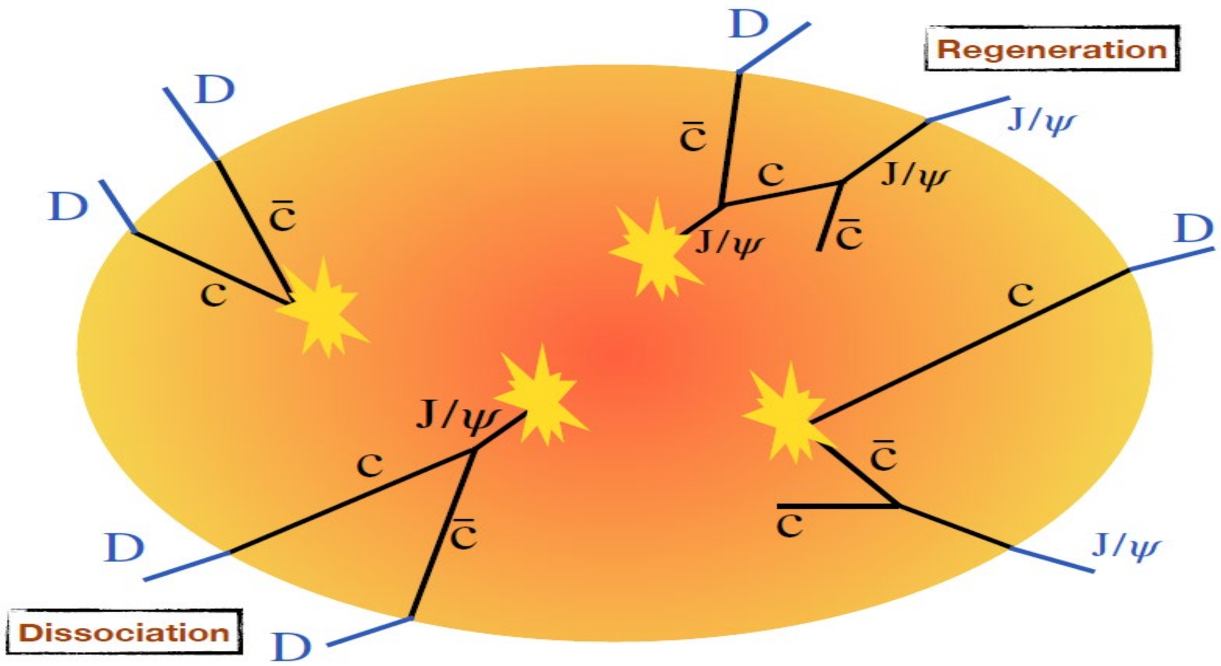
with $m_D(T)$ the Debye mass   Karsch, Mehr & Satz, Z. Phys. C (1988)



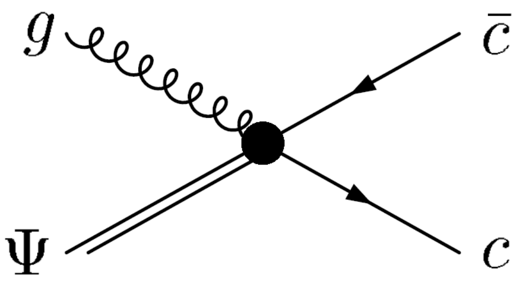



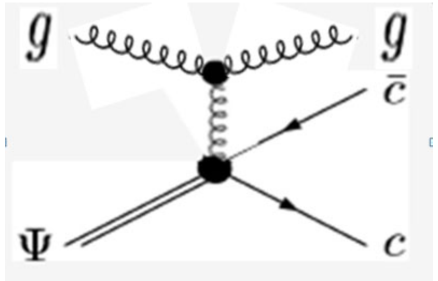
# Part 1: Quarkonium dynamical reactions in QGP




Forward reaction
Dissociation
$\psi \rightarrow c + \bar{c}$

Backward reaction
Regeneration
$c + \bar{c} \rightarrow \psi$




**2→2 LO:** $\Psi + g \leftrightarrow Q + \bar{Q}$

**2→3 NLO:** $\Psi + g \leftrightarrow Q + \bar{Q} + g$

# Interactions of quarkonium in QGP & Feynman rules

- Effective Hamiltonian

$$H_{\rm eff} = H_0 + H_I \quad \begin{cases} H_0 = \dfrac{\vec{p}^2}{m_Q} + V_1(|\vec{r}|) + \sum_a \dfrac{\lambda^a}{2}\dfrac{\bar{\lambda}^a}{2} V_2(|\vec{r}|) \\ H_I = V_{SO} + V_{OO} + V_{3g} + V_{q\bar{q}g} \end{cases}$$

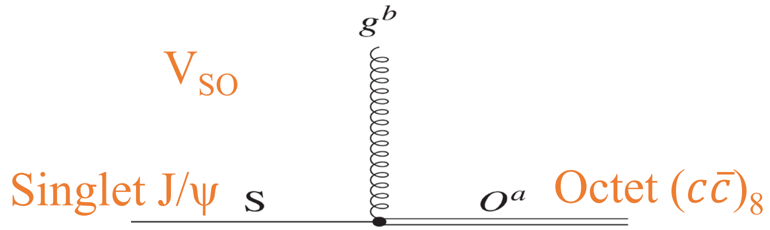
$Q\bar{Q}$ system

$Q\bar{Q}$ coupling to external gluons + light quark/gluon system

- Soft gluon wavelength λ >> bound state size r
→ multipole expansion → pNRQCD color-electric dipole (E1) coupling

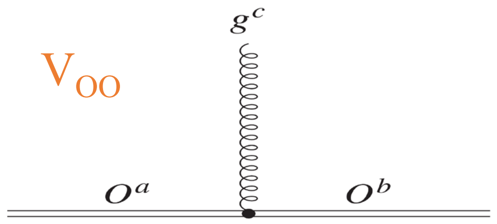
Peskin, NPB(1979), Brambilla et al., RMP(2005)

$V_{SO}$



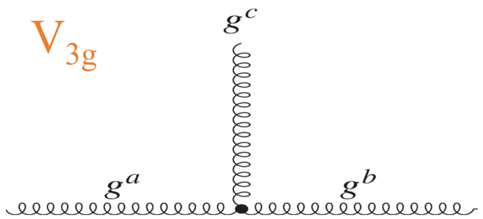

$$\langle O, a|V_{SO}|S\rangle = \frac{g_s}{\sqrt{2N_c}} \vec{E}^a(t,\vec{x}) \cdot \langle O|\vec{r}|S\rangle$$

$V_{OO}$



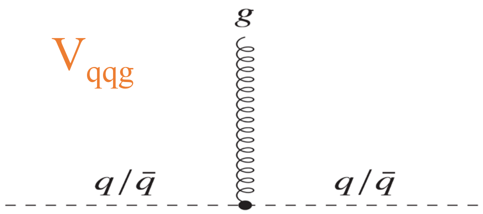

$$\langle O, a|V_{OO}|O, b\rangle = \frac{ig_s}{2} d^{abc} \vec{E}^c(t,\vec{x}) \cdot \langle O|\vec{r}|O\rangle$$

$V_{3g}$




$$V_{3g} \sim \int d^3\vec{x} \frac{1}{2} \vec{B}^a \cdot \vec{B}^a = (-\frac{g_s}{2}) f^{abc} \int d^3\vec{x} (\nabla \times \vec{A}^a) \cdot (\vec{A}^b \times \vec{A}^c)$$

$V_{qqg}$




$$V_{q\bar{q}g} = g_s \int d^3x \psi^{i\dagger}(x) \vec{\alpha} \cdot \vec{A}^a(x) (\frac{\lambda^a}{2})^{ij} \psi^j(x)$$

# LO & NLO dissociation amplitudes

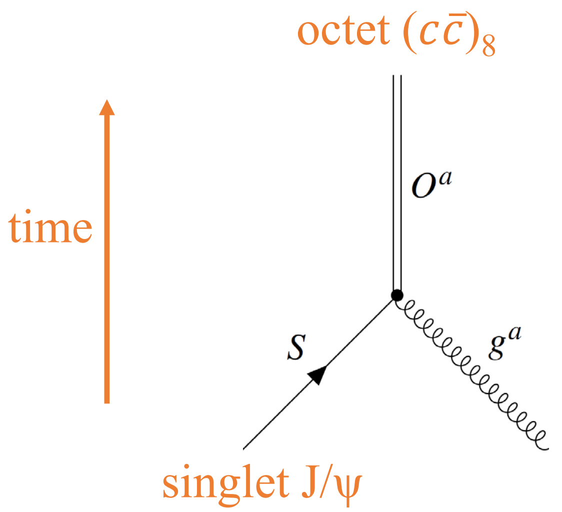
- Transition matrix elements (OFPT): LO + NLO

$$T_{fi} = \langle f|H_{\rm int}|i\rangle + \sum_m \frac{\langle f|H_{\rm int}|m\rangle\langle m|H_{\rm int}|i\rangle}{E_i - E_m + i\epsilon} = T_{fi}^{\rm LO} + T_{fi}^{\rm NLO}.$$

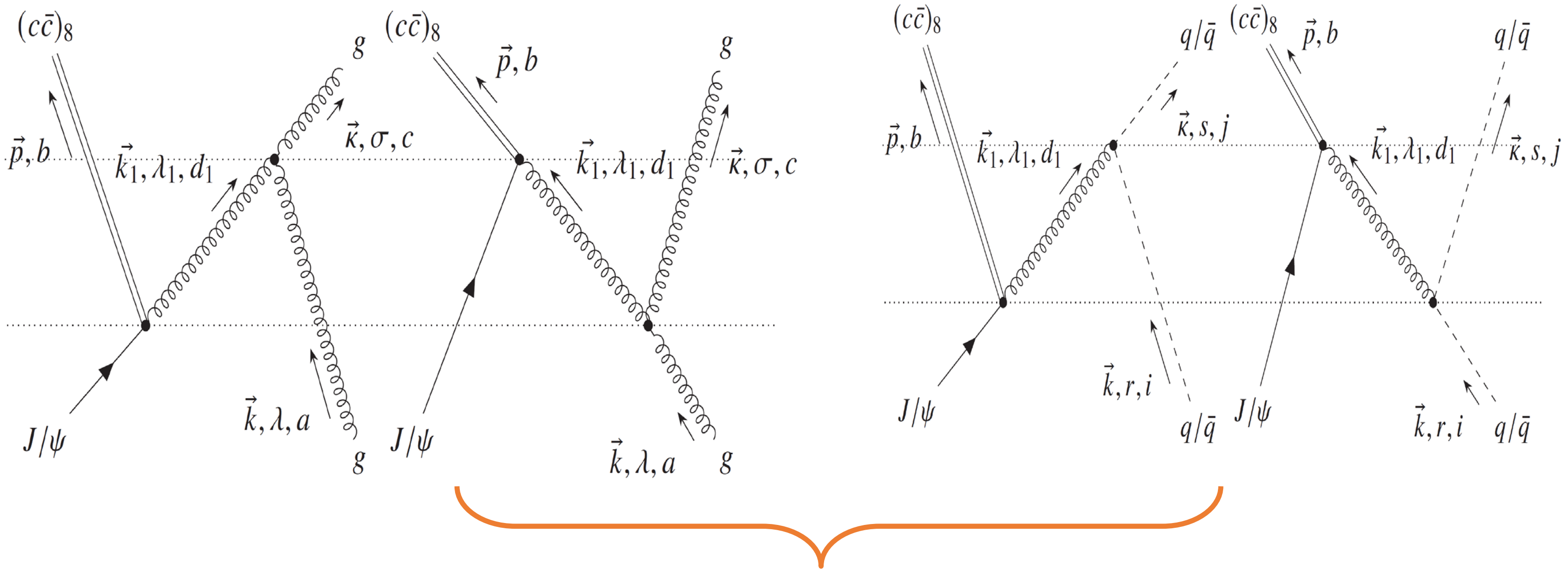
- Time-ordered Feynman diagrams

$g + \psi \to c + \bar{c}$

$g/q(\bar{q}) + \psi \to g/q(\bar{q}) + c + \bar{c}$



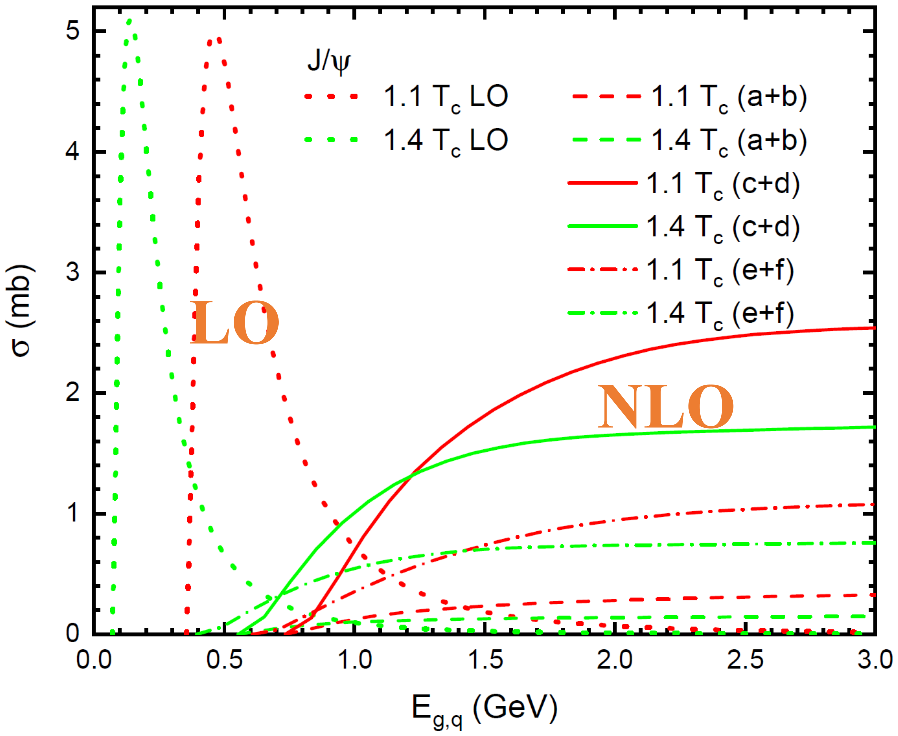

**LO: O($g_s$r)**




**NLO: O($g_s^2$r)**

# J/ψ dissociation cross sections & rates

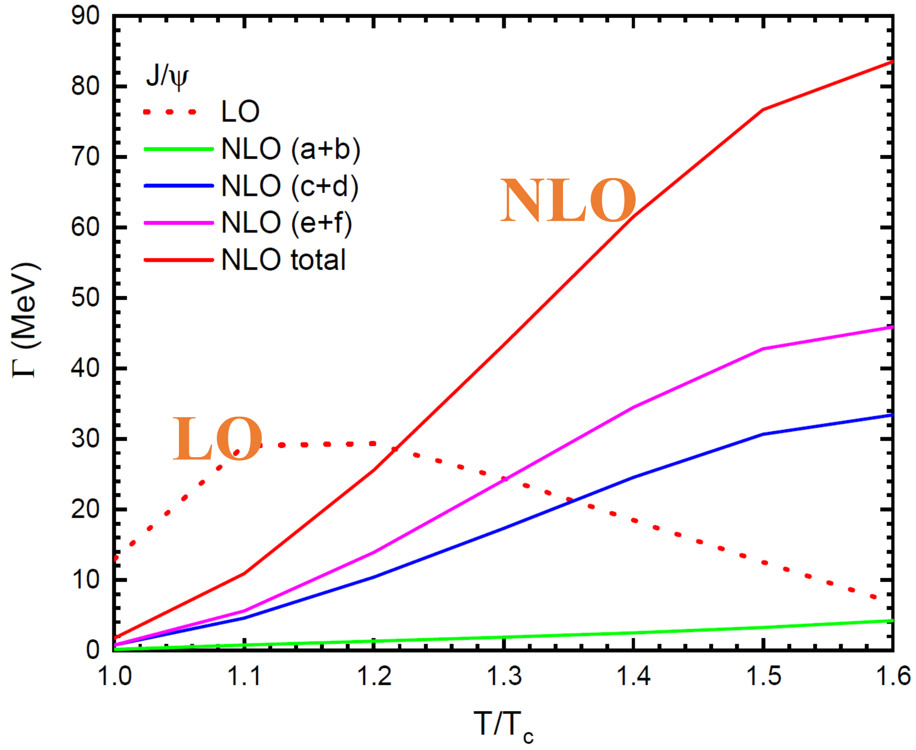
## Cross section




- LO peaking at low incident $E_g$ (~ $E_B$)
  & falling off thereafter

- NLO growing with $E_{g/q}$ and saturating eventually
  ← outgoing g/q carrying away the excess energy

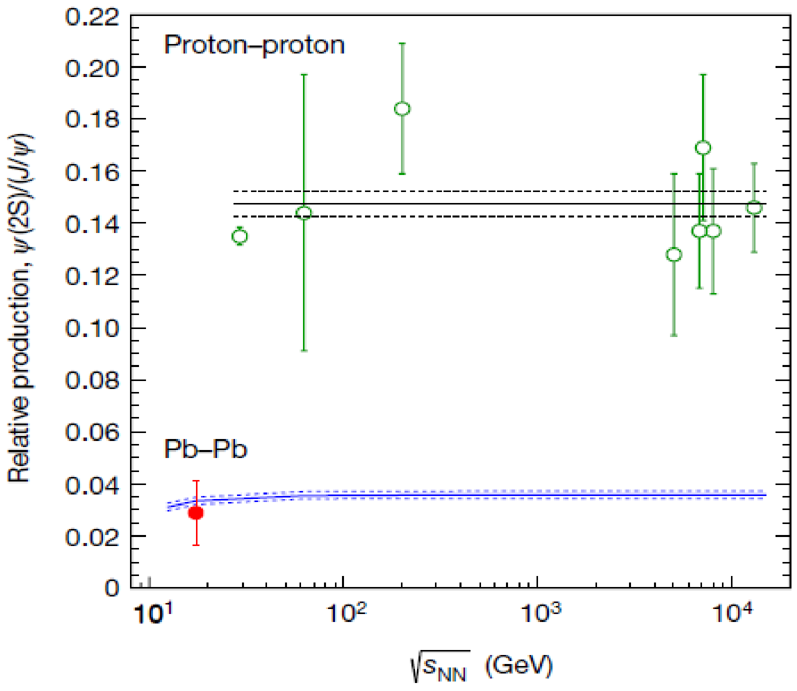
## Diss. rate $\Gamma(T) = d_p \int \frac{d^3k}{(2\pi)^3} f_p(E_p, T) v_{rel} \sigma(k_p, T)$




- LO rate dominating at lower T

- NLO rate increasing with T & taking over at higher T
  ← new phase space opening up.

# Part 2: Quarkonium transport & equilibration in QGP




P. B. Munzinger et al., Nature, 561 (2018)

Proton-proton: NRQCD

Pb-Pb: statistical hadronization = equilibrium

➔ How is the quarkonia equilibrium achieved?
What's the underlying dynamics?

- Do the LO & NLO reactions lead to the quarkonium equilibration (both chemical & kinetic)?

➔ Simulate $J/\psi$ dissociation & regeneration in a static & uniform QGP box, with 10 pairs of $c\bar{c}$

# Equilibrium limit for heavy quarkonium

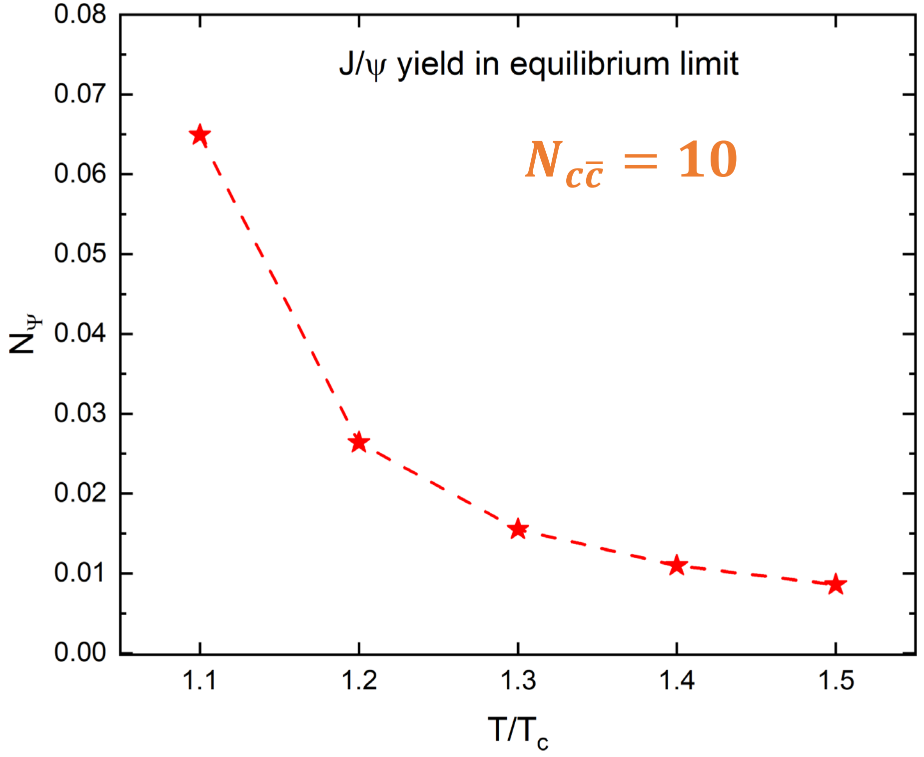
- Relative chemical equilibrium between open $c$-$\bar{c}$ & hidden J/ψ

→ statistical balance equation

$$N_{Q\bar{Q}} = N_Q^{\rm open} + N_\Psi \quad \begin{cases} N_Q^{\rm open} = d_Q \gamma_Q V \dfrac{m_Q^2 T}{2\pi^2} K_2\left(\dfrac{m_Q}{T}\right) \\ N_\Psi = d_\Psi \gamma_Q^2 V \dfrac{m_\Psi^2 T}{2\pi^2} K_2\left(\dfrac{m_\Psi}{T}\right) \end{cases}$$

- Fugacity $\gamma_Q$ characterizing the deviation from absolute chemical equilibrium

- < 1% of $c\bar{c}$ existing in J/ψ
- At lower T, more $c\bar{c}$ existing in bound states

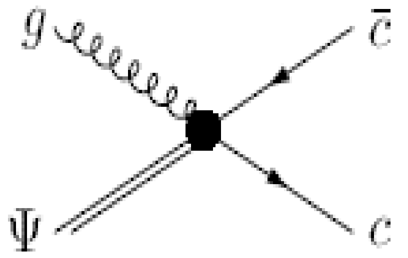


# J/ψ transport in QGP: Boltzmann equation

- Time evolution of J/ψ phase space distributions:

$$\frac{\mathrm{d}}{\mathrm{d}t} f_\Psi(\vec{x},\vec{p},t) = (\frac{\partial}{\partial t} + \frac{\partial \vec{x}}{\partial t}\cdot\nabla_{\vec{x}}) f_\Psi(\vec{x},\vec{p},t) = C_{22}(\Psi + g \rightleftharpoons Q + \bar{Q}) + C_{23}(\Psi + p \rightleftharpoons Q + \bar{Q} + p)$$

- LO 2→2: $g + J/\psi \leftrightarrow c + \bar{c}$



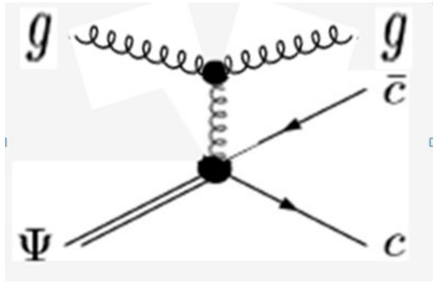

$$C_{22} = C_{22}[\mathrm{gain}] - C_{22}[\mathrm{loss}]$$

reaction amplitude

$$= \frac{1}{2E_\Psi(\vec{p})} \int \frac{\mathrm{d}^3 p_2}{2E_g(2\pi)^3} \frac{\mathrm{d}^3 p_3}{2E_Q(2\pi)^3} \frac{\mathrm{d}^3 p_4}{2E_{\bar{Q}}(2\pi)^3} \sum |\mathcal{M}^{\mathrm{LO}}|^2 (2\pi)^4 \delta^4(p + p_2 - p_3 - p_4)$$

$$\times [\frac{d_\Psi}{d_Q d_{\bar{Q}}} f_Q(\vec{x},\vec{p}_3,t) f_{\bar{Q}}(\vec{x},\vec{p}_4,t)\,(1 + f_g(\vec{x},\vec{p}_2,t)) - f_\Psi(\vec{x},\vec{p},t) f_g(\vec{x},\vec{p}_2,t)].$$

- NLO 2→3: $p + J/\psi \leftrightarrow p + c + \bar{c}\ ,\ \ p = g, q/\bar{q}$




$$C_{23} = C_{23}[\mathrm{gain}] - C_{23}[\mathrm{loss}]$$

$$= \frac{1}{2E_\Psi(\vec{p})} \int \frac{\mathrm{d}^3 p_2}{2E_p(\vec{p}_2)(2\pi)^3} \frac{\mathrm{d}^3 p_3}{2E_Q(2\pi)^3} \frac{\mathrm{d}^3 p_4}{2E_{\bar{Q}}(2\pi)^3} \frac{\mathrm{d}^3 p_5}{2E_p(\vec{p}_5)(2\pi)^3} \sum |\mathcal{M}^{\mathrm{NLO}}|^2 (2\pi)^4 \delta^4(p + p_2 - p_3 - p_4 - p_5)$$

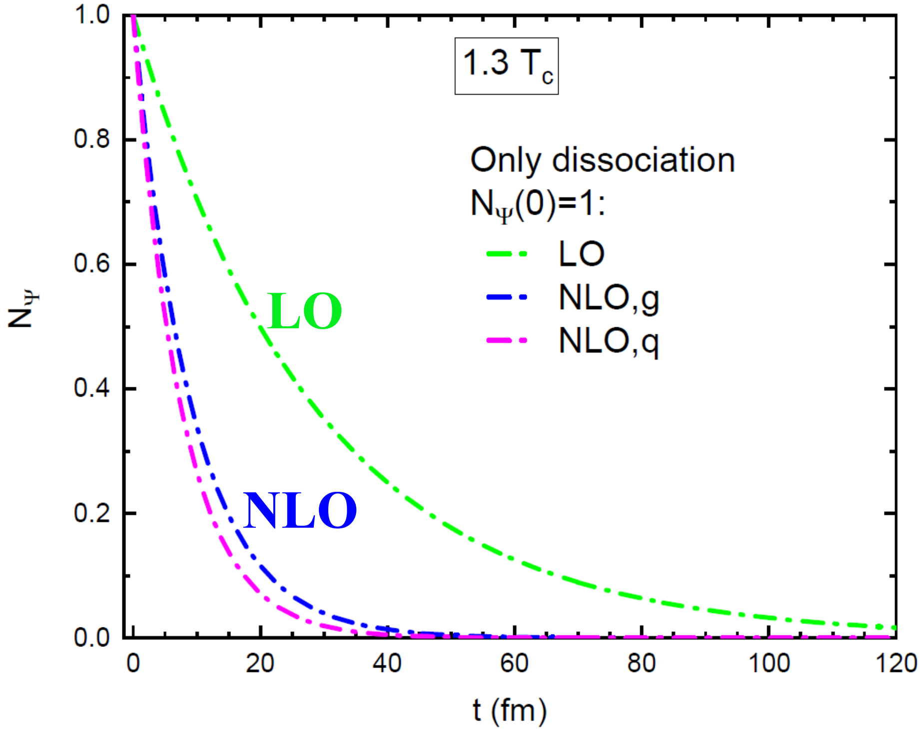
$$\times [\frac{d_\Psi}{d_Q d_{\bar{Q}}} f_Q(\vec{x},\vec{p}_3,t) f_{\bar{Q}}(\vec{x},\vec{p}_4,t) f_p(\vec{x},\vec{p}_5,t)\,(1 \pm f_p(\vec{x},\vec{p}_2,t)) - f_\Psi(\vec{x},\vec{p},t) f_p(\vec{x},\vec{p}_2,t)\,(1 \pm f_p(\vec{x},\vec{p}_5,t))]$$

- T + P symmetry <=> detailed balance principle: $|M_{diss}|^2 = |M_{reg}|^2$

# Simulation of J/ψ dissociation and regeneration

- Only dissociation: rate equation for the loss

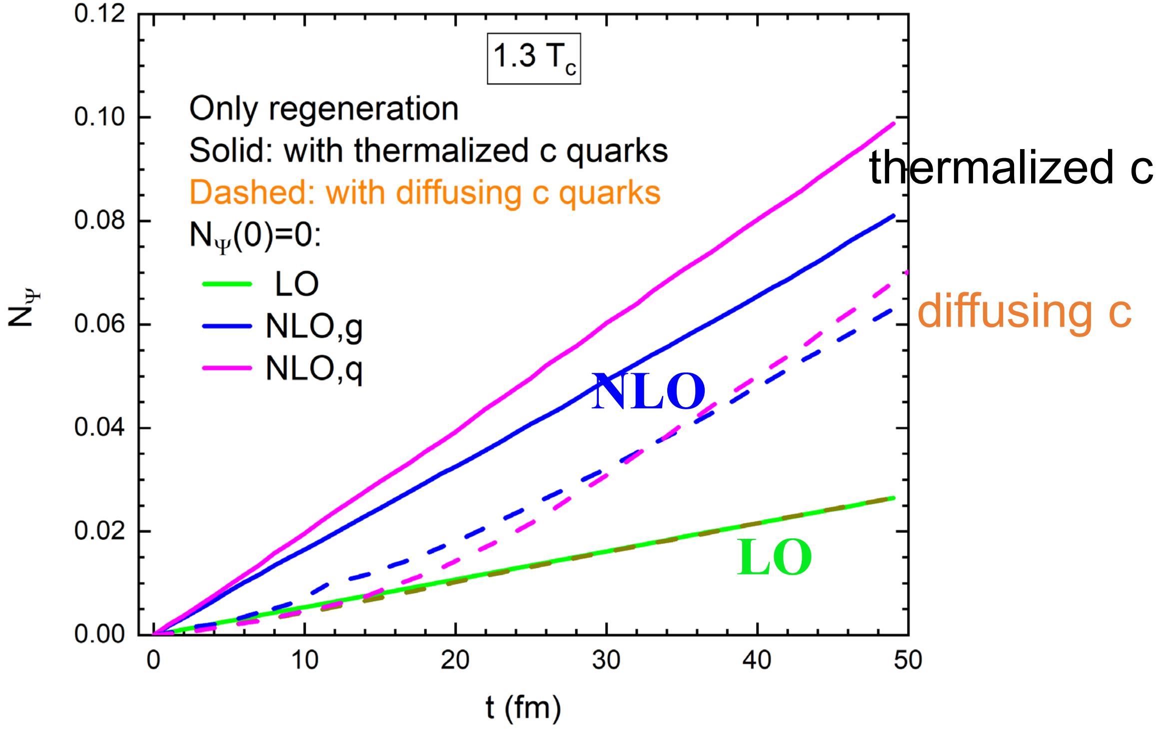
$$\frac{d}{dt} f_\psi(\vec{p},t)[loss] = -\Gamma(p,T) f_\psi(\vec{p},t)$$




- Faster NLO dissociation than LO

- Only regeneration: off-diagonal $c\bar{c}$ recombination

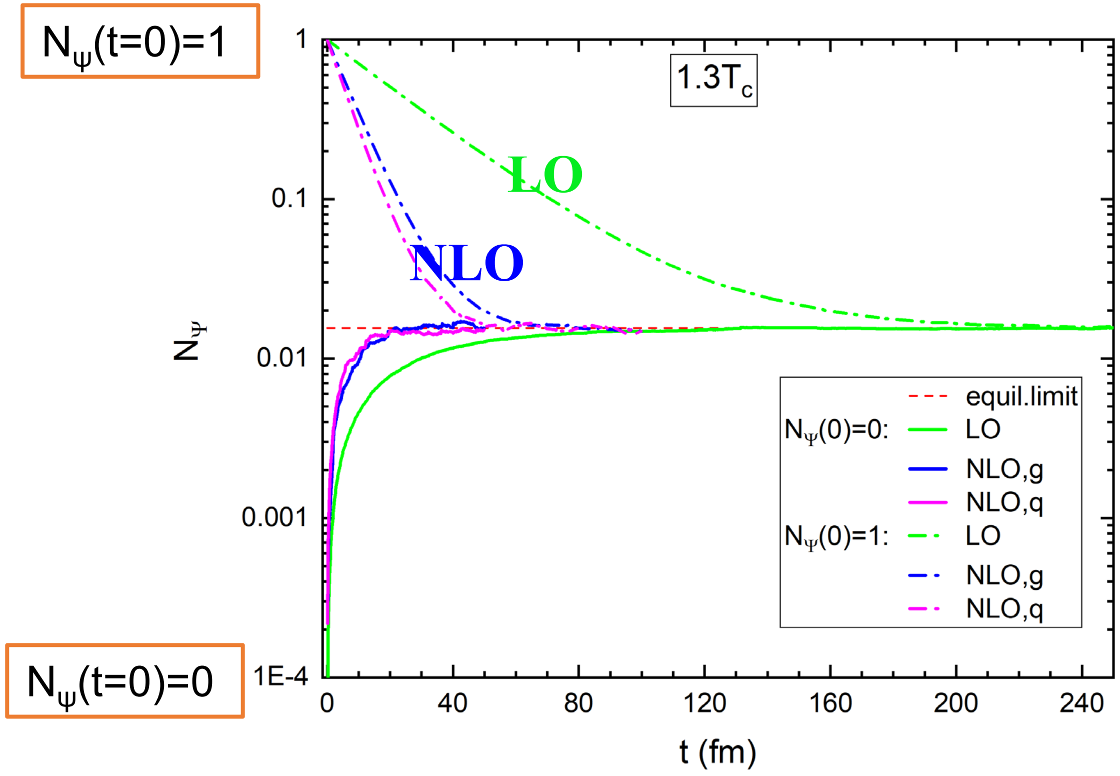
$$\frac{dN}{dp_\Psi dt} \propto \sum_{n_Q=1}^{10} \sum_{n_{\bar{Q}}=1}^{10} |\mathcal{M}|^2 \delta^3(\vec{p}_Q - \vec{p}_{nQ}) \delta^3(\vec{p}_{\bar{Q}} - \vec{p}_{n\bar{Q}})$$




- Faster NLO regeneration than LO
- In early stage, rate w/ diffusing c small eventually, equal to rate w/ thermalized c

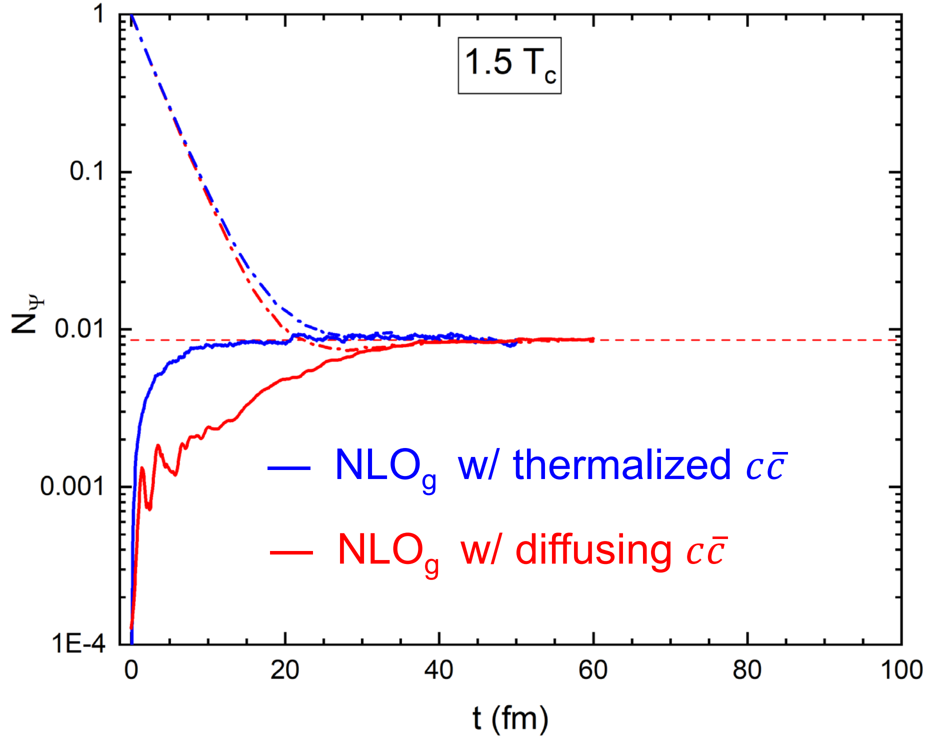
# Dissociation + regeneration → J/ψ equilibration

## Thermalized $c$ & $\bar{c}$




- Faster equilibration for NLO than LO
- N(0)=1 slower equilibration than N(0)=0

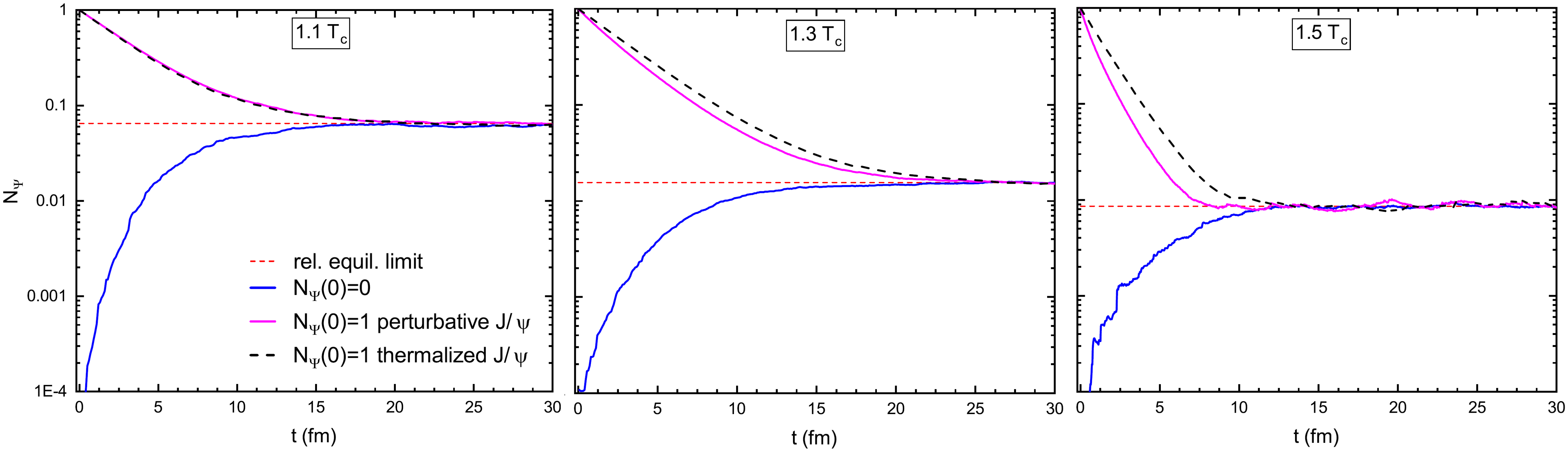
## Thermalized vs. diffusing $c$ & $\bar{c}$




- Slower J/ψ equilibration with diffusing c-quarks
- J/ψ equilibration lagging behind c thermalization

# J/ψ equilibration with total rates & diffusing c

LO + NLO reactions & diffusing $c$ & $\bar{c}$ w/ $\gamma = 0.4$ fm$^{-1}$

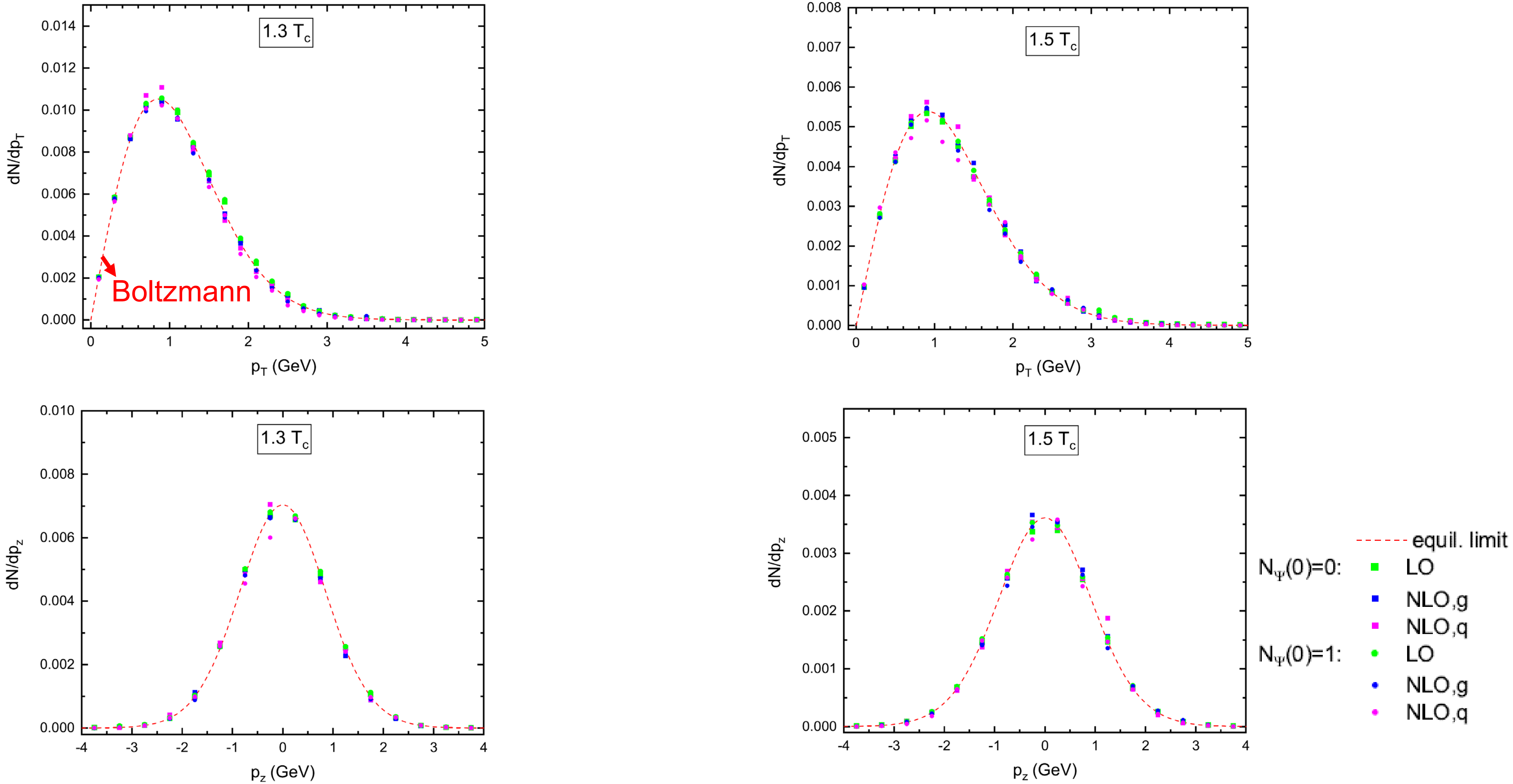



- J/ψ chemical equilibration time ~ 10-15 fm at 1.1-1.5T$_c$ ≈ QGP lifetime in central Pb-Pb collisions
- Equilibration times insensitive to initial conditions (Initially harder J/ψ equilibrating a bit faster)

# Kinetic equilibration: $p_T$ & $p_z$ spectra

$dN/dp_T$

$dN/dp_z$




- J/ψ $p_T$ and $p_z$ distributions reaching Boltzmann distri. ➔ both chemical & kinetic equilibrium achieved!

# Summary & Outlook

- Heavy quarkonium reaction cross sections & rates were computed, using the color-electric dipole (E1) coupling of quarkonium to external gluons

  - NLO was found to dominate at high incident gluon energy and high T

- J/ψ chemical & kinetic equilibration was demonstrated via simulation of dissociation & regeneration

  - Off-diagonal recombination of thermalizing $c\bar{c}$ was instrumental

  - NLO reactions accelerate the equilibration ➔ final equilibration time ~ 10-15 fm/c

- Our work provides a baseline for realistic phenomenological applications & a dynamical way of understanding the rationale of SHM for charmonia production in HIC

# Back-ups

# Description of quarkonia dynamics in QGP

**Subsystem: non-unitary, time-irreversible evolution**

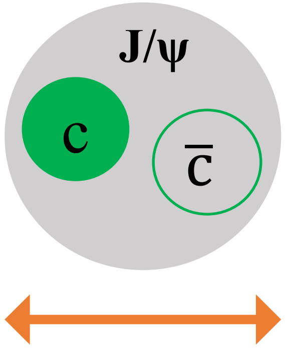
$$\mathrm{Tr}_E\left[U(t,0)(\rho_S \otimes \rho_E)U^\dagger(t,0)\right]$$

**Markovian (weak coupling)**

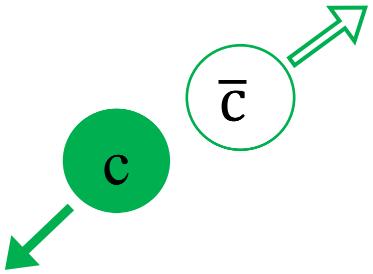
| Quantum optical limit (low T) | Quantum Brownian motion (high T) |
|---|---|
| **Lindblad equation** | **Lindblad equation** |
| **Boltzmann equation** | **Langevin/Fokker-Planck equation** |

$$\frac{d\rho_S(t)}{dt} = -i\left[H_{S,\mathrm{eff}}, \rho_S(t)\right] + \sum_n \left(L_n \rho_S(t) L_n^\dagger - \frac{1}{2}\{L_n^\dagger L_n, \rho_S(t)\}\right)$$

**Wigner transform (smearing for positivity)**
**Semiclassical (gradient expansion)**




dipole apprx.




wavefn. decoherence

8

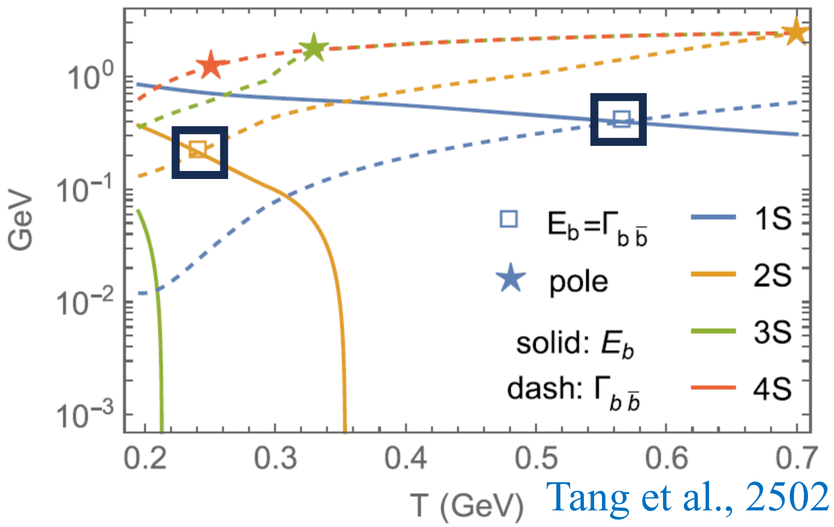
taken from X. Yao slides

# Static HQ potential at finite temperature

* Re $V_{Q\bar{Q}}$ → binding energy $\epsilon_B$

* Im $V_{Q\bar{Q}}$, → thermal width $\Gamma(T) \sim \langle Im\ V \rangle$, arising from the self-energy of exchanged space-like gluon

* Dissociation temperature definition: $\Gamma(T_{diss}) = \epsilon_B$,
rather than $\epsilon_B \sim 0$, or $\langle r \rangle \sim \infty$



Tang et al., 2502.09044

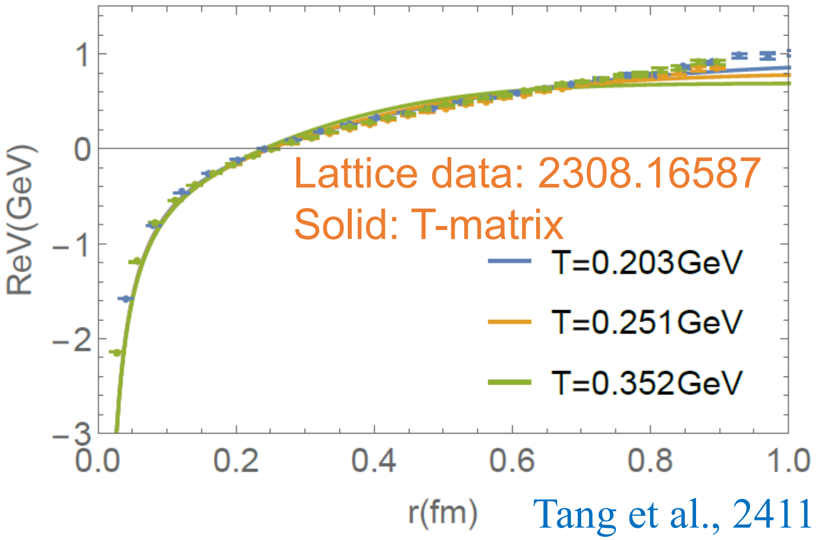
About Re $[V_{Q\bar{Q}}]$

* LO perturbation (Laine et al., 0611300) → screened

* Lattice computation (HotQCD Collaboration, 2308.16587)→ unscreened

* Beyond LO perturbation (Carrington et al., 2407.00310)→ unscreened

* T-matrix approach (Tang et al., 2411.09132)→ weak screening

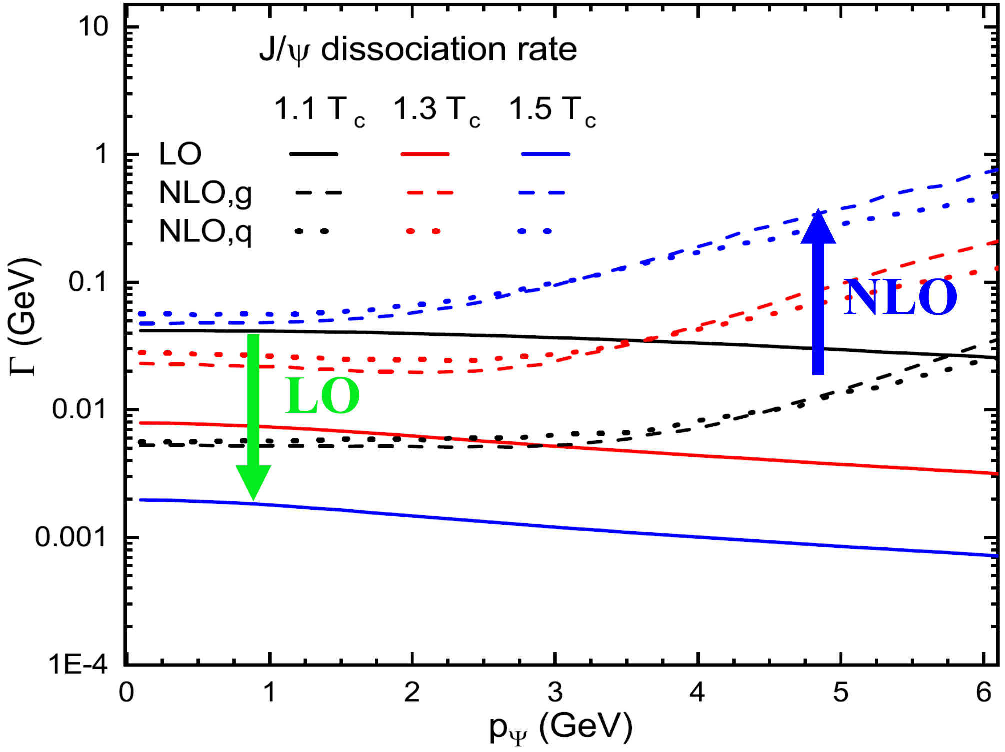


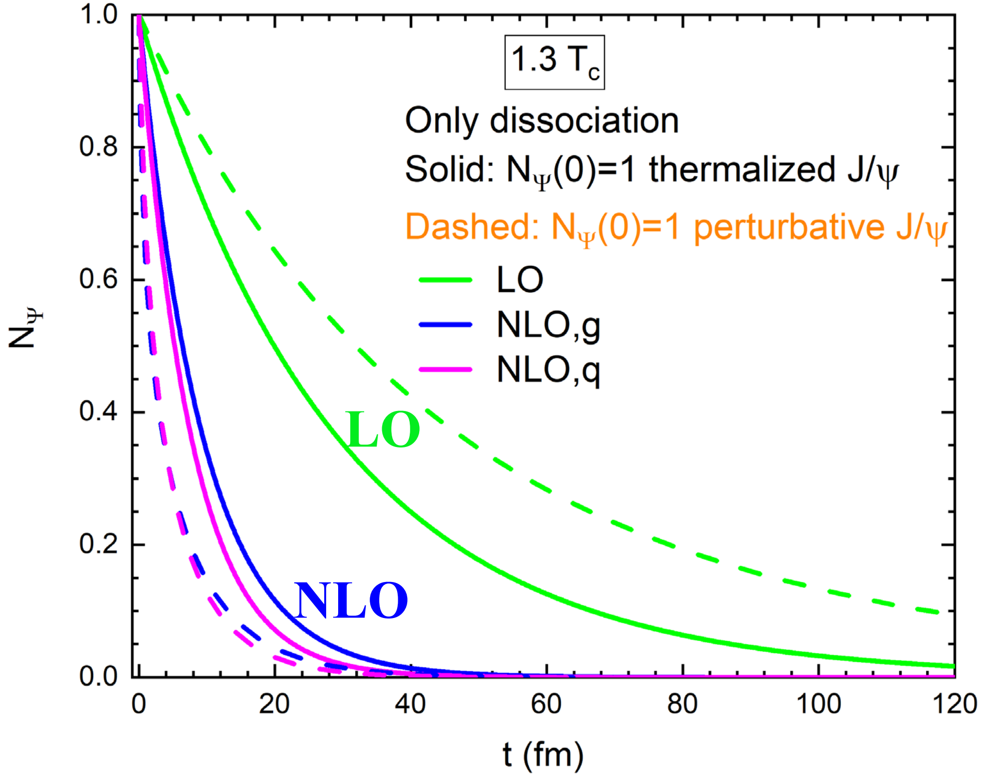
Tang et al., 2411.09132

# J/ψ dissociation in a static QGP box

- Iteration scheme: differential rate equation for the loss/dissociation of J/ψ

$$\frac{d}{dt} f_\psi(\vec{p}, t)[loss] = -\Gamma(p, T) f_\psi(\vec{p}, t)$$




- NLO prevailing over LO at higher T & larger p
- Average momenta: $\langle p \rangle_{\text{perturbative}} > \langle p \rangle_{\text{thermalized}}$

# From Boltzmann eq. to rate equation

* Only if single heavy quarks are fully thermalized, for LO,

$$
\begin{aligned}
&\frac{d_\Psi}{d_Q d_{\bar{Q}}} f_Q^{\mathrm{eq}}(\vec{p}_3) f_{\bar{Q}}^{\mathrm{eq}}(\vec{p}_4)\left(1+f_g(\vec{p}_2)\right) \\
=& d_\Psi \gamma_Q e^{-E_Q(\vec{p}_3)/T} \gamma_{\bar{Q}} e^{-E_{\bar{Q}}(\vec{p}_4)/T}\left(1+\frac{1}{e^{E_g(\vec{p}_2)/T}-1}\right) \\
=& d_\Psi \gamma_Q^2 e^{\left(-E_Q(\vec{p}_3)-E_{\bar{Q}}(\vec{p}_4)+E_g(\vec{p}_2)\right)/T} \frac{1}{e^{E_g(\vec{p}_2)/T}-1} \\
=& d_\Psi \gamma_Q^2 e^{-E_\Psi(\vec{p})/T} \frac{1}{e^{E_g(\vec{p}_2)/T}-1} \\
=& f_\Psi^{\mathrm{eq}}(\vec{p}) f_g(\vec{p}_2),
\end{aligned} \tag{15}
$$

Energy conservation

* Boltzmann eq. ➔ rate eq.

Loss Gain

$$
\frac{\mathrm{d}}{\mathrm{d}t} f_\Psi(\vec{x},\vec{p},t) = -\Gamma^{\mathrm{LO}}(p, T(\vec{x}))\left[f_\Psi(\vec{x},\vec{p},t) - f_\Psi^{\mathrm{eq}}(\vec{x},\vec{p})\right]. \tag{16}
$$

➔ Momentum spectra thermalizing

$$
\frac{\mathrm{d}}{\mathrm{d}t} N_\Psi(t) = -\Gamma(\langle p\rangle, T)\left[N_\Psi(t) - N_\Psi^{\mathrm{eq}}\right]
$$

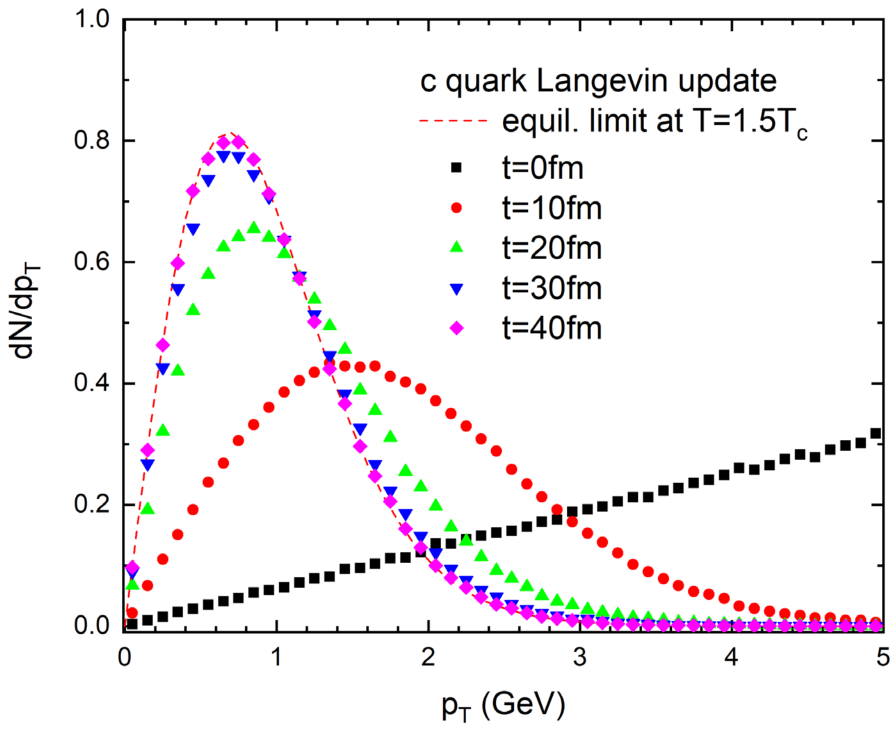
➔ Integrated yield equilibrating

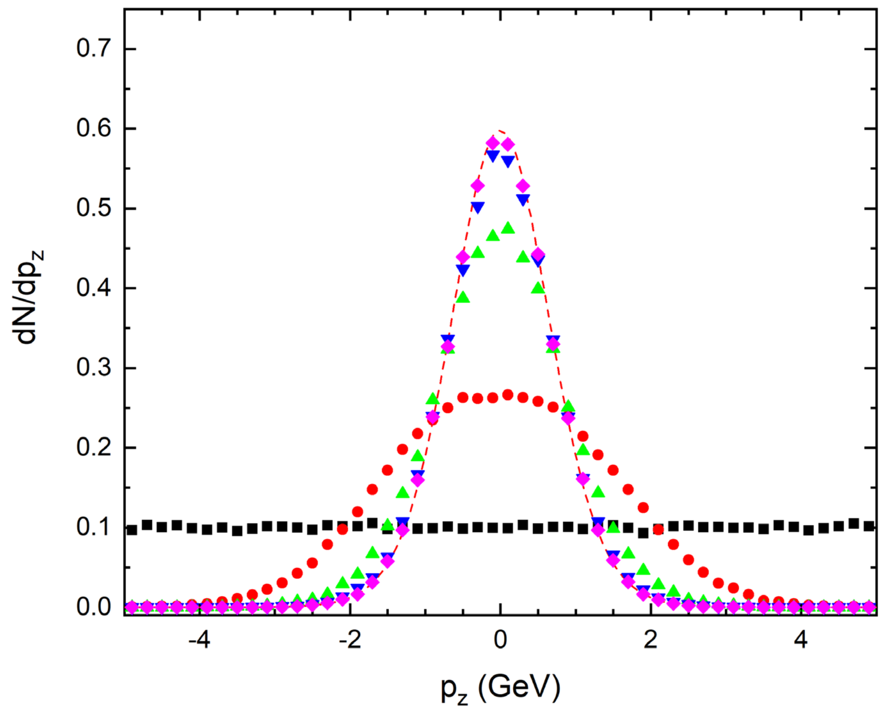
# Charm quark Langevin diffusion

- Coupling the J/ψ Boltzmann transport to the single HQ diffusion

$$d\vec{p} = -\gamma \vec{p} dt + \sqrt{2Ddt}\vec{\rho}$$

$\gamma = 0.1\ fm^{-1}$




- c-quark thermalization time ~ 30 fm/c for single HQ thermal relaxation rate $\gamma$=0.1 $fm^{-1}$