Cosmological Gravitational Particle Production: Sterile Neutrinos as Dark Matter candidates

*13/04/2026 @ **GUTPC-HIAS***

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*Based on **JHEP 04 (2024) 027** & **JCAP 03 (2026) 030***

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CONTENTS

Intro

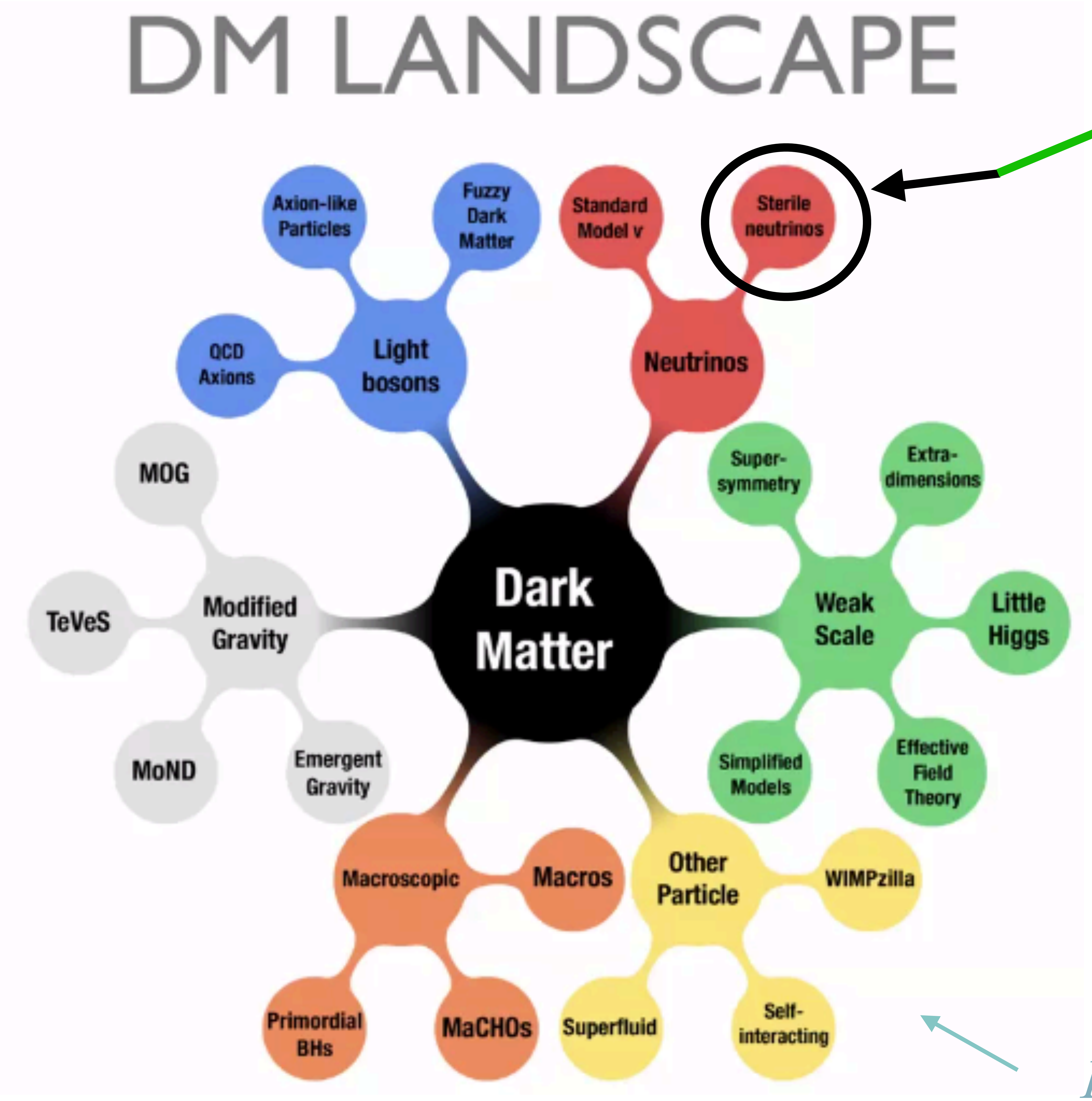
- Compelling evidence for the existence of Dark Matter (DM) on different astrophysical scales (galactic, clusters of galaxies, cosmological scale,…)

- ~ 84% of the matter in the Universe is DARK

- DM candidate: **stable** (lifetime must be long compared to cosmological timescales), (dominantly) **Non-relativistic**, **electrically neutral** and **colorless**. (Only?) **gravitational interactions**

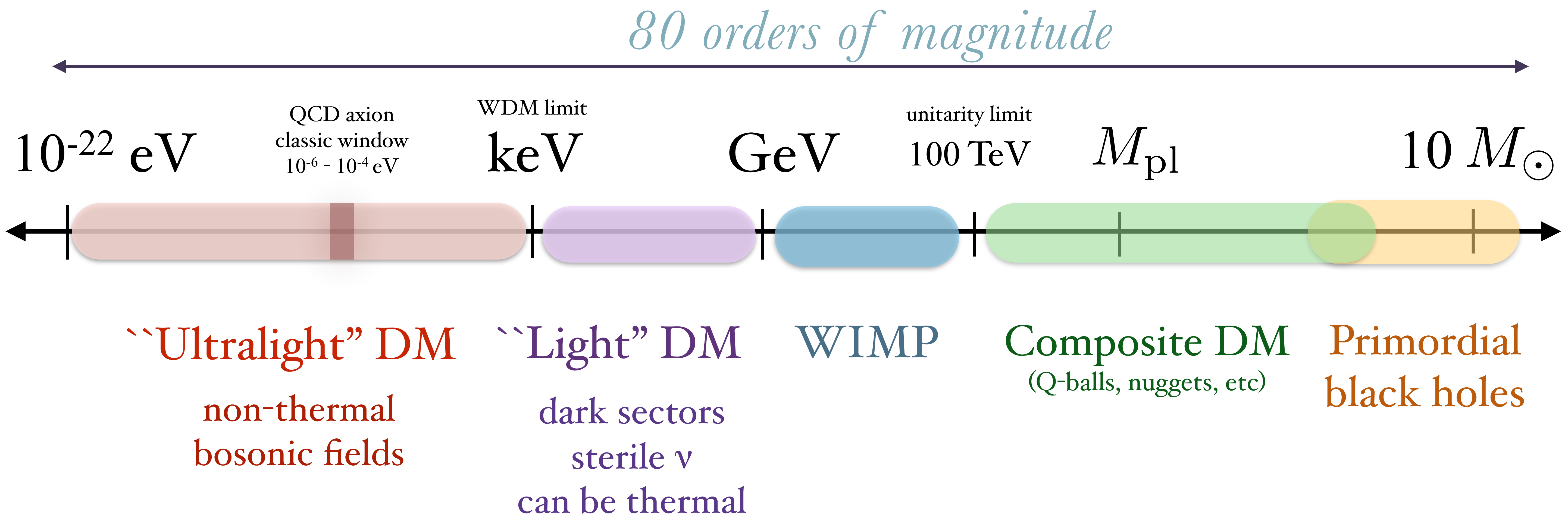
- What is DM? How was produced?

Intro



Gravitationally produced Sterile Neutrinos as non-thermal Dark matter

A. Boyarsky, O. Ruchayskiy and M. Shaposhnikov, '09
A. Boyarsky, M. Drewes, T. Lasserre, S. Mertens and O. Ruchayskiy, '19



Bertone and Tait, Nature '18

Tongyan Lin '19
TASI lectures on DM

BASICS OF THE MODEL (INFLATIONARY PART)

The basics of the model

- Starting point: A conformally invariant fermionic action of a field Ψ with mass M (breaking scale invariance) on a FRW conformally flat metric

$$ds^2 = a(x_0)^2\,\eta_{\mu\nu}dx^\mu dx^\nu \qquad S_\Psi = \int d^4x\sqrt{|g|}\,\bar{\Psi}(i\gamma^\alpha\nabla_\alpha - M)\,\Psi \longrightarrow (i\gamma^\alpha\nabla_\alpha - M)\,\Psi = 0$$

$$g_{\mu\nu} = \Omega^2\tilde{g}_{\mu\nu}\,, \quad \Psi = \Omega^{-3/2}\tilde{\Psi} \qquad e^\mu_\alpha = \Omega^{-1}\tilde{e}^\mu_\alpha \qquad x_0 \equiv \eta$$

$$(i\gamma^\mu\partial_\mu - a(\eta)M)\,\Psi = 0$$

- Flat space Dirac equation with a **time-dependent mass** causing **particle production**

V. Mukhanov, 2005

L. Parker '69, A. A. Grib and S. G. Mamaev '69

S. G. Mamaev, V. M. Mostepanenko and A. A. Starobinsky, '76

- Next step, however a non-trivial task, is to solve the e.o.m. revealing the particle production effect

The basics of the model

$$\Psi(x) = \sum_i \left(a_i U_i + b_i^\dagger V_i \right)$$

$$U_{\mathbf{k},s}(\eta, \mathbf{x}) = \frac{e^{i\mathbf{k}\cdot\mathbf{x}}}{(2\pi)^{3/2}} \begin{pmatrix} u_{A,k}(\eta) \\ s\, u_{B,k}(\eta) \end{pmatrix} \otimes h_s(\hat{\mathbf{k}})$$

$$k \equiv |\mathbf{k}|, \; \hat{\mathbf{k}} = \mathbf{k}/|\mathbf{k}|$$

$$V_{\mathbf{k},s}(\eta, \mathbf{x}) = -\frac{e^{-i\mathbf{k}\cdot\mathbf{x}}}{(2\pi)^{3/2}} \begin{pmatrix} -u^*_{B,k}(\eta) \\ s\, u^*_{A,k}(\eta) \end{pmatrix} \otimes h_s(-\hat{\mathbf{k}})\, e^{i\phi}$$

$$(U_i, U_j) = (V_i, V_j) = \delta_{ij} \;, \; (U_i, V_j) = 0$$

$$h_s^\dagger(\hat{\mathbf{k}})\, h_r(\hat{\mathbf{k}}) = \delta_{rs}$$

$$i\partial_\eta \begin{pmatrix} u_A \\ u_B \end{pmatrix} = \begin{pmatrix} aM & k \\ k & -aM \end{pmatrix} \begin{pmatrix} u_A \\ u_B \end{pmatrix}$$

$$|u_A|^2 + |u_B|^2 = 1$$

The basics of the model

$$a_i|0\rangle = b_i|0\rangle = 0$$ ***BD-vacuum***

T. S. Bunch and P. C. W. Davies, '78

$$\begin{pmatrix} u_A \\ u_B \end{pmatrix}^{\text{in}} \xrightarrow{\eta \to -\infty} \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix} e^{-ik\eta}$$

$$\begin{pmatrix} u_A \\ u_B \end{pmatrix}^{\text{out}} \xrightarrow{\eta \to \infty} \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{-i\int \omega(\eta) d\eta}$$

$$\tilde{U}_{\mathbf{k},s} = \alpha_{\mathbf{k},s} U_{\mathbf{k},s} + \beta_{\mathbf{k},s} V_{-\mathbf{k},s}$$

Bogolyubov coefficient

$$\beta_{\mathbf{k},s} = \text{phase} \times (u_{A,k}\tilde{u}_{B,k} - u_{B,k}\tilde{u}_{A,k})$$

$$\beta'_{\mathbf{k},s} = 0 \longrightarrow \eta \sim \eta_e \equiv 0$$

Average number of particles at the IN-vacuum

$$\langle \tilde{N}_{\mathbf{k},s} \rangle \equiv \langle 0|\tilde{a}^\dagger_{\mathbf{k},s}\tilde{a}_{\mathbf{k},s}|0\rangle = |\beta_{\mathbf{k},s}|^2$$

$$n = \sum_s \int \frac{d^3\mathbf{k}}{(2\pi)^3 a^3} |\beta_{\mathbf{k},s}|^2$$

Other scenarios

CGPP

N. Herring and D. Boyanovsky, Phys. Rev. D 101, no.12, 123522 (2020)

J. Klaric, A. Shkerin and G. Vacalis, JCAP 02, 034 (2023).

Edward W. Kolb et al., *Higgs Inflation: Particle Factory*, arXiv: 2510.24651 (2025)

Edward W. Kolb, Andrew J. Long et al., *Creation of spin-3/2 dark matter via cosmological gravitational particle production*, arXiv: 2512.16976 (2025)

Graviton Exchange Mechanism (GEM)

N. Bernal, M. Dutra, Y. Mambrini, K. Olive, M. Peloso and M. Pierre, Spin-2 PortalDark Matter,Phys.Rev.D 97 (2018)115020

Y. Ema, K. Nakayama and Y.Tang, Production of Purely Gravitational Dark Matter, JHEP 09 (2018)135

Y. Mambrini and K.A. Olive, Gravitational Production of Dark Matter during Reheating, Phys.Rev.D 103 (2021)115009

B. Barman, S. Clery, R. T. Co, Y. Mambrini and K. A. Olive, JHEP 12, 072 (2022)

INFLATIONARY PRODUCTION

The setup

- Focus on a Radiation Dominated universe after Inflation

$$a(\eta) = \left\{ \left(\frac{1}{a_e H_e} - \eta \right)^{-1} H_e^{-1} \quad \text{for} \quad \eta \leq 0 \quad , \quad a_e^2 H_e \left(\eta + \frac{1}{a_e H_e} \right) \quad \text{for } \eta > 0 \right\}$$

- Assume 3 right-handed **Dirac** neutrinos. 2 responsible for neutrino masses and the lightest is the DM. For a time-dependent fermion mass $M \equiv M(\eta)$

$$u_A'' + \left[i(Ma)' + a^2 M^2 + k^2 \right] u_A = 0 \,,$$
$$u_B'' + \left[-i(Ma)' + a^2 M^2 + k^2 \right] u_B = 0$$

- The standard Higgs field h with quartic coupling λ_h **takes on a large value** in the Early Universe

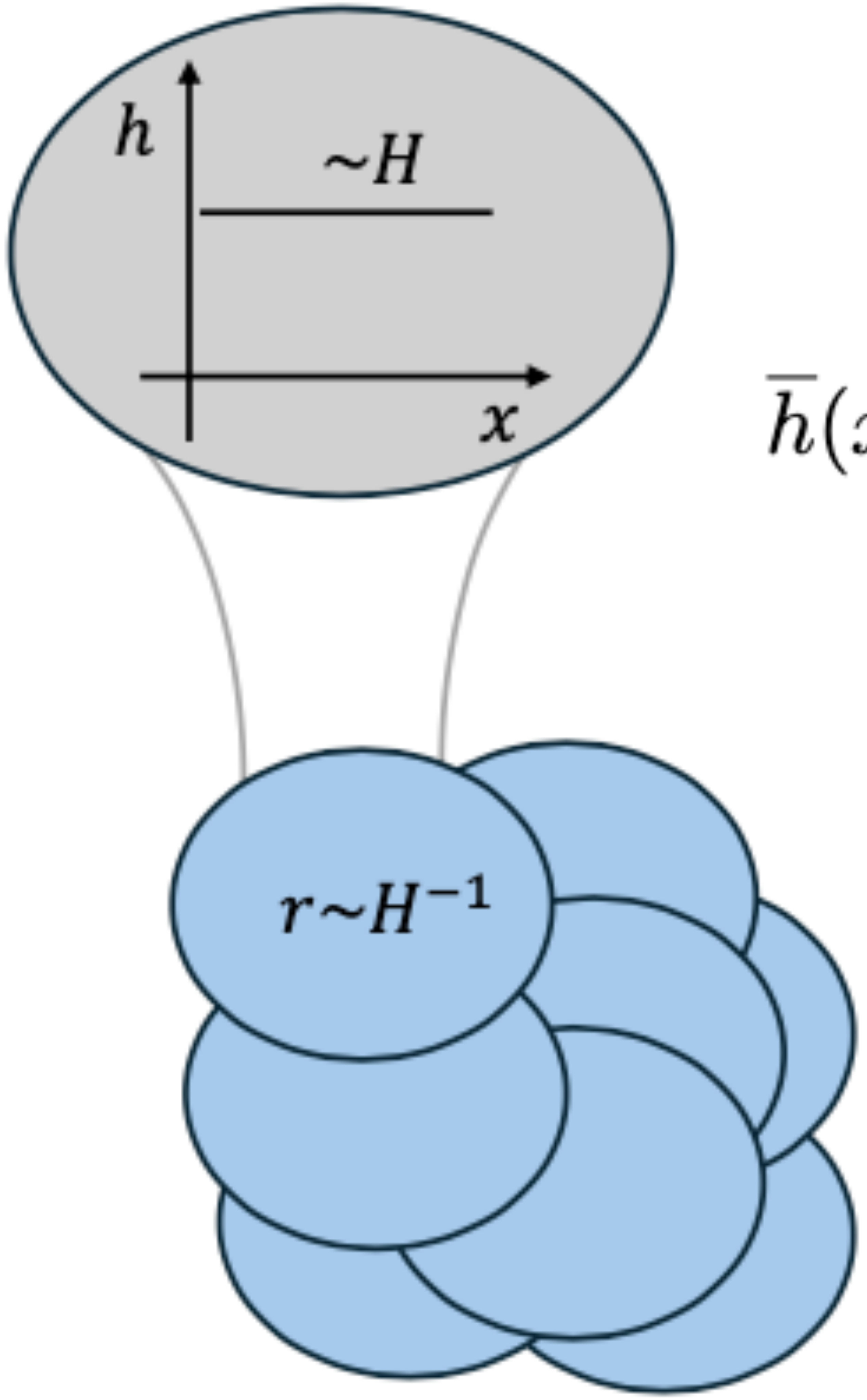
$$\langle h^2 \rangle \sim 0.1 \frac{H^2}{\sqrt{\lambda_h}} \rightarrow \bar{h} = \sqrt{\langle h^2 \rangle} \approx H_e \qquad M(\eta) \sim \mathscr{Y}_R \langle h \rangle + m$$

A. A. Starobinsky and J. Yokoyama, Phys. Rev. D 50, 6357-6368 (1994)

The setup

$$h(x) = \bar{h}(x) + \text{fluctuations}$$

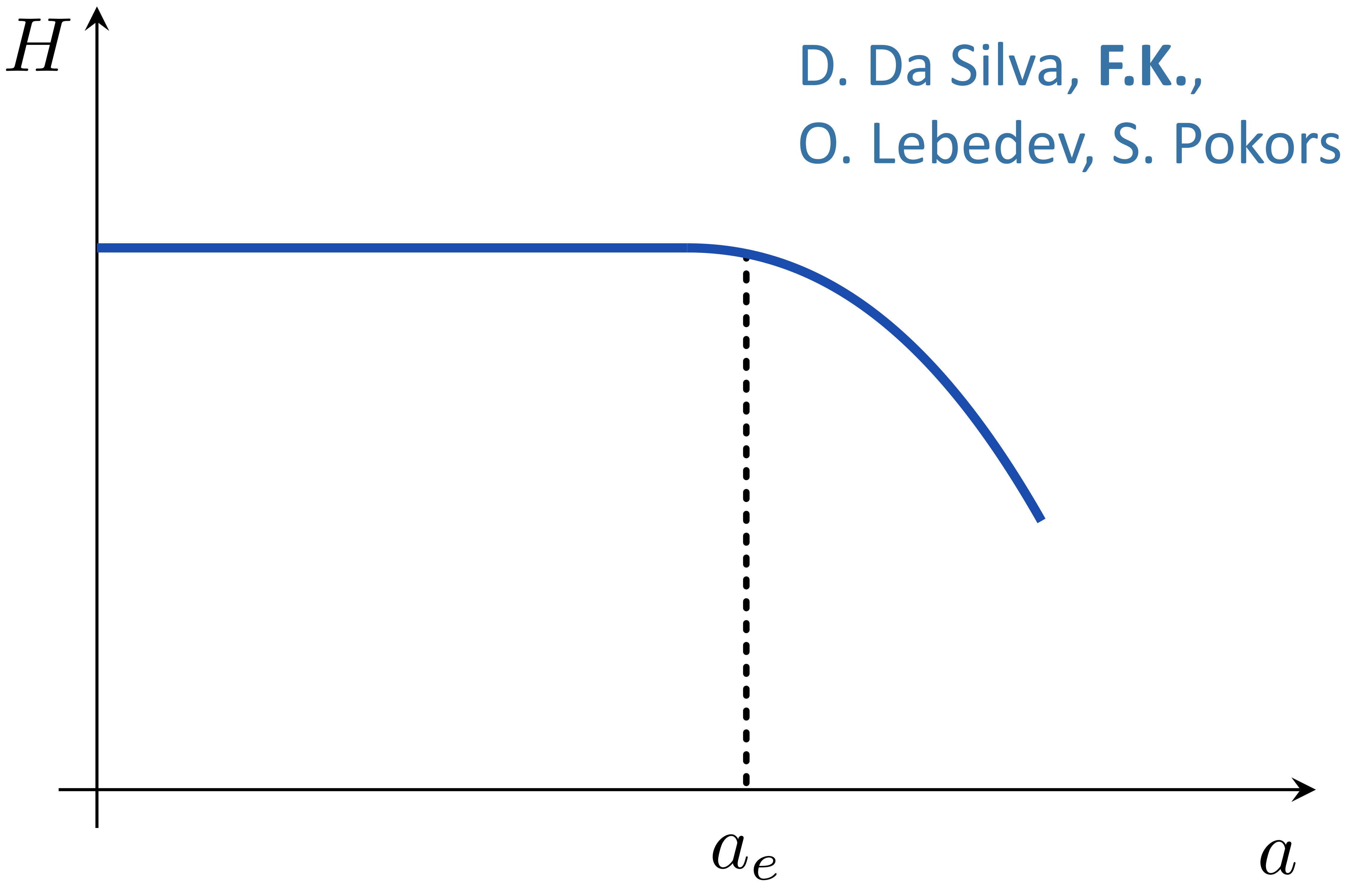
$$\bar{h}(x): \quad k/a \ll H$$



$\bar{h}(x) \simeq const$ in each Hubble patch

$$\dot{\bar{h}}(x) \simeq 0$$

The setup



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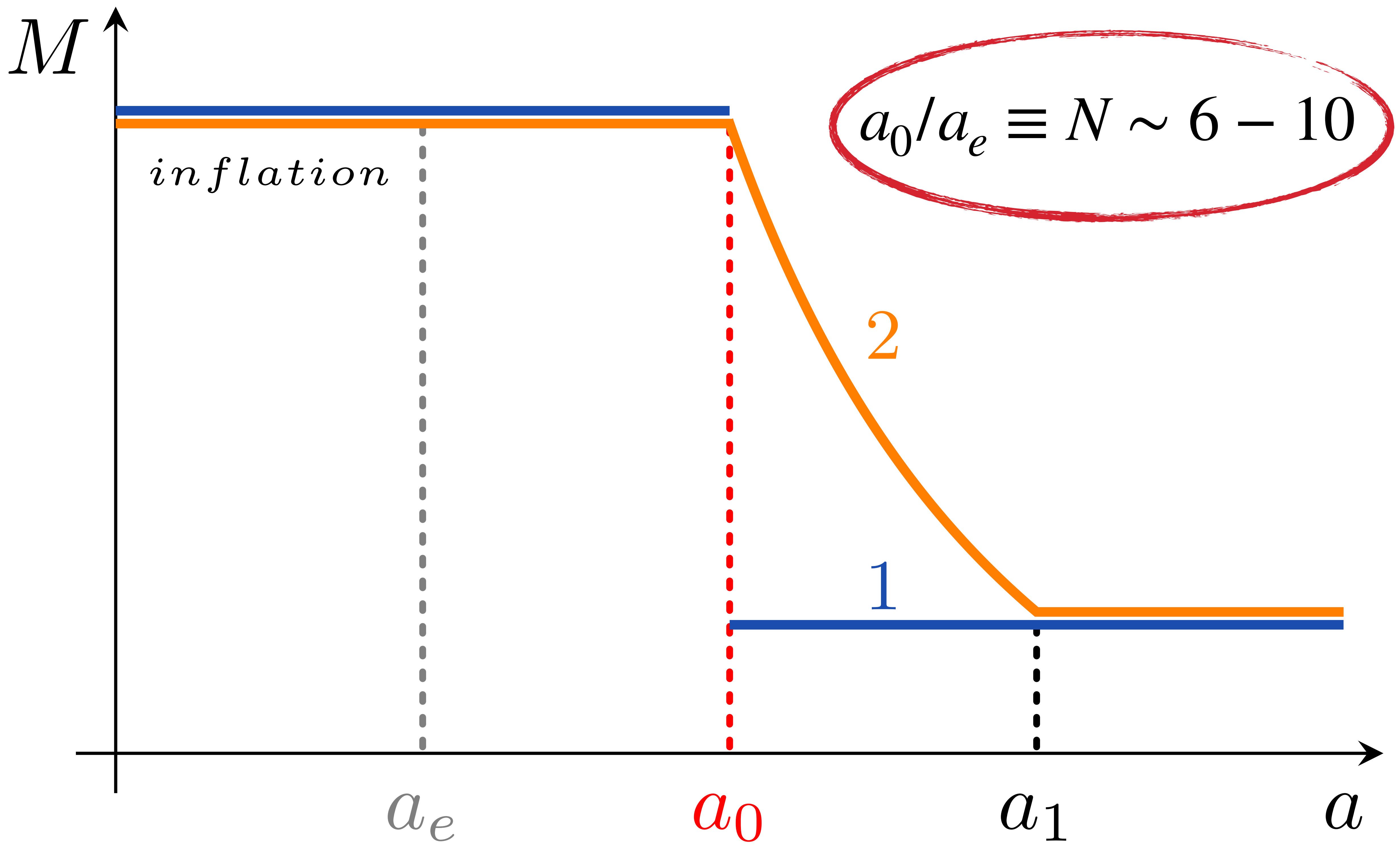


- Two possibilities 1) Step-function mass and 2) Smoothed from thermal effects $M \propto T \propto 1/a$

- **3 mechanisms for SN** production: i) Gravitational production, ii) Higgs condensate decay, iii) Freeze-in

Step-function mass

$$M\,\theta(\eta_0-\eta)+m\,\theta(\eta-\eta_0)$$

$$-\infty<\eta<\eta_0 \quad : \quad M(\eta)=M=\mathscr{Y}_R\langle h\rangle$$

$$\eta_0<\eta<\infty \quad : \quad M(\eta)=m$$

In-wavefunction

$$\eta^2 u_A''+\left(k^2\eta^2+\left[\frac{iM}{H_e}+\frac{M^2}{H_e^2}\right]\right)u_A=0$$

For u_B e.o.m. just set $M\to -M$

BC

$$\begin{pmatrix}u_A\\u_B\end{pmatrix}^{\rm in}\xrightarrow{\eta\to-\infty}\begin{pmatrix}1/\sqrt{2}\\1/\sqrt{2}\end{pmatrix}e^{-ik\eta}$$

$$u_A^{in}(a)=\sqrt{\frac{\pi k}{4aH_e}}\,e^{i\frac{\pi}{2}(1-iM/H_e)}\,H^{(1)}_{1/2-iM/H_e}\left(\frac{k}{aH_e}\right)$$

$$u_B^{in}(a)=\sqrt{\frac{\pi k}{4aH_e}}\,e^{i\frac{\pi}{2}(1+iM/H_e)}\,H^{(1)}_{1/2+iM/H_e}\left(\frac{k}{aH_e}\right)$$

$$\frac{k}{a_eH_e}\ll 1$$

$$u_{A,k}^{in}\simeq\frac{1}{\sqrt{2}}\times e^{i\frac{M}{H_e}\ln\frac{k}{a_eH_e}}\,,$$

$$u_{B,k}^{in}\simeq\frac{1}{\sqrt{2}}\times e^{-i\frac{M}{H_e}\ln\frac{k}{a_eH_e}}$$

Step-function mass

Out-wavefunction: $\eta > \eta_0$

$$u''_A + \left(k^2 + i m a_e^2 H_e + \eta^2\, m^2 a_e^4 H_e^2\right)\, u_A = 0$$

BC $\begin{pmatrix} u_A \\ u_B \end{pmatrix} \xrightarrow{\eta\to\infty} \begin{pmatrix} 1 \\ \frac{k}{2am} \end{pmatrix} e^{-i\int \omega(\eta) d\eta}$

$$C = \frac{k^2}{2 m a_e^2 H_e}$$

$$u^{out}_{A,k} = e^{-\frac{\pi}{4}C} D_{-iC}\left(e^{i\pi/4}\sqrt{\frac{2m}{H(\eta)}}\right) \times \text{phase}$$

$$u^{out}_{B,k} = \sqrt{C}\, e^{-\frac{\pi}{4}C + \frac{i\pi}{4}} D_{-1-iC}\left(e^{i\pi/4}\sqrt{\frac{2m}{H(\eta)}}\right) \times \text{phase}$$

Out-wavefunction: $\eta \leq \eta_0$

$m \to M$

$k^2 \ll M a_e^2 H_e$

$$u^{out}_{A,k}(0) \simeq e^{-\pi C/4} 2^{-iC/2} \sqrt{\cosh\frac{\pi C}{2}} \times \left(1 + i\frac{N^2}{2}\frac{M}{H_e} + \mathcal{O}\left(\frac{k}{a_e H_e}\right)\right)$$

$$u^{out}_{B,k}(0) \simeq e^{-\pi C/4} 2^{-iC/2} \sqrt{\sinh\frac{\pi C}{2}} \times \left(1 - i\frac{N^2}{2}\frac{M}{H_e} + \mathcal{O}\left(\frac{k}{a_e H_e}\right)\right)$$

Step-function mass

$$|\beta_{\mathbf{k}}| = |u^{in}_{A,k} u^{out}_{B,k} - u^{in}_{B,k} u^{out}_{A,k}|$$

$$|\beta_k| \simeq \frac{1}{2}\frac{M}{H_e}\left|N^2 - 2\ln\frac{k}{a_e H_e}\right|$$

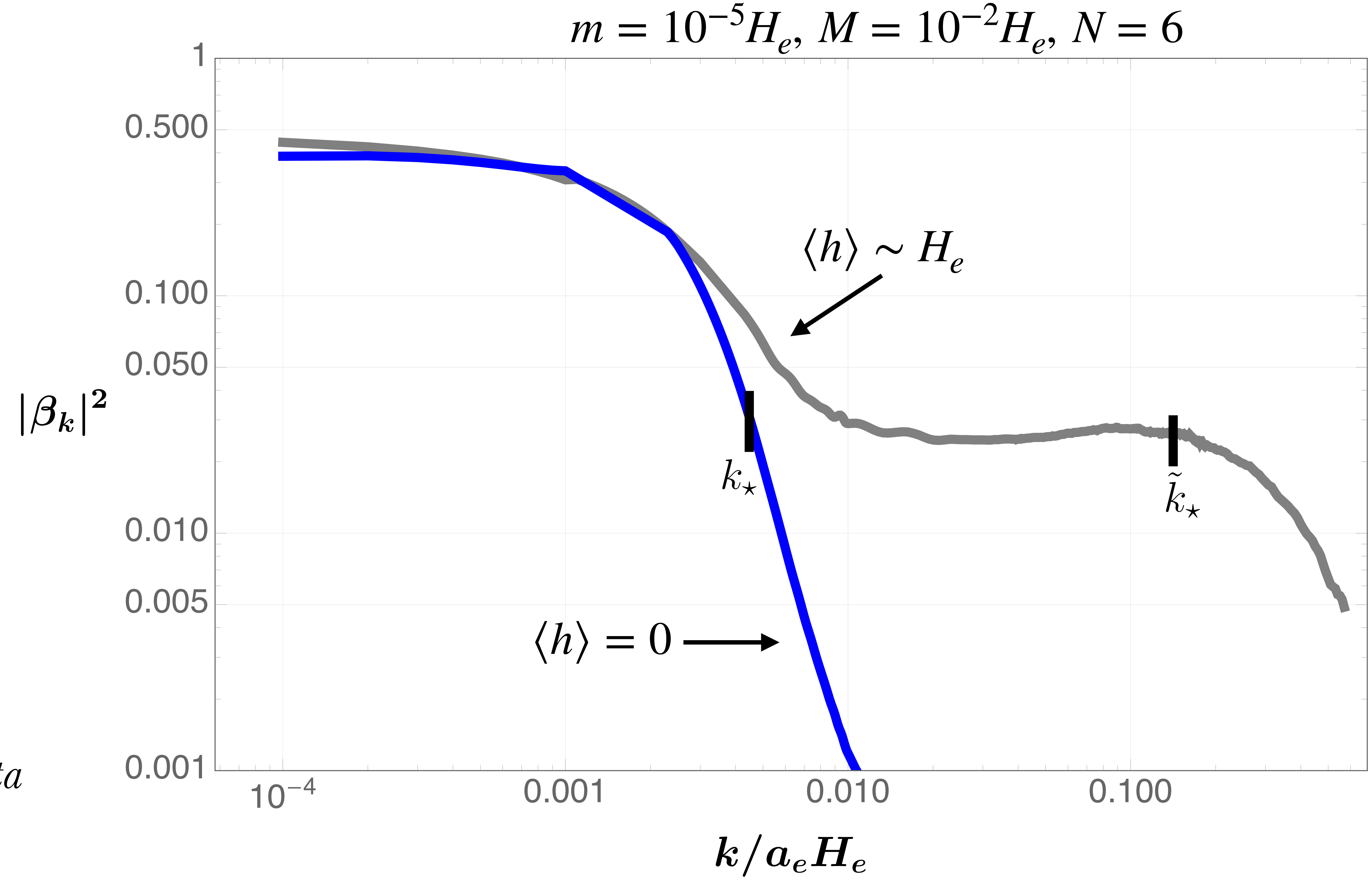
Cut-offs

$$k_* = \sqrt{2ma_e^2 H_e}$$

$$\tilde{k}_* = \sqrt{2Ma_e^2 H_e}$$

At $H(a_M) = M \rightarrow \tilde{k}_* = a_M M$
Only particles with physical 3-momenta $\tilde{k}/a < M$ are created
Non-relativistic

Particle number



Step-function mass

Particle number

$$n \sim 4 \times \int \frac{d^3\mathbf{k}}{(2\pi)^3 a^3}\,\theta(\tilde{k}_* - k)\,|\beta_k|^2 \longrightarrow n \sim \frac{2}{3\pi^2}\,\tilde{k}_*^3\,\frac{1}{a^3}\,|\beta_k|^2 = \frac{\sqrt{2}}{3\pi^2} N^4 \frac{M^{7/2}}{H_e^{1/2}}\frac{a_e^3}{a^3}$$

Particle abudnance

$$Y = \frac{n}{s_{\rm SM}} \quad , \quad s_{\rm SM} = \frac{2\pi^2 g_*}{45} T^3$$

$$Y_{\nu_R}^{\rm step} = 10^{-3} \times N^4 \frac{M^{7/2}}{M_{\rm Pl}^{3/2} H_e^2}$$

DM observational constraint

$$Y \leq 4.4 \times 10^{-10}\,\frac{\rm GeV}{M}$$

$$M \sim 10^9\ {\rm GeV} \rightarrow \mathcal{Y}_R \sim 10^{-4}$$

Sterile Neutrino production during inflation is ***insignificant*** *unless it is very heavy. Also* $Y_{\nu_R}^{\rm step} \ll Y_{\nu_R}^{\rm Higgs} \ll Y_{\nu_R}^{\rm FI}$

Slow effective mass decrease

Particle number

$$n \sim \frac{1}{3\pi^2} N^3 M^3 \frac{a_e^3}{a^3}$$

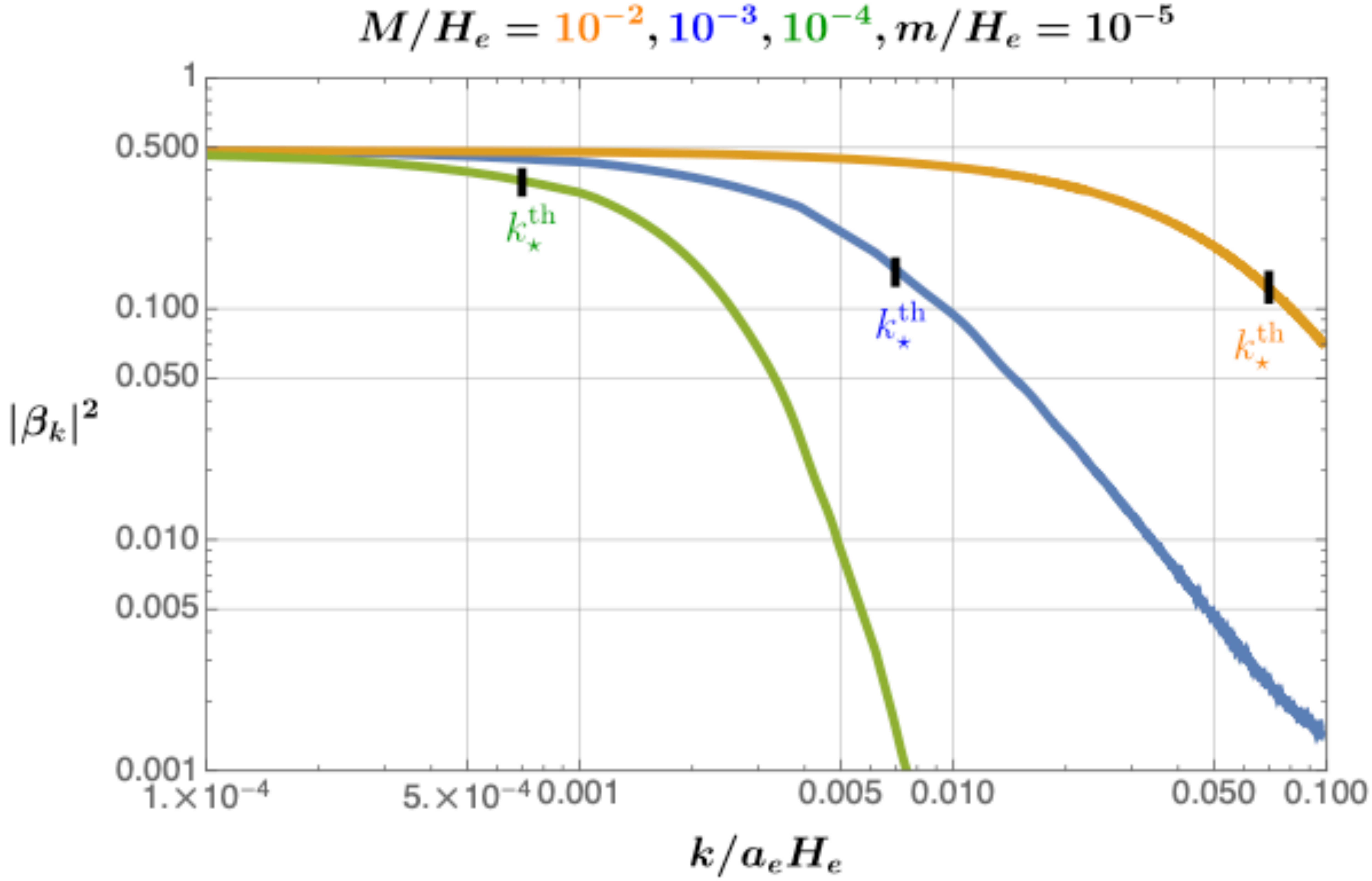
Thermal cut-off

$$k_*^{\rm th} = a_0\, M$$

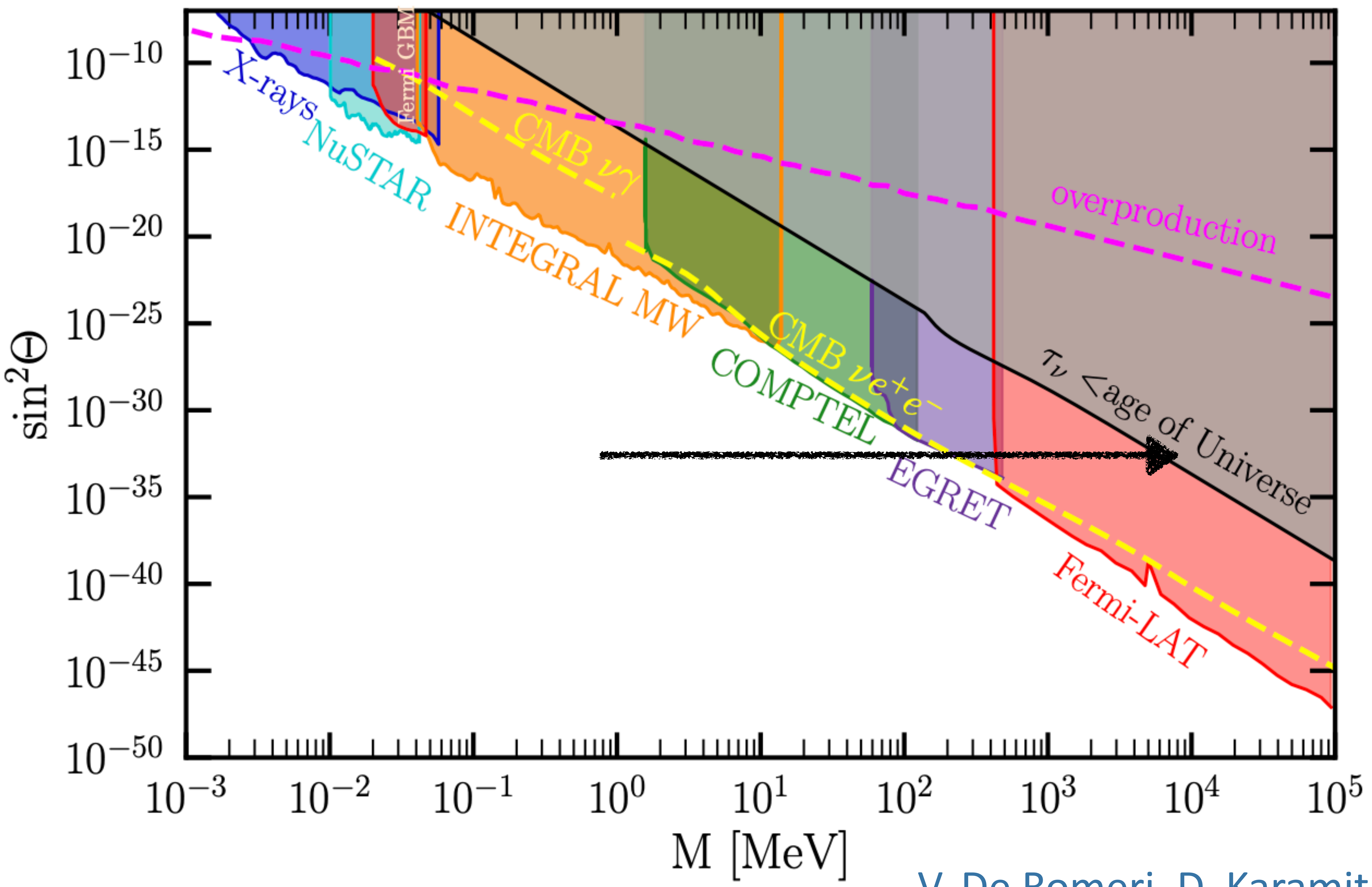
Particle abudnance

$$Y_{\nu_R}^{\rm slow} \sim 10^{-3} \times N^3 \frac{M^3}{(M_{\rm Pl} H_e)^{3/2}}$$

$$Y_{\rm fast} \sim Y_{\rm slow} \times N \sqrt{\frac{M}{H_e}} \ll Y_{\rm slow}$$



The constrains



V. De Romeri, D. Karamitros, O. Lebedev and T. Toma, JHEP 10 (2020) 137

POST-INFLATIONARY PRODUCTION

Oscillating period

- Inflation completes and the Inflaton field ϕ oscillates around its minimum

Classical time-dependent background which naturally leads to particle production

- Quantum (and classical) gravity effects induces gauge invariant **Planck-suppressed operators** among various fields including the Inflaton

Particle production takes place even in the absence of direct renormalizable couplings between the inflaton and other fields

- For $\phi_0 < M_{\rm Pl}$ after Inflation an EFT description is on giving in leading order the dim-5 operator

$$\boxed{\frac{\mathcal{C}}{M_{\rm Pl}}\phi^2\,\bar{\Psi}\Psi}$$

under $\phi \to -\phi$ and P conservation

~~$\phi\bar{\Psi}\Psi$~~ ~~$\phi^2\bar{\Psi}\gamma_5\Psi$~~

Oscillating period

- The Wilson coefficient C

1) Demands a complete quantum gravity theory, e.g. via an n-point function in string theory

W. Buchmuller, K. Hamaguchi, O. Lebedev and M. Ratz, Phys. Rev. Lett. 96 (2006)

2) Different structure compared to those generated by graviton exchange, no relation to $T_{\mu\nu}$

3) Dim-5 operator breaks CFT as classical gravity does but C ***is not*** *proportional to M*

Y. Ema, R. Jinno, K. Mukaida and K. Nakayama, Phys. Rev. D 94 (2016) 063517

- There are no general arguments to naturally suppress C and is treated as free parameter under $C \leq 1$

Oscillating period

Fermion production rate

$$\phi^2(t) = \sum_{n=-\infty}^{\infty} \zeta_n e^{-\mathrm{i}n\omega t}$$

ω is the oscillation frequency and ζ_n are slow functions of time

$$\underbrace{-i\int_{-\infty}^{\infty} dt \langle f|V(t)|i\rangle}_{\mathcal{M}} = -i\,\frac{\mathcal{C}}{M_{\mathrm{Pl}}}\,(2\pi)^4\delta(\mathbf{p}+\mathbf{q})\sum_{n=1}^{\infty}\zeta_n\,\delta(E_p+E_q-n\omega)\,\bar{u}v$$

$$\sum_{\mathrm{spin}}|\mathcal{M}_n|^2 = \frac{\mathcal{C}^2}{M_{\mathrm{Pl}}^2}\,2(n\,\omega)^2\,|\zeta_n|^2 \longrightarrow$$

$$\Gamma = \sum_{n=1}^{\infty}\Gamma_n = \sum_{n=1}^{\infty}\int\left(\sum_{\mathrm{spin}}|\mathcal{M}_n|^2\right)d\Pi$$

$$= \frac{\mathcal{C}^2}{4\pi M_{\mathrm{Pl}}^2}\omega^2\sum_{n=1}^{\infty}n^2\,|\zeta_n|^2$$

Oscillating period

Relic abundance

The particle density for Dirac fermions is found via the Boltzmann equation

$$\dot{n} + 3Hn = 2\Gamma \longrightarrow n\, a^3 = \int da\, a^2\, \frac{2\Gamma}{H}$$

Quadratic Inflaton potential

$$V = \frac{1}{2} m_\phi^2 \phi^2\,, \quad \phi(t) = \frac{\phi_0}{a^{3/2}}\cos m_\phi t$$

$$\Gamma = \frac{\mathcal{C}^2 m_\phi^2}{16\pi M_{\rm Pl}^2}\,\frac{\phi_0^4}{a^6} \qquad H = \frac{m_\phi \phi_0}{\sqrt{6} M_{\rm Pl}\, a^{3/2}} \qquad n(t) = \frac{\mathcal{C}^2 m_\phi}{2\sqrt{6}\pi M_{\rm Pl}}\,\frac{\phi_0^3}{a^3}$$

Defining the dilution factor $\Delta_{\rm NR} = \left(\frac{H_e}{H_R}\right)^{1/2} = a_R^{3/4} \simeq T_R^{\rm inst}/T_R$

$$Y_{\nu_R}^{\rm QG} = 10^{-1}\, \mathcal{C}^2\, \frac{H_e^{3/2}\, M_{\rm Pl}^{1/2}}{\Delta_{\rm NR}\, m_\phi^2}$$

Oscillating period

Constraints and implications

For $Y_{\nu_R}^{\rm QG} \lesssim Y$

$$\mathcal{C} \lesssim 10^{-4}\ \Delta_{\rm NR}^{1/2}\ \frac{M_{\rm Pl}^{3/4} H_e^{1/4}}{\phi_0} \sqrt{\frac{\rm GeV}{M}}$$

$H_e \sim 10^{-5} M_{\rm Pl}$ → $\phi_0 \sim 0.1 M_{\rm Pl}$

$$\mathcal{C} \leq 10^{-5}\ \Delta_{\rm NR}^{1/2} \sqrt{\frac{\rm GeV}{M}}$$

Unless $\Delta_{\rm NR} \gg 1$, $C \ll 1$ *for* $M \geq$ GeV *otherwise the Universe would be too dark*

Quartic Inflaton potential

$$V = \frac{1}{4}\lambda_\phi \phi^4 \qquad \omega \simeq 0.85\sqrt{\lambda_\phi}\phi_0/a \qquad \zeta_1 \simeq 0.25\phi_0^2/a^2$$

$$Y_{\nu_R}^{\rm QG} \simeq 1.2 \times 10^{-2}\ \mathcal{C}^2\ \frac{\phi_0^2}{M_{\rm Pl}^{3/2} H_e^{1/2}}$$

No dependence on reheating temperature!!! (*or* $\equiv \Delta_{\rm NR} = 1$)

Results

- For both cases $\mathcal{C}(M \sim \text{keV}) \simeq 10^{-2} - 10^{-1}$

*The QG dim-5 operator with a **small** Wilson coefficient can generate **all of the dark matter** even in the* keV *regime*

- SN are **cold DM** in contrast to the Dodelson-Widrow mechanism (E_{ν_R} related to $T_{\rm SM}$)

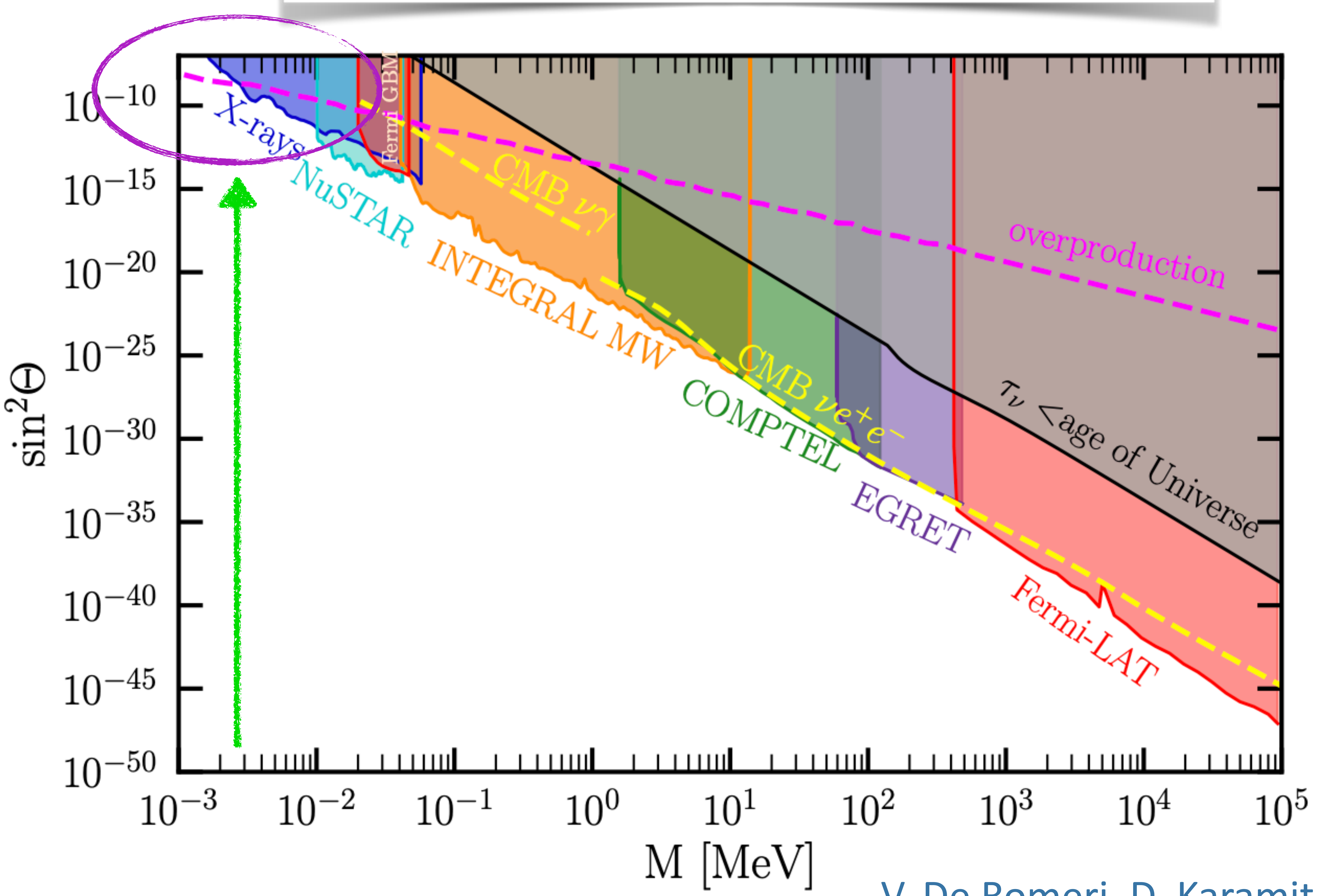
$$E_{\nu_R} \sim m_\phi \ll V^{1/4} \sim T_{\rm SM}$$

SN become non-relativistic at $T_{\rm SM} \gg M$ *and are **cold** at the stage of structure formation*

- The gravitational production mechanism is operative **irrespective** of the **active-sterile** mixing angle Θ

$\nu_R \to \nu_a \gamma$
$\nu_R \to \nu_a e^- e^+$
→ *X-ray and gamma ray emission as well as CMB distortion* → *Signatures of decaying dark matter as monochromatic photons*

The message



V. De Romeri, D. Karamitros, O. Lebedev and T. Toma, JHEP 10 (2020) 137

Conclusions

- Connection with GUTs: Sorry for that but not explicitly
- DM in the form of SN is produced **only** gravitationally via Inflaton **during** and **after** Inflation
- **During** Inflation the relic density of the produced (cold) Dirac SN, even though enhanced due to the Higgs condensate, is extremely suppressed for $M \ll 6 \times 10^9$ GeV
- At the **post**-inflationary regime **quantum gravity** operators break conformal invariance and couple the SN with the Inflaton. At leading order a dim-5 operator with a relatively small Wilson coefficient can generate **all of the dark matter** of the Universe even in the keV regime with observable signatures
- Future plans: Connection with baryogenesis via leptogenesis, further investigation of majorana SN production under the presence of a spectator scalar field and extension to vector field case among others

THANK YOU!!!