# Gravitational transition radiation: a new gravitational wave production mechanism

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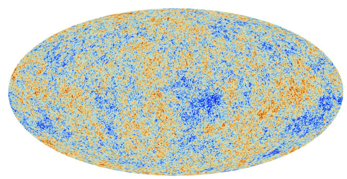
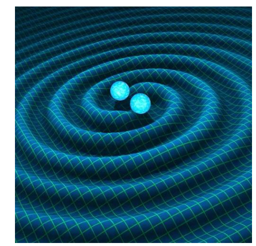
**GUTPC, HIAS (April 9-14, 2026)**

# Why GWs are important?

eV MeV Energy scale of the early Universe







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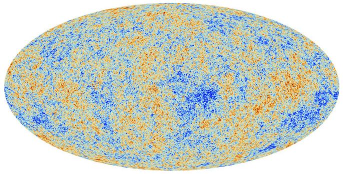
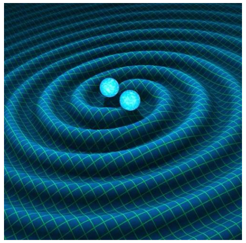
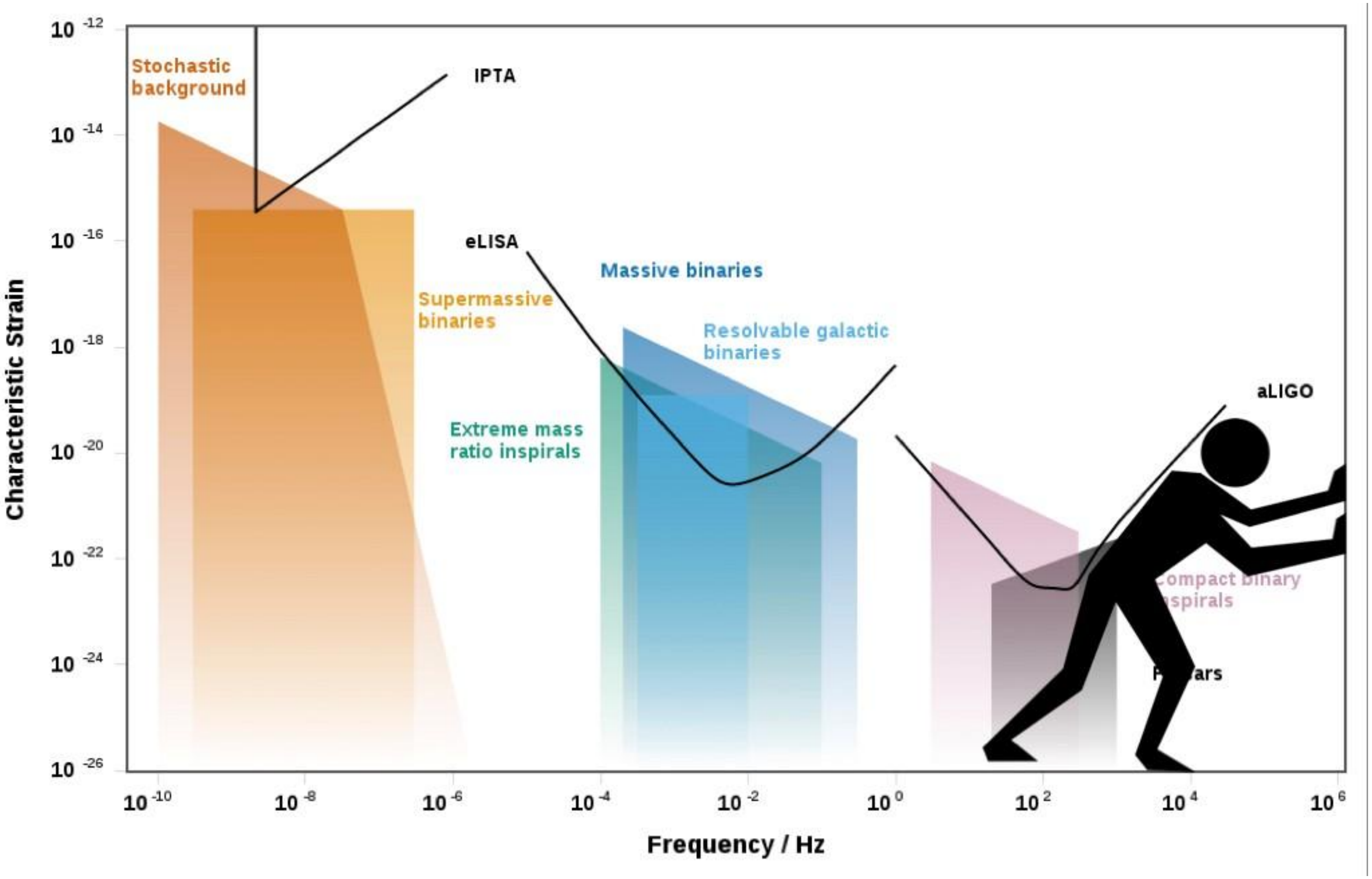


GWs as a probe of the early Universe → Need to understand all production channels

# Pushing to the high-freqency range of GWs

No known astro foregrounds → likely new physics (except minimal CGMB)




[Ultra-High-Frequency Gravitational Waves Initiative]

# Plan

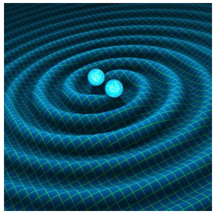
- GWs from graviton production
- Gravitational transition radiation: new high-frequency GWs from bubble expansion
- Outlook and summary

# GWs from graviton production

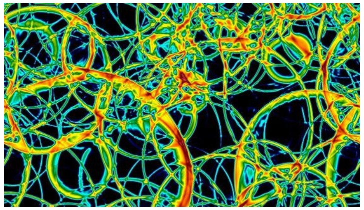
# Representative GW sources (macroscopic)

- Compact binary coalescences

  supermassive BHs, regular BHs, compact stars



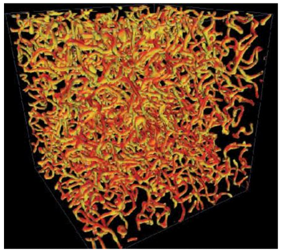
[NASA]



[David J. Weir '18]

- Cosmic first-order phase transitions (FOPTs)

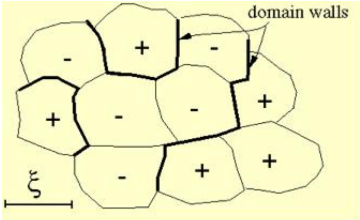
  bubble collisions, sound waves, turbulence



[Paul Shellard]

- Cosmic strings

- Cosmic domain walls



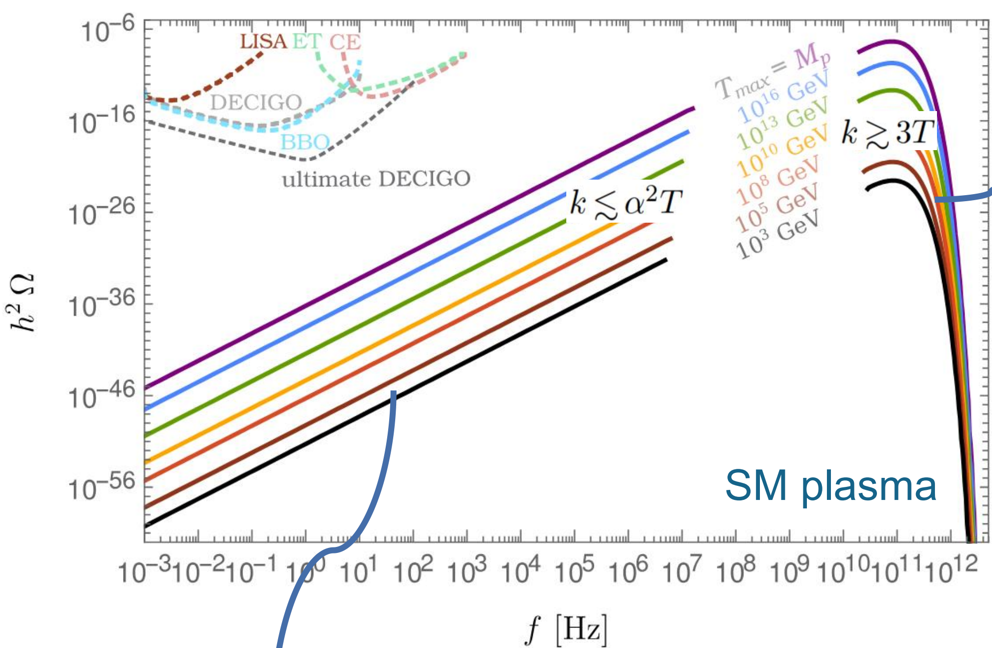

[CTC]

# GW sources (graviton production)

- Cosmic gravitational microwave background

[Ghiglieri & Laine ‘15; Ghiglieri, Jackson, Laine & Zhu ‘20; Ringwald, Schütte-Engel & Tamarit ‘20; Ghiglieri, Schütte-Engel & Speranza ‘24]

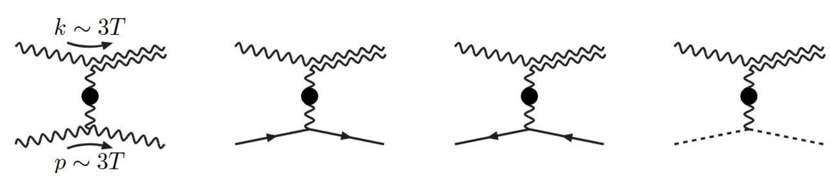
may be thought of as graviton freeze-in




[Ringwald, Schütte-Engel & Tamarit ‘20]

$$\frac{\mathrm{d}\rho_{\mathrm{GW}}}{\mathrm{d}t\,\mathrm{d}^3\mathbf{k}} = \frac{4\pi G}{(2\pi)^3}\sum_{\lambda}\epsilon^{\mathrm{TT}(\lambda)}_{ij,\mathbf{k}}\,\epsilon^{\mathrm{TT}(\lambda)*}_{mn,\mathbf{k}}\int_{\mathcal{X}} e^{i\mathcal{K}\cdot\mathcal{X}}\langle\, T^{ij}(0)\,T^{mn}(\mathcal{X})\,\rangle$$

particle scatterings, strongly interacting particles dominate, Boltzmann suppressed for very large graviton energy

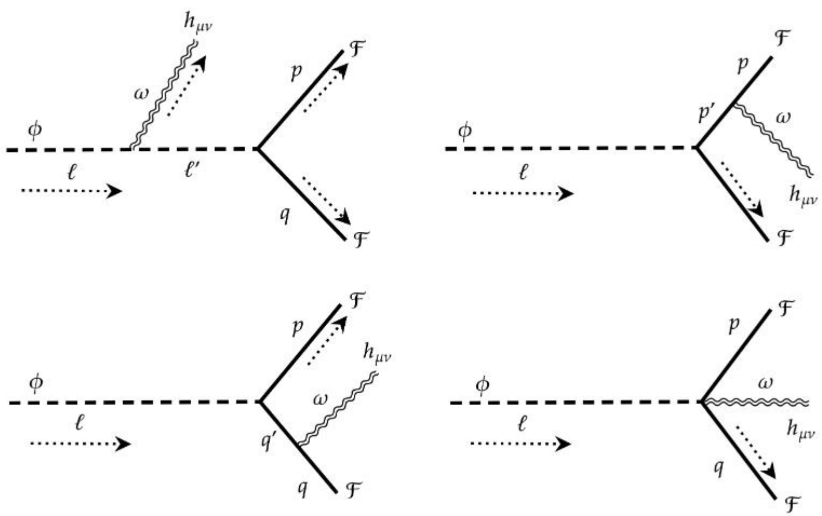



[Ghiglieri & Laine ‘15]

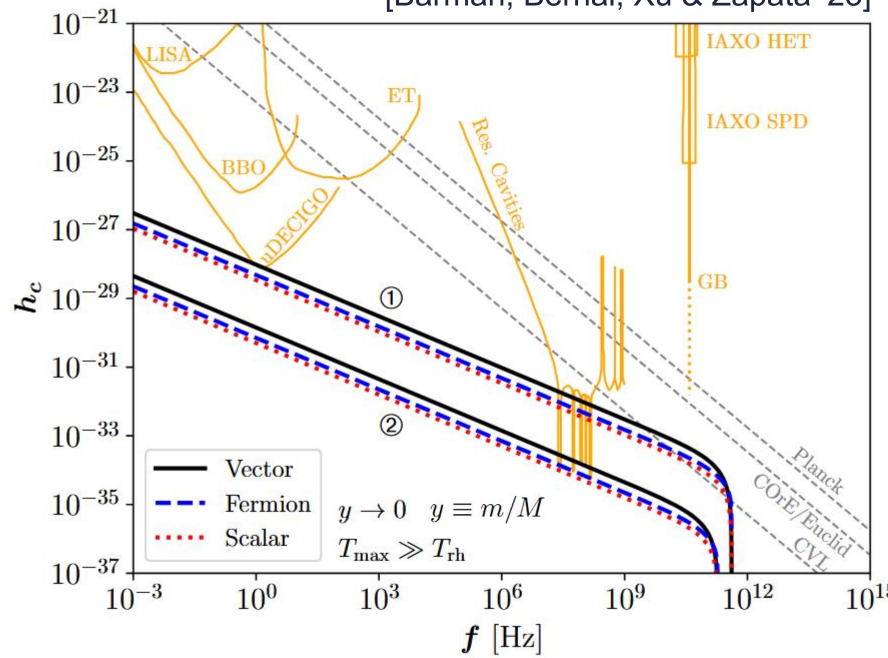
hydrodynamic fluctuations, inversly proportional to cross section, weakly interacting particles dominate

# GW sources (graviton production)

- Gravitational bremsstrahglung [Nakayama & Tang '18; Barman, Bernal, Xu & Zapata '23; Kanemura & Kaneta '23]




[Barman, Bernal, Xu & Zapata '23]




$$h_c(f) = \frac{1}{f}\sqrt{\frac{3\,H_0^2\,\Omega_{\rm GW}(f)}{2\pi^2}} = 1.26\times 10^{-18}\left(\frac{\rm Hz}{f}\right)\sqrt{h^2\,\Omega_{\rm GW}(f)}$$

① $M = M_P/10,\ T_{\rm rh} = 5.5\times 10^{15}$ GeV

② $M = M_P/10^3,\ T_{\rm rh} = M_P/(2\times 10^4)$

*See also: Phys.Lett.B 853 (2024) 138695 (Anna Tokareva)

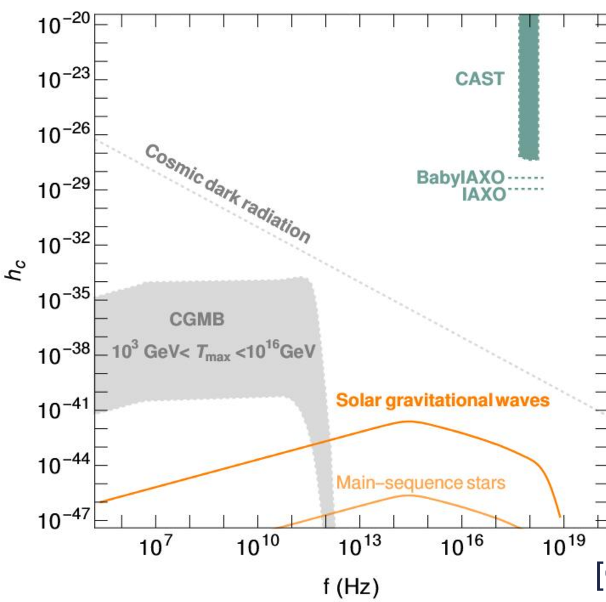
# GW sources (graviton production)

- Graviton emission from PBH evaporation [Dolgov, Naselsky & Noviko ‘00; Anantua, Easther & Giblin ‘09; Dolgov & Ejlli ‘11]


[Generated by Gemini]

- Graviton emission from the Sun

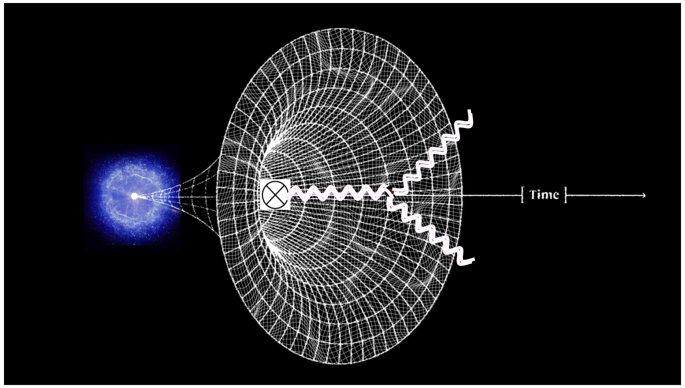
[Weinberg ‘65, García-Cely & Ringwald ‘24]




[García-Cely & Ringwald ‘24]

- Tensor perturbations from inflation

may be thought of as graviton pair production




[Adapted from Ben Gibson]

# Gravitational transition radiation: new high-frequency GWs from FOPTs

# Electromagnetic transition radiation (EMTR)

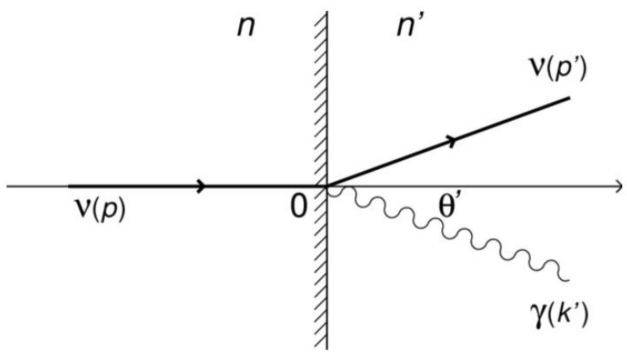
## Transition radiation

文A 8 languages

Article Talk Read Edit View history Tools

From Wikipedia, the free encyclopedia

**Transition radiation** (**TR**) is a form of electromagnetic radiation emitted when a charged particle passes through inhomogeneous media, such as a boundary between two different media. This is in contrast to Cherenkov radiation, which occurs when a charged particle passes through a homogeneous dielectric medium at a speed greater than the phase velocity of electromagnetic waves in that medium.




EMTR can also occur if the particle changes charge across the boundary

Is there a gravitational analogue?

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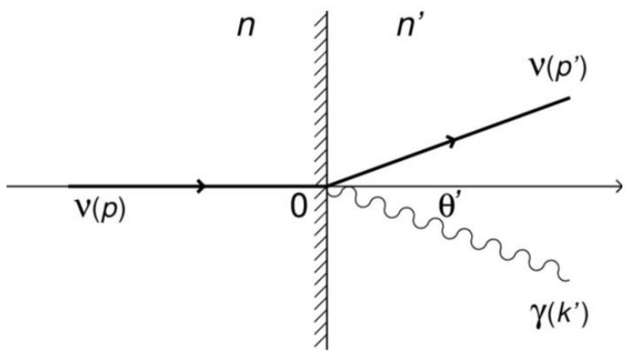


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Is there a gravitational analogue? → mass change across a boundary

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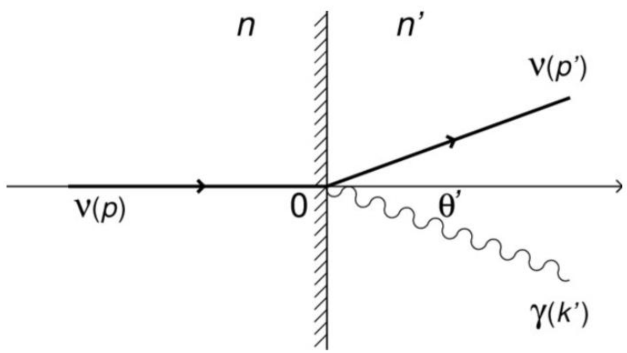
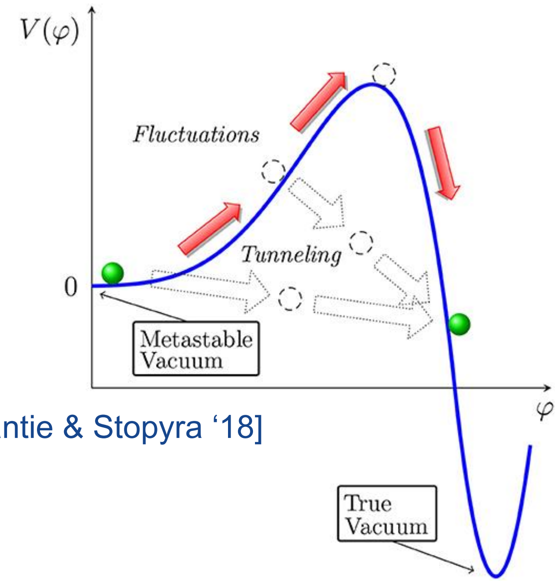



EMTR can also occur if the particle changes charge across the boundary

Is there a gravitational analogue? ➡ mass change across a boundary

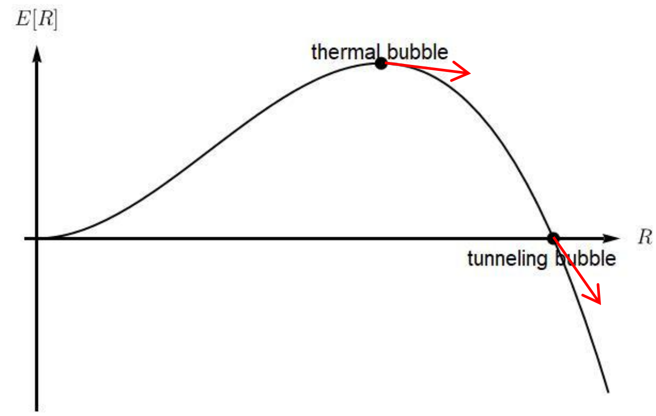
**Naturally occurs near a bubble wall (or domain wall)!**

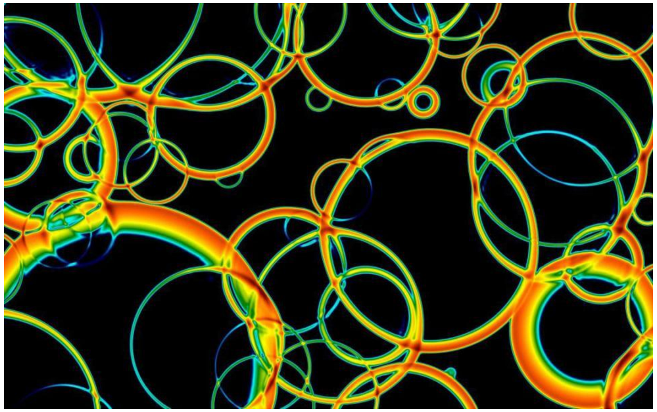
# Bubble nucleation and expansion




[Markkanen, Rajantie & Stopyra '18]

$$E(R) = 4\pi R^2\sigma - 4\pi R^3\epsilon/3$$



[David Weir]

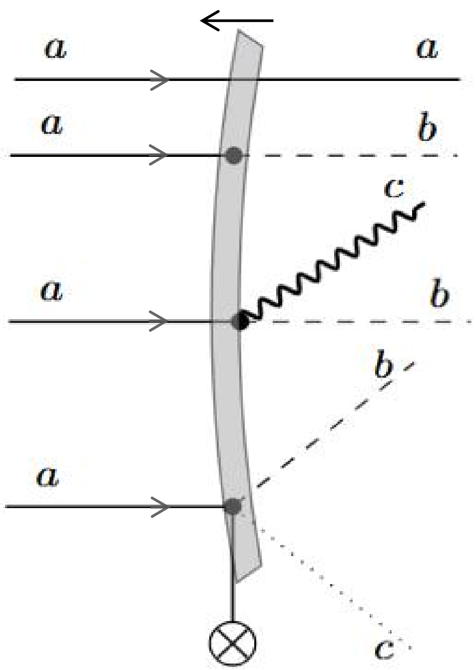
How fast the bubble can expand?

# Bubble walls in the ultra-relativistic limit

Generally, bubble wall dynamics is super complicated...

hydrodynamics, kinetic theory (needs to solve Boltzmann equations)...

But if the bubble is super fast (ultra-relativistic), it becomes simple.




- ☐ 1-to-1 elementary transition

  $\mathcal{P}_{\text{friction}} \propto \gamma_w^0$ [Bodeker & Moore '09]

- ☐ 1-to-1 mixing transition

  $\mathcal{P}_{\text{friction}} \propto \gamma_w^0$ [Azatov & Vanvlasselaer '20]

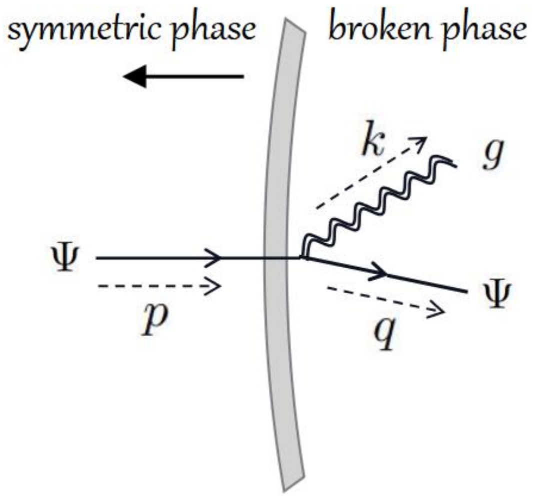
- ☐ **Transition radiation**

  $\mathcal{P}_{\text{friction}} \propto \gamma_w$ [Bodeker & Moore '17; Gouttenoire, Jinno & Sala '21 ]

- ☐ 1-to-2 scalar particle production

  $\mathcal{P}_{\text{friction}} \propto \log(1 + \#\gamma_w)$ [Baldes, Gouttenoire & Sala '21; WA '23]

# Gravitational transition radiation




(in the rest frame of the wall)

$$\text{incoming } \Psi : \quad p = (p^0, \mathbf{p}_\perp, p^z)$$
$$\text{outgoing } \Psi : \quad q = (q^0, \mathbf{q}_\perp, q^z)$$
$$\text{graviton } g : \quad k = (k^0, \mathbf{k}_\perp, k^z)$$

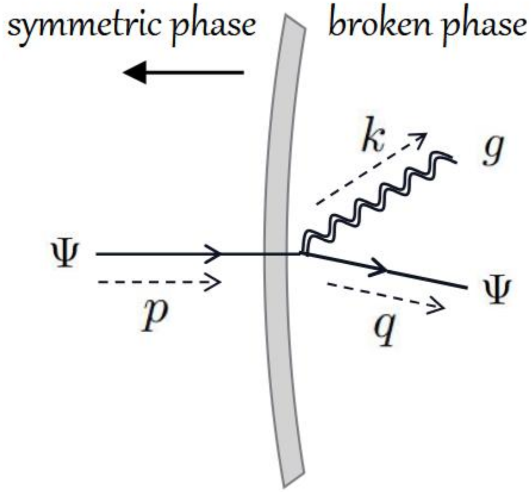
produced graviton number density in the plasma frame:

$$n_{\mathrm{g}}^{\mathrm{gen}} = \underbrace{\frac{1}{v_w \gamma_w}}_{\text{wall frame to plasma frame}} \underbrace{\int \frac{\mathrm{d}^3\mathbf{p}_s}{(2\pi)^3} \frac{p_s^z}{p^0} f_{\Psi;s}^{(\mathrm{wall})}(\mathbf{p}_s, T)}_{\text{flux of } \Psi \text{ particles}} \times \underbrace{\mathrm{d}\mathbb{P}_{\Psi\to\Psi+g}}_{\text{differential probability}}$$

where

$$\mathrm{d}\mathbb{P}_{\Psi\to\Psi+g} = \frac{1}{2p_s^z} \int \frac{\mathrm{d}^3\mathbf{q}_b}{(2\pi)^3 2q^0} \int \frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^3 2k^0} |\mathcal{M}|^2 \, (2\pi)^3 \delta(p^0 - q^0 - k^0) \delta^2(\mathbf{p}_\perp - \mathbf{q}_\perp - \mathbf{k}_\perp)$$

# Gravitational transition radiation

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we'll discuss it carefully later

# Graviton energy density

GW energy density (at production) in the plasma frame

$$\rho_{\rm GW}^{\rm gen} = \frac{1}{v_w \gamma_w} \int \frac{\mathrm{d}^3 \mathbf{p}_s}{(2\pi)^3} \frac{p_s^z}{p^0} f_{\Psi;s}^{(\rm wall)}(\mathbf{p}_s, T) \times \frac{1}{2p_s^z} \int \frac{\mathrm{d}^3 \mathbf{q}_b}{(2\pi)^3 2q^0} \int \frac{\mathrm{d}^3 \mathbf{k}}{(2\pi)^3 2k^0}$$
$$\times \tilde{k}^0 \times |\mathcal{M}|^2 (2\pi)^3 \delta(p^0 - q^0 - k^0) \delta^2(\mathbf{p}_\perp - \mathbf{q}_\perp - \mathbf{k}_\perp)$$

$\tilde{k}^0 \equiv \gamma_w (k^0 - v_w k^z) \approx k^0 / 2\gamma_w$ (graviton energy in the plasma frame)

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After some simplification, we obtain:

$$\frac{{\rm d}\rho_{\rm GW}^{\rm gen}(k^0)}{{\rm d}\ln k^0} \approx \frac{\sqrt{2}T^2}{512\pi^3}\frac{1}{\gamma_w^2} \times \int {\rm d}|\mathbf{k}_\perp|\,|\mathbf{k}_\perp| \frac{(k^0)^2}{\sqrt{(k^0)^2 - |\mathbf{k}_\perp|^2}}\frac{\left|\mathcal{M}(k^0, |\mathbf{k}_\perp|)\right|^2}{\sqrt{G(k^0, |\mathbf{k}_\perp|)}}$$

where

$$G(k^0, |\mathbf{k}_\perp|) = \left(\frac{3\gamma_w T}{2} - k^0\right)^2 - |\mathbf{k}_\perp|^2 - m_{\Psi,b}^2$$

with kinematic constraint:

$$\begin{cases} k^0 \leq \dfrac{3\gamma_w T}{2} - m_{\Psi,b}\,, \\ 0 \leq |\mathbf{k}_\perp| \leq \min\left\{k^0, \sqrt{\left(\dfrac{3\gamma_w T}{2} - k^0\right)^2 - m_{\Psi,b}^2}\right\} \end{cases}$$

[Bodeker & Moore ‘17]

# Bödeker-Moore method on computing $\mathcal{M}$

An example: $-\mathcal{L} \supset v(z)\,\chi_1\chi_2\chi_3$

Free-field quantization:

$$\chi_1^{(\mathrm{free})}(x) = \int \frac{\mathrm{d}^3\mathbf{p}_s}{(2\pi)^3\,2p^0}\left(\hat{a}_{\mathbf{p}_s}\,\zeta_{\mathbf{p}_s}(z)\,\mathrm{e}^{-\mathrm{i}\left(p^0 t - \mathbf{p}_\perp\cdot\mathbf{x}_\perp\right)} + \hat{a}^\dagger_{\mathbf{p}_s}\,\zeta^*_{\mathbf{p}_s}(z)\,\mathrm{e}^{\mathrm{i}\left(p^0 t - \mathbf{p}_\perp\cdot\mathbf{x}_\perp\right)}\right)$$

$$\Rightarrow \quad \mathrm{i}\mathcal{M}_{\chi_1(p)\to\chi_2(q)\chi_3(k)} = (-\mathrm{i})\int \mathrm{d}z\,V(z;\mathbf{p}_s,\mathbf{q}_b,\mathbf{k}_b)\,\zeta_{\mathbf{p}_s}(z)\zeta^*_{\mathbf{q}_b}(z)\zeta^*_{\mathbf{k}_b}(z)$$

[Bodeker & Moore '17]

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---

Mode function satisfies $\left[-(p^0)^2 + \mathbf{p}_\perp^2 - \partial_z^2 + m^2_{\chi_1}(z)\right]\zeta_{\mathbf{p}_s}(z) = 0$

[Bodeker & Moore ‘17]

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WKB regime: $p^0, q^0, k^0 \sim \gamma_w T \gg \dfrac{1}{L_w} \sim T$

[Bodeker & Moore '17]

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WKB regime: $p^0, q^0, k^0 \sim \gamma_w T \gg \dfrac{1}{L_w} \sim T$

$$\zeta_{\mathbf{p}_s}(z) \simeq \sqrt{\frac{p_s^z}{p^z(z)}}\, \mathrm{e}^{\mathrm{i}\int_0^z \mathrm{d}z'\, p^z(z')},$$

$$\zeta_{\mathbf{q}_b}(z) \simeq \sqrt{\frac{q_b^z}{q^z(z)}}\, \mathrm{e}^{\mathrm{i}\int_0^z \mathrm{d}z'\, q^z(z')} \text{ (similarly for } \mathbf{k}_b)$$

$$p^z(z') = \sqrt{(p^0)^2 - \mathbf{p}_\perp^2 - m_{\chi_1}^2(z')} \approx p^0 - \frac{1}{2} \frac{\mathbf{p}_\perp^2 + m_{\chi_1}^2(z')}{p^0}$$

[Bodeker & Moore '17]

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$$\zeta_{\mathbf{p}_s}(z) \simeq \sqrt{\frac{p_s^z}{p^z(z)}}\,\mathrm{e}^{\mathrm{i}\int_0^z \mathrm{d}z'\, p^z(z')},$$

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where $x = q^0/p^0$

[Bodeker & Moore '17]

# Bödeker-Moore method on computing $\mathcal{M}$

An example: $-\mathcal{L} \supset v(z)\,\chi_1\chi_2\chi_3$

Free-field quantization:

$$\chi_1^{(\text{free})}(x) = \int \frac{\mathrm{d}^3\mathbf{p}_s}{(2\pi)^3\,2p^0}\left(\hat{a}_{\mathbf{p}_s}\,\zeta_{\mathbf{p}_s}(z)\,\mathrm{e}^{-\mathrm{i}\left(p^0 t-\mathbf{p}_\perp\cdot\mathbf{x}_\perp\right)} + \hat{a}^\dagger_{\mathbf{p}_s}\,\zeta^*_{\mathbf{p}_s}(z)\,\mathrm{e}^{\mathrm{i}\left(p^0 t-\mathbf{p}_\perp\cdot\mathbf{x}_\perp\right)}\right)$$

$$\Rightarrow \quad \mathrm{i}\mathcal{M}_{\chi_1(p)\to\chi_2(q)\chi_3(k)} = (-\mathrm{i})\int \mathrm{d}z\, V(z;\mathbf{p}_s,\mathbf{q}_b,\mathbf{k}_b)\,\zeta_{\mathbf{p}_s}(z)\zeta^*_{\mathbf{q}_b}(z)\zeta^*_{\mathbf{k}_b}(z)$$

---

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where $x = q^0/p^0$

$$\int \mathrm{d}z'\,A(z') \approx \Theta(-z)A_s\,z + \Theta(z)A_b\,z$$

Similarly, $V(z) \approx \Theta(-z)V_s + \Theta(z)V_b$

[Bodeker & Moore '17]

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Similarly, $V(z) \approx \Theta(-z)V_s + \Theta(z)V_b$

$$\Rightarrow\quad \boxed{\mathrm{i}\mathcal{M} = -\mathrm{i}\left[2\mathrm{i}p^0\left(\frac{V_b}{A_b}-\frac{V_s}{A_s}\right)\right]}$$

# Graviton-matter vertices

Compute

Scalar: $$-\delta\mathcal{L}_{gs} = \frac{\kappa}{2} h_{\mu\nu} \left\{ (\partial^\mu \chi)(\partial^\nu \chi) - \eta^{\mu\nu} \left[ \frac{1}{2}(\partial_\rho \chi)\partial^\rho \chi - \frac{1}{2} m_\chi^2(z)\chi^2 \right] \right\}$$

$$\Rightarrow \quad V^{(gs)}(z;p,q,k) \approx \frac{\kappa}{2}\epsilon^*_{\mu\nu}(k) \left\{ p^\mu q^\nu + p^\nu q^\mu - \eta^{\mu\nu} \left[ p \cdot q - m_\chi^2(z) \right] \right\}$$

graviton poloraisation sum $$\sum_{\lambda=\pm} \epsilon^{\lambda *}_{\mu\nu}(k)\epsilon^{\lambda}_{\alpha\beta}(k) = \frac{1}{2}\left( \hat{\eta}_{\mu\alpha}\hat{\eta}_{\nu\beta} + \hat{\eta}_{\mu\beta}\hat{\eta}_{\nu\alpha} - \hat{\eta}_{\mu\nu}\hat{\eta}_{\alpha\beta} \right)$$

$$\hat{\eta}_{\mu\nu} = \eta_{\mu\nu} - \frac{k_\mu \bar{k}_\nu + \bar{k}_\mu k_\nu}{k \cdot \bar{k}}$$ (watch out for gravitational Ward identiy)

---

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---

Fermion: $$V^{(gf)}(z; p, q, k) \approx \frac{\kappa}{8} \epsilon^*_{\mu\nu}(k) \bar{u}^{s'}(q) \left[ (p+q)^\mu \gamma^\nu + (p+q)^\nu \gamma^\mu - 2\eta^{\mu\nu} \left( \not{p} + \not{q} - 2m_\psi(z) \right) \right] u^s(p)$$

Vector: $$\begin{aligned} V^{(gA)}(z; p, q, k) \approx \frac{\kappa}{2} \epsilon^*_\beta(q) \epsilon^*_{\mu\nu}(k) \Big[ & \eta^{\mu\nu} \eta^{\alpha\beta} (p \cdot q - m_A^2(z)) - \eta^{\mu\nu} p^\beta q^\alpha + \eta^{\nu\alpha} p^\beta q^\mu - \eta^{\alpha\beta} p^\nu q^\mu + \eta^{\mu\beta} p^\nu q^\alpha \\ & - \eta^{\nu\alpha} \eta^{\mu\beta} (p \cdot q - m_A^2(z)) + \eta^{\nu\beta} p^\mu q^\alpha - \eta^{\alpha\beta} p^\mu q^\nu + \eta^{\mu\alpha} p^\beta q^\nu - \eta^{\nu\beta} \eta^{\mu\alpha} \left( p \cdot q - m_A^2(z) \right) \Big] \epsilon_\alpha(p) \end{aligned}$$

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Compute

Scalar: $$-\delta\mathcal{L}_{gs} = \frac{\kappa}{2} h_{\mu\nu}\left\{(\partial^\mu\chi)(\partial^\nu\chi) - \eta^{\mu\nu}\left[\frac{1}{2}(\partial_\rho\chi)\partial^\rho\chi - \frac{1}{2}m_\chi^2(z)\chi^2\right]\right\}$$

$$\Rightarrow\quad V^{(gs)}(z;p,q,k) \approx \frac{\kappa}{2}\epsilon^*_{\mu\nu}(k)\left\{p^\mu q^\nu + p^\nu q^\mu - \eta^{\mu\nu}\left[p\cdot q - \cancel{m_\chi^2(z)}\right]\right\}$$

graviton poloraisation sum $$\sum_{\lambda=\pm}\epsilon^{\lambda *}_{\mu\nu}(k)\epsilon^{\lambda}_{\alpha\beta}(k) = \frac{1}{2}\left(\hat\eta_{\mu\alpha}\hat\eta_{\nu\beta} + \hat\eta_{\mu\beta}\hat\eta_{\nu\alpha} - \hat\eta_{\mu\nu}\hat\eta_{\alpha\beta}\right)$$

$$\hat\eta_{\mu\nu} = \eta_{\mu\nu} - \frac{k_\mu\bar k_\nu + \bar k_\mu k_\nu}{k\cdot\bar k}$$ (watch out for gravitational Ward identiy)

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Vector: $$V^{(gA)}(z;p,q,k) \approx \frac{\kappa}{2}\epsilon^*_\beta(q)\epsilon^*_{\mu\nu}(k)\Big[\eta^{\mu\nu}\eta^{\alpha\beta}(p\cdot q - m_A^2(z)) - \eta^{\mu\nu}p^\beta q^\alpha + \eta^{\nu\alpha}p^\beta q^\mu - \eta^{\alpha\beta}p^\nu q^\mu + \eta^{\mu\beta}p^\nu q^\alpha$$
$$- \eta^{\nu\alpha}\eta^{\mu\beta}(p\cdot q - m_A^2(z)) + \eta^{\nu\beta}p^\mu q^\alpha - \eta^{\alpha\beta}p^\mu q^\nu + \eta^{\mu\alpha}p^\beta q^\nu - \eta^{\nu\beta}\eta^{\mu\alpha}\left(p\cdot q - m_A^2(z)\right)\Big]\epsilon_\alpha(p)$$

$$\boxed{\mathrm{i}\mathcal{M} = -\mathrm{i}\left[2\mathrm{i}p^0\left(\frac{V_b}{A_b} - \frac{V_s}{A_s}\right)\right]}$$

# Gravitational power spectrum

(modified if there is a strong supercooling)

$$\Omega_{\rm GW}(f) = \frac{1}{\rho_c}\left(\frac{g_{\star,s}(T_0)}{g_{\star,s}(T)}\right)^{\frac{4}{3}}\left(\frac{T_0}{T}\right)^4 \frac{{\rm d}\rho_{\rm GW}^{\rm gen}(\tilde{k}^0)}{{\rm d}\ln\tilde{k}^0}\bigg|_{\tilde{k}^0=\left(\frac{g_{\star,s}(T)}{g_{\star,s}(T_0)}\right)^{\frac{1}{3}}\left(\frac{T}{T_0}\right)2\pi f}$$

We find

$$\boxed{\Omega_{\rm GW}(f)h^2 \simeq 3.6\times 10^{-10}\left(\frac{100}{g_{\star,s}(T)}\right)^{\frac{4}{3}}\left(\frac{m_{\chi,b}}{m_{\rm pl}}\right)^2 I\left(\frac{f}{T_0};g_{\star,s}(T)\right)}$$

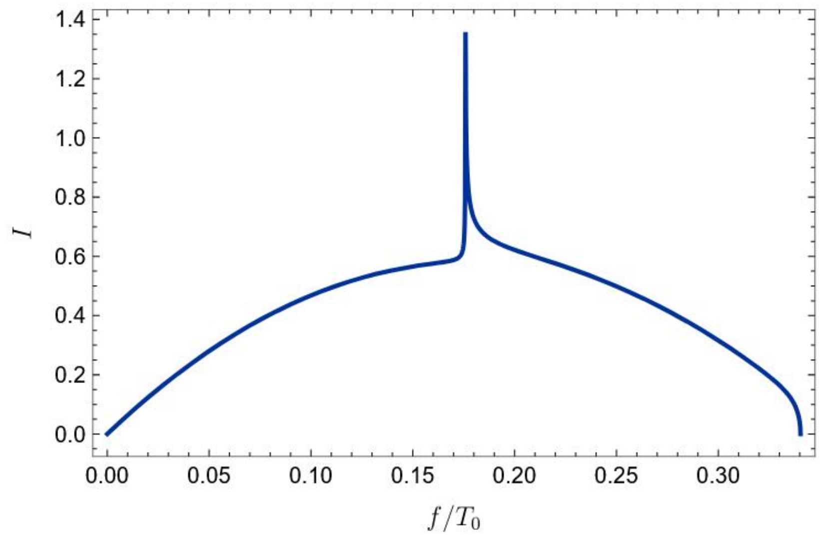
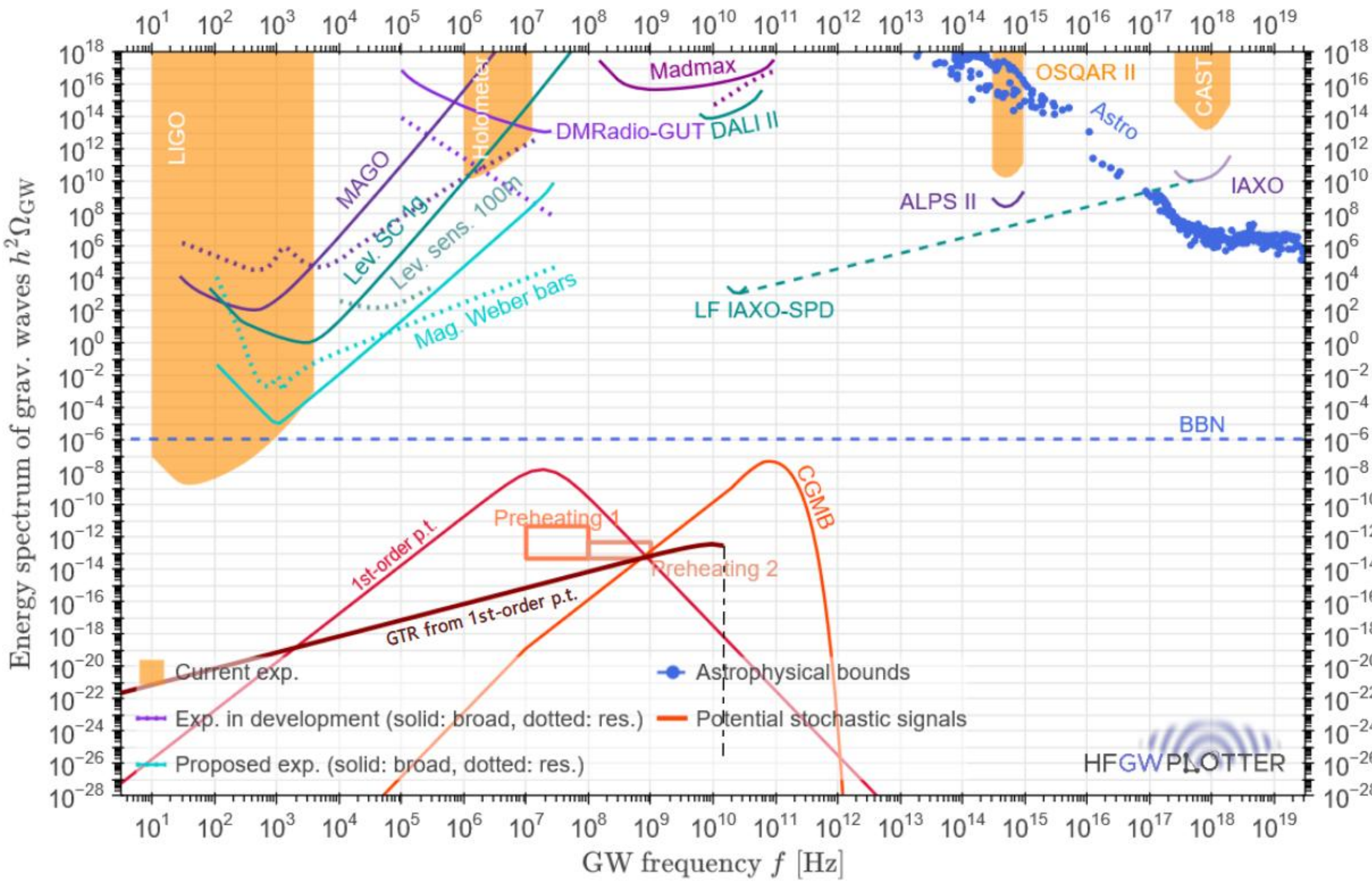



FIG. 2. The shape of the power spectrum for $g_{\star,s}(T)=100$. The peak frequency today is at $f_{\rm peak}\approx 0.176T_0$.

why spike? $\frac{{\rm d}\rho_{\rm GW}^{\rm gen}(k^0)}{{\rm d}\ln k^0} \approx \frac{\sqrt{2}T^2}{512\pi^3}\frac{1}{\gamma_w^2}\times\int {\rm d}|\mathbf{k}_\perp|\,|\mathbf{k}_\perp|\frac{(k^0)^2}{\sqrt{(k^0)^2-|\mathbf{k}_\perp|^2}}\frac{|\mathcal{M}(k^0,|\mathbf{k}_\perp|)|^2}{\sqrt{G(k^0,|\mathbf{k}_\perp|)}}$

# A benchmark point (subject to model building!)

$$T = 10^{14}\,\mathrm{GeV},\ \gamma_w = 500,\ m_{\chi,b} = 50\,T,$$
$$T_{\max} = m_{\rm pl},\ \beta/H = 20,\ \alpha_n = 1,\ v_w \to 1$$




[GW plotter: Muia, Ringwald &Tamarit]

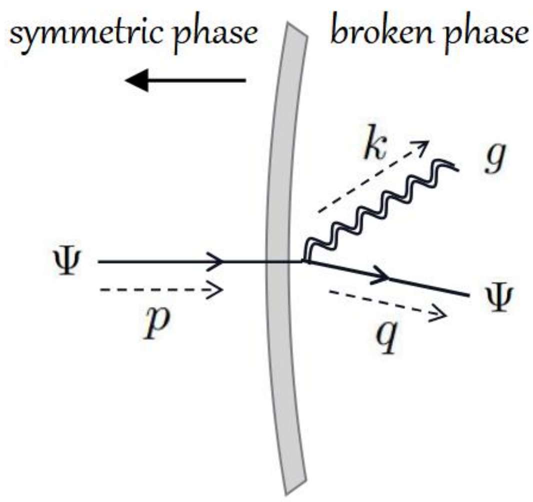
# Outlook and summary

# Outlook

- One can consider gravitational transition radiation from fermion and gauge bosons (enhanced GW power spectrum?)

- For domain walls, which are non-transient source, one might have more significant GWs from this mechanism

# Summary




- Gravitational transition radiation provides a mechanism of GW production. May deserve further investigation.

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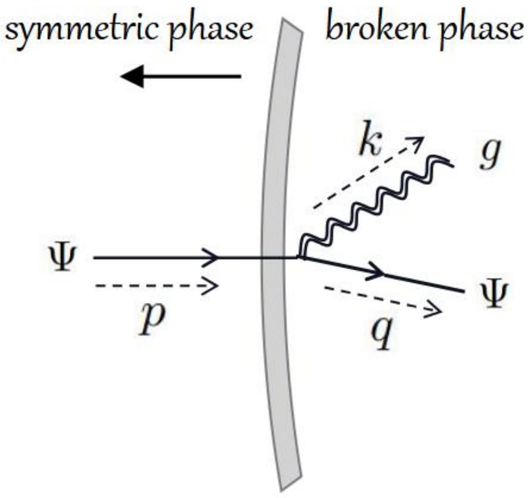


- Gravitational transition radiation provides a mechanism of GW production. May deserve further investigation.

# Thank you!