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Probing quark-lepton correlation in GUTs with high precision neutrino measurements

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arXiv: 2511.16196

Collaborate with Gao-Xiang Fang and Ye-Ling Zhou

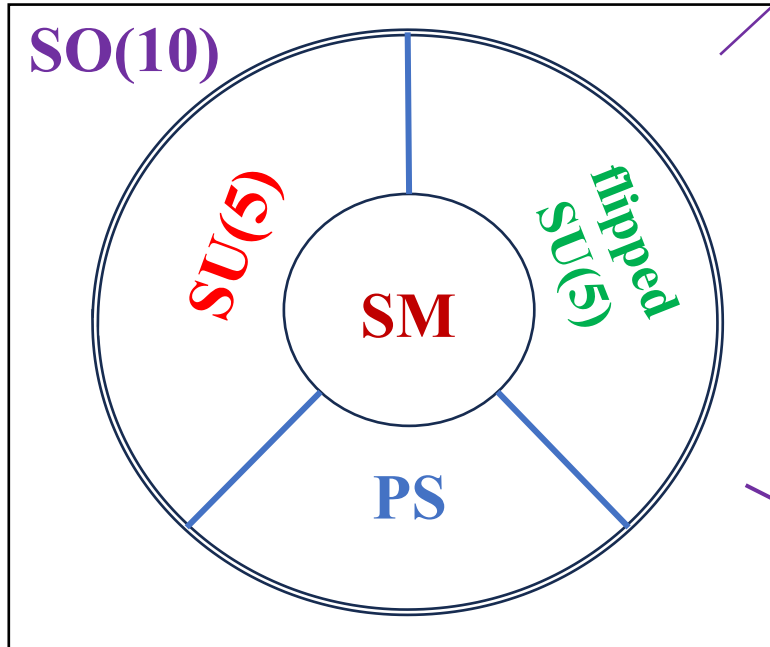
Contents

- **Introduction** Motivation in GUTs & Precision era of neutrino physics
- **Quark-Lepton correlation** unsuccessful & representative models: M1, M2, M3
- **Numerical Analysis** prediction & benchmarks
- **Conclusion**

Introduction: Motivation in GUTs

Grand unified theory (GUT)

- GUT groups

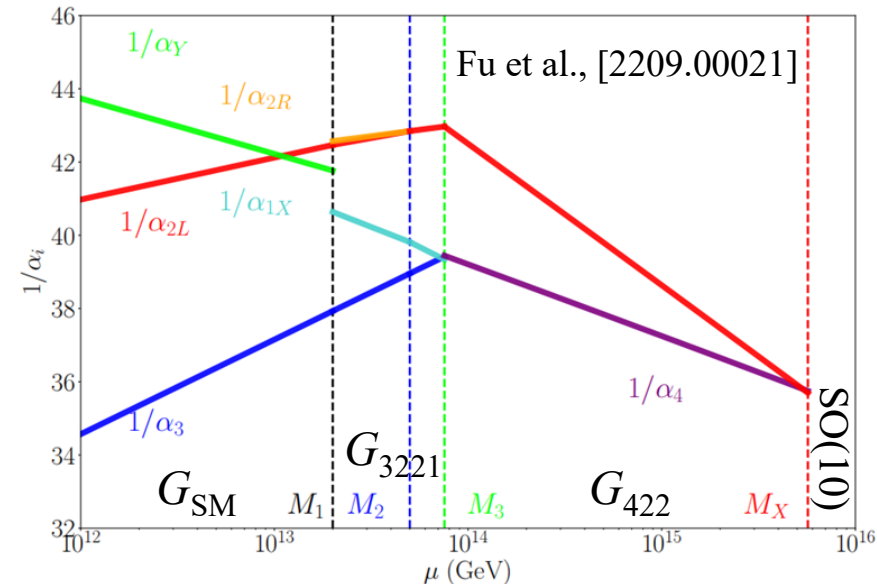


SO(10) GUT introduces:

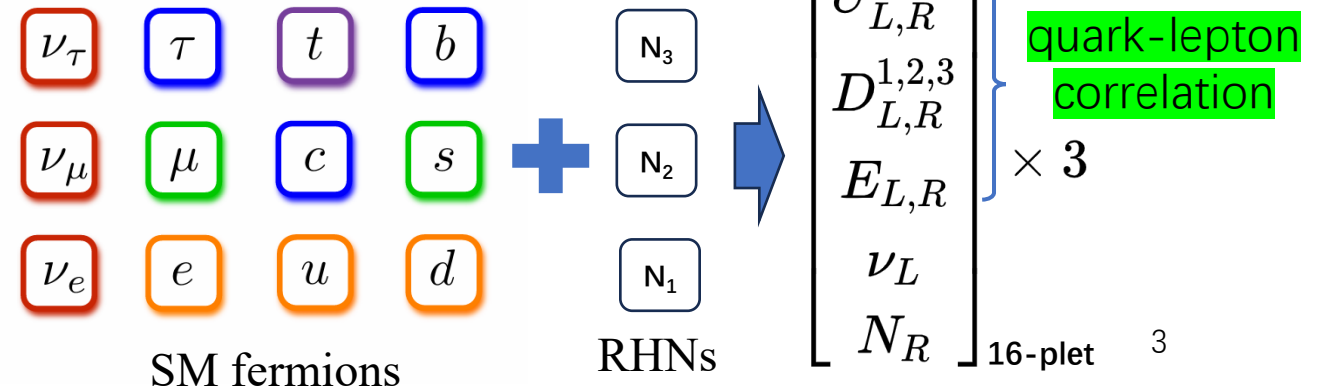
- Fermion 16-plet 16_F
- 126_H introduce $M_R \rightarrow$ Seesaw M_ν
- $10_H, 120_H, 126_H \rightarrow$ Fermion masses

Unifies gauge symmetries & couplings

$$SO(10) \supset G_{SM} \equiv SU(3)_C \times SU(2)_L \times U(1)_Y$$



Unifies the fermion matter



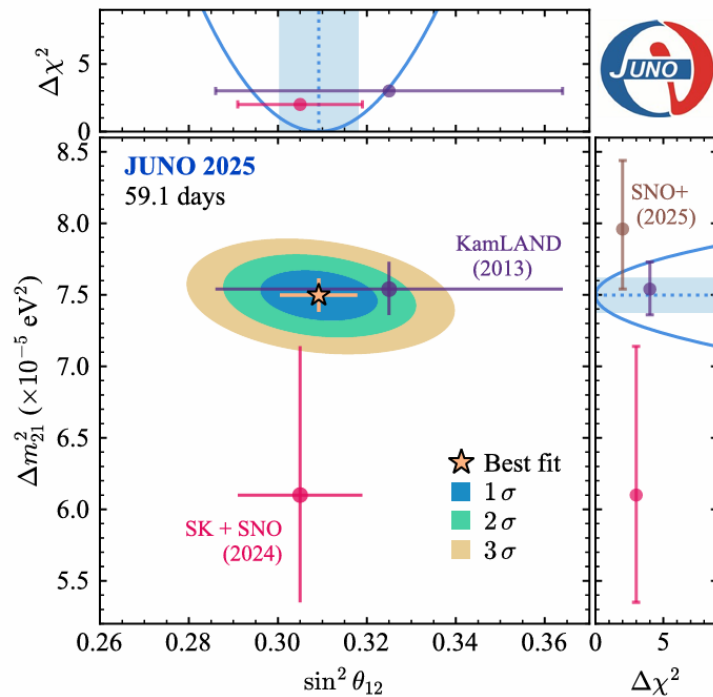
Introduction: Precision era of neutrino physics

JUNO data first release

$$\sin^2 \theta_{12} = 0.3092 \pm 0.0087$$

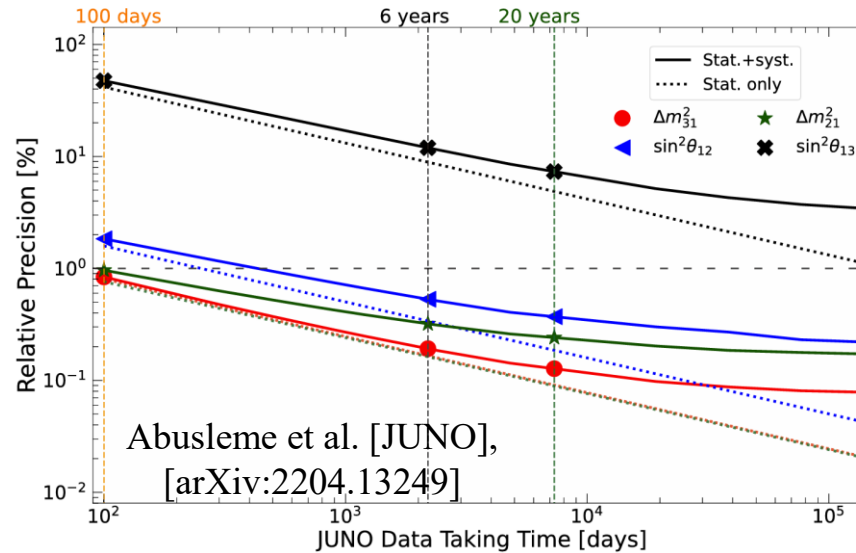
$$\Delta m_{21}^2 = (7.50 \pm 0.12) \times 10^{-5} \text{ eV}^2$$

59.1 days data 2511.14593



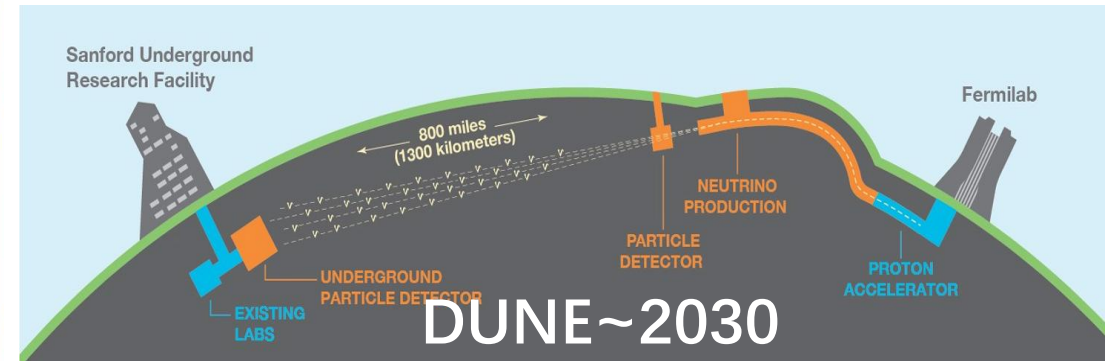
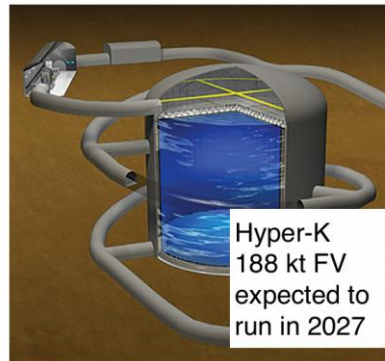
Precision improved by ~1.6
compare to all previous measurements !

For the future:



**Precision expectation:
1 order than today's**

- NO & IO ?
- CP δ ?
- θ_{23} octant ?



- Era of neutrino high precision measurements is coming !!!

Help us examine GUT models in lepton sector

Quark-Lepton correlation: General Yukawas

16-dim representation: $16_F = \{u_1^c, d_1^c, d_1, u_1, \nu^c, e^c, d_2, u_2, u_2^c, d_2^c, d_3, u_3, u_3^c, d_3^c, e, \nu\}_L$

$$16 \times 16 = 10_S + 126_S + 120_A \begin{cases} 10_H \supset (1, 2, 2) : \{h_{10}^u, h_{10}^d\} & \text{in PS (422): SM(321)} \\ 120_H \supset (1, 2, 2) + (15, 2, 2) : \{h_{120_1}^u, h_{120_1}^d, h_{120_{15}}^u, h_{120_{15}}^d\} \\ 126_H \supset (15, 2, 2) : \{h_{126}^u, h_{126}^d\} \end{cases}$$

GUT: $-\mathcal{L}_Y = 16_F [Y_{10} 10_H + Y_{126} \overline{126}_H + Y_{120} 120_H] 16_F + \text{h.c.}$ $+ \tilde{Y}_{10}^* 16_F 16_F 10_H^* + \tilde{Y}_{120}^* 16_F 16_F 120_H^*$

After getting their VEVs:

$$\{\langle h_{10}^u \rangle, \langle h_{10}^d \rangle, \langle h_{120_1}^u \rangle, \langle h_{120_1}^d \rangle\} = \{c_{10}^u, c_{10}^d, c_{120_1}^u, c_{120_1}^d\} \times \frac{v_{EW}}{\sqrt{2}}$$

$$\{\langle h_{126}^u \rangle, \langle h_{126}^d \rangle, \langle h_{120_{15}}^u \rangle, \langle h_{120_{15}}^d \rangle\} = \{c_{126}^u, c_{126}^d, c_{120_{15}}^u, c_{120_{15}}^d\} \times \sqrt{\frac{3}{2}} v_{EW}$$

Fermion masses

$$v_L = \langle \{\overline{10}, 3, 1\}_{126} \rangle$$

$$v_S = \langle \{\overline{10}, 1, 3\}_{126} \rangle$$

Neutrino masses



SM: $-\mathcal{L}_Y = Y_u \overline{Q} \tilde{h}_{SM} u_R + Y_d \overline{Q} h_{SM} d_R + Y_\nu \overline{L} \tilde{h}_{SM} \nu_R + Y_e \overline{L} h_{SM} e_R + \text{h.c.}$

$$Y_u = c_{10}^u Y_{10} + c_{126}^u Y_{126} + (c_{120_1}^u + c_{120_{15}}^u) Y_{120}$$

$$Y_d = c_{10}^d Y_{10} + c_{126}^d Y_{126} + (c_{120_1}^d + c_{120_{15}}^d) Y_{120}$$

$$Y_e = c_{10}^d Y_{10} - 3c_{126}^d Y_{126} + (c_{120_1}^d - 3c_{120_{15}}^d) Y_{120}$$

$$Y_\nu = c_{10}^u Y_{10} - 3c_{126}^u Y_{126} + (c_{120_1}^u - 3c_{120_{15}}^u) Y_{120}$$

$$M_L = v_L Y_{126}, \quad M_R = v_S Y_{126}$$

$$+ (c_{10}^d)^* \tilde{Y}_{10} + ((c_{120_1}^d)^* + (c_{120_{15}}^d)^*) \tilde{Y}_{120}$$

$$+ (c_{10}^u)^* \tilde{Y}_{10} + ((c_{120_1}^u)^* + (c_{120_{15}}^u)^*) \tilde{Y}_{120}$$

$$+ (c_{10}^u)^* \tilde{Y}_{10} + ((c_{120_1}^u)^* - 3(c_{120_{15}}^u)^*) \tilde{Y}_{120}$$

$$+ (c_{10}^d)^* \tilde{Y}_{10} + ((c_{120_1}^d)^* - 3(c_{120_{15}}^d)^*) \tilde{Y}_{120}$$

Quark-Lepton correlation

Forbidden by $U(1)_{PQ}$

K. S. Babu et al., [hep-ph/9209215]

B. Bajc et al., [hep-ph/0510139]

Type-I dominant: $M_\nu = -Y_\nu M_R^{-1} Y_\nu^T v^2 = m_0 Y_\nu Y_{126}^{-1} Y_\nu^T$

Quark-Lepton correlation: Unsuccessful Higgs contents

$$\begin{aligned}
 Y_u &= c_{10}^u Y_{10} + c_{126}^u Y_{126} + (c_{120_1}^u + c_{120_{15}}^u) Y_{120} \\
 Y_d &= c_{10}^d Y_{10} + c_{126}^d Y_{126} + (c_{120_1}^d + c_{120_{15}}^d) Y_{120} \\
 Y_e &= c_{10}^d Y_{10} - 3c_{126}^d Y_{126} + (c_{120_1}^d - 3c_{120_{15}}^d) Y_{120} \\
 Y_\nu &= c_{10}^u Y_{10} - 3c_{126}^u Y_{126} + (c_{120_1}^u - 3c_{120_{15}}^u) Y_{120} \\
 M_\nu &= -Y_\nu M_R^{-1} Y_\nu^T v^2 \quad (\text{We select } Y_{126} \text{ diagonal basis})
 \end{aligned}$$

Too many DoFs !

- A single Higgs contents**

10_H

120_H

$\overline{126}_H$

$O_u^\dagger O_d = \mathbf{I}$
No CKM!

$$\begin{aligned}
 Y_u &= c_{10}^u Y_{10}, \\
 Y_d &= c_{10}^d Y_{10}, \\
 Y_e &= c_{10}^d Y_{10}, \\
 Y_\nu &= c_{10}^u Y_{10},
 \end{aligned}$$

Degenerate masses!

$$\begin{aligned}
 Y_u &= (c_{120_1}^u + c_{120_{15}}^u) Y_{120}, \\
 Y_d &= (c_{120_1}^d + c_{120_{15}}^d) Y_{120}, \\
 Y_e &= (c_{120_1}^d - 3c_{120_{15}}^d) Y_{120}, \\
 Y_\nu &= (c_{120_1}^u - 3c_{120_{15}}^u) Y_{120},
 \end{aligned}$$

$Y_{126} \rightarrow M_R \checkmark$
Seesaw $M\nu \checkmark$

$$\begin{aligned}
 Y_u &= c_{126}^u Y_{126}, \\
 Y_d &= c_{126}^d Y_{126}, \\
 Y_e &= -3c_{126}^d Y_{126}, \\
 Y_\nu &= -3c_{126}^u Y_{126},
 \end{aligned}$$

$O_u^\dagger O_d = \mathbf{I}$
No CKM!

No $Y_{126} \rightarrow M_R$!
No Seesaw $M\nu$

Degenerate fermion masses, no flavor mixing, light neutrino massless



How much Higgs content is needed for a successful fit?



Minimal
One? Two?

A complete list
($10_H, \overline{126}_H, 120_H$)

Quark-Lepton correlation: Unsuccessful Higgs contents

- Two Higgs contents

$$10_H^{\mathbb{R},\mathbb{C}} + 120_H^{\mathbb{R},\mathbb{C}}$$

$$\begin{aligned} Y_u &= c_{10}^u Y_{10} + (c_{120_1}^u + c_{120_{15}}^u) Y_{120} \\ Y_d &= c_{10}^d Y_{10} + (c_{120_1}^d + c_{120_{15}}^d) Y_{120} \\ Y_e &= c_{10}^d Y_{10} + (c_{120_1}^d - 3c_{120_{15}}^d) Y_{120} \\ Y_\nu &= c_{10}^u Y_{10} + (c_{120_1}^u - 3c_{120_{15}}^u) Y_{120} \end{aligned}$$

**No $Y_{126} \rightarrow M_R!$
No Seesaw M_ν**

E. Witten, [PLB 91, 1980]

$$\left| \frac{c_{10}^{u*}}{c_{10}^d} \right| = 1 \approx \frac{m_c(m_\tau - m_b) - m_t(m_\mu - m_s)}{m_s m_\tau - m_\mu m_b} \approx \frac{m_t}{m_b}$$

**Nearly degenerate m_b & $m_t!$
Contradict with experiments.**

$$10_H^{\mathbb{C}} + \overline{126}_H^{\mathbb{C}}$$

B. Bajc et al., [hep-ph/0510139]

$$(c_{10}^u)^* \neq c_{10}^d$$

$$10_H^{\mathbb{R}} + \overline{126}_H^{\mathbb{C}}$$

$$\begin{aligned} Y_u &= c_{10}^{d*} Y_{10} + c_{126}^u Y_{126}, \\ Y_d &= c_{10}^d Y_{10} + c_{126}^d Y_{126}, \\ Y_e &= c_{10}^d Y_{10} - 3c_{126}^d Y_{126}, \\ Y_\nu &= c_{10}^{d*} Y_{10} - 3c_{126}^u Y_{126}, \end{aligned}$$



$$120_H^{\mathbb{R},\mathbb{C}} + \overline{126}_H^{\mathbb{R},\mathbb{C}}$$

$$\begin{aligned} Y_u &= c_{126}^u Y_{126} + (c_{120_1}^u + c_{120_{15}}^u) Y_{120}, \\ Y_d &= c_{126}^d Y_{126} + (c_{120_1}^d + c_{120_{15}}^d) Y_{120}, \\ Y_e &= -3c_{126}^d Y_{126} + (c_{120_1}^d - 3c_{120_{15}}^d) Y_{120}, \\ Y_\nu &= -3c_{126}^u Y_{126} + (c_{120_1}^u - 3c_{120_{15}}^u) Y_{120}, \end{aligned}$$

$\tan \beta$	Comment	χ_{NH}^2	χ_{IH}^2	
-	no RGE	103	910	} Non-SUSY
-	RGE	200	3859	
10	no RGE	247	1861	} SUSY
10	RGE	224	4358	

Unable fit all fermion masses, mixing
Dueck et al., [1306.4468]

Quark-Lepton correlation: Successful models

General fermion Yukawas and M_ν :

$$Y_u = c_{10}^u Y_{10} + c_{126}^u Y_{126} + (c_{120_1}^u + c_{120_{15}}^u) Y_{120}$$

$$Y_d = c_{10}^d Y_{10} + c_{126}^d Y_{126} + (c_{120_1}^d + c_{120_{15}}^d) Y_{120}$$

$$Y_e = c_{10}^d Y_{10} - 3c_{126}^d Y_{126} + (c_{120_1}^d - 3c_{120_{15}}^d) Y_{120}$$

$$Y_\nu = c_{10}^u Y_{10} - 3c_{126}^u Y_{126} + (c_{120_1}^u - 3c_{120_{15}}^u) Y_{120}$$

$$M_\nu = m_0 Y_\nu Y_{126}^{-1} Y_\nu^T$$

In Y_{126} real diagonal basis \rightarrow diagonal M_R

Input model parameters:

Model	Main features	n_{para}
M1	Only Y_{10}, Y_{126}	19
M2	Real Y_{10}, Y_{126}, Y_{120}	19
M3	Hermitian Y_u, Y_d, Y_e, Y_ν	18

Comparable \checkmark

In sum 18 Observables:

- 6 quark yukawa
- 3 lepton yukawa
- 3+1 CKM parameters
- 3 PMNS mixing angle
- 2 ν mass square difference

$$n_{\text{obs}} = 18$$

M1 $10_H^C + \overline{126}_H^C$ with $U(1)_{\text{PQ}}$

$$Y_u = S + D \quad (c_{10}^u)^* \neq c_{10}^d$$

$$Y_d = r_2 S + e^{i\omega} r_1 D,$$

$$Y_e = r_2 S - 3e^{i\omega} r_1 D,$$

$$Y_\nu = S - 3D,$$

$$M_R = v_R D,$$

K. S. Babu et al., [1805.10631]

M2 $10_H^R + \overline{126}_H^C + 120_H^R$ (CP symmetry)

$$Y_u = e^{i\alpha} S + D + e^{i\lambda} A, \quad \rightarrow \text{real}$$

$$Y_d = e^{-i\alpha} S + e^{i\omega} r_1 D + e^{-i\lambda} A,$$

$$Y_e = e^{-i\alpha} S - 3e^{i\omega} r_1 D + e^{-i\gamma} r_4 A,$$

$$Y_\nu = e^{i\alpha} S - 3D + e^{i\gamma} r_4 A,$$

$$M_R = v_R D.$$

K. S. Babu, B. Bajc et al., [1612.04329]

K. S. Babu, C.S. Fong, et al., [2508.14969]

M3 $(10_H^C, \overline{126}_H^C, 120_H^C)$ $U(1)_{\text{PQ}} \times Z_2^P$

$$Y_u = S + D + iA, \quad \rightarrow \text{real}$$

$$Y_d = r_2 S + r_1 D + ir_3 A,$$

$$Y_e = r_2 S - 3r_1 D + ir_4 A,$$

$$Y_\nu = S - 3D + ir_5 A,$$

$$M_R = v_R D.$$

B. Fu et al., [2209.00021]

Numerical Analysis: NO & IO comparison

We run SciPy Differential Evolution (DE) optimization in HIAS cluster with 3000 CPU cores.

Step 1 – Random Initialization

- **Generating 15,000 random points**
- Compute Fermion masses and mixings
- Initial $\chi^2 \sim O(10^8)$



Step 2 – Differential Evolution Minimization

- 15,000 independent chains (MPI parallel)
- Population = 15/chain, up to **50,000 generations**
- **best1bin** mutation (F = 0.7)



Step 3 – Selection

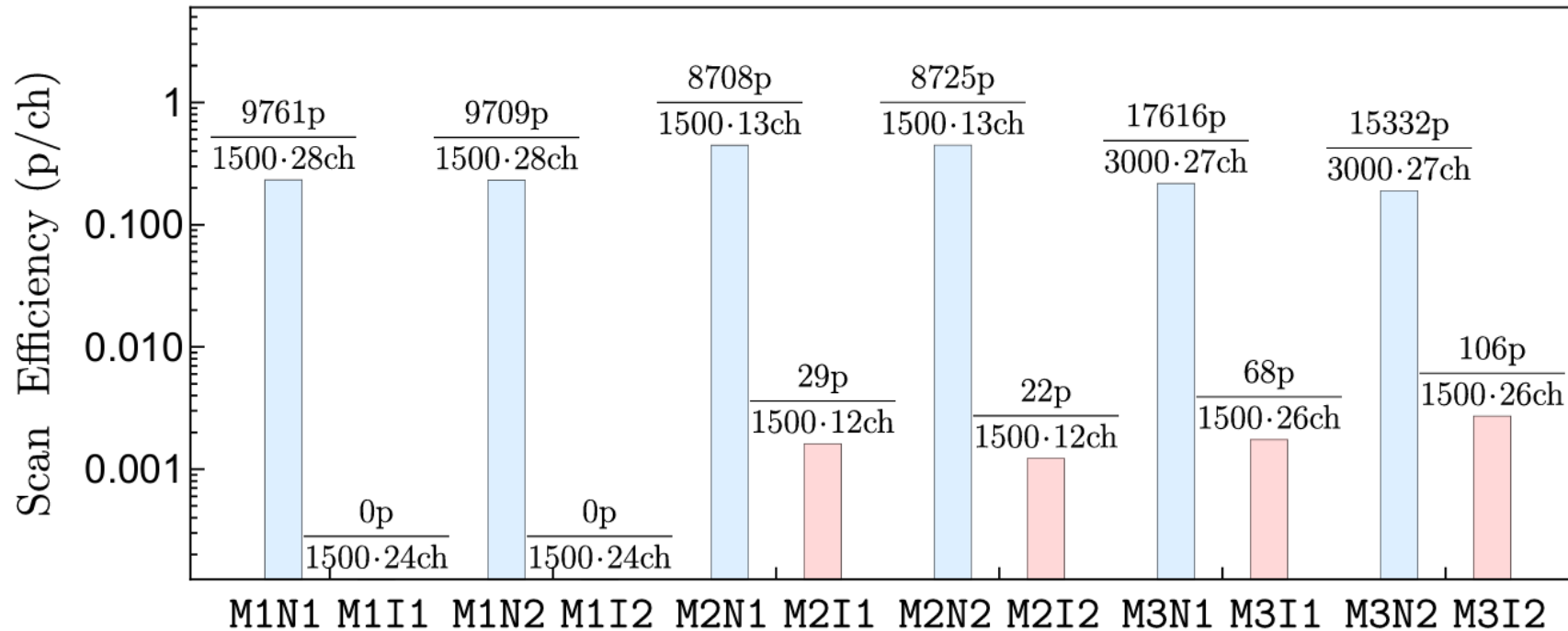
- Keep only points with $\chi^2 / n_{\text{obs}} < 10$

GitHub Link: https://github.com/YeLingZhou/GUT_Yukawa_Fit

We note ν fitting data for M1 as: NuFIT 6.0 (2024)

	Normal Ordering ($\Delta\chi^2 = 0.6$)		Inverted Ordering (best fit)		
	bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range	
IC19 without SK atmospheric data	$\sin^2 \theta_{12}$	$0.307^{+0.012}_{-0.011}$	0.275 → 0.345	$0.308^{+0.012}_{-0.011}$	0.275 → 0.345
	$\theta_{12}/^\circ$	$33.68^{+0.73}_{-0.70}$	31.63 → 35.95	$33.68^{+0.73}_{-0.70}$	31.63 → 35.95
	$\sin^2 \theta_{23}$	$0.561^{+0.012}_{-0.015}$	0.430 → 0.596	$0.562^{+0.012}_{-0.015}$	0.437 → 0.597
	$\theta_{23}/^\circ$	$48.5^{+0.7}_{-0.9}$	41.0 → 50.5	$48.6^{+0.7}_{-0.9}$	41.4 → 50.6
	$\sin^2 \theta_{13}$	0.02195	M1N2 0.02376	0.02224	M1I2 0.02397
	$\theta_{13}/^\circ$	8.52	8.87	8.58	8.91
	$\delta_{CP}/^\circ$	177^{+19}_{-20}	96 → 422	285^{+25}_{-28}	201 → 348
	$\frac{\Delta m_{21}^2}{10^{-5} \text{ eV}^2}$	$7.49^{+0.19}_{-0.19}$	6.92 → 8.05	$7.49^{+0.19}_{-0.19}$	6.92 → 8.05
	$\frac{\Delta m_{3\ell}^2}{10^{-3} \text{ eV}^2}$	$+2.534^{+0.025}_{-0.023}$	+2.463 → +2.606	$-2.510^{+0.024}_{-0.025}$	-2.584 → -2.438
	IC24 with SK atmospheric data	$\sin^2 \theta_{12}$	$0.308^{+0.012}_{-0.011}$	0.275 → 0.345	$0.308^{+0.012}_{-0.011}$
$\theta_{12}/^\circ$		$33.68^{+0.73}_{-0.70}$	31.63 → 35.95	$33.68^{+0.73}_{-0.70}$	31.63 → 35.95
$\sin^2 \theta_{23}$		$0.470^{+0.017}_{-0.013}$	0.435 → 0.585	$0.550^{+0.012}_{-0.015}$	0.440 → 0.584
$\theta_{23}/^\circ$		$43.3^{+1.0}_{-0.8}$	41.3 → 49.9	$47.9^{+0.7}_{-0.9}$	41.5 → 49.8
$\sin^2 \theta_{13}$		0.02215	M1N1 0.02388	0.02231	M1I1 0.02409
$\theta_{13}/^\circ$		8.56	8.89	8.59	8.93
$\delta_{CP}/^\circ$		212^{+26}_{-41}	124 → 364	274^{+22}_{-25}	201 → 335
$\frac{\Delta m_{21}^2}{10^{-5} \text{ eV}^2}$		$7.49^{+0.19}_{-0.19}$	6.92 → 8.05	$7.49^{+0.19}_{-0.19}$	6.92 → 8.05
$\frac{\Delta m_{3\ell}^2}{10^{-3} \text{ eV}^2}$		$+2.513^{+0.021}_{-0.019}$	+2.451 → +2.578	$-2.484^{+0.020}_{-0.020}$	-2.547 → -2.421

Numerical Analysis: NO & IO comparison



Preference

NO



IO



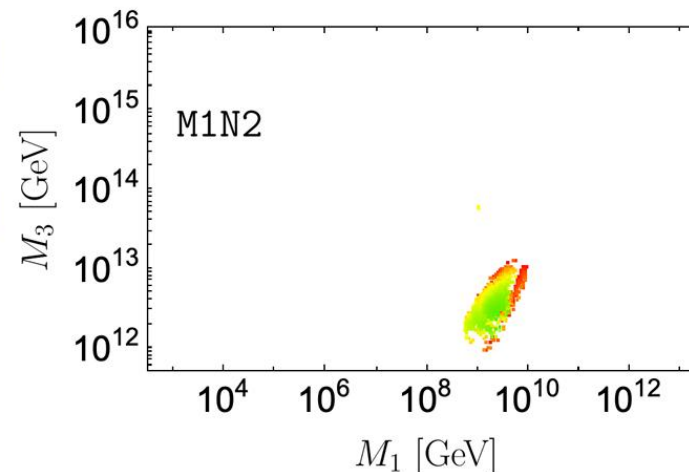
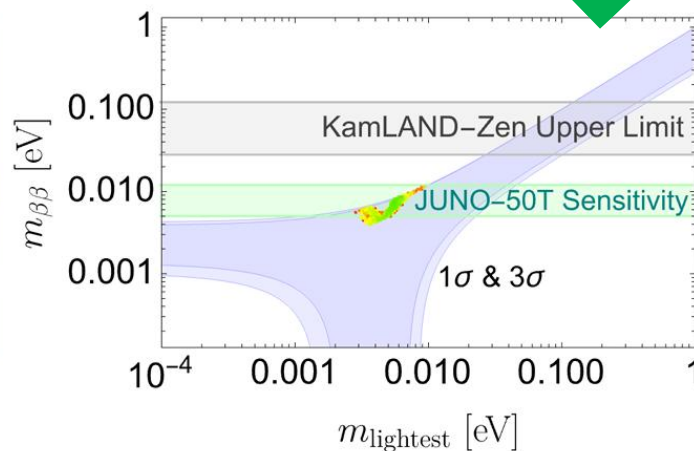
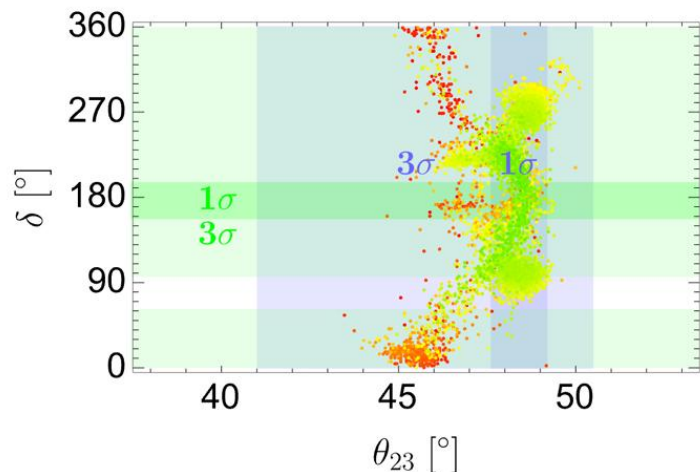
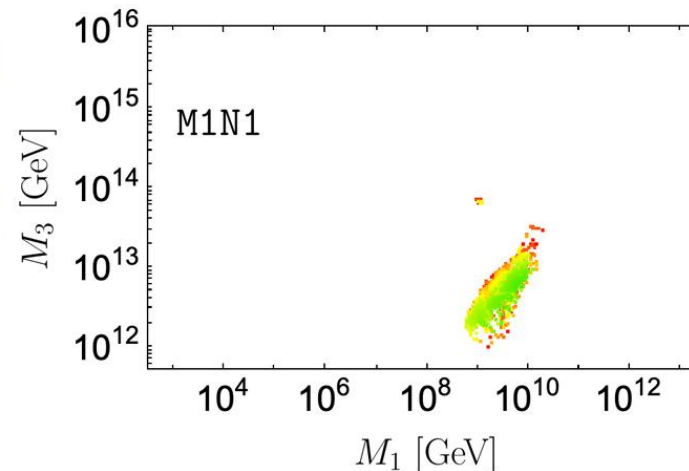
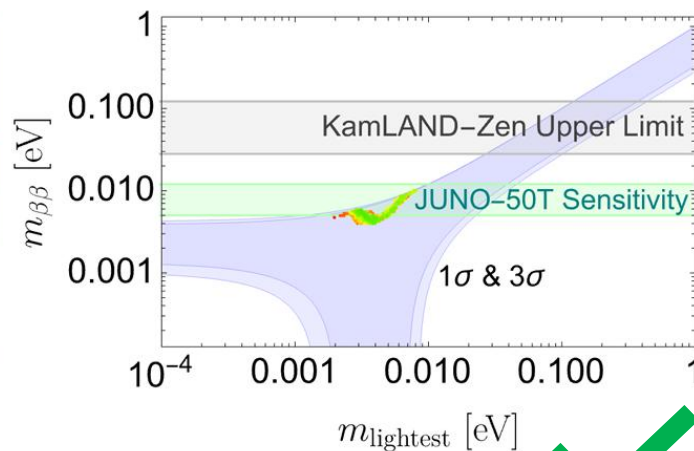
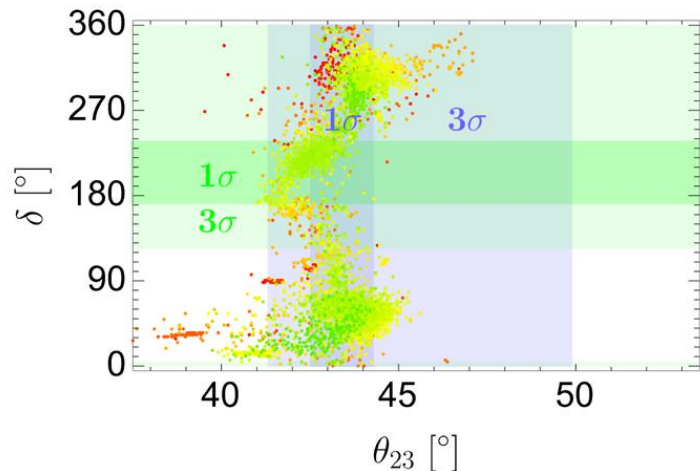
Very rare: we only show IO in backup slides.

FIG. 1: Viable points (p) per core hour (ch) obtained in M1, M2 and M3 with $\chi^2/n_{\text{obs}} < 10$ in each scan.

Numerical Analysis: χ^2 fitting at the GUT scale

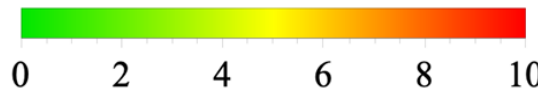
Fitting data from K. S. Babu, B. Bajc, et al., [1612.04329]
 who run Yukawas to the GUT scale. We fit at GUT scale.

M1



Nearly flatly distributed

χ^2/n_{obs}



Small hierarchy

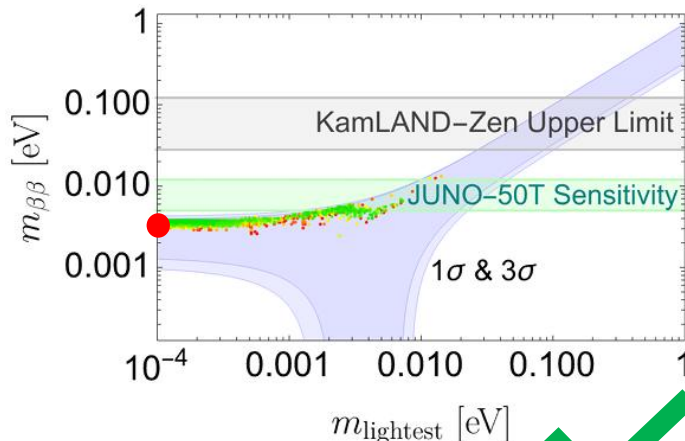
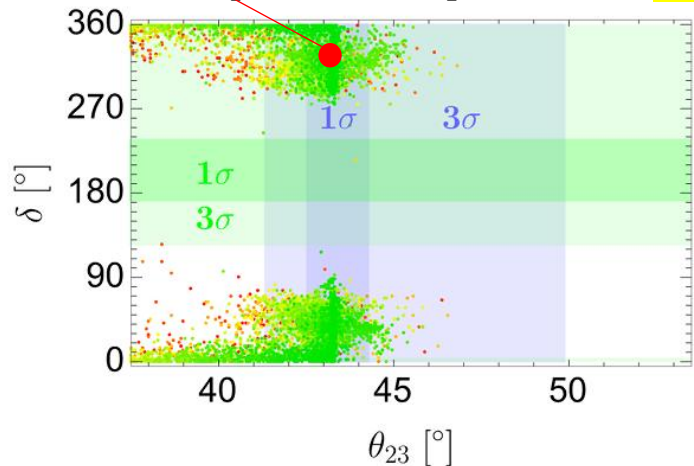
- N_1 leptogenesis \checkmark

Numerical Analysis: χ^2 fitting at the GUT scale

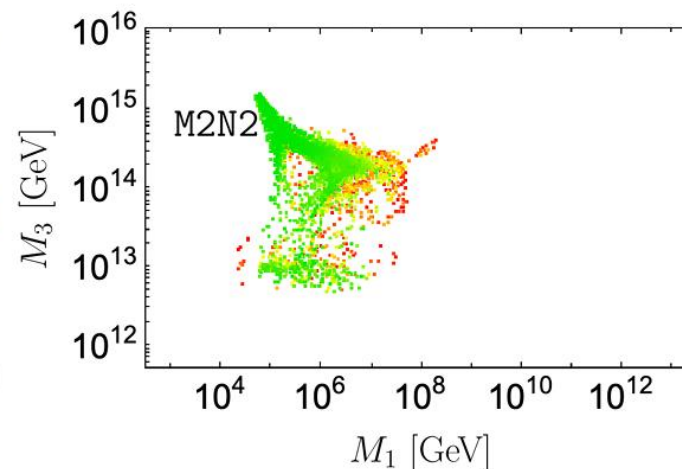
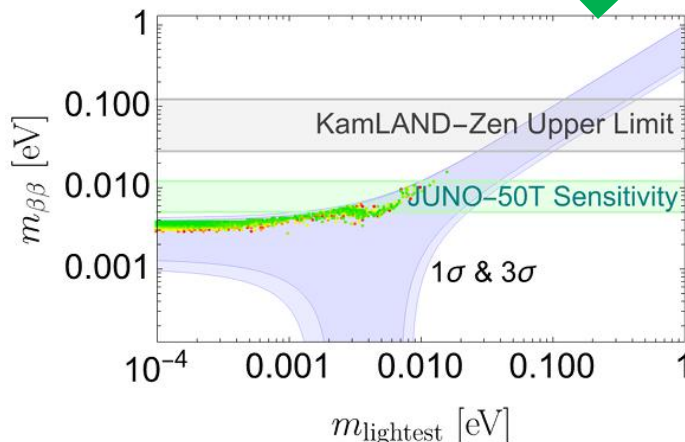
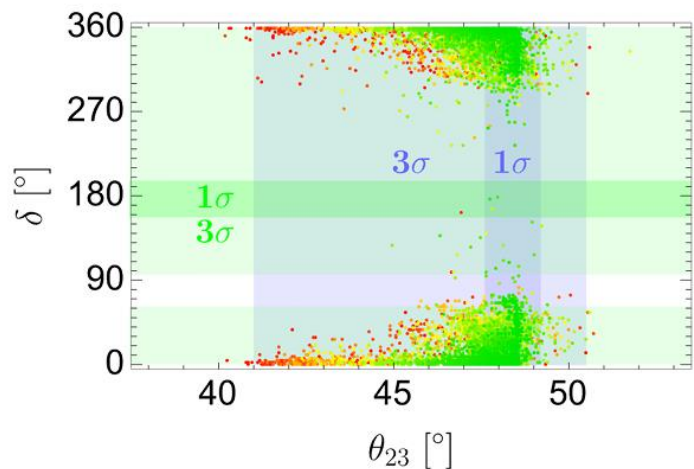
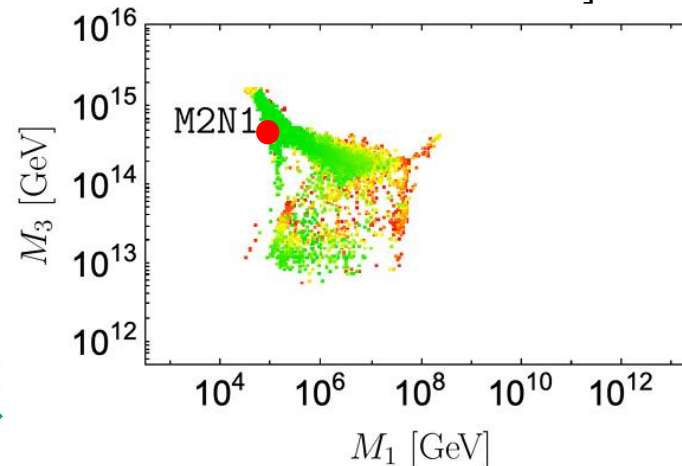
• K. S. Babu, C. S. Fong, et al., [2508.14969]

- Very large $M_3/M_1 \sim O(10^9)$ checked ✓
- More parameter space ✓

M2



[Flipped-SU(5) prediction see Prof. X.G H's talk]



$\delta \in [-70^\circ, +70^\circ]$ χ^2/n_{obs}

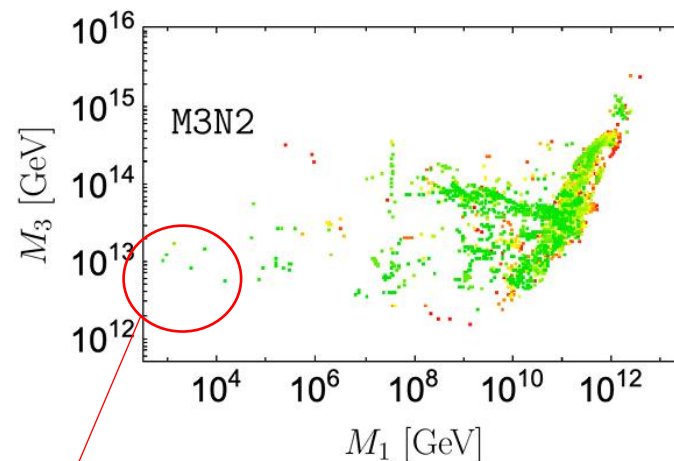
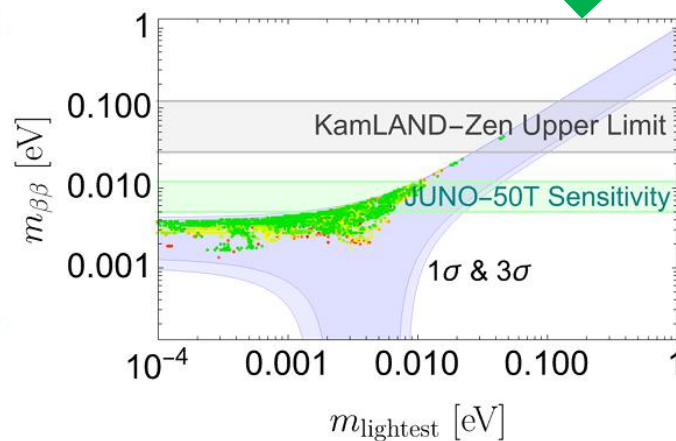
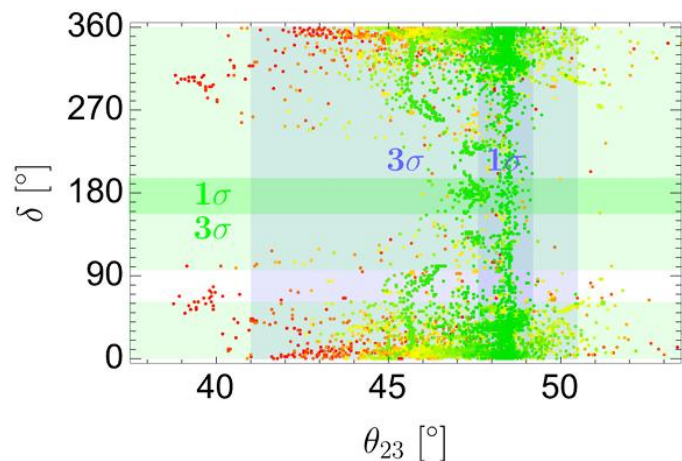
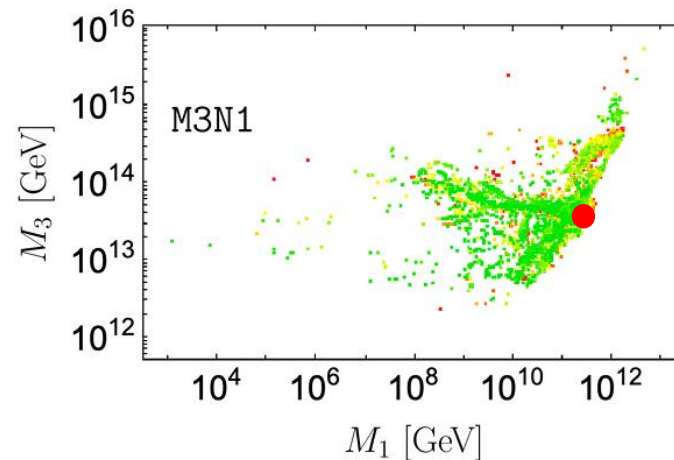
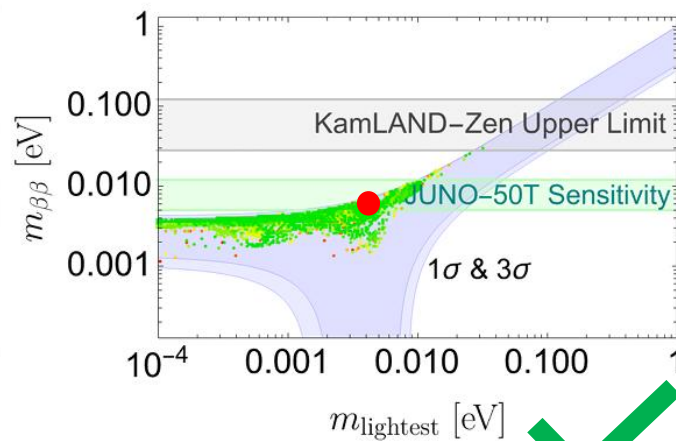
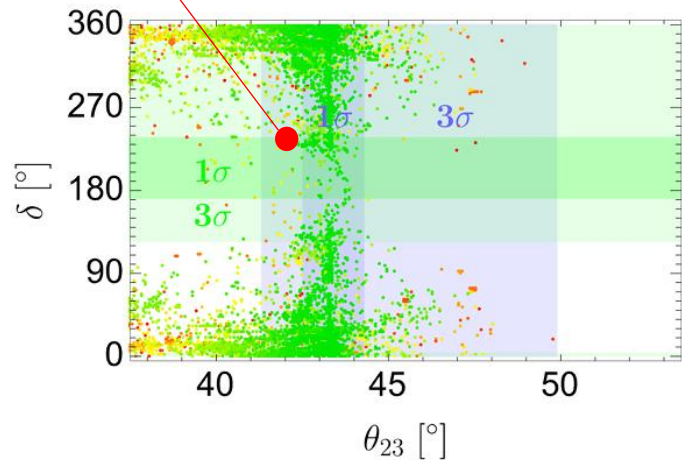
Large hierarchy:

- N_1 leptogenesis \times
 - N_2 leptogenesis \checkmark
- P. Di Bari, et al., [hep-ph/0502082]
O. Vives, et al., [hep-ph/0512160]

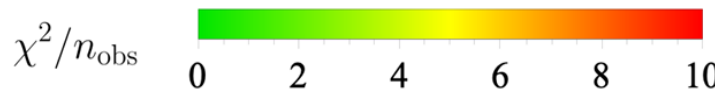
Numerical Analysis: χ^2 fitting at the GUT scale

M3

- B. Fu et al., [2209.00021] →
- We find large hierarchy for $M_3/M_1 \sqrt{\nu}$
 - More parameter space ✓



$\delta \in [-90^\circ, +90^\circ]$

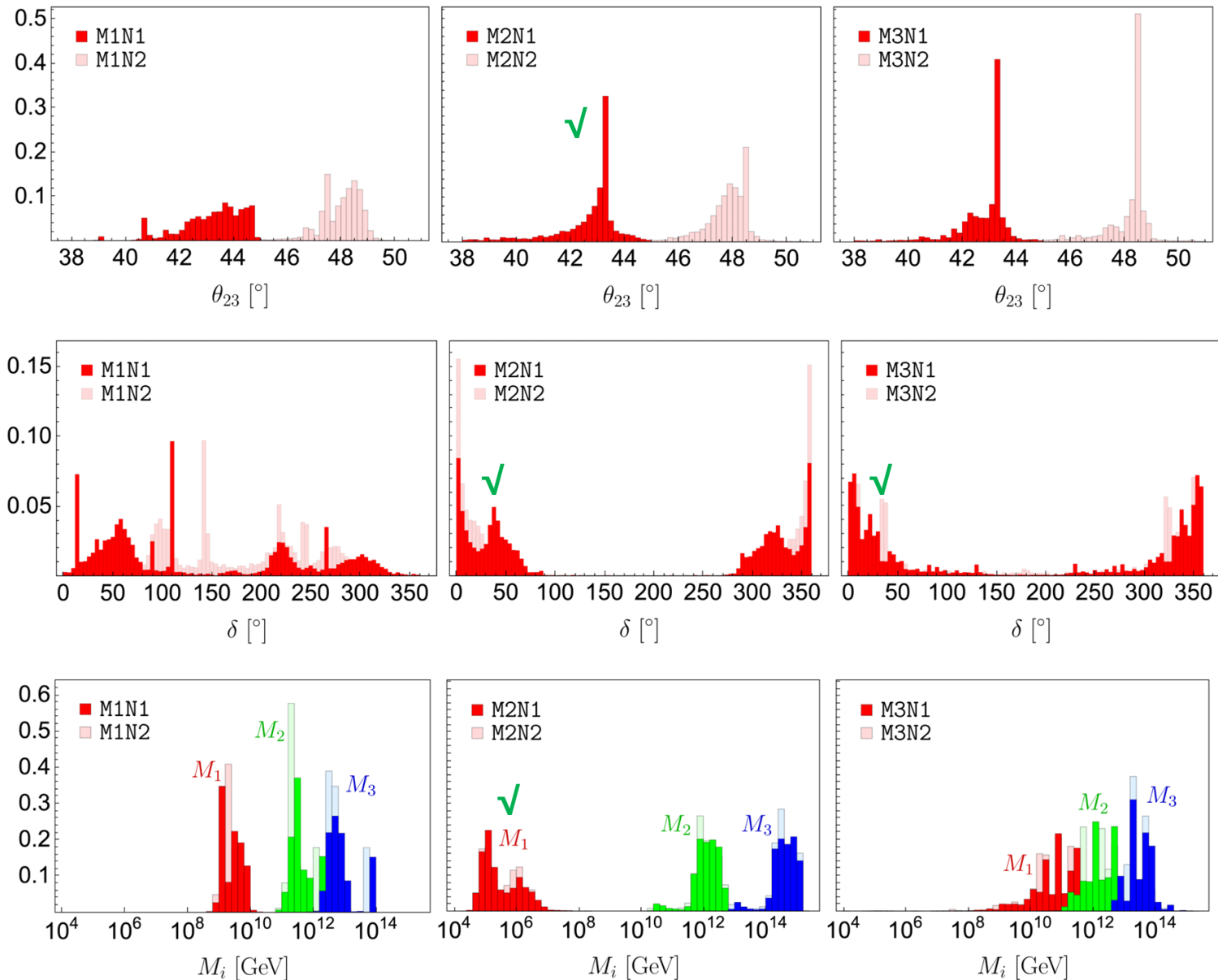


TeV Collider Searches?

Small + large hierarchy:

- N_1 leptogenesis ✓
- N_2 leptogenesis ✓

Numerical Analysis: statistical histogram



Histograms normalized
by weight $e^{-\chi^2/2}$

- M2: θ_{23} slightly prefer Oct.1
- δ mainly in $[-70^\circ, +70^\circ]$ in M2,
and $[-90^\circ, +90^\circ]$ in M3, flat in M1
- Large hierarchy obvious in M2

Quark-lepton correlation can be increasingly important to test GUTs in the precision era of neutrino physics:

- Clear preference: NO \surd & IO
- M1-M3 can partially be tested by future $0\nu\beta\beta$ experiments
- In M2, large hierarchy of RHN masses \surd , more parameter space are found
- PMNS $\delta \in [-90^\circ, 90^\circ]$ in M2 and M3, nearly flatly distributed in M1.
- M2 prefers θ_{23} lies in 1st octant, M1, M3 not significant

Thanks!

Three models: M1, M2, M3

Parameterization

$$\begin{aligned}
 c_{10}^u &= |c_{10}^u|e^{i\alpha}, \quad c_{10}^d = |c_{10}^d|e^{i\beta}, \\
 c_{126}^u &= |c_{126}^u|, \quad c_{126}^d = |c_{126}^d|e^{i\omega}, \quad v_S = |v_S|, \\
 c_{120_1}^u + c_{120_{15}}^u &= |c_{120_1}^u + c_{120_{15}}^u|e^{i\lambda}, \\
 c_{120_1}^d + c_{120_{15}}^d &= |c_{120_1}^d + c_{120_{15}}^d|e^{i\rho}, \\
 c_{120_1}^u - 3c_{120_{15}}^u &= |c_{120_1}^u - 3c_{120_{15}}^u|e^{i\gamma}, \\
 c_{120_1}^d - 3c_{120_{15}}^d &= |c_{120_1}^d - 3c_{120_{15}}^d|e^{i\eta},
 \end{aligned}$$

$$\begin{aligned}
 D &= |c_{126}^u|Y_{126}, \quad S = |c_{10}^u|Y_{10}, \quad A = |c_{120_1}^u + c_{120_{15}}^u|Y_{120}, \\
 v_R &= \frac{|v_S|}{|c_{126}^u|}, \quad r_1 = \frac{|c_{126}^d|}{|c_{126}^u|}, \quad r_2 = \frac{|c_{10}^d|}{|c_{10}^u|}, \quad r_3 = \frac{|c_{120_1}^d + c_{120_{15}}^d|}{|c_{120_1}^u + c_{120_{15}}^u|}, \\
 r_4 &= \frac{|c_{120_1}^d - 3c_{120_{15}}^d|}{|c_{120_1}^u + c_{120_{15}}^u|}, \quad r_5 = \frac{|c_{120_1}^u - 3c_{120_{15}}^u|}{|c_{120_1}^u + c_{120_{15}}^u|}.
 \end{aligned}$$

χ^2 analysis: $\chi^2 = \sum_{i=1}^{n_{\text{obs}}} \left(\frac{O_i - O_i^{\text{bf}}}{\sigma_{O_i}} \right)^2$ **Observables $n_{\text{obs}}=18$** NO:

$$y_u = (2.54 \pm 0.86) \cdot 10^{-6}, \quad y_d = (6.56 \pm 0.65) \cdot 10^{-6},$$

$$y_c = (1.37 \pm 0.04) \cdot 10^{-3}, \quad y_s = (1.24 \pm 0.06) \cdot 10^{-4},$$

$$y_t = (0.428 \pm 0.003), \quad y_b = (5.7 \pm 0.005) \cdot 10^{-3},$$

$$\theta_{12}^q = (0.227 \pm 0.0006), \quad \theta_{13}^q = (4.202 \pm 0.13) \cdot 10^{-3},$$

$$\theta_{23}^q = (4.858 \pm 0.06) \cdot 10^{-2}, \quad \delta^q = (1.207 \pm 0.054);$$

$$y_e = (2.703 \pm 0.027) \cdot 10^{-6}, \quad y_\mu = (5.707 \pm 0.057) \cdot 10^{-4},$$

$$y_\tau = (9.702 \pm 0.0097) \cdot 10^{-3}, \quad \text{Babu, Bajc, Saad, 1612.04329}$$

With RG effect running up to

$$M_{\text{GUT}} = 2 \times 10^{16} \text{ GeV}$$

NuFIT 6.0 $\sin^2 \theta_{23} = 0.561 \pm 0.014$, w/o SK atm.

$$\sin^2 \theta_{13} = 0.02195 \pm 0.00056,$$

$$\Delta m_{31}^2 = (2.534 \pm 0.024) \times 10^{-3} \text{ eV}^2,$$

$\sin^2 \theta_{23} = 0.470 \pm 0.015$, w/ SK atm.

$$\sin^2 \theta_{13} = 0.02215 \pm 0.00057$$

$$\Delta m_{31}^2 = (2.513 \pm 0.020) \times 10^{-3} \text{ eV}^2,$$

JUNO $\Delta m_{21}^2 = (7.50 \pm 0.12) \times 10^{-5} \text{ eV}^2$,

$$\sin^2 \theta_{12} = 0.3092 \pm 0.00087,$$

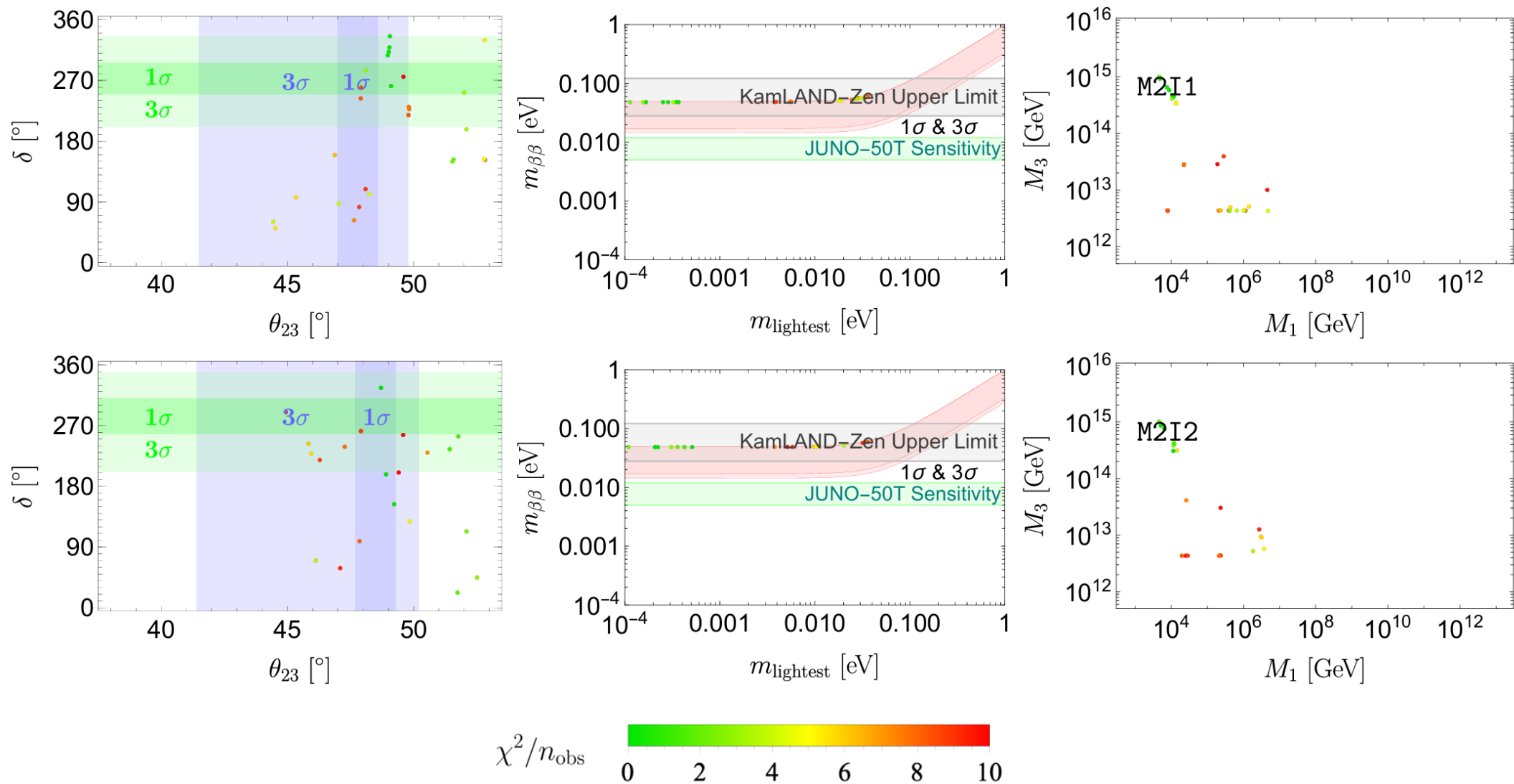
Fitting Datas

Data of IO are not shown here

Benchmarks

Physical Quantity	M1		M2				M3			
	M1N1B1	M1N2B1	M2N1B1	M2N1B2	M2N2B1	M2N2B2	M3N1B1	M3N1B2	M3N2B1	M3N2B2
$y_u/10^{-6}$	2.65	2.75	2.55	3.07	2.53	4.85	0.637	2.54	0.744	2.54
$y_c/10^{-3}$	1.40	1.39	1.37	1.35	1.37	1.37	1.37	1.37	1.37	1.37
y_t	0.429	0.427	0.428	0.429	0.428	0.430	0.428	0.428	0.428	0.428
$y_d/10^{-6}$	3.87	3.13	6.56	7.29	6.56	7.14	5.28	6.56	5.28	6.56
$y_s/10^{-4}$	1.37	1.32	1.24	1.17	1.24	1.11	1.24	1.24	1.25	1.24
$y_b/10^{-2}$	0.571	0.571	0.570	0.570	0.570	0.568	0.569	0.570	0.569	0.570
$y_e/10^{-6}$	2.69604	2.70309	2.70346	2.70190	2.70339	2.70629	2.70318	2.70346	2.70345	2.70341
$y_\mu/10^{-4}$	5.65981	5.68560	5.70708	5.72116	5.70713	5.70725	5.71839	5.70706	5.71432	5.70698
$y_\tau/10^{-2}$	0.97358	0.97054	0.97021	0.96623	0.97020	0.97211	0.97154	0.97020	0.97124	0.97020
θ_{12}^q	0.22750	0.22749	0.22739	0.22760	0.22739	0.22721	0.22745	0.22739	0.22745	0.22739
$\theta_{23}^q/10^{-2}$	4.904	4.872	4.858	4.876	4.858	4.835	4.858	4.858	4.856	4.858
$\theta_{13}^q/10^{-3}$	4.252	4.215	4.202	4.201	4.202	4.237	4.202	4.202	4.202	4.202
δ^q	1.269	1.251	1.207	1.253	1.207	1.227	1.201	1.207	1.203	1.207
$\Delta m_{21}^2/10^{-5} \text{ eV}^2$	7.53	7.50	7.50	7.50	7.50	7.48	7.50	7.50	7.50	7.50
$\Delta m_{3\ell}^2/10^{-3} \text{ eV}^2$	2.515	2.535	2.513	2.512	2.534	2.539	2.513	2.513	2.534	2.534
$\sin^2 \theta_{12}^l$	0.313	0.309	0.309	0.303	0.309	0.308	0.309	0.309	0.309	0.309
$\sin^2 \theta_{23}^l$	0.456	0.561	0.470	0.460	0.561	0.540	0.471	0.470	0.561	0.561
$\sin^2 \theta_{13}^l$	0.02152	0.02180	0.02215	0.02235	0.02195	0.02223	0.02217	0.02215	0.02194	0.02195
$\chi^2/(n_{\text{obs}} = 18)$	1.54	1.71	$5.88 \cdot 10^{-6}$	0.312	$7.60 \cdot 10^{-6}$	0.958	0.495	$3.41 \cdot 10^{-7}$	0.462	$3.39 \cdot 10^{-7}$
m_l/eV	$3.34 \cdot 10^{-3}$	$5.60 \cdot 10^{-3}$	$5.29 \cdot 10^{-5}$	$7.81 \cdot 10^{-4}$	$5.41 \cdot 10^{-5}$	$1.52 \cdot 10^{-3}$	$3.82 \cdot 10^{-3}$	$5.46 \cdot 10^{-4}$	$3.44 \cdot 10^{-3}$	$1.03 \cdot 10^{-3}$
m_β/eV	$9.40 \cdot 10^{-3}$	$1.05 \cdot 10^{-2}$	$8.85 \cdot 10^{-3}$	$8.88 \cdot 10^{-3}$	$8.85 \cdot 10^{-3}$	$9.02 \cdot 10^{-3}$	$9.64 \cdot 10^{-3}$	$8.87 \cdot 10^{-3}$	$9.49 \cdot 10^{-3}$	$8.91 \cdot 10^{-3}$
$m_{\beta\beta}/\text{eV}$	$4.94 \cdot 10^{-3}$	$7.18 \cdot 10^{-3}$	$3.70 \cdot 10^{-3}$	$4.22 \cdot 10^{-3}$	$3.67 \cdot 10^{-3}$	$4.76 \cdot 10^{-3}$	$6.51 \cdot 10^{-3}$	$4.09 \cdot 10^{-3}$	$6.17 \cdot 10^{-3}$	$4.42 \cdot 10^{-3}$
$\delta/^\circ$	38.816	210.46	43.046	11.532	341.29	6.4034	186.19	14.663	189.07	351.61
M_1/GeV	$7.48 \cdot 10^9$	$2.56 \cdot 10^9$	$9.95 \cdot 10^4$	$7.36 \cdot 10^6$	$8.43 \cdot 10^4$	$7.13 \cdot 10^6$	$1.01 \cdot 10^3$	$1.37 \cdot 10^{10}$	$9.09 \cdot 10^2$	$2.71 \cdot 10^{10}$
M_2/GeV	$4.95 \cdot 10^{11}$	$2.14 \cdot 10^{11}$	$2.14 \cdot 10^{12}$	$6.96 \cdot 10^{10}$	$3.64 \cdot 10^{12}$	$9.07 \cdot 10^{10}$	$6.36 \cdot 10^{11}$	$2.57 \cdot 10^{11}$	$5.60 \cdot 10^{11}$	$5.01 \cdot 10^{11}$
M_3/GeV	$5.61 \cdot 10^{12}$	$2.48 \cdot 10^{12}$	$7.51 \cdot 10^{14}$	$2.29 \cdot 10^{13}$	$1.02 \cdot 10^{15}$	$2.97 \cdot 10^{13}$	$1.36 \cdot 10^{13}$	$9.45 \cdot 10^{12}$	$1.19 \cdot 10^{13}$	$2.09 \cdot 10^{13}$

IO results M2: Very rare points



IO results M3: Very rare points

