Flavor Physics and CP from Non-Invertible Selection Rules

Hajime Otsuka (Kyushu University)

Outline :

1. Coupling selection rules (review)
2. Yukawa textures
3. Application to MSSM
4. CP
5. String compactifications
6. Conclusions

GUTPC 2026, Hangzhou, April 12, 2026

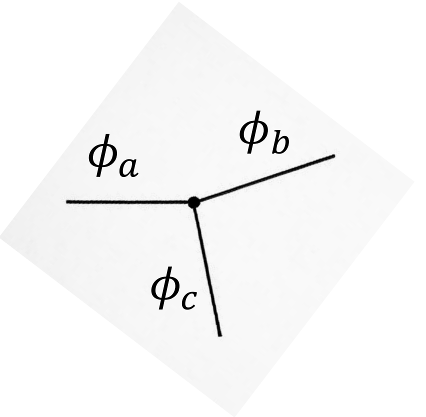
In a 4D theory with group-like symmetries,
a field corresponds to (a rep. of) a group element

- Transformations of fields:

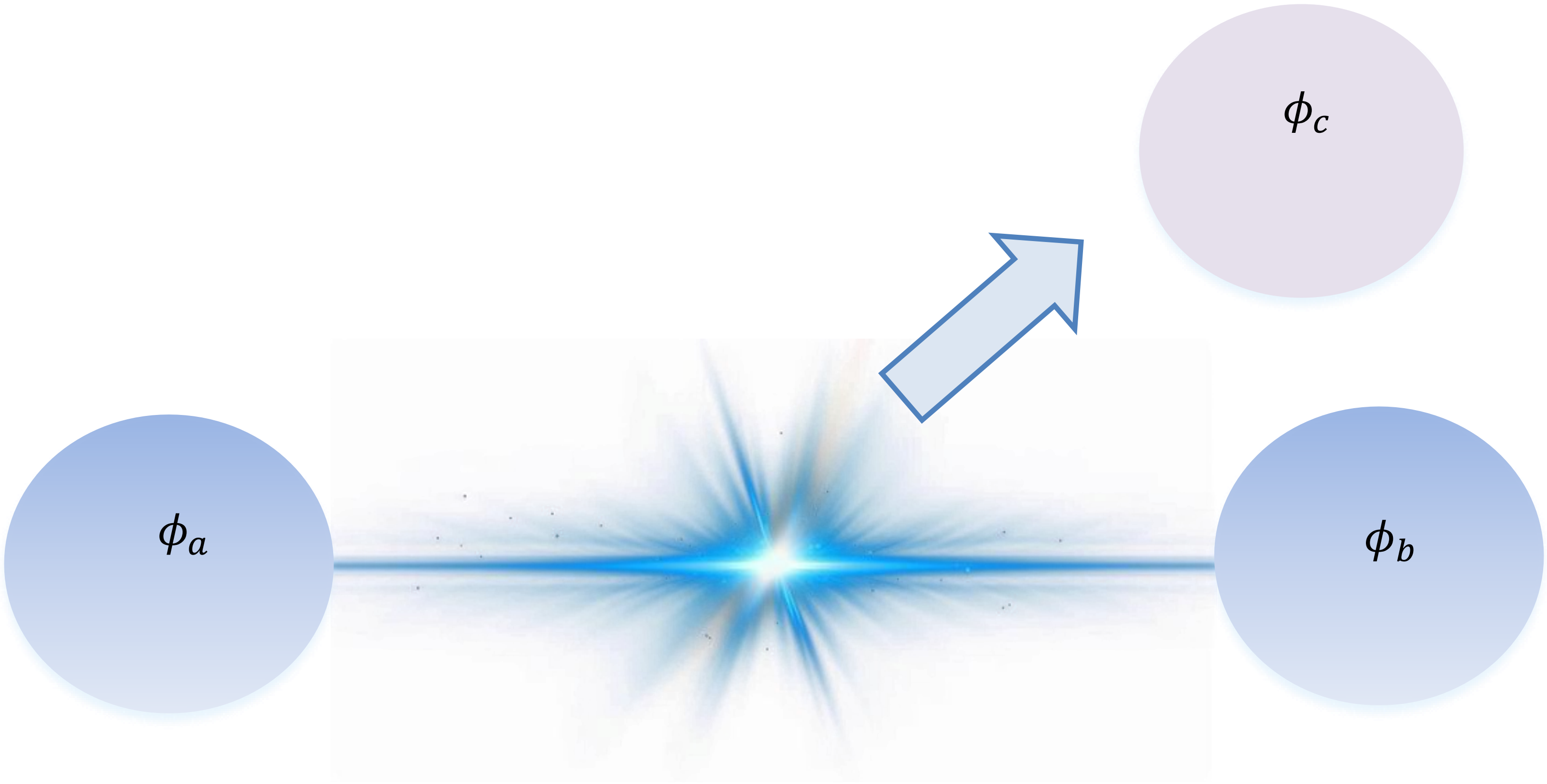
$$\phi \overset{g}{\to} \rho(g)\phi \qquad g \in G$$

- Suppose that fields $\{\phi_a, \phi_b, \phi_c\}$ correspond to a rep. of G

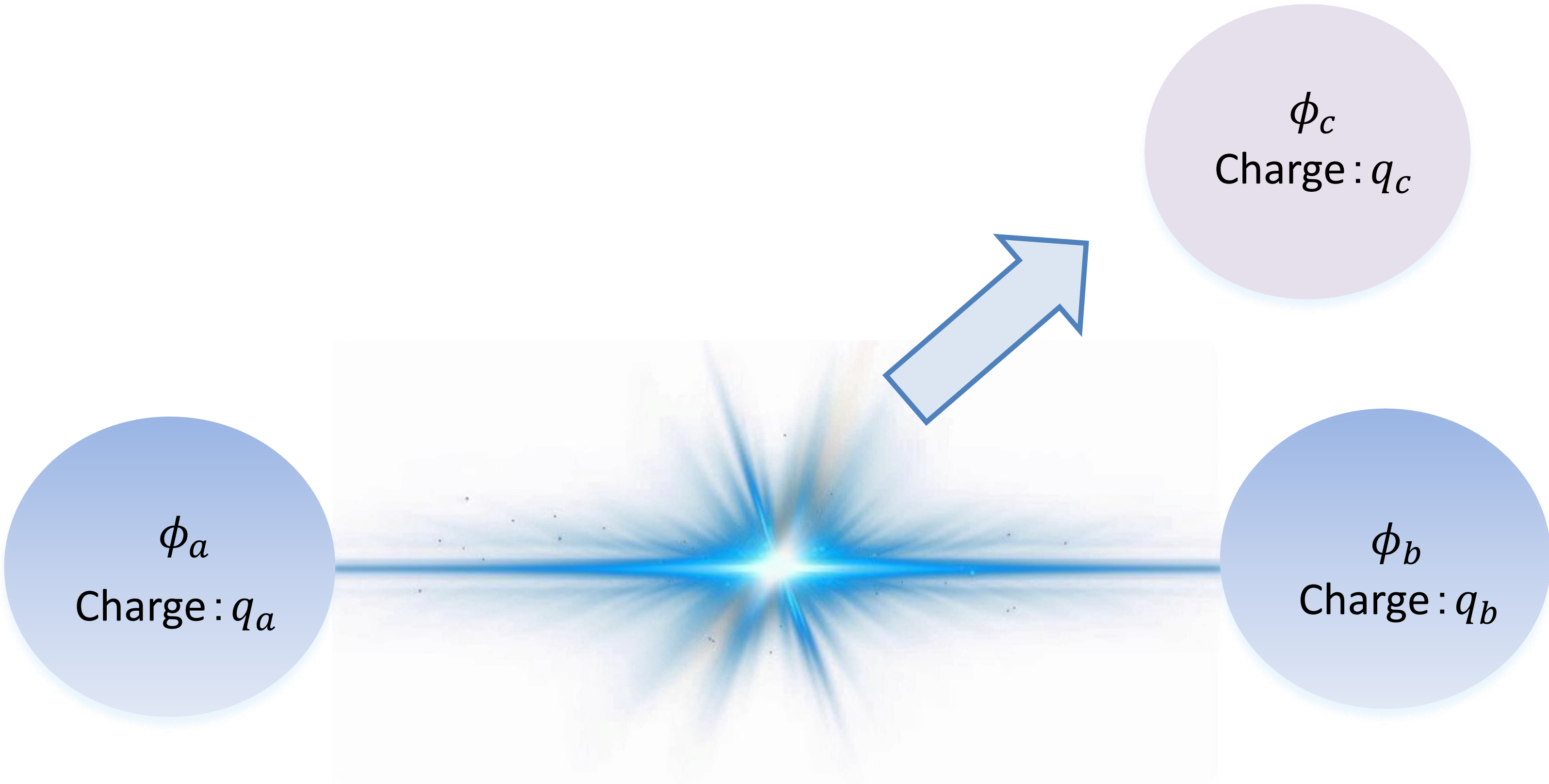
if $\rho_a \otimes \rho_b \otimes \rho_c \supset \mathbb{I}$ $\to$ $\phi_a\phi_b\phi_c$ is allowed
if $\rho_a \otimes \rho_b \otimes \rho_c \not\supset \mathbb{I}$ $\to$ $\phi_a\phi_b\phi_c$ is forbidden



"*Coupling selection rules*"
(When G is Abelian, it is the charge conservation law)



"Selction rules " based on a symmetry determine the presence or absence of interactions



Charge conservation law : $q_a + q_b = q_c$

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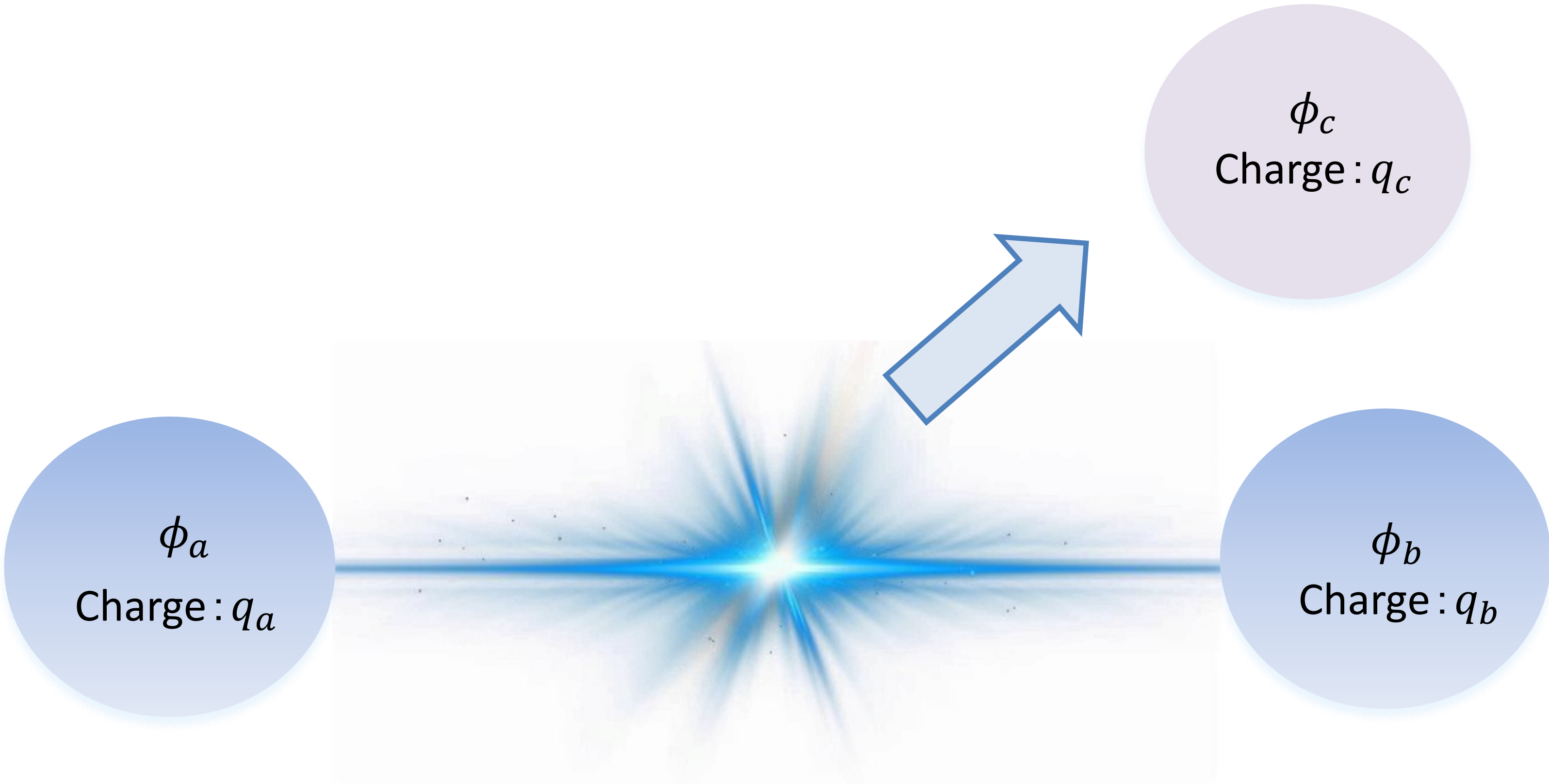
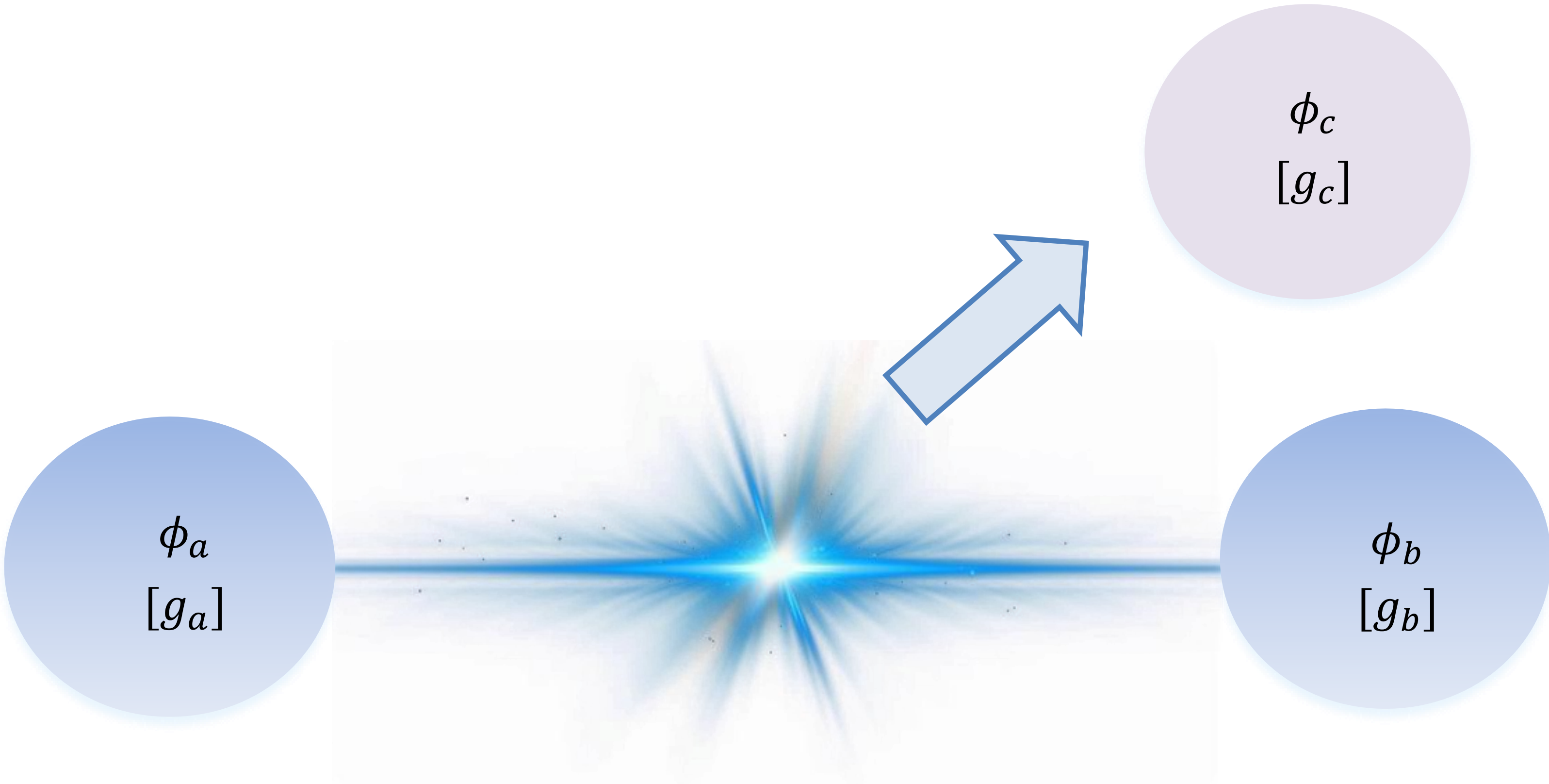


Charge conservation law : $q_a + q_b = q_c$

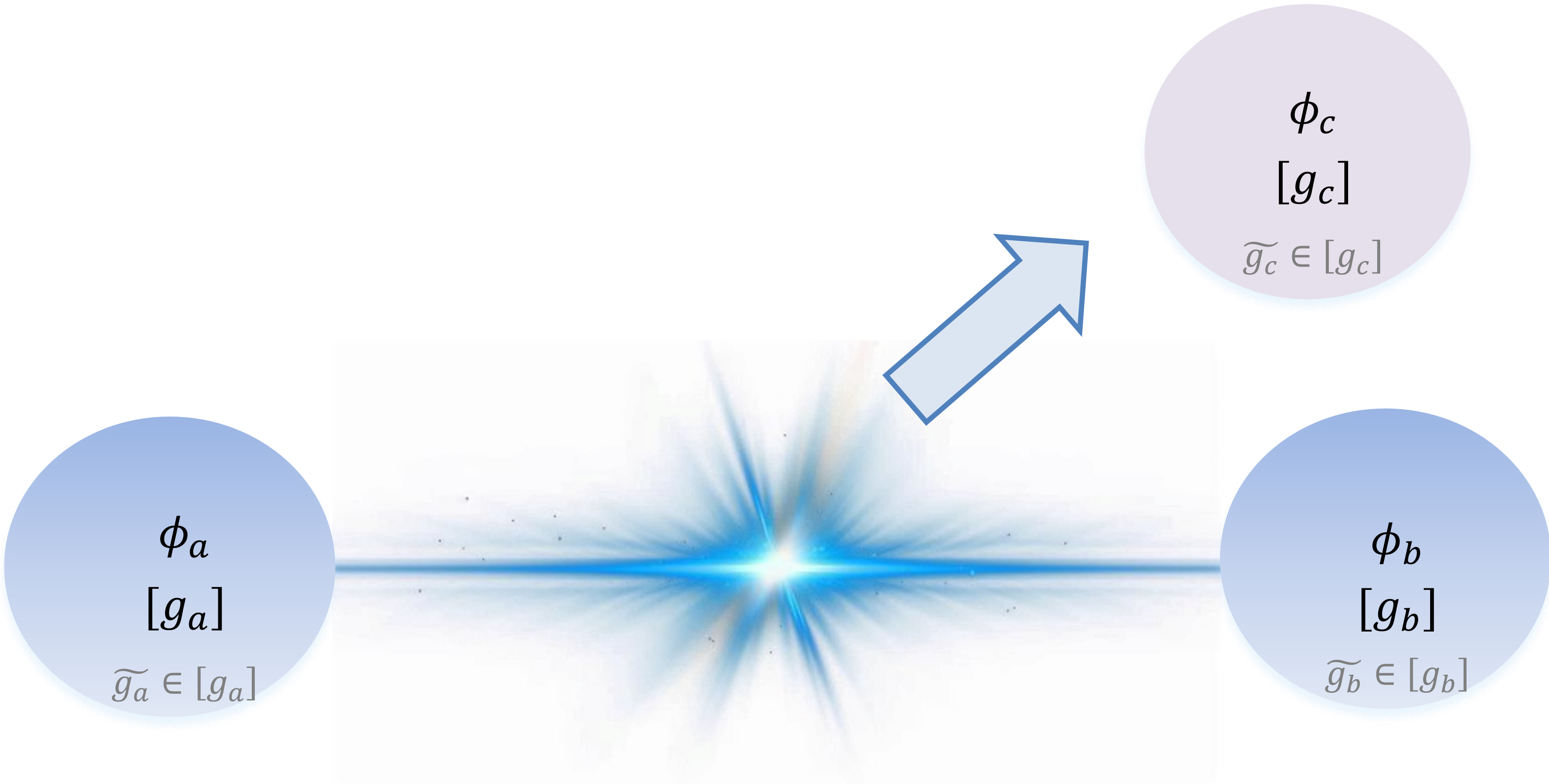
"Selction rules " based on a symmetry determine the presence or absence of interactions

It was associated with conservation laws derived from group-theoretical symmetry
→ Selection rules based on non-invertible symmetries have been developing.



Ex., when fields are labeled by a conjugacy class of a group G, i.e., $[g_a] = \{hg_ah^{-1}|h \in G\}$

The tensor product of conjugacy classes are non-invertible, ex., $[g_a] \cdot [g_b] = \sum_c N_{ab}^c [g_c]$



Interactions $\phi_a \phi_b \phi_c$ are allowed when $\widetilde{g_a}\widetilde{g_b}\widetilde{g_c} = e$ (identity)

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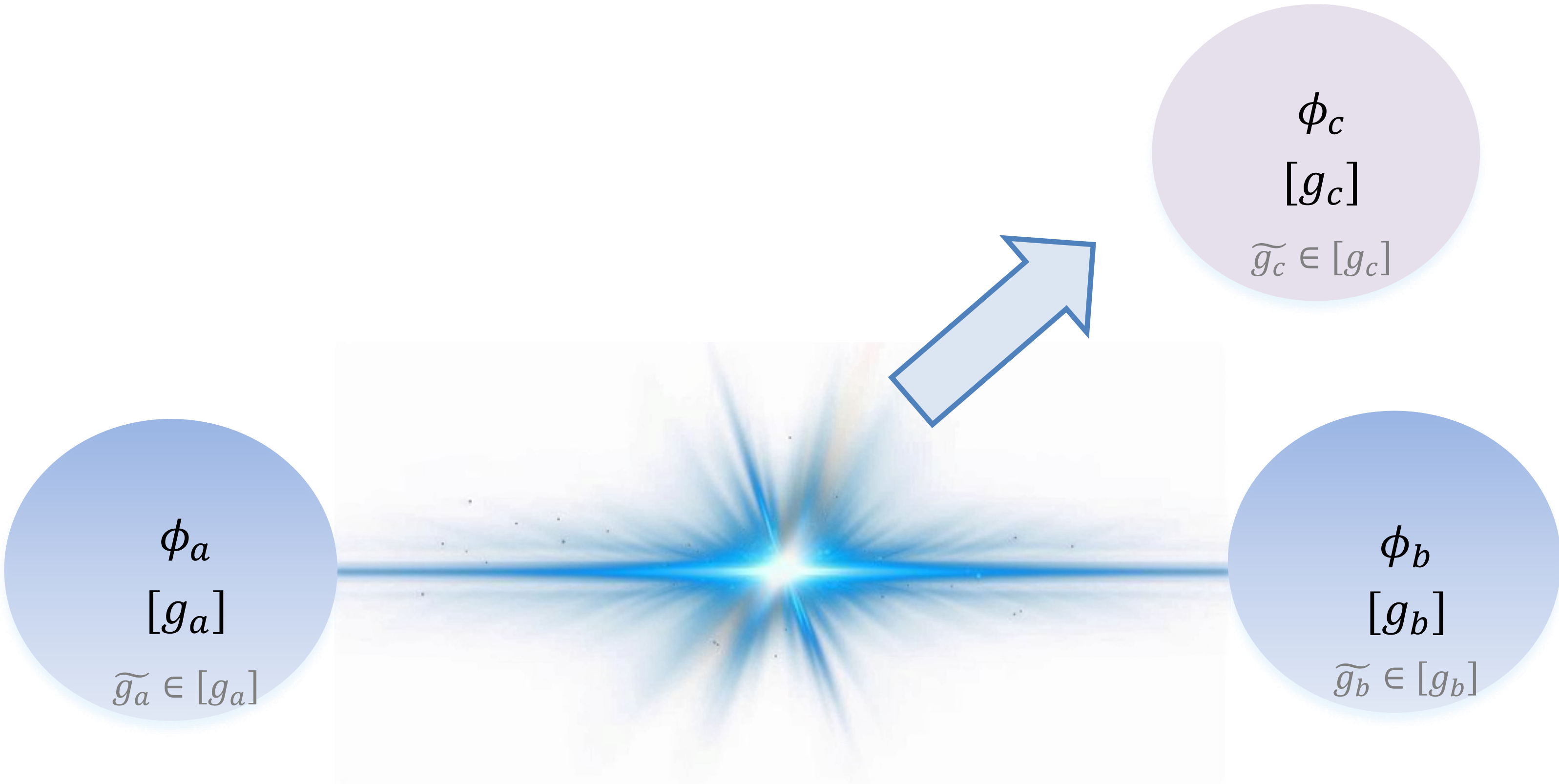

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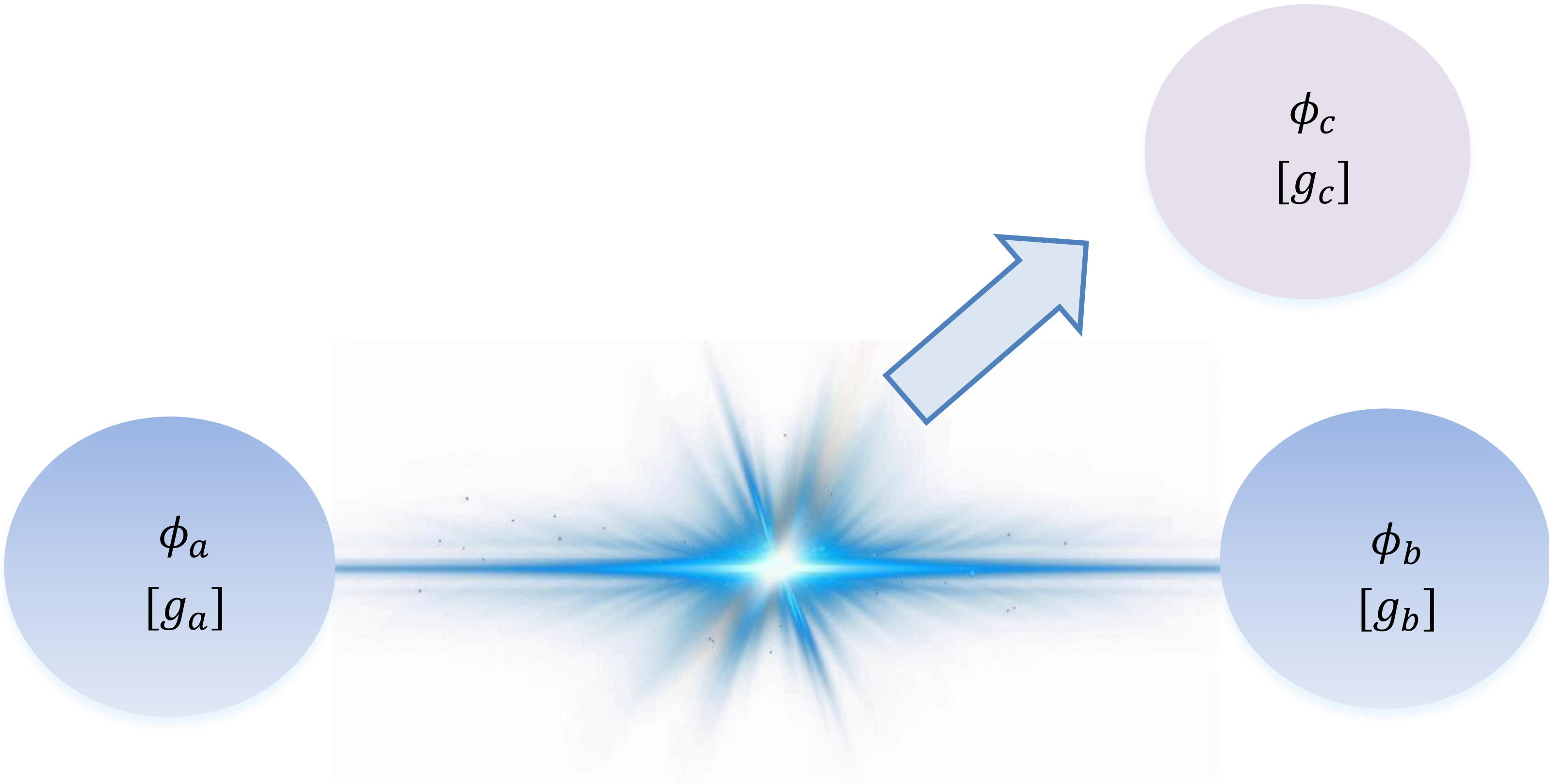
Selection rules would be different from group-like ones since a field is labeled by a class



Main message :

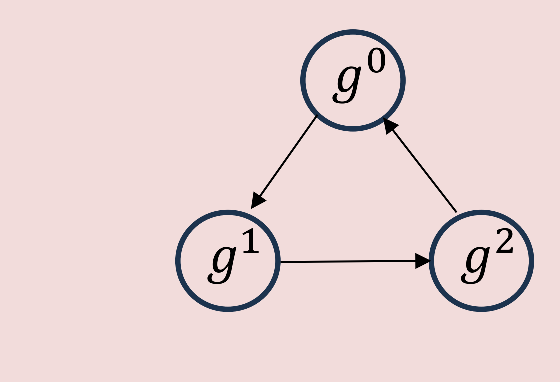
1. Application of fusion rules ($[g_a] \cdot [g_b] = \sum_c N_{ab}^{c} [g_c]$)
   → Novel Yukawa textures and CP

2. String compactifications → non-invertible selection rules

Non-invertible selection rules

arising from gauging outer automorphisms of a group

- Let us start with $\mathbb{Z}_3$ *symmetry (generator :* $g = e^{2\pi i/3}$*)*



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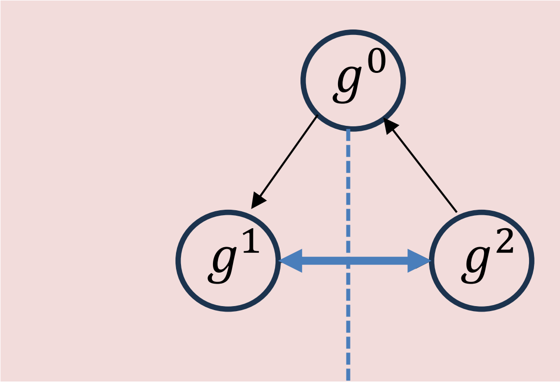


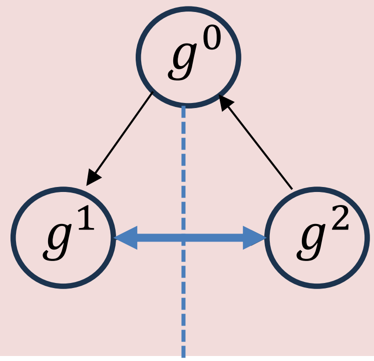
$\mathbb{Z}_2$ *gauging (automorphism of $\mathbb{Z}_3$)*

(corresponding to the orbifold $\mathbb{Z}_2$ twist)

- Let us start with $\mathbb{Z}_3$ *symmetry (generator : $g = e^{2\pi i/3}$)*

$[g_0] = \{g^0\}$

$[g_1] = \{g^1, g^2\}$



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$[g_k]$: $\mathbb{Z}_2$ *invariant conjugacy class of* $D_3 \cong \mathbb{Z}_3 \rtimes \mathbb{Z}_2$

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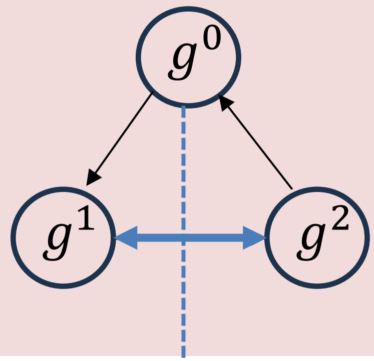

$\mathbb{Z}_2$ gauging (automorphism of $\mathbb{Z}_3$)

$[g_k]$: *$\mathbb{Z}_2$ invariant conjugacy class of $D_3 \cong \mathbb{Z}_3 \rtimes \mathbb{Z}_2$*

Fibonacci fusion rule :

$$[g_0] \cdot [g_0] = [g_0]$$
$$[g_0] \cdot [g_1] = [g_1]$$
$$[g_1] \cdot [g_1] = [g_0] \oplus [g_1]$$

$$\mathbb{I} \otimes \mathbb{I} = \mathbb{I}$$
$$\mathbb{I} \otimes \tau = \tau \otimes \mathbb{I} = \tau$$
$$\tau \otimes \tau = \mathbb{I} + \tau$$

- $[g_1]$ does not have the inverse

Fields Φ_i in 4D QFT :

Φ_i : labeled by the class $[g_i]$

$\tilde{g}^i \in [g_i]$

Selection rules :

An interaction in the classical Lagrangian

$$\mathcal{L} \supset \Phi_1 \cdots \Phi_n$$

is allowed when

$$\tilde{g}^1 \cdots \tilde{g}^n = \text{identity}$$

4D QFT with the Fibonacci fusion rule

$$[g_0]\cdot[g_0]=[g_0]$$
$$[g_0]\cdot[g_1]=[g_1]$$
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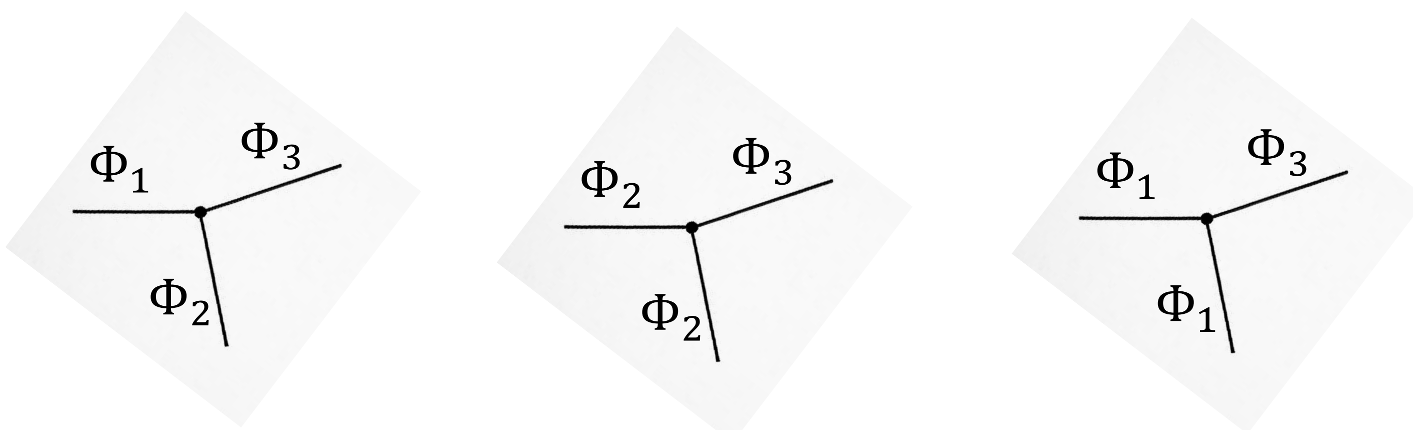

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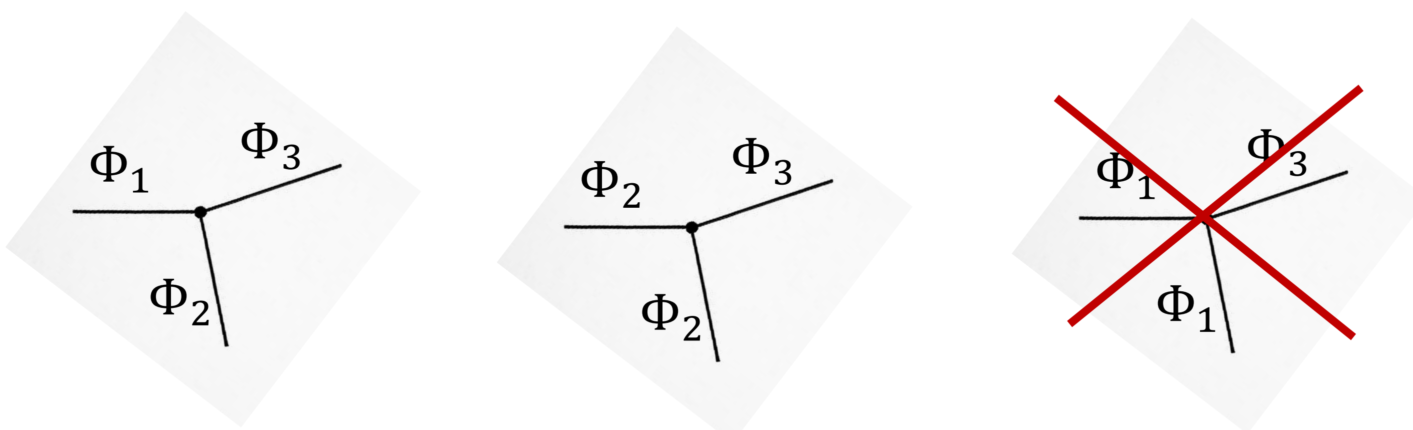


2. *Yukawa textures* with two families

$$m_{\mathrm{down}} = \begin{pmatrix} 0 & * \\ * & * \end{pmatrix}$$

S. Weinberg (79)

Texture zeros approach

Two generations of quarks

- In the basis in which the up-type quark mass matrix is diagonal,

$$m_{\text{down}} = \begin{pmatrix} 0 & m \\ m & M \end{pmatrix}$$

$$\xrightarrow{M \gg m} \begin{pmatrix} m^2/M & 0 \\ 0 & M \end{pmatrix} = \begin{pmatrix} m_d & 0 \\ 0 & m_s \end{pmatrix}$$

The Cabibbo angle is successfully predicted to be

$$\sin\theta_c \sim \tan\theta_c = m/M = \sqrt{m_d/m_s}$$

S. Weinberg (1977)

- *Two generations of fermions* $Y_{ij}\psi_i^L\psi_j^R H$ *(i,j=1,2):*

Left : ($[g_0],[g_1]$*)*
Right : ($[g_0],[g_1]$*)*
Higgs : $[g_1]$

$$Y_{ij} = \begin{pmatrix} 0 & * \\ * & * \end{pmatrix}$$

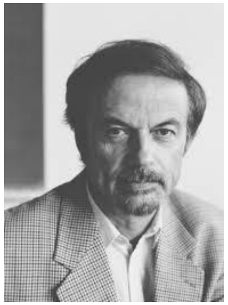
$*$: non-zero entries

Fibonacci fusion rule

$$[g_0]\cdot[g_0] = [g_0]$$
$$[g_0]\cdot[g_1] = [g_1]$$
$$[g_1]\cdot[g_1] = [g_0]\oplus[g_1]$$

One cannot derive Weinberg texture from group-based symmetries, ex., U(1) or $\mathbb{Z}_M$ *for any M*

Yukawa textures with three families



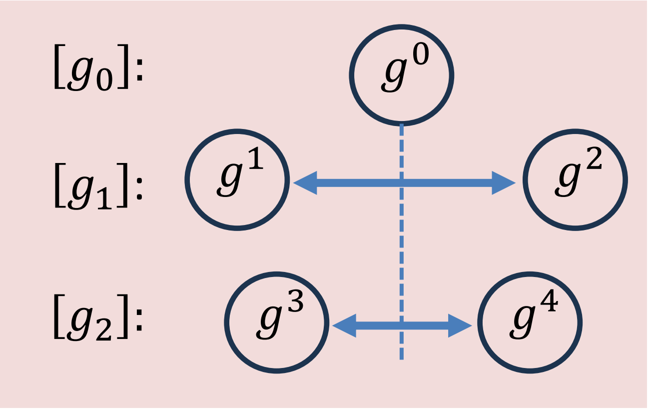
$$m^{(\text{Fritzsch,NNI})} = \begin{pmatrix} 0 & * & 0 \\ * & 0 & * \\ 0 & * & * \end{pmatrix}$$

H. Fritzsch (78), G.C.Branco, L.Lavoura, F.Mota (89),...

$M = 5$ ($\mathbb{Z}_2$ *gauging of* $\mathbb{Z}_5$)

T. Kobayashi, H.O., M. Tanimoto, 2409.05270 [hep-ph]

- Three different classes :



$\mathbb{Z}_2$ gauging (automorphism of $\mathbb{Z}_5$)

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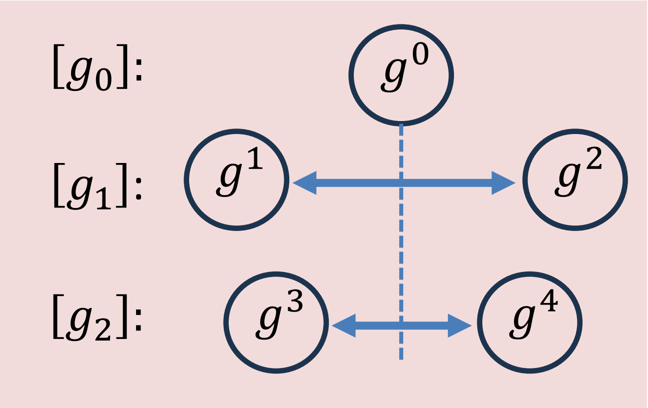


- *Yukawa couplings (3 generations) :*

Left : $([g_0], [g_1], [g_2])$
Right : $([g_0], [g_1], [g_2])$
Higgs : $[g_1]$

$$Y = \begin{pmatrix} 0 & * & 0 \\ * & 0 & * \\ 0 & * & * \end{pmatrix}$$

$*$: non-zero entries

Fusion rules :
$[g_0][g_0] = [g_0]$
$[g_1][g_1] = [g_0] + [g_2]$
$[g_2][g_2] = [g_0] + [g_1]$
$[g_1][g_2] = [g_1] + [g_2]$

"Nearest Neighbor Intereraction"

Yukawa Textures for Leptons

- We classify the Yukawa textures of leptons with Weinberg op.:

$$Y_{ij}\bar{L}_i H e_j + \frac{C_{ij}}{\Lambda} L_i H L_j H$$

Ex.,

$$Y = \begin{pmatrix} 0 & 0 & * \\ 0 & * & * \\ * & * & * \end{pmatrix}, \begin{pmatrix} 0 & * & 0 \\ * & 0 & * \\ 0 & * & * \end{pmatrix} \qquad C = \begin{pmatrix} * & 0 & 0 \\ 0 & * & 0 \\ 0 & 0 & * \end{pmatrix}$$

$*$: non-zero entries

- *Consistent with the charged lepton masses and PMNS matrix*

- *Other textures are discussed in*

T. Kobayashi, H.O., M. Tanimoto, H. Uchida, 2505.07262 [hep-ph]

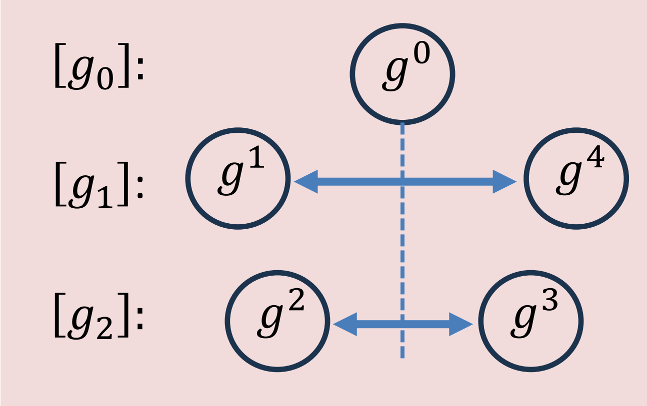
3. *MSSM with non-invertible selection rules*

MSSM with non-invertible selection rules

Y. Nakai, H.O., Y. Shigekami, Z. Zhang, 2512.21509 [hep-ph]

$$W = Y_u Q H_u U + Y_d Q H_d D + Y_e L H_d E + Y_\nu L H_u N + M_N N N + \mu H_u H_d$$

$(\mathbb{Z}_2$ *gauging of* $\mathbb{Z}_5) \times (\mathbb{Z}_2$ *gauging of* $\mathbb{Z}_5)$



$$Y_u = \begin{pmatrix} * & 0 & 0 \\ 0 & * & 0 \\ 0 & 0 & * \end{pmatrix}, \quad Y_d = \begin{pmatrix} 0 & * & 0 \\ * & * & * \\ 0 & * & * \end{pmatrix}, \quad Y_e = \begin{pmatrix} * & 0 & 0 \\ 0 & * & 0 \\ 0 & 0 & * \end{pmatrix}, \quad Y_\nu = \begin{pmatrix} 0 & * & 0 \\ * & 0 & * \\ 0 & * & * \end{pmatrix}$$

- *Consistent with the flavor structure of quarks and leptons*
- *We check that the Yukawa textures remain stabile under RG effects.*

MSSM with non-invertible selection rules

Y. Nakai, H.O., Y. Shigekami, Z. Zhang, 2512.21509 [hep-ph]

$$\mathcal{L}_{\text{soft}} \supset -\sum_f M_f^2 \tilde{f}^\dagger \tilde{f} - \left(A_u \tilde{Q} H_u \tilde{u} + A_d \tilde{Q} H_d \tilde{d} + A_e \tilde{L} H_d \tilde{e} + \text{c.c.} \right)$$

- *All sfermion masses are diagonalized due to the selection rule*
  → suppressing dangerous FCNC processes

$$M_f^2 \simeq M_{\text{SUSY}}^2 \, \text{diag}\left(O(1), O(1), O(1)\right)$$

$$A_f \simeq M_{\text{SUSY}} \, O(1) \, Y_f \qquad f = u, d, e$$

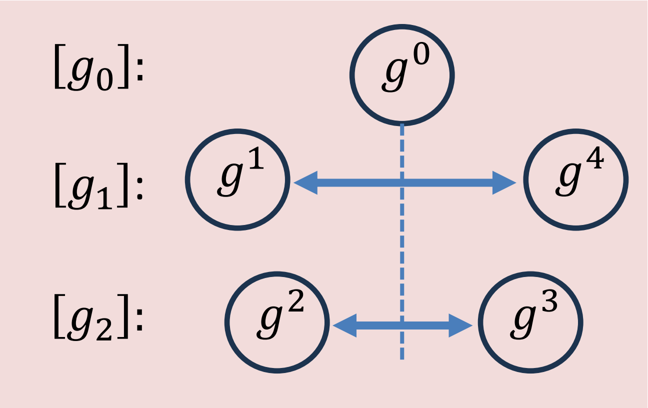
Soft terms remain stabile under RG effects

→ *Non-invertible selection rules can address the flavor structure and FCNC problems*

4. CP from non-invertible selection rules

Group-like symmetries in non-invertible selection rules

Ex. $\mathbb{Z}_2$ *gauging of* $\mathbb{Z}_5$



Fusion rules :

$[g_0][g_0] = [g_0]$

$[g_1][g_1] = [g_0] + [g_2]$

$[g_2][g_2] = [g_0] + [g_1]$

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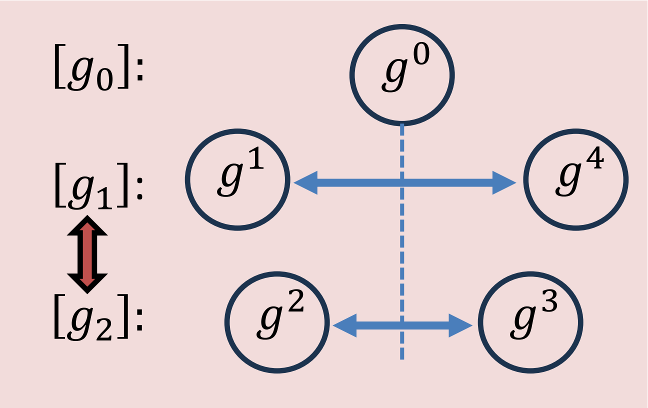
Group-like symmetries in non-invertible selection rules

Ex. $\mathbb{Z}_2$ *gauging of* $\mathbb{Z}_5$

$\mathbb{Z}_2$ *symmetry :* $[g_1] \leftrightarrow [g_2]$



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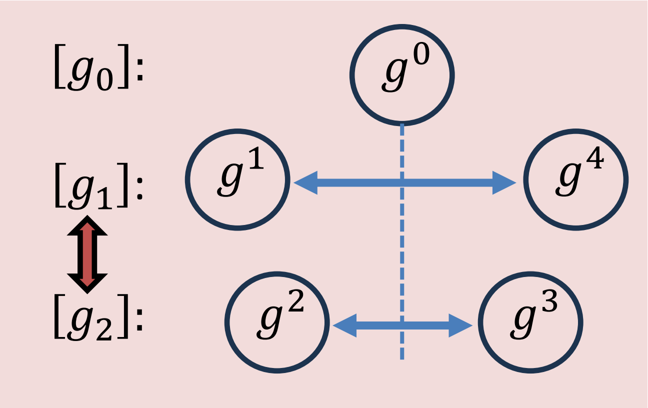

$\mathbb{Z}_2$ *symmetry :* $[g_1] \leftrightarrow [g_2]$

$\mathbb{Z}_2$ *flavor symmetry :* $\Phi_1 \leftrightarrow \Phi_2$

Fusion rules :

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$[g_1][g_1] = [g_0] + [g_2]$
$[g_2][g_2] = [g_0] + [g_1]$
$[g_1][g_2] = [g_1] + [g_2]$

Q : What is a class of Φ^*(the conjugate of Φ) ?

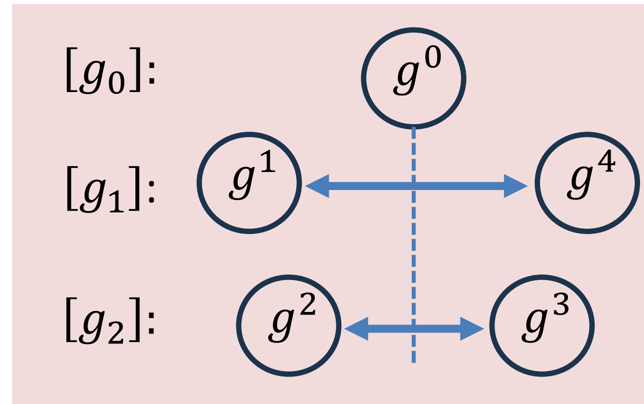
A : Φ^* is labeled by the inverse class $[g_i^{-1}]$

Ex. $\mathbb{Z}_2$ *gauging of* $\mathbb{Z}_5$

$$[g_1] = \{g^1, g^4\}$$

$$(g^1)^{-1} = g^4$$
$$(g^4)^{-1} = g^1$$

$$[g_1^{-1}] = \{g^4, g^1\} = [g_1]$$



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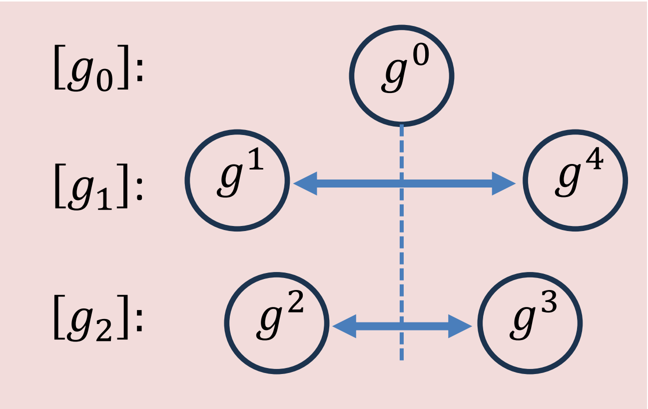


$[g_i^{-1}] = [g_i]$ for $\mathbb{Z}_2$ *gauging of* $\mathbb{Z}_N$

- Φ and Φ^* are labeled by the same class

Q : Can we consider the different class between Φ and Φ^* ?

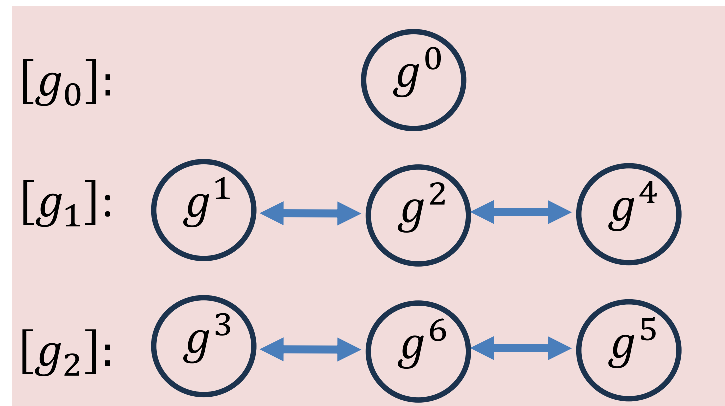
A : Different gauging of $\mathbb{Z}_N$

CP in non-invertible selection rules

T. Kobayashi, H.O., 2512.16376 [hep-ph]

Ex. $\mathbb{Z}_3$ *gauging of* $\mathbb{Z}_7$

$g \equiv e^{2\pi i/7}$



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$[g_2][g_2] = 2[g_1] + [g_2]$

$[g_1][g_2] = 3[g_0] + [g_1] + [g_2]$

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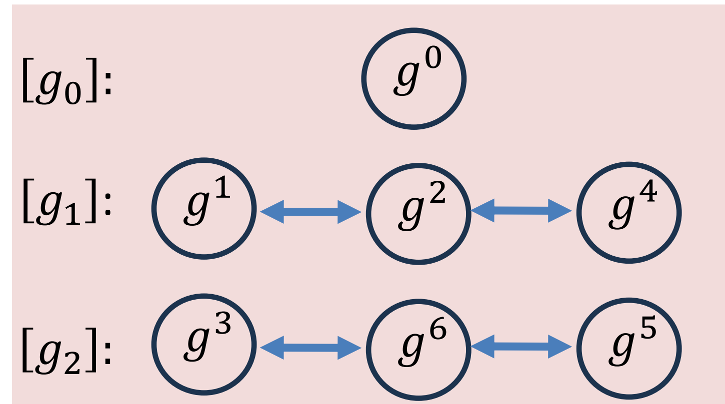

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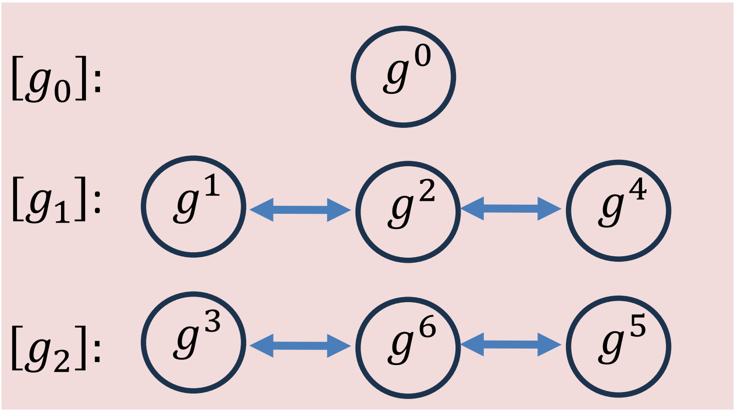

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- Φ and Φ^* are labeled by a different class, ex.,
  $\Phi : [g_1]$
  $\Phi^* : [g_2]$

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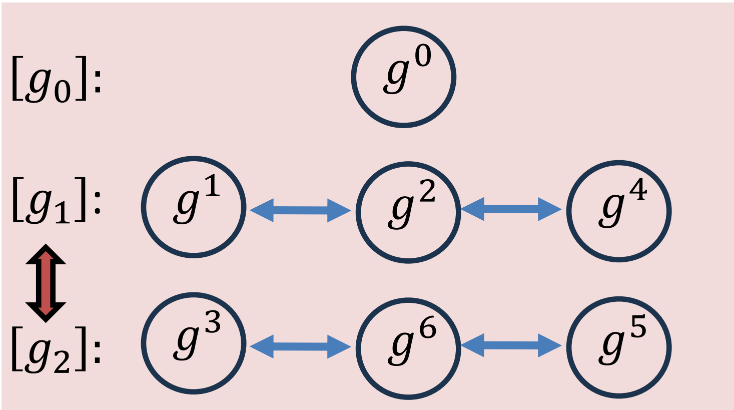

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$[g_1][g_2]=3[g_0] + [g_1] + [g_2]$

$\mathbb{Z}_2$ *symmetry :* $[g_1] \leftrightarrow [g_2]$

- Φ and Φ^* are labeled by a different class, ex.,

  $\Phi : [g_1]$

  $\Phi^*: [g_2]$

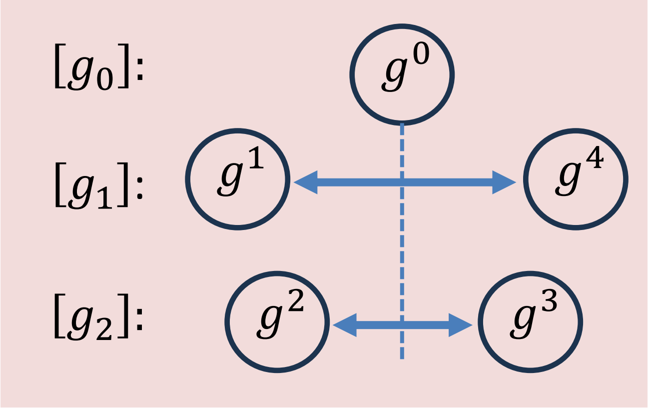
  CP transformation : $\Phi \leftrightarrow \Phi^*$

Flavor and CP in non-invertible selection rules

T. Kobayashi, H.O., 2512.16376 [hep-ph]

Combining two selection rule :

$\mathbb{Z}_2$ *gauging of* $\mathbb{Z}_5$



Yukawa Textures

$$Y = \begin{pmatrix} 0 & * & 0 \\ * & 0 & * \\ 0 & * & * \end{pmatrix}$$

Flavor and CP in non-invertible selection rules

T. Kobayashi, H.O., 2512.16376 [hep-ph]

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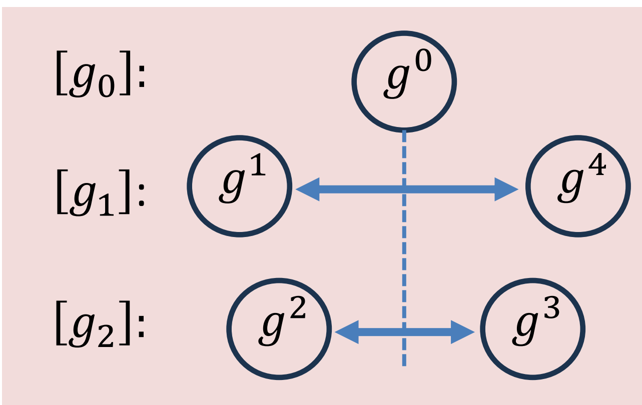
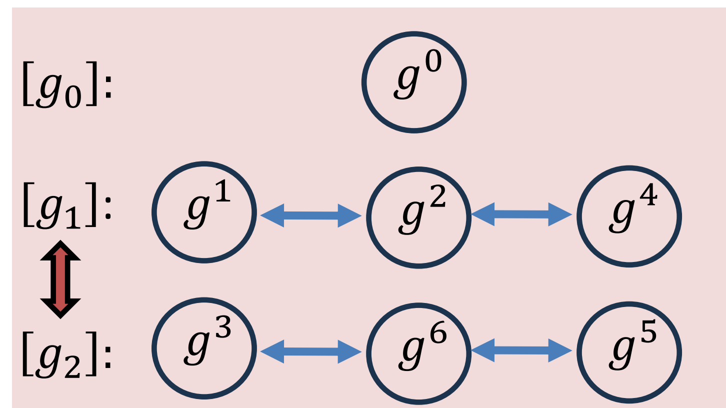


$\mathbb{Z}_3$ *gauging of* $\mathbb{Z}_7$



Yukawa Textures

$$Y = \begin{pmatrix} 0 & * & 0 \\ * & 0 & * \\ 0 & * & * \end{pmatrix}$$

CP (real couplings)

$$Y = Y^*$$

Spontaneous CP violation

T. Kobayashi, H.O., 2512.16376 [hep-ph]

Ex. $\mathbb{Z}_3$ *gauging of* $\mathbb{Z}_7$

Fusion rules :

$[g_0][g_0]=[g_0]$
$[g_1][g_1]=[g_1]+2[g_2]$
$[g_2][g_2]=2[g_1]+[g_2]$
$[g_1][g_2]=3[g_0]+[g_1]+[g_2]$

Φ : SM singlet field labeled by $[g_1]$

$$V = m^2\Phi\Phi^* + \xi_1\Phi\Phi^*(\Phi+\Phi^*) + \xi_2(\Phi^3+\Phi^{*3}) + \lambda_1(\Phi\Phi^*)^2 + \lambda_2\Phi\Phi^*(\Phi^2+\Phi^{*2}) + \lambda_3(\Phi^4+\Phi^{*4}).$$

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T. Kobayashi, H.O., 2512.16376 [hep-ph]

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Spontaneous CP violation by Φ with $m^2, \xi_1 < 0, \quad \xi_2, \lambda_i > 0$

$$\mathcal{L}_{\text{quark}} = -(Y_u)_{ij}\bar{Q}_{L,i}Hu_{R,j} - (Y_d)_{ij}\bar{Q}_{L,i}H^c d_{R,j} - (Y_u')_{ij}\frac{\Phi}{\Lambda}\bar{Q}_{L,i}Hu_{R,j} - (Y_d')_{ij}\frac{\Phi}{\Lambda}\bar{Q}_{L,i}H^c d_{R,j}$$

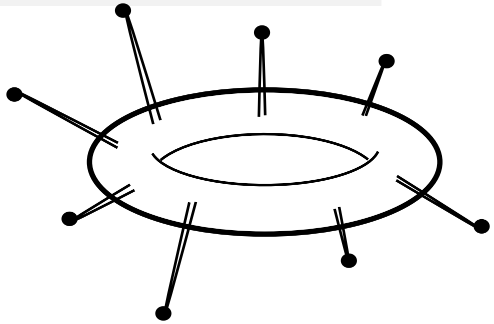
For some models, one can realize the same texture for $Y_{u,d}$ and $Y'_{u,d}$

5. String compactifications → Non-invertible selection rules

i) Heterotic string on toroidal orbifolds (2 slide)

ii) Type IIB magnetized D-brane models on toroidal orbifolds

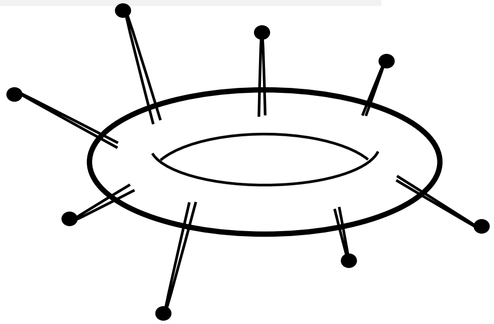
Heterotic string on toroidal orbifolds (1/2)



- Orbifold geometry requires for a string to be closed up to space group action $h = (\theta, v) \in S$

θ : Point group action
v : Lattice translation

Heterotic string on toroidal orbifolds (1/2)



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- Consider a complexified worldsheet boson Z

$$Z(\sigma + \pi) = hZ(\sigma) = \theta Z + v$$

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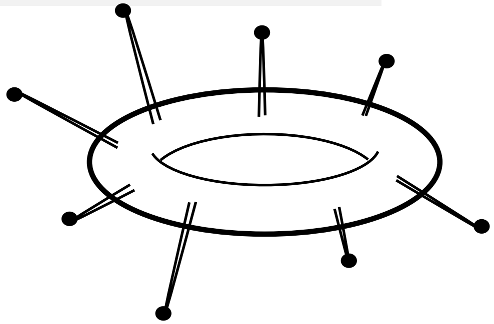

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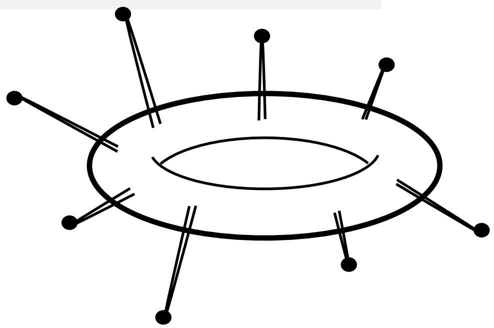

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$$\rightarrow Z(\sigma + \pi) = (g^{-1}hg)Z(\sigma)$$

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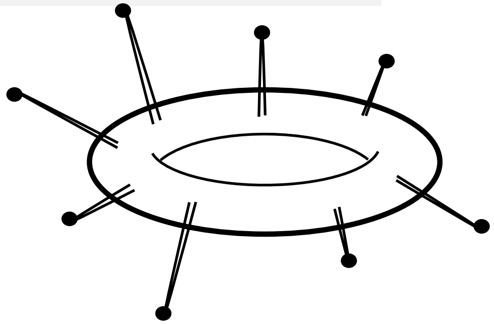

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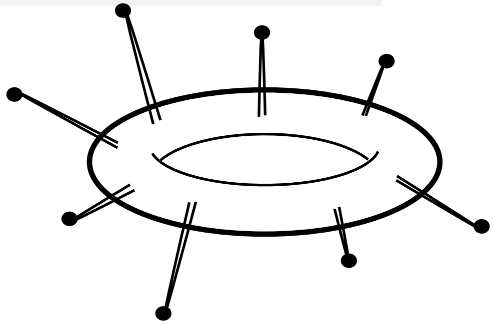

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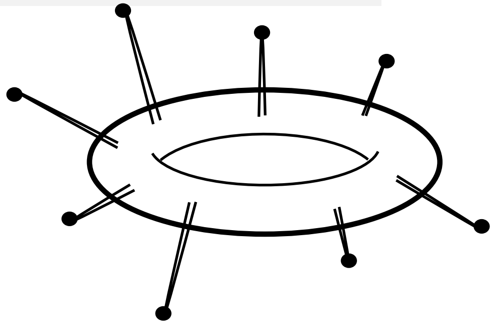
$$\rightarrow Z(\sigma + \pi) = (g^{-1}hg)Z(\sigma)$$

- The Hilbert space is specified by

$$\mathcal{H}_{[h]} = \{Z |\, Z(\sigma + \pi) = hZ(\sigma), h \sim g^{-1}hg, \qquad g, h \in S\}$$

Heterotic string on toroidal orbifolds (2/2)



- Twisted string is labeled by conjugacy group of the space group S :

$$\mathcal{H}_{[h]} = \{Z | \, Z(\sigma+\pi) = hZ(\sigma), h \sim g^{-1}hg, \qquad g, h \in S\}$$

- Selection rules of twisted string :

Fusion : $[g_i] \cdot [g_j] = \sum_k d_{ij}^k [g_k]$

→ Non-invertible selection rules on (non) Abelian orbifolds

($\mathbb{Z}_N, S_3, T_7$ orbifolds by T. Kobayashi, R. Nishida, H.O., 2509.10019[hep-th])

4. String compactifications → Non-invertible selection rules

i) Heterotic string on toroidal orbifolds (2 slide)
ii) Type IIB magnetized D-brane models on toroidal orbifolds (1 slide)

Overview

T. Kobayashi, H.O., 2408.13984 [hep-th]

(i) $\mathbb{Z}_N$ *symmetry* :

$$\phi^j \to g^{k}{}_j \phi^j$$

(ii) Gauging $\mathbb{Z}_2$ *(*the automorphism of $\mathbb{Z}_N$*),*
i.e., $D_N \cong \mathbb{Z}_N \rtimes \mathbb{Z}_2$

$$[g_k] = \{hg^k h^{-1} \mid h = e, r\}$$

$[g_k]$ includes g^k and $g^{-k} = g^{M-k}$

(iii) $\mathbb{Z}_2$ *invariant modes labeled by* $[g^k]$

$$\Phi_j = \phi_j + \phi_{M-j}$$

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Symmetry of chiral zero modes on higher-dimensional Super Yang-Mills on T^2 with magnetic fluxes

(EFT of type IIB Magnetized D-brane models)

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Symmetry of chiral zero modes on higher-dimensional Super Yang-Mills on T^2 with magnetic fluxes
(EFT of type IIB Magnetized D-brane models)

$T^2/\mathbb{Z}_2$ orbifold with gauged $\mathbb{Z}_2$
(leave all operators invariant under $\mathbb{Z}_2$*)*

Fusion rules of momentum ops.:

$$\hat{U}_i^{(\lambda)} \hat{U}_i^{(\lambda')} = \hat{U}_i^{(\lambda+\lambda')} + \hat{U}_i^{(\lambda-\lambda')}$$

$$i = 1,2$$

$\mathbb{Z}_2$ invariant modes on $T^2/\mathbb{Z}_2$

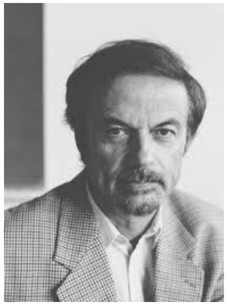
Non-invertible symmetries
→ Flavor symmetries of chiral zero modes (quarks/leptons)

Conclusions

Main message : Fusion rules : $[g_a] \cdot [g_b] = \sum_c N_{ab}^c [g_c]$
→ Novel Yukawa textures and CP

- leading to the non-trivial structure of mass matrices including

$$m_{\text{down}} = \begin{pmatrix} 0 & * \\ * & * \end{pmatrix}$$



$$m^{(\text{Fritzsch,NNI})} = \begin{pmatrix} 0 & * & 0 \\ * & 0 & * \\ 0 & * & * \end{pmatrix}$$

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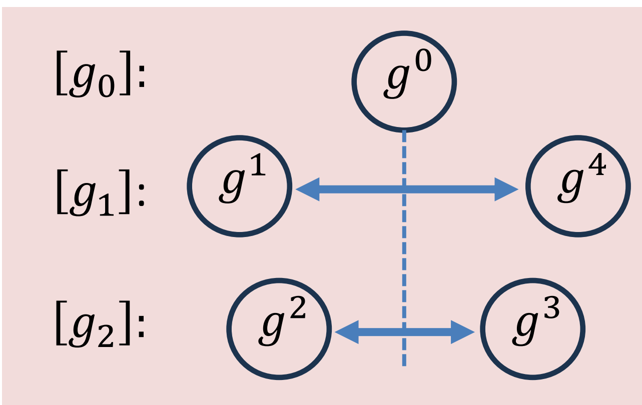
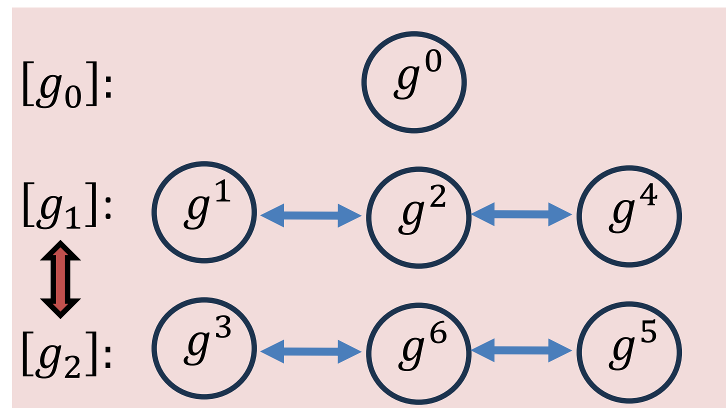
$\mathbb{Z}_2$ gauging of $\mathbb{Z}_5$



$\mathbb{Z}_3$ gauging of $\mathbb{Z}_7$



Yukawa Textures

$$Y=\begin{pmatrix}0 & * & 0\\ * & 0 & *\\ 0 & * & *\end{pmatrix}$$

CP (real couplings)

$$Y=Y^*$$

Discussions

1. Origin of non-invertible selection rules

- Heterotic string on toroida orbifolds and Calabi-Yau threefolds
  (Twisted states are labeled by conjugacy classes)

  T. Kobayashi, H.O., R. Nishida, 2504.09773, 2509.10019

- Type IIB magnetized D-brane models on toroidal orbifolds

  T. Kobayashi, H.O., 2408.13984 [hep-th]

2. Other non-invertible selection rules ?

- Selection rules of conjugacy classes are summalized in

  J. Dong, T. Jeric, T. Kobayashi, R. Nishida, and H.O., 2507.02375 [hep-th]

3. Other phenomenological applications ?

- Radiative seesaw models — T. Kobayashi, H.O., T. Nomura, H. Okada, O. Popov, Y. Shigekami,.. ('25)
- Strong CP problem — Q. Liang, T. T. Yanagida ('25), T. Kobayashi, H.O., T. T. Yanagida ('25)
- GUT-like models — T. Kobayashi, H.O., T. T. Yanagida ('25)

Thank you!