Gravitational particle production

Oleg Lebedev

UNIVERSITY OF HELSINKI

University of Helsinki

– **cosmological gravitational particle production:**

- *always there*
- *important for very weakly coupled particles*
- *non-thermal dark matter*

– **production of $s = 0, \frac{1}{2}, 1$ states**

- *qualitatively different features*

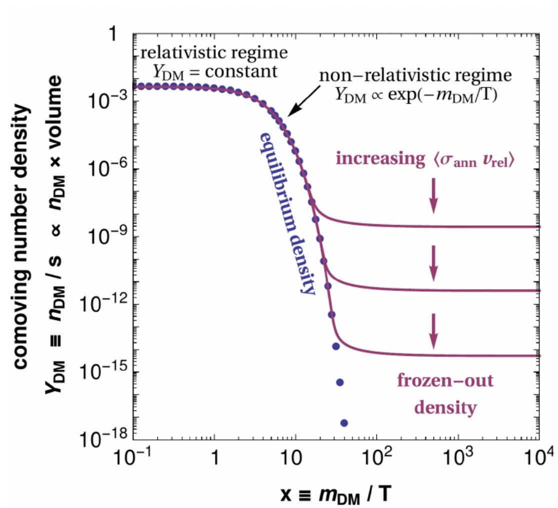
– **quantum gravity effects**

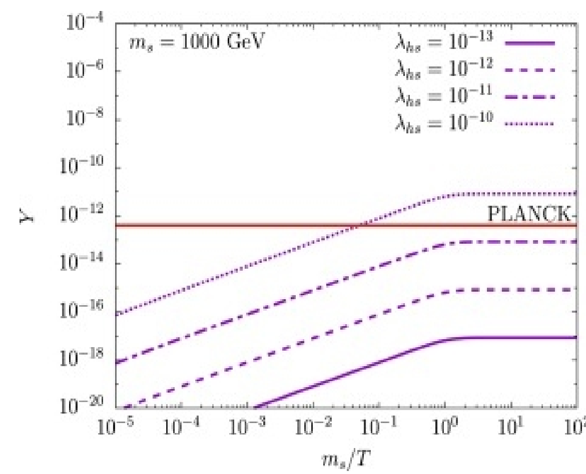
- *as important as classical gravity*

Dark Matter Models

thermal

non-thermal





No memory

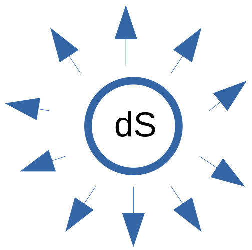
Memory

(“attractor solution”)

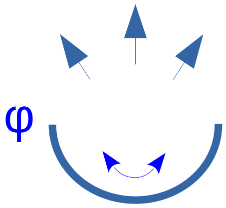
OL 2210.02293

Particle production in the Early Universe:

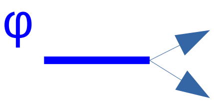
- during inflation



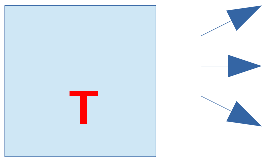
- via inflaton oscillations



- inflaton decay



- thermal emission (freeze-in)



Stable (very) <u>weakly</u> coupled particles accummulate!

Particle production via classical gravity

Parker, Grib, Zeldovich, Starobinsky,... 60-70's

Expanding Friedmann Universe :

$$g_{\mu\nu} = a^2(\eta)\,\mathrm{diag}(1,-1,-1,-1)$$

(*conformal time* η : $dt = a\,d\eta$)

Free scalar field :

$$\mathcal{L} = \frac{1}{2} g^{\mu\nu}\partial_\mu\Phi\,\partial_\nu\Phi - V(\Phi)$$

$$V(\Phi) = \frac{1}{2}m^2\Phi^2$$

Rescaled scalar :

$$\phi = a(\eta)\,\Phi$$

Mode expansion and EOM :

$$\phi(\eta,\mathbf{x}) = \int \frac{d^3\mathbf{k}}{(2\pi)^3}\left[a_{\mathbf{k}}\,\chi_k(\eta)\,e^{i\mathbf{k}\cdot\mathbf{x}} + a^\dagger_{\mathbf{k}}\,\chi^*_k(\eta)\,e^{-i\mathbf{k}\cdot\mathbf{x}}\right]$$

$$\chi''_k + \omega_k^2\,\chi_k = 0 \quad , \quad \omega_k^2(\eta) = k^2 + a^2(\eta)\,m^2 + \left(\frac{1}{6}-\xi\right)a^2(\eta)\,R(\eta)$$

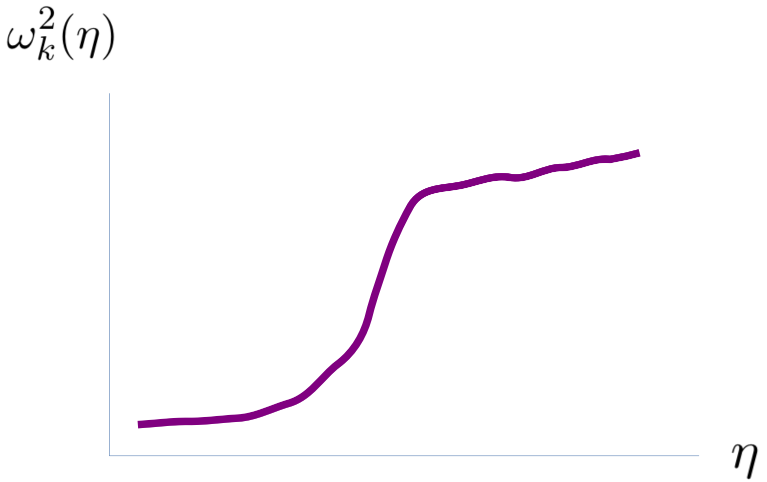
particle production!

To define a particle, need adiabatically flat space-time and Fock space vacuum

$$\chi_k'' + \omega_k^2\,\chi_k = 0$$

$$\eta \to -\infty \quad : \qquad \omega_k^2(\eta) \;\to\; k^2$$

$$\eta \to +\infty \quad : \qquad \omega_k^2(\eta) \;\to\; a^2(\eta)\,m^2$$



Asymptotic solutions :
("Minkowski-like")

$$\chi_k^{\rm in}(\eta \to -\infty) \to \frac{e^{-i\int^\eta d\eta'\omega_k(\eta')}}{\sqrt{2\omega_k(\eta)}}\,,$$

$$\chi_k^{\rm out}(\eta \to +\infty) \to \frac{e^{-i\int^\eta d\eta'\omega_k(\eta')}}{\sqrt{2\omega_k(\eta)}}\,.$$

Bunch-Davies vacuum *:*
(no particles initially)

$$a_{\mathbf{k}}^{\rm in}\,|0^{\rm in}\rangle = 0$$

Keeping the field intact, can choose different expansion modes,

$$\phi(\eta,\mathbf{x}) = \int \frac{d^3\mathbf{k}}{(2\pi)^3}\left[a_{\mathbf{k}}\,\chi_k(\eta)\,e^{i\mathbf{k}\cdot\mathbf{x}} + a^\dagger_{\mathbf{k}}\,\chi^*_k(\eta)\,e^{-i\mathbf{k}\cdot\mathbf{x}}\right]$$

Bogolyubov transform :

$$\begin{cases} \chi_k^{\rm in}(\eta) = \alpha_k\,\chi_k^{\rm out}(\eta) + \beta_k\,\chi_k^{{\rm out}\,*}(\eta) \\ a^{\rm in}_{\mathbf{k}} = \alpha^*_k\,a^{\rm out}_{\mathbf{k}} - \beta^*_k\,a^{{\rm out}\,\dagger}_{-\mathbf{k}} \end{cases}$$

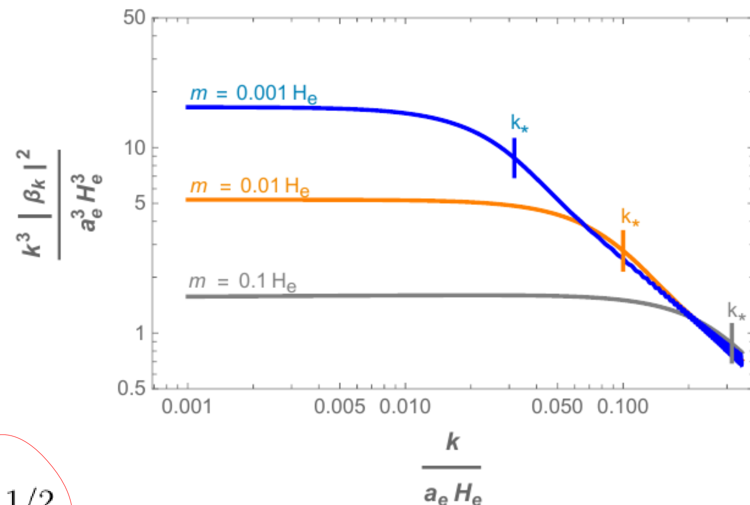
$\Rightarrow$ $a^{\rm out}_{\mathbf{k}}\,|0^{\rm in}\rangle \neq 0$

Particle number at late times:

Feiteira, OL' 25

$$\langle 0^{\rm in}|N^{\rm out}|0^{\rm in}\rangle = V\int\frac{d^3\mathbf{k}}{(2\pi)^3}\,|\beta_k|^2$$

$$\beta_k = i(\chi_k^{\rm out\,\prime}\chi_k^{\rm in} - \chi_k^{\rm out}\chi_k^{\rm in\,\prime})$$

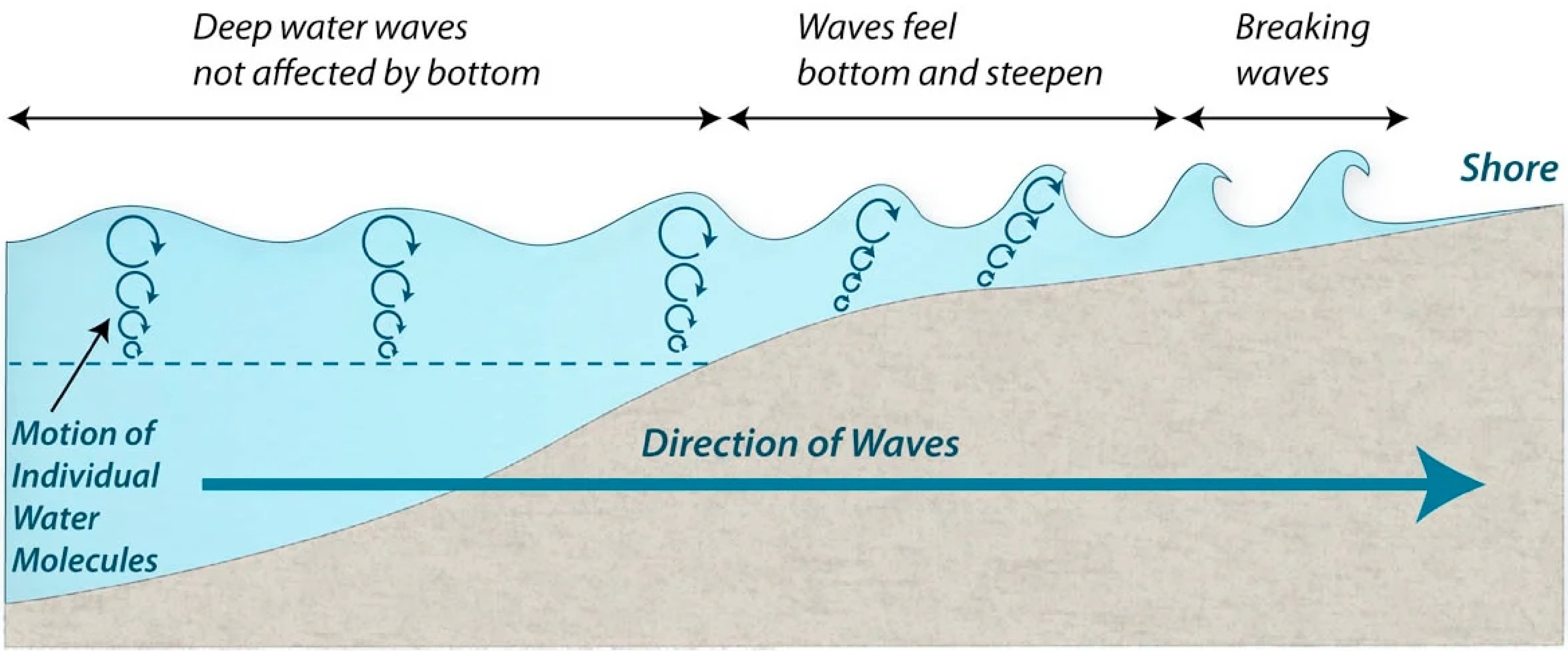


<u>*Total*</u>: $$a^3 n = \frac{3\kappa^2\,a_e^3}{4\pi^2}\,\frac{H_e^{11/2}}{m^{5/2}}$$

huge!

Inflationary particle production:

fluctuations → *particles*



Starobinsky ~ Bogolyubov

$\overline{\Phi}$ → particles when **H ~ m**

particle abundance

$$Y = \frac{n}{s_{\rm SM}} \quad , \quad s_{\rm SM} = \frac{2\pi^2 g_*}{45} T^3$$

radiation domination:

$$Y \simeq 0.07 \times \frac{\overline{\Phi}^2}{m^{1/2} M_{\rm Pl}^{3/2}}$$

$$m \ll \text{ eV}$$

matter domination:

$$Y \simeq 0.07 \times \frac{1}{\Delta} \frac{H_e^{1/2}\, \overline{\Phi}^2}{m M_{\rm Pl}^{3/2}}$$

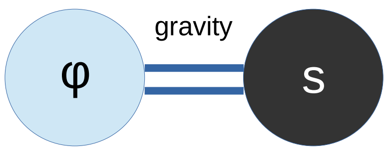
$$\Delta \equiv \sqrt{\frac{H_e}{H_R}} \simeq \frac{T_{\rm inst}}{T_R} \gg 1$$

$$T_R \lesssim \text{GeV}$$

Quantum gravity effects

OL,Yoon' 22
OL 2210.02293

Quantum gravity induces all gauge invariant operators
(*with unknown coefficients*)



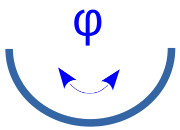
Effective field theory regime ***after inflation*** $\varphi \lesssim M_{\rm Pl}$ →

Lowest order inflaton (φ) – dark relic (s) couplings:

C_i = unknown Wilson coefficients ;
omit dim-5 by φ-parity

$$\Delta\mathcal{L}_6 = \frac{C_1}{M_{\rm Pl}^2}\,(\partial_\mu\phi)^2 s^2 + \frac{C_2}{M_{\rm Pl}^2}\,(\phi\partial_\mu\phi)(s\partial^\mu s) + \frac{C_3}{M_{\rm Pl}^2}\,(\partial_\mu s)^2\phi^2 - \frac{C_4}{M_{\rm Pl}^2}\,\phi^4 s^2 - \frac{C_5}{M_{\rm Pl}^2}\,\phi^2 s^4$$

Oscillating inflaton background → **efficient particle production**



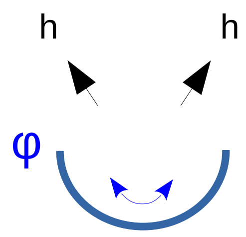
Dolgov, Kirilova '90
Traschen, Brandenberger '90

Example: perturbative calculation (*no collective effects*)

Take

$$\Delta V = \frac{1}{4}\lambda_{\phi h}\phi^2 h^2$$

$$\phi^2(t) = \sum_{n=-\infty}^{\infty} \zeta_n e^{-in\omega t}$$



Transition amplitude $|0\rangle \to |p,q\rangle$

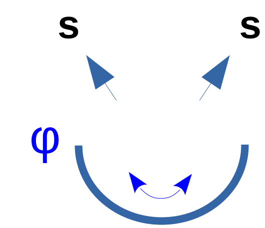
$$-i\int_{-\infty}^{\infty} dt\langle f|V(t)|i\rangle = -i\,\frac{\lambda_{\phi h}}{2}\,(2\pi)^4\delta(\mathbf{p}+\mathbf{q})\sum_{n=1}^{\infty}\zeta_n\delta(E_p+E_q-n\omega)$$

Reaction rate per unit volume

$$\Gamma = \sum_{n=1}^{\infty}\Gamma_n = \sum_{n=1}^{\infty}\frac{1}{2}\int|\mathcal{M}_n|^2 d\Pi_n = \frac{\lambda_{\phi h}^2}{64\pi}\sum_{n=1}^{\infty}|\zeta_n|^2\sqrt{1-\left(\frac{2m_h}{n\omega}\right)^2}\,\theta(n\omega-2m_h)$$

Same applies to gravity-induced operators

$$\frac{\phi^4 s^2}{M_{\rm Pl}^2}\ ,\quad \frac{\phi^6 s^2}{M_{\rm Pl}^4}\ ,\quad \frac{\phi^8 s^2}{M_{\rm Pl}^6}\ ,\ \dots$$



Take $\frac{C_4}{M_{\rm Pl}^2}\,\phi^4 s^2$:

– compute *s*-production rate Γ

– solve the Boltzmann equation $\dot{n} + 3Hn = 2\Gamma$

– apply bound on dark relic abundance $Y \leq 4.4 \times 10^{-10}\,\frac{\rm GeV}{m_s}$

$$|C_4| < 10^{-3}\,\Delta_{\rm NR}^{1/2}\,\frac{H_{\rm end}^{5/4}\,M_{\rm Pl}^{11/4}}{\phi_0^4}\sqrt{\frac{\rm GeV}{m_s}}$$

dilution factor

$$\Delta_{\rm NR} \equiv \left(\frac{H_{\rm end}}{H_{\rm reh}}\right)^{1/2}$$

Typical bound strength :

($\phi_0 \sim M_{\rm Pl}$, $H_{\rm end} \sim 10^{14}$ GeV , $\Delta_{\rm NR} \sim 1$, $m_s \gtrsim 1$ GeV)

$$|C_4| < 10^{-8}$$

??

Koutroulis, OL, Pokorski ‘24

Fermions

Lowest order operator = dim-5:

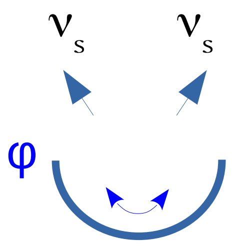
$$\frac{\mathcal{C}}{M_{\rm Pl}}\,\phi^2\,\bar\Psi\Psi$$

Bound from the dark relic abundance:

$$\mathcal{C} \lesssim 10^{-4}\,\Delta_{\rm NR}^{1/2}\,\frac{M_{\rm Pl}^{3/4} H_e^{1/4}}{\phi_0}\,\sqrt{\frac{\rm GeV}{M}}$$

Typical bound:

$$\mathcal{C} \leq 10^{-5}$$



*produces viable **COLD keV sterile neutrino DM** (C ~ 0.1)*

Quark-lepton production by classical gravity:

By conformal transformation:

Parker ‘ 71
Chung, Everett, Yoo, Zhou ‘11

$$(i\gamma^\mu \partial_\mu - a(\eta) M)\,\Psi = 0 \quad \Rightarrow \quad Y \propto \left(\frac{M}{M_{\rm Pl}}\right)^{3/2}$$

Seems tiny for standard fermion masses...

But the Higgs field is **large**:

Starobinsky, Yokoyama ‘94

$$\langle h^2 \rangle \to 0.1 \frac{H^2}{\sqrt{\lambda_h}}$$

Feiteira, Koutroulis, OL, Pokorski ‘25

$$Y^{\rm SM} \sim 10^{-3} \times \frac{Y_f^{7/2}}{\lambda_h} \left(\frac{H_e}{M_{\rm Pl}}\right)^{3/2}$$

up to 10^{20} above naive estimate

Non-reheating SM production

Bottom line: gravitationally produced relics are (over) abundant

How to get rid of them?

inflaton energy density ~ a^{-3}

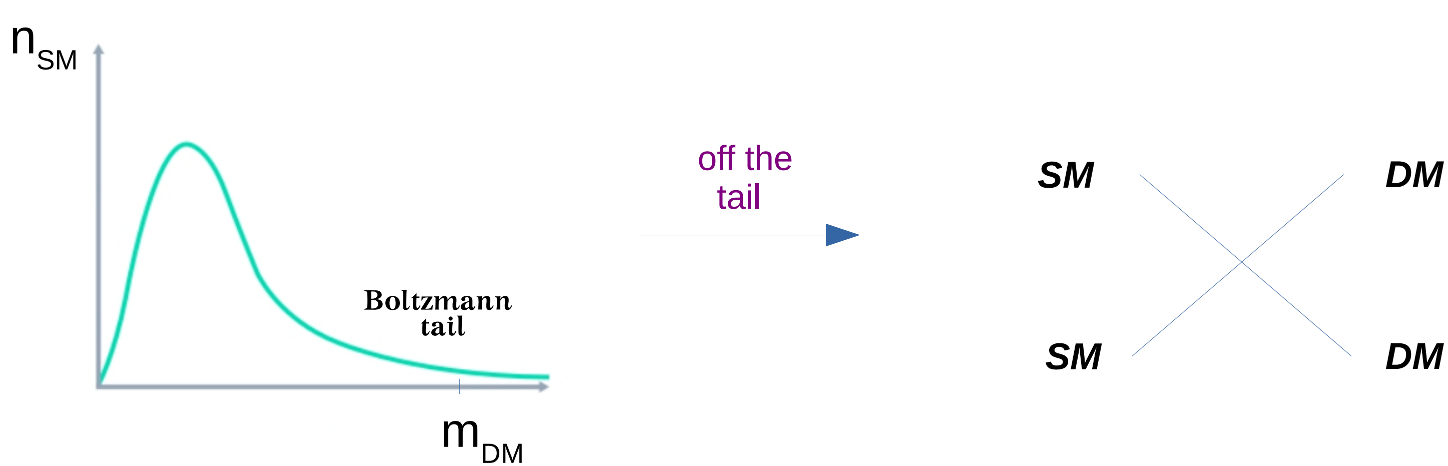
rel. relic energy density ~ a^{-4}

↓

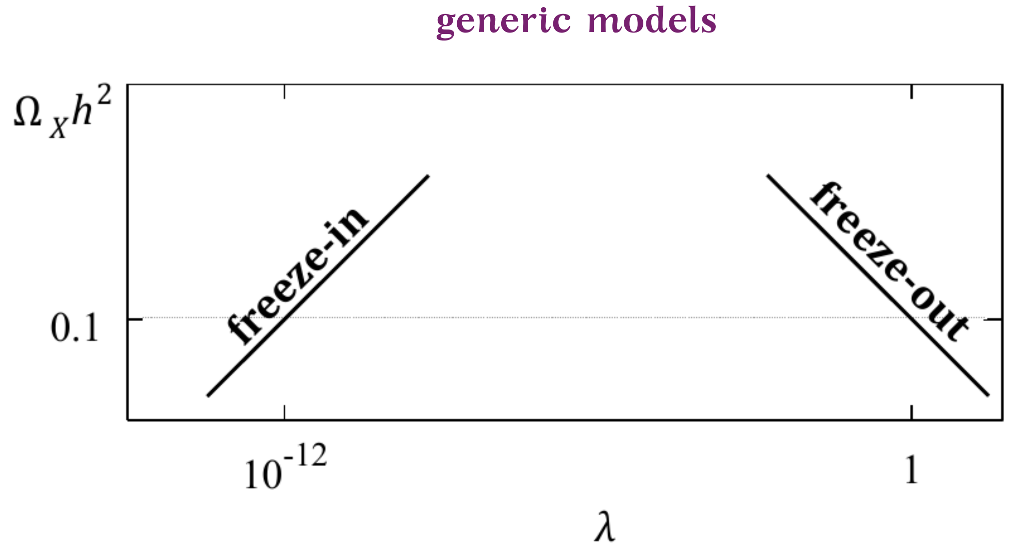
low T_R

What if $T_R < m_{DM}$?

Cosme,Costa,OL ‘23



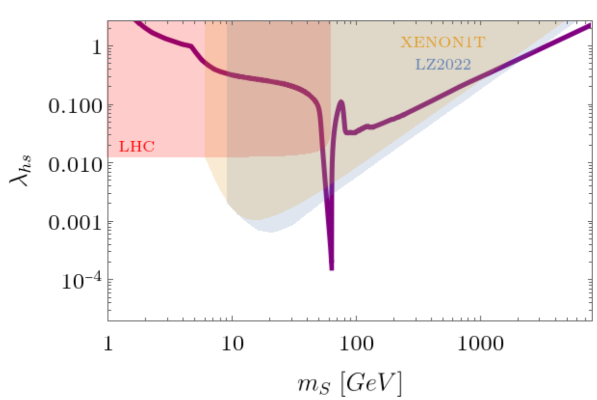
Cosme,Costa,OL ‘23



freeze-in at stronger coupling

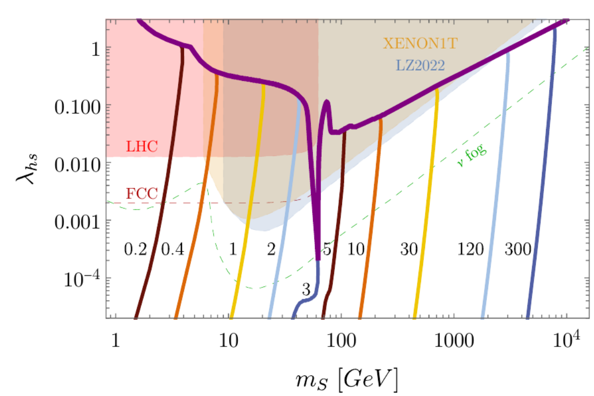
Arcadi,Costa,Goudelis,OL ‘24

“excluded”



→

viable



On spin-1 particle production: some subtleties

A massive vector with non-minimal couplings to gravity:

$$S = \int d^4x \sqrt{|g|} \left(-\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \frac{1}{2} m_A^2 g^{\mu\nu} A_\mu A_\nu - \frac{1}{2} \xi_1 R g^{\mu\nu} A_\mu A_\nu - \frac{1}{2} \xi_2 R^{\mu\nu} A_\mu A_\nu \right)$$

PHYSICAL REVIEW LETTERS **133,** 061602 (2024)

Editors' Suggestion

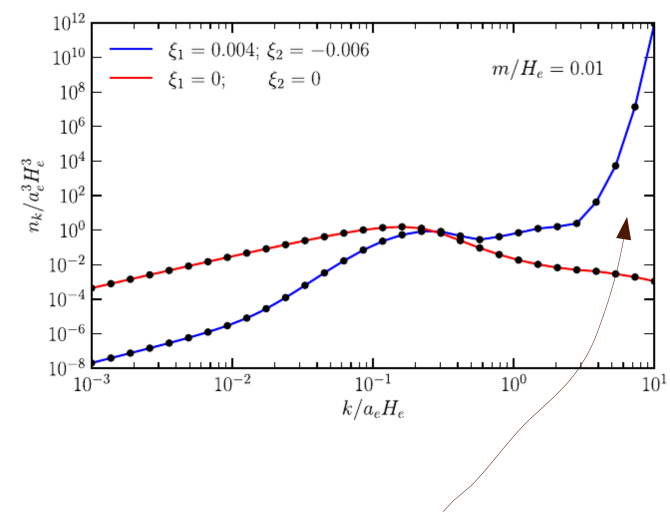
Runaway Gravitational Production of Dark Photons

Christian Capanelli,[1,*] Leah Jenks,[2,†] Edward W. Kolb,[3,‡] and Evan McDonough[4,§]

[1] *Department of Physics and Trottier Space Institute, McGill University, Montréal, Quebec H3A 2T8, Canada*
[2] *Kavli Institute for Cosmological Physics, The University of Chicago, Chicago, Illinois 60637, USA*
[3] *Kavli Institute for Cosmological Physics and Enrico Fermi Institute, The University of Chicago, Chicago, Illinois 60637, USA*
[4] *Department of Physics, University of Winnipeg, Winnipeg, Manitoba R3B 2E9, Canada*

(Received 10 May 2024; revised 7 July 2024; accepted 12 July 2024; published 9 August 2024)

We demonstrate that gravitational particle production of a massive, Abelian, vector (Proca) field during inflation in the presence of nonminimal coupling to gravity may suffer from an instability which leads to runaway production of high-momentum modes. This is untenable unless there is some mechanism to regulate the runaway. We discuss the parameter space of the particle mass and nonminimal couplings where such a runaway occurs and possible ways to tame the runaway. We find that there is no obvious way to resolve the runaway in a UV completion or with kinetic mixing to the standard model.



runaway production of high-momentum modes

… however, this is an effective theory (UV-incomplete):

A A* → *graviton graviton scattering :

$$|\mathcal{A}| \propto |\xi_{1,2}| \, \frac{E^4}{m_A^2 M_{\rm Pl}^2}$$

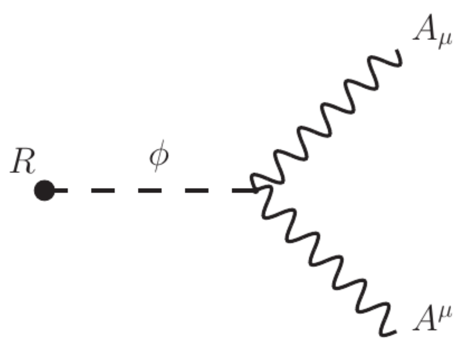
⬇

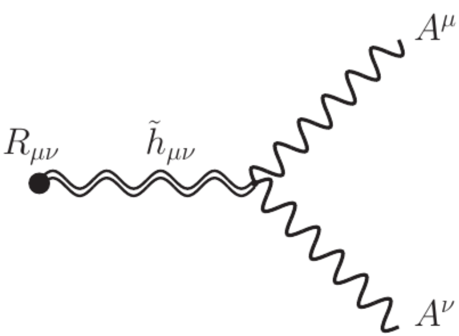
$$p_{\rm max} \lesssim \frac{\sqrt{m_A M_{\rm Pl}}}{|\xi_{1,2}|^{1/4}}$$

consistent with gauge invariance !

To study the high-energy behavior, **need a UV completion**

Generate effective ξ couplings by integrating out heavy spin-0 and spin-2 states :





$$\mathcal{L}_{\rm sc} = \overline{D_\mu \Phi}\, D^\mu \Phi - \frac{1}{2}\xi\, R\, |\Phi|^2 - V(\Phi)$$

$$\mathcal{L}_{\xi_1} = -\frac{1}{2}\xi\, \frac{m_A^2}{m_s^2}\, R\, A_\mu A^\mu$$

$$p \ll p_{\rm max} \sim \sqrt{\frac{\xi}{\xi_1}} m_A \quad , \quad |\xi R|/2 \ll m_s^2$$

$$|\xi_1| \ll \frac{1}{6}\frac{m_A^2}{H^2}$$

$$S = \int d^5x\, \frac{1}{\hat{\kappa}^2}\sqrt{|\hat{g}|}\hat{R} + \int d^5x \sqrt{|\hat{g}|}\, \mathcal{L}_{\rm mat}\, \delta(x_5)$$

$$S_{\rm loc} = \epsilon \int d^5x\, \frac{1}{\kappa^2}\sqrt{|\hat{g}|}\, R(\hat{g}_{\mu\nu})\, \delta(x_5)$$

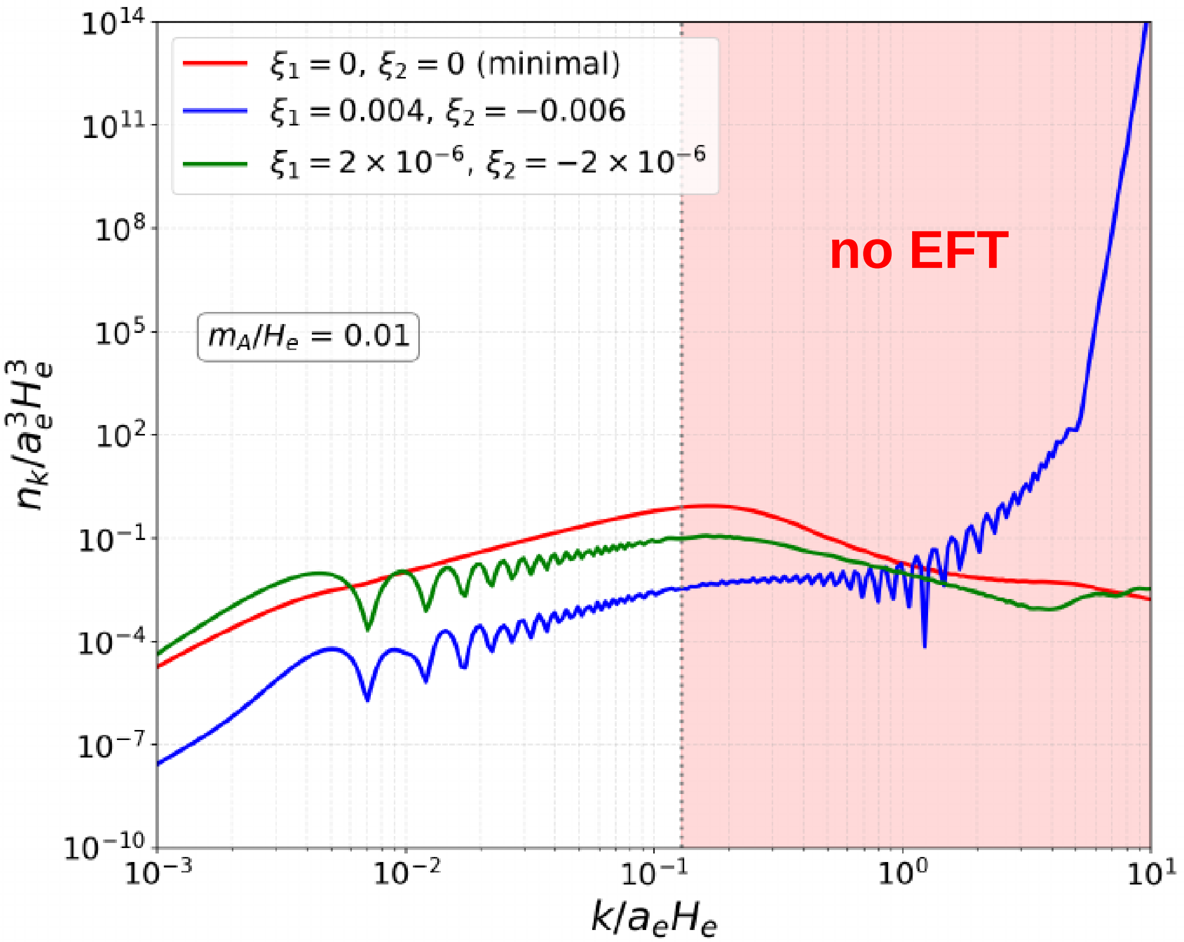
$$\mathcal{L}_{\xi_2} \simeq -\epsilon\, \frac{m_A^2}{m_1^2}\, R_{\mu\nu} A^\mu A^\nu$$

$$p_{\rm max} \lesssim \frac{m_A}{\sqrt{|\xi_2|}}$$

$$|\xi_{1,2}| \ll \frac{m_A^2}{H^2}$$

Result:

no ghosts , no runaway production in UV complete models



CONCLUSION

- *gravity is efficient in particle production*
- *dark relics are (over)produced during/after inflation*
- *traditional freeze-in models problematic*
- *EFT constraints*