GUT Seesaw Parameters from Low Energy Flavor Physics Inputs

Xiao-Gang He

Tsung-Dao Lee Institute

Shanghai Jiao Tong University

The 3rd Grand Unified Theory, Phenomenology and Cosmology

HIAS, Hangzhou, 9-14/4, 2026

1. SM Low Energy Flavor Physics Parameters

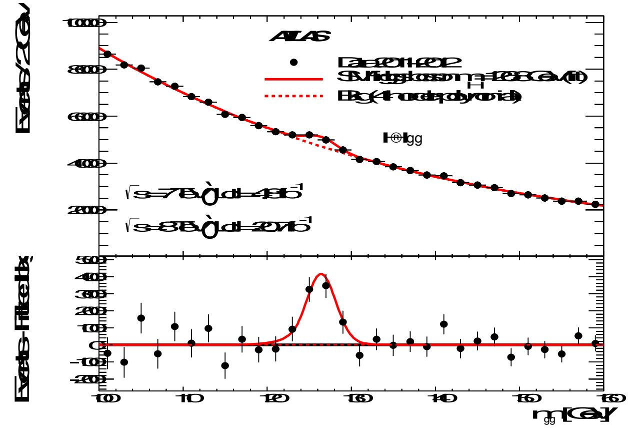
2. GUT Model Reducing SM Free Parameters

3. Seesaw Parameters From Low Energy Inputs

4. Conclusions

1. SM Low Energy Flavor Physics Parameters

Flavor of particles: The SM of Elementary Particles



Fundamental Interactions:

Electromagnetic Interaction, mediator: Photon γ

Weak Interaction, mediators: W and Z bosons

Strong interaction, mediator: gluon g

Gravitational Interaction, mediator: Graviton G(?)

Particle mass generating mechanis:

Higgs Mechanism (God particle reveals it); h

Quarks: The building block of Hadrons

u c t (electric charge +2/3 e) Quarks are elementary particles

d s b (electric charge -1/3 e) Three generations/families

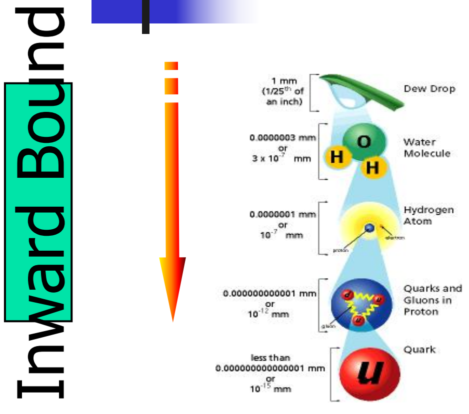
Leptons: Particles have no strong interaction

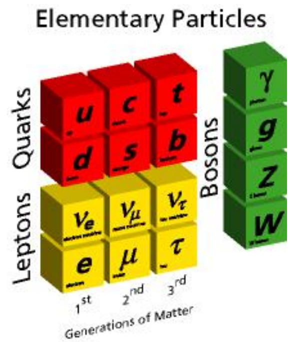
ν_e ν_μ ν_τ (electric charge 0 e) Leptons are elementary particles

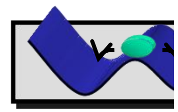
e μ τ (electric charge -1 e) Three generations/families

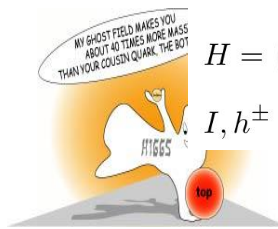
The SM of strong and electroweak interactions

SU(3) x SU(2) x U(1) gauge theory for strong and electroweak interaction
Glashow, Weinberg and Salam









$G:(8,1,0),\quad W:(1,3,0),\quad B:(1,1,0);$

$Q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix} : (3,2,1/6),\quad u_R(3,1,2/3),\quad d_R)3,1,-1/3);$

$L_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} : (1,2,-1/2),\quad e_R(1,1,-1);$

$H = \begin{pmatrix} (v+h+iI)/\sqrt{2} \\ h^- \end{pmatrix} : (1,2,-1/2),$

$I, h^\pm$ "eaten" by Z and $W^\pm$.

Parity violation, a conner stone: 70 years of pairty violation, 1956, Lee and Yang.

SPCS 2026: 70th anniversary of the discovery of Parity Nonconservtion and prospects in Frontier Physics

Spce-Time: TDLI, June 22-23 https://indico-tdli.sjtu.edu.cn/event/4784/

Can one negeclects gravitation interaction when studying particle interactions?
The coulomb force between two protons: $F_c = e^2/r^2$,
And Gravitational force: $F_g = -Gm^2/r^2$ $|F_g|/|F_c| = 7\times 10^{-38}$

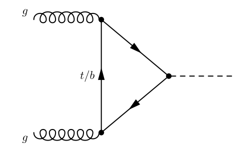
Gravitational force is much weaker than electromagnetism!

This is a model for flavor for the known basics building blocks of our Universe!

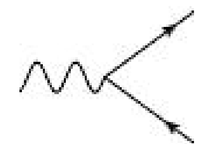
The number of genrations

In the SM, only 3 generations of quarks and leptons are allowed.

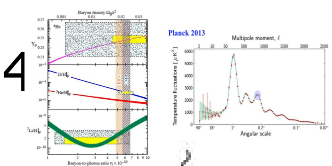
gg -> Higgs ~ (number of heavy quarks)^2, if fourth generation exist, their mass should be large, 9 times bigger production of Higgs. LHC data ruled out more than 3 generations of quarks.



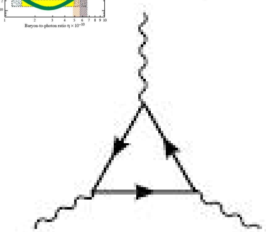
LEP already ruled out more than 3 neutrinos with mass less than $m_Z/2$.



Cosmology and astrophysics, number of light neutrinos also less than 4



SM, triangle anomaly cancellation: equal number of quarks and leptons!



There are only three generations of sequential quarks and leptons!

Why 3 generations? How do they mix with each other?

Beyond SM, conclusions may change, X-G He and G. Valencia, PPLB707 (2012)

Weak Interactions

Parameters in the standard model with 3 generations

Gauge boson couplings and masses: $g_1=g'$, $g_2=g$, $g_3=g_s$, m_γ, m_W, m_Z

Fermion Masses: m_e, m_μ, m_τ, $m_{\nu e}$, $m_{\nu\mu}$, $m_{\nu\tau}$

m_u, m_d, m_c, m_s, m_t, m_b

Higgs boson mass and couplings: m_h or λ, m_i/v to ith fermion

(Weak mixing angle θ_W: $\tan\theta_W = g_2/g_1$, $e = g_2 \sin\theta_W$)

$\alpha_{em} = e^2/4\pi$, $\alpha_2=g_2^2/4\pi$, $\alpha_3=\alpha_s=g_s^2/4\pi$; $G_F = g_2^2/(\sqrt{2}\, m_W^2)$

Mixing: quark mixing (3 mixing angles + 1 Dirac-phase)

Neutrino mixing (3 mixing angles +1 Dirac-phase + 2 Majorana-phases)

1 possible strong CP violating parameter θ : $\mathcal{L} = \theta \frac{1}{16\pi^2} F^a_{\mu\nu} \tilde{F}^{\mu\nu a}$

Total independent model parameters: 18 +1 without neutrino masses.

Another 9 if include neutrino masses at low energies or more.

(3 gauge couplings + 1 W or Z mass + 1 Higgs coupling or Higgs mass + (6 quark + 3 charged lepton masses)
+ 3 quark mixing angle + 1 Dirac-phase, 1 strong phase,
and 3+4+2+... neutrino masses, mixing angles and phases) Rich flavor physics!

What do we know about the SM parameters?

Many are well measured

α_{em}=1/137.035999084(21) $\sin^2\theta_W$=0.23121(4) α_3=0.1179(9) (G_F = 1.1663788(6)x10^{-5} GeV^{-2})

m_Z=91.1876(21) GeV m_h=125.25(0.17) GeV m_W=80.357(6) GeV

Charged lepton masses:

m_e=0.51099895000(15) MeV m_μ=105.6583755(23) MeV m_τ= 1776.86(12) MeV

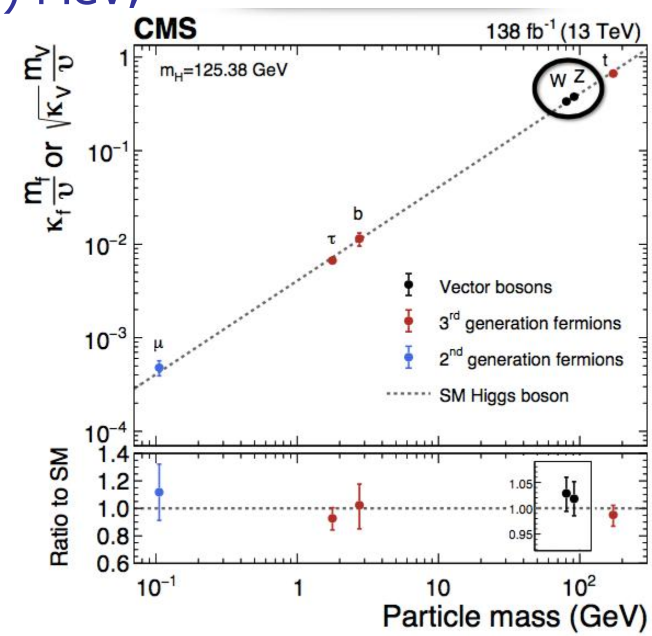
Quark masses:

light flavor: m_u =1.16(+0.49, -0.26) MeV m_d=4.67(+0.48, -0.17) MeV,

m_s=93.4(+8.6, -3.4) MeV,

Heavy flavor: m_c=1.27(0.02) GeV, m_b=4.18(+0.03,-0.02) GeV,

m_t= 172.69(0.30) GeV



Higgs sector, all data with agree SM Higgs. No nned of more Higgs!

Strong CP violating phase $\theta < 10^{-9}$

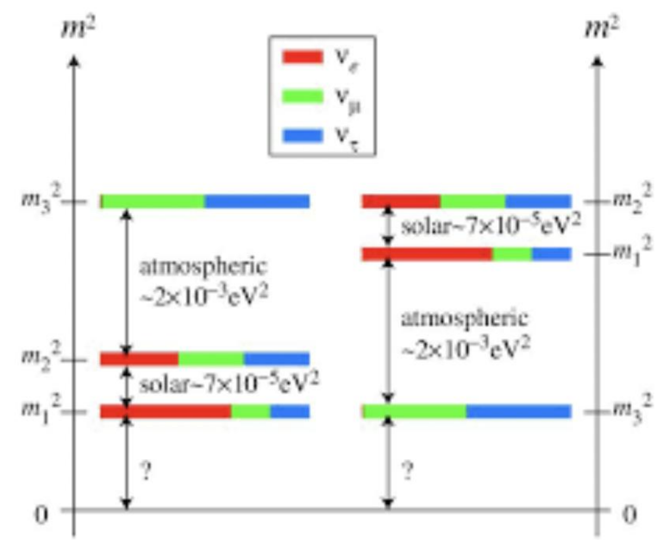
What about quark and neutrino mixing angles and CP violating phases, and neutrino masses?

Still unknown Neutrino Physics parameters

Neutrino mass hierarchy

Juno finally mission: neutrino

mass hirarchies: NH or IH, another ~ 5 years!?



CP violating phase

DUNE and hyperK: CP violation. Another N ~ 8 years1?

Absolute neutrino mass scale

Katrine and cosmology

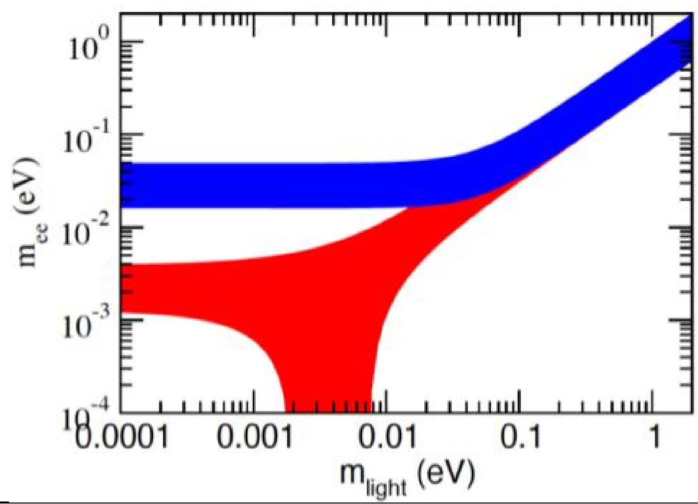
$$m_{\nu_e}^{\text{eff}} < 0.8 \text{ eV}$$

Type of neutrino: Dirac or Majorana

Neutrinoless double beta decays: PandaX XT, CDEX....

A long time to wait!!

$$T_{1/2}^{0\nu} > 1.8 \times 10^{26} \text{ yr, at } 90\% \text{ CL}$$



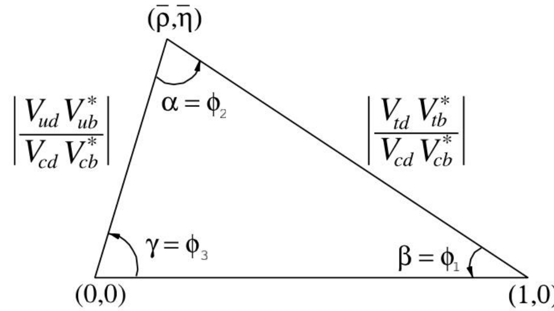
Compatible with inverted hierarchy. Good by neutrinoless double beta decay measurement!

Quark and Lepton mixing patterns

The mis-match of weak and mass eigen-state bases lead quark and lepton mix within generations.

Quark mixing the Cabibbo -Kobayashi-Maskawa (CKM) matrix $V_{\rm CKM}$,

lepton mixing the Pontecorvo -Maki-Nakawaga-Sakata (PMNS) matrix $U_{\rm PMNS}$



$$L = -\frac{g}{\sqrt{2}}\overline{U}_L\gamma^\mu V_{\rm CKM} D_L W^+_\mu - \frac{g}{\sqrt{2}}\overline{E}_L\gamma^\mu U_{\rm PMNS} N_L W^-_\mu + H.C.\ ,$$

$U_L = (u_L, c_L, t_L, ...)^T$, $D_L = (d_L, s_L, b_L, ...)^T$, $E_L = (e_L, \mu_L, \tau_L, ...)^T$, and $N_L = (\nu_1, \nu_2, \nu_3, ...)^T$

For n-generations, $V = V_{\rm CKM}$ or $U_{\rm PMNS}$ is an $n \times n$ unitary matrix.

A commonly used form of mixing matrix for three generations of fermions is given by $V_{KM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23} \end{pmatrix} \approx \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

where $s_{ij} = \sin\theta_{ij}$ and $c_{ij} = \cos\theta_{ij}$ are the mixing angles and δ is the CP violating phase.

If neutrinos are of Majorana type, for the PMNS matrix one should include an additional diagonal

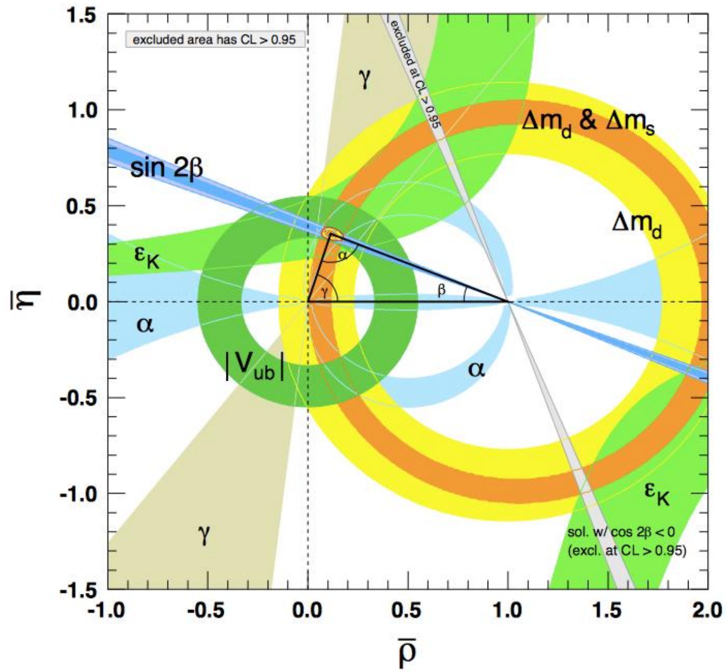
matrix with two Majorana phases ${\rm diag}(e^{i\alpha_1/2}, e^{i\alpha_2/2}, 1)$ multiplied to the matrix from right in the above.

Status of Quark Mixing

Quark Mixing

PDG

$$\begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho-i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$



$$\lambda = 0.22500 \pm 0.00067\,, \qquad A = 0.826^{+0.018}_{-0.015}\,,$$

$$\bar{\rho} = 0.159 \pm 0.010\,, \qquad \bar{\eta} = 0.348 \pm 0.010\,.$$

$$\sin\theta_{12} = 0.22500 \pm 0.00067\,, \qquad \sin\theta_{13} = 0.00369 \pm 0.00011\,,$$

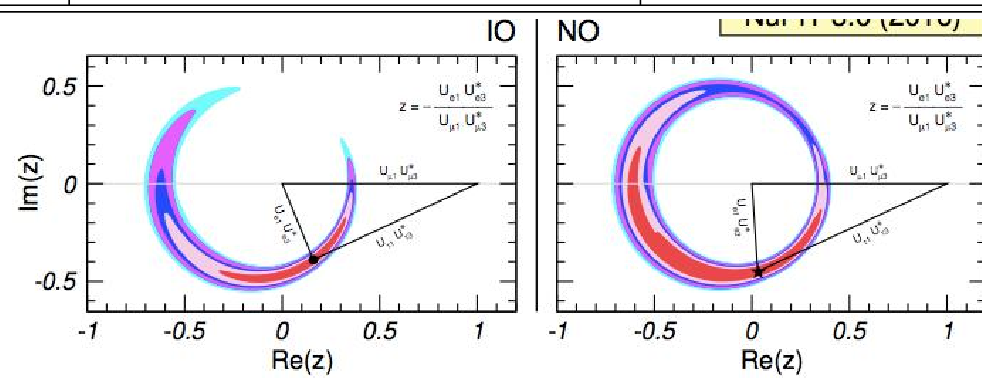
$$\sin\theta_{23} = 0.04182^{+0.00085}_{-0.00074}\,, \qquad \delta = 1.144 \pm 0.027\,.$$

Status of Lepton Mixing

Neutrino Mixing

PDG

IC24 with SK atmospheric data		Normal Ordering (best fit)		Inverted Ordering ($\Delta\chi^2 = 6.1$)	
		bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range
	$\sin^2\theta_{12}$	$0.308^{+0.012}_{-0.011}$	$0.275 \to 0.345$	$0.308^{+0.012}_{-0.011}$	$0.275 \to 0.345$
	$\theta_{12}/^\circ$	$33.68^{+0.73}_{-0.70}$	$31.63 \to 35.95$	$33.68^{+0.73}_{-0.70}$	$31.63 \to 35.95$
	$\sin^2\theta_{23}$	$0.470^{+0.017}_{-0.013}$	$0.435 \to 0.585$	$0.550^{+0.012}_{-0.015}$	$0.440 \to 0.584$
	$\theta_{23}/^\circ$	$43.3^{+1.0}_{-0.8}$	$41.3 \to 49.9$	$47.9^{+0.7}_{-0.9}$	$41.5 \to 49.8$
	$\sin^2\theta_{13}$	$0.02215^{+0.00056}_{-0.00058}$	$0.02030 \to 0.02388$	$0.02231^{+0.00056}_{-0.00056}$	$0.02060 \to 0.02409$
	$\theta_{13}/^\circ$	$8.56^{+0.11}_{-0.11}$	$8.19 \to 8.89$	$8.59^{+0.11}_{-0.11}$	$8.25 \to 8.93$
	$\delta_{CP}/^\circ$	212^{+26}_{-41}	$124 \to 364$	274^{+22}_{-25}	$201 \to 335$
	$\frac{\Delta m^2_{21}}{10^{-5}\ \text{eV}^2}$	$7.49^{+0.19}_{-0.19}$	$6.92 \to 8.05$	$7.49^{+0.19}_{-0.19}$	$6.92 \to 8.05$
	$\frac{\Delta m^2_{3\ell}}{10^{-3}\ \text{eV}^2}$	$+2.513^{+0.021}_{-0.019}$	$+2.451 \to +2.578$	$-2.484^{+0.020}_{-0.020}$	$-2.547 \to -2.421$



2. GUT Model Reducing SM Free Parameters

SM has many free parameters. Possible to reduce them?

Extensions of SM usually introduce more paramters in the model!

SUSY, Multi-Higgs, New symmetries, usually, introduce more parameters (some of them may reduce the parameter in certain sectors)...

Relating masses to mixing angles: Vus = $\sqrt{d/s}$ The Weinberg ansatz works well!

PHYSICAL REVIEW D   VOLUME 41, NUMBER 5   1 MARCH 1990

Relating the long B lifetime to a very heavy top quark

Xiao-Gang He
School of Physics, University of Melbourne, Parkville 3052, Australia

Wei-Shu Hou*
Max-Planck-Institut für Physik, Föhringer Ring 6, 8000 München 40, Federal Republic of Germany

(Received 30 June 1989)

The long B lifetime is related to the heaviness of the top quark by a particular mass-mixing ansatz. The u-type quark mass matrix is of the Fritzsch form, while for the d type it is diagonal except in the d-s plane, which generates the Cabibbo rotation. We predict $m_t \gtrsim 200$ GeV from $V_{cb} \lesssim 0.06$. One gets "maximal CP violation," and the relations $|V_{ub}/V_{cb}| = \sqrt{m_u/m_c}$, $|V_{td}/V_{cb}| = \sqrt{m_d/m_s}$, and $|V_{ts}| = |V_{cb}|$ are close to exact. An interesting Wolfenstein pattern emerges. We discuss the viability and implications of having such a heavy top quark (such as lower M_Z), and the possibility of a vanishing V_{ub} and its impact on CP violation.

$$m_U \sim \begin{pmatrix} 0 & a & 0 \\ a & 0 & b \\ 0 & b & c \end{pmatrix} \qquad m_D \sim \begin{pmatrix} 0 & \tilde{a} & 0 \\ \tilde{a} & \tilde{b} & 0 \\ 0 & 0 & \tilde{c} \end{pmatrix}$$

$$V \sim \begin{pmatrix} 1 & \sqrt{d/s} - \sqrt{u/c}\, e^{i\sigma} & -\sqrt{u/t}\, e^{i\sigma} \\ -\sqrt{d/s} + \sqrt{u/c}\, e^{-i\sigma} & 1 & \sqrt{c/t} \\ \sqrt{d/s}\sqrt{c/t} & -\sqrt{c/t} & 1 \end{pmatrix}$$

Small Vcb predicted a top quark mass about 200 GeV!

Still too much guess involved!

$$D = \begin{pmatrix} 0 & F & 0 \\ F & E & 0 \\ 0 & 0 & D \end{pmatrix}, \quad E = \begin{pmatrix} 0 & F & 0 \\ F & -3E & 0 \\ 0 & 0 & D \end{pmatrix}$$

Later people expanded with leptons and also GUT model with susy.

Dimopulous, Hall and Raby, PRL 68 (1992) 1984.

GUT Model Reducing SM Free Parameters

GUT can do more

Unification is one wayto try: Unify forces - reduce gauge couplings,

g_1, g_2, g_3 all become equal at some high scale g_U

Unify representation - Quarks and leptons are put into GUT representations

reduce Yukawa coupling, relate masses of particles and etc...

Attenpts have also been make to explain the number of generations

Example: 16 of SO(10) one generation.

SO(15) -> SO(10)XSp(4): 128= (16,4)+ (bar-16,4)   4-generations?

SO(16) -> SO10)XSU(4): 128 =(16,4) + (bar-16, bar-4) 4-generation?

Sp(4) and SU(4) as horizontal groups. But....

Have more particles with higher masses scale than electroweak scale...

but a progress for us looking at electroweak scale physics.

SM has many free parameters. Possible to reduce them?

Examples: SO(10)

Gauge boson in 45 representation, Fermions in 16,
Higgs fields 10 and 120, anti-126, 210...

$10 \to 5 + \bar{5}$

$16 \to 10 + \bar{5} + 1$

$45 \to 24 + 10 + \overline{10} + 1$

$54 \to 15 + \overline{15} + 24$

$120 \to 5 + \bar{5} + 10 + \overline{10} + 45 + \overline{45}$

$126 \to 1 + \bar{5} + 10 + \overline{15} + 45 + \overline{50}$

$210 \to 1 + 5 + \bar{5} + 10 + \overline{10} + 24 + 40 + \overline{40} + 75$

16 => $1_F = \nu^c + \bar{\mathbf{5}}_F = \begin{pmatrix} d_1^c \\ d_2^c \\ d_3^c \\ e \\ -\nu \end{pmatrix} + \mathbf{10}_F = \begin{pmatrix} 0 & u_3^c & -u_2^c & u_1 & d_1 \\ -u_3^c & 0 & u_1^c & u_2 & d_2 \\ u_2^c & -u_1^c & 0 & u_3 & d_3 \\ -u_1 & -u_2 & -u_3 & 0 & e_R \\ -d_1 & -d_2 & -d_3 & -e_R & 0 \end{pmatrix}$

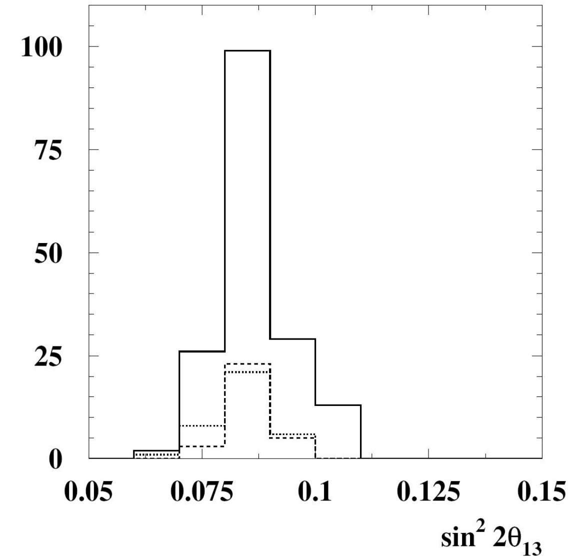
Right handed neutrino naturally included!

Yukawa couplings: 16X16 = 10s + 120a + 160 s

In 3 Higgs representations 10, 120, bar-126 can coupling to fermions!

SO(10) Predictions

$$16_F(Y_{10}10_H + Y_{\overline{126}}\overline{126}_H + Y_{120}120_H)16_F$$



Minimal SO(10) Model without 120

$$\mathcal{L}_{\text{Yukawa}} = Y_{10}\,\mathbf{16}\,\mathbf{16}\,\mathbf{10}_H + Y_{126}\,\mathbf{16}\,\mathbf{16}\,\overline{\mathbf{126}}_H$$

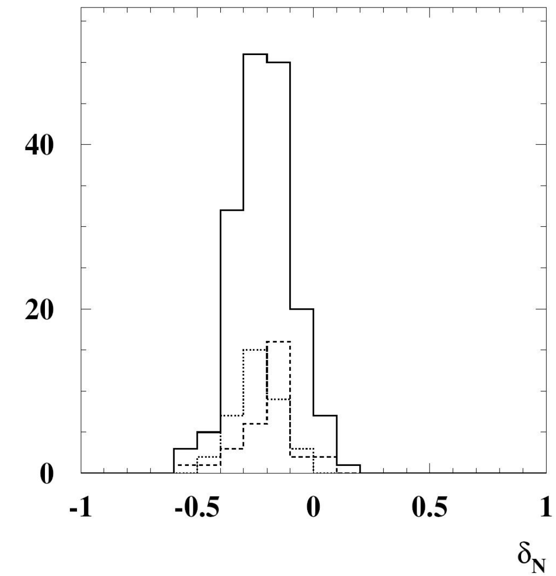
Two Yukawa matrices determine all fermion masses and mixings, including the neutrinos

$$\begin{aligned} M_u &= \kappa_u Y_{10} + \kappa'_u Y_{126} \\ M_d &= \kappa_d Y_{10} + \kappa'_d Y_{126} \\ M_\nu^D &= \kappa_u Y_{10} - 3\kappa'_u Y_{126} \\ M_l &= \kappa_d Y_{10} - 3\kappa'_d Y_{126} \end{aligned}$$

$$\begin{aligned} M_{\nu R} &= \langle \Delta_R \rangle Y_{126} \\ M_{\nu L} &= \langle \Delta_L \rangle Y_{126} \end{aligned}$$

Good prediction for θ_{13}

δ Away from $-\pi/2$!!! Tobe tested!!



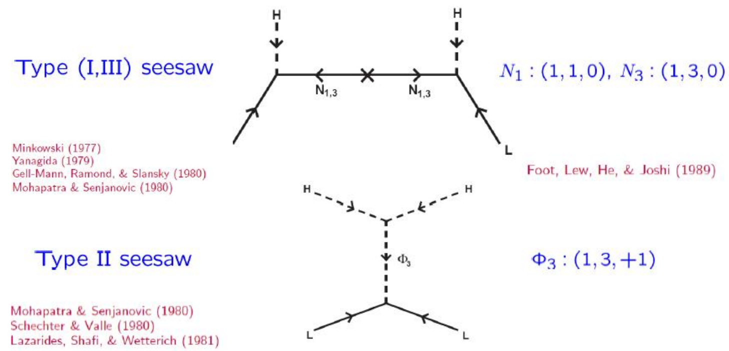
Model has only 11 real parameters plus 7 phases

Babu, Mohapatra (1993)
Fukuyama, Okada (2002)
Bajc, Melfo, Senjanovic, Vissani (2004)
Fukuyama, Ilakovac, Kikuchi, Meljanac,
Okada (2004)
Aulakh et al (2004)
Bertolini, Frigerio, Malinsky (2004)
Babu, Macesanu (2005)
Bertolini, Malinsky, Schwetz (2006
Dutta, Mimura, Mohapatra (2007)
Bajc, Dorsner, Nemevsek (2009)
Jushipura, Patel (2011).

A good time to revise the numbers before measurement of CP violating phase!!

3. Seesaw Parameters From Low Energy Inputs

Majorana Neutrinos and Seesaw Mechanism



$$\mathcal{L}_{\text{eff}} = \frac{LLHH}{M} \Rightarrow m_\nu \sim \frac{v^2}{M}$$

The Seesaw Mechanism $L = \bar{\nu}_L (Y_\nu v/\sqrt{2})\nu_R + \bar{\nu}_R^c M_R \nu_R/2$

The Seesaw mechanism refers to the neutrino mass matrix of the form

$$L_m = -\frac{1}{2}(\nu_L^c, \nu_R)\begin{pmatrix} 0 & m_D \\ m_D^T & M_R \end{pmatrix}\begin{pmatrix} \nu_L \\ \nu_R^c \end{pmatrix}.$$

For one generation, if $M_R >> m_D$, the eigenmasses are

$$m_\nu \approx -m_D M_R^{-1} m_D^T, \quad m_N \approx M_R$$

A very nice way to explain why light neutrino masses are so much lighter than their charged lepton partners.

M_D and M_R arbitrary, now predictions can be made.

Efforts have been made to make M_R to be expressed in terms of known low energy parameters $M_{u,d,\,e}$, V_{KM}, V_{PMNS} …

X–G He, S. Low and R Volkas, PRD 78 (2008) 113001

$$M_R \simeq \widehat{m}_e \, U^*_{\text{PMNS}} \, \widehat{m}_\nu^{-1} \, U^\dagger_{\text{PMNS}} \, \widehat{m}_e \quad M_R \simeq \widehat{m}_{d,u} \, U^*_{\text{PMNS}} \, \widehat{m}_\nu^{-1} \, U^\dagger_{\text{PMNS}} \, \widehat{m}_{d,u}$$

$$M_R \simeq \widehat{m}_d \, U^T_{\text{CKM}} \, \widehat{m}_\nu^{-1} \, U_{\text{CKM}} \, \widehat{m}_d$$

Neutrino Mixing after JUNO first result

New exciting data from Juno, last Nov. the most preciese data for θ_{12}

arXiv: 511.14593 Already 43 citations: $\sin^2\theta_{12} = 0.3092 \pm 0.0087$ and $\Delta m^2_{21} = (7.50 \pm 0.12) \times 10^{-5}\ \mathrm{eV}^2$

$$V_{TB} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$$

Popular model in early 2000,
ruled out by Dayabay Data

$$V_b = \begin{pmatrix} \frac{2c}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{2s}{\sqrt{6}}, e^{i\alpha} \\ -\frac{c}{\sqrt{6}} - \frac{s}{\sqrt{2}}e^{-i\alpha} & \frac{1}{\sqrt{3}} & \frac{c}{\sqrt{2}} - \frac{s}{\sqrt{6}}e^{i\alpha} \\ -\frac{c}{\sqrt{6}} - \frac{s}{\sqrt{2}}e^{-i\alpha} & \frac{1}{\sqrt{3}} & -\frac{c}{\sqrt{2}} - \frac{s}{\sqrt{6}}e^{i\alpha} \end{pmatrix},$$

Simple and popular candidate,
now 3.6 tension with data

$$V_c = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{c}{\sqrt{3}} & \frac{s}{\sqrt{3}}e^{i\alpha} \\ -\frac{1}{\sqrt{6}} & \frac{c}{\sqrt{3}} - \frac{s}{\sqrt{2}}e^{-i\alpha} & \frac{c}{\sqrt{2}} + \frac{s}{\sqrt{3}}e^{i\alpha} \\ -\frac{1}{\sqrt{6}} & \frac{c}{\sqrt{3}} + \frac{s}{\sqrt{2}}e^{-i\alpha} & -\frac{c}{\sqrt{2}} + \frac{s}{\sqrt{3}}e^{i\alpha} \end{pmatrix}$$

Compatible with data!

$$\sin^2\theta_{12} = \frac{1 - 3\sin^2\theta_{13}}{3(1-\sin^2\theta_{13})}$$

X–G He, PLB 874 (2026) 140270

$$\delta_{CP} = \arg\left(\frac{c^2}{2}e^{-i\alpha} - \frac{s^2}{3}e^{i\alpha}\right).$$

$$c(NH) = 0.96569 \pm 0.00087\ , \quad \cos\alpha(NH) = -0.1432 \pm 0.0740\ ,$$

$$c(IH) = 0.96547 \pm 0.00088\ , \quad \cos\alpha(IH) = 0.238 \pm 0.0586\ .$$

A GUT model to realize: $M_R \simeq \widehat{m}_u\, U^*_{\rm PMNS}\, \widehat{m}_\nu^{-1}\, U^\dagger_{\rm PMNS}\, \widehat{m}_u$

PHYSICAL REVIEW D **78,** 113001 (2008)

Determining the heavy seesaw neutrino mass matrix from low-energy parameters

Xiao-Gang He,[1,*] Sandy S. C. Law,[2,†] and Raymond R. Volkas[2,‡]
[1] *Department of Physics, Center for Theoretical Sciences and LeCosPA Center, National Taiwan University, Taipei 10617, Taiwan, Republic of China*
[2] *School of Physics, The University of Melbourne, Victoria 3010, Australia*
(Received 13 October 2008; published 2 December 2008)

We explore how the seesaw sector in neutrino mass models may be constrained through symmetries to be completely determined in terms of low-energy mass, mixing angle and *CP*-violating phase observables.

A. Relating m_ν^D to $\hat{m}_u$ via a flipped $SU(5)$ model

We consider a flipped $SU(5)$ group [14] augmented by A_4 flavor symmetry [22,23]:

$$G_1 = SU(5) \otimes U(1)_X \times A_4, \qquad (4.1)$$

$$\supset SU(3)_c \otimes SU(2)_L \otimes \underbrace{U(1)_T \otimes U(1)_X}_{U(1)_Y} \times A_4, \qquad (4.2)$$

$$\psi_{L\alpha} = \begin{pmatrix} u_R^{1c} \\ u_R^{2c} \\ u_R^{3c} \\ e_L \\ -\nu_L \end{pmatrix} \sim (\bar{5}, -3)(\underline{3});$$

$$\chi_L^{\alpha\beta} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & d_R^{3c} & -d_R^{2c} & -u_L^1 & -d_L^1 \\ -d_R^{3c} & 0 & d_R^{1c} & -u_L^2 & -d_L^2 \\ d_R^{2c} & -d_R^{1c} & 0 & -u_L^3 & -d_L^3 \\ u_L^1 & u_L^2 & u_L^3 & 0 & -\nu_R^c \\ d_L^1 & d_L^2 & d_L^3 & \nu_R^c & 0 \end{pmatrix} \sim (10, 1)(\underline{1} \oplus \underline{1}' \oplus \underline{1}''); \qquad e_R^c \sim (1, 5)(\underline{1} \oplus \underline{1}' \oplus \underline{1}'');$$

$$\Phi^\sigma_{(3)} = \begin{pmatrix} h_{1d} \\ h_{2d} \\ h_{3d} \\ \phi^{0*}_{(3)} \\ -\phi^+_{(3)} \end{pmatrix} \sim (5, -2)(\underline{3}); \qquad \Phi^\sigma_{(1\oplus 1'\oplus 1'')} \sim (5, -2)(\underline{1} \oplus \underline{1}' \oplus \underline{1}''); \qquad \Delta^{\alpha\beta\gamma\delta} \sim (\overline{50}, 2)(\underline{1} \oplus \underline{1}' \oplus \underline{1}''),$$

$$-\mathcal{L} = Y_{\lambda 1}\bar{\psi}_L \Phi^*_{(3)} e_R + \sqrt{2} Y_{\lambda 2}\bar{\psi}_L \chi^c_L \Phi_{(3)} + \frac{Y_{\lambda 3}}{4}(\bar{\chi}_L)_{\alpha\beta}(\chi^c_L)_{\gamma\delta}(\Phi^*_{(1\oplus 1'\oplus 1'')})_\sigma \epsilon^{\alpha\beta\gamma\delta\sigma} + Y_{\lambda 4}(\bar{\chi}_L)_{\alpha\beta}(\chi^c_L)_{\gamma\delta}\Delta^{\alpha\beta\gamma\delta} + \text{H.c.},$$

$$\langle\phi^0_{(3)}\rangle \equiv \langle\phi^{0*}_{(3)}\rangle = (v_{(3)}, v_{(3)}, v_{(3)})$$

Neutral compoments

$$Y_{\lambda 1}\bar{e}_L\langle\phi^0_{(3)}\rangle e_R - Y_{\lambda 2}(\bar{u}_L\langle\phi^{0*}_{(3)}\rangle u_R + \bar{\nu}_L\langle\phi^{0*}_{(3)}\rangle \nu_R) + \frac{Y_{\lambda 3}}{2}(\bar{d}^c_R d^c_L + \bar{d}_L d_R)\langle\phi^0_{(1\oplus 1'\oplus 1'')}\rangle + Y_{\lambda 4}\bar{\nu}^c_R \nu_R\langle\Delta^0_{(1\oplus 1'\oplus 1'')}\rangle + \text{H.c.}.$$

$$m_e\colon\ \lambda_1(\bar{e}_L\langle\phi^0_{(3)}\rangle)_{\underline{1}} e_R + \lambda'_1(\bar{e}_L\langle\phi^0_{(3)}\rangle)_{\underline{1}'} e''_R + \lambda''_1(\bar{e}_L\langle\phi^0_{(3)}\rangle)_{\underline{1}''} e'_R + \text{H.c.};$$

$$m_u\colon\ -\lambda_2\bar{u}_L(\langle\phi^{0*}_{(3)}\rangle u_R)_{\underline{1}} - \lambda'_2\bar{u}''_L(\langle\phi^{0*}_{(3)}\rangle u_R)_{\underline{1}'} - \lambda''_2\bar{u}'_L(\langle\phi^{0*}_{(3)}\rangle u_R)_{\underline{1}''} + \text{H.c.};$$

$$m^D_\nu\colon\ -\lambda_2(\bar{\nu}_L\langle\phi^{0*}_{(3)}\rangle)_{\underline{1}}\nu_R - \lambda'_2(\bar{\nu}_L\langle\phi^{0*}_{(3)}\rangle)_{\underline{1}'}\nu''_R - \lambda''_2(\bar{\nu}_L\langle\phi^{0*}_{(3)}\rangle)_{\underline{1}''}\nu'_R + \text{H.c.}.$$

$$m_e = U_\omega \hat{m}_e; \qquad m_u = -\hat{m}_u U_\omega; \qquad m_\nu^D = -U_\omega \hat{m}_u;$$

$$\hat{m}_{e,u} = \mathrm{diag}(\sqrt{3}\lambda_{1,2} v_{(3)}, \sqrt{3}\lambda'_{1,2} v_{(3)}, \sqrt{3}\lambda''_{1,2} v_{(3)}). \qquad U_\omega = \frac{1}{\sqrt{3}}\begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix}$$

$$V_{eL}^\dagger = U_\omega, \qquad V_{uL}^\dagger = V_{eR} = I, \qquad V_{uR} = -U_\omega, \qquad m_\nu^D = -V_{eL}^\dagger \hat{m}_u.$$

$$\hat{m}_\nu \simeq V_\nu m_\nu^D M_R^{-1} (m_\nu^D)^T V_\nu^T.$$

$$\hat{m}_\nu \simeq V_\nu V_{eL}^\dagger \hat{m}_u M_R^{-1} (V_{eL}^\dagger \hat{m}_u)^T V_\nu^T = U_{\mathrm{PMNS}}^\dagger \hat{m}_u M_R^{-1} \hat{m}_u U_{\mathrm{PMNS}}^*,$$

$$M_R \simeq \hat{m}_u U_{\mathrm{PMNS}}^* \hat{m}_\nu^{-1} U_{\mathrm{PMNS}}^\dagger \hat{m}_u.$$

M_R is completely determined by low energy physics inputs

m_ν and M_R are rank 3 matrices.

Sample solutions using most recent inputs

		Normal Ordering (best fit)		Inverted Ordering ($\Delta\chi^2 = 5.9$)	
		bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range
IC24 with SK atmospheric data	$\sin^2\theta_{12}$	$0.3088^{+0.0067}_{-0.0066}$	$0.2893 \to 0.3295$	$0.3088^{+0.0067}_{-0.0066}$	$0.2893 \to 0.3295$
	$\theta_{12}/^\circ$	$33.76^{+0.42}_{-0.41}$	$32.54 \to 35.03$	$33.76^{+0.42}_{-0.41}$	$32.54 \to 35.03$
	$\sin^2\theta_{23}$	$0.470^{+0.017}_{-0.014}$	$0.435 \to 0.584$	$0.550^{+0.013}_{-0.016}$	$0.439 \to 0.584$
	$\theta_{23}/^\circ$	$43.29^{+0.96}_{-0.79}$	$41.27 \to 49.86$	$47.90^{+0.73}_{-0.92}$	$41.51 \to 49.83$
	$\sin^2\theta_{13}$	$0.02248^{+0.00055}_{-0.00059}$	$0.02064 \to 0.02418$	$0.02262^{+0.00057}_{-0.00056}$	$0.02093 \to 0.02441$
	$\theta_{13}/^\circ$	$8.62^{+0.11}_{-0.11}$	$8.26 \to 8.95$	$8.65^{+0.11}_{-0.11}$	$8.32 \to 8.99$
	$\delta_{CP}/^\circ$	212^{+26}_{-36}	$125 \to 365$	274^{+22}_{-25}	$203 \to 335$
	$\frac{\Delta m^2_{21}}{10^{-5}\ \mathrm{eV}^2}$	$7.537^{+0.094}_{-0.10}$	$7.236 \to 7.823$	$7.537^{+0.094}_{-0.10}$	$7.236 \to 7.822$
	$\frac{\Delta m^2_{3\ell}}{10^{-3}\ \mathrm{eV}^2}$	$+2.511^{+0.021}_{-0.020}$	$+2.450 \to +2.576$	$-2.483^{+0.020}_{-0.020}$	$-2.547 \to -2.421$

Quark masses at scale m_Z

```
mt = 172.1;

mc = 3.7*10^(-3)*mt;

mu = 2.2*10^(-3)*mc;
```

No Majorana phase case

NH: M_R= (in GeV)

$$\begin{pmatrix} 1.3345\times10^{6} + 456.629\, i & -2.39867\times10^{8} - 3.39607\times10^{7}\, i & 8.95408\times10^{10} - 9.74346\times10^{9}\, i \\ -2.39867\times10^{8} - 3.39607\times10^{7}\, i & 4.84425\times10^{10} + 1.19671\times10^{10}\, i & -1.68088\times10^{13} - 5.3156\times10^{11}\, i \\ 8.95408\times10^{10} - 9.74346\times10^{9}\, i & -1.68088\times10^{13} - 5.3156\times10^{11}\, i & 6.43154\times10^{15} - 1.29007\times10^{15}\, i \end{pmatrix}$$

Mi in GeV =

$$\{498157.,\ 1.44922\times10^{10},\ 7.45365\times10^{15}\}$$

$m_{\nu1} = 10^{-3}$ eV

IH:M_R = (in GeV)

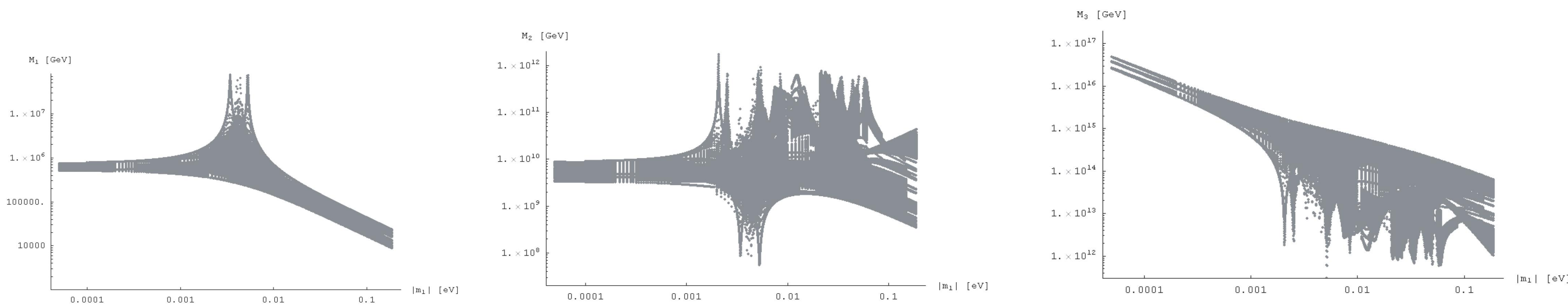
$$\begin{pmatrix} -16946.3 - 6178.04\, i & 6.73963\times10^{6} - 9.95557\times10^{7}\, i & 1.66143\times10^{9} - 2.43123\times10^{10}\, i \\ 6.73963\times10^{6} - 9.95557\times10^{7}\, i & 2.20666\times10^{11} + 1.2827\times10^{7}\, i & 5.24977\times10^{13} + 2.33634\times10^{9}\, i \\ 1.66143\times10^{9} - 2.43123\times10^{10}\, i & 5.24977\times10^{13} + 2.33634\times10^{9}\, i & 1.32363\times10^{16} + 3.76133\times10^{11}\, i \end{pmatrix}$$

Mi in GeV =

$$\{28349.9,\ 1.24781\times10^{10},\ 1.3237\times10^{16}\}$$

$m_{\nu3} = 10^{-3}$ eV

Plots of $M_{1,2,3}$ vs. $|m_1|$ in the $\widehat{m}_f = \widehat{m}_u$ case with normal hierarchy



Can Leptogenesis work ?!

Time to update details.

5. Conclusions

GUT Model Reducing SM Free Parameters

Seesaw Parameters can be determined by Low Energy Inputs

Thank you for your attentions