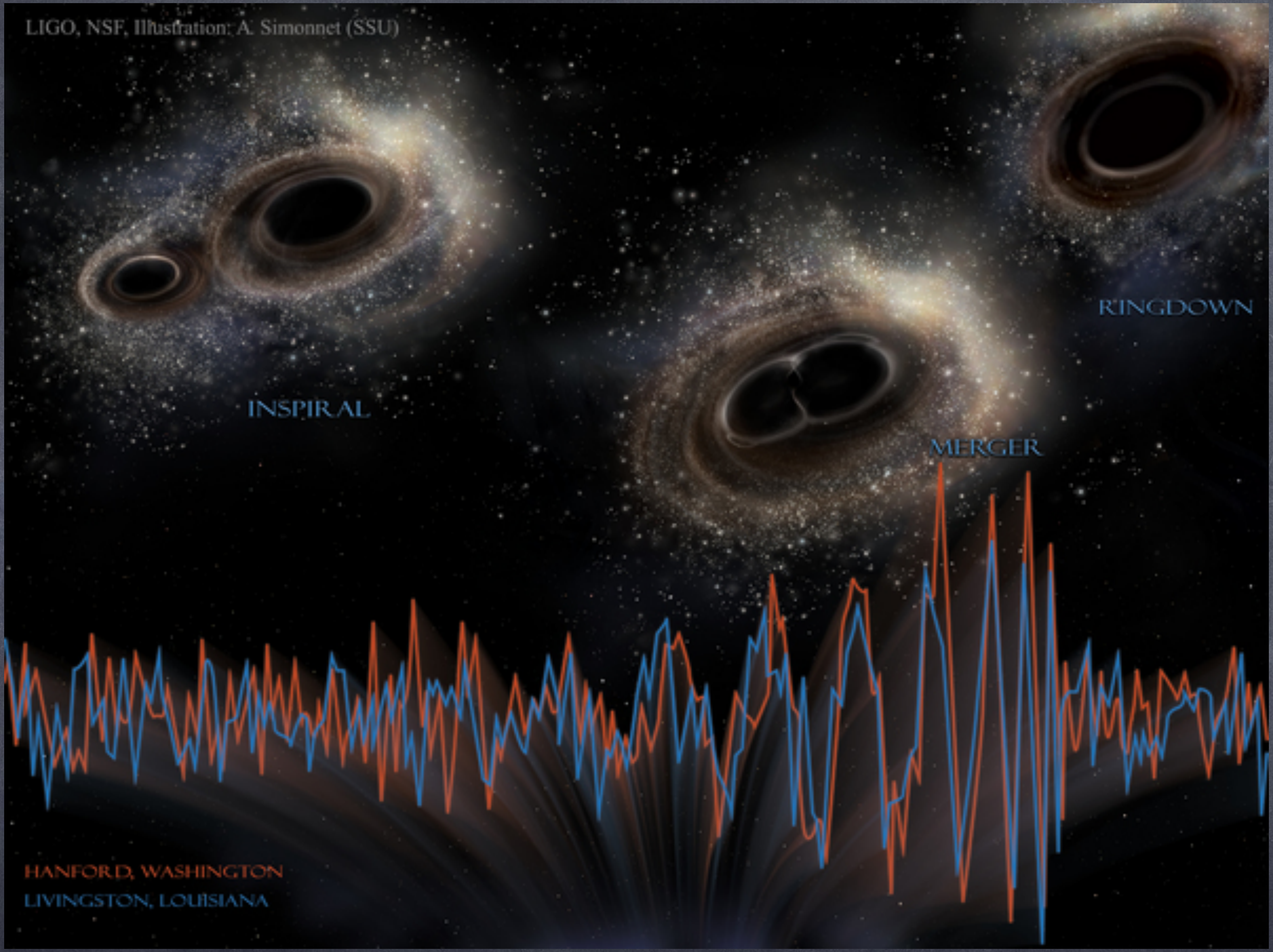
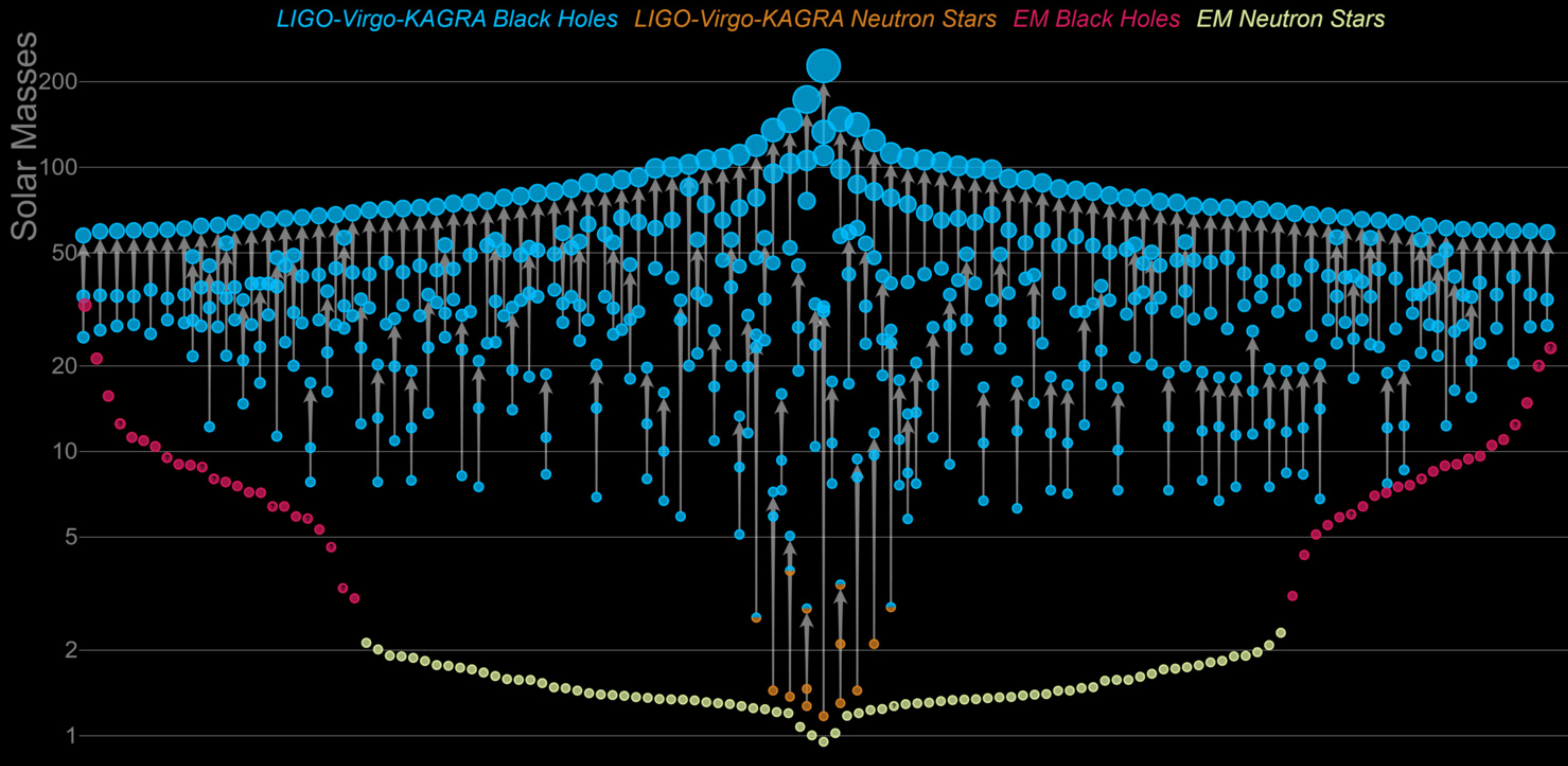
Prominence: A discriminator of gravitational wave signals

Dawn of gravitational wave astronomy



- Detection of GW150914 on September 14, 2015 by LIGO
- First observation of heavy black hole binaries

Masses in the Stellar Graveyard as of 09/25



LIGO-Virgo-KAGRA | Aaron Geller | Northwestern

Pulsar Timing Arrays

- Galactic size GW detector made of millisecond pulsar array
- Precise rotation periods and radio pulses from poles of pulsars make them ultra-precise clocks

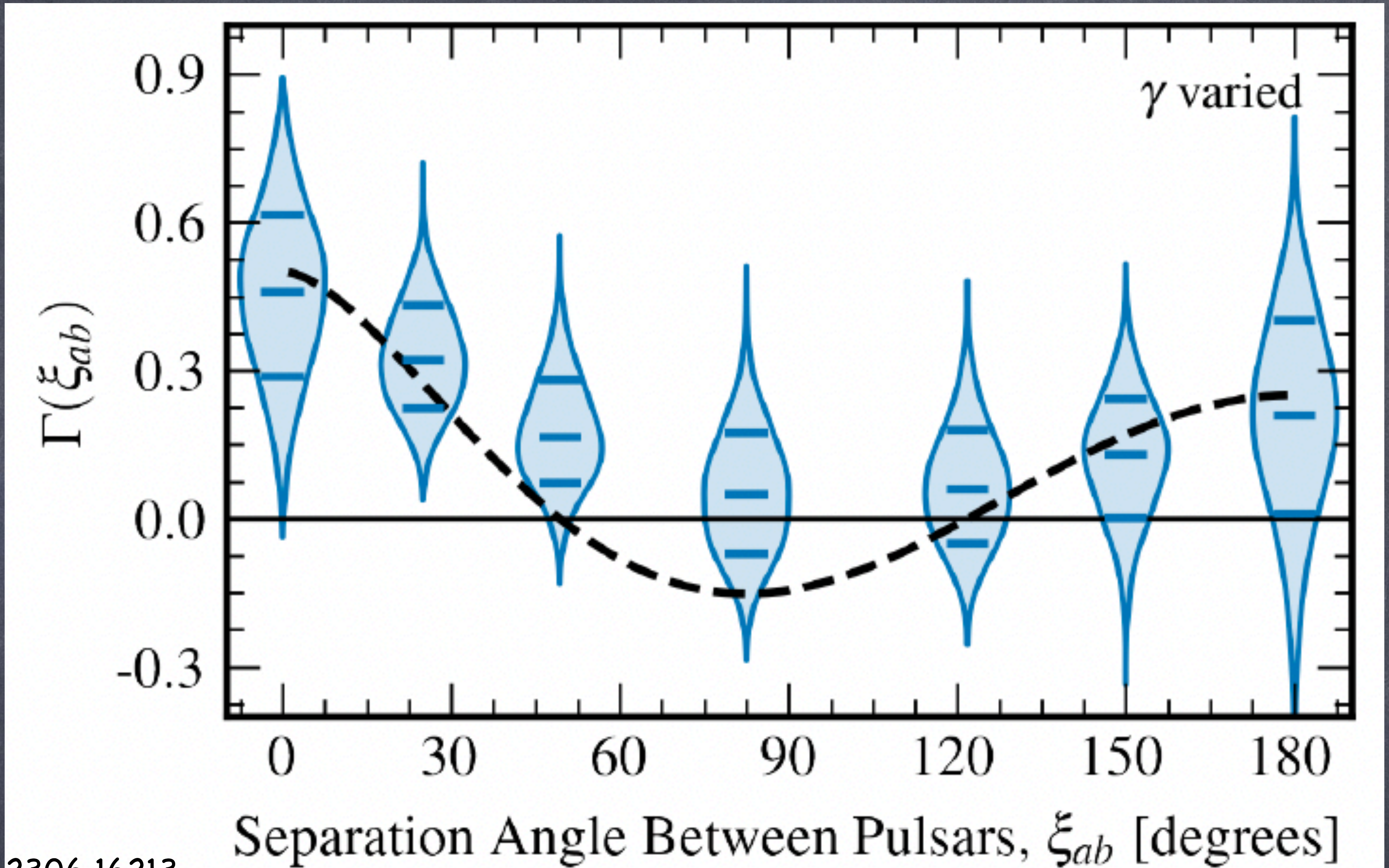
- GW perturbs spacetime between the pulsar and Earth and changes the time of arrival (TOA) of pulses

- Measure residuals in TOA: $R^a = TOA^a_{measured} - TOA^a_{model}$

- Cross-correlate timing residuals of pairs of pulsars separated by angle ξ_{ab}

- Sensitive to frequencies between 1/(total observation time) and 1/cadence i.e., [1/years, 1/weeks]

Hellings-Downs curve



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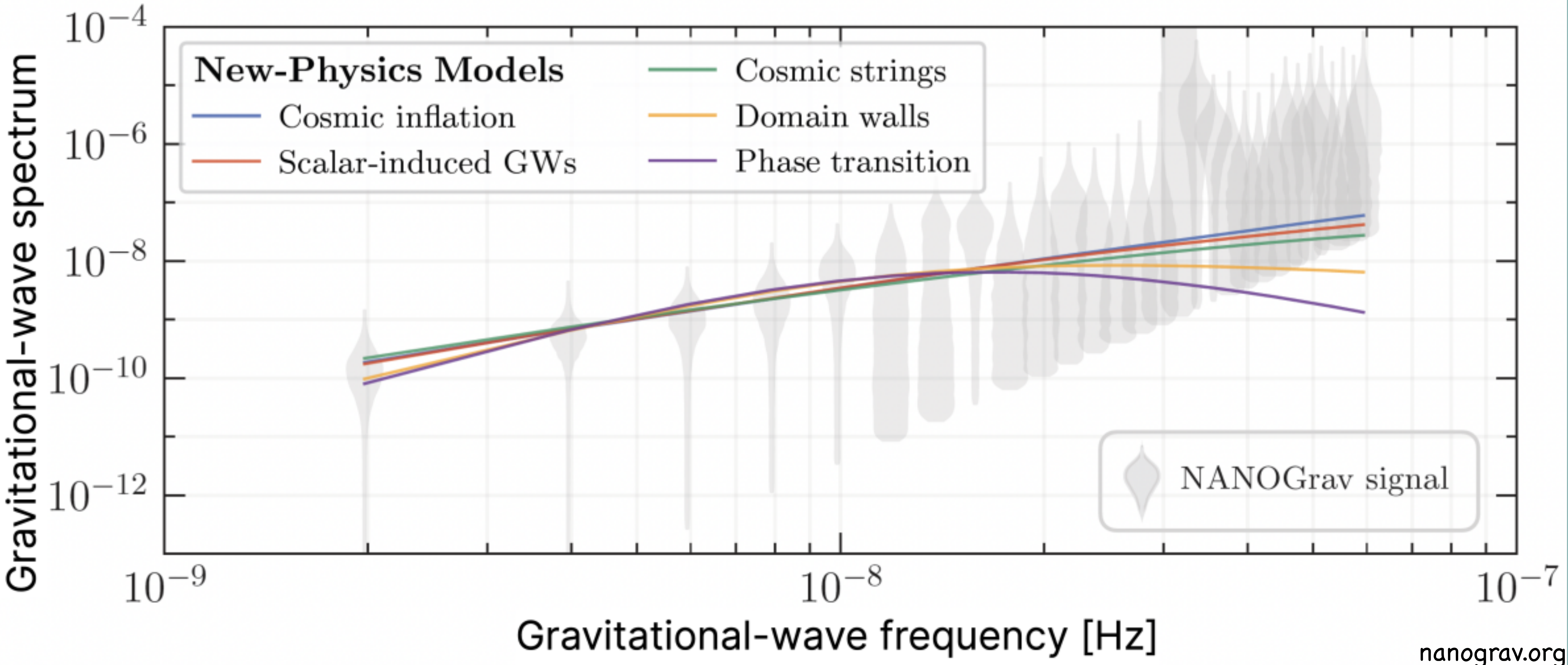
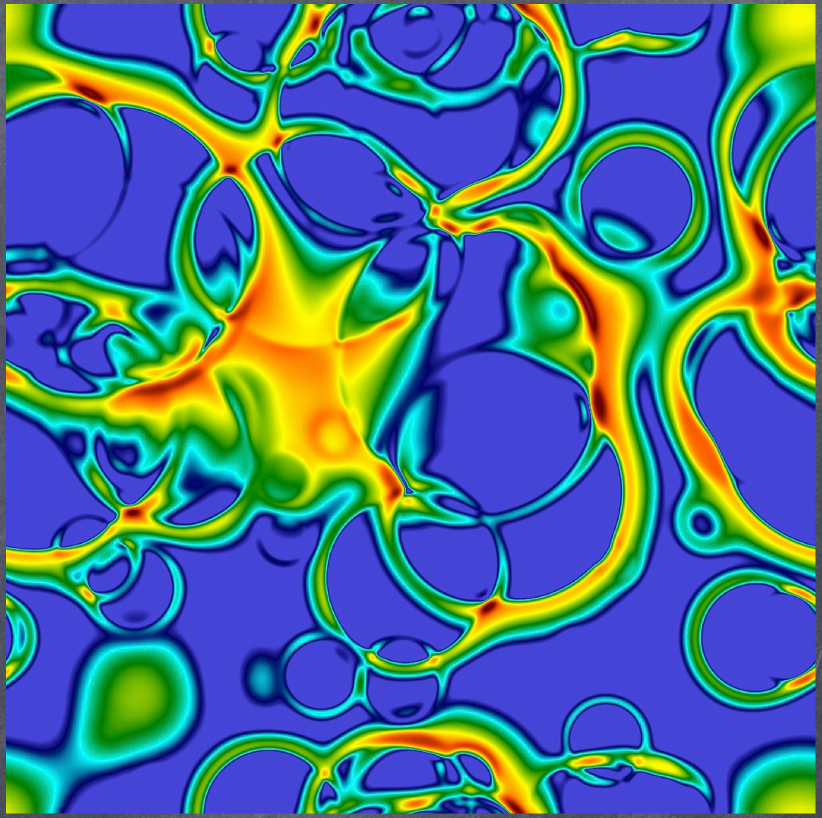
- PTAs see $\sim 3\sigma$ quadrupole correlation of timing residuals
- Smoking gun signal of stochastic GW background



nanograv.org

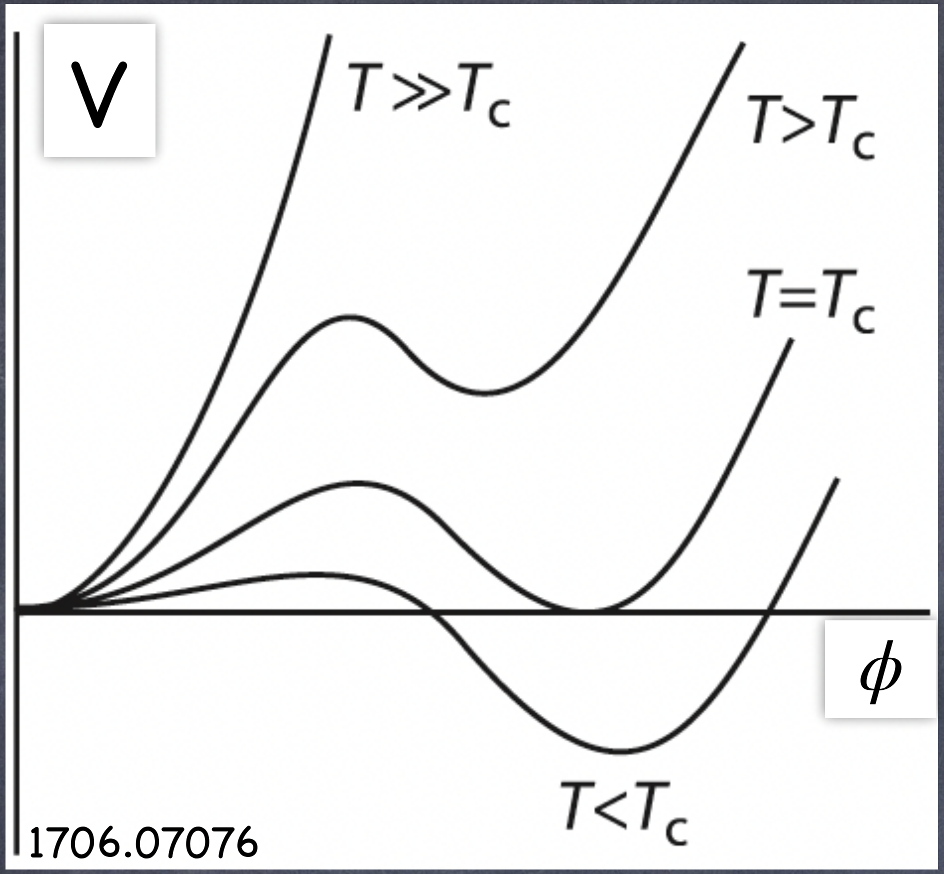
Sources of gravitational waves

Phase transitions



- Change in physical state (phase) of a system
- Change in vacuum structure as Universe cools
- Caused by spontaneous breaking of symmetries

First order PT



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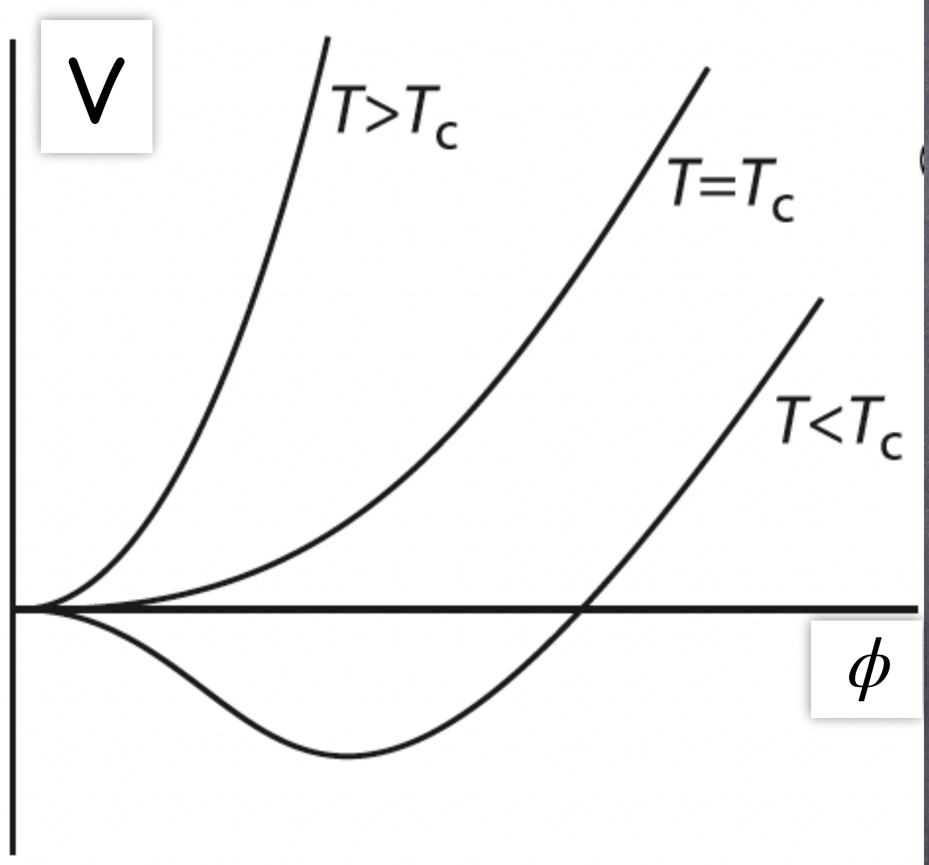
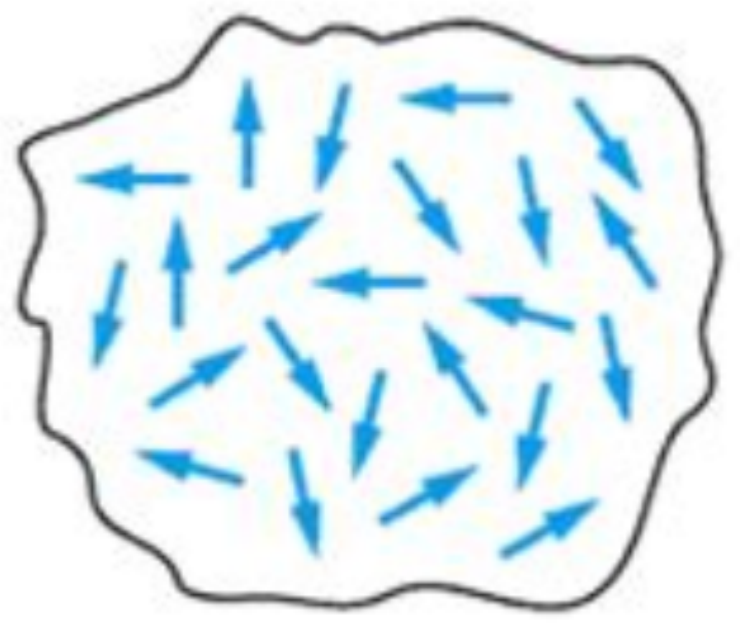
Second order PT

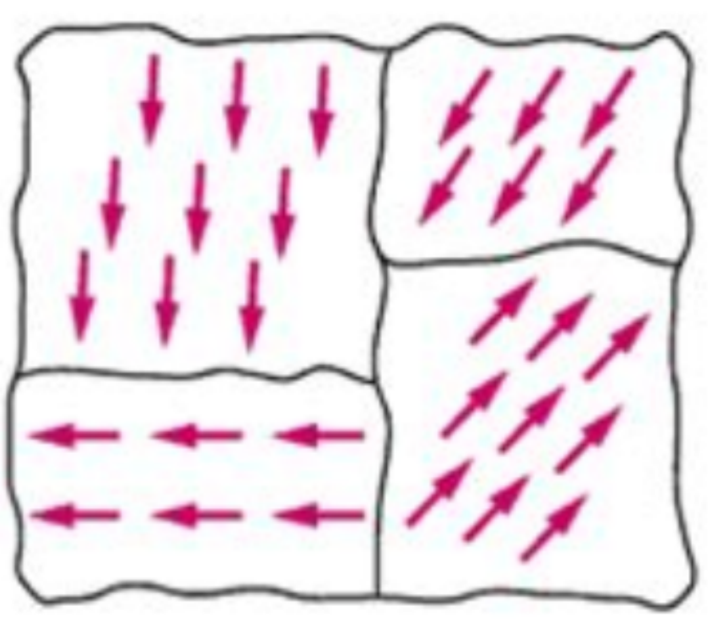


- FOPT associated with potential barrier and involves latent heat
- Thermal jump/quantum tunneling leads to nucleation of true vacuum bubbles
- Bubbles expand, transfer energy to plasma, collide thereby producing GWs

Cosmic defects



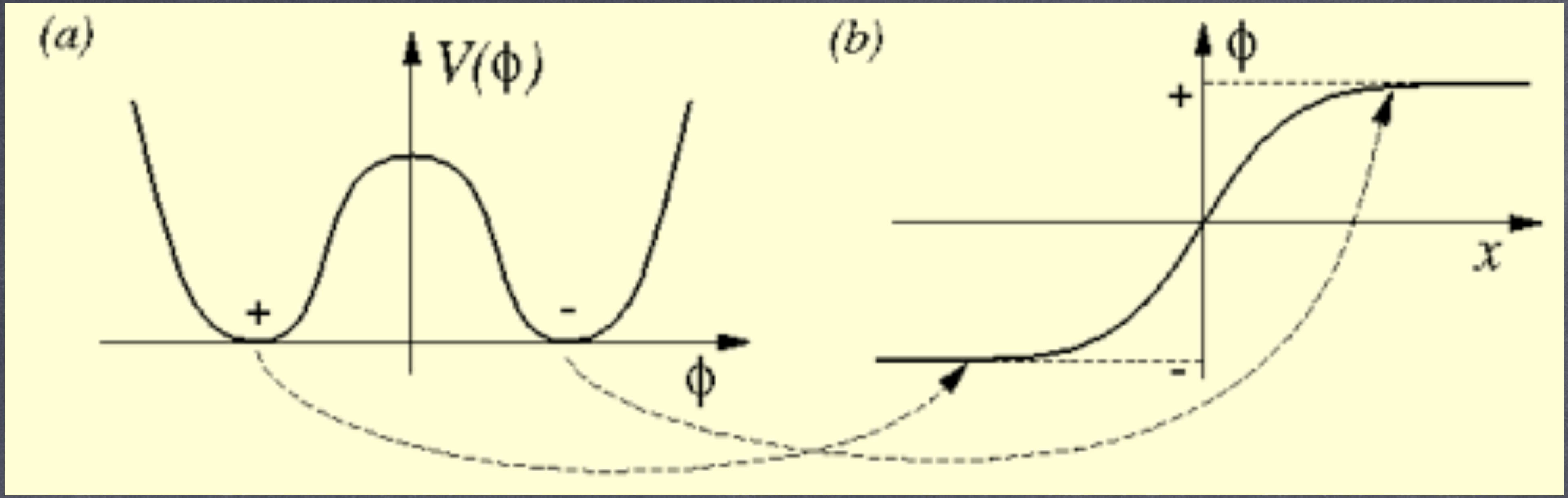
Above T_c



Below T_c

- As temperature is lowered below Curie temperature paramagnetic material becomes ferromagnetic
- Internal spin symmetry spontaneously broken
- Magnetic domains separated by stable domain walls

Cosmic domain walls

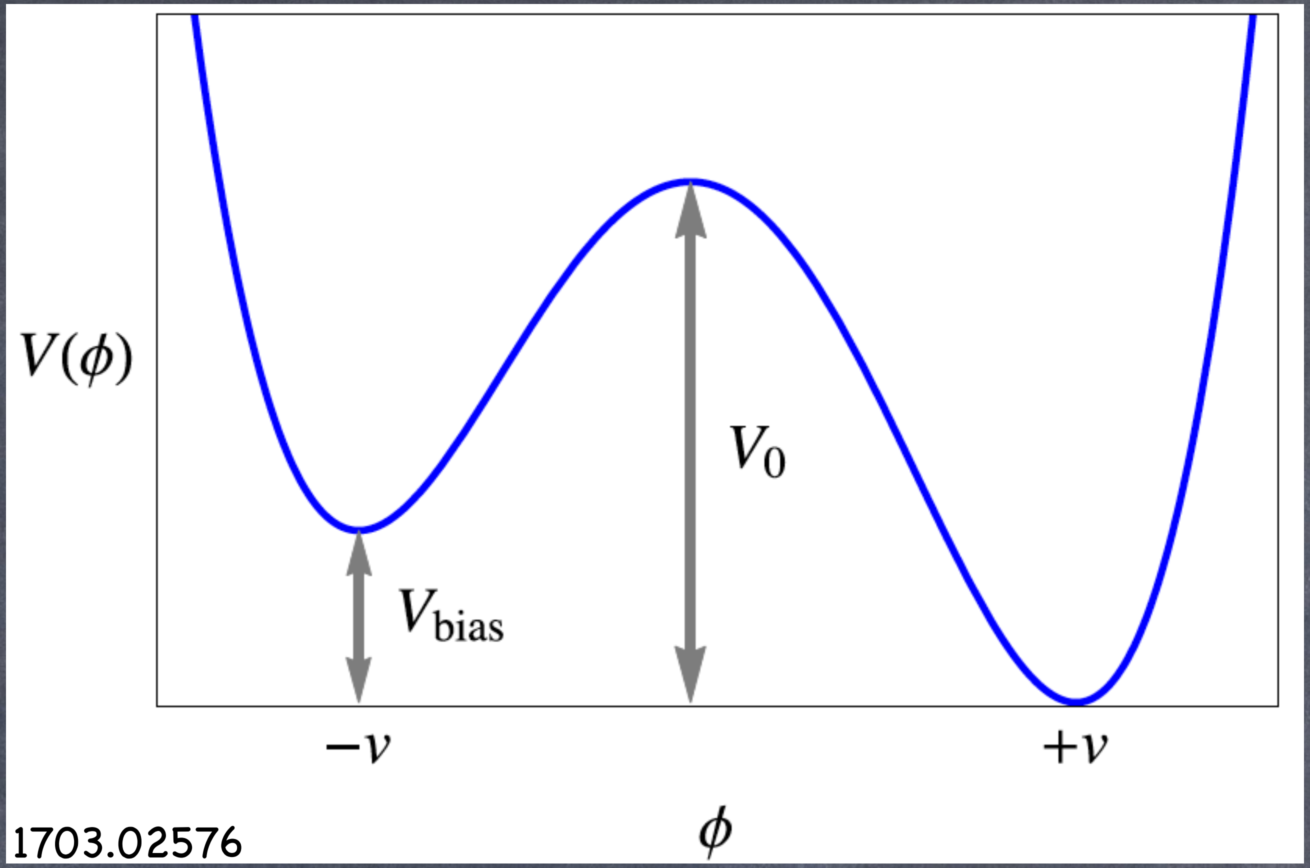


- Spontaneous breaking of $\phi \to -\phi$ symmetry

- + and - domains separated by domain wall which has surface energy density (called surface tension)

- Symmetry unbroken at center of wall $\phi = 0$

- Stable domain walls can dominate the energy density of the Universe since $\rho_{walls} \sim 1/t \sim H \implies \Omega_{walls} \sim 1/H \sim t$



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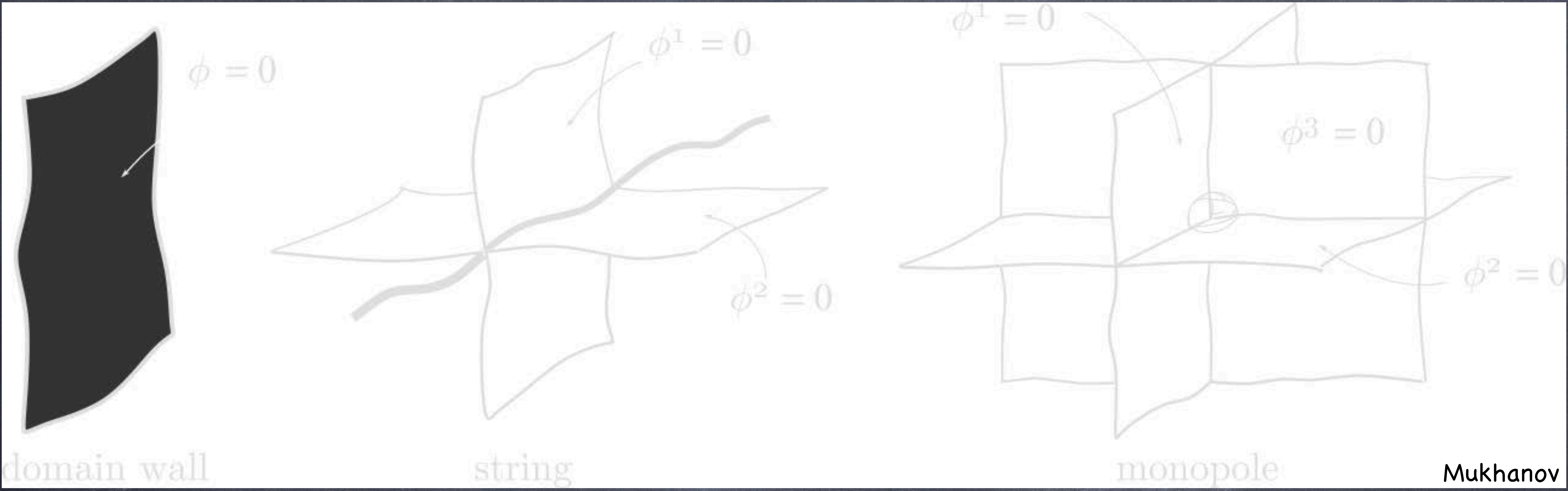
- Solution to domain wall problem is to break the symmetry
- Energy difference produces pressure $\sim V_{\text{bias}}$ on the walls
- Collapsing domain walls produce GWs

Other cosmic defects

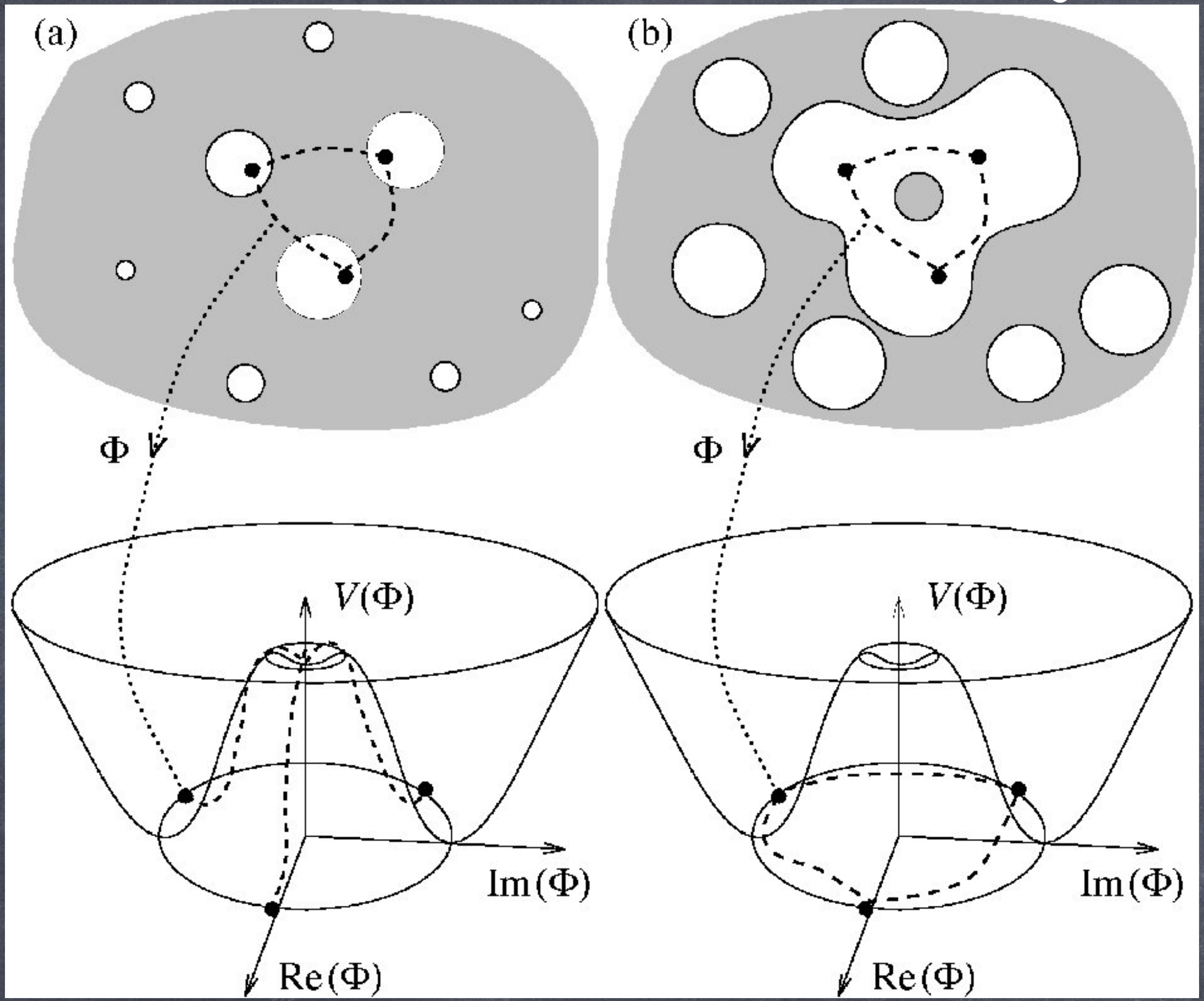


Mukhanov

Spontaneous symmetry breaking in a higher dimensional field space

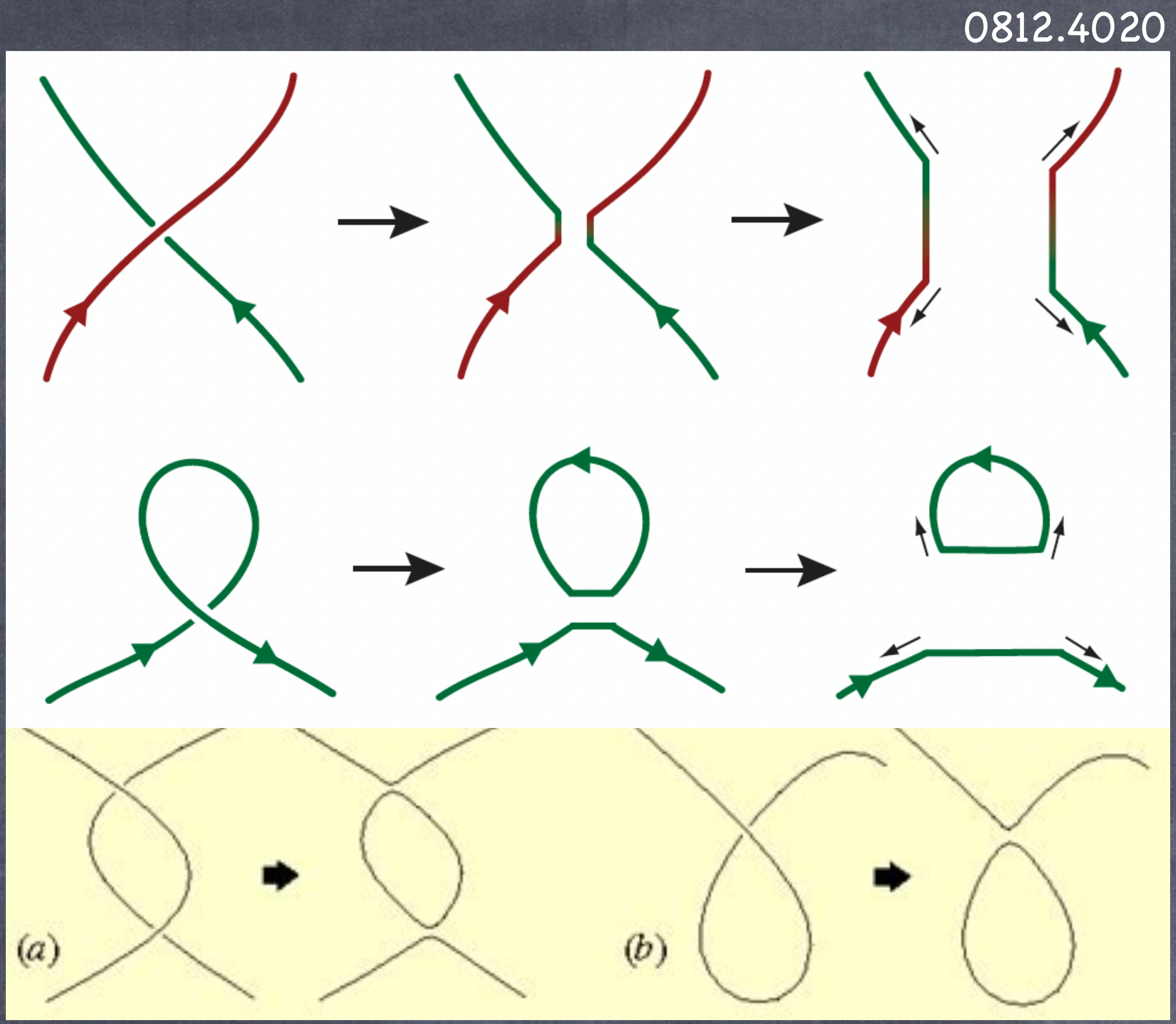
Cosmic strings

Jeong, Smoot



- Can have field theoretic origin, e.g., from spontaneous U(1) symmetry breaking
- Can be fundamental strings of superstring theory stretched to cosmic scales

String interactions



- Strings collide and reconnect (with segments exchanged) with intercommutation probability
- Produce loops by self-interactions or pair-wise interactions
- Oscillating loops lose energy mainly by GW emission

Scalar-induced GWs

- At linear order in perturbation theory, scalar and tensor modes evolve independently
- At second order these modes become coupled
- Large first-order scalar perturbations generate second-order tensor perturbations
- These tensor perturbations appear as GWs when scalar modes reenter the horizon at the end of inflation

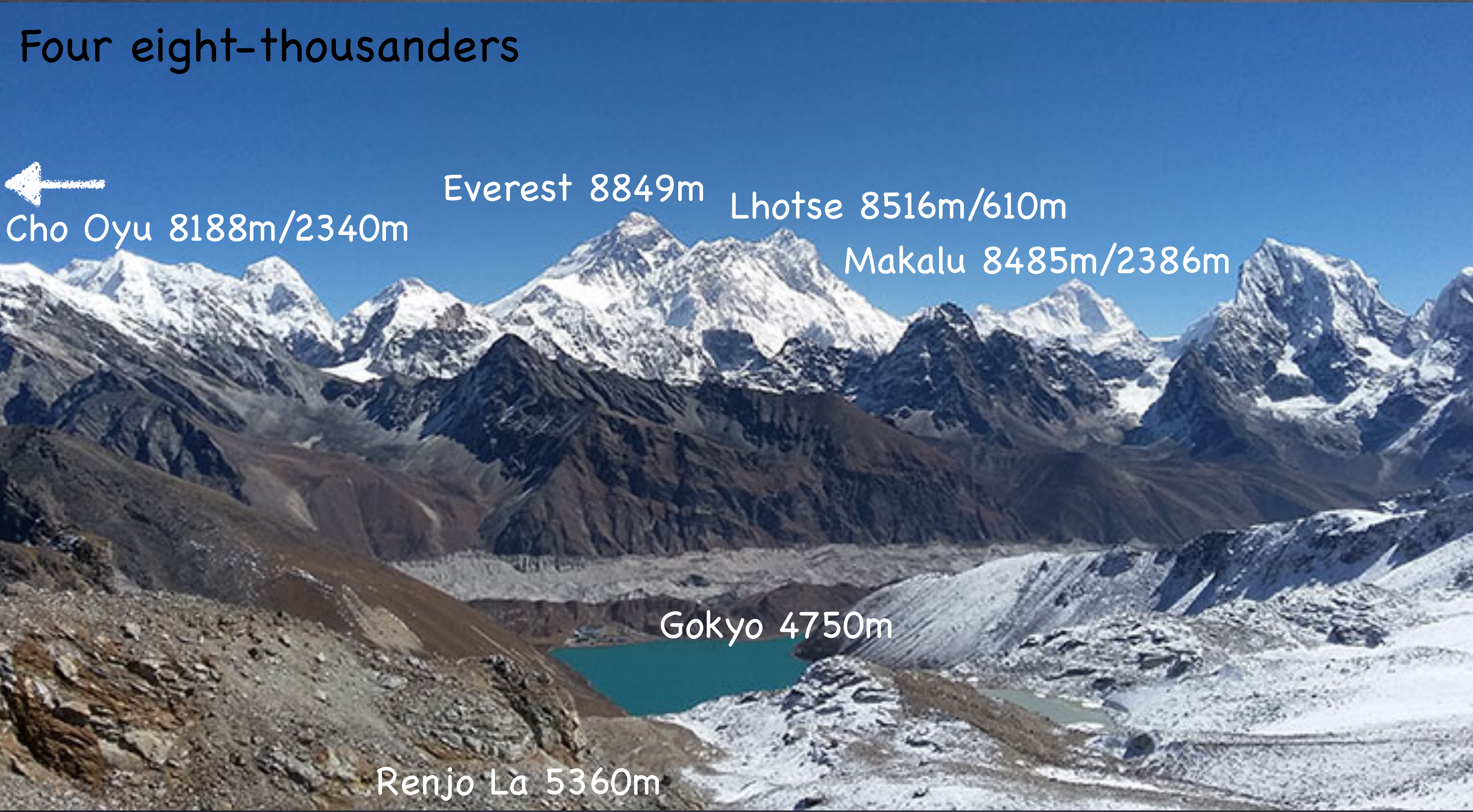
Discriminating between GW sources

Signal-to-Noise Ratio

$$\mathrm{SNR} = \sqrt{T_{\mathrm{obs}} \int_{f_{\mathrm{min}}}^{f_{\mathrm{max}}} df \, \frac{\Omega^2(f)}{\Omega^2_{\mathrm{Sens}}(f)}}$$

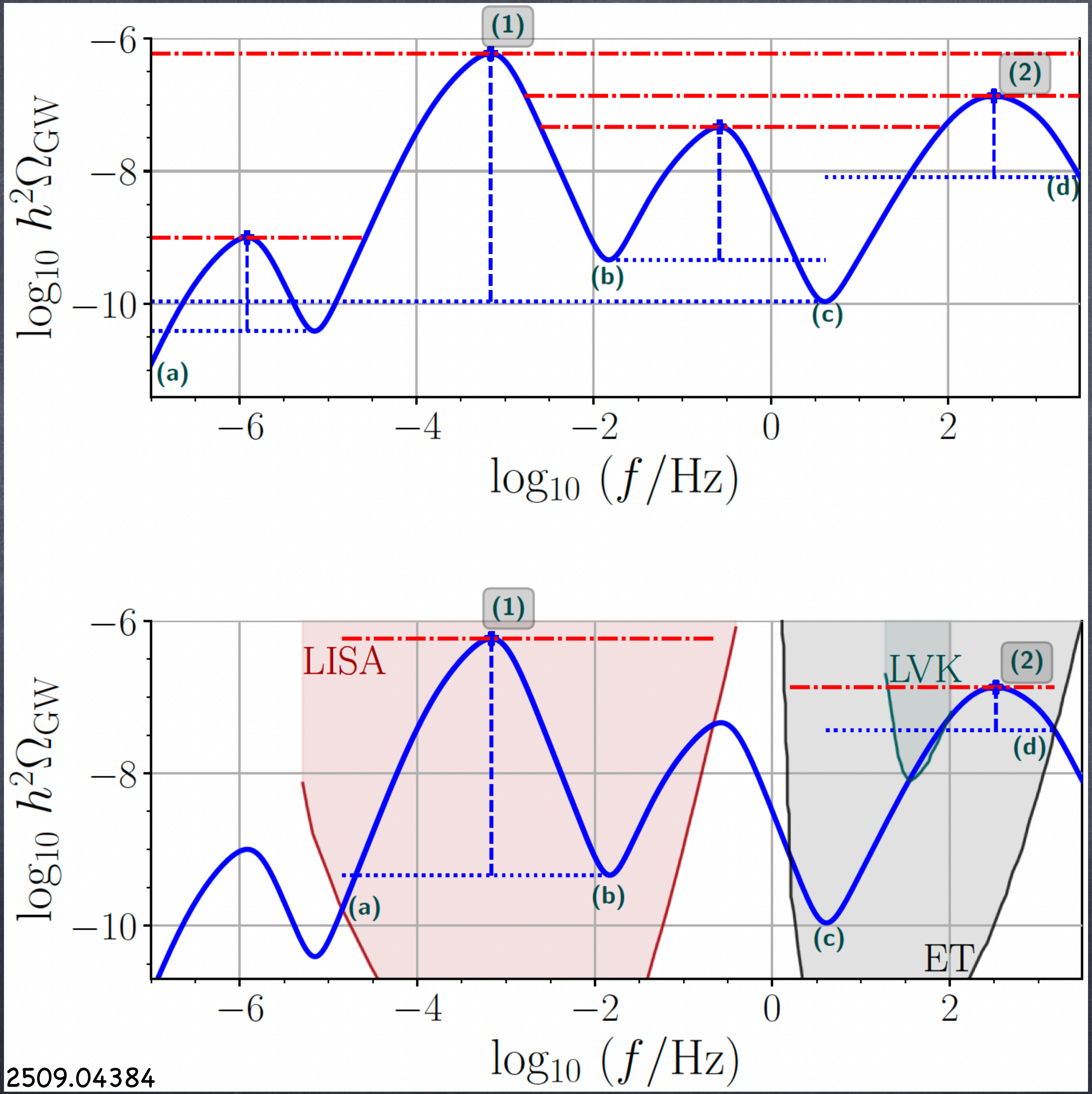
- Used in most theoretical studies
- Spectral information of signal and background washed out because SNR is an integrated measure
- Would like observable that utilizes the shape of the signal to discriminate between sources

Prominence is a measure of how much a peak rises above the surrounding terrain



Lhotse has greater elevation than Makalu, but is much less prominent

Prominence



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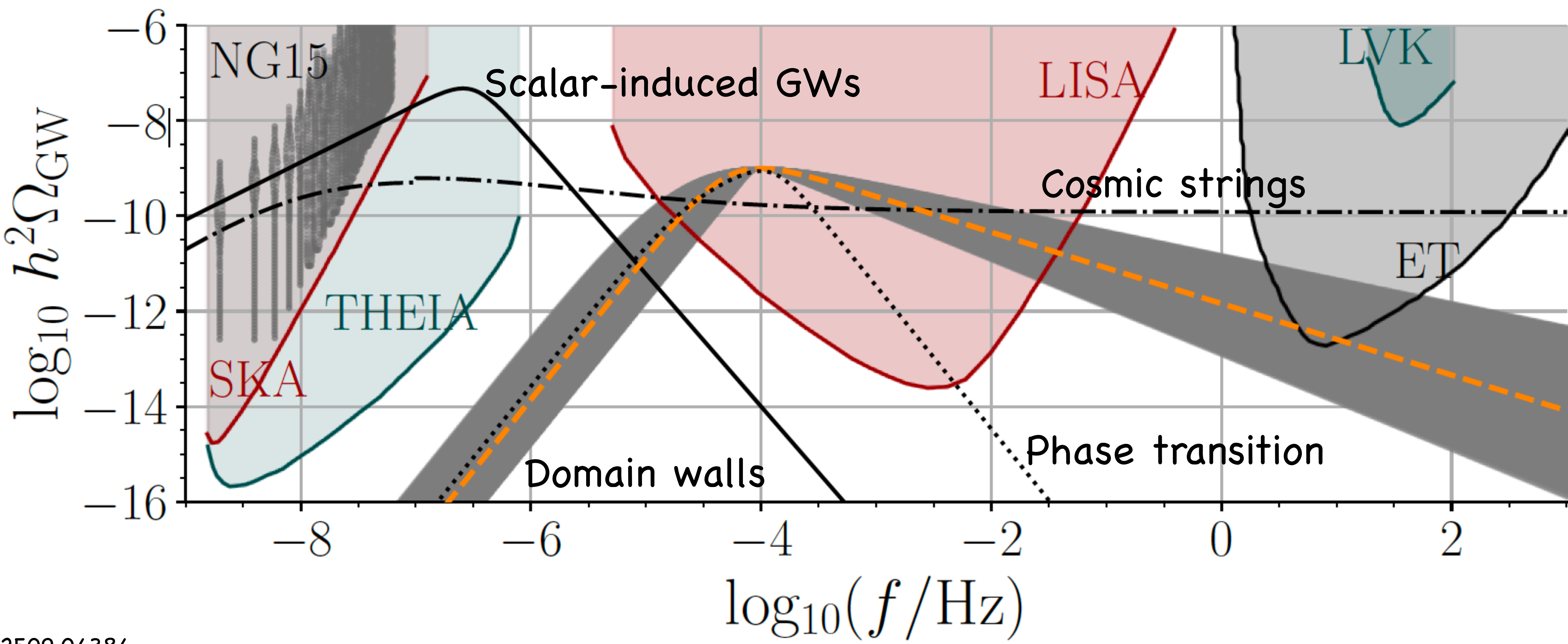
$$\mathscr{P}_i = \log_{10}\left[h^2\Omega_{\rm GW}^{\rm peak}\right]_i - \log_{10}\left[h^2\Omega_{\rm GW}^{\rm base}\right]_i$$

GW signals



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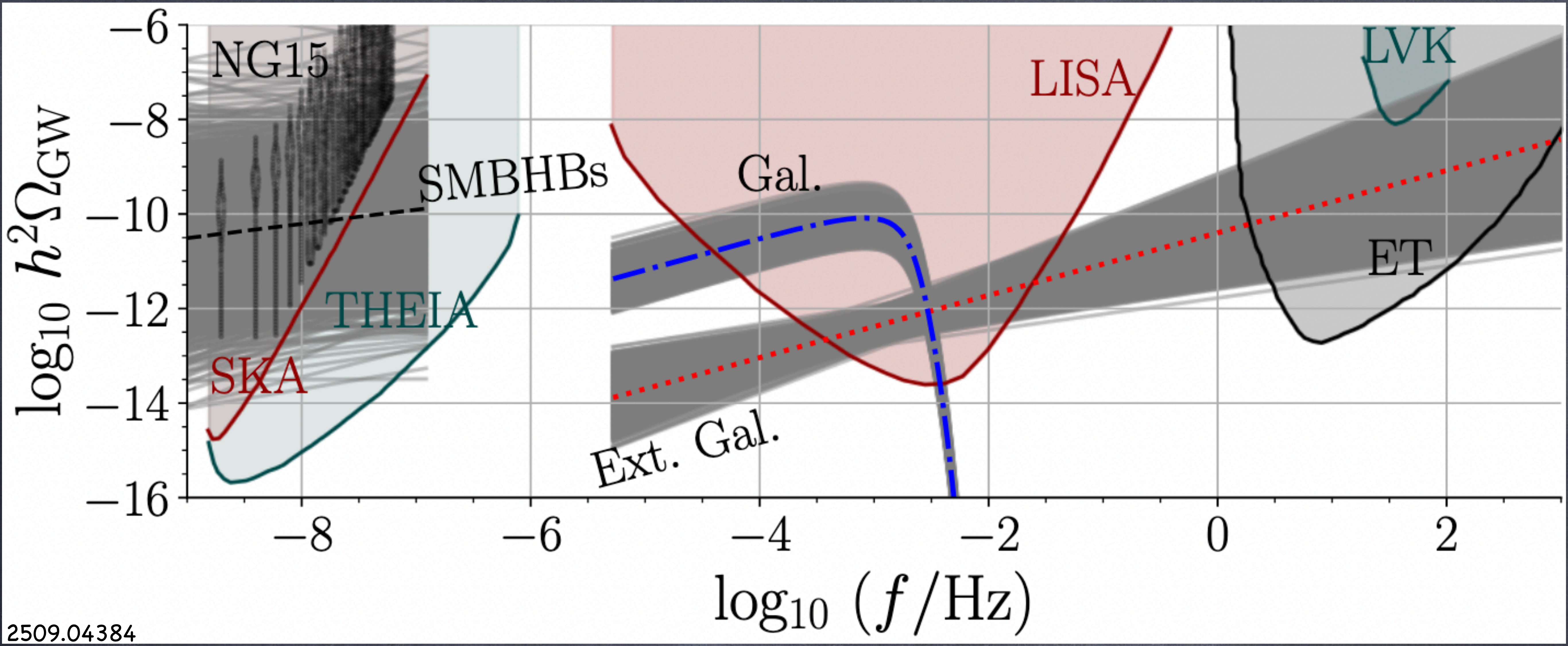
- DW spectrum is broader than FOPT spectrum
- DW and FOPT spectra are similar below peak frequency
- CS spectrum in interferometer band is almost flat
- SIGWs only relevant in PTA band

Backgrounds



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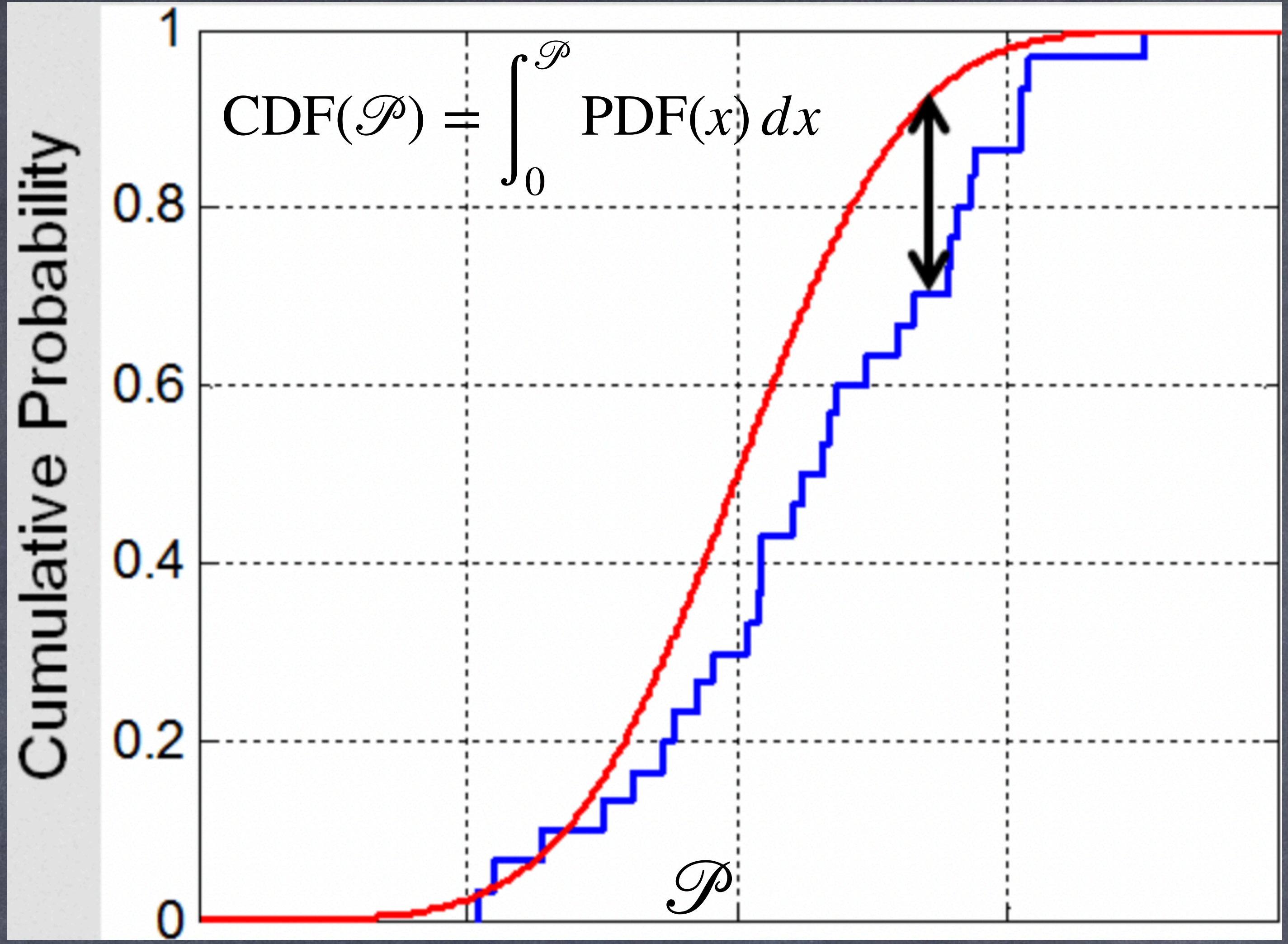
- SMBHB spectrum extends to higher frequencies and falls off above 1 HZ

- Interferometer bkg templates subtract individually resolvable binaries

- We analyze PTA and interferometer bands separately

Assumptions and procedure to illustrate how $\mathscr{P}$ works ...

- Observation time: 4 years
- Detection threshold: SNR = 10
- Spectrum must have at least one peak
- CS contribution in interferometer band is subdominant to FOPT and DW peaks
- Peak amplitude and frequency of FOPT and DW spectra are identical (most conservative case)
- Simulate datasets with backgrounds varied within ranges determined by simulations
- For each peak construct PDF of $\mathscr{P}$



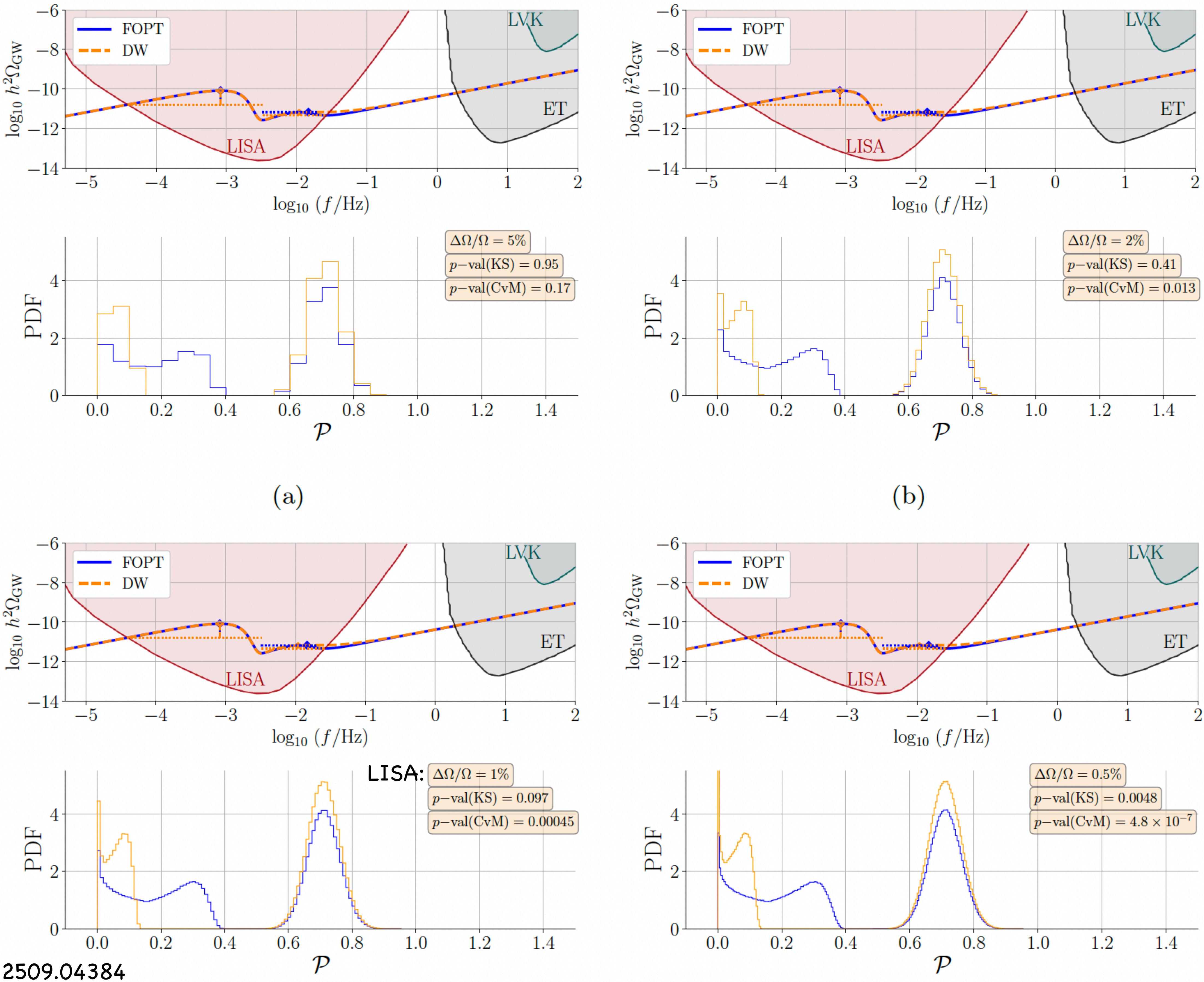
Kolmogorov-Smirnov test:

$$\mathrm{KS} = \max_{\mathscr{P}} \left| \mathrm{CDF}_1(\mathscr{P}) - \mathrm{CDF}_2(\mathscr{P}) \right|$$

Cramer-von Mises test (measures the aggregate difference in CDFs):

$$\mathrm{CvM} = \int_0^{\infty} \left[\mathrm{CDF}_1(\mathscr{P}) - \mathrm{CDF}_2(\mathscr{P})\right]^2 \; \mathrm{dCDF}_1(\mathscr{P})$$

Impact of amplitude uncertainties



LISA:

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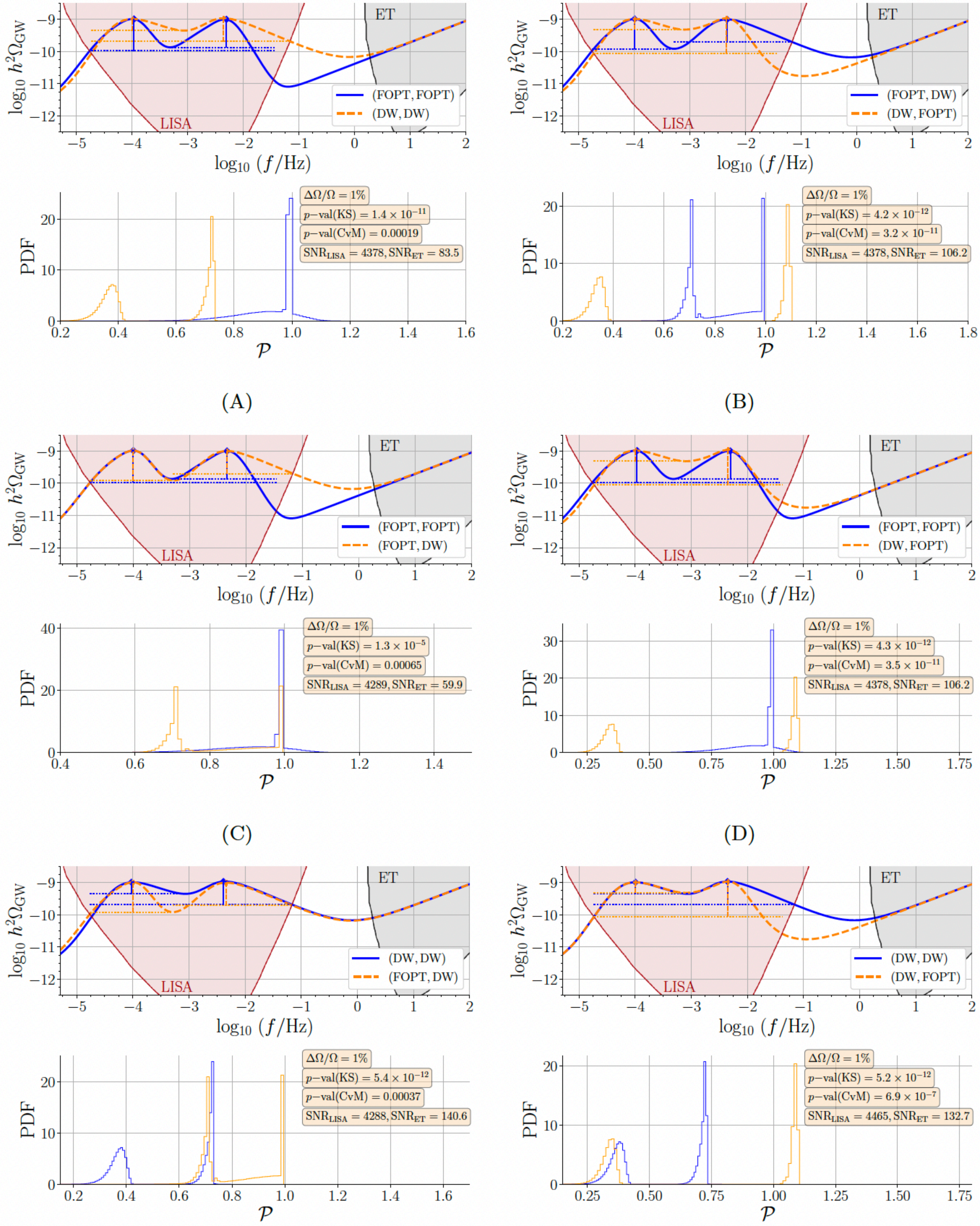
Bin size determined by measurement uncertainty in $\Omega_{\rm GW}$

Two-peaked spectra in the LISA band

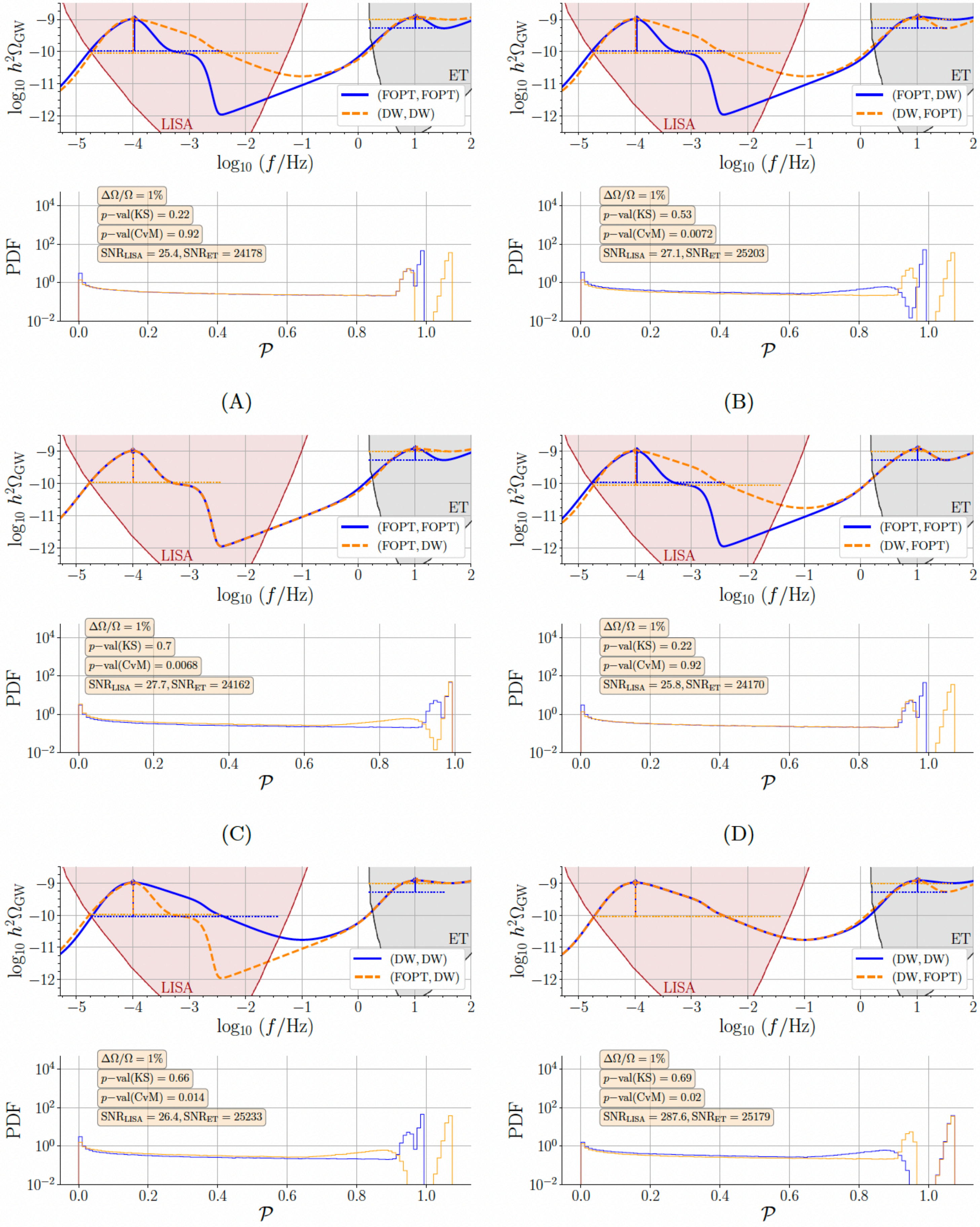


(A) (B)

(C) (D)

>3.4σ discrimination with CvM test. Much more sensitivity with KS test

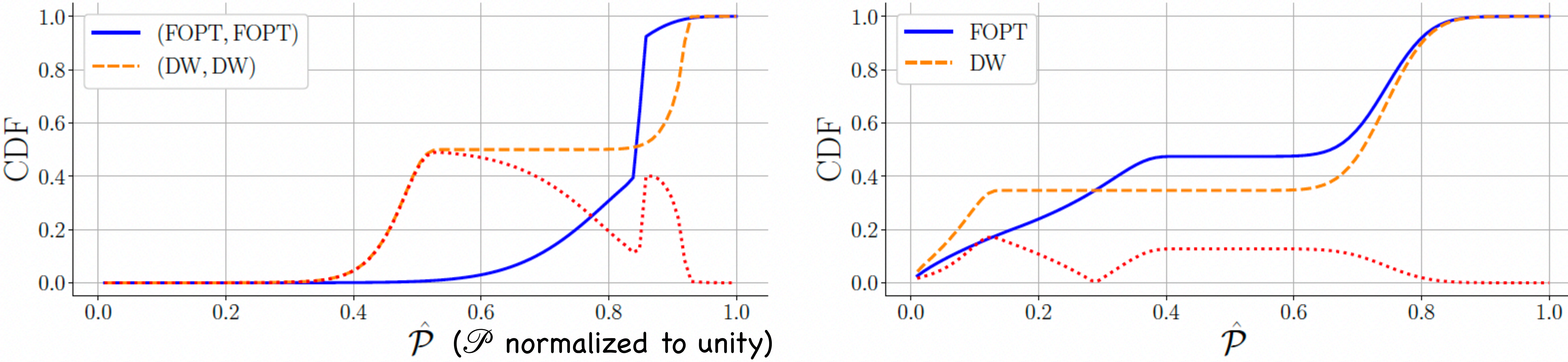
Two-peaked spectra with peaks in the LISA and ET bands



(A) (B)

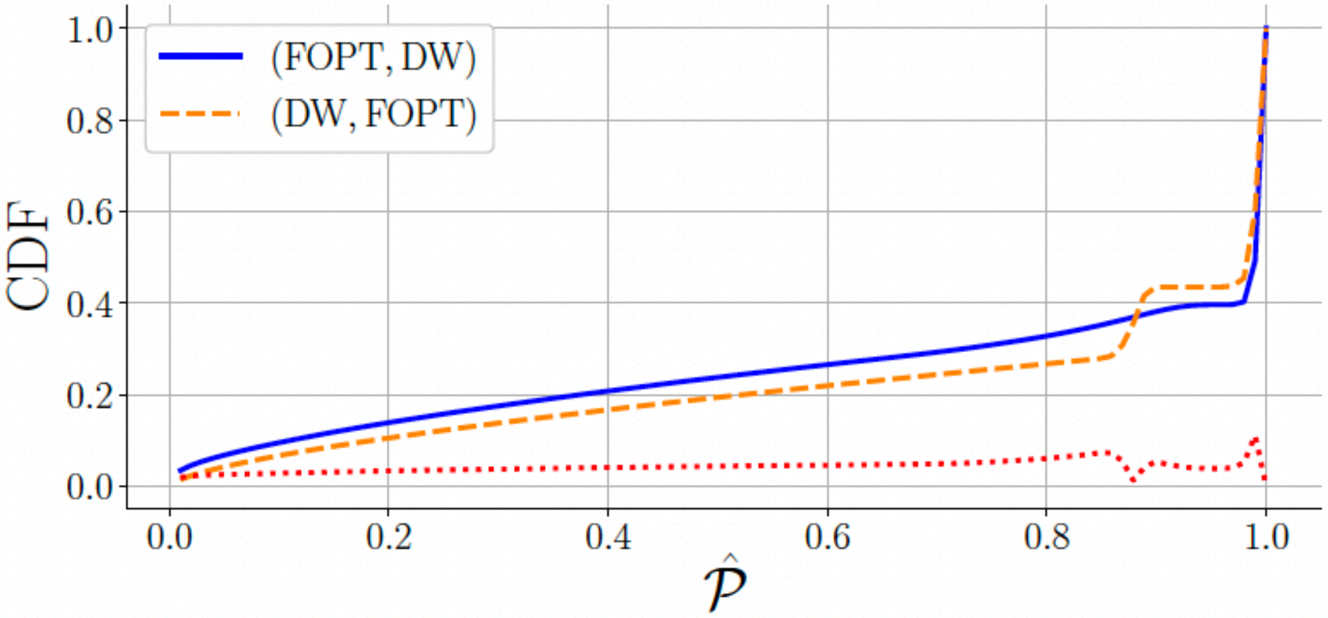
(C) (D)

Poor discrimination because of large extrgalactic bkg in the ET band



(a) p-val(KS) $= 1.4 \times 10^{-11}$, p-val(CvM) $= 0.00019$. (b) p-val(KS) $= 0.097$, p-val(CvM) $= 0.00045$.

dotted = |solid - dashed|



(c) p-val(KS) $= 0.53$, p-val(CvM) $= 0.0072$.

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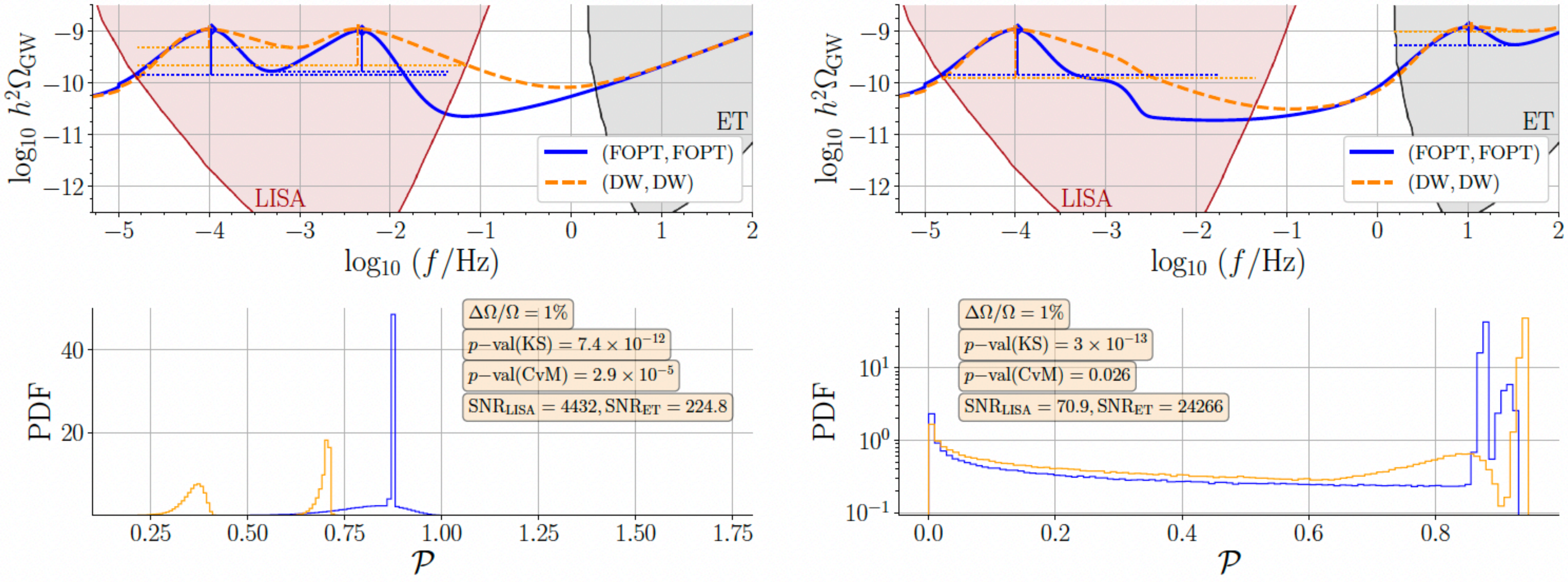
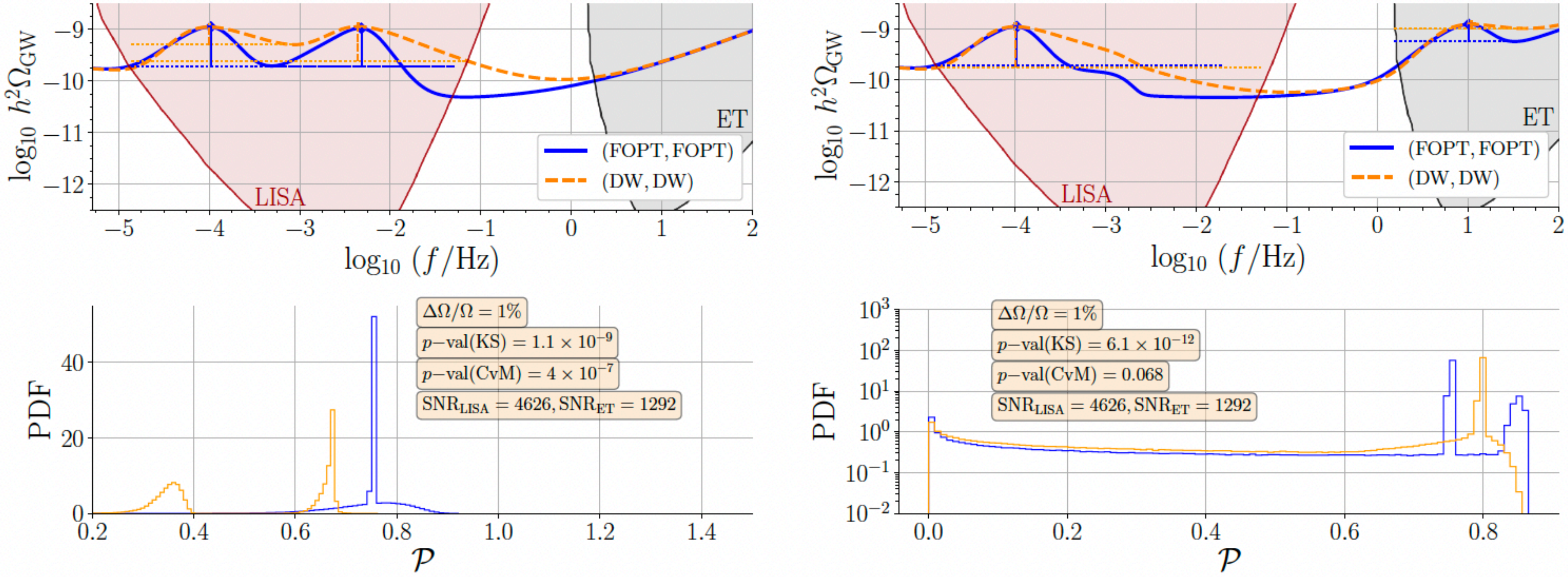
CvM is usually more conservative than KS, but can give smaller p-values if the CDFs

1) do not have sharp features
2) exhibit a systematic shift or persistent differences

Impact of cosmic strings



(a) $G\mu = 10^{-14}$ (CS contribution < Bkg) (b) $G\mu = 10^{-14}$



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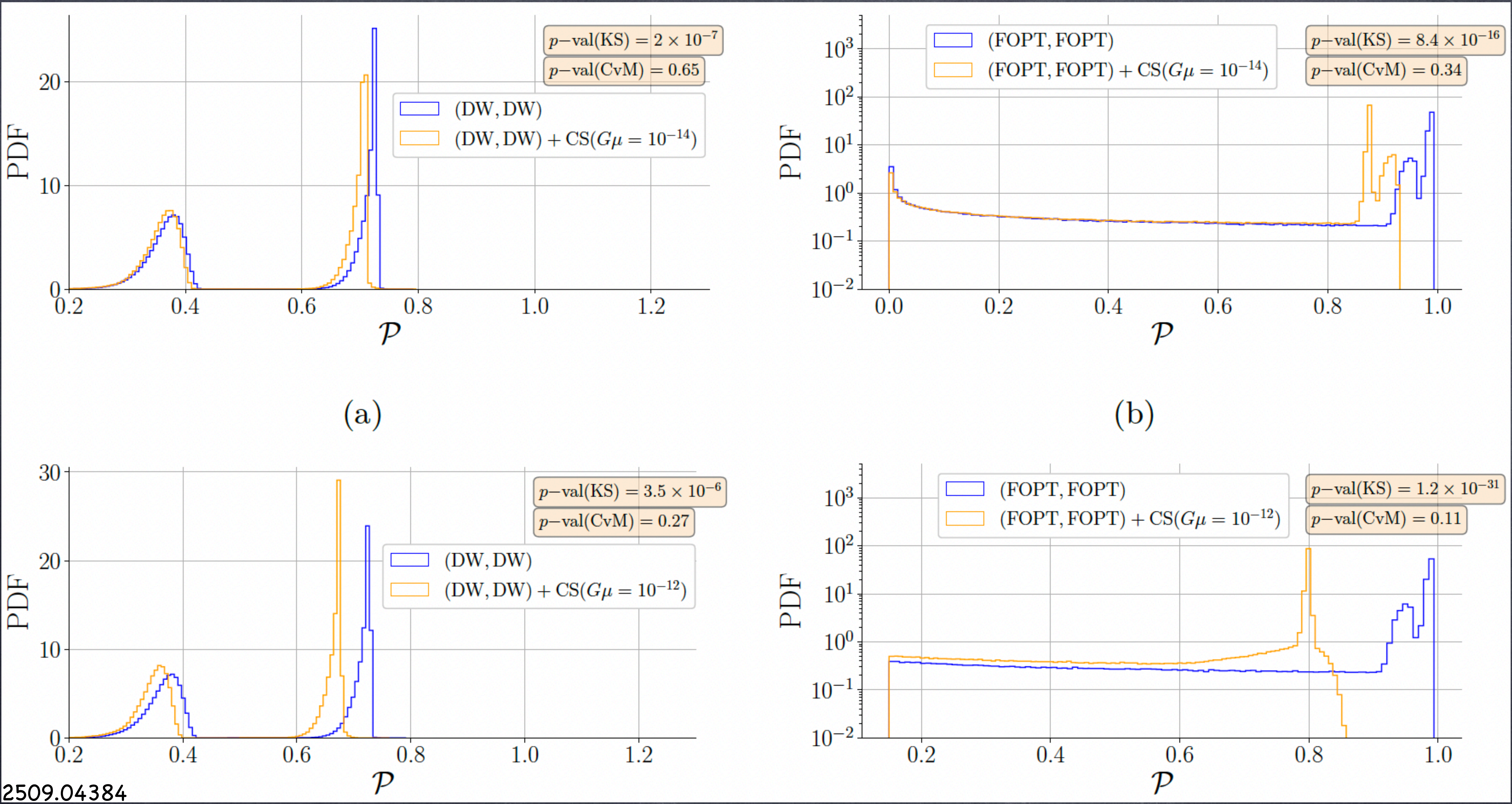
(c) $G\mu = 10^{-12}$ (CS contribution $\gtrsim$ Bkg) (d) $G\mu = 10^{-12}$

CSs can enhance the discrimination power of $\mathscr{P}$



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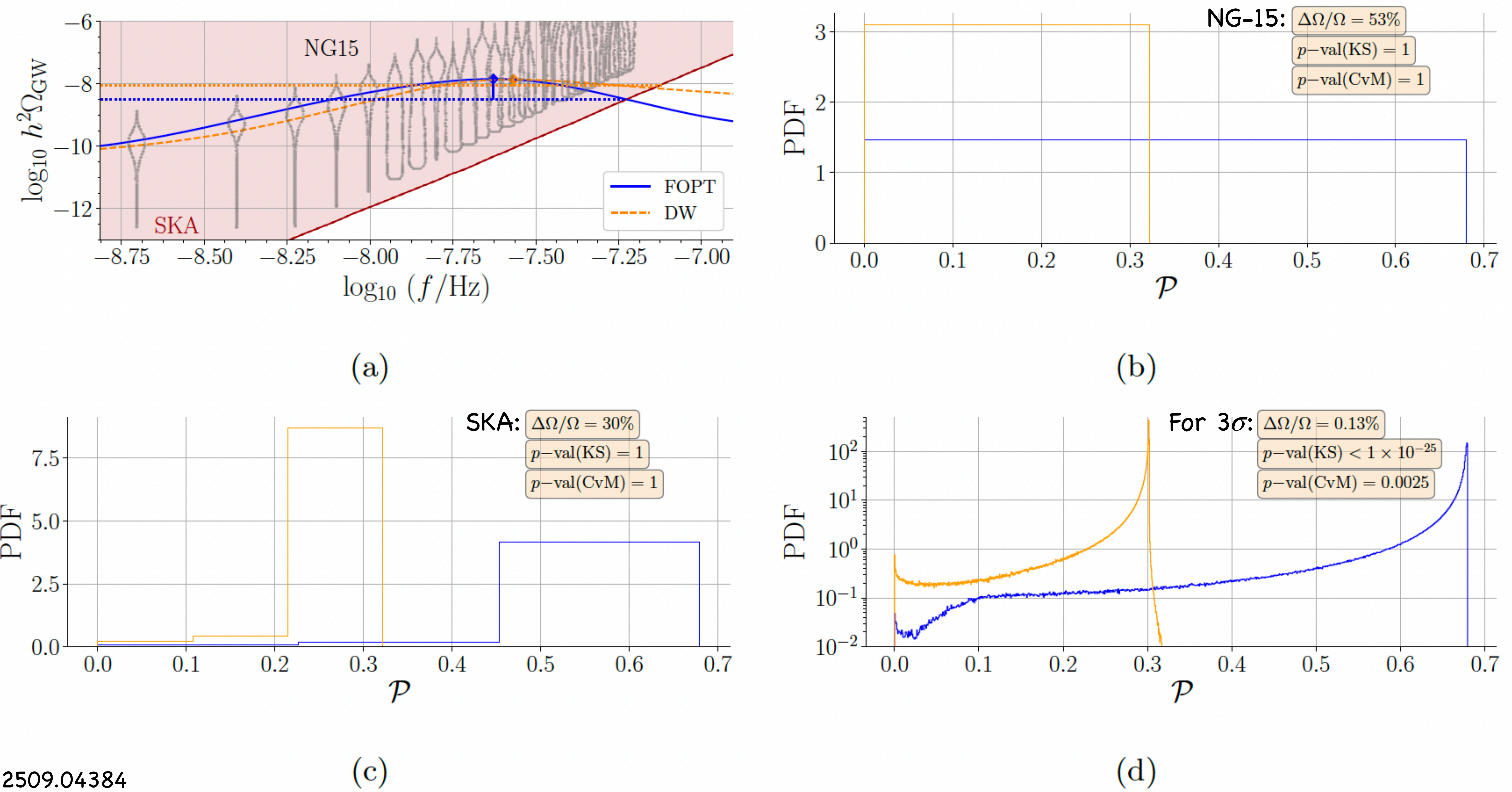
CSs can bury the bkg and produce large local differences giving small p-values for the KS test

Bigger effect for FOPTs than DWs because spectrum is narrower

Signals in the PTA band



(a) (b)

(c) (d)

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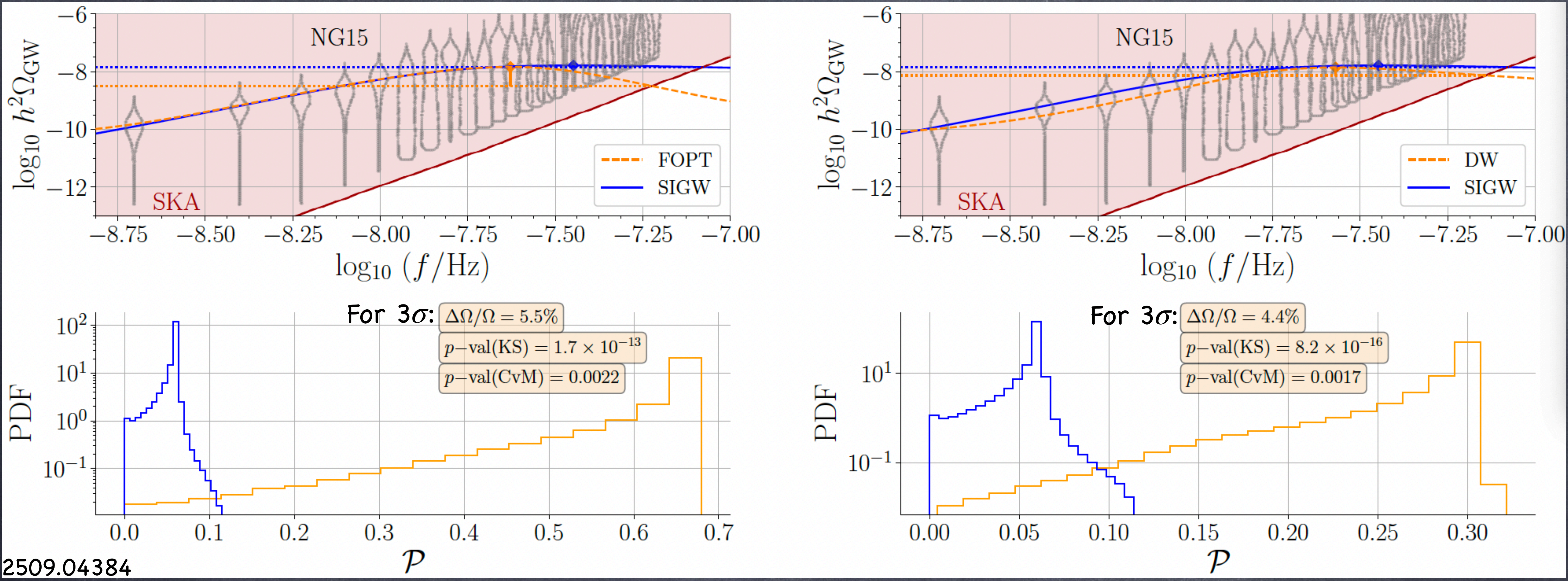
FOPT, DW, SIGW parameters set by fitting to NANOGrav-15 data

Gauge CSs do not fit these data well



For 3σ: $\Delta\Omega/\Omega = 5.5\%$

p–val(KS) $= 1.7 \times 10^{-13}$

p–val(CvM) $= 0.0022$

For 3σ: $\Delta\Omega/\Omega = 4.4\%$

p–val(KS) $= 8.2 \times 10^{-16}$

p–val(CvM) $= 0.0017$

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Easier to discriminate between FOPT/SIGW, DW/SIGW than FOPT/DW

Prominence in real-world settings

Prominence vs SNR-based signal discrimination

$$\Omega_i = \Omega_{\mathrm{SIG_i}} + \Omega_{\mathrm{Bkg}}$$

$$\langle \Omega_1 | \Omega_2 \rangle = T_{\mathrm{obs}} \int_{f_{\mathrm{min}}}^{f_{\mathrm{max}}} df \, \frac{\Omega_1(f)\Omega_2(f)}{\Omega^2_{\mathrm{Sens}}(f)}$$

If $\Omega_1 = \Omega_2 \equiv \Omega$,

$$\mathrm{SNR} = \sqrt{\langle \Omega | \Omega \rangle} = \sqrt{T_{\mathrm{obs}} \int_{f_{\mathrm{min}}}^{f_{\mathrm{max}}} df \, \frac{\Omega^2(f)}{\Omega^2_{\mathrm{Sens}}(f)}}$$

Signal-space statistic:

$$\chi^2 = \langle \Omega_1 - \Omega_2 | \Omega_1 - \Omega_2 \rangle$$

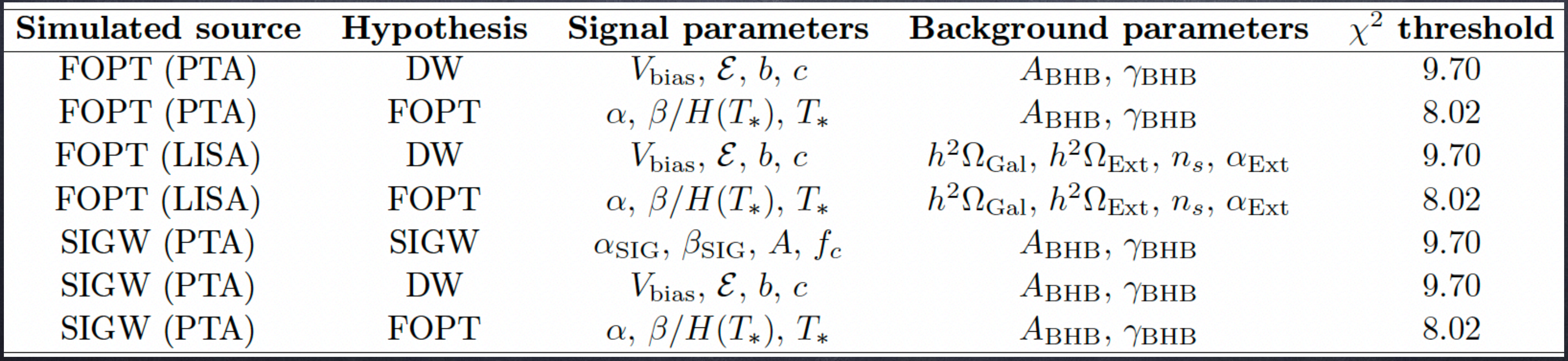
$$\text{Mismatch :} \quad \mathscr{M}_{12} \equiv 1 - \frac{\langle \Omega_1 | \Omega_2 \rangle}{\mathrm{SNR}_1 \, \mathrm{SNR}_2}$$

$$\therefore \; \chi^2 = (\mathrm{SNR}_1 - \mathrm{SNR}_2)^2 + 2\mathscr{M}_{12} \, \mathrm{SNR}_1 \, \mathrm{SNR}_2$$

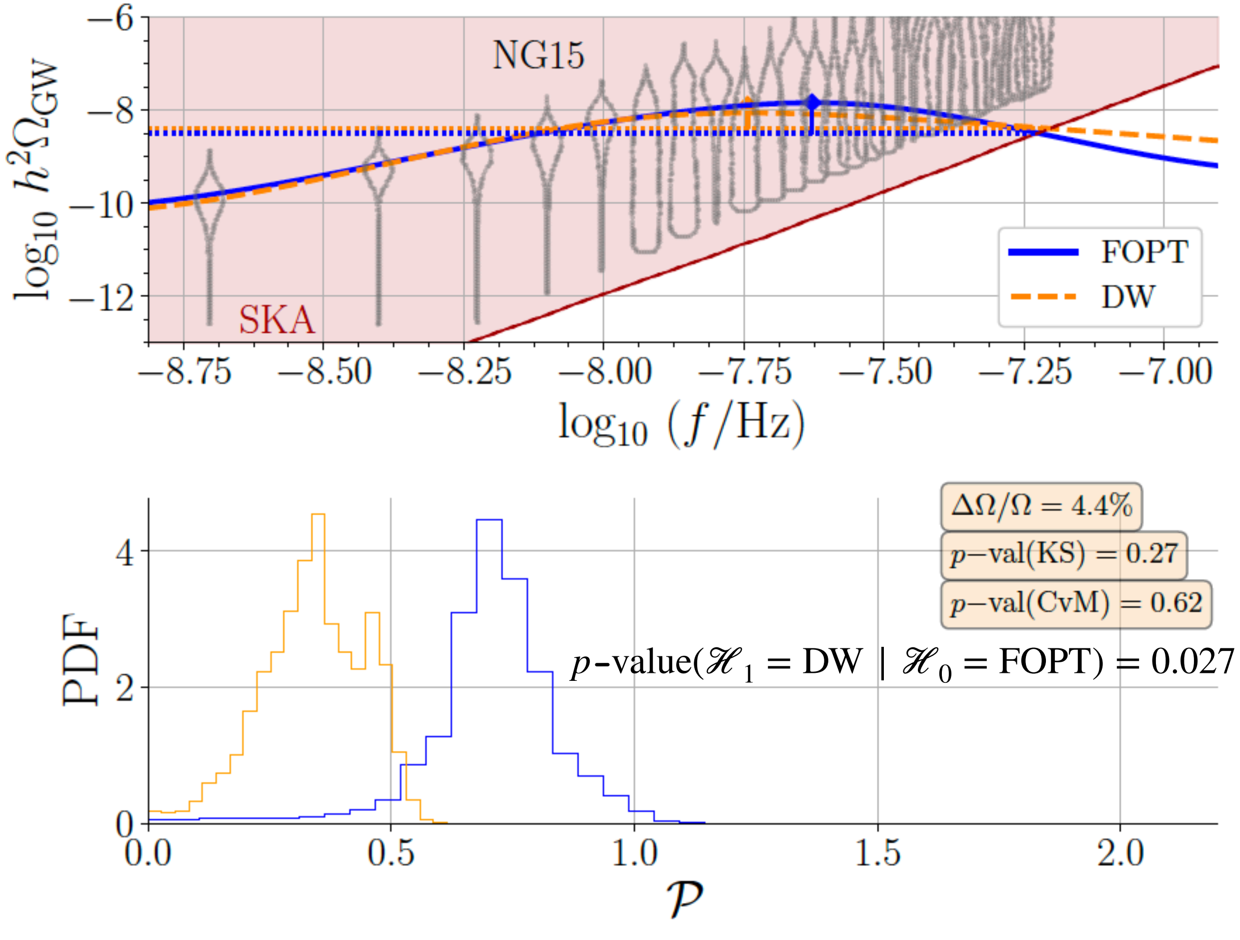
$$\text{For identical signals, } \mathscr{M}_{12} = 0 \implies \chi^2 = 0$$

$$\text{For no signal overlap, } \mathscr{M}_{12} = 1 \implies \chi^2 = \mathrm{SNR}_1^2 + \mathrm{SNR}_2^2$$

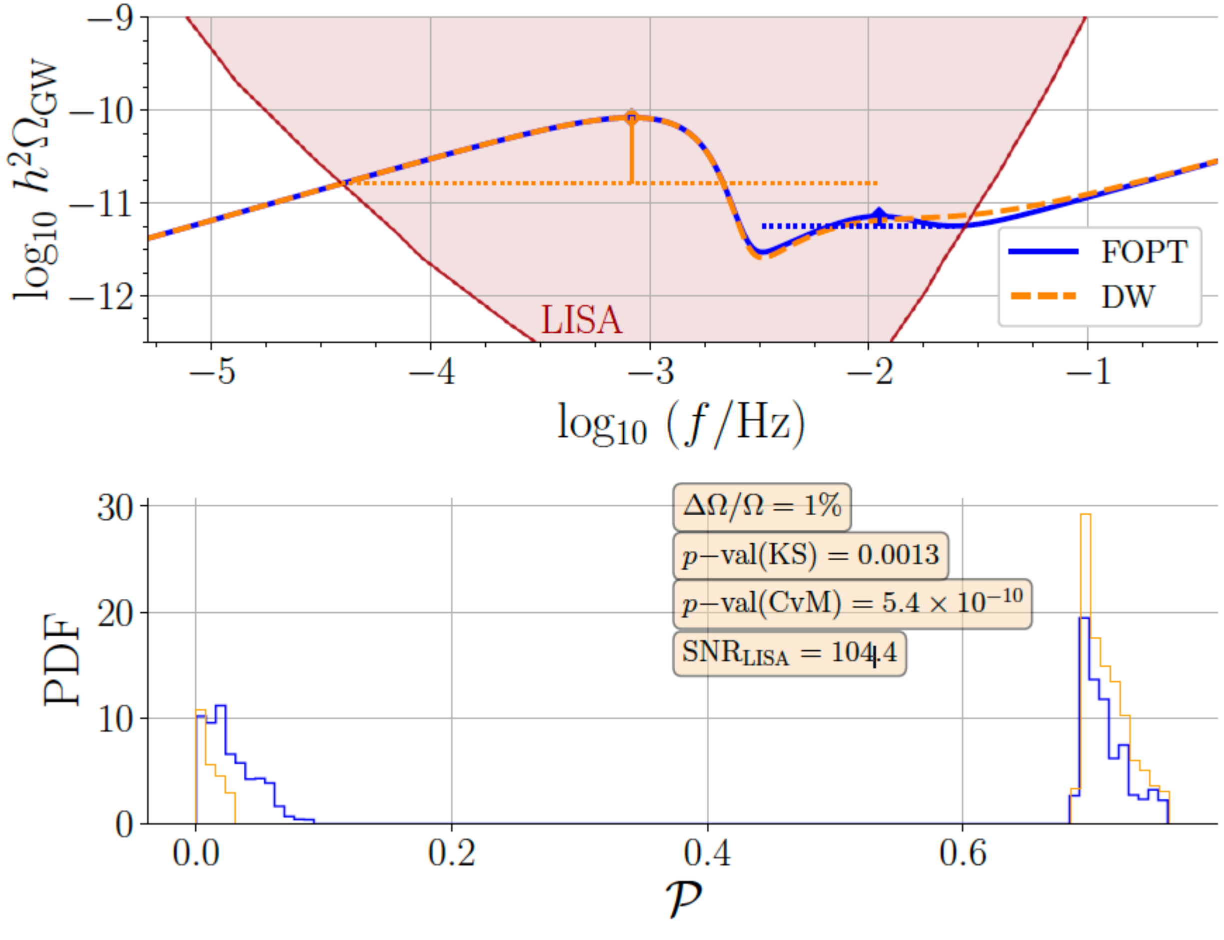
- Fix Ω_1 (simulating experimental spectrum)
- Fit with Ω_2 by varying Ω_{SIG_2} and Ω_{Bkg}
- Scan over parameters in table for simulated source and hypothetical source
- Select points for each source within 2σ as per SNR
- Produce CDFs for $\mathscr{P}$ and apply KS and CvM tests



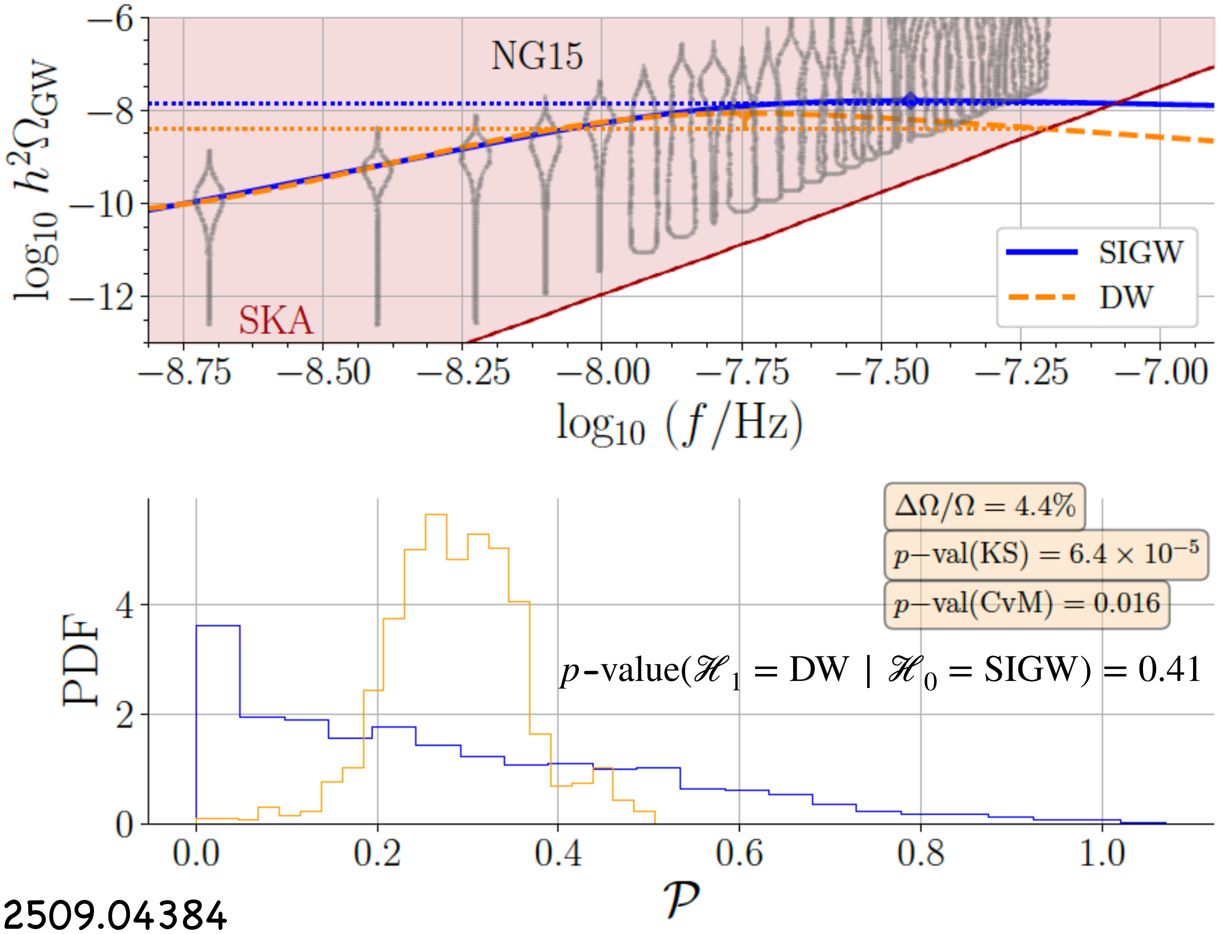
Simulated source	**Hypothesis**	**Signal parameters**	**Background parameters**	**χ^2 threshold**
FOPT (PTA)	DW	$V_{\text{bias}},\, \mathcal{E},\, b,\, c$	$A_{\text{BHB}},\, \gamma_{\text{BHB}}$	9.70
FOPT (PTA)	FOPT	$\alpha,\, \beta/H(T_*),\, T_*$	$A_{\text{BHB}},\, \gamma_{\text{BHB}}$	8.02
FOPT (LISA)	DW	$V_{\text{bias}},\, \mathcal{E},\, b,\, c$	$h^2\Omega_{\text{Gal}},\, h^2\Omega_{\text{Ext}},\, n_s,\, \alpha_{\text{Ext}}$	9.70
FOPT (LISA)	FOPT	$\alpha,\, \beta/H(T_*),\, T_*$	$h^2\Omega_{\text{Gal}},\, h^2\Omega_{\text{Ext}},\, n_s,\, \alpha_{\text{Ext}}$	8.02
SIGW (PTA)	SIGW	$\alpha_{\text{SIG}},\, \beta_{\text{SIG}},\, A,\, f_c$	$A_{\text{BHB}},\, \gamma_{\text{BHB}}$	9.70
SIGW (PTA)	DW	$V_{\text{bias}},\, \mathcal{E},\, b,\, c$	$A_{\text{BHB}},\, \gamma_{\text{BHB}}$	9.70
SIGW (PTA)	FOPT	$\alpha,\, \beta/H(T_*),\, T_*$	$A_{\text{BHB}},\, \gamma_{\text{BHB}}$	8.02

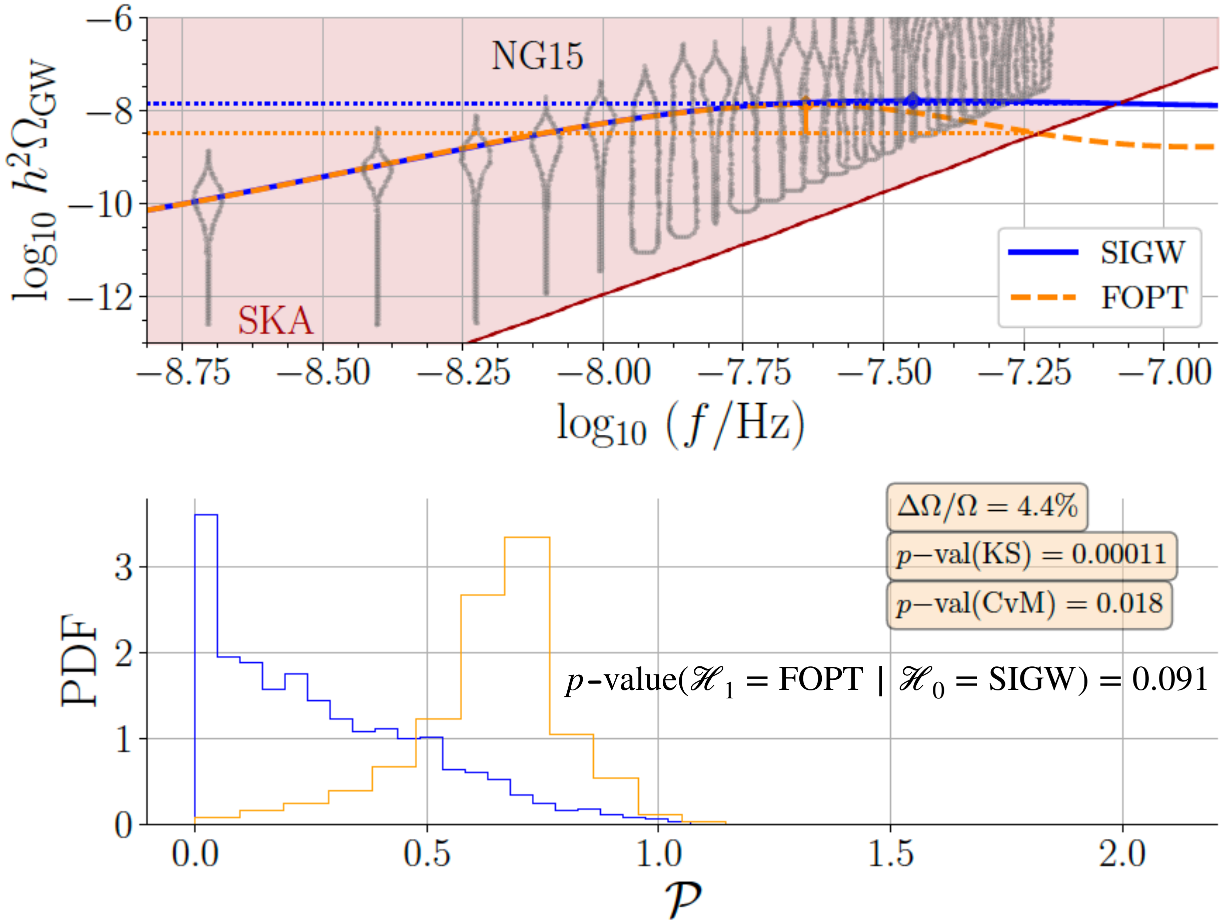


(a)



(b)





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Summary

- Prominence is sensitive not only to the peak amplitude of the GW spectrum, but also its shape

- $\mathscr{P}$ is especially useful when signals are similar and have relatively small SNR

- Amplitude uncertainty at PTAs needs to be at the sub-percent to percent level for $\mathscr{P}$ to distinguish signals at 3σ

- Uncertainty at LISA and ET will be small enough ($\sim 1\%$) that $\mathscr{P}$ can play a central role in signal discrimination

- Signals that cannot be distinguished at 2σ using SNR can be distinguished using $\mathscr{P}$ at much $>3\sigma$ in some cases