Cosmology in warped massive gravity

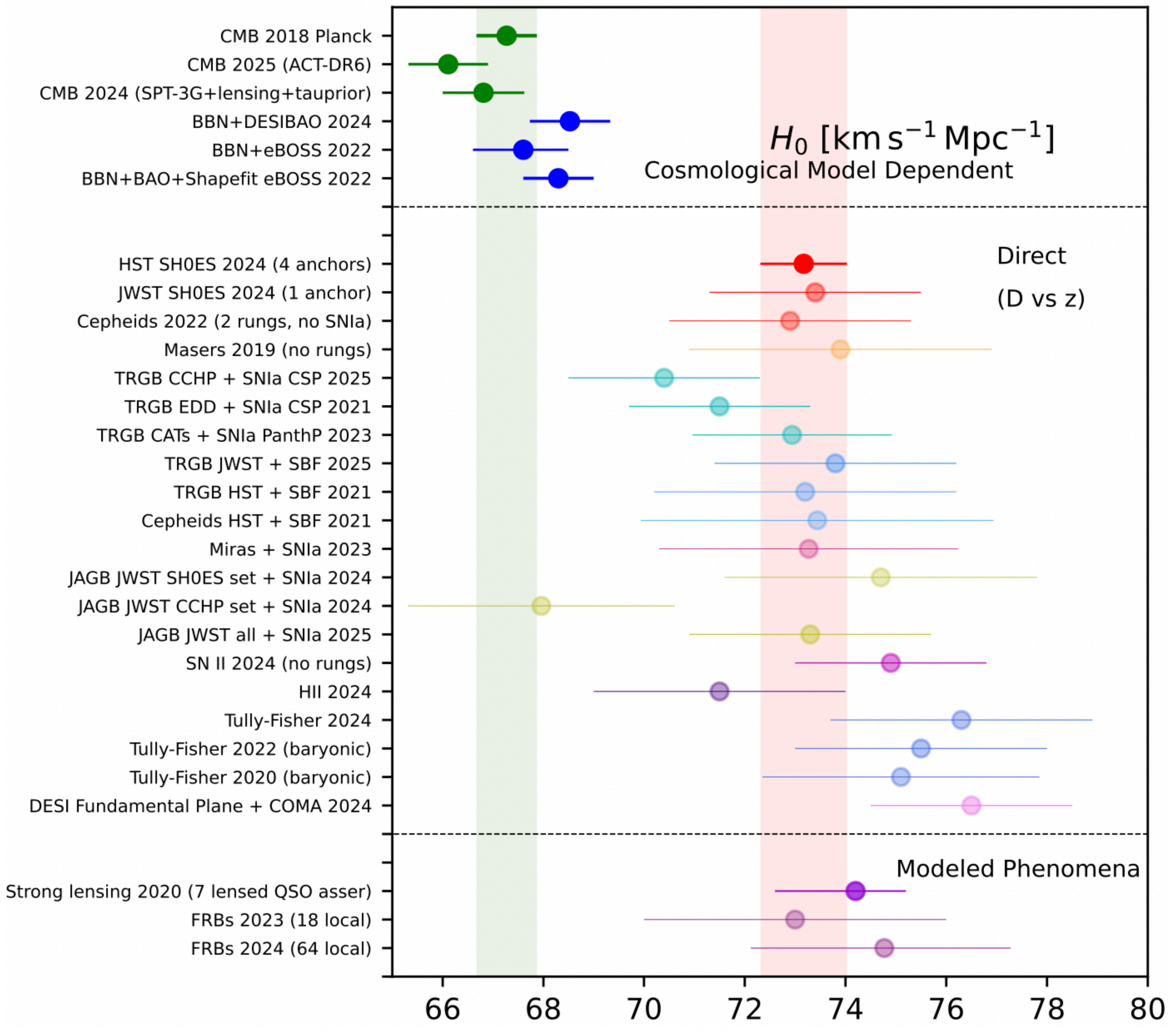
Based on **2509.09270** with Y. Wei, X. Zhou

Sebastian Garcia-Saenz
SUSTech

The case for modified gravity

- “Widening cracks are appearing in the ΛCDM model”

Leauthaud, Riess (2025)



CosmoVerse white paper (2025)

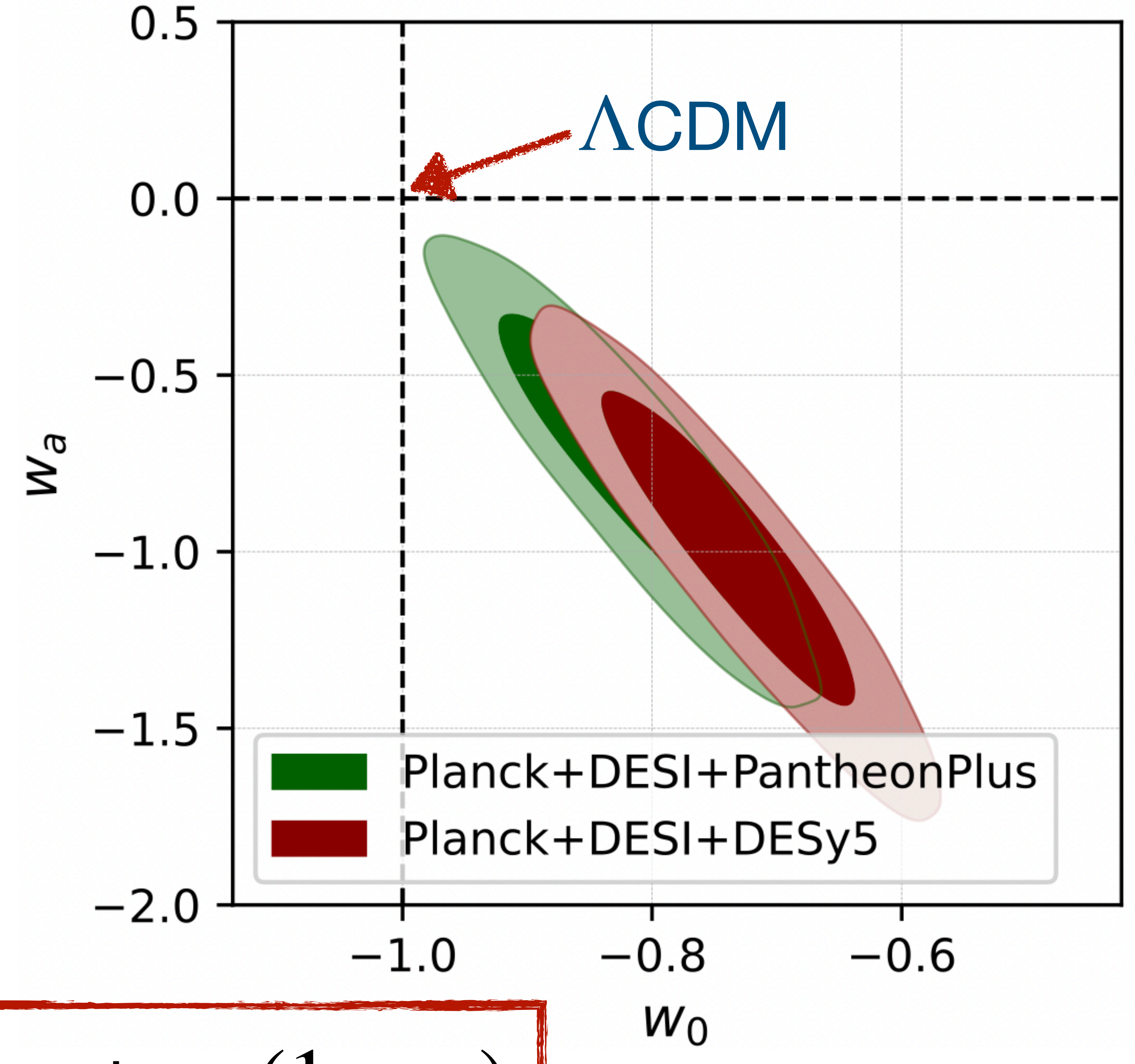
$5-6\sigma$ Hubble tension (crisis?)

Silvia Galli’s tensiometer

The case for modified gravity

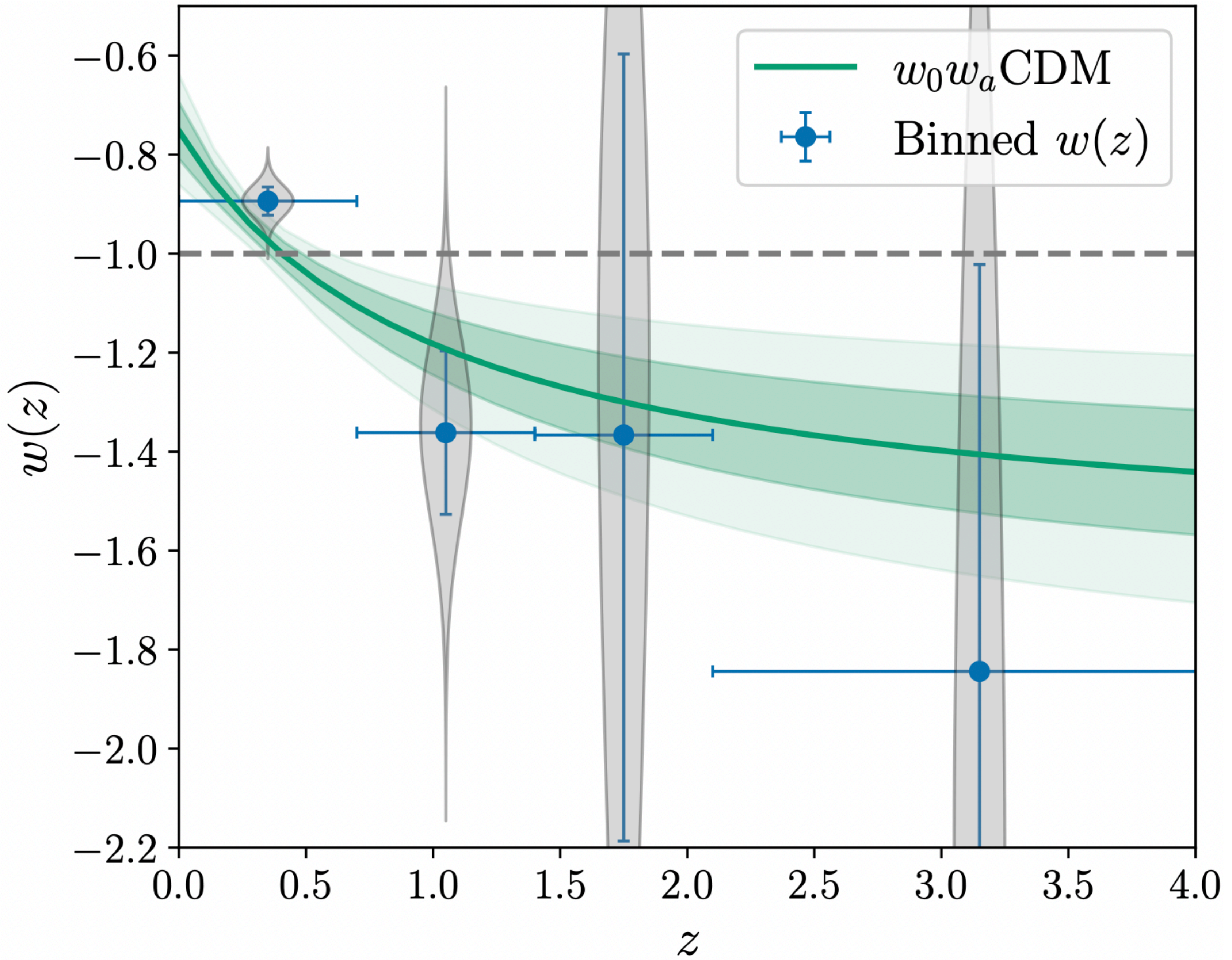
- Mounting evidence that dark energy is dynamical
- Observational hints from large-scale structure   DESI DR2 (2025)
- Cosmological constant long challenged by fundamental theory
- Modified gravity: from alternative to only explanation?

CosmoVerse white paper (2025)



$$w = w_0 + w_a(1 - a)$$

DESI (2025)



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Massive gravity

- Small but non-zero graviton mass offers a compelling approach to modify GR in the infrared
- Requiring the theory to describe only a massive spin-2 particle makes the problem challenging

Boulware, Deser (1972)

- One solution is given by the ghost-free dRGT theory

De Rham, Gabadadze, Tolley (2010)

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De Rham, Gabadadze, Tolley (2010)

$$S[g] = \frac{M_{\rm Pl}^2}{2} \int d^4x \sqrt{-g} \left[R + 2m^2 U(S_{(4)}) \right]$$

graviton mass

dRGT potential

$$U(S_{(4)}) = e_2(S_{(4)}) + \alpha_3 e_3(S_{(4)}) + \alpha_4 e_4(S_{(4)})$$

$$S_{(4)} \equiv \mathbf{1}_{(4)} - \sqrt{g^{-1} f}$$

fiducial metric (non-dynamical)

Massive gravity

- Notorious issue in massive gravity: very low strong-coupling scale

$$\Lambda_* \equiv \left(M_{\rm Pl} m^2\right)^{1/3} \lesssim 10^{-19}\,\mathrm{MeV} \qquad \Rightarrow \qquad \Lambda_*^{-1} \gtrsim 10^3\,\mathrm{km}$$

- Problem can be remedied in AdS background, but this does not seem realistic in cosmology

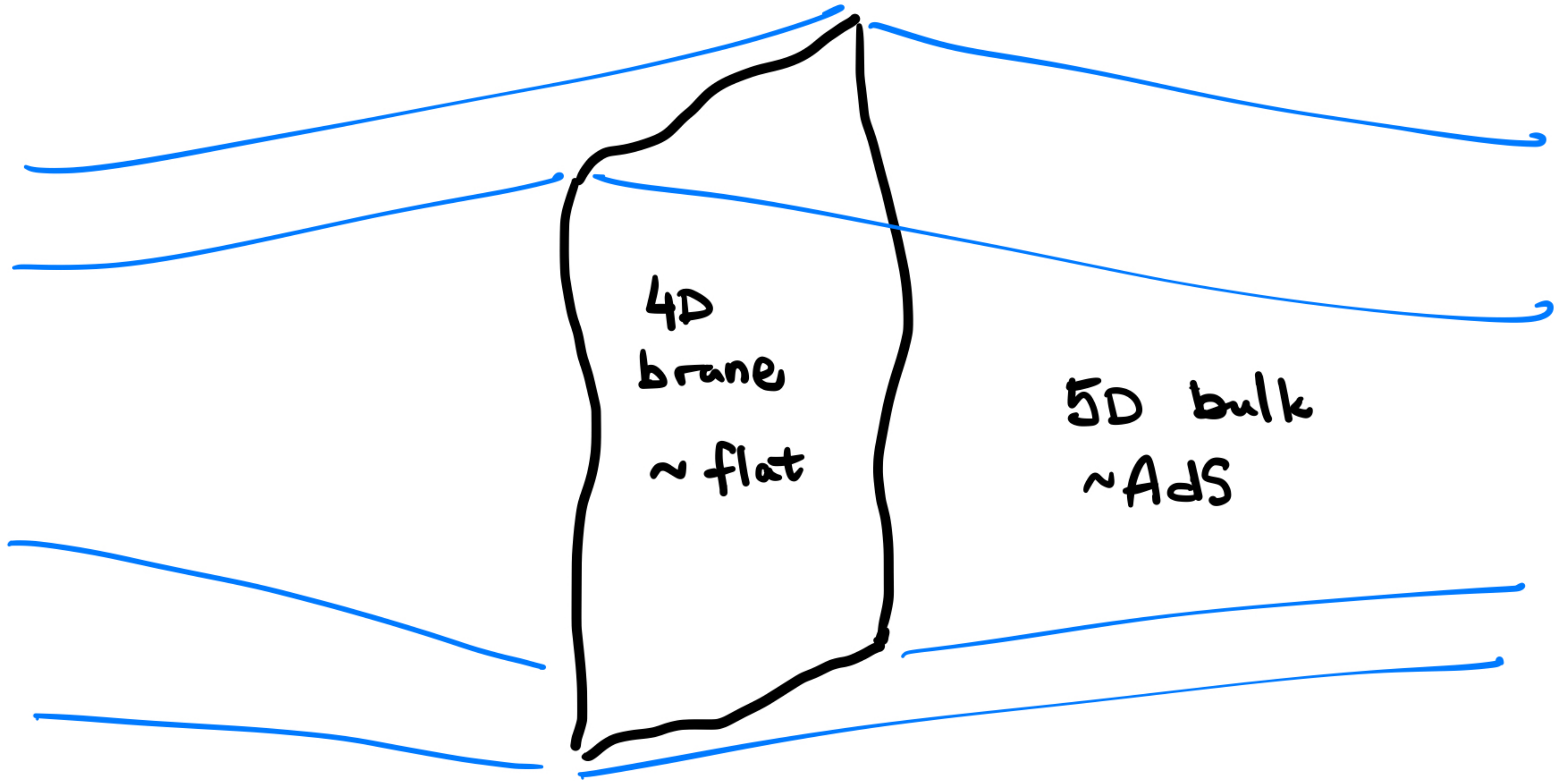
Kogan et al. (2000)
Porrati (2000)

- Key insight: AdS space of massive gravity may be higher dimensional, while the effective 4D theory may resemble our universe

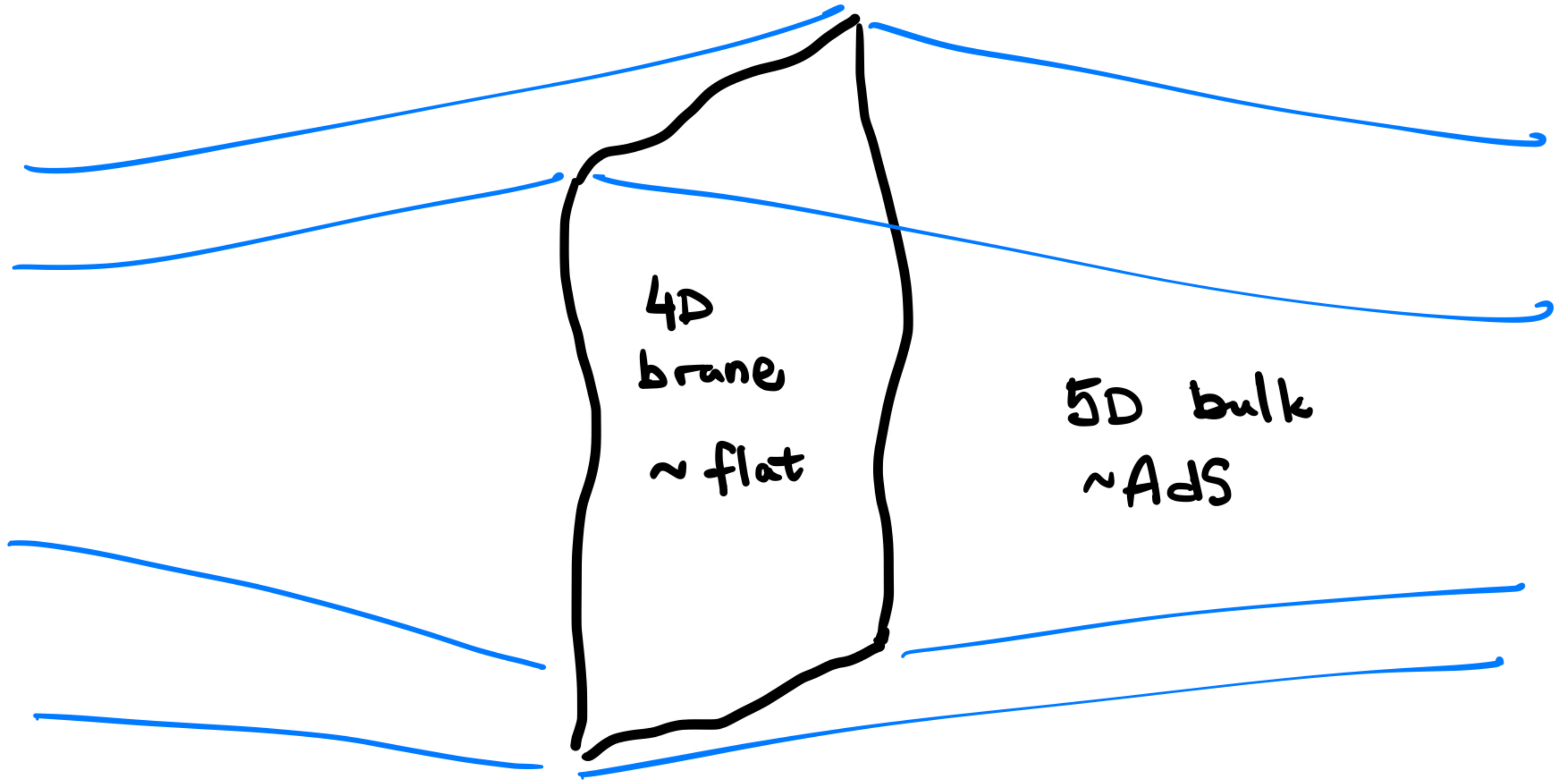
Gabadadze (2017)

Warped massive gravity

- Brane-world gravity provides a concrete realization of the idea



Warped massive gravity



- Brane-world gravity provides a concrete realization of the idea

- Warped massive gravity is defined by 5D and 4D dRGT interactions

$$
\begin{aligned}
S = \frac{M_5^3}{2}\int \mathrm{d}^5X\sqrt{-g_{(5)}}\left(R_{(5)}-2\Lambda_5\right) + M_5^3 m_5^2\int \mathrm{d}^5X\sqrt{-g_{(5)}}\,U_{(5)}(S_{(5)}) \\
+\frac{M_4^2}{2}\int \mathrm{d}^4x\sqrt{-g}\,R + M_4^2 m_4^2\int \mathrm{d}^4x\sqrt{-g}\,U(S_{(4)}) + S_{\mathrm{brane}}\,.
\end{aligned}
$$

Gabadadze (2017)

Warped massive gravity

$$S = \frac{M_5^3}{2}\int \mathrm{d}^5X\sqrt{-g_{(5)}}\left(R_{(5)} - 2\Lambda_5\right) + M_5^3 m_5^2 \int \mathrm{d}^5X\sqrt{-g_{(5)}}\,U_{(5)}(S_{(5)})$$
$$+ \frac{M_4^2}{2}\int \mathrm{d}^4x\sqrt{-g}\,R + M_4^2 m_4^2 \int \mathrm{d}^4x\sqrt{-g}\,U(S_{(4)}) + S_{\mathrm{brane}}\,.$$

4D matter

M_4, M_5 : 4D and 5D Planck scales

m_4, m_5 : 4D and 5D graviton masses

→ massive version of DGP model

Dvali, Gabadadze, Porrati (2000)

5D fiducial metric: AdS

$$\mathrm{d}s^2_{f(5)} = \frac{L^2}{(Y+L)^2}\left(\eta_{\mu\nu}\mathrm{d}X^\mu \mathrm{d}X^\nu + \mathrm{d}Y^2\right)$$

4D fiducial metric: flat

$$\mathrm{d}s^2_{f(4)} = \eta_{\mu\nu}\mathrm{d}X^\mu \mathrm{d}X^\nu$$

$Y = 0$ → brane location

Cosmology in WMG

- Problem with MG with flat fiducial metric: no flat and closed FLRW solutions
- Open FLRW solutions are possible
- To this end, define an open FLRW chart

D'Amico et al. (2011)

Gumrukcuoglu, Lin, Mukohyama (2011)

$$X^0 = f(t,y)\sqrt{1+K|\boldsymbol{x}|^2}\,, \qquad X^i = f(t,y)\sqrt{K}x^i\,, \qquad Y = g(t,y)$$

3D curvature

Stückelberg "fields"

$$\mathrm{d}s^2_{g(5)} = -n^2(t,y)\mathrm{d}t^2 + a^2(t,y)\gamma_{ij}\mathrm{d}x^i\mathrm{d}x^j + b^2(t,y)\mathrm{d}y^2$$

$$\mathrm{d}s^2_{g(4)} = -\mathrm{d}t^2 + a^2(t)\gamma_{ij}\mathrm{d}x^i\mathrm{d}x^j$$

$y = 0$ → brane location

Cosmology in WMG

- Key question: can one obtain a decoupled equation for the scale factor of the 4D FLRW metric?

Binetruy et al. (2000)
Deffayet (2000)

In DGP: yes 🙂

In WMG: not in general ☹️

- However, we have found two particular classes that do give decoupled 4D cosmological dynamics, while also satisfying the dRGT constraint

Neumann model : Neumann boundary conditions for Stückelberg fields on the brane

Special model : special tuning between 4D and 5D dRGT coefficients

WMG Neumann model

Note: jump function

$$[X] \equiv X(y=0^+) - X(y=0^-)$$

$$X' \equiv \partial_y X\,,\ \dot{X} \equiv \partial_t X$$

- Assume Neumann boundary conditions for the Stückelberg fields:

$$[f'] = [g'] = 0$$

- This results in a modified Raychaudhuri equation in 4D

$$\frac{\ddot{a}}{a} + H^2 - \frac{K}{a^2} - \frac{1}{4}\left[J_1^2 + J_1 J_2\right] + m_5^2\left[B_1\frac{\dot{a}}{\sqrt{K}} + B_2\right] = 0$$

"GR terms"

"DGP terms" → from Israel junction condition

"WMG terms"

WMG Neumann model

- Modified Raychaudhuri equation

$$\frac{\ddot{a}}{a} + H^2 - \frac{K}{a^2} - \frac{1}{4}\left[J_1^2 + J_1 J_2\right] + m_5^2\left[B_1\frac{\dot{a}}{\sqrt{K}} + B_2\right] = 0$$

constants

"GR terms"

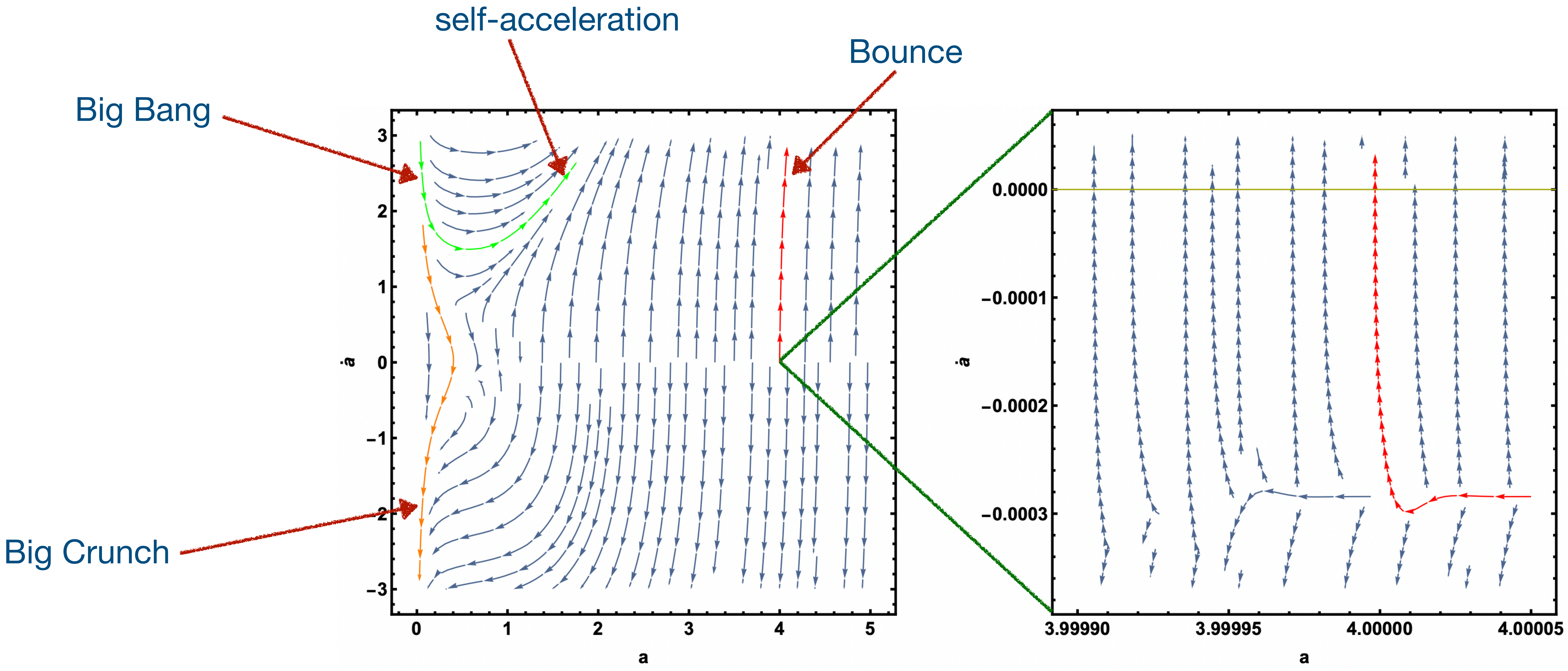
"DGP terms"

"WMG terms"

- "WMG terms" introduce genuinely novel features
  - Non-perturbative in curvature constant K
  - No globally defined first integral, i.e. no Friedmann-like equation $H^2(a)$

WMG Neumann model

- Model includes Big Bang, Big Crunch and bounce solutions, without exotic matter



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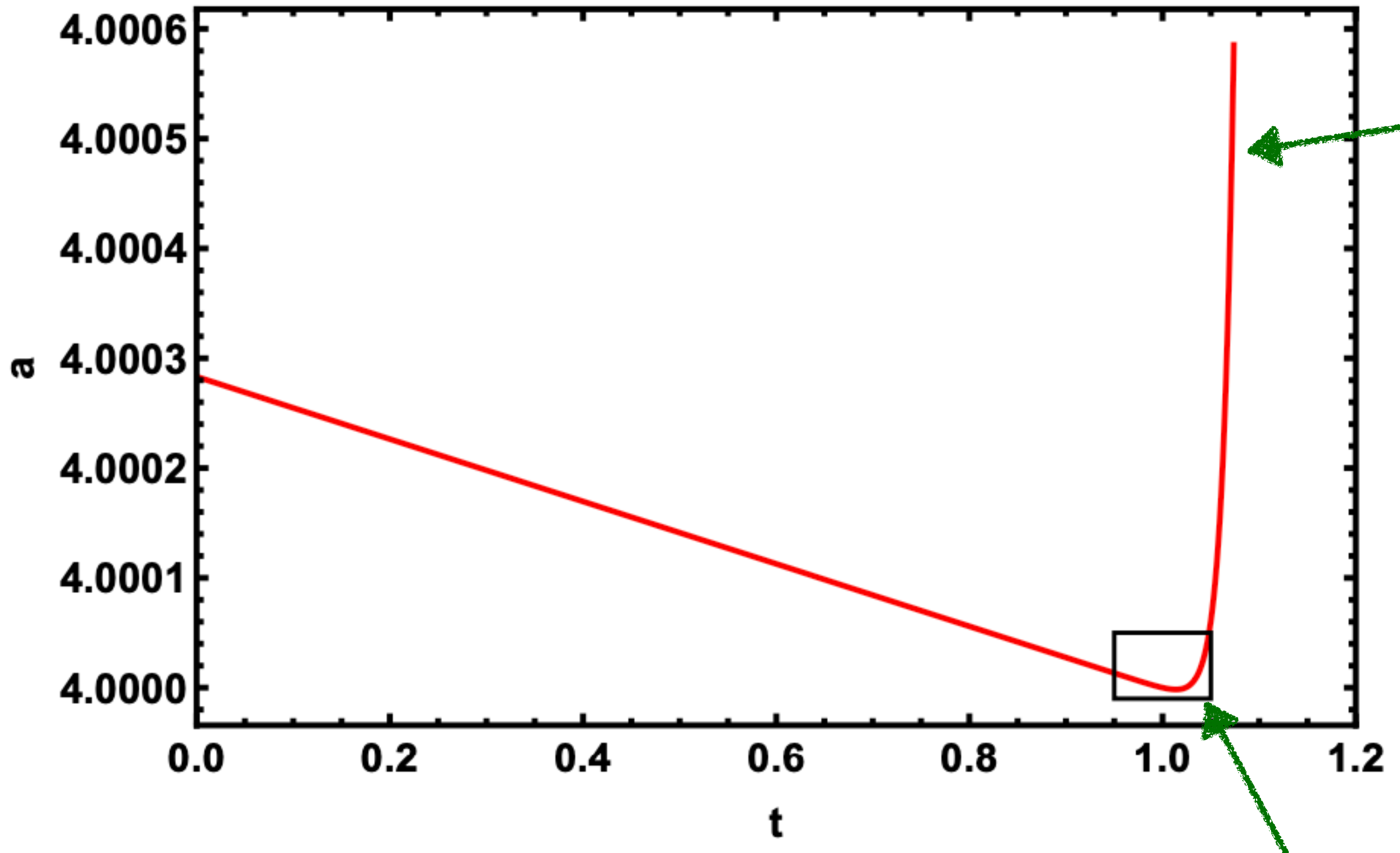
WMG Neumann model

- Bounce models explored so far also have a rip singularity



$$a(t) = \frac{36}{\lambda}\left(t_{\rm rip} - t\right)^{-3}$$

$$\lambda \equiv \frac{2\mu^2 m_5^2 \mathcal{B}_1}{\kappa^2 \sqrt{K}}$$

$$a \simeq a_{\rm b} + \frac{K}{2a_{\rm b}}\left(t - t_{\rm b}\right)^2$$

WMG special model

- Another way to satisfy the dRGT constraints is by imposing a relation between the 5D and 4D potential coefficients:

Massive gravity potentials:

$$U_{(5)}(S_{(5)}) = e_2(S_{(5)}) + \beta_3 e_3(S_{(5)}) + \beta_4 e_4(S_{(5)}) + \beta_5 e_5(S_{(5)})\,,$$

$$U(S_{(4)}) = e_2(S_{(4)}) + \alpha_3 e_3(S_{(4)}) + \alpha_4 e_4(S_{(4)})\,,$$

Special relation:

$$\beta_3 = \alpha_3 - \frac{\alpha_4 - \beta_4}{1 + 2\alpha_3 - 2\sigma\sqrt{1 + \alpha_3 + \alpha_3^2 - \alpha_4}}\,,$$

$$\beta_5 = -1 - \left(1 + \alpha_3 + \alpha_3^2 - \beta_4\right)\left(1 + 2\alpha_3 - 2\sigma\sqrt{1 + \alpha_3 + \alpha_3^2 - \alpha_4}\right)$$
$$+ \alpha_3(\alpha_4 - \beta_4) + \alpha_4\left(1 + \alpha_3 - \frac{\alpha_4 - \beta_4}{1 + 2\alpha_3 - 2\sigma\sqrt{1 + \alpha_3 + \alpha_3^2 - \alpha_4}}\right)$$

Note: further simplification when $\beta_4 = \alpha_4$

WMG special model

- In the special model the unusual “WMG terms” are absent

Special relation $\rightarrow$ $B_1 = 0$

- The result is a modified Friedmann equation, similar to the DGP equation, but with differences

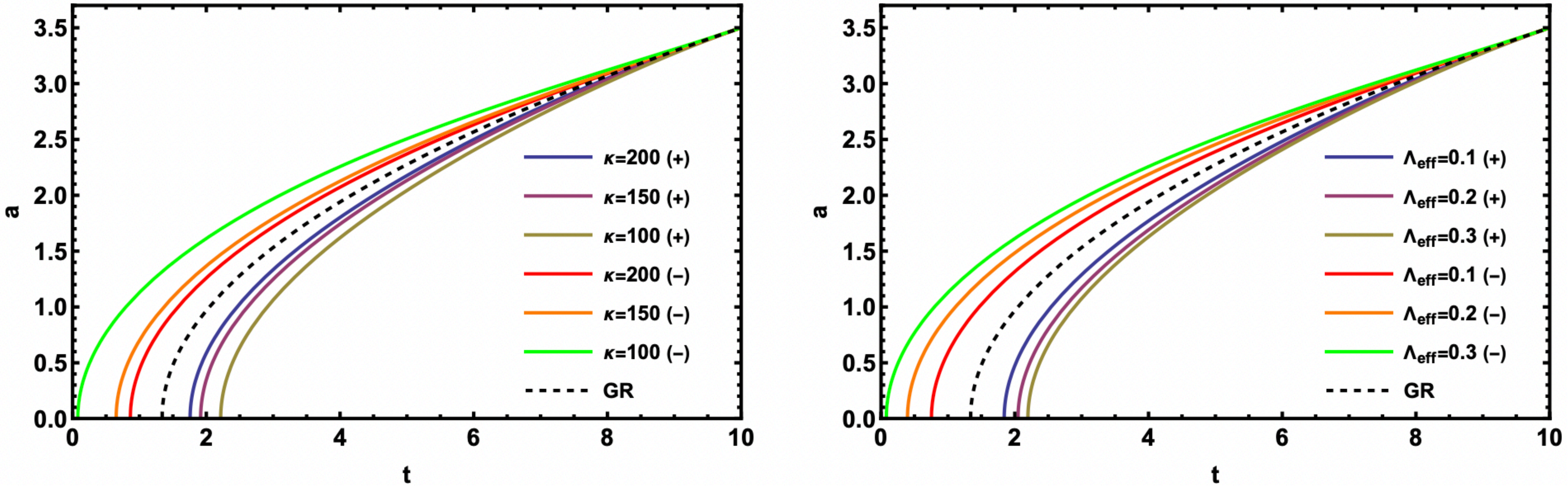
$$\frac{H^2}{H_0^2} - 1 = \frac{\Omega_\mathrm{r}}{a^4} + \frac{\Omega_\mathrm{m}}{a^3} - (\Omega_\mathrm{r} + \Omega_\mathrm{m}) + 2M\left[\sqrt{N + \frac{\Omega_\mathrm{r}}{a^4} + \frac{\Omega_\mathrm{m}}{a^3}} - \sqrt{N + \Omega_\mathrm{r} + \Omega_\mathrm{m}}\right]$$

H : Hubble parameter

$\Omega_{\mathrm{r,m}}$: matter density parameters

$K = \mathscr{C} = 0$ has been set

WMG special model



$$\frac{H^2}{H_0^2} - 1 = \frac{\Omega_\mathrm{r}}{a^4} + \frac{\Omega_\mathrm{m}}{a^3} - (\Omega_\mathrm{r} + \Omega_\mathrm{m}) + 2M\left[\sqrt{N + \frac{\Omega_\mathrm{r}}{a^4} + \frac{\Omega_\mathrm{m}}{a^3}} - \sqrt{N + \Omega_\mathrm{r} + \Omega_\mathrm{m}}\right]$$

$$M \equiv \pm\frac{\mu}{\kappa H_0}\,, \qquad N \equiv \frac{\kappa^2}{\mu^2 H_0^2}\left(\frac{\mu^2}{\kappa^2} - \frac{m_4^2\mathcal{A}}{6}\right)^2 - \frac{\Lambda_\mathrm{eff}}{6H_0^2}$$

$\mu/\kappa \equiv M_5^3/M_4^2$

Note: $N > 0$ in DGP, but may have either sign in WMG

$$\Lambda_\mathrm{eff} = \Lambda_5 - 3m_5^2\mathcal{B}_2 + \frac{1}{6}\left(\frac{\kappa m_4^2}{\mu}\mathcal{A}\right)^2$$

WMG special model

H : Hubble parameter

$\Omega_{\mathrm{r,m}}$: matter density parameters

a : scale factor

- Peculiar models:

set $\frac{\mu^2}{\kappa^2} - \frac{m_4^2 \mathcal{A}}{6} = 0$ and $\Lambda_{\text{eff}} = 0$ ⟶ $$\frac{H^2}{H_0^2} = \frac{2\mu\sqrt{\Omega_{\mathrm{m}}}}{\kappa H_0} a^{-3/2} + \mathcal{O}\left(a^{-5/2}\right)$$

accelerated expansion ($w_{\text{eff}} = -1/2$) with pure dust and no cosmological constant

case with pure radiation ⟶ $$\frac{H^2}{H_0^2} = \frac{\Omega_{\mathrm{r}}}{a^4} \pm \frac{2\mu\sqrt{\Omega_{\mathrm{r}}}}{\kappa H_0} \frac{1}{a^2}$$

effective spatial curvature term

WMG special model

- Model fits data roughly as good as ΛCDM
- Potential to alleviate Hubble tension, but at the price of a tension in Ω_m
- Ongoing work focused on perturbations

different priors

Bayesian criteria

Dataset	Model	H_0	Ω_m	M	N	$\Delta - \ln B$	ΔDIC	ΔWAIC
CMB	ΛCDM	67.21 ± 0.46	0.3158 ± 0.0063	0	0	0	0	0
	M	58 ± 8	$0.455^{+0.078}_{-0.17}$	$0.29^{+0.20}_{-0.32}$	$--$	0.55	0.54	0.81
	M(+)	56^{+9}_{-6}	$0.473^{+0.059}_{-0.15}$	< 0.426	$--$	-2.03	0.14	0.05
	M(−)	$70.7^{+1.3}_{-3.5}$	$0.287^{+0.028}_{-0.014}$	> -0.0950	$--$	-1.75	0.77	0.57
	M(−0.05)	$72.3^{+1.0}_{-3.0}$	$0.273^{+0.023}_{-0.011}$	> -0.126	$--$	-1.13	1.44	1.54
PPS	ΛCDM	73.5 ± 1.0	0.332 ± 0.018	0	0	0	0	0
	M	73.6 ± 1.0	$0.298^{+0.059}_{-0.13}$	0.22 ± 0.40	> 0.388	1.62	0.46	0.77
	M(+)	73.5 ± 1.0	$0.238^{+0.048}_{-0.068}$	< 0.599	$--$	0.53	0.02	0.04
	M(−)	73.7 ± 1.0	$0.416^{+0.038}_{-0.083}$	> -0.285	> 0.418	1.31	1.13	1.06
	M(−0.05)	73.7 ± 1.0	$0.429^{+0.034}_{-0.086}$	> -0.309	> 0.438	0.97	1.13	1.03

SNIa data

WMG special model

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SNIa data

Thank you for your attention

Back-up

$$\frac{[n']}{b} = \frac{\kappa}{3}(2\rho + 3p) + \frac{\kappa}{\mu}\left(2\frac{\ddot{a}}{a} - H^2 + \frac{K}{a^2}\right) + \frac{\kappa m_4^2}{3\mu}\mathcal{A}\,,$$

$$\frac{[a']}{ab} = -\frac{\kappa}{3}\rho + \frac{\kappa}{\mu}\left(H^2 - \frac{K}{a^2}\right) + \frac{\kappa m_4^2}{3\mu}\mathcal{A}\,,$$

$$\mathcal{A} \equiv \frac{(1+\alpha_3)\left(2+\alpha_3+2\alpha_3^2-3\alpha_4\right)+2\sigma\left(1+\alpha_3+\alpha_3^2-\alpha_4\right)^{3/2}}{(\alpha_3+\alpha_4)^2}$$

Back-up

$$\mathcal{B}_1 = \frac{A_\pm}{3}\Big[-(4+6\beta_3+4\beta_4+\beta_5)+3A_\pm(1+3\beta_3+3\beta_4+\beta_5)$$
$$-3A_\pm^2(\beta_3+2\beta_4+\beta_5)+A_\pm^3(\beta_4+\beta_5)\Big]\,,$$

$$\mathcal{B}_2 = -\frac{1}{3}\Big[-(10+10\beta_3+5\beta_4+\beta_5)+3A_\pm(4+6\beta_3+4\beta_4+\beta_5)$$
$$-3A_\pm^2(1+3\beta_3+3\beta_4+\beta_5)+A_\pm^3(\beta_3+2\beta_4+\beta_5)\Big]\,.$$

$$A_\pm \equiv \frac{1+2\alpha_3+\alpha_4\pm\sqrt{1+\alpha_3+\alpha_3^2-\alpha_4}}{\alpha_3+\alpha_4}$$