IS THERE A UV FREE E_6 GUT?

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Babu, BB, Susič, in progress

Introduction

We heard many talks on SU(5) and SO(10).

There is just one group with complex representations and so chiral fields on top of SU(N) and SO(2N) - it is

$$E_6$$

Not so known: rank 6, contains the SM

Few facts about E_6:

- fundamental representation: 27^i
- upper and lower indices $i = 1, \ldots, 27$
- anti-fundamental representation: $\overline{27}_i$
- adjoint: 78^i_j (with some constraints)
- invariant tensor d_{ijk} or d^{ijk} symmetric
- invariants by summing upper and lower indices ($27^i \overline{27}_i$ or $78^j_i 78^i_j$)

Last year in this conference: E_6 with large representations (27, 351$'$, 650)

- motivation 1: dark matter candidate (inert doublet in 27_H)
- motivation 2: good Yukawa structure (10 in 27_H complex)
- realistic model

Babu, BB, Susič, 2403.20278

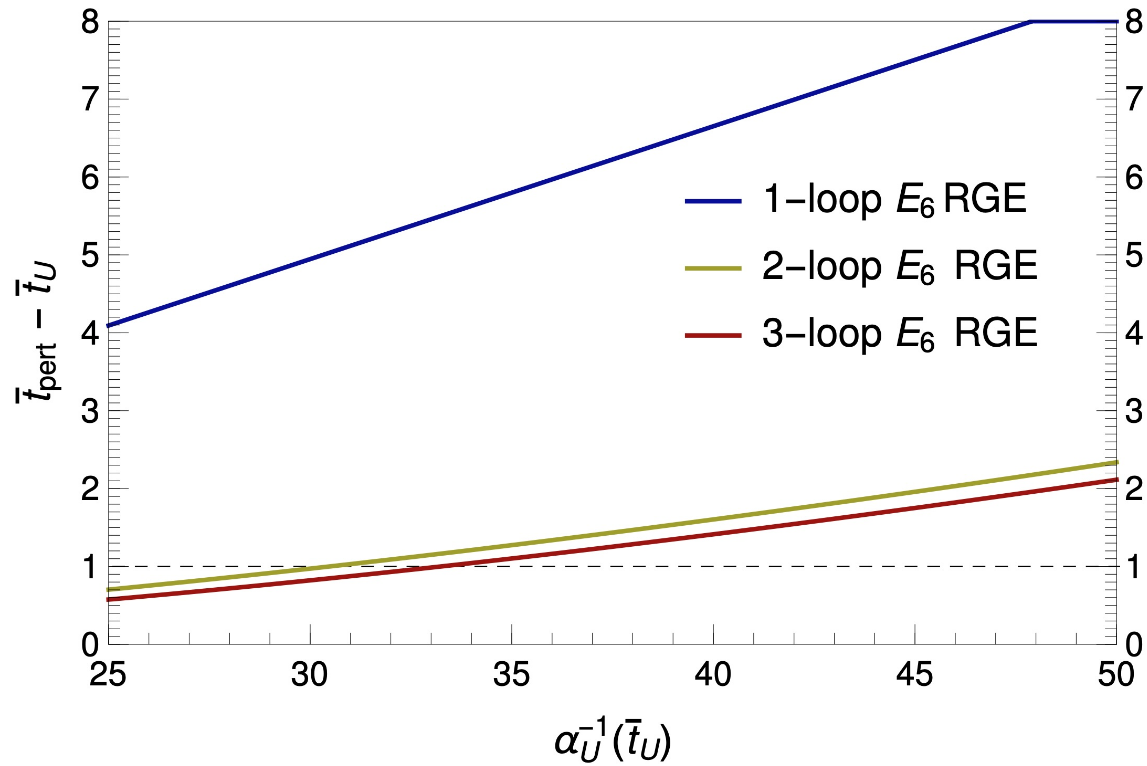
But large representations $\rightarrow$ possible problems

Perturbativity

$$\frac{d}{dt}\alpha^{-1} = -\frac{1}{2\pi}\left(a + b\left(\frac{\alpha}{4\pi}\right) + c\left(\frac{\alpha}{4\pi}\right)^2 + \dots\right)$$

$$a = 16 \quad , \quad b = 11956 \quad , \quad c = 560730$$

a anomalously small $\rightarrow$ 2-loop important, higher loops less



typically $\alpha_U^{-1} \approx 40$

$\rightarrow$ Landau pole at 1 to 2 orders of magnitude above GUT

- perturbative unitarity: $Re(a_0) \leq 0.5$

largest-magnitude eigenvalue for the partial-wave coefficient

$$(a_0)^{max} = \frac{\alpha_U}{2\sqrt{78}} \sqrt{\frac{3\pi^2}{4} 27 C(27)^2 + \sum_R \zeta(R) dim(R) C(R)^2}$$

$C(R) \ldots$ Casimir of R

$\zeta(R) = 1/2$ (real) or 1 (complex) irrep R

In our case $(a_0)^{max} = 0.69$ but with the approximation of massless particles

$\rightarrow \approx$ 1 order of magnitude above M_{GUT} the theory is non-unitary

The reason is large representations of $351'$ and 650

Is it possible to live without them?

Lowest E_6 irreducible representations below dimension 1000:

$$\mathbf{27}, \quad \mathbf{78}, \quad \mathbf{351}, \quad \mathbf{351'}, \quad \mathbf{650}$$

So we should survive with $\mathbf{27}$, $\overline{\mathbf{27}}$, $\mathbf{78}$

Fermions:

$$27_F = \underbrace{16}_{chiral} + \underbrace{10}_{vectorlike} + 1$$

Natural to have 3 generations of 27_F

The only possible Yukawa

$$\sum_{ij} Y_{ij} 27_{F\,i} 27_{F\,j} 27_H + h.c. \qquad (d_{abc} 27^a 27^b 27^c)$$

gives no mixing (one Yukawa only)

Two 27_H allow mixing but still

$$M_D = M_E$$

Down quarks and charged lepton masses equal (like multiple 10 Higgses only in SO(10))

We need an extra vectorlike fermionic pair $27_F + \overline{27}_F$, with extra Yukawa

$$\sum_i \eta_i 27_{F\,i} \overline{27}_F 78_H$$

Since 78_H has different Clebsches for down quarks and charged leptons $\to M_D \neq M_E$ (once we break Pati-Salam)

Fermionic sector: $4 \times 27_F + \overline{27}_F$

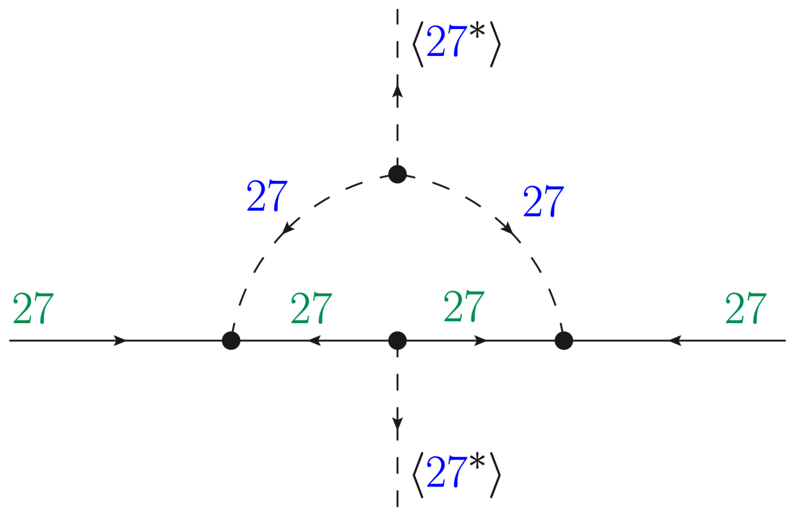
This should be enough for the charged fermion sector

In fact in SO(10):

$$126_H + 4 \times 16_F + \overline{16}_F$$

Babu, BB, Saad, 1605.05116

Neutrinos get mass only at loop level



To be done

Asymptotic freedom

We thus have

$$
\begin{aligned}
1\times 78 \quad & \ldots \quad \text{vector} \\
3+2N_F\times 27 \quad & \ldots \quad \text{fermions} \\
1\times 78+N_H\times 27 \quad & \ldots \quad \text{Higgses}
\end{aligned}
$$

The E_6 gauge beta function at 1-loop:

$$
b_1=\frac{11}{3}\times 12-\frac{2}{3}(3+2N_F)3-\frac{1}{6}\left(12+2N_H3\right)
$$

we will take $N_F=1$ and $N_H=1$ so $b_1=31$

So $b_1>0$ and at least in principle could be UV free.

But we must include also Yukawa and Higgs couplings:

$$\begin{aligned}\mathcal{L}_{Yukawa} &= \frac{1}{2}\sum_{i,j=1}^{4} Y_{ij} {27_F}^i {27_F}^j 27_H + \sum_{i=1}^{4} \eta_i {27_F}^j 78_H \overline{27_F} \\ &+ \frac{1}{2}\bar{y}\overline{27}_F\overline{27}_F 27^*_H + h.c.\end{aligned}$$

$\rightarrow \mathbf{Y}, \boldsymbol{\eta}, \bar{y}$

$$\begin{aligned}\mathcal{L}_{Higgs} = \lambda \left({78_H}^2\right)^2 + \lambda'_1 \left(27_H 27^*_H\right)^2 + \lambda'_2\, 27_H 27_H 27^*_H 27^*_H \\ + \lambda''_1 (27_H 27^*_H) {78_H}^2 + \lambda''_2 27^*_H 78^2_H 27_H\end{aligned}$$

$\rightarrow \lambda, \lambda'_1, \lambda'_2, \lambda''_1, \lambda''_2$

Higgs:

$$16\pi^2\ \frac{d}{dt}\lambda = 27g^4 - 144g^2\lambda + 172\lambda^2 + 27\lambda_1''^2 + 2\lambda_1''\lambda_2'' + \frac{1}{12}\,\lambda_2''^2 + 4\lambda\,(\boldsymbol{\eta}^\dagger\boldsymbol{\eta}) - \frac{1}{6}\,(\boldsymbol{\eta}^\dagger\boldsymbol{\eta})^2,$$

$$16\pi^2\ \frac{d}{dt}\lambda_1' = \frac{44}{3}g^4 - 104g^2\lambda_1' + 124\lambda_1'^{\,2} + 160\lambda_1'\lambda_2' + 160\lambda_2'^{\,2} + 39\lambda_1''^2 + \frac{26}{9}\,\lambda_1''\lambda_2'' + \frac{11}{81}\,\lambda_2''^2 + 20\lambda_1'\,\mathrm{Tr}(\mathbf{Y}^\dagger\mathbf{Y}) - 10\,\mathrm{Tr}(\mathbf{Y}^\dagger\mathbf{Y}\mathbf{Y}^\dagger\mathbf{Y}) + 80\lambda_1'\,|\bar{y}|^2 - 160\,|\bar{y}|^4$$

$$16\pi^2\ \frac{d}{dt}\lambda_2' = 2g^4 - 104g^2\lambda_2' + 24\lambda_1'\lambda_2' - 24\lambda_2'^{\,2} + \frac{1}{54}\,\lambda_2''^2 + 20\lambda_2'\,\mathrm{Tr}(\mathbf{Y}^\dagger\mathbf{Y}) + 4\,\mathrm{Tr}(\mathbf{Y}^\dagger\mathbf{Y}\mathbf{Y}^\dagger\mathbf{Y}) + 80\lambda_2'\,|\bar{y}|^2 + 64\,|\bar{y}|^4$$

$$16\pi^2\ \frac{d}{dt}\lambda_1'' = 12g^4 - 124g^2\lambda_1'' + 160\lambda\lambda_1'' + \frac{52}{9}\,\lambda\lambda_2'' + 112\lambda_1'\lambda_1'' + 4\lambda_1'\lambda_2'' + 80\lambda_2'\lambda_1'' + \frac{8}{3}\,\lambda_2'\lambda_2'' + 4\lambda_1''^2 + \frac{1}{9}\,\lambda_2''^2 + 10\lambda_1''\,\mathrm{Tr}(\mathbf{Y}^\dagger\mathbf{Y}) + 2\lambda_1''\,(\boldsymbol{\eta}^\dagger\boldsymbol{\eta}) + 40\lambda_1''\,|\bar{y}|^2$$

$$16\pi^2\ \frac{d}{dt}\lambda_2'' = 108g^4 - 124g^2\lambda_2'' + 4\lambda\lambda_2'' + 4\lambda_1'\lambda_2'' + 8\lambda_2'\lambda_2'' + 8\lambda_1''\lambda_2'' + \frac{41}{9}\,\lambda_2''^2 + 10\lambda_2''\,\mathrm{Tr}(\mathbf{Y}^\dagger\mathbf{Y}) + 2\lambda_2''\,(\boldsymbol{\eta}^\dagger\boldsymbol{\eta}) + 40\lambda_2''\,|\bar{y}|^2$$

Yukawa:

$$16\pi^2 \frac{d}{dt}\mathbf{Y} = 5(\mathbf{Y}\mathbf{Y}^\dagger\mathbf{Y} + \mathbf{Y}\mathbf{Y}^\dagger\mathbf{Y}) + \frac{13}{3}\left(\boldsymbol{\eta}\boldsymbol{\eta}^\dagger\mathbf{Y} + \mathbf{Y}\boldsymbol{\eta}^*\boldsymbol{\eta}^T\right)$$
$$- \frac{26}{3}(\boldsymbol{\eta}\boldsymbol{\eta}^T)\,\bar{y}^* + 5\,\mathrm{Tr}(\mathbf{Y}^\dagger\mathbf{Y})\,\mathbf{Y} + 5|\bar{y}|^2\,\mathbf{Y} - 52g^2\,\mathbf{Y}$$

$$16\pi^2 \frac{d}{dt}\bar{y} = (15\,\bar{y}^*\bar{y} + \frac{26}{3}\,\boldsymbol{\eta}^\dagger\boldsymbol{\eta})\,\bar{y} - \frac{26}{3}(\boldsymbol{\eta}^T\mathbf{Y}^*\boldsymbol{\eta}) + 5\,\mathrm{Tr}(\mathbf{Y}^\dagger\mathbf{Y})\,\bar{y} - 52\,g^2\,\bar{y}$$

$$16\pi^2 \frac{d}{dt}\boldsymbol{\eta} = 5\,(\mathbf{Y}\mathbf{Y}^\dagger)\boldsymbol{\eta} + 5\,|\bar{y}|^2\,\boldsymbol{\eta} + 20(\boldsymbol{\eta}^\dagger\boldsymbol{\eta})\,\boldsymbol{\eta} - 10\bar{y}\,\mathbf{Y}\,\boldsymbol{\eta}^* - 52\,g^2\boldsymbol{\eta}$$

Gauge:

$$16\pi^2 \frac{d}{dt}g = -31g^3$$

For large $t = \log \mu$ can we find an approximate free solution?

Ansatz:

$$\mathbf{Y} = diag(0, 0, Y_3, Y_4) \quad , \quad \eta = (0, 0, 0, \eta_4)$$

$$\left(g^2, |Y_3|^2|Y_4|^2, |\bar{y}|^2, |\eta_4|^2\right) = \frac{16\pi^2}{t}\left(\alpha, \alpha_3, \alpha_4, \bar{\alpha}, \alpha_\eta\right)$$

Procedure: solve first for gauge and Yukawa $(\alpha, \alpha_3, \alpha_4, \bar{\alpha}, \alpha_\eta)$

$$\text{solution 1} \quad : \quad (\alpha_3, \alpha_4, \bar{\alpha}, \alpha_\eta) = x \left(\frac{1}{5}, \frac{1}{5}, \frac{1}{5}, \frac{1}{4} \right)$$

$$\text{solution 2} \quad : \quad (\alpha_3, \alpha_4, \bar{\alpha}, \alpha_\eta) = x \left(0, \frac{1}{4}, \frac{1}{4}, \frac{1}{4} \right)$$

$$\text{solution 3} \quad : \quad (\alpha_3, \alpha_4, \bar{\alpha}, \alpha_\eta) = x \left(0, \frac{15 + 2\sqrt{14}}{116}, \frac{15 - 2\sqrt{14}}{116}, \frac{15}{116} \right)$$

$$\text{solution 4} \quad : \quad (\alpha_3, \alpha_4, \bar{\alpha}, \alpha_\eta) = x \left(0, \frac{15 - 2\sqrt{14}}{116}, \frac{15 + 2\sqrt{14}}{116}, \frac{15}{116} \right)$$

$$x = \frac{1}{10} \left(104\alpha - 1 \right) = \frac{21}{310}$$

Plugging each of these into eqs. for Higgs with ansatz

$$(\lambda, \lambda_1', \lambda_2', \lambda_1'', \lambda_2'') \propto \frac{1}{t}$$

we find NO solution

First conclusion:

There seem to be NO UV free solution.

However we could have still a perturbative behaviour till Planck scale, from the practical point of view it would be the same.

From now on we would like to check this second possibility

Symmetry breaking

Take one

$78_H = \Phi$ and one $27_H = \Psi$

The renormalisable potential is

$$V(\Phi,\Psi) = V_{78}(\Phi) + V_{27}(\Psi) + V_{mix}(\Phi,\Psi)$$

$$\begin{aligned} V_{78}(\Phi) &= -\tfrac{1}{2} M_{78}^2 \,\mathrm{Tr}\,\Phi^2 + \lambda\,(\mathrm{Tr}\,\Phi^2)^2, \\ V_{27}(\Psi) &= -M_{27}^2 \;\Psi_i^* \Psi^i + m_{27} \Psi^i\,\Psi^j\,\Psi^k\; d_{ijk} \\ &\quad + \lambda_1' (\Psi_i^* \Psi^i)^2 + \lambda_2'\, d_{ijm}\, d^{klm}\; \Psi^i \Psi^j \Psi_k^* \Psi_l^* + h.c., \\ V_{mix}(\Phi,\Psi) &= m''\; \Psi_i^* \Phi^i{}_j \Psi^j + \lambda_1''\; (\Psi_i^* \Psi^i)(\mathrm{Tr}\,\Phi^2) + \lambda_2''\; \Psi_i^* \Phi^i{}_j \Phi^j{}_k \Psi^k. \end{aligned}$$

$SU(3)_C \times SU(3)_L \times SU(3)_R$ decomposition of irrep 27 of E_6:

$$\underbrace{\begin{pmatrix} u_1 & d_1 & d'_1 \\ u_2 & d_2 & d'_2 \\ u_3 & d_3 & d'_3 \end{pmatrix}}_{(\mathbf{3},\mathbf{3},\mathbf{1})} \quad , \quad \underbrace{\begin{pmatrix} \nu'^c & e' & e \\ e'^c & \nu' & \nu \\ e^c & \nu^c & n \end{pmatrix}}_{(\mathbf{1},\bar{\mathbf{3}},\mathbf{3})} \quad , \quad \underbrace{\begin{pmatrix} u^c_1 & u^c_2 & u^c_3 \\ d^c_1 & d^c_2 & d^c_3 \\ d'^c_1 & d'^c_2 & d'^c_3 \end{pmatrix}}_{(\bar{\mathbf{3}},\mathbf{1},\bar{\mathbf{3}})}$$

- 15 SM fermions of one generation
- 2 right-handed neutrinos per generation - possible nonzero vevs

$$\langle \nu^c \rangle \quad , \quad \langle n \rangle = V_2$$

- vector-like heavy lepton doublet and down-type quark pairs

We will simplify the analyses by breaking only to LR:

$SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B_L}$ $(\rightarrow \langle \nu^c \rangle = 0)$

Vevs which preserves LR:

From 27: V_2

From 78: a_3, b_3

Equation of motion:

$$
\begin{aligned}
M_{27}^2 &= 2\left|V_2\right|^2 \lambda_1' + \frac{1}{9}\left({a_3}^2(9\lambda_1'' - \lambda_2'') + {b_3}^2(9\lambda_1'' - \lambda_2'') + 2a_3 b_3 \lambda_2''\right) \\
M_{78}^2 &= 4{a_3}^2\lambda_1 + 4{b_3}^2\lambda_1 + 2\left|V_2\right|^2 \lambda_1'' \\
m'' &= \frac{2}{3}(a_3 - b_3)\lambda_2''
\end{aligned}
$$

Higgs masses

Mass square at tree level in $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ must be all positive (minimum) $\rightarrow$

constraints on the model parameters

Surprise: some masses are exactly zero, no matter what the input parameters

This follows from more symmetry of the potential

$$V(78) = -(78)^2 + ((78)^2)^2$$

The only quartic invariant is the quadratic squared:

$$\begin{aligned} (78)^2 &= \sum_{i=1}^{78} \psi_i^2 \\ ((78)^2)^2 &= \left(\sum_{i=1}^{78} \psi_i^2 \right)^2 \end{aligned}$$

This means that the real symmetry of the potential is

$$SO(78)$$

Of course the gauge and Yukawa sector have only E_6 symmetry

Also some terms in the potential have less symmetry but they do not contribute to the potential

So we have pseudo-Goldstone bosons

If 78 alone:

$$E_6 \xrightarrow{\langle 78 \rangle} SO(10) \times U(1)$$

$78 - (45+1) = 32 \quad (= 16 + \overline{16})$ would-be Goldstones

But in the potential

$$SO(78) \xrightarrow{\langle 78 \rangle} SO(77)$$

$\frac{78\times 77}{2} - \frac{77\times 76}{2} = 77$ massless scalars

So we have

$77 - 32 = 45$ pseudo Goldstones (the 45-dimensional irrep of SO(10))

Our system is $78 + 27$

$\langle 27 \rangle$ breaks more: if we break to LR we remain with

$$(8, 1, 1, 0) + (1, 3, 1, 0) + (1, 1, 3, 0)$$

massless LR irreps

These scalars will get mass at 1-loop. The whole issue is:

Are these 1-loop masses square positive or negative?

This issue is well know in SO(10) with 45-dimensional Higgs. Here also if you break to SM there remain tree order massless $(8,1,0)+(1,3,0)$. Only 1-loop give a nonzero contribution

An even more elementary example is SU(3) with octet ϕ and a Z_2 simmetry $\phi\to-\phi$:

$$V(\phi)=-\frac{m^2}{2}Tr\left(\phi^2\right)+\frac{\lambda}{4}\left(Tr\left(\phi^2\right)\right)^2$$

In fact

$$Tr\left(\phi^4\right)\propto\left(Tr\left(\phi^2\right)\right)^2$$

Not enough invariants: 3×3 traceless matrices have only $Tr\left(\phi^2\right)$ and $Tr\left(\phi^3\right)$

Back to our E_6 case

Are the 1-loop masses square of LR $(8,1,1,0)$, $(1,3,1,0)$ (due to LR same as $(1,1,3,0)$) positive?

The contributions are typically complicated, so we start with simplest cases:

- gauge contribution only: always one mass positive, one negative

$$m_8^2 > 0 \quad , \quad m_3^2 < 0 \quad \text{or} \quad m_8^2 < 0 \quad , \quad m_3^2 > 0$$

- gauge contribution+ Yukawa contribution: both masses square can be positive, providing a large enough Yukawa contribution. Since $\alpha(M_{GUT}) \approx 1/40$ one finds that
  $|\eta| \gtrsim 2$
  Is this otherwise acceptable?

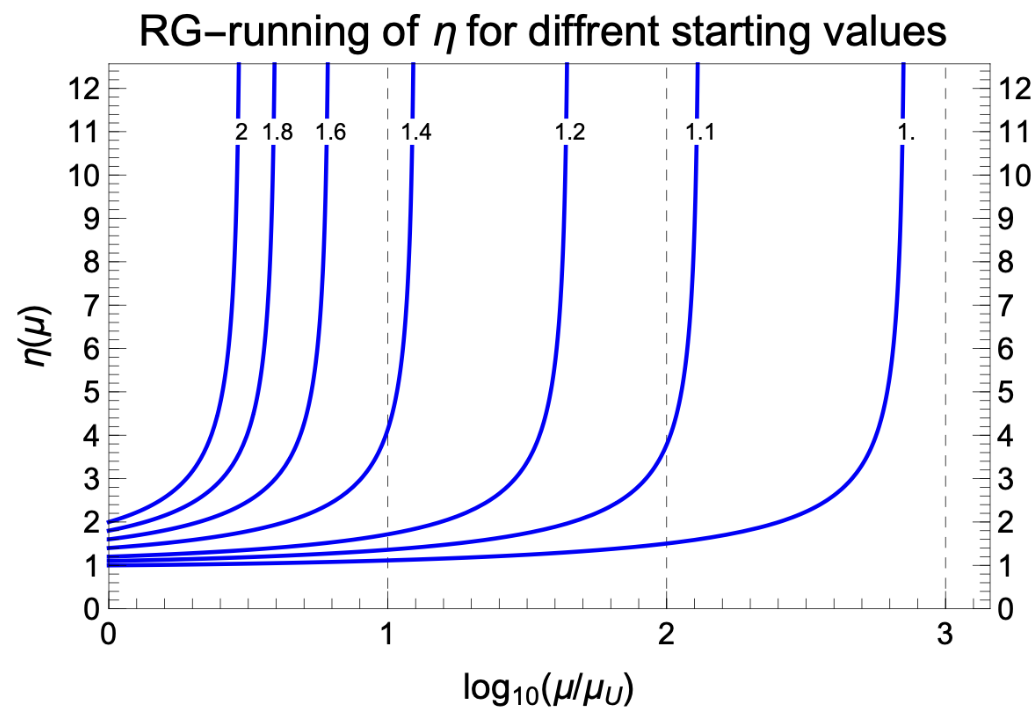
RGEs

We assume that in the above RGEs most of the couplings are negligible, except one $\bar{y}$, a top Yukawa in the 3rd family (not matching with $\bar{y}$) and η of the fourth family (we are mostly interested in the latter):

$$\mathbf{Y} = \mathrm{diag}\,(0,0,y,0)$$
$$\boldsymbol{\eta} = (0,0,0,\eta)$$

We assume all quantities are real. The RGEs simplify to

$$16\pi^2\,\frac{d}{dt}y = \left(15y^2 - 52g^2\right)y$$
$$16\pi^2\,\frac{d}{dt}\bar{y} = \left(15\bar{y}^2 + \frac{26}{3}\eta^2 - 52g^2\right)\bar{y}$$
$$16\pi^2\,\frac{d}{dt}\eta = \left(5\bar{y}^2 + 20\eta^2 - 52g^2\right)\eta$$



We see that with $\eta(M_{GUT}) = 2$ the Landau pole is approx half order of magnitude above te GUT scale!

To arrive safely to the Planck scale one would need $\eta(M_{GUT}) \approx 1$.

Gauge and Yukawa not enough $\rightarrow$ Higgs contributions needs to be included!

Higgs contribution to PGB mass

This can be in principle got from Coleman-Weinberg

$$m^2_{PGB} = \frac{\partial^2 V_{CW}(\phi)}{\partial \phi^2_{PGB}}$$

with

$$V_{CW} = \frac{1}{64\pi^2} \sum_i c_i m_i^4(\phi) \log\left(\frac{m_i^2(\phi)}{\mu^2}\right)$$

$$c_{spin\,0} = 1 \quad , \quad c_{spin\,1/2} = -2 \quad , \quad c_{spin\,1} = 3$$

In fact we used this one for the contributions due to gauge and Yukawa

There are various issues with the Higgs contributions:

- presence of pseudo-Goldstones and would-be Goldstones lead to apparent $\log(0)$ terms
- the matrices involved are rather large and usually not diagonalisable analytically

For this reason it seems more appropriate to use directly Feynman diagrams (Weinberg). Work in progress

Conclusions

- Large representations of E_6 lead soon to the Landau pole
- It seems that there are no asymptotically free E_6 GUTs
- There is a possible candidate for a perturbative E_6 till the GUT scale $(4 \times 27_F + \overline{27}_F + N_H \times 27_H + 78_H)$
- to be still checked:
  1. Higgs contribution to the pseudo-Goldstone masses at 1-loop (should relax the large value of η)
  2. complete symmetry breaking (scales, LR$\rightarrow$ SM)
  3. SM fermion masses and mixings (tree level charged fermions, 1-loop neutrinos)