Bounce vs Inflation: blue vs red tilts

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Mainly based on recent papers
with Sravan Kumar, Alexei Starobinsky, Anna Tokareva,

and works in progress
with A.Addazi, Yi-Fu Cai, A.Naskar, L.Rachwal, Shi Pi, A.Tokareva, Ye Xuan, and my students

Motivation and Introduction

A long-standing Grand Problem – quantizing gravity

- In an attempt to understand how to modify the Einstein's gravity which is not UV-complete we turn to more fundamental approaches

Strings

- Strings and especially string field theory should in principle give a chance to understand quantum gravity

Strings feature non-local interactions in the form of higher- and even infinite-derivative form factors

- Aref'eva, Barvinskiy, Biswas, Koivisto, Krasnikov, Kuz'min, Mazumdar, Modesto, Sen, Siegel, Shapiro, Tomboulis, Witten, Zwiebach, . . .

Some old references

- **Classic one:**
  M. Ostrogradski, Mem. Ac. St. Petersburg, VI 4, 385–517 (1850)
- **Mathematical:**
  H.T. Davis, Ann. of Math. 2, no. 4, 686–714 (1931)
- H.T. Davis, The Theory of Linear Operators from the Standpoint of Differential Equations of Infinite Order (Indiana, the Principia Press, 1936)
- R.D. Carmichael, Bull. Amer. Math. Soc. 42, 193–218 (1936)
- L. Carleson, Math. Scand. 1, 31–38 (1953)
- **Turned out to be quite surprising to me:**
  A. Pais and G.E. Uhlenbeck, Phys. Rev. 79, 145–165 (1950)

Action to study [1602.08475, 1606.01250, 1711.08864]

$$S = \int d^4x\sqrt{-g}\Bigg(\frac{M_P^2 R}{2} - \Lambda + \frac{\lambda}{2}\Big(R\mathcal{F}_R(\Box)R + W_{\mu\nu\lambda\sigma}\mathcal{F}_W(\Box)W^{\mu\nu\lambda\sigma}\Big)\Bigg)$$

Here $\mathcal{F}_X(\Box) = \sum_{n\geq 0} f_{X_n}\Box^n$ with all f_{X_n} constants and we often use $\mathcal{F} \equiv \mathcal{F}_R$

We assume that $\Box$ enters form-factors in a combination $\Box/\mathcal{M}_s^2$ where the mass parameter is the non-locality scale. We put $\mathcal{M}_s = 1$ for a while.

This is the most general action to study linear perturbations around MSS (in 4 dimensions).

We name it Analytic Infinite Derivative (AID) gravity.

Covariant spin-2 propagator on MSS:

$$S_2 = \frac{1}{2}\int d^4x\sqrt{-\bar{g}}\; h^{\perp}_{\nu\mu}\left(\bar{\Box} - \frac{\bar{R}}{6}\right)\left[\mathcal{P}(\bar{\Box})\right] h^{\perp\,\mu\nu}$$

$$\mathcal{P}(\bar{\Box}) = 1 + \frac{2}{M_P^2}\lambda f_{R_0}\bar{R} + \frac{2}{M_P^2}\lambda\mathcal{F}_W\left(\bar{\Box} + \frac{\bar{R}}{3}\right)\left(\bar{\Box} - \frac{\bar{R}}{3}\right)$$

The Stelle's case corresponds to $\mathcal{F}_W = 1$ such that

$$\mathcal{P}(\bar{\Box})_{Stelle} = 1 + \frac{2}{M_P^2}\lambda f_{R_0}\bar{R} + \frac{2}{M_P^2}\lambda\cdot 1\cdot\left(\bar{\Box} - \frac{\bar{R}}{3}\right)$$
$$= \frac{2\lambda}{M_P^2}(\bar{\Box} - m^2)$$

This is an obvious second pole which will be a ghost.

Physical propagators around Minkowski, AID form-factors:

$$\mathcal{O}_s = \frac{(6\lambda\Box\mathcal{F}(\Box) - M_P^2)(2\lambda\Box\mathcal{F}_W(\Box)/M_P^2 + 1)}{6\lambda(\mathcal{F}(\Box) + \frac{1}{3}\mathcal{F}_W(\Box))}$$
$$= (\Box - \mu^2)e^{2\sigma(\Box)}$$

$$\mathcal{O}_t = \Box(2\lambda\Box\mathcal{F}_W(\Box)/M_P^2 + 1) = \Box e^{2\omega(\Box)}$$

Then, avoiding all odds $\omega = \sigma + \text{const}$**:**

$$\mathcal{F}_W(\Box) = M_P^2 \frac{e^{2\omega(\Box)} - 1}{2\lambda\Box}$$

$$\mathcal{F}(\Box) = \frac{M_P^2}{6\lambda\Box}\left[\left(\frac{\Box}{\mu^2} - 1\right)e^{2\omega(\Box)} + 1\right]$$

What else can AID quadratic action serve for?

- **If we just start with the above proposed quadratic in curvature action it can accommodate many interesting solutions without requiring any other more general gravity model.**
- **For example, any conformally flat metric which satisfies $\Box R = r_1 R$ with constant r_1 is a solution.**
- **In particular, Starobinsky inflation is an exact solution here.**
- **Solution representing a ghost-free bouncing scenarios also were found.**

Starobinsky inflation in non-local gravity

[1604.03127, 1711.08864] and the recent development [2209.02515]

For any scalar curvature satisfying:

$$\Box R = r_1 R + r_2$$

with r_1, r_2 constants we have a solution of AID gravity if:

$$\mathcal{F}^{(1)}(r_1) = 0, \ \frac{r_2}{r_1}(\mathcal{F}_1 - f_0) = -\frac{M_P^2}{2\lambda} + 3r_1\mathcal{F}_1, \ \mathcal{F}_1 \equiv \mathcal{F}(r_1),$$

$$4\Lambda r_1 = -r_2 M_P^2, \ \text{but for us } \Lambda = 0 \Rightarrow r_2 = 0$$

For a wide range of assumptions it exhuasts all the space of solution.

Notice that the Weyl part does not contribute to the background because the configuration of interest is conformally flat.

Tensor to scalar ratio r

$$r = \frac{2|\delta_h|^2}{|\delta_{\mathcal{R}}|^2} = 48\frac{\dot{H}^2}{H^4}e^{2\omega(\bar{R}/6)}$$

All quantities here are at the horizon crossing $k = Ha$.

Analogously

$$N = \int_{t_i}^{t_f} H dt = \frac{1}{2\epsilon_1} \Rightarrow r = 48\epsilon_1^2 e^{2\omega(\bar{R}/6)} = \frac{12}{N^2}e^{2\omega(\bar{R}/6)}$$

We have gained an extra factor $e^{2\omega(\bar{R}/6)}$ compared to the local R^2 inflation.

Non-Gaussianities, briefly [2003.00629] and recently [2210.16459]

$$f_{NL} = \frac{5}{12}(1 - n_s) + \text{corrections}\{\omega(\bar{R}/4), \omega'(\bar{R}/4)\}$$

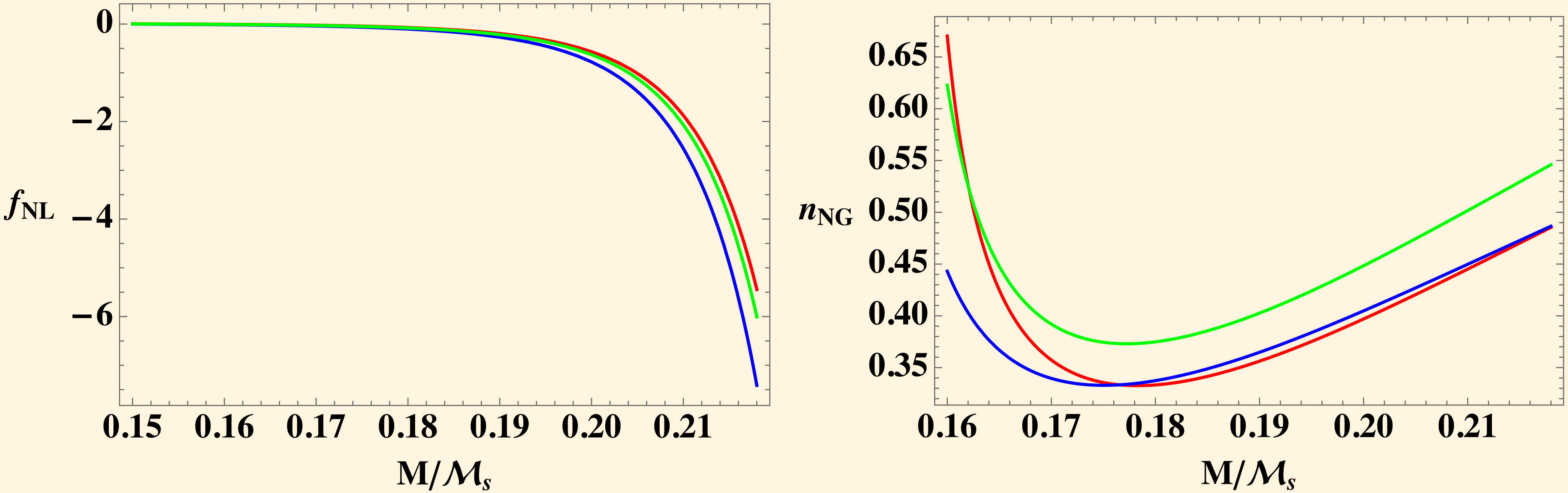

Figure 1: In the left panel we plot $f_{\rm NL}^{\rm sq}$, $f_{\rm NL}^{\rm eq}$, $f_{\rm NL}^{\rm orth}$ versus $M/\mathcal{M}_s$ (in red, blue and green coloured lines respectively) and in the right panel we plot the corresponding running of $f_{\rm NL}$. In these plots we recover the predictions of local R^2 gravity in the limit $\frac{M}{\mathcal{M}_s} \to 0$.

Non-Gaussianities, recap

- f_{NL} *can* be made large.
- Maldacena consistency relation known to hold for a single scalar field model of inflation is violated here.
- Different shapes differ on contrary to a single-field inflation
- We have constraints on f_{NL} from observations and this crucially shows up in our AID gravity model as the constraint on the scale of non-locality which we denote $\mathcal{M}_s$
- Namely

$$\mathcal{M}_s > 10^{-4} M_P$$

- Moreover, f_{NL} can be of either sign based on the lowest power of polynomial of our entire function σ, i.e. whether it is odd or even.

Gravitational waves production [2211.02070]

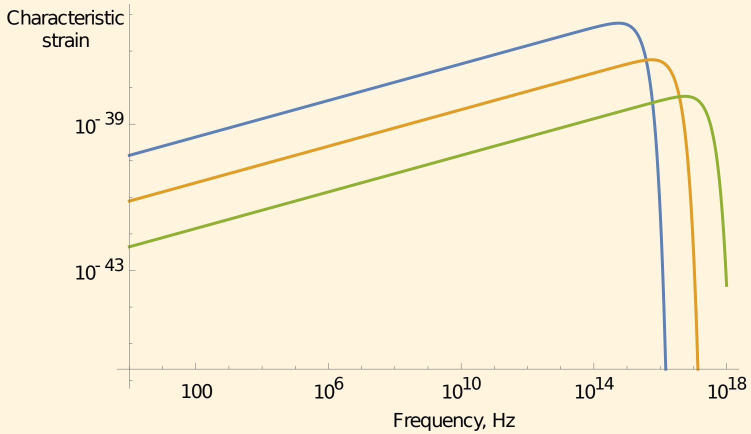

Figure 2: The blue curve shows the gravitational wave signal for $T_{reh} = 10^{10}$ GeV, orange for $T_{reh} = 10^{9}$ GeV and green for $T_{reh} = 10^{8}$ GeV. In all cases we assume $\Gamma_{GW}/\Gamma_{SM} = 10^{-3}$. The lowest characteristic strain available for future gravitational wave detectors is 10^{-24} for the frequencies $1 - 10^{6}$ Hz.

Even though these predictions are not immediately encouraging for possible observatinal signatures they are:

First: do not contradict exsiting observation

Second: prove our ability to do these involved computations in a higher (infinite) derivative gravity theories

Third: various effects, including a possibility for a lower T_{reh} yet to be explored

Can it be a Non-local scalar field [arxiv:2103.01945]

Consider Analytic Infinite Derivative (AID) scalar field action:

$$L = \frac{1}{2}\phi(\Box - m^2) f^{-1}(\Box)\phi - V(\phi)$$

We demand the form-factor to be an exponent of an entire function $\sigma(z)$

$$f(z) = \exp(2\sigma(z))$$

This is required to have no extra poles in the perturbative vacuum.

We also normalize it as $f(0) = f(m^2) = 1$ to preserve the local answers in the IR limit.

Non-local scalar field, continued

Several arguments to consider the above action:

- **It naturally appears in SFT and in p-adic strings**
- **It was proven to be unavoidable in order to build unitary and renormalizable diffeomorphism invariant gravity**
- **This construction can make any arbitrary potential renormalizable**
- **Surely, some other benefits**

Namely, we can adjust the fall rate of the propagator for large momenta by choosing the form-factor. Power-counting convergence requires the fall faster than $\sim 1/p^2$.

New excitations – Half of them are ghosts!

Linearization around a background solution ϕ_0:

$$L = \frac{1}{2}\psi\left[(\Box - m^2)f^{-1}(\Box) - V''(\phi_0)\right]\psi$$

Let's assume $V''(\phi_0) = v \approx \text{const} \neq 0$.

- **In general there is an infinite number of new excitations with perhaps complex conjugate masses squared**
- **The kinetic operator is again an entire function and obeys the Weierstrass decomposition**

$$(\Box - m^2)f^{-1}(\Box) - v^2 \sim \prod_i (\Box - \mu_i^2)e^{\sigma_v(\Box)}$$

- **Each μ_i corresponds to a mass of a distinct excitation.**

Conclusions and Outlook

- **A class of analytic infinite derivative (AID) theories has been considered targeting the goal of constructing a UV complete and unitary gravity. These models have clear connection with SFT.**
- **This gravity model features many nice properties, like native embedding of the Starobinsky inflation, finite Newtonian potential at the origin, presence of a non-singular bounce, etc.**
- **Corrections to the inflationary observables are explicitly computed in terms of model parameters**
- **In particular we see correction to r, we observe different values for different shapes of NG-s and allow for a different sign of f_{NL}**

Open Questions

- **A non-local scalar field coupled to gravity is a purely ad hoc model.**
- **Non-local gravity action does not allow for a simple frame-changing by a conformal transformation. A most natural one is mathematically spoiled by the fact that we have to invert $\mathcal{F}(\Box)$ which is apparently is not invertible on the whole complex plane.**
- **We cannot fix an all-around good form-factor for all backgrounds. Like we cannot make all vacua of a non-local scalar field model ghost-free. We thus must either modify our gravity model further [2209.22515, 2210.16459, 2305.18716] or assume new excitations as a reality [work in progress].**

Thank you for listening!