Northeastern University

Non-Abelian Domain walls

“oreo” and CP violation

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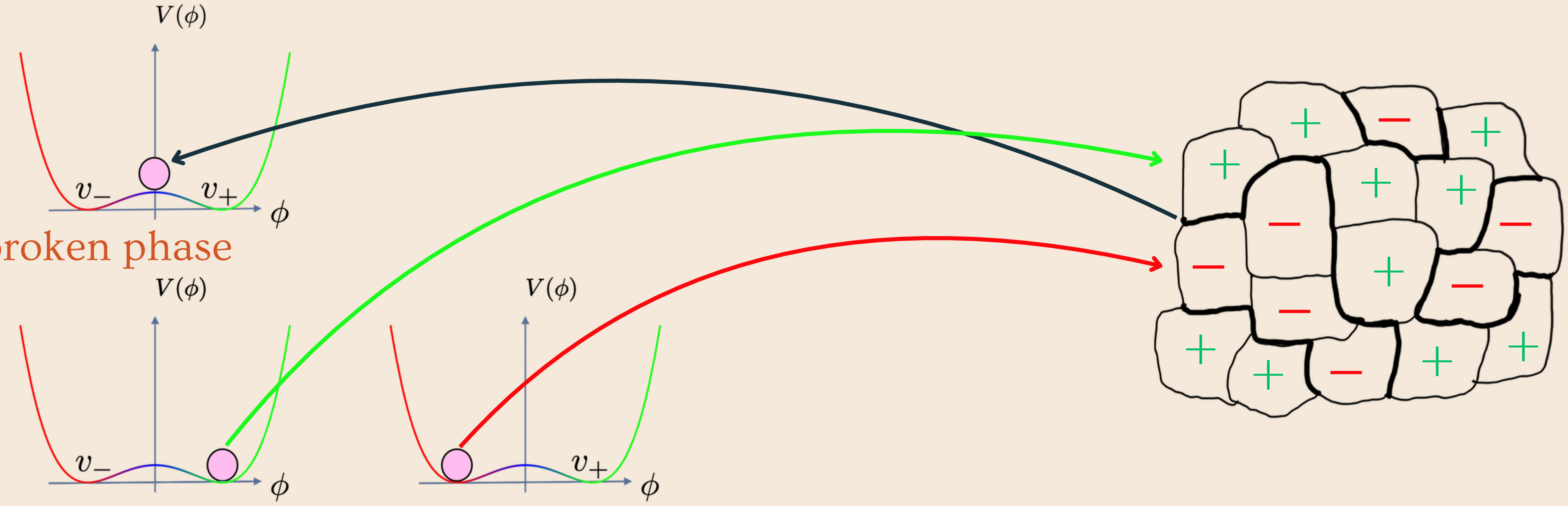
BF, S. F. King, L. Marsili, S. Pascoli, J. Turner, Y-L. Zhou, 2409.16359
BF, S. F. King, L. Marsili, J. Turner, Y-L. Zhou, 2512.13784

Domain walls

false vacuum remnants from early universe symmetry-breaking phase transitions

Example of Z_2-symmetric case

unbroken phase

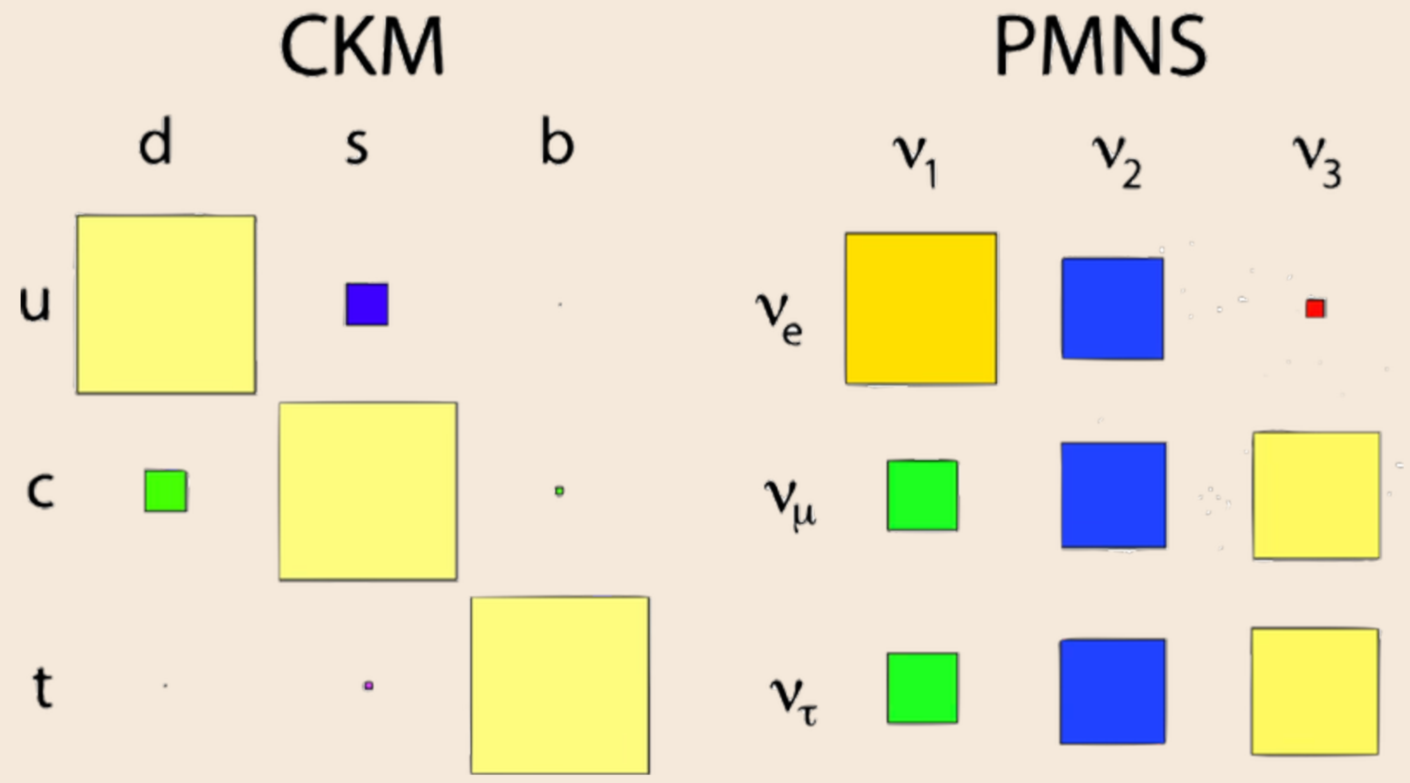


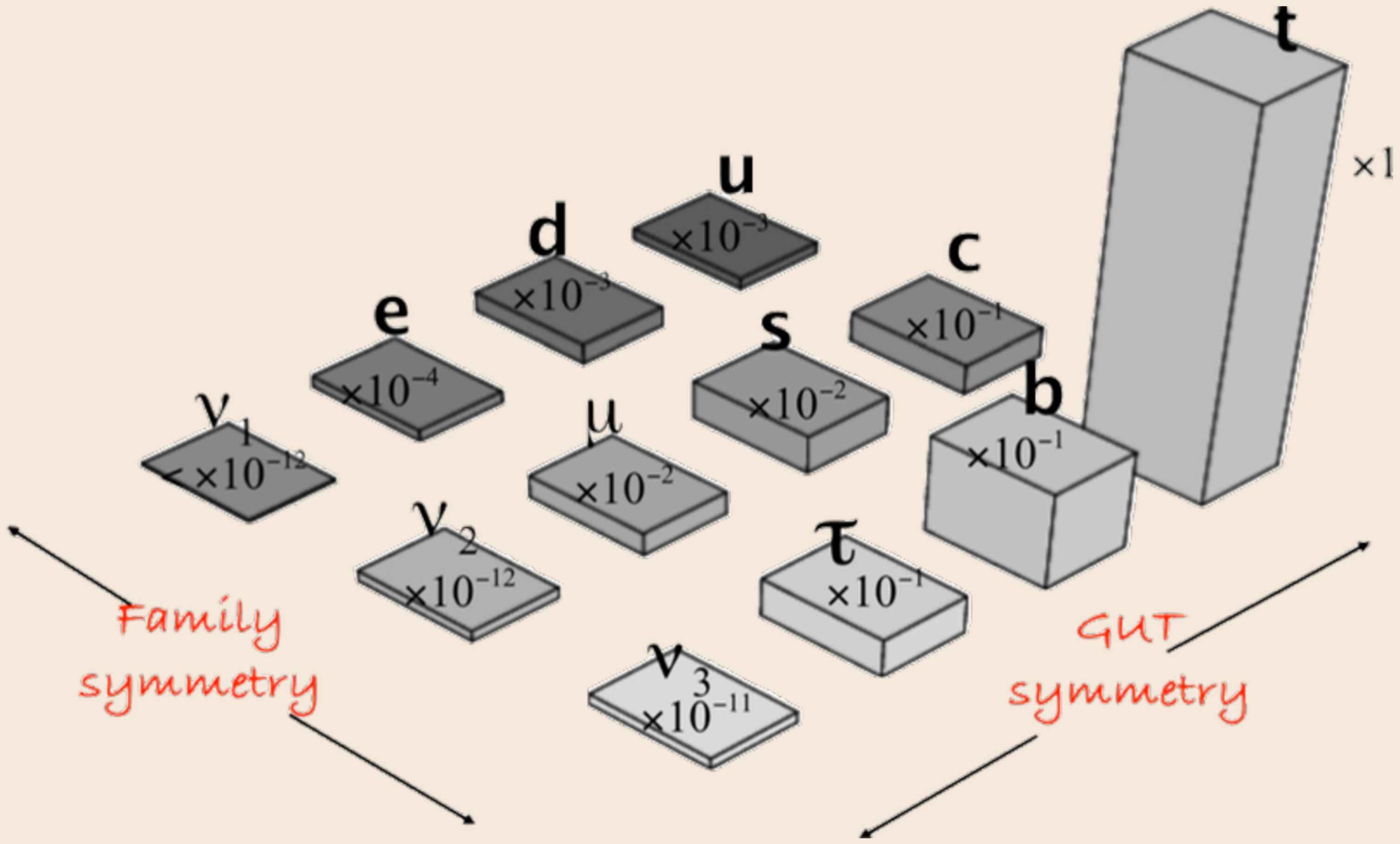
Symmetry beyond Z_2

Z_2 symmetry is the simplest discrete symmtry, but is it enough?

Flavour problem

The origin of quark and lepton mixing?





Flavour symmetry

A different direction of unification

Symmetry beyond Z_2

Z_2 symmetry is the simplest discrete symmtry, but is it enough?

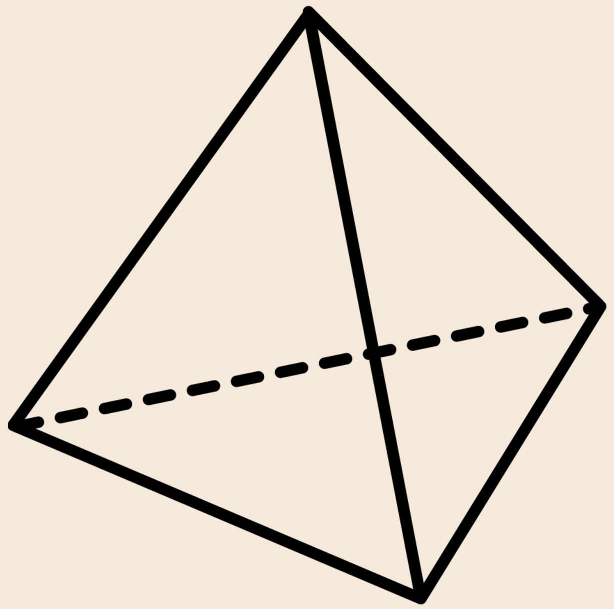
Lepton triplets

Embed leptons of different flavours into a triplet representation

$$\ell = \begin{pmatrix} \ell_1 \\ \ell_2 \\ \ell_3 \end{pmatrix}$$

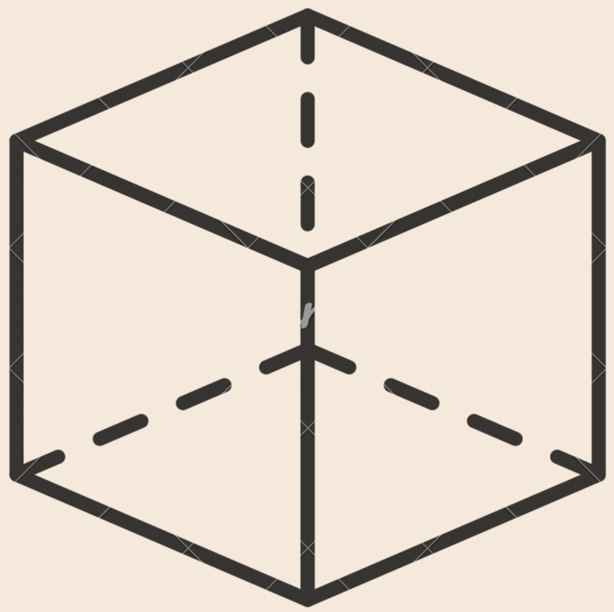
A_4

rigid rotation group of a tetrahedron



S_4

rigid rotation group of a cube



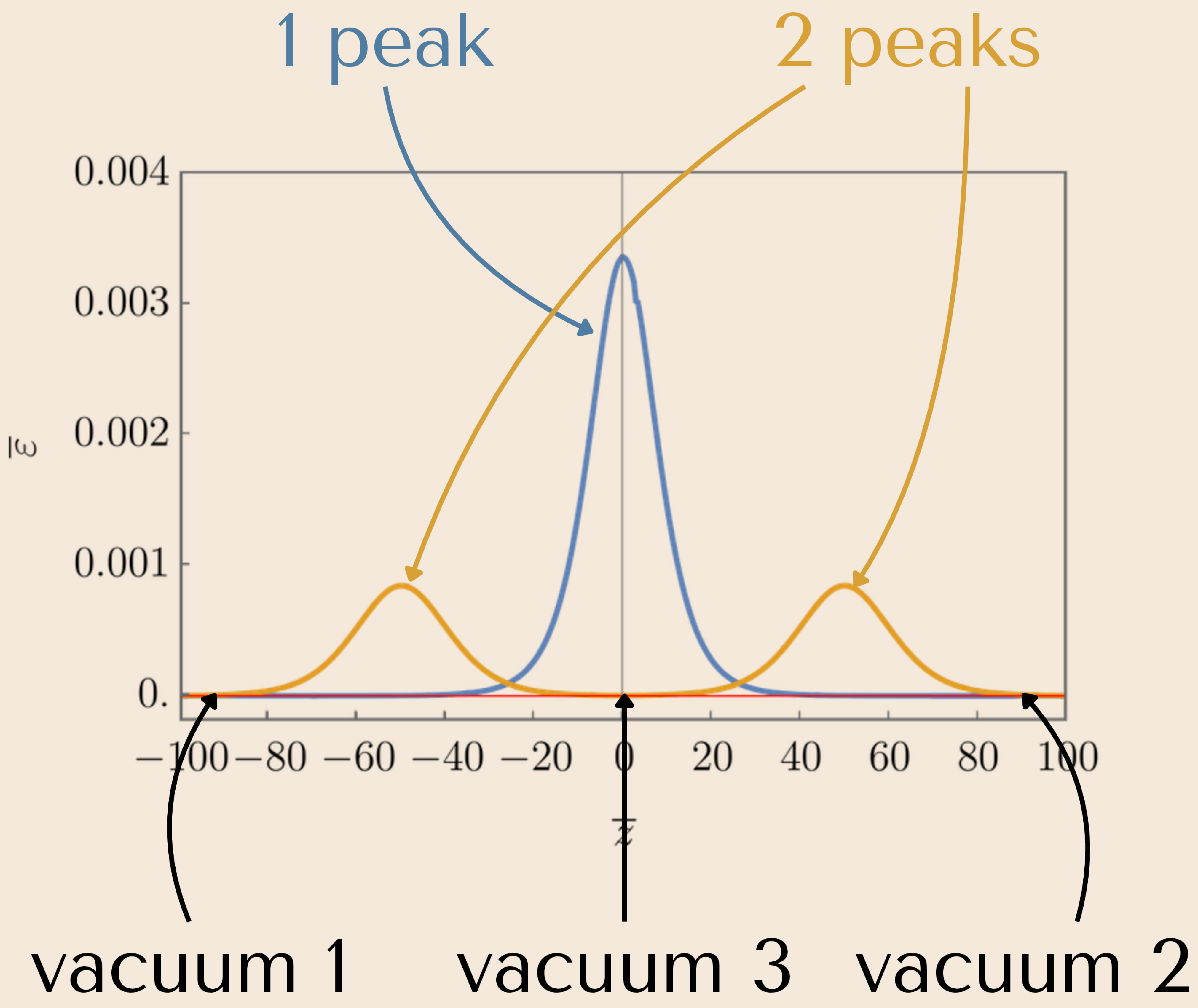
Composite Domain Walls

consequence of vacuum structure

Multiple solutions

When there are multiple solutions between two vacua, the energy of the solutions can be different.

The solution with two peaks can have a different vacuum between the peaks.



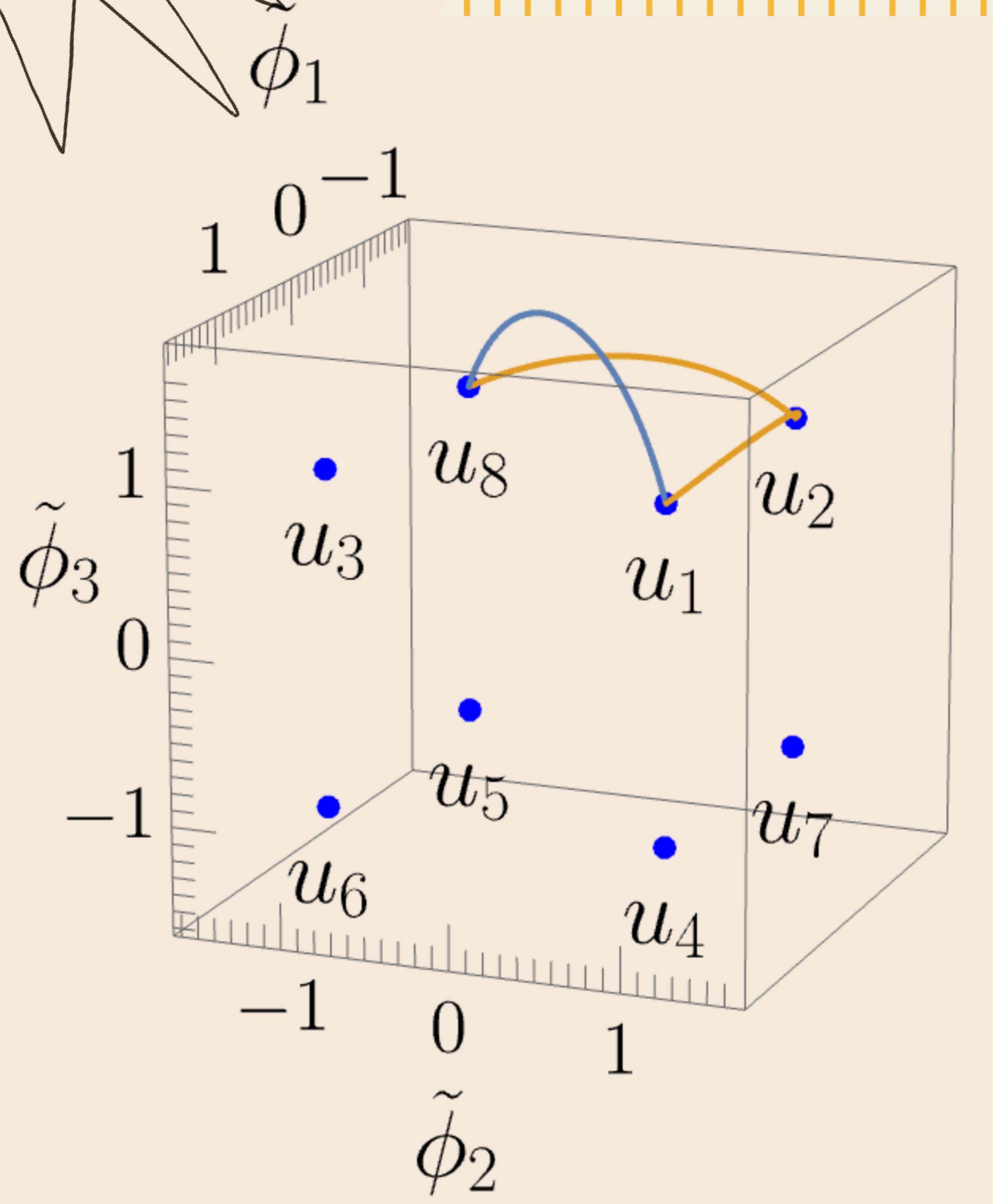
Unstable Domain walls

S_4

$$V(\phi) = -\frac{\mu^2}{2} I_1 + \frac{g_1}{4} I_1^2 + \frac{g_2}{2} I_2$$

$$I_1 = \phi_1^2 + \phi_2^2 + \phi_3^2\,, \quad I_2 = \phi_1^2\phi_2^2 + \phi_2^2\phi_3^2 + \phi_3^2\phi_1^2$$

$$\beta = \frac{g_2}{g_1} = -0.01$$



Unstable Domain walls

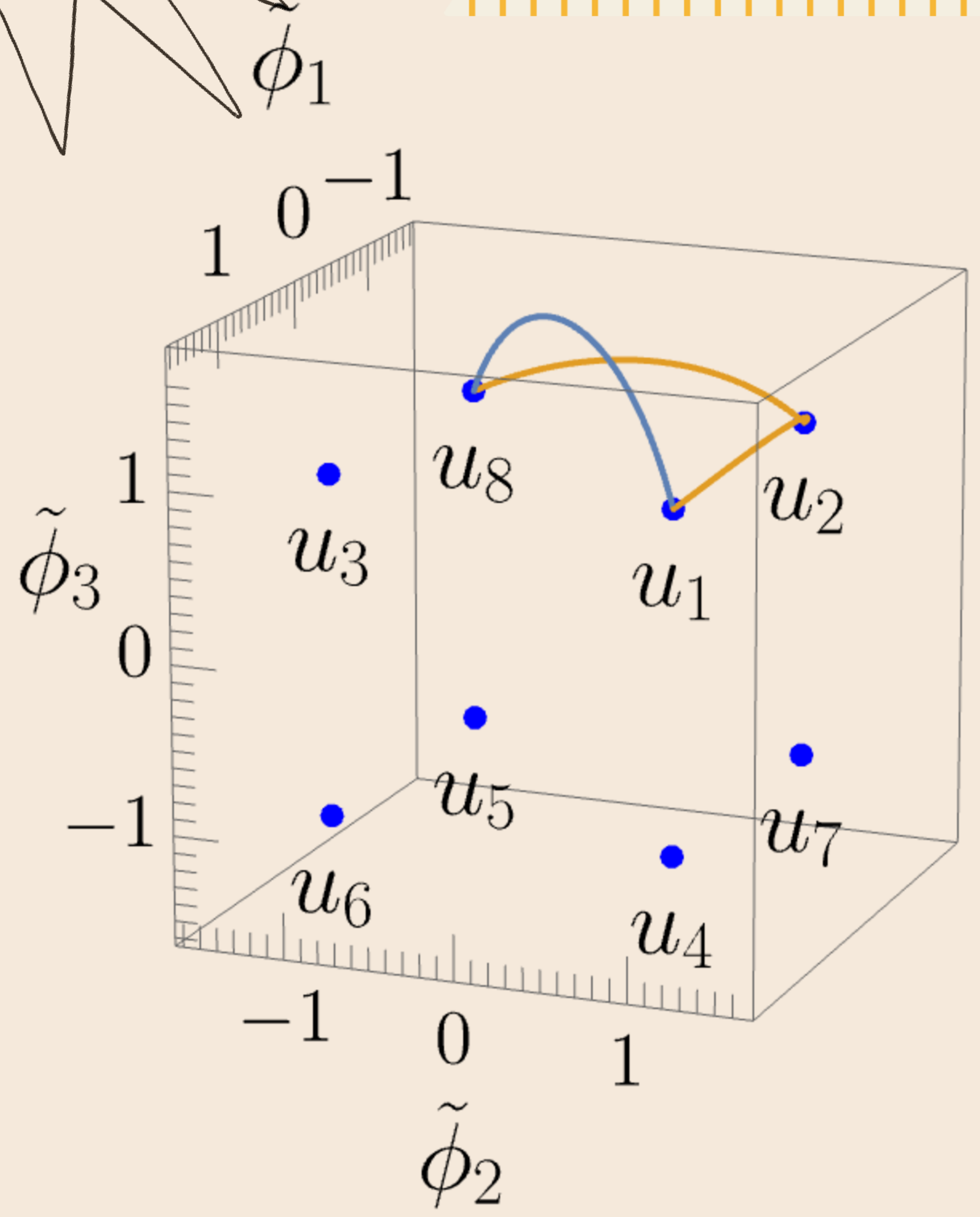
$S_4 \to A_4$

$$V(\phi) = -\frac{\mu^2}{2} I_1 + \frac{g_1}{4} I_1^2 + \frac{g_2}{2} I_2 + A\phi_1\phi_2\phi_3$$

$$I_1 = \phi_1^2 + \phi_2^2 + \phi_3^2\,, \quad I_2 = \phi_1^2\phi_2^2 + \phi_2^2\phi_3^2 + \phi_3^2\phi_1^2$$

$$\beta = \frac{g_2}{g_1} = -0.01$$

$$a = \frac{A}{2\mu\sqrt{3g_1 + 2g_2}} = 0$$



Unstable Domain walls

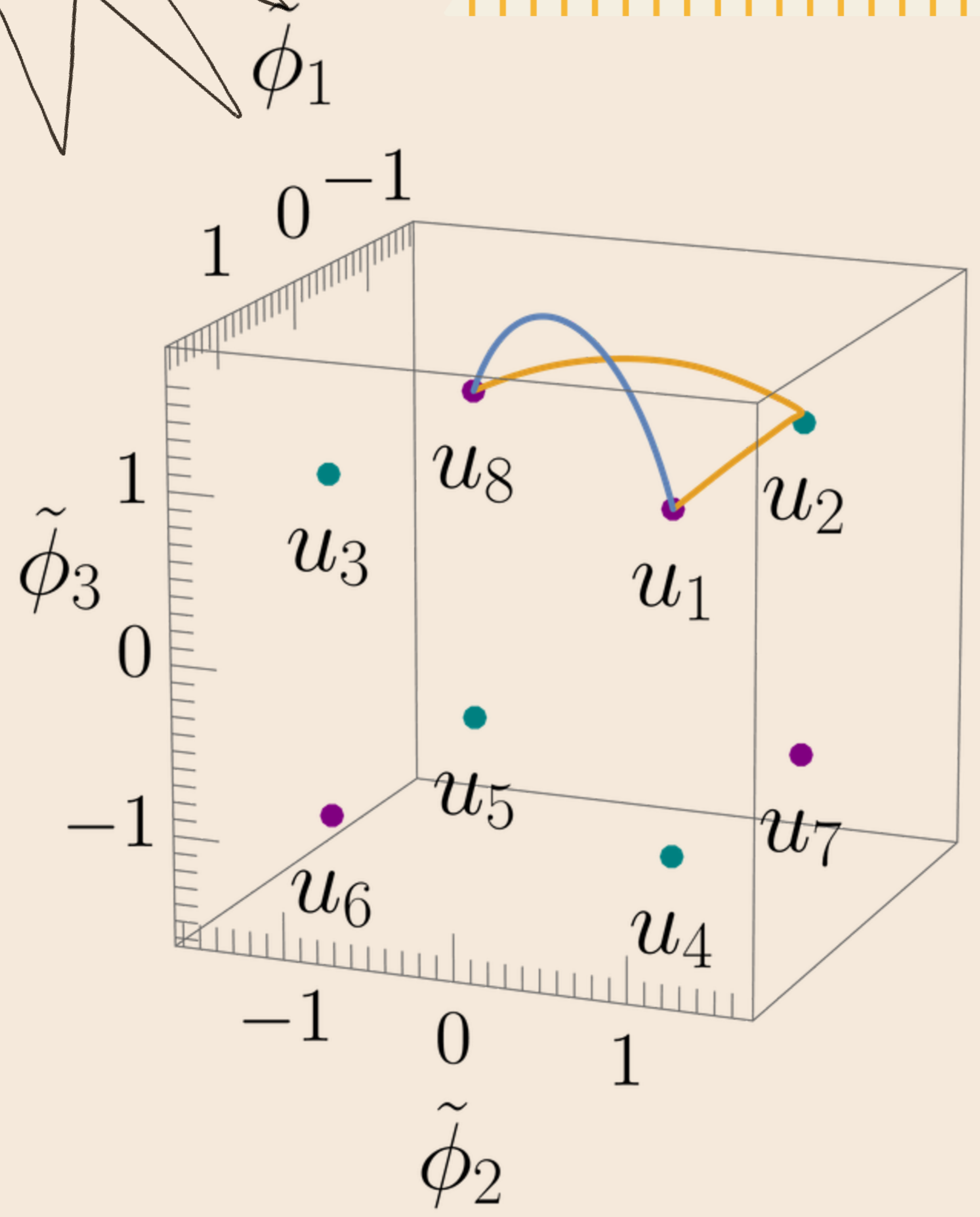
$S_4 \to A_4$

$$V(\phi) = -\frac{\mu^2}{2} I_1 + \frac{g_1}{4} I_1^2 + \frac{g_2}{2} I_2 + A\phi_1\phi_2\phi_3$$

$$I_1 = \phi_1^2 + \phi_2^2 + \phi_3^2\,, \quad I_2 = \phi_1^2\phi_2^2 + \phi_2^2\phi_3^2 + \phi_3^2\phi_1^2$$

$$\beta = \frac{g_2}{g_1} = -0.01$$

$$a = \frac{A}{2\mu\sqrt{3g_1 + 2g_2}} = -0.00003$$



Unstable Domain walls

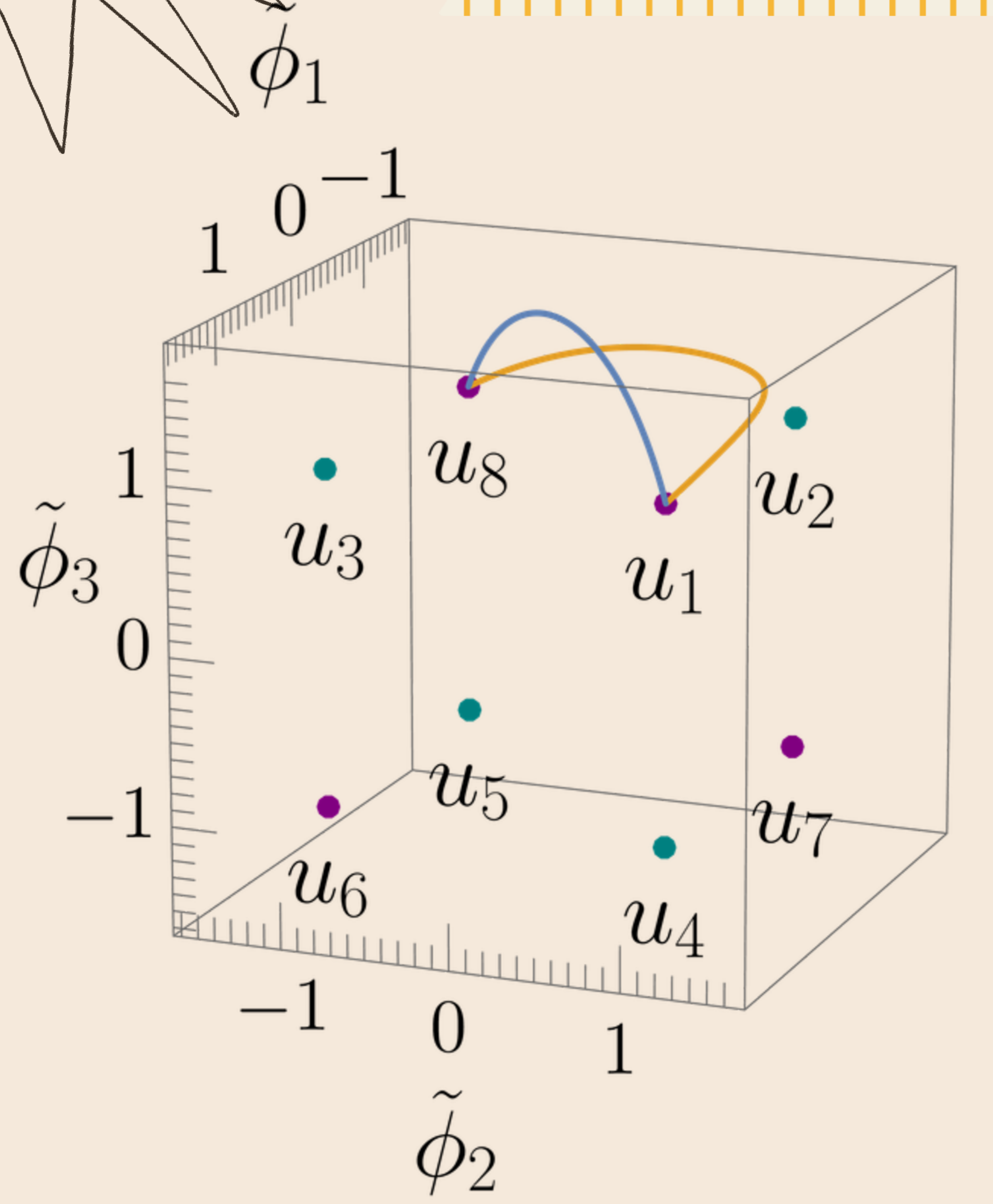
$S_4 \to A_4$

$$V(\phi) = -\frac{\mu^2}{2} I_1 + \frac{g_1}{4} I_1^2 + \frac{g_2}{2} I_2 + A\phi_1\phi_2\phi_3$$

$$I_1 = \phi_1^2 + \phi_2^2 + \phi_3^2, \quad I_2 = \phi_1^2\phi_2^2 + \phi_2^2\phi_3^2 + \phi_3^2\phi_1^2$$

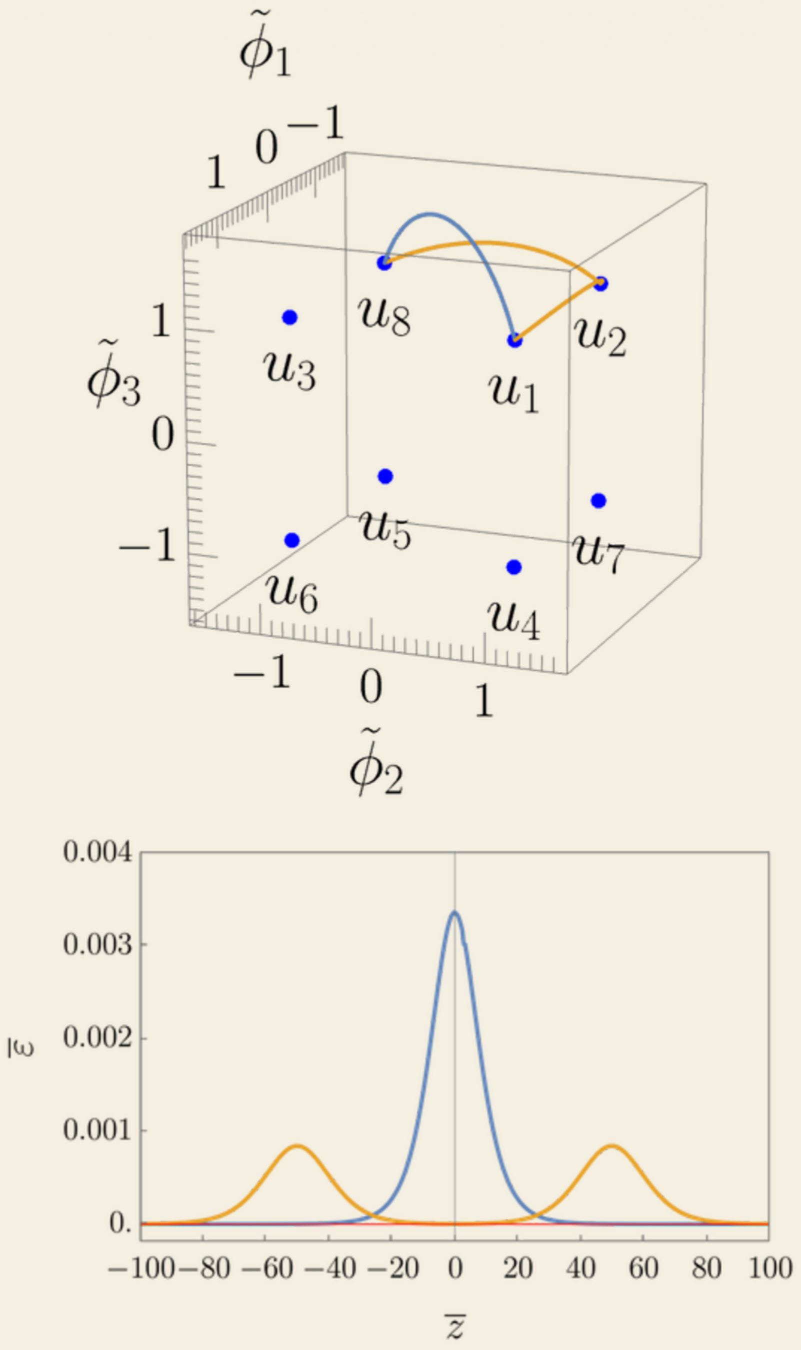
$$\beta = \frac{g_2}{g_1} = -0.01$$

$$a = \frac{A}{2\mu\sqrt{3g_1 + 2g_2}} = -0.0003$$

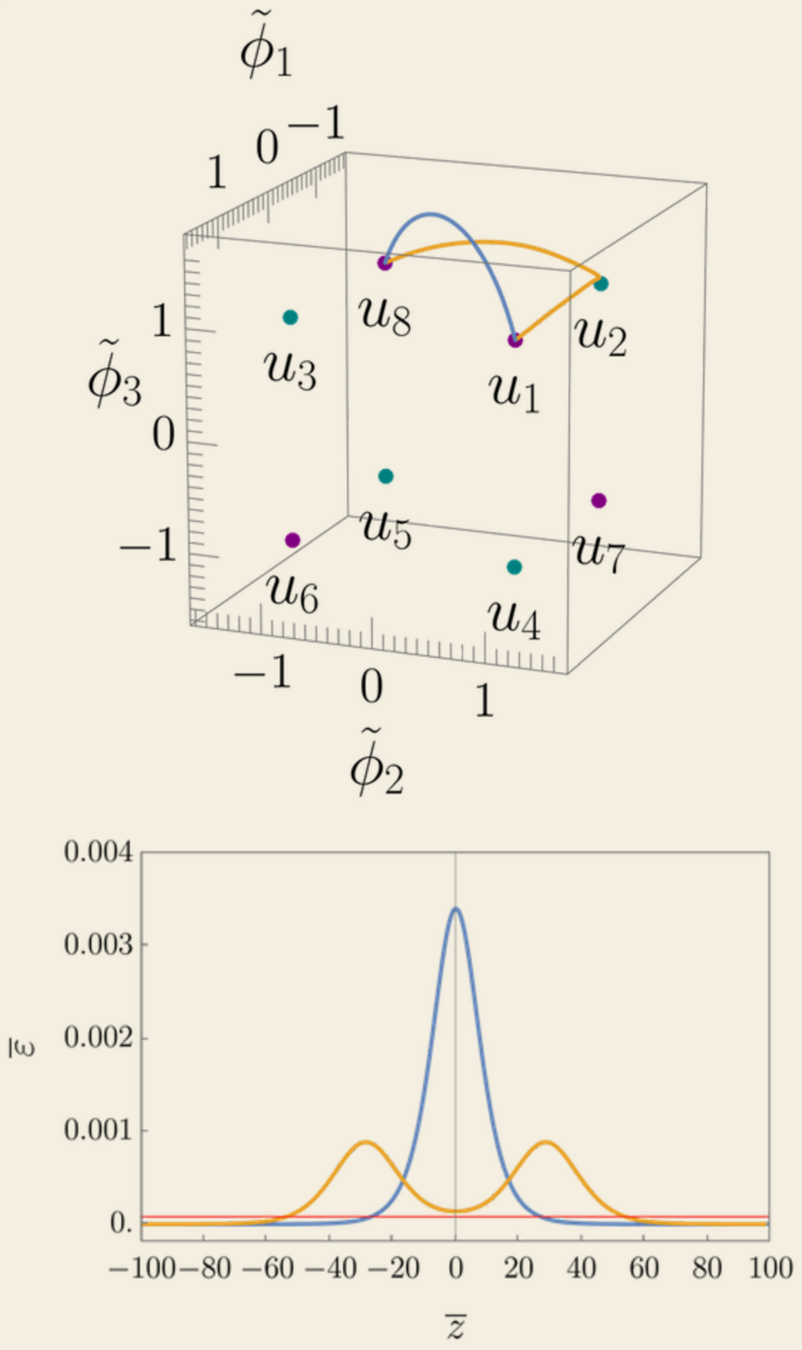


Path and Energy Density

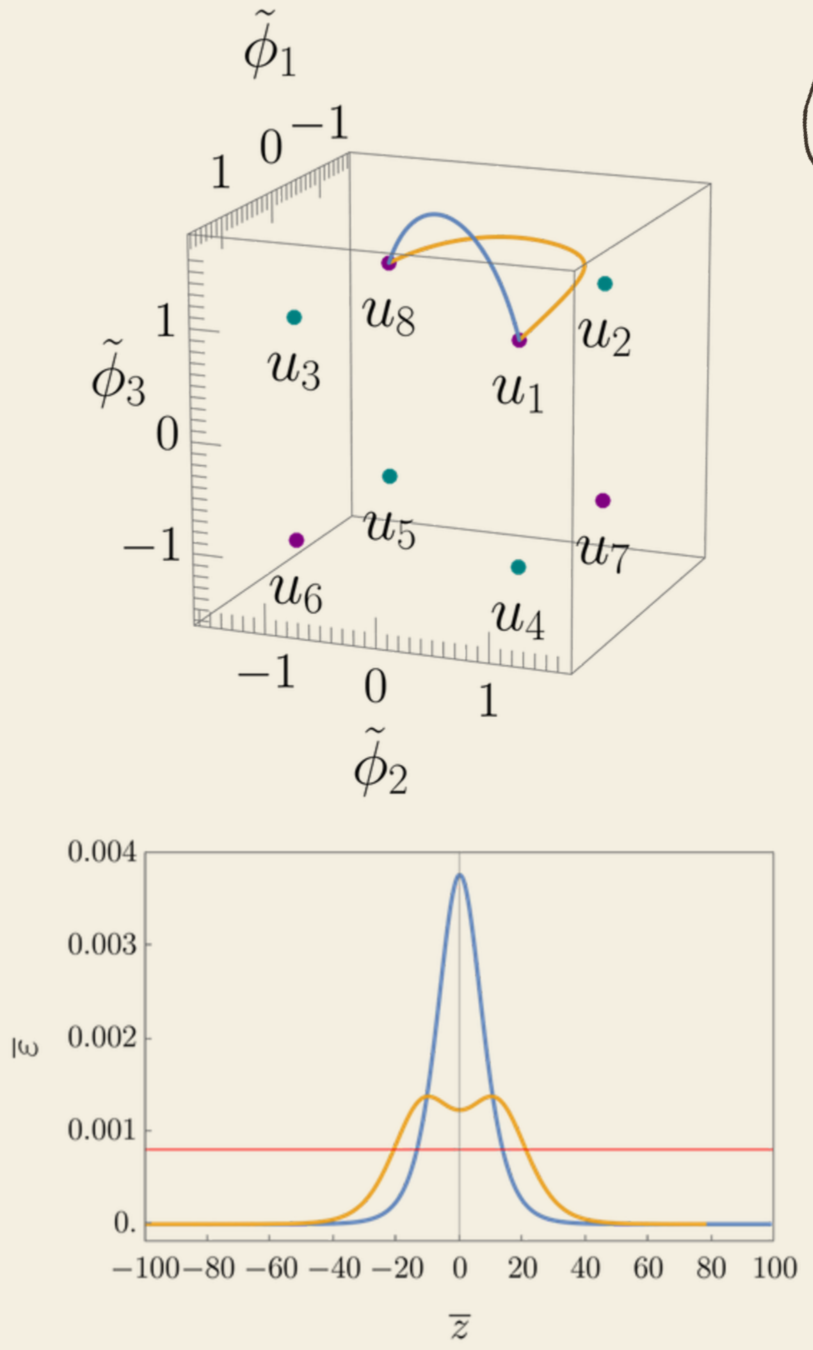
As “a” become larger, the path in field space departs from u_2 and the energy density in the “valley” grows.



a=0



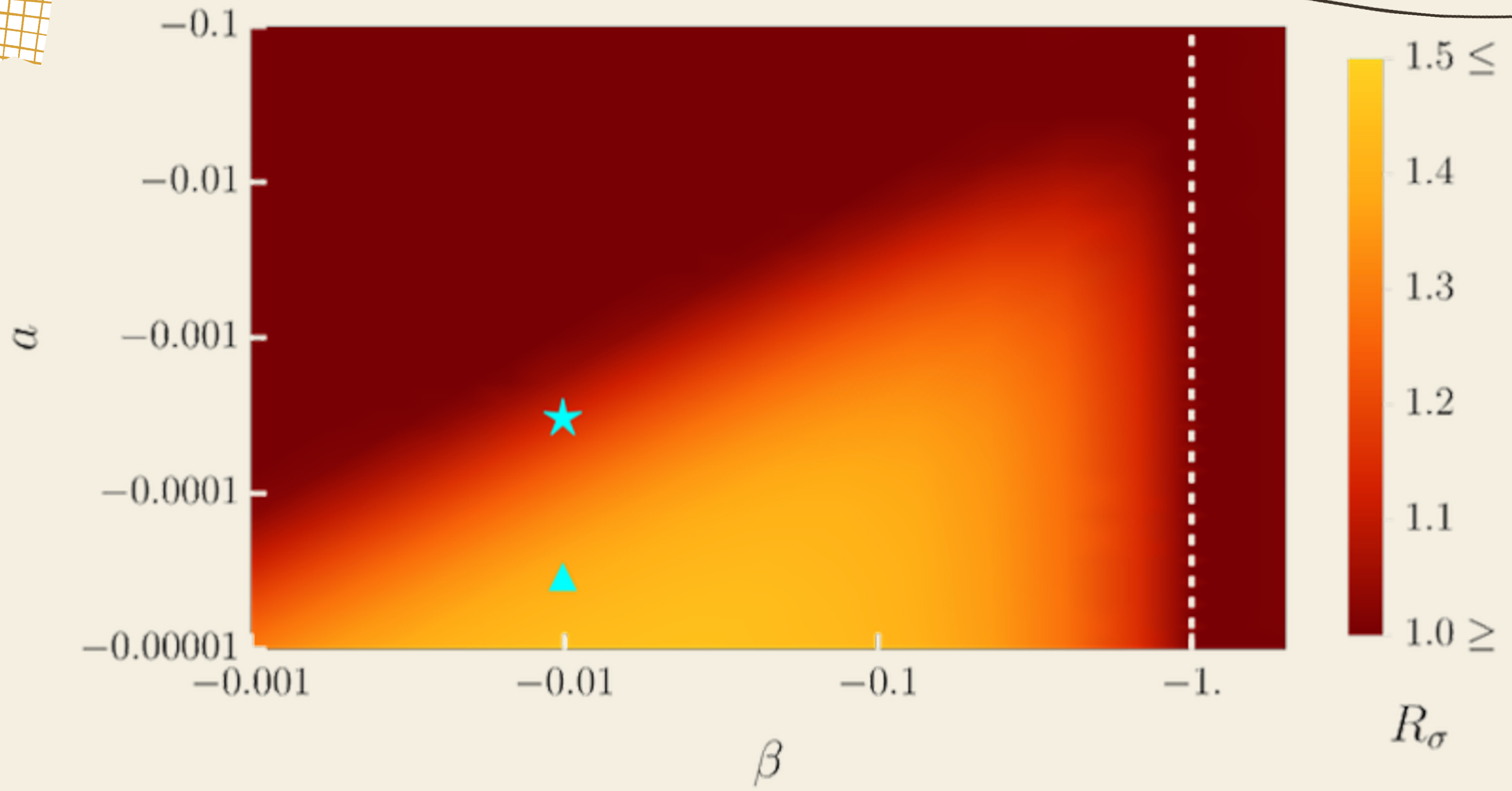
a=-3x10^{-5}



a=-3x10^{-4}

1 peak or 2 peaks?

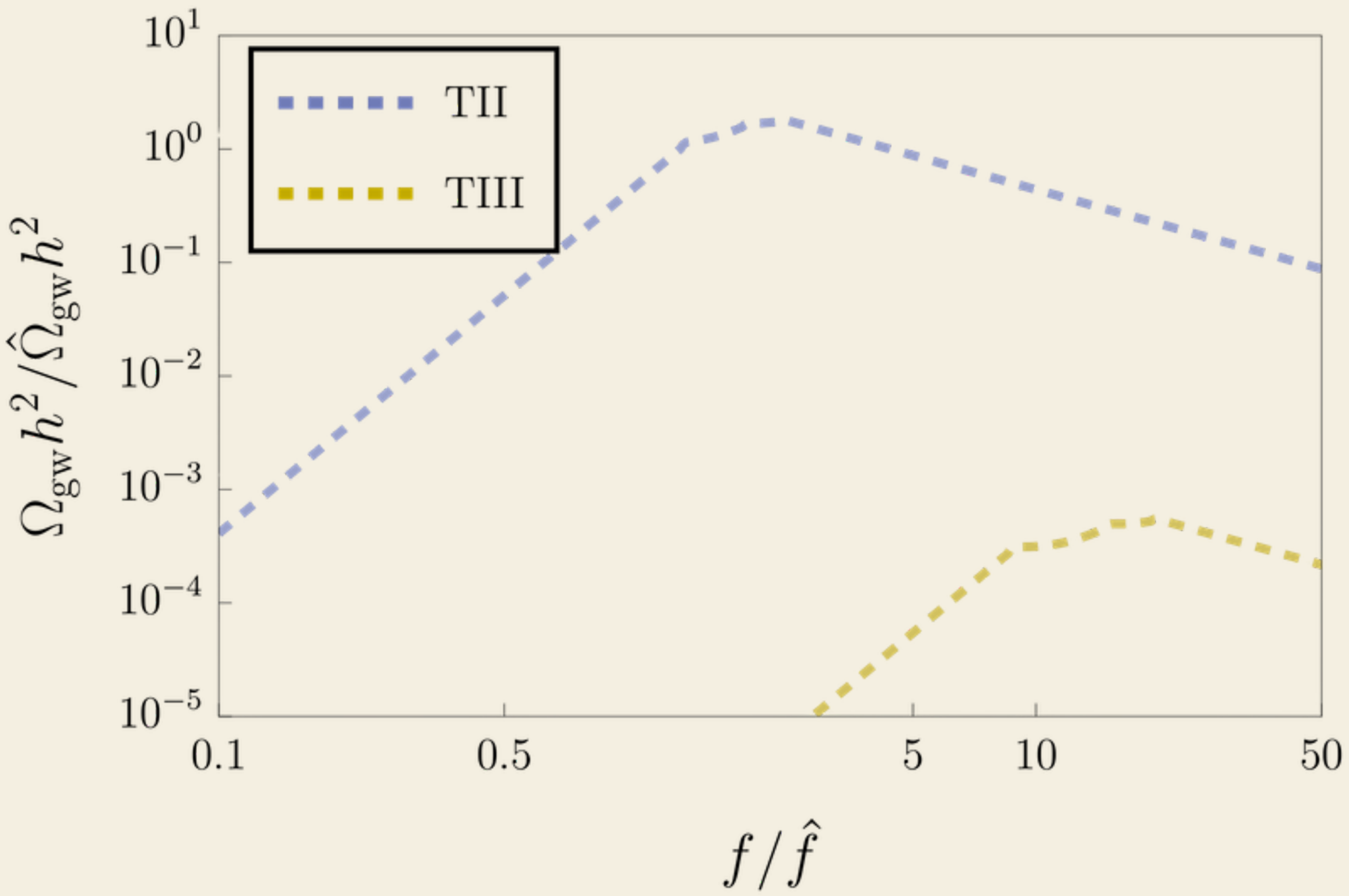
The stability of the solution depends on the parameter choice.



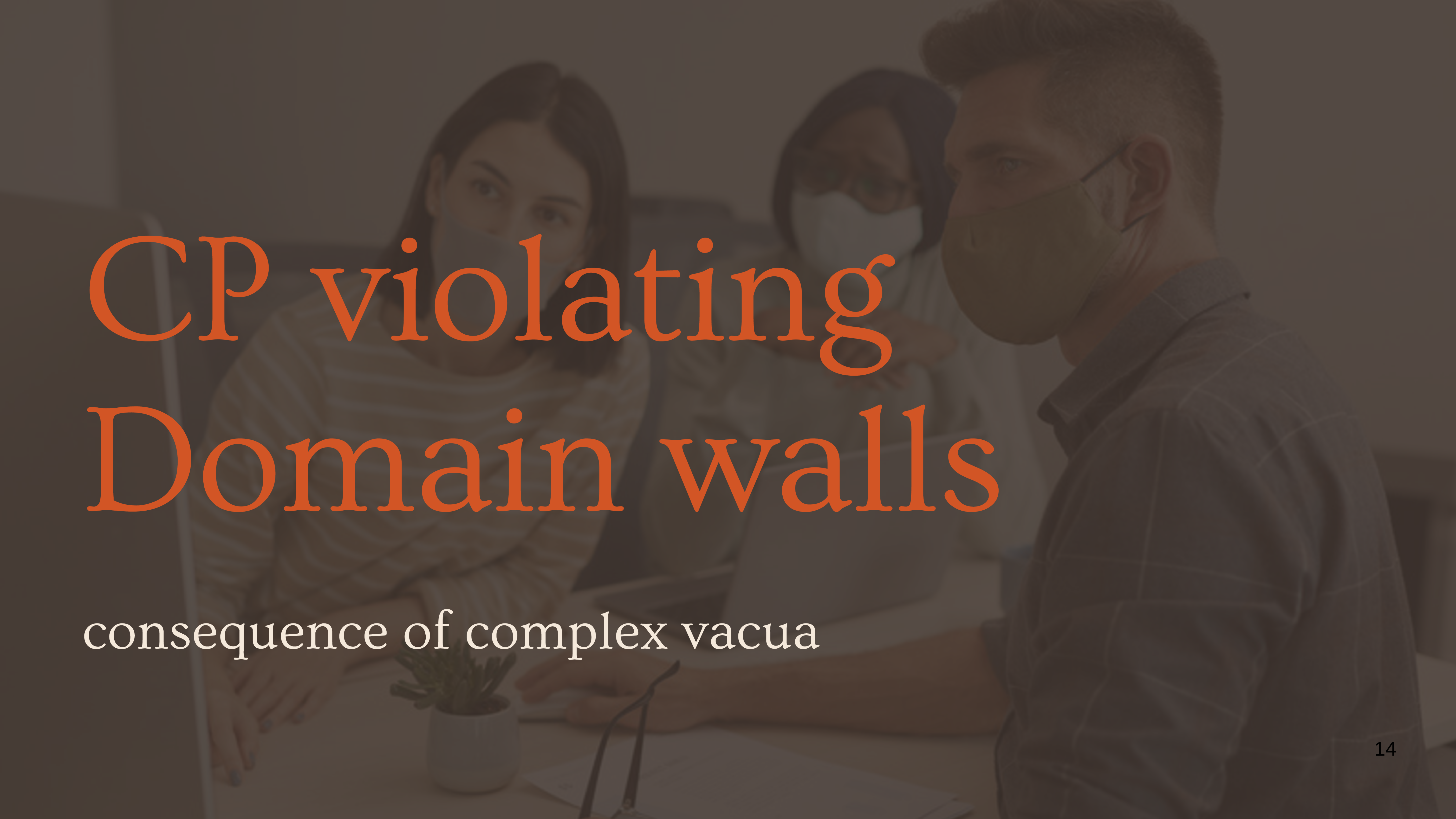
$$R_\sigma = \sigma_{1p}/\sigma_{2p}$$

Gravitational wave

The cubic term in acts as an effective bias for the walls. As it is allowed by the symmetry, we expect it to be larger than the explicit symmetry breaking term.



$\Omega_{\rm gw} \propto \sigma^4 V_{\rm bias}^{-2}$



CP violating Domain walls

consequence of complex vacua

Spontaneous CP violation

The concept of spontaneous CP violation (SCPV) was first proposed by the Chinese-American physicist T. D. Lee in 1973.

A Theory of Spontaneous T Violation*

T. D. Lee
Department of Physics, Columbia University, New York, New York 10027
(Received 11 April 1973)

Two Higgs Doublet Model

$$\langle \phi_1 \rangle = \begin{pmatrix} 0 \\ v_1 \end{pmatrix} \quad \langle \phi_2 \rangle = \begin{pmatrix} 0 \\ v_2 e^{i\theta} \end{pmatrix}$$

The two Higgs doublets get VEVs with a relevant phase.

Spontaneous CP violation in A_4 symmetric model

If the VEVs of different scalars in the scalar triplet show phase differences, the CP is spontaneously broken.

complex scalar model with $A_4 \times U(1) \times \mathrm{CP}$

$$V(\varphi,\varphi^*) = -\mu_1^2 \mathcal{I}_{11} + g_{11} \mathcal{I}_{11}^2 + 2g_{21} \mathcal{I}_{21} + g_{22}(\mathcal{I}_{22} + \text{h.c.})$$

$$\mathcal{I}_{11} = |\varphi_1|^2 + |\varphi_2|^2 + |\varphi_3|^2$$
$$\mathcal{I}_{21} = |\varphi_1|^2|\varphi_2|^2 + |\varphi_2|^2|\varphi_3|^2 + |\varphi_3|^2|\varphi_1|^2,$$
$$\mathcal{I}_{22} = \varphi_1^2(\varphi_2^2)^* + \varphi_2^2(\varphi_3^2)^* + \varphi_3^2(\varphi_1^2)^*$$

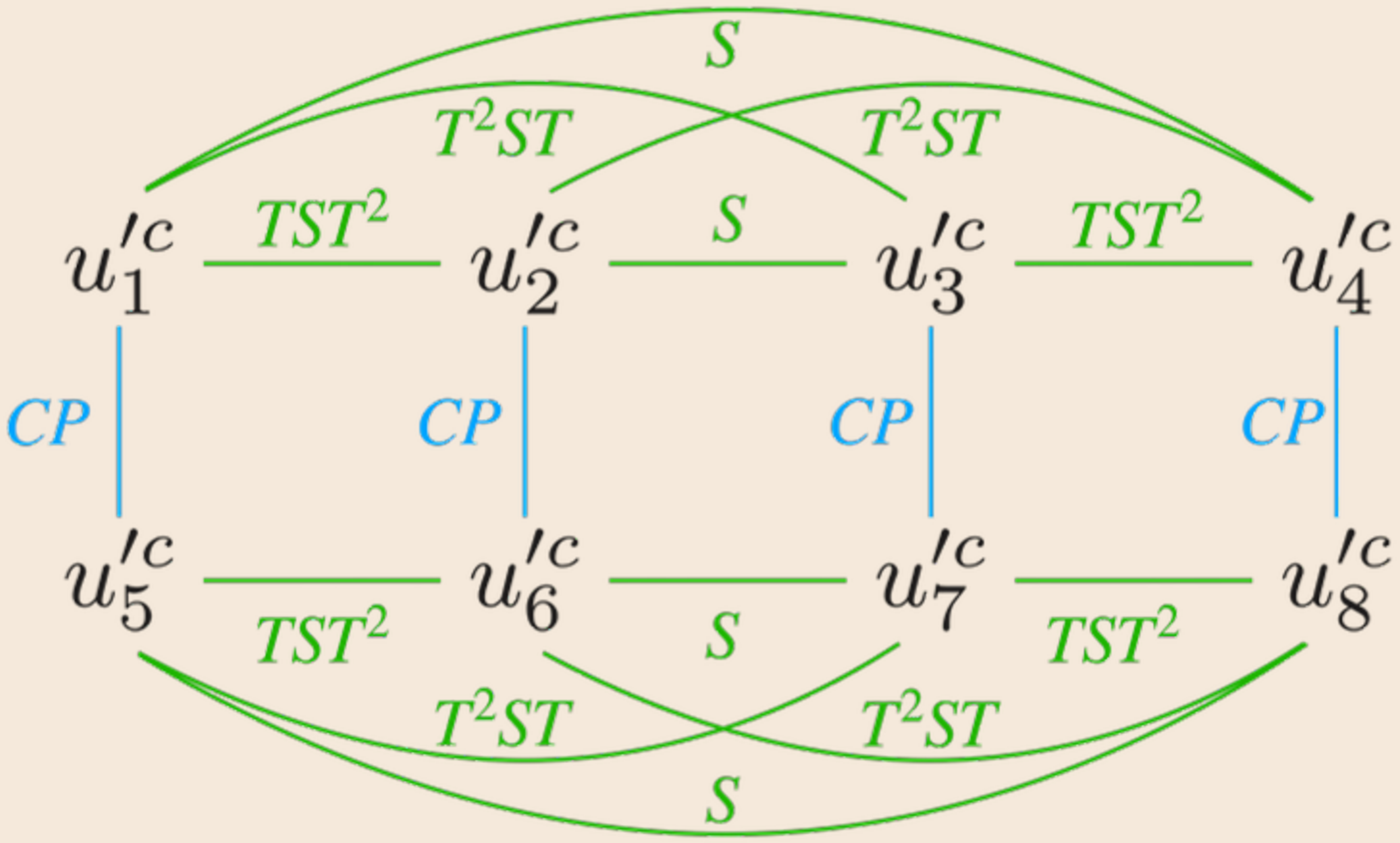
$$\{u_1'^c, u_2'^c, u_3'^c, u_4'^c\} = \left\{ \begin{pmatrix} 1 \\ \omega \\ \omega^* \end{pmatrix}, \begin{pmatrix} 1 \\ -\omega \\ \omega^* \end{pmatrix}, \begin{pmatrix} \omega^* \\ 1 \\ -\omega \end{pmatrix}, \begin{pmatrix} -\omega \\ \omega^* \\ 1 \end{pmatrix} \right\} u'^c e^{i\alpha}$$

$$\{u_5'^c, u_6'^c, u_7'^c, u_8'^c\} = \left\{ \begin{pmatrix} 1 \\ \omega^* \\ \omega \end{pmatrix}, \begin{pmatrix} 1 \\ -\omega^* \\ \omega \end{pmatrix}, \begin{pmatrix} \omega \\ 1 \\ -\omega^* \end{pmatrix}, \begin{pmatrix} -\omega^* \\ \omega \\ 1 \end{pmatrix} \right\} u'^c e^{i\alpha}$$

$$\omega = e^{i2\pi/3} \qquad u'^c = \frac{\mu}{\sqrt{3g_1 + 2g_2 - g_2'}}$$

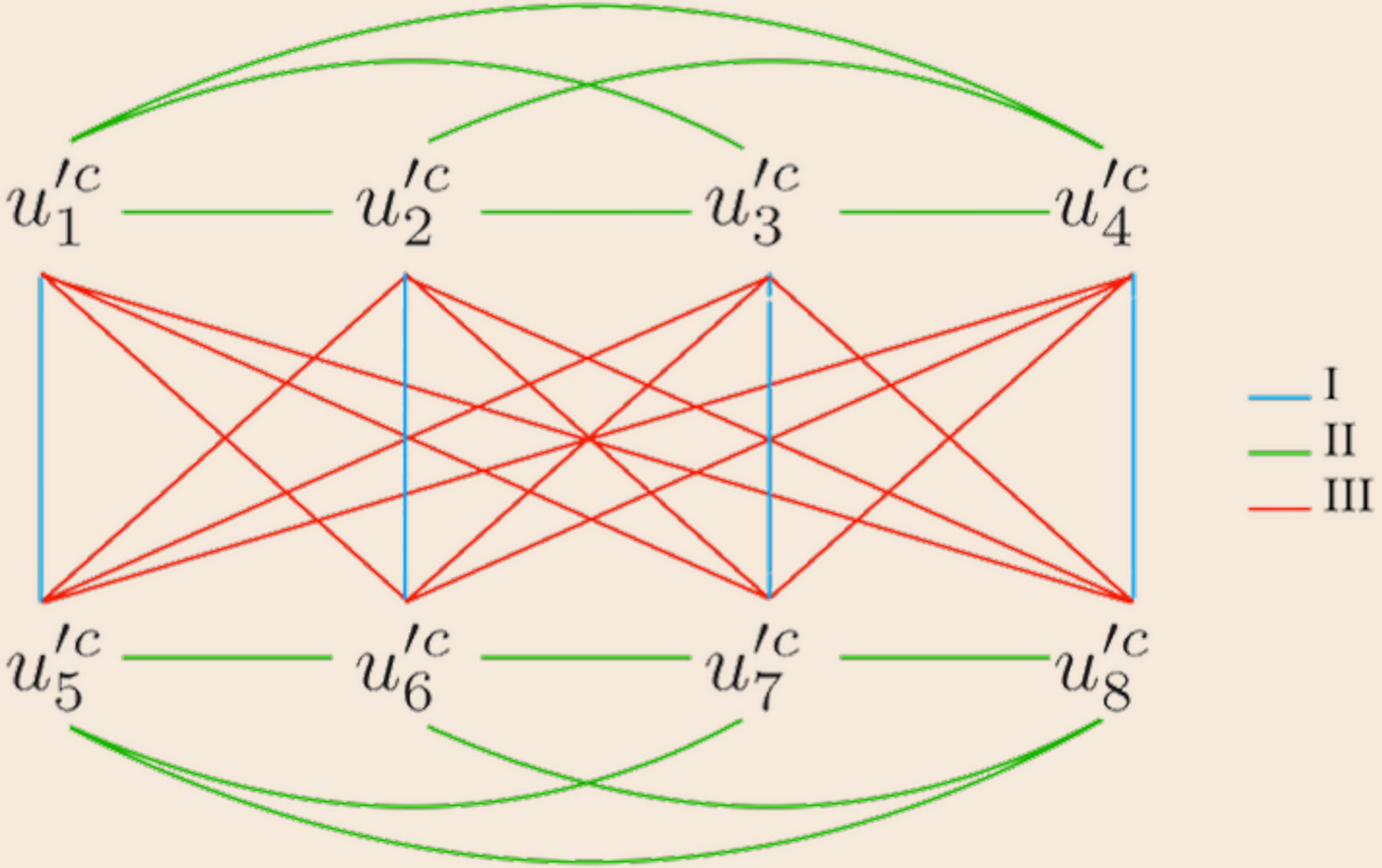
Relation between the vacua

The vacua can be related by A_4 transformations.



Three types of domain walls

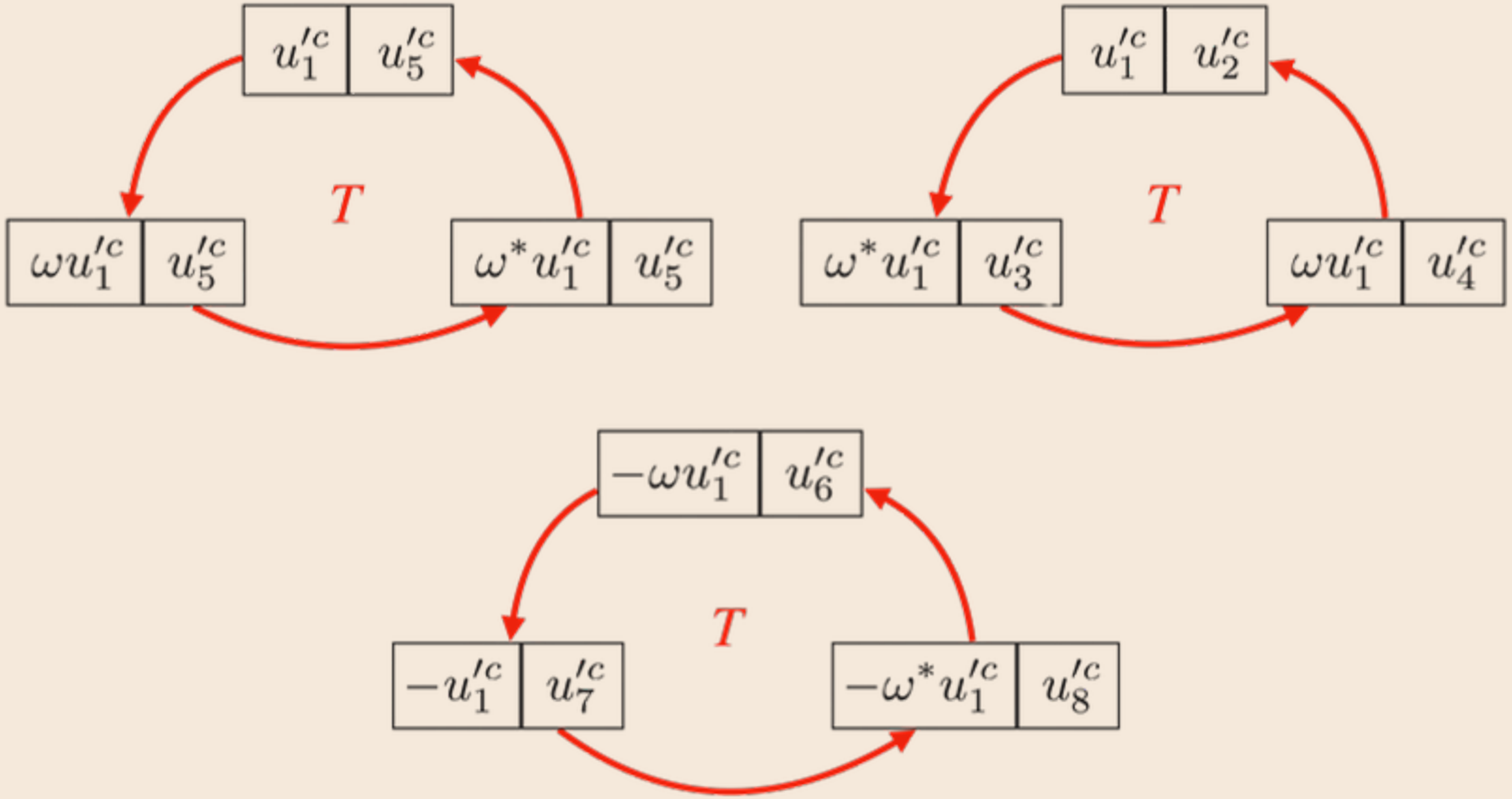
As the vacua are related by A_4 transformations, some of the domain walls are equivalent and have identical properties. As a result, there are only three types of domain walls.



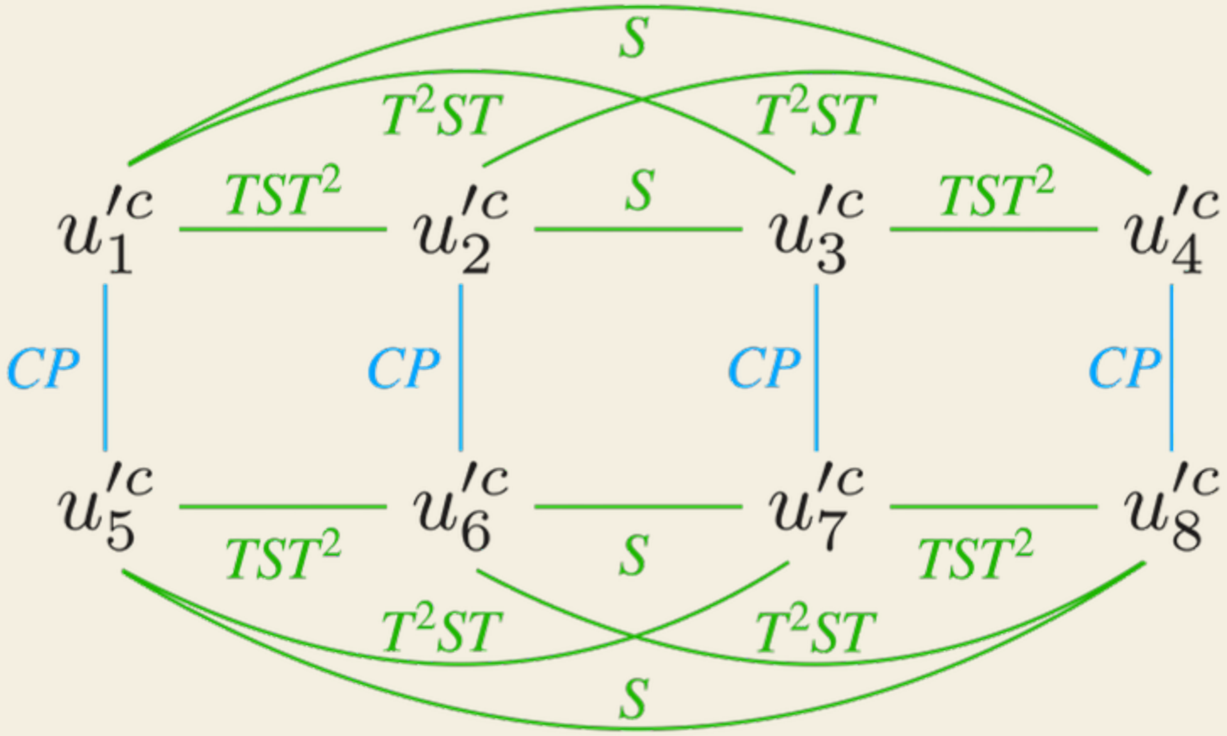
Equavalent Domain Walls

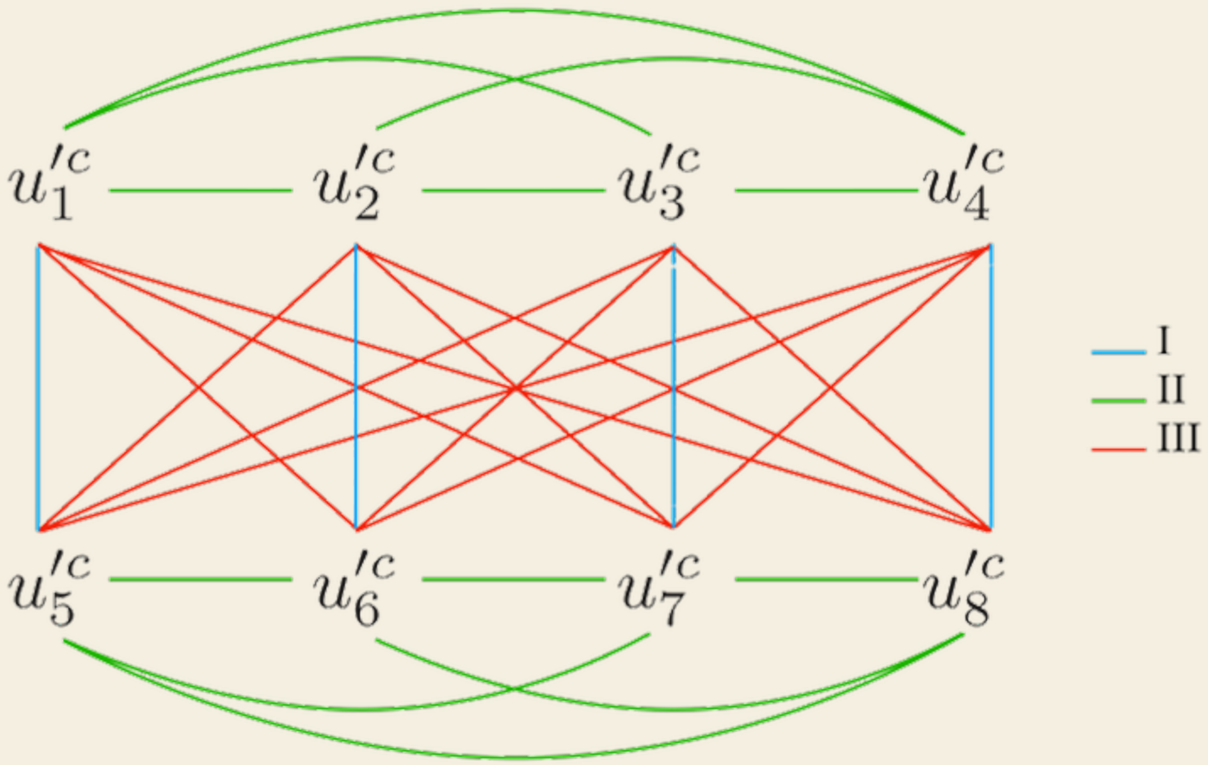
The domain walls of the same type can be related by A_4 transformations.

For example, under T transformation



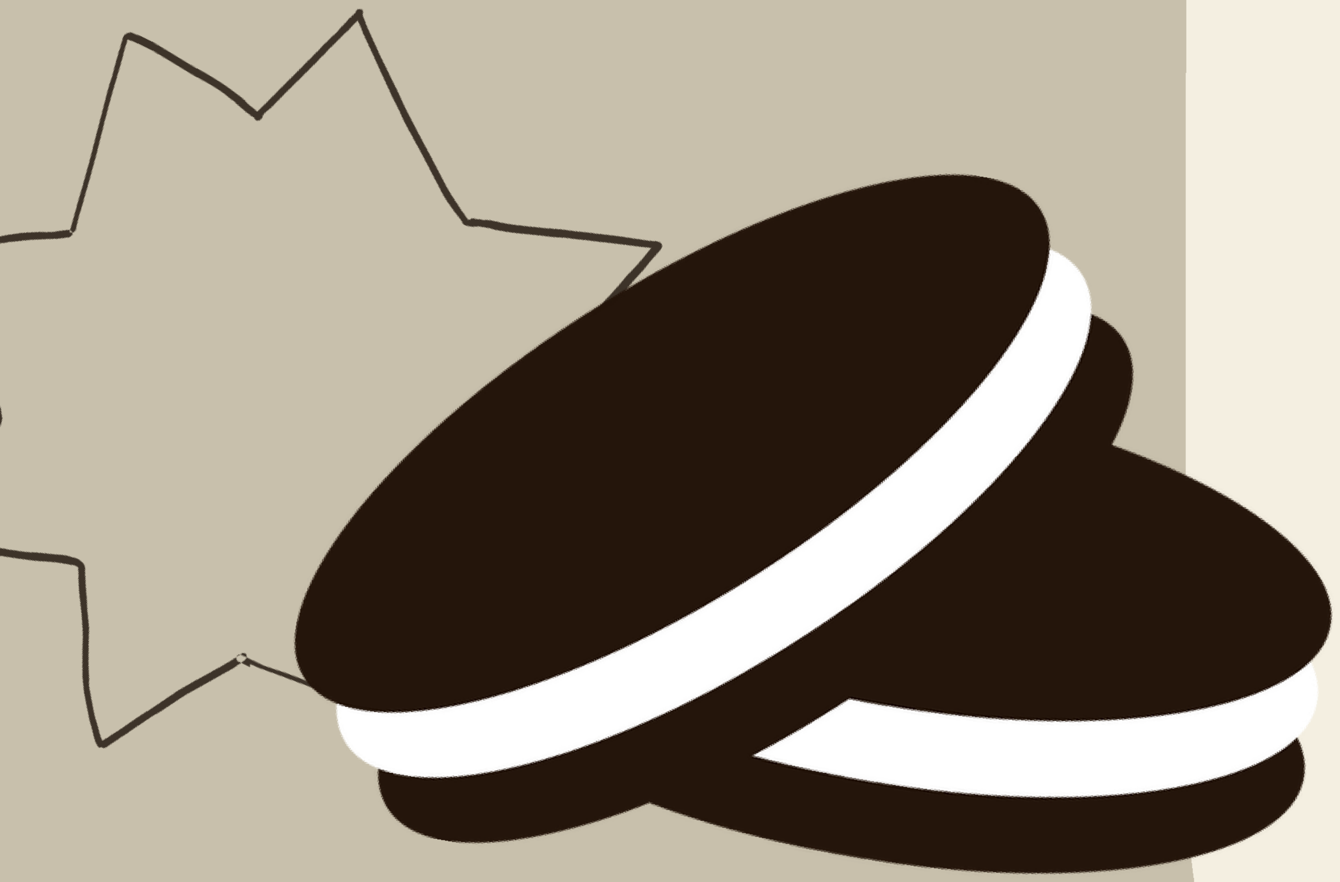
CP-violating domain walls





We refer the walls between two CP-conjugated vacua as the CP-violating domain walls as CP is violated differently on the two sides of the wall.

SUMMARY



MOTIVATED BY FLAVOUR PHYSICS

Flavour physics provide strong motivation to consider spontaneous breaking of non-Abelian symmetries and there consequence in the ealry universe. Domain wall provide us a good phenomena that can be use to test the flavour models.

ABUNDANT VARIETIES

In non-Abelian symmetric models, the domain wall can have non-trivial internal structures, like an “oreo”.

SPONTANEOUS CP VIOLATION

In models with non-Abelian symmetries, the CP symmetry can be spontaneously broken, and the walls between CP conjugate vacua further provide a difference in the CP phase.