河南師範大學
HENAN NORMAL UNIVERSITY

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Model Building with Non-Invertible Selection Rules: Two Examples in Lepton Physics

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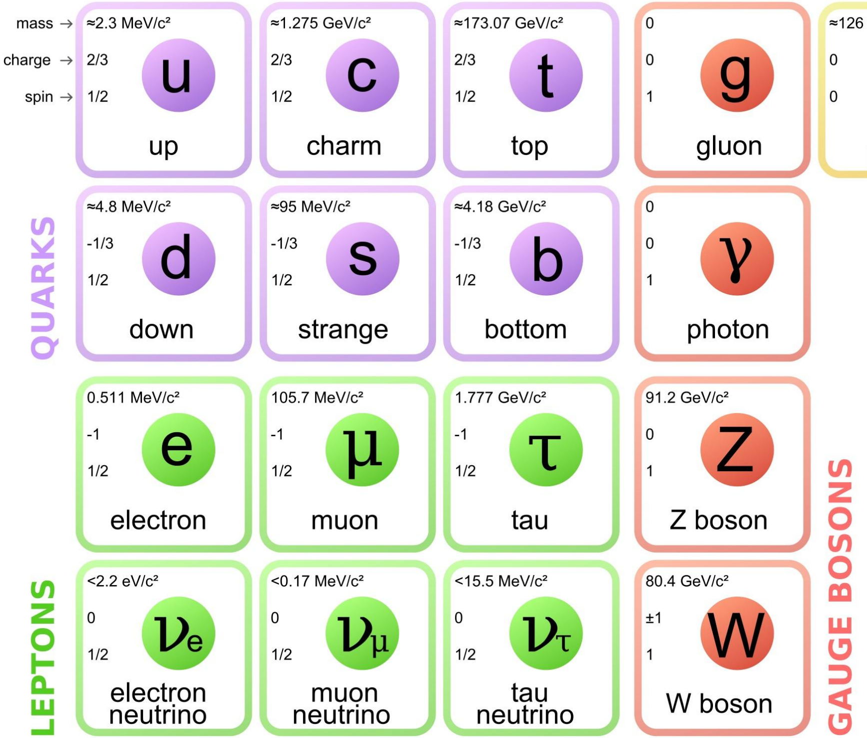
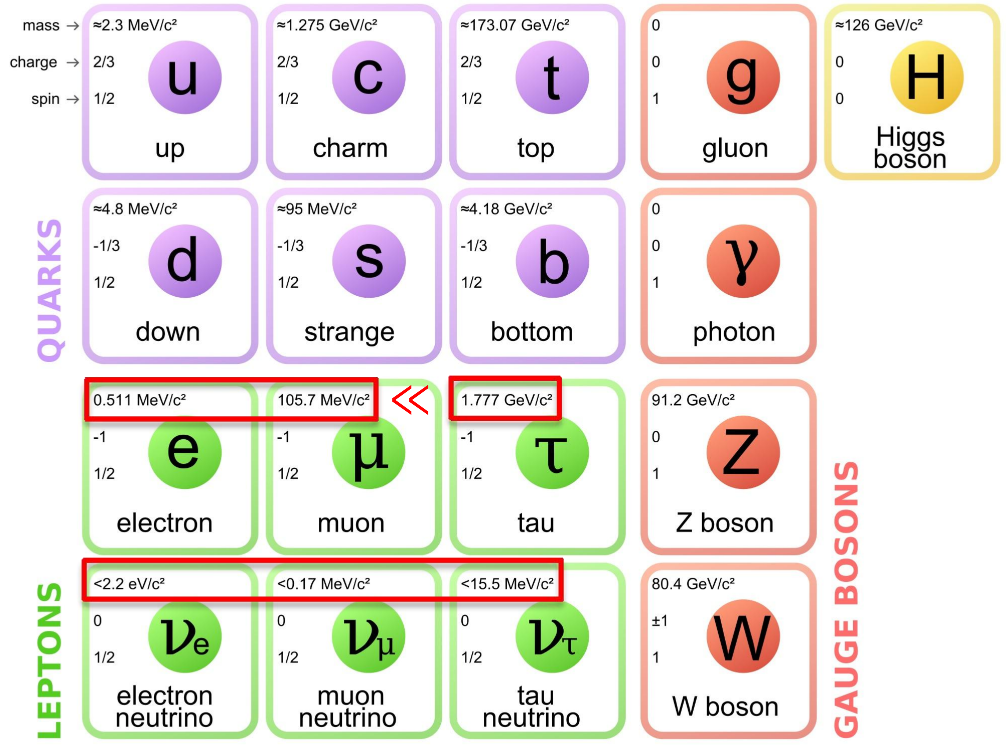
- Hiroshi Okada and Jia-Jun Wu, [arXiv:2603.17587 [hep-ph]].
- Jingqian Chen, Chao-Qiang Geng, Hiroshi Okada, and Jia-Jun Wu, [arXiv:2507.11951[hep-ph]].

The Standard Model and Beyond



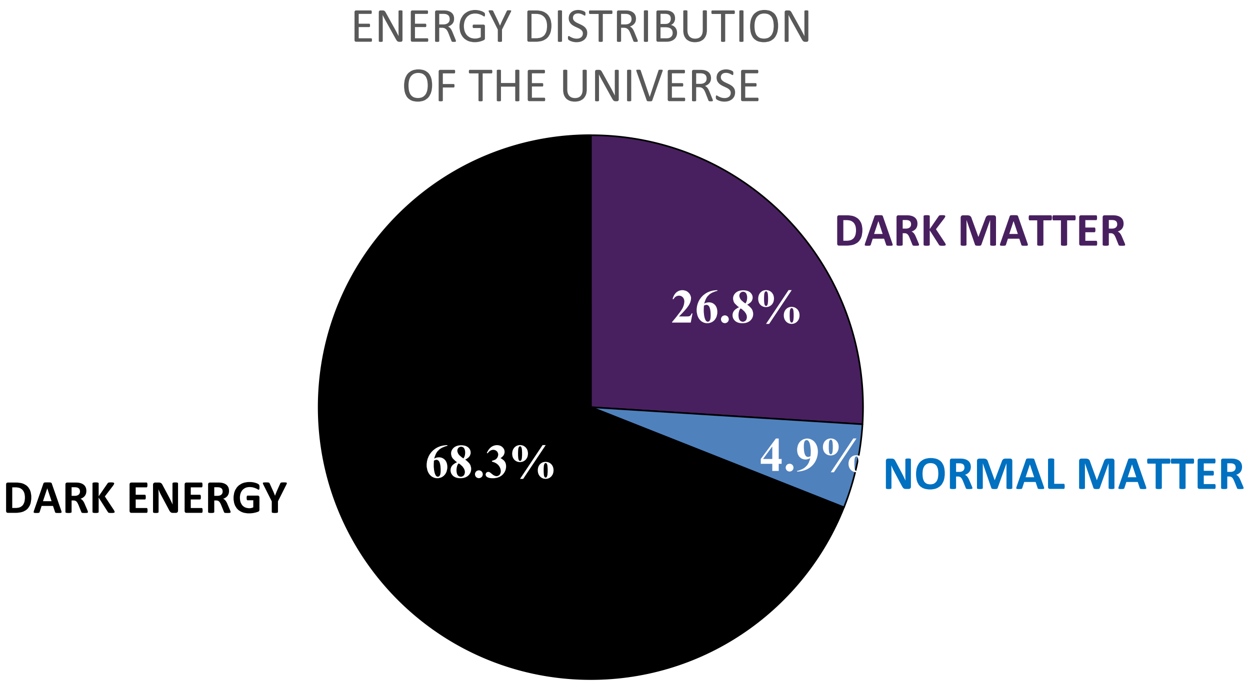
- The Origin of Neutrino Mass
- The Nature of Dark Matter
- Flavor Hierarchies
- Possible Anomalies
- … … …

Lepton Mass： Neutrino Mass & Lepton Mass Hierarchy Problem

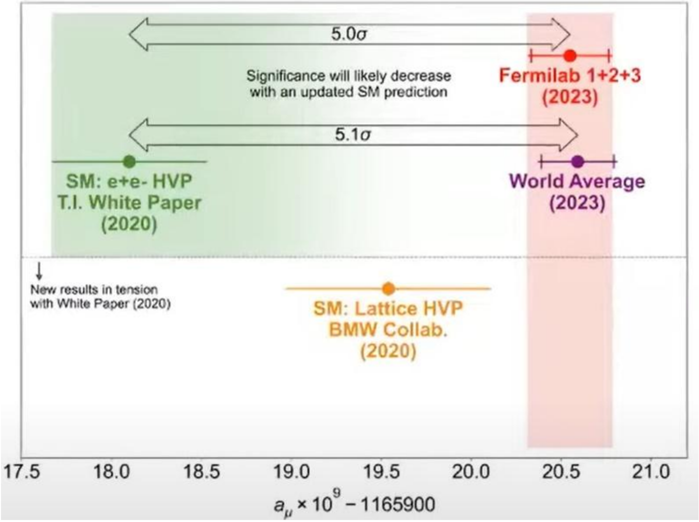


- In the Standard Model, neutrinos are massless, whereas experimental observations have revealed non-zero neutrino masses.
- The τ lepton has a significantly larger mass than other charged leptons.
- Neutrino masses are much smaller than charged-lepton masses

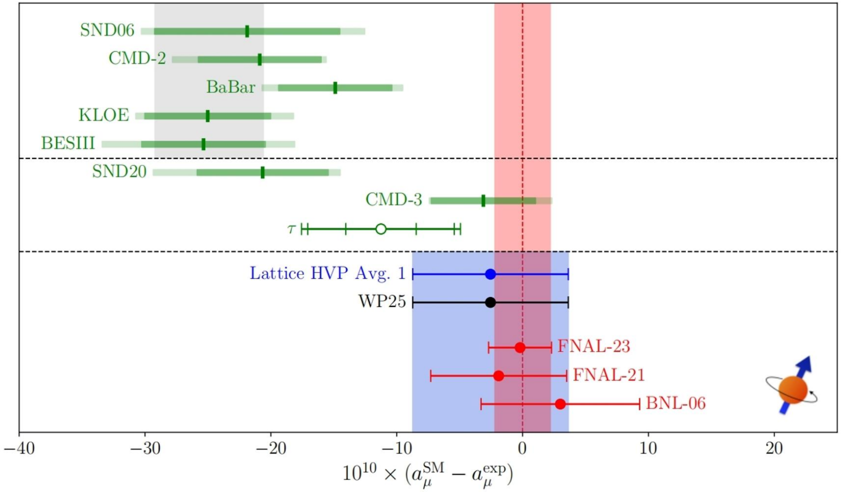
Dark Matter


ENERGY DISTRIBUTION
OF THE UNIVERSE
DARK MATTER
26.8%
4.9%
NORMAL MATTER
68.3%
DARK ENERGY

Muon g-2 Anomaly



August 2023



June 2025

- D.~P.~Aguillard, et al. Measurement of the Positive Muon Anomalous Magnetic Moment to 127 ppb. arXiv:2506.03069 [hep-ex].
- Aliberti, R., et al. "The anomalous magnetic moment of the muon in the Standard Model: an update." arXiv:2505.21476 (2025).

Non-invertible Selection Rules

Physical origin and background:

- S. H. Shao, [arXiv:2308.00747 [hep-th]].
- L. Bhardwaj and Y. Tachikawa, [arXiv:1704.02330 [hep-th]].
- C. M. Chang, Y. H. Lin, S. H. Shao, Y. Wang and X. Yin, [arXiv:1802.04445 [hep-th]].
- J. Kaidi, Y. Tachikawa and H. Y. Zhang, [arXiv:2402.00105 [hep-th]].
- Y. Choi, H. T. Lam and S. H. Shao, [arXiv:2205.05086 [hep-th]].
- … … …

Recent applications to particle physics and phenomenology:

- T. Kobayashi, H. Otsuka, M. Tanimoto and H. Uchida, [arXiv:2505.07262 [hep-ph]].
- M. Suzuki and L. X. Xu, [arXiv:2503.19964 [hep-ph]].
- T. Kobayashi, H. Otsuka and T. T. Yanagida, [arXiv:2508.12287 [hep-ph]].
- H. Okada and Y. Shigekami, [arXiv:2507.16198 [hep-ph]].
- T. Kobayashi, H. Otsuka and M. Tanimoto, [arXiv:2409.05270 [hep-ph]].
- … … …

Non-invertible Selection Rules

A Group： $g_1 g_2 = g_3,$

The product on the right-hand side is a single element, and every element has an inverse.

Non-invertible selection rules: $U_i U_j = \sum_k c_{ij}^k U_k,$

The product on the right-hand side can be a linear combination of several elements, and the elements may not have inverses.

Ising fusion rules (IFR): $\epsilon \otimes \epsilon = \mathbb{I}, \quad \sigma \otimes \sigma = \mathbb{I} \oplus \epsilon, \quad \sigma \otimes \epsilon = \epsilon \otimes \sigma = \sigma,$

$$\{\epsilon, \sigma\} \otimes \mathbb{I} = \mathbb{I} \otimes \{\epsilon, \sigma\} = \{\epsilon, \sigma\}.$$

Fibonacci fusion rules (FFR): $\mathbb{I} \otimes \tau = \tau \otimes \mathbb{I} = \tau, \quad \tau \otimes \tau = \mathbb{I} \oplus \tau.$

σ and τ are elements without an inverse

Model 1

2 Generations (E_R, E_L, N_R)

	$L_{L_{e,\mu}}$	L_{L_τ}	ℓ_R	E_R	E_L	N_R	H	η	S
$SU(2)_L$	$\mathbf{2}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{2}$	$\mathbf{1}$
$U(1)_Y$	$-\frac{1}{2}$	$-\frac{1}{2}$	-1	-1	-1	0	$\frac{1}{2}$	$\frac{1}{2}$	0
IFR	ϵ	$\mathbb{I}$	$\mathbb{I}$	σ	σ	σ	$\mathbb{I}$	σ	σ

$$(\epsilon \otimes \epsilon = \mathbb{I}, \quad \sigma \otimes \sigma = \mathbb{I} \oplus \epsilon, \quad \sigma \otimes \epsilon = \epsilon \otimes \sigma = \sigma)$$

Jingqian Chen, Chao-Qiang Geng, Hiroshi Okada, and Jia-Jun Wu, [arXiv:2507.11951[hep-ph]].

Lagrangian for the Lepton Sector:

$$-\mathcal{L}_\ell = \sum_{\ell=e,\mu} \sum_{a=1,2} y_{\ell a} \overline{L_{L_\ell}} \eta E_{R_a} + \sum_{a=1,2} \sum_{i=1}^{3} y_{E_{ai}} \overline{E_{L_a}} \ell_{R_i} S + \sum_{a=1}^{3} y_{\tau a} \overline{L_{L_\tau}} H \ell_{R_a} + \sum_{a=1,2} h_{\tau a} \overline{L_{L_\tau}} \eta E_{R_a}$$
$$+ \sum_{\ell=e,\mu} \sum_{a=1,2} y_{\eta_{\ell a}} \overline{L_{L_\ell}} \tilde{\eta} N_{R_a} + \sum_{a=1,2} y_{\eta_{\tau a}} \overline{L_{L_\tau}} \tilde{\eta} N_{R_a} + M_{E_a} \overline{E_{L_a}} E_{R_a} + M_{N_a} \overline{N^C_{R_a}} N_{R_a} + \text{h.c.},$$

Model 1

The Scotogenic Model

	$L_{L_{e,\mu}}$	L_{L_τ}	ℓ_R	E_R	E_L	N_R	H	η	S
$SU(2)_L$	**2**	**2**	**1**	**1**	**1**	**1**	**2**	**2**	**1**
$U(1)_Y$	$-\frac{1}{2}$	$-\frac{1}{2}$	-1	-1	-1	0	$\frac{1}{2}$	$\frac{1}{2}$	0
IFR	ϵ	$\mathbb{I}$	$\mathbb{I}$	σ	σ	σ	$\mathbb{I}$	σ	σ

$$(\epsilon\otimes\epsilon=\mathbb{I},\quad \sigma\otimes\sigma=\mathbb{I}\oplus\epsilon,\quad \sigma\otimes\epsilon=\epsilon\otimes\sigma=\sigma)$$

Lagrangian for the Lepton Sector:

$$-\mathcal{L}_\ell=\sum_{\ell=e,\mu}\sum_{a=1,2} y_{\ell a}\overline{L_{L_\ell}}\eta E_{R_a}+\sum_{a=1,2}\sum_{i=1}^{3} y_{E_{ai}}\overline{E_{L_a}}\ell_{R_i}S+\sum_{a=1}^{3} y_{\tau a}\overline{L_{L_\tau}}H\ell_{R_a}+\sum_{a=1,2} y_{\tau a}\overline{L_{L_\tau}}\eta E_{R_a}$$
$$+\sum_{\ell=e,\mu}\sum_{a=1,2} y_{\eta_{\ell a}}\overline{L_{L_\ell}}\tilde{\eta}N_{R_a}+\sum_{a=1,2} y_{\eta_{\tau a}}\overline{L_{L_\tau}}\tilde{\eta}N_{R_a}+M_{E_a}\overline{E_{L_a}}E_{R_a}+M_{N_a}\overline{N^C_{R_a}}N_{R_a}+\text{h.c.},$$

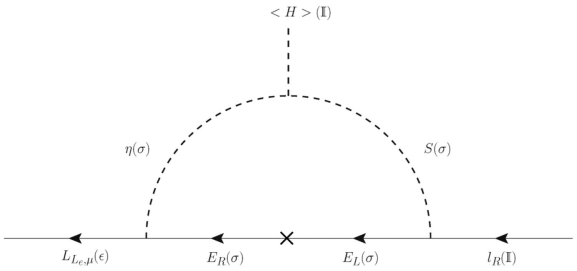
The $\overline{L_{L_{e,\mu}}}\ell_R H$ term is forbidden by the IFR, and therefore no tree-level mass can be generated:

$$\epsilon\otimes\mathbb{I}\otimes\mathbb{I}=\epsilon\neq\mathbb{I}$$

The electron and muon masses are generated at one loop, analogously to the neutrino mass generation mechanism.

Lepton Masses

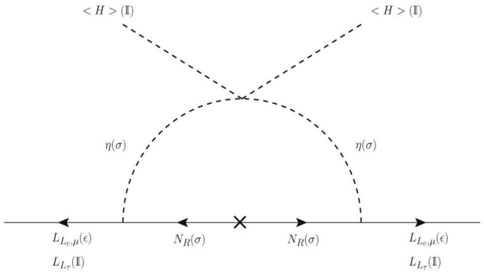
Generation of electron and muon Masses:



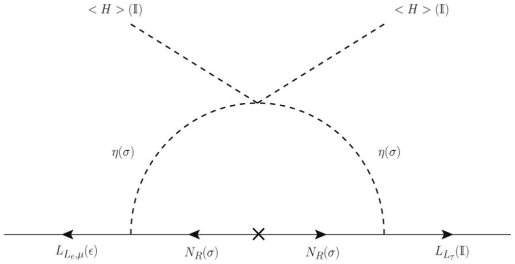
$$\epsilon \otimes \mathbb{I} \otimes \mathbb{I} = \epsilon \neq \mathbb{I}$$

IFR dynamically broken at the loop

Generation of neutrino Masses:



IFR Invariant

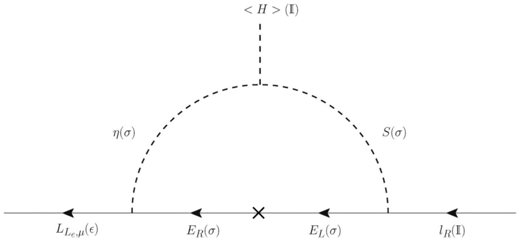


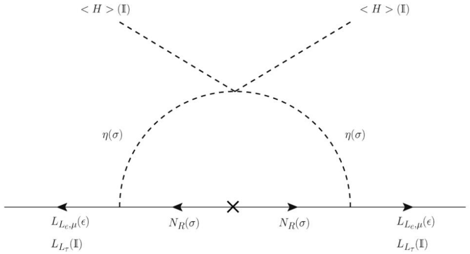
IFR broken

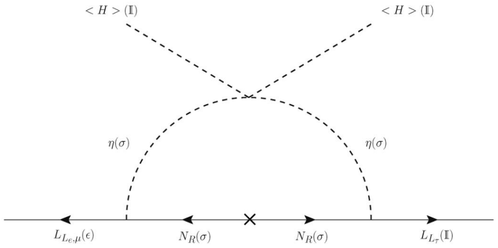
Lepton Masses

Mixing Terms in the Scalar Potential:

$$
\begin{aligned}
V_{\text{mix}} = {} & \lambda_3 (H^\dagger H)(\eta^\dagger \eta) + \lambda_4 (H^\dagger \eta)(\eta^\dagger H) + \frac{\lambda_5}{2}\left[(H^\dagger \eta)^2 + \text{h.c.}\right] \\
& + \lambda_{HS}(H^\dagger H) S^* S + \lambda'_{HS}\left[(H^\dagger H) S^2 + \text{h.c.}\right] \\
& + \lambda_{\eta S}(\eta^\dagger \eta) S^* S + \lambda'_{\eta S}\left[(\eta^\dagger \eta) S^2 + \text{h.c.}\right] + \boxed{\left(\mu H^\dagger \eta S + \text{h.c.}\right)}.
\end{aligned}
$$







$$\sigma \otimes \sigma \sim \mathbb{I}$$

In these loop diagrams, the Ising fusion rule is effectively "groupified" into a Z_2-like symmetry with a conserved σ-parity. As a result, the lightest σ-type particle becomes stable and can serve as a dark matter candidate.

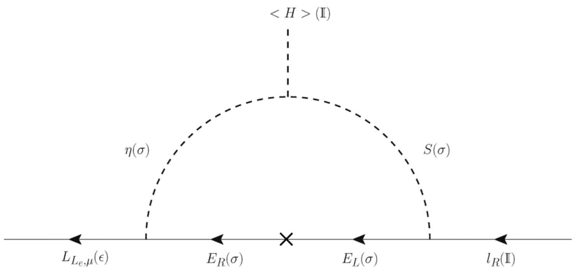
- J. Kaidi, Y. Tachikawa and H. Y. Zhang, [arXiv:2402.00105 [hep-th]].
- M. Suzuki and L. X. Xu, [arXiv:2503.19964 [hep-ph]].

Lepton Masses

Charged Lepton Masses (tree level):

$$\mathcal{M}_\ell^{\text{tree}} = \frac{v_H}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ y_{\tau 1} & y_{\tau 2} & y_{\tau 3} \end{pmatrix},$$

Charged Lepton Masses (one-loop level):



$$\left(\mu H^\dagger \eta S + \text{h.c.}\right)$$

$$\eta \equiv [\eta^+, (\eta_R + i\eta_I)/\sqrt{2}]^T$$

$$S \equiv (S_R + iS_I)/\sqrt{2}$$

⇨

$$\eta_R = c_\theta H_1 + s_\theta H_2, \quad \eta_I = c_\theta A_1 + s_\theta A_2,$$

$$S_R = -s_\theta H_1 + c_\theta H_2, \quad S_I = -s_\theta A_1 + c_\theta A_2,$$

⇩

$$\sum_{\ell=e,\mu} \sum_{a=1,2} \frac{y_{\ell a}}{\sqrt{2}} \overline{\ell'_{L_\ell}} E_{R_a} (c_\theta H_1 + s_\theta H_2) + i \sum_{\ell=e,\mu} \sum_{a=1,2} \frac{y_{\ell a}}{\sqrt{2}} \overline{\ell'_{L_\ell}} E_{R_a} (c_\theta A_1 + s_\theta A_2)$$
$$+ \sum_{a=1,2} \sum_{i=1}^{3} \frac{y_{E_{ai}}}{\sqrt{2}} \overline{E_{L_a}} \ell_{R_i} (-s_\theta H_1 + c_\theta H_2) + i \sum_{a=1,2} \sum_{i=1}^{3} \frac{y_{E_{ai}}}{\sqrt{2}} \overline{E_{L_a}} \ell_{R_i} (-s_\theta A_1 + c_\theta A_2) + \text{h.c.},$$

Lepton Masses

Charged Lepton Masses (one-loop level):

$$(\delta m)_{\ell i} = \sum_{a=1,2} \frac{y_{\ell a} M_{E_a} y_{E_{ai}}}{2(4\pi)^2} s_\theta c_\theta \left[F_I(H_1, H_2, E_a) - F_I(A_1, A_2, E_a)\right], \quad \text{其中，} F_I(b_1, b_2, f) = \left[\frac{m_{b_1}^2}{m_f^2 - m_{b_1}^2} \ln\left(\frac{m_{b_1}^2}{m_f^2}\right) - \frac{m_{b_2}^2}{m_f^2 - m_{b_2}^2} \ln\left(\frac{m_{b_2}^2}{m_f^2}\right)\right],$$

Complete charged-lepton mass matrix:

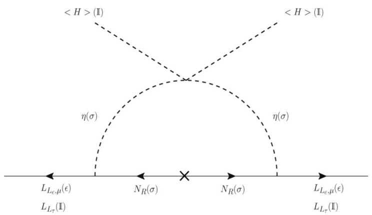
$$\mathcal{M}_\ell \equiv \mathcal{M}_\ell^{\text{tree}} + \delta m \quad \Rightarrow \quad \frac{v_H}{\sqrt{2}} \begin{pmatrix} \sqrt{2}\frac{\delta m_{e1}}{v_H} & \sqrt{2}\frac{\delta m_{e2}}{v_H} & \sqrt{2}\frac{\delta m_{e3}}{v_H} \\ \sqrt{2}\frac{\delta m_{\mu 1}}{v_H} & \sqrt{2}\frac{\delta m_{\mu 2}}{v_H} & \sqrt{2}\frac{\delta m_{\mu 3}}{v_H} \\ y_{\tau 1} & y_{\tau 2} & y_{\tau 3} \end{pmatrix}$$

$$\Downarrow$$

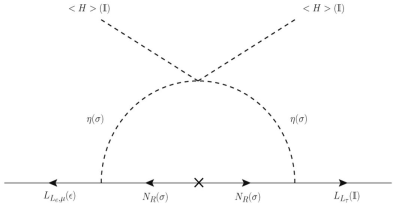
$$\mathcal{M}_\ell \mathcal{M}_\ell^\dagger \sim \begin{pmatrix} (\delta m \delta m^\dagger)_{2\times 2} & (\delta m \mathcal{M}_\ell^{\text{tree},\dagger})_{2\times 1} \\ (\mathcal{M}_\ell^{\text{tree}} \delta m^\dagger)_{1\times 2} & (\mathcal{M}_\ell^{\text{tree}} \mathcal{M}_\ell^{\text{tree},\dagger})_{1\times 1} \end{pmatrix} \quad \Rightarrow \quad m_e, m_\mu \ll m_\tau$$

Lepton Masses

Neutrino Masses:



IFR Invariant



IFR Broken

$$(m_\nu)_{ij} = \sum_{a=1}^{2} \frac{(y_\eta)_{ia} M_{N_a} (y_\eta^T)_{aj}}{2(4\pi)^2} \left[c_\theta^2 F_I(H_1, A_1, N_a) + s_\theta^2 F_I(H_2, A_2, N_a) \right] \equiv \sum_{a=1}^{2} (y_\eta)_{ia} D_{N_a} (y_\eta^T)_{aj}.$$

Casas–Ibarra parameterization： $y_\eta = U_\nu^* \sqrt{D_\nu} O_N \sqrt{D_N^{-1}}$

$$\text{NH}: \; O_N = \begin{bmatrix} 0 & 0 \\ \cos z & -\sin z \\ \sin z & \cos z \end{bmatrix}, \quad \text{IH}: \; O_N = \begin{bmatrix} \cos z & -\sin z \\ \sin z & \cos z \\ 0 & 0 \end{bmatrix}$$

$(\text{NH}): \sum D_\nu \sim \sqrt{\Delta m_{\text{atm}}^2} \sim \mathcal{O}(50)$ meV $\quad \Delta m_{\text{atm}}^2 \equiv |D_{\nu_3}^2 - D_{\nu_1}^2|$

$(\text{IH}): \sum D_\nu \sim 2\sqrt{\Delta m_{\text{atm}}^2} \sim \mathcal{O}(100)$ meV $\quad \Delta m_{\text{atm}}^2 \equiv |D_{\nu_2}^2 - D_{\nu_3}^2|$

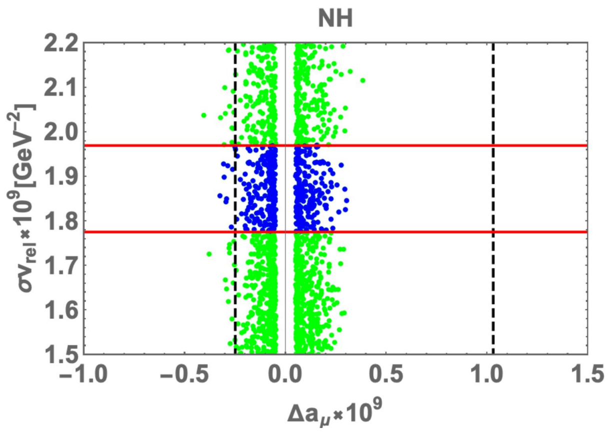
Cosmological： $\sum D_\nu \leq 120$ meV

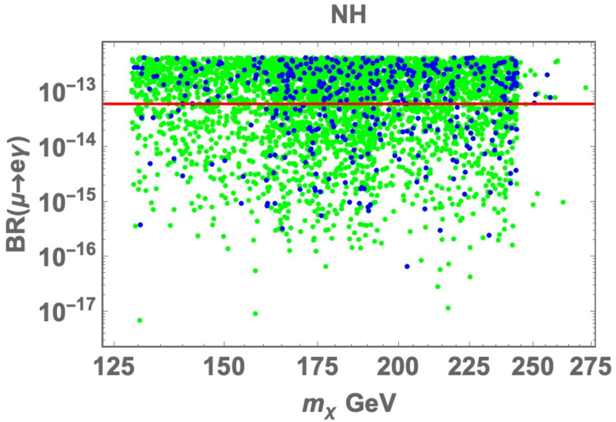
DESI + CMB： $\sum D_\nu \leq 72$ meV

Numerical Results

Taking into account the LFV constraints and requiring the muon g-2 deviation to be around 1σ, we take H_2 as the dark matter candidate:

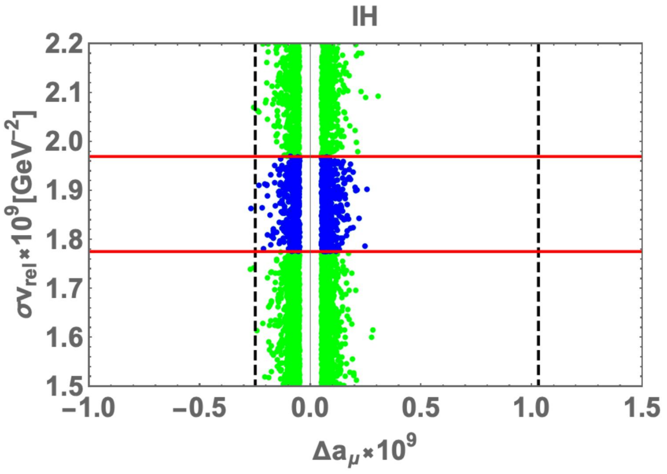
The NH case：

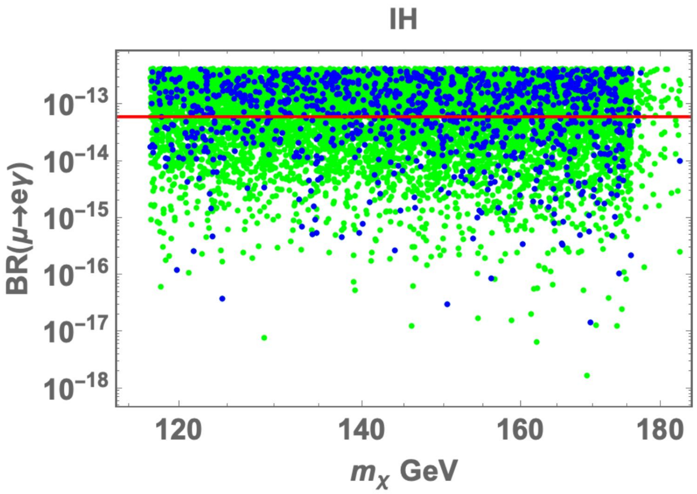




Numerical Results

The IH case：





Model 2

3 Generations of $SU(2)_L$ quintet fermions $\Sigma_R \equiv \left[\Sigma_1^{++}, \Sigma_1^{+}, \Sigma^0, \Sigma_2^{-}, \Sigma_2^{--}\right]_R^T$

	Lepton Fields			Scalar Fields	
	L_L	e_R	Σ_R	H	ϕ_4
$SU(2)_L$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{5}$	$\mathbf{2}$	$\mathbf{4}$
$U(1)_Y$	$-\frac{1}{2}$	-1	0	$\frac{1}{2}$	$\frac{1}{2}$
FFR	τ	τ	τ	$\mathbb{I}$	τ

$(\, \mathbb{I} \otimes \tau = \tau \otimes \mathbb{I} = \tau, \quad \tau \otimes \tau = \mathbb{I} \oplus \tau \,)$

Hiroshi Okada and Jia-Jun Wu, [arXiv:2603.17587 [hep-ph]].

One $SU(2)_L$ quartet scalar
$\phi_4 = \left[\varphi^{++}, \varphi_2^{+}, \varphi^0, \varphi_1^{-}\right]^T$

Relevant Lagrangian for the Lepton Sector:

$$-\mathcal{L}_Y = (y_\ell)_{ii}\overline{L_{L_i}} H e_{R_i} + (y_\nu)_{ij}[\overline{L_{L_i}}(i\tau_2)\phi_4^* \Sigma_{R_j}] + (M_R)_i[\overline{\Sigma_{R_i}^C}\Sigma_{R_i}] + \text{h.c.}$$

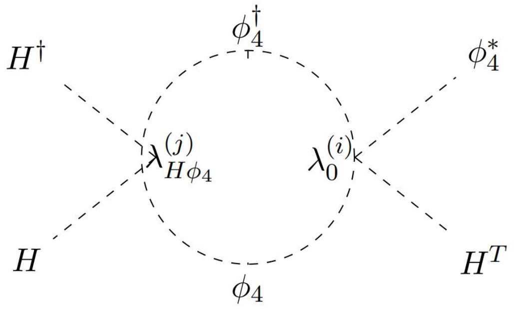
Inside the brackets, the $SU(2)_L$ indices are contracted to form a singlet.

Higgs potential: $\mathcal{V} = \mathcal{V}_2 + \mathcal{V}_4$

$$\mathcal{V}_2 = -\mu_H^2 H^\dagger H + \mu_4^2 \phi_4^\dagger \phi_4,$$

$$\mathcal{V}_4 = \lambda_0^{(i)} [\phi_4^\dagger H \phi_4^\dagger \phi_4]_i + \lambda_{H\phi_4}^{(i)} [H^\dagger H \phi_4^\dagger \phi_4]_i + \text{c.c.}$$
$$+ \lambda_H |H|^4 + \lambda_{\phi_4}^{(i)} [\phi_4^\dagger \phi_4 \phi_4^\dagger \phi_4]_i$$

$H^\dagger \phi_4^* H^T H$ → This term, which would induce the ϕ_4 VEV at tree level, is forbidden by FFR.



$\delta\lambda_{ij} [H^\dagger \phi_4^* H^T H]_{ij}$ term is generated at one-loop level through $\lambda_0^{(i)} [\phi_4^\dagger H \phi_4^\dagger \phi_4]_i$ and $\lambda_{H\phi_4}^{(i)} [H^\dagger H \phi_4^\dagger \phi_4]_i$

Dynamically violates FFR in the loop

$$\mathbb{I} \otimes \tau = \tau \otimes \mathbb{I} = \tau, \quad \tau \otimes \tau = \mathbb{I} \oplus \tau$$

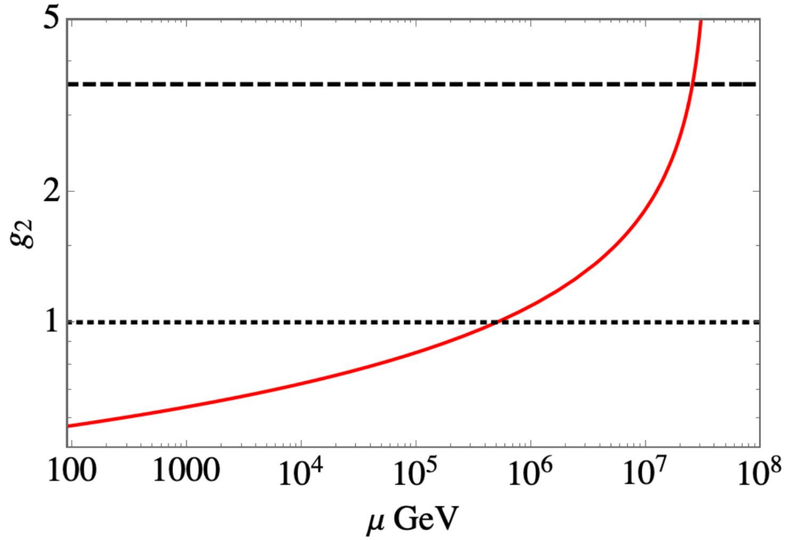
The quartet VEV is not put in by hand; It is induced radiatively.

The same basic logic: tree-level forbidden, loop-level generated.

Dynamical determination of the cutoff scale

In generic loop-induced VEV models, the cutoff scale is usually introduced by hand.
Here, we instead determine it in a more dynamical and physically motivated way.

Running of g_2 and the EFT cutoff



Higher $SU(2)_L$ multiplets drive g_2 upward in the UV.

We estimate the EFT cutoff using two benchmark criteria:

$g_2 = 1 \rightarrow \Lambda \simeq 5.05 \times 10^5$ GeV

$g_2 = \sqrt{4\pi} \rightarrow \Lambda \simeq 2.56 \times 10^7$ GeV

Assuming small scalar quartics and moderate y_ν, the gauge sector sets the lowest cutoff scale.

Neutrino masses and typical scales

$$m_\nu \sim v_4^2\, y_\nu\, M_R^{-1}\, y_\nu^T$$

- For $\Lambda \sim 10^5$–10^7 GeV, we obtain $v_4 \sim 0.07$–0.1 GeV.
- Then the neutrino Yukawa couplings can be of order 10^{-3} for $M_R \sim 1$ TeV.
- Compared with conventional seesaw setups, this may offer a somewhat more natural parameter choice.

The tiny neutrino scale comes from a loop-induced small v_4, rather than ultra-tiny Yukawa couplings.

Summary:

- Non-invertible selection rules provide a new way to control allowed couplings beyond ordinary group-based symmetries.

- The first example is an Ising-fusion-rule radiative lepton model, where the electron and muon masses are generated at one loop and neutrino masses also arise radiatively.

- The second example is a Fibonacci-fusion-rule neutrino-mass model, where a loop-induced small VEV is generated and the EFT cutoff is fixed dynamically by the running of $SU(2)_L$ gauge coupling.

- Overall, these results show that non-invertible selection rules can serve as useful tools in particle-physics model building, and that non-invertibility itself can provide new structures beyond conventional symmetry-based constructions.

THANKS!

$$\delta\lambda_{ij} \approx -\frac{\lambda_0^{(i)}\lambda_{H\phi_4}^{(j)}}{(4\pi)^2}\left[\frac{2-2r+(1+r)\ln(r)}{1-r}\right], \qquad r \equiv m_H^2/\Lambda^2$$

$$\begin{aligned} v_4 &\sim v_H^3 \sum_{i,j=1}^{2} \frac{\delta\lambda_{ij}}{2\mu_4^2+(\lambda_{H\phi_4}^{(1)}+\lambda_{H\phi_4}^{(2)})v_H^2} \\ &= -\frac{v_H^3}{(4\pi)^2}\left[\frac{2-2r+(1+r)\ln(r)}{1-r}\right]\sum_{i,j=1}^{2}\frac{\lambda_0^{(i)}\lambda_{H\phi_4}^{(j)}}{2\mu_4^2+(\lambda_{H\phi_4}^{(1)}+\lambda_{H\phi_4}^{(2)})v_H^2} \\ &\simeq -\frac{v_H}{2(4\pi)^2}\frac{v_H^2}{\mu_4^2}\left[\frac{2-2r+(1+r)\ln(r)}{1-r}\right]\sum_{i,j=1}^{2}\lambda_0^{(i)}\lambda_{H\phi_4}^{(j)}, \end{aligned}$$

- once $\delta\lambda_{ij}$ is generated, it induces a suppressed v_4.
- the dependence on the cutoff is weak.

Dynamical determination of the cutoff scale

In generic loop-induced VEV models, the cutoff scale is usually introduced by hand; here we aim to determine it in a less arbitrary, more physically motivated way.

$$\frac{1}{g_2^2(\mu)} = \frac{1}{g_2^2(m_{in.})} - \frac{b_g^{SM}}{(4\pi)^2} \ln\left[\frac{\mu^2}{m_{in.}^2}\right] - \theta(\mu - m_{th.})\frac{\Delta b_g^{\Sigma_R}}{(4\pi)^2} \ln\left[\frac{\mu^2}{m_{th.}^2}\right] - \theta(\mu - m_{th.})\frac{\Delta b_g^{\Phi_4}}{(4\pi)^2} \ln\left[\frac{\mu^2}{m_{th.}^2}\right],$$

$$\Delta b_g^{\Sigma_R} = 3 \times \frac{20}{3}, \quad \Delta b_g^{\Phi_4} = \frac{5}{3}.$$

- large SU(2)L multiplets make g_2 grow in the UV;
- we identify the strong-coupling scale with the physical cutoff Λ.