UV behavior and symmetry breaking of 5D asymptotic GUTs

Roman Pasechnik

Lund University, Sweden

Based on

Phys. Rev. D **111** (2025) 095013, `arXiv:2409.16137`

Eur. Phys. J. C **85** (2025) 496, `arXiv:2501.13118`

`arXiv:2601.00453`

GUTPC 2026, Hangzhou

Physics context

- Asymptotic grand unification replaces conventional fixed-scale unification by a common **UV-attractive flow** of gauge couplings.
- Five-dimensional orbifold gauge theories provide a natural arena for such constructions:
  - gauge symmetry breaking by boundary conditions,
  - chiral zero modes in the 4D effective theory,
  - possible ultraviolet fixed points above the compactification scale.
- However, viability is highly nontrivial:
  - the orbifold vacuum can be destabilized by gauge-scalar dynamics,
  - minimal symmetry-breaking patterns are sharply constrained,
  - UV fixed points for gauge, Yukawa and scalar couplings are not automatic.

Aim of this talk

Establish a coherent criterion for viable 5D aGUT model building:

orbifold GUT $\Longrightarrow$ stable vacuum

$\Longrightarrow$ UV consistency

$\Longrightarrow$ viable 4D theory.

Central question

Which 5D orbifold GUTs are simultaneously

stable and UV consistent ?

Background: A. Hebecker and J. March-Russell, Nucl. Phys. B625 (2002) 128; L.J. Hall, H. Murayama and Y. Nomura, Nucl. Phys. B645 (2002) 85.

Roadmap

Structure of the talk

1. **5D orbifold setup and vacuum stability**
   - orbifold breaking on $S^1/(\mathbb{Z}_2 \times \mathbb{Z}_2')$
   - gauge-scalars from A_5 and Wilson-line dynamics
2. **Classification of stable symmetry breaking**
   - ordinary groups: $SU(N)$, $Sp(2N)$, $SO(N)$
   - stable vs unstable orbifold vacua
3. **Minimal viable aGUT candidates**
   - unique $SU(6)$ route to the SM
   - $SU(8)$ with intermediate Pati–Salam stage
4. **Exceptional groups**
   - stable orbifolds do not yield a minimal exceptional aGUT
5. **UV behavior of 5D gauge-Yukawa theories**
   - one-loop renormalization
   - fixed points as a consistency criterion

Take-home message

A viable 5D aGUT must satisfy *all* of the following:

stable orbifold

acceptable zero modes

UV fixed points

Guiding logic

classification $\Longrightarrow$ selection

selection $\Longrightarrow$ viable models

viable models $\Longrightarrow$ UV consistency test

5D orbifold setup

Minimal framework

A 5D gauge theory compactified on

$$S^1/(\mathbb{Z}_2 \times \mathbb{Z}_2'),$$

with two parities acting at the boundaries of the interval.

Parity action on gauge fields

For $k = 1, 2$, the orbifold parities act as

$$A_\mu(r_k(x^5)) = P_k\, A_\mu(x^5)\, P_k, \quad A_5(r_k(x^5)) = -\, P_k\, A_5(x^5)\, P_k\,.$$

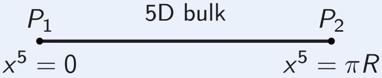
P_1 and P_2 break the bulk gauge group G to

$$G \;\to\; H_1 \cap H_2 \equiv H\,.$$

Key observation

The parity assignments for A_μ and A_5 are opposite. Therefore, components that are odd for A_μ can generate **massless gauge-scalar zero modes** from A_5.

Orbifold picture



Zero modes

$(+,+)$ for A_μ $\Longrightarrow$ 4D GB zero modes

$(-,-)$ for A_μ $\Longrightarrow$ $(+,+)$ for A_5

$\Longrightarrow$ gauge-scalar zero modes

Physical role

These A_5 zero modes may acquire a VEV and further rearrange the orbifold vacuum.

Orbifold GUT setup: A. Hebecker, J. March-Russell, NPB625 (2002) 128.

Why an orbifold can be unstable

Gauge-scalar dynamics

If the orbifold projection leaves zero modes in the extra-dimensional gauge field A_5, they behave as **massless gauge-scalars** in the 4D effective theory. Their potential is generated radiatively at one loop.

Hosotani / Wilson-line mechanism

$\langle A_5 \rangle \neq 0$ is equivalent to a non-trivial Wilson line along the compact dimension (L.J. Hall et al, NPB645 (2002) 85.)

$$W = \mathcal{P} \exp\left(i \oint dx^5 \, \langle A_5 \rangle \right) .$$

At special discrete values, this vacuum can be mapped by a large gauge transformation to a *different orbifold parity assignment*.

Stability criterion

An orbifold is **stable** only if the effective potential is minimized at the origin of the gauge-scalar field space. If the global minimum occurs at a maximal Wilson-line VEV, the theory dynamically prefers a **different orbifold vacuum**.

Vacuum selection

Starting orbifold (P_1, P_2)

↓

Gauge-scalar zero mode $\langle A_5 \rangle \neq 0$

↓

Equivalent orbifold (P'_1, P'_2)

Interpretation

local min at $\langle A_5 \rangle = 0 \Rightarrow$ (meta)stable

global min at max VEV $\Rightarrow$ not preferred!

Main lesson

Orbifold symmetry breaking must be checked *dynamically*.

One-loop potential and stability criterion

Universal structure of the gauge-scalar potential

For a background gauge-scalar VEV, the KK spectrum is shifted as

$$m_n \sim \frac{n+a}{R} \qquad \text{or} \qquad m_n \sim \frac{n+\frac{1}{2}+a}{R},$$

so that the one-loop effective potential is built from two functions:

$$F_+(a) = \frac{3}{2}\sum_{n=1}^{\infty}\frac{\cos(2\pi na)}{n^5}, \qquad F_-(a) = \frac{3}{2}\sum_{n=1}^{\infty}(-1)^n\frac{\cos(2\pi na)}{n^5}.$$

Origin of the two contributions

$$(\pm,\pm)\ \text{parity sectors} \implies F_+(a)$$
$$(\pm,\mp)\ \text{parity sectors} \implies F_-(a)$$

The full potential is a representation-dependent sum of such terms over the bulk field content.

Minima pattern

$$-F_+(a)\ \text{is minimized at}\ a=0$$
$$-F_-(a)\ \text{is minimized at}\ a=\frac{1}{2}$$

Physical meaning

$$(\pm,\pm)\ \text{modes} \Rightarrow \text{stabilize}$$
$$(\pm,\mp)\ \text{modes} \Rightarrow \text{destabilize}$$

Bottom line

$$\boxed{\text{stable orbifold} \iff V_{\rm eff}^{\rm min}\ \text{is at}\ a=0}$$

$$a=0 \iff \text{original orbifold}$$

$$a=\frac{1}{2} \iff \text{shifted orbifold}$$

Classification of stable orbifolds

General pattern

The one-loop gauge contribution to the gauge-scalar potential selects only a **restricted subset** of orbifold breakings as dynamically stable.

Key lessons

- Stability already provides a strong **model-selection principle**.
- For $SU(N)$:
  - two-block orbifolds are always stable,
  - three-block orbifolds are stable only if $p \geq N/2$,
  - four-block orbifolds are always unstable.
- Analogously for $Sp(2N)$, $SO(2N)$.

Stable symmetry-breaking patterns

Model	**Breaking pattern**	**Stability criteria**
$\mathrm{SU}(N)$	$\mathrm{SU}(N) \to \mathrm{SU}(A) \times \mathrm{SU}(N-A) \times \mathrm{U}(1)$	stable $\forall$ A
	$\mathrm{SU}(N) \to \mathrm{SU}(p) \times \mathrm{SU}(q) \times \mathrm{SU}(s) \times \mathrm{U}(1)^2$	$p \geq N/2$
$\mathrm{Sp}(2N)$	$\mathrm{Sp}(2N) \to \mathrm{Sp}(2A) \times \mathrm{Sp}(2(N-A))$	stable $\forall$ A
	$\mathrm{Sp}(2N) \to \mathrm{Sp}(2p) \times \mathrm{Sp}(2q) \times \mathrm{Sp}(2s)$	$p \geq N/2$
	$\mathrm{Sp}(2N) \to \mathrm{SU}(p) \times \mathrm{SU}(q) \times \mathrm{U}(1)^2$	stable $\forall p, q$
$\mathrm{SO}(2N)$	$\mathrm{SO}(2N) \to \mathrm{SO}(2A) \times \mathrm{SO}(2(N-A))$	stable $\forall$ A
	$\mathrm{SO}(2N) \to \mathrm{SO}(2p) \times \mathrm{SO}(2q) \times \mathrm{SO}(2s)$	$p \geq N/2$
	$\mathrm{SO}(2N) \to \mathrm{SU}(p) \times \mathrm{SU}(q) \times \mathrm{U}(1)^2$	stable $\forall p, q$

Interpretation

The bulk gauge group can be broken to at most **three non-Abelian subgroups** in a stable minimal orbifold vacuum.

Stability criterion in practice: non-physical $SU(6)$ illustration

Gauge-sector illustrative representative

An attractive but unphysical orbifold:

$$SU(6) \to SU(3)_c \times SU(2)_L \times U(1)^2, \qquad \phi_{A_5} = (1,2)_{1/2,\,3}$$

with three-block structure $(p,q,s) = (2,3,1)$. The upper panel: the one-loop potential shifts the vacuum away from the origin.

Gauge-transformed representative

The bottom panel: gauge-transformed representative

$$SU(6) \to SU(4) \times U(1) \times U(1), \qquad \phi_{A_5} = 4_{5,\,1}$$

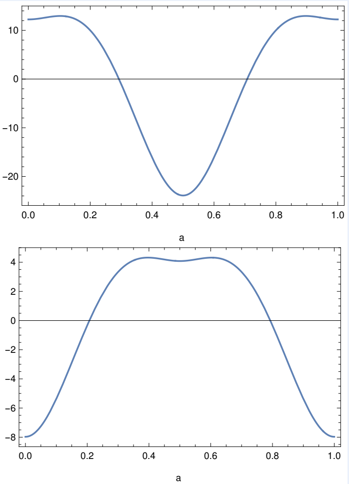
which, in the canonical ordering, belongs to the stable class, related to the upper panel by a large gauge transformation.

Physical message

The algebraically appealing orbifold is not necessarily the true vacuum:

large gauge transformation $\Longleftrightarrow$ different stable parity realization.

Gauge contribution to the effective potential



Physically viable scenario: minimal $SU(6)$ aGUT

Stable orbifold

The minimal viable SM-oriented model is based on the stable three-block pattern

$$SU(6) \to SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)_Z$$

with $(p, q, s) = (3, 2, 1)$. A convenient parity choice is

$$P_1 = \mathrm{diag}(+, +, +, +, +, -),$$
$$P_2 = \mathrm{diag}(+, +, +, -, -, -).$$

Why this model survives

- stable orbifold vacuum,
- chiral SM zero modes from minimal bulk matter,
- attractive fixed points for gauge and bulk Yukawa couplings,
- viable only for $n_g = 3$ bulk gen.-s.

Bulk matter embedding: two antisym 15-plets

$$\Psi_{\mathbf{15}}^{(+,-)} = (3,2)^{(+,+)}_{\mathbf{1/6},\,\mathbf{1}} \oplus (1,2)^{(-,-)}_{\mathbf{1/2},\,\mathbf{-2}} \oplus (\bar{3},1)^{(+,-)}_{\mathbf{-2/3},\,\mathbf{1}} \oplus (1,1)^{(+,-)}_{\mathbf{1},\,\mathbf{1}} \oplus (3,1)^{(-,+)}_{\mathbf{-1/3},\,\mathbf{-2}}$$

$$\Psi_{\mathbf{15}}^{(-,-)} = (3,1)^{(-,-)}_{\mathbf{2/3},\,\mathbf{-1}} \oplus (1,1)^{(-,-)}_{\mathbf{-1},\,\mathbf{-1}} \oplus (\bar{3},1)^{(+,+)}_{\mathbf{1/3},\,\mathbf{2}} \oplus (\bar{3},2)^{(-,+)}_{\mathbf{-1/6},\,\mathbf{-1}} \oplus (1,2)^{(+,-)}_{\mathbf{-1/2},\,\mathbf{2}}$$

Zero modes

Only the $(+,+)$ and $(-,-)$ components generate zero modes:

$$Q_L = (3,2)_{1/6,\,1}, \qquad L_L = (\bar{3},1)_{1/3,\,2},$$

$$u_R = (3,1)_{2/3,\,-1}, \qquad e_R = (1,1)_{-1,\,-1}, \qquad \ell_R = (1,2)_{1/2,\,-2}.$$

One SM family is reproduced, up to $U(1)_Z$ charge.

Takeaway

This is the **unique minimal stable ordinary-group route** that directly yields the SM gauge structure in 5D aGUT model building.

Second viable candidate: stable $SU(8)$ with Pati–Salam stage

Stable orbifold

A second viable minimal candidate

$$SU(8) \to SU(4) \times SU(2)_L \times SU(2)_R \times U(1)^2$$

via the stable pattern $(p, q, s) = (4, 2, 2)$. The gauge-scalar transforms as

$$\phi_0 = (4, 1, 2)_{1,0} \oplus (\bar{4}, 1, 2)_{-1,0},$$

so it can further break Pati–Salam.

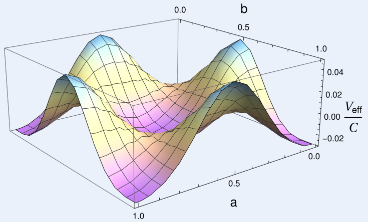
Minimal bulk matter

The SM fermions can be embedded in the antisym. 28-rep, whose decomposition contains the PS multiplets, $(4, 2, 1)_{0,1} \oplus (4, 1, 2)_{0,-1}$, together with additional states that can be lifted once $U(1)$'s are broken.

Status

- stable orbifold vacuum, with viable Pati–Salam stage,
- up to 3 bulk gen.-s, with non-perturbative Yukawa FP for $n_g = 3$.

Gauge-scalar V_{eff} for $SU(8)$



Degenerate gauge-equivalent minima

$$(a, b) = (0, 0) \quad \text{and} \quad \left(\frac{1}{2}, \frac{1}{2}\right)$$

Takeaway

Unlike $SU(6)$, the stable $SU(8)$ route does not go directly to the SM, but yields an intermediate PS.

Ordinary groups: what survives?

Outcome of the stability analysis

Combining orbifold vacuum stability, chiral zero-modes and FP viability, the minimal candidates are reduced to a short list.

Viable and non-viable routes

- ✓ $SU(6) \to SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)_Z$: minimal stable route connected to the SM.
- ✓ $SU(8) \to SU(4) \times SU(2)_L \times SU(2)_R \times U(1)^2$: viable stable route with an intermediate Pati–Salam stage.
- △ $SO(10) \to SU(4) \times SU(2)_L \times SU(2)_R$: minimal realization fails to remain fully satisfactory once Yukawa/FP constraints are imposed.
- × Generic larger sets of algebraically allowed orbifolds are removed already by the one-loop vacuum selection.

Shortlist

$SU(6) \to$ SM-like

$SU(8) \to$ PS stage

$SO(10) \to$ near miss

Main lesson

Orbifold stability is not a minor point.

It acts as a **dynamical filter** removing most constructions.

Transition

This raises the next question:

Can exceptional groups do better?

Exceptional groups: strategy of the analysis

Why a new strategy is needed

For the classical groups $SU(N)$, $Sp(2N)$ and $SO(N)$, stable orbifolds can be classified by explicitly studying the relative alignment of the parity matrices.

For the exceptional groups, this direct approach rapidly becomes cumbersome. The analysis is therefore reorganized in terms of the **maximal common subgroup** of a pair $P_i \times P_j$.

Working criterion

Each $\mathbb{Z}_2$ breaks the bulk G to a maximal subgroup H_i, the candidate 4D gauge group is $H = H_i \cap H_j$. The stable orbifold is identified by selecting the maximal common subgroup that is *not destabilized* by the gauge-scalar potential.

Scope

The relevant exceptional groups are E_6, E_7, while G_2 and F_4 do not accommodate the SM gauge structure, and E_8 does not lead to useful minimal chiral constructions.

Logic

$$G \xrightarrow{P_i} H_i, \quad G \xrightarrow{P_j} H_j$$

$$\Longrightarrow \quad H_i \cap H_j \quad \Longrightarrow \quad \text{test stability}$$

Typical possibilities

For a given $P_i \times P_j$:

- one maximal subgroup may be unique,
- or several candidates may exist,
- only one is generally selected by $V_{\rm eff}$.

Observation

Phenomenologically attractive exceptional breakings need *not* coincide with the dynamically stable ones.

No minimal exceptional 5D aGUT from stable orbifolds

The E_6 case

For the parity pair $P_1 \times P_2$, one attractive candidate

$$SU(4) \times SU(2)_L \times SU(2)_R \times U(1),$$

containing PS and a gauge-scalar capable of SSB to the SM.

However, the one-loop potential does *not* select this vacuum. The dynamically stable orbifold $SU(5) \times U(1) \times U(1)$, is not a minimal viable exceptional aGUT route.

The E_7 case

A similar phenomenon occurs for E_7: the phenomenologically attractive candidate does not coincide with the stable one.

The stable orbifold $SO(10) \times U(1) \times U(1)$ is selected rather than the more useful $SU(6) \times SU(2) \times U(1)$.

Main result

Under the minimal assumptions,

no minimal exceptional 5D aGUT

Why the exceptional route fails

- stable and phenomenologically attractive breakings do not coincide,
- the gauge-scalar potential selects the "wrong" maximal subgroup,
- minimality is too restrictive.

Implication

Go beyond the minimal setup, through e.g.

- supersymmetry,
- non-minimal bulk matter,
- or modified gauge-scalar dynamics.

UV behavior of 5D gauge-Yukawa theories

Why UV consistency is nontrivial

In five dimensions, the bulk couplings carry negative mass dimension:

$$[g_5] = [y_5] = m^{-1/2}, \qquad [\lambda_5] = m^{-1}.$$

Hence, the usual expectation: 5D theories lose predictivity above $1/R$.

Effective dimensionless couplings

After compactification, one defines dimensionless effective couplings such as

$$\tilde{\alpha} = \mu R\, \alpha, \qquad \tilde{\alpha}_y = \mu R\, \alpha_y,$$

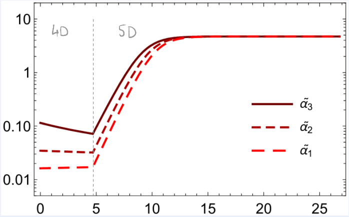
whose RG evolution exhibits **power-law running** (H. Gies, PRD68 (2003) 085015).

Key idea

Power-law running is not automatically pathological!

If the effective couplings flow to perturbative UV fixed points, the 5D theory can remain well behaved. Assuming perturbative FPs exist, **can the theory be renormalized consistently in the bulk and on the boundaries?**

From 4D logs to 5D FPs



Gauge FP at one loop

The one-loop 5D beta function,

$$2\pi \frac{d\tilde{\alpha}}{d\ln\mu} = 2\pi\,\tilde{\alpha} - b_5\,\tilde{\alpha}^2,$$

so that a UV FP exists for

$$b_5 > 0, \qquad \tilde{\alpha}^* = \frac{2\pi}{b_5}.$$

One-loop renormalization and final picture

Bulk

At one loop, the divergent operator basis in 5D is the same as in a 4D-renormalizable theory:

$$\mathcal{L}_5 = -\frac{1}{4}F_{MN}^2 + i\bar{\Psi}\Gamma^M D_M\Psi + (D_M\Phi)^\dagger(D^M\Phi) - y_5\,\bar{\Psi}\Psi\Phi - \lambda_5(\Phi^\dagger\Phi)^2.$$

The novelty is the **power-law** sensitivity, not new bulk operators.

Boundary

Orbifolding breaks translation invariance along x^5, so loops induce **localized divergences** at the fixed points.

At one loop, these are absorbed by a finite set of 4D-renormalizable boundary operators collected in $\mathcal{L}_4$.

Key conclusion

finite counterterms + UV perturbative FPs

$\Longrightarrow$ controlled 5D theory at one loop.

Renormalization logic

Bulk divs: same operator basis as in 4D

↓

Boundary divs: finite localized counterterms

↓

UV-consistent aGUT: if FPs are perturbative

What remains after both filters

A viable 5D aGUT must satisfy

stable vacuum and UV consistency.

Minimal survivors:

$SU(6) \to$ SM-like, $SU(8) \to$ PS stage.