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Paths to Solving the Strong CP Problem and the Footprints Left Behind

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Based on: JHEP 04 (2022), JHEP 12 (2022), JHEP (2025), JHEP (2026), PLB (2026),
with Fujikura, Girmohanta, Hor, Lee, Nakagawa, Qiu, Sato, Suzuki, L. Wang, Y. Wang, Yamada

Strong CP Problem

QCD Lagrangian for strong interactions allows

$$\mathcal{L}_\theta = \theta \frac{g_s^2}{32\pi^2} G^{a\mu\nu} \widetilde{G}^a_{\mu\nu}$$

explicitly violating **CP** symmetry.

The physical strong CP phase : $\bar{\theta} \equiv \theta - \arg\det\left(M_u M_d\right)$

The current upper bound on the neutron electric dipole moment

$$\Rightarrow \ |\bar{\theta}| < 10^{-11}$$

Why is $\bar{\theta}$ so small ??

Some shifts of $\bar{\theta}$ would not provide a visible change in our world.

Potential Solutions

- **Massless up quark**

  Existing global U(1) axial symmetry can rotate the θ term to zero.

  Ruled out by the lattice result.

- **Axion**

  Axion dynamically cancels the θ term at the minimum of its potential.

- **Spontaneous CP (or P) violation**

  CP is an exact symmetry of the Lagrangian but broken spontaneously at the vacuum.

  CKM phase is generated without reintroducing the θ term.

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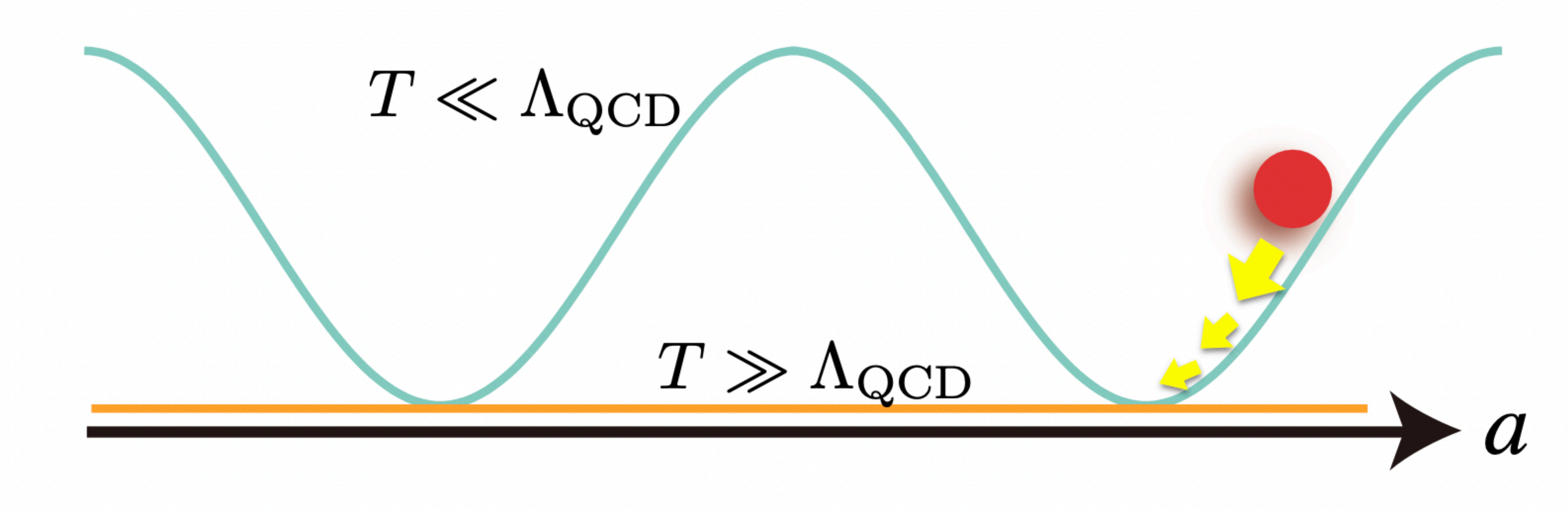
Axion Solution

The most common explanation is **the Peccei-Quinn mechanism** that the strong CP phase is promoted to a dynamical variable.

$$\mathcal{L}_\theta = \left(\theta + \frac{a}{f_a}\right) \frac{g_s^2}{32\pi^2} G^{a\mu\nu} \tilde{G}^a_{\mu\nu}$$



Fuminobu Takahashi slide

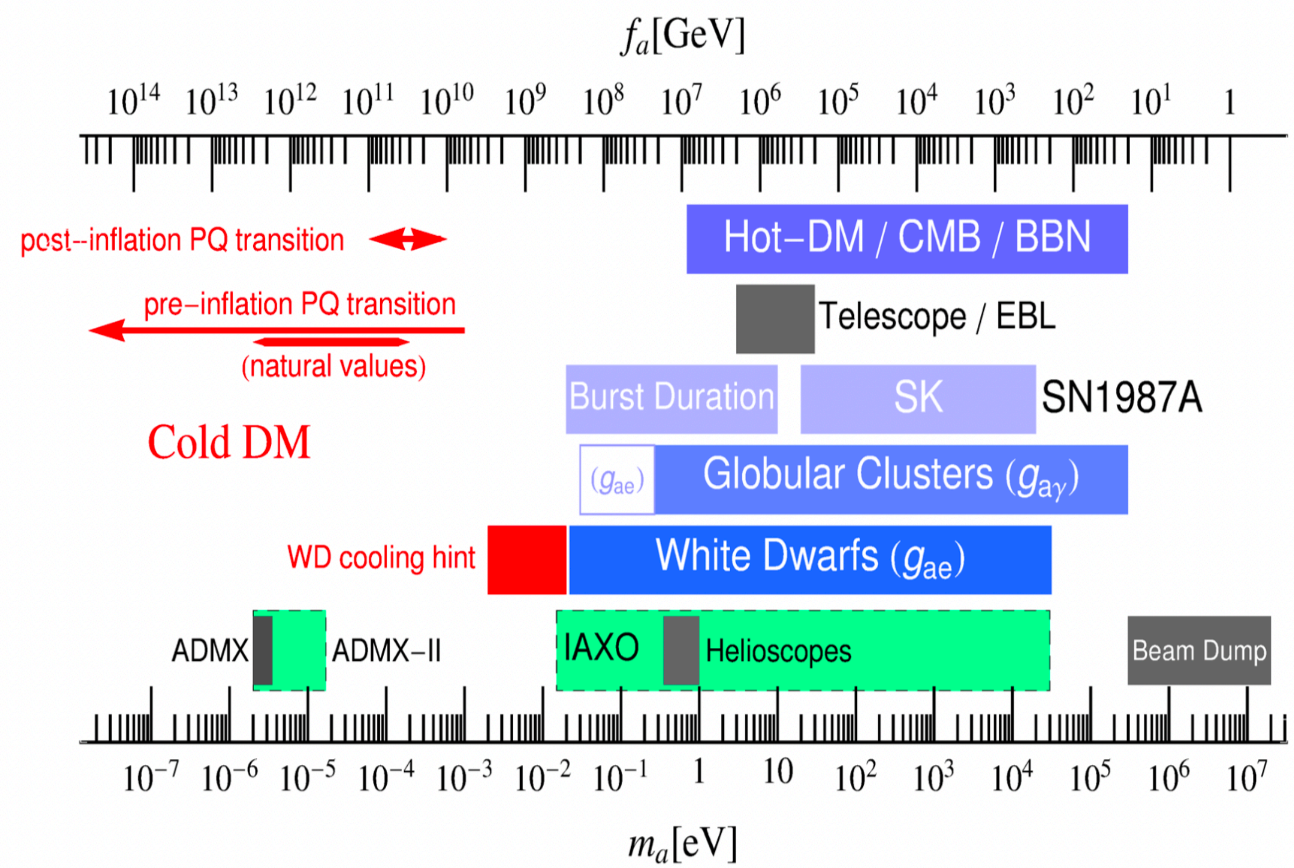
The **axion *a*** dynamically cancels the strong CP phase !

Axion Solution

Axion is a pseudo-Nambu-Goldstone boson associated with spontaneous breaking of **a global U(1)PQ symmetry.**

Non-perturbative QCD effects break the U(1)PQ explicitly and generate the axion potential :

$$V(a) \sim m_\pi^2 f_\pi^2 \cos\left(\theta + \frac{a}{f_a}\right)$$



PDG

Axion can be a dominant component of dark matter :

$$10^8\ \mathrm{GeV} \lesssim f_a \lesssim 10^{12}\ \mathrm{GeV}$$

Naturalness problem of the intermediate scale

Composite Axion

Axion emerges as **a pion-like bound state** of a new gauge dynamics.

Chiral symmetry breaking :

$$\mathrm{SU}(4)_L \times \mathrm{SU}(4)_R \to \underline{\mathrm{SU}(4)_V}$$
$$\supset \mathrm{SU}(3)_c$$

One of pNGBs is identified as axion.

Fields	$[SU(3)_c]$	$[SU(N)]$	$U(1)_{\mathrm{PQ}}$
Q	$\mathbf{3}$	$\mathbf{N}$	1
η	$\mathbf{1}$	$\mathbf{N}$	-3
$\bar{Q}$	$\bar{\mathbf{3}}$	$\bar{\mathbf{N}}$	0
$\bar{\eta}$	$\mathbf{1}$	$\bar{\mathbf{N}}$	0

Anomaly coefficient :

$$A(\mathrm{U}(1)_{\mathrm{PQ}} - \mathrm{SU}(3)_c{}^2) = \underline{N}$$ ➡ Axion coupling to QCD

↓ **Domain wall number**

Dimensional transmutation naturally explains <u>the hierarchically small PQ breaking scale</u>.

Axion Cosmology

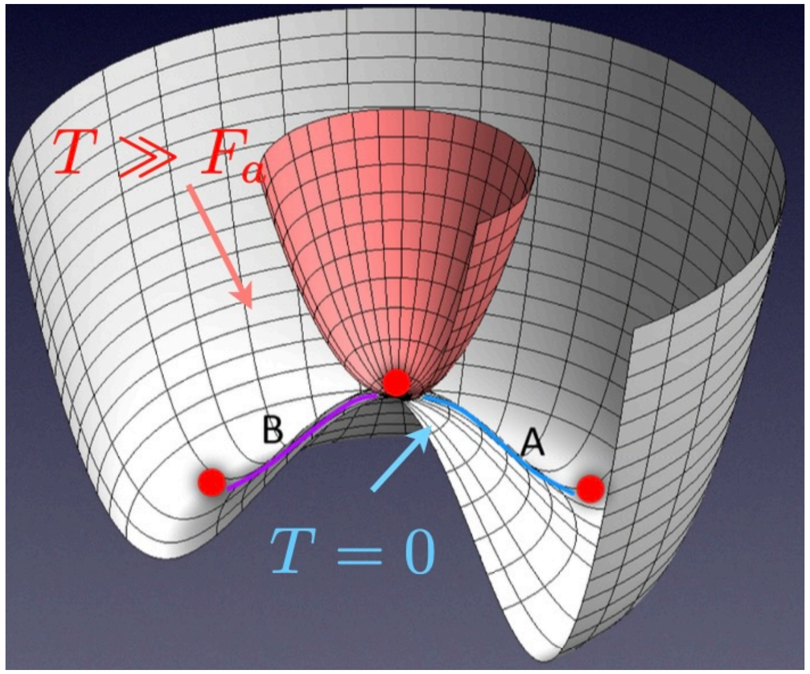
- Cosmological consequences :

★ U(1)PQ breaking **before** inflation → **Isocurvature perturbations**

Constraint on the inflation scale

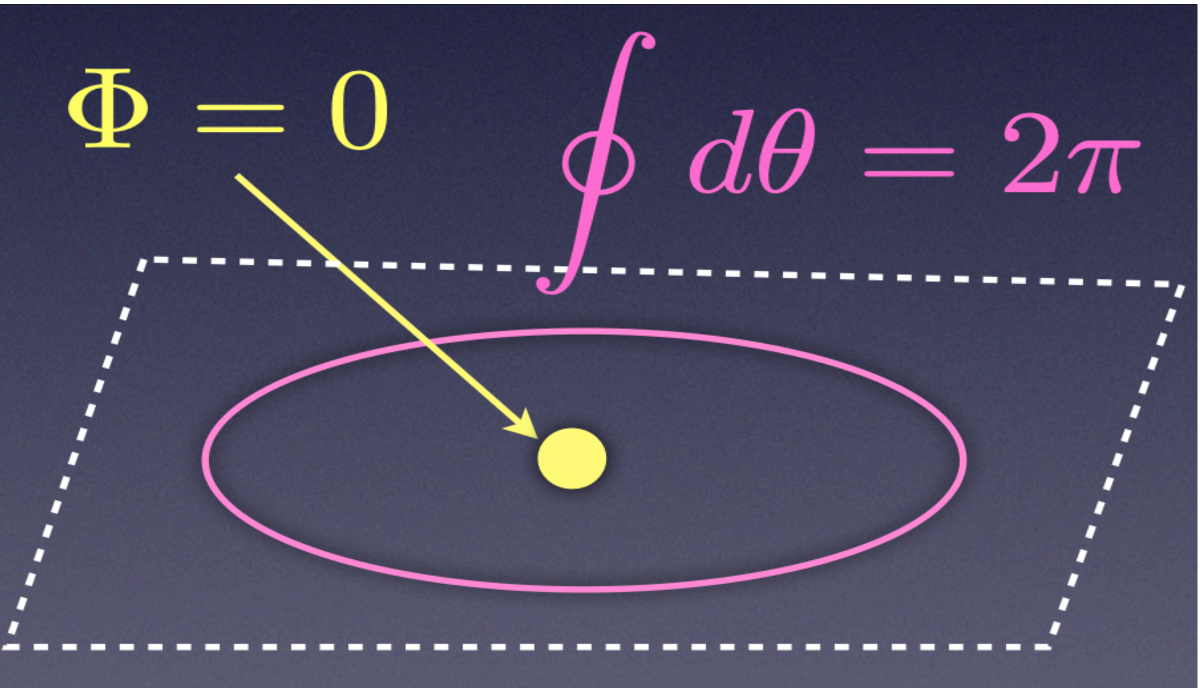
★ U(1)PQ breaking **after** inflation

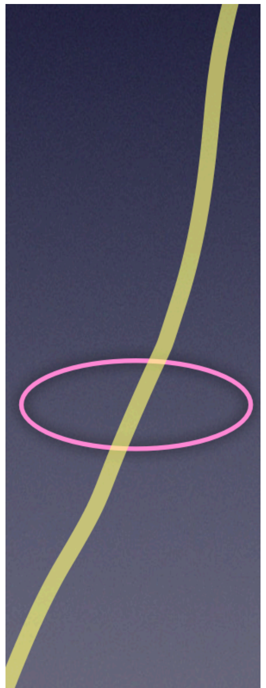
Axion field acquires **spatial variations** across the Universe.



Masahiro Kawasaki slide

$$\Phi \sim v_{\rm PQ} e^{i\theta_a}$$





Formation of **cosmic strings**

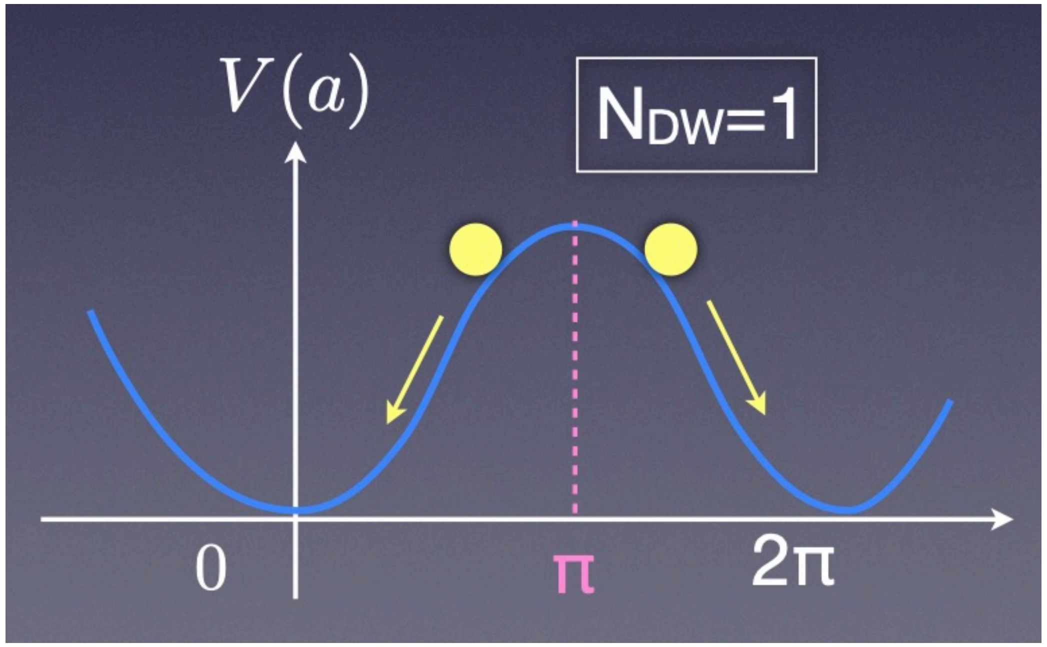
Domain Walls

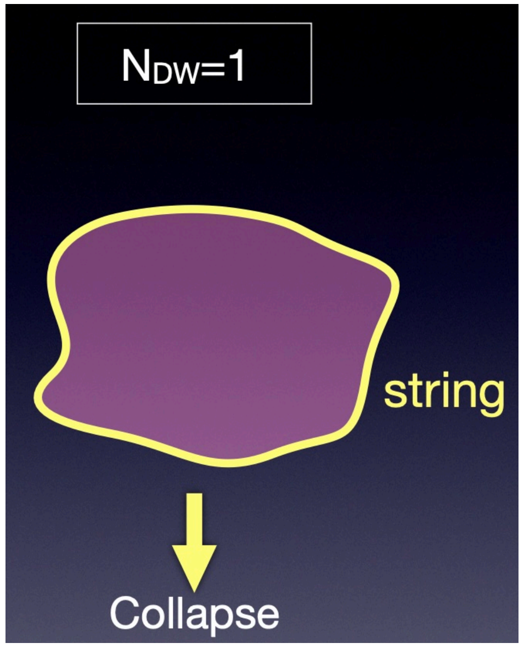
As the Universe expands and cools,
the axion gets a potential with $N_{\rm DW}$ minima.

Domain wall number (associated with color anomaly)

N_{DW} = 1

Domain walls form as disk-like structures attached to strings and eventually collapse due to their tension.





Masahiro Kawasaki slide

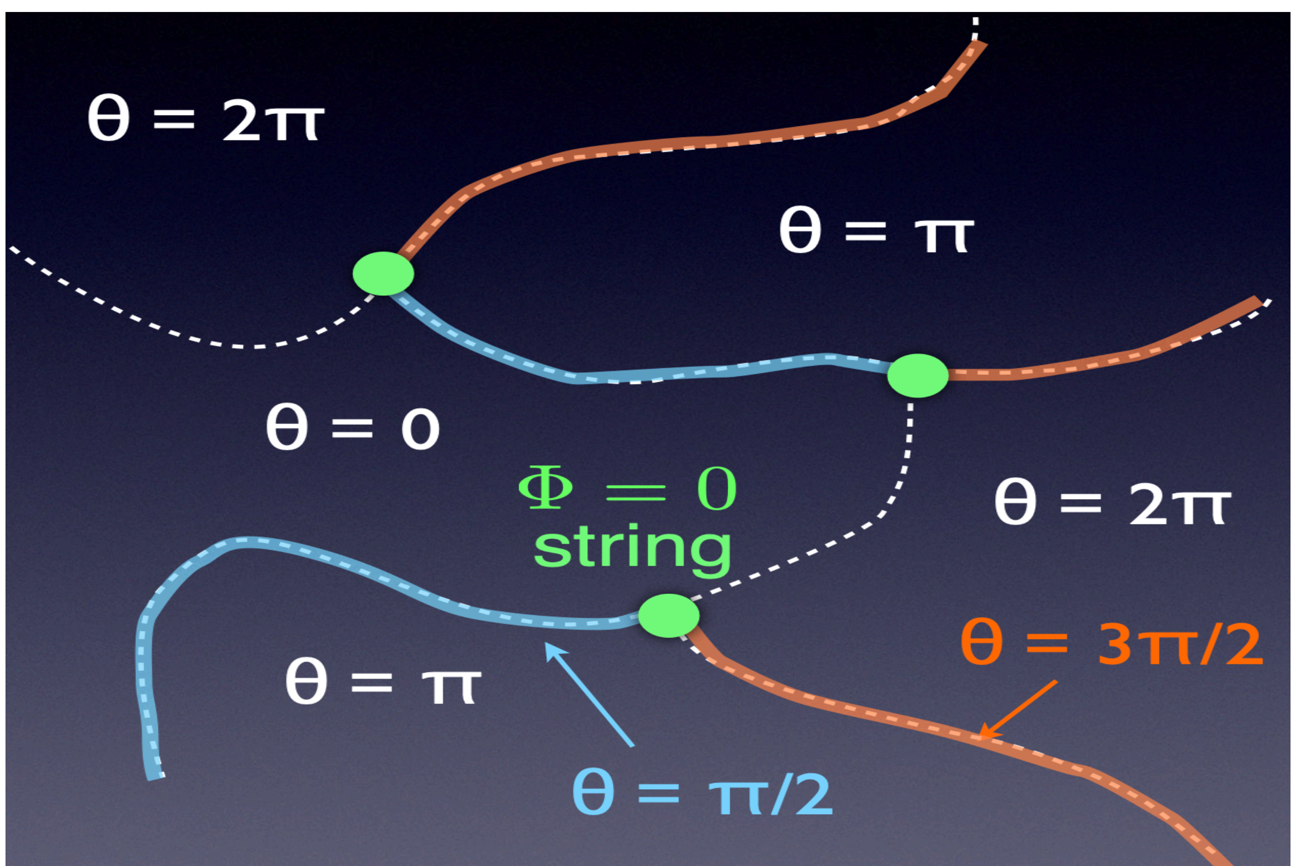
Decay of the string-domain wall network produces a lot of axions which dominate the axion DM abundance.

Domain Walls

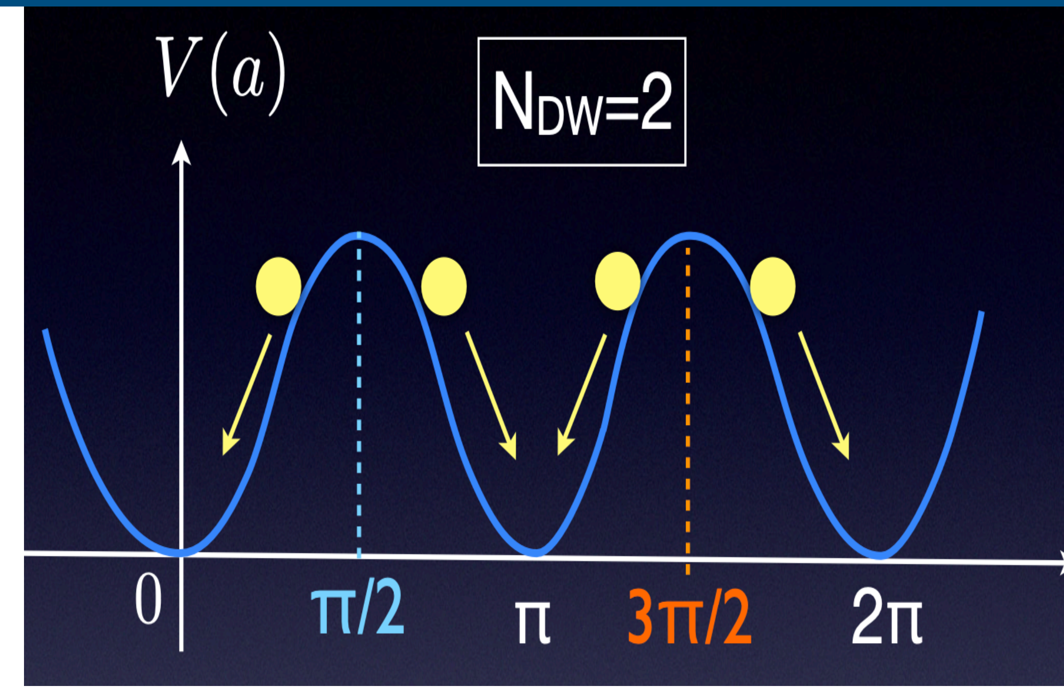
$N_{DW} > 1$

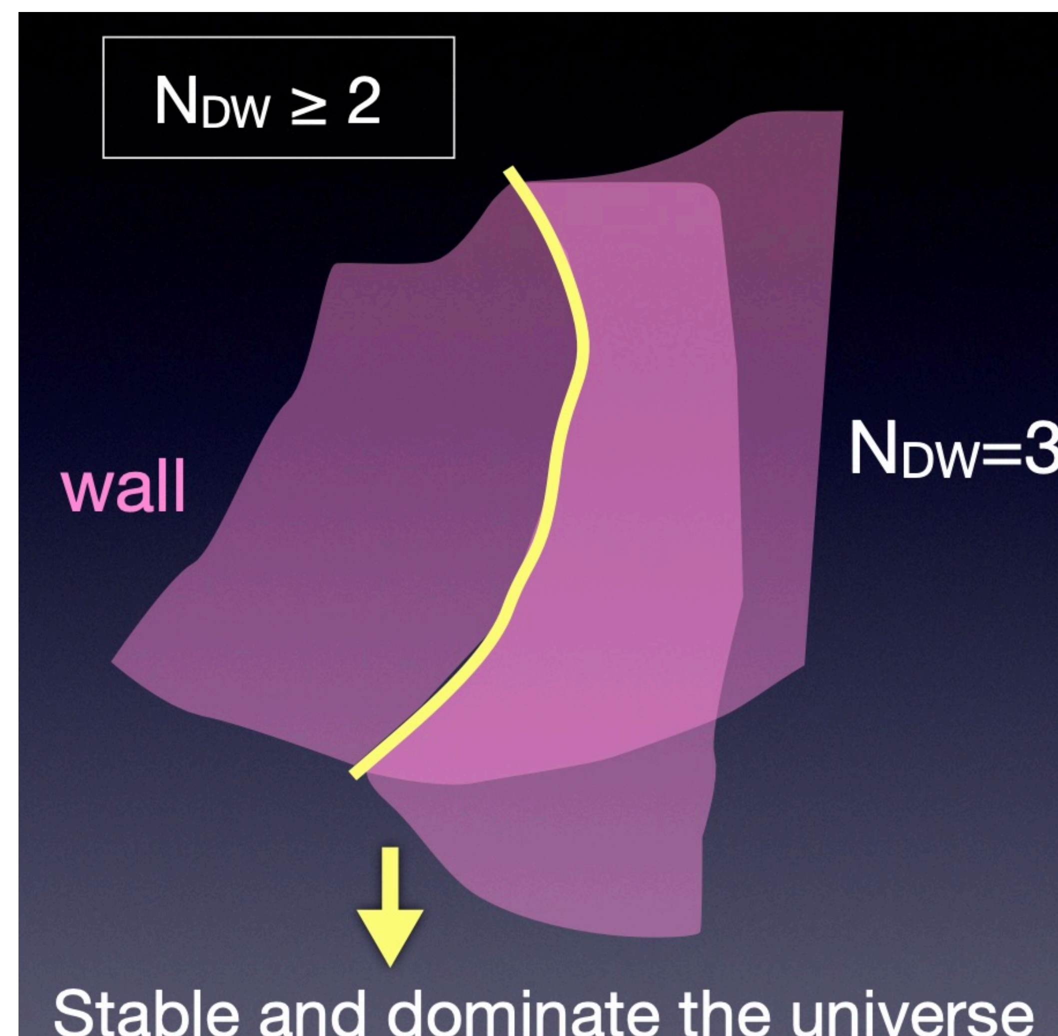
Domain walls form a **stable** network that eventually dominates the Universe.

➡ **Domain wall problem**



Masahiro Kawasaki slide





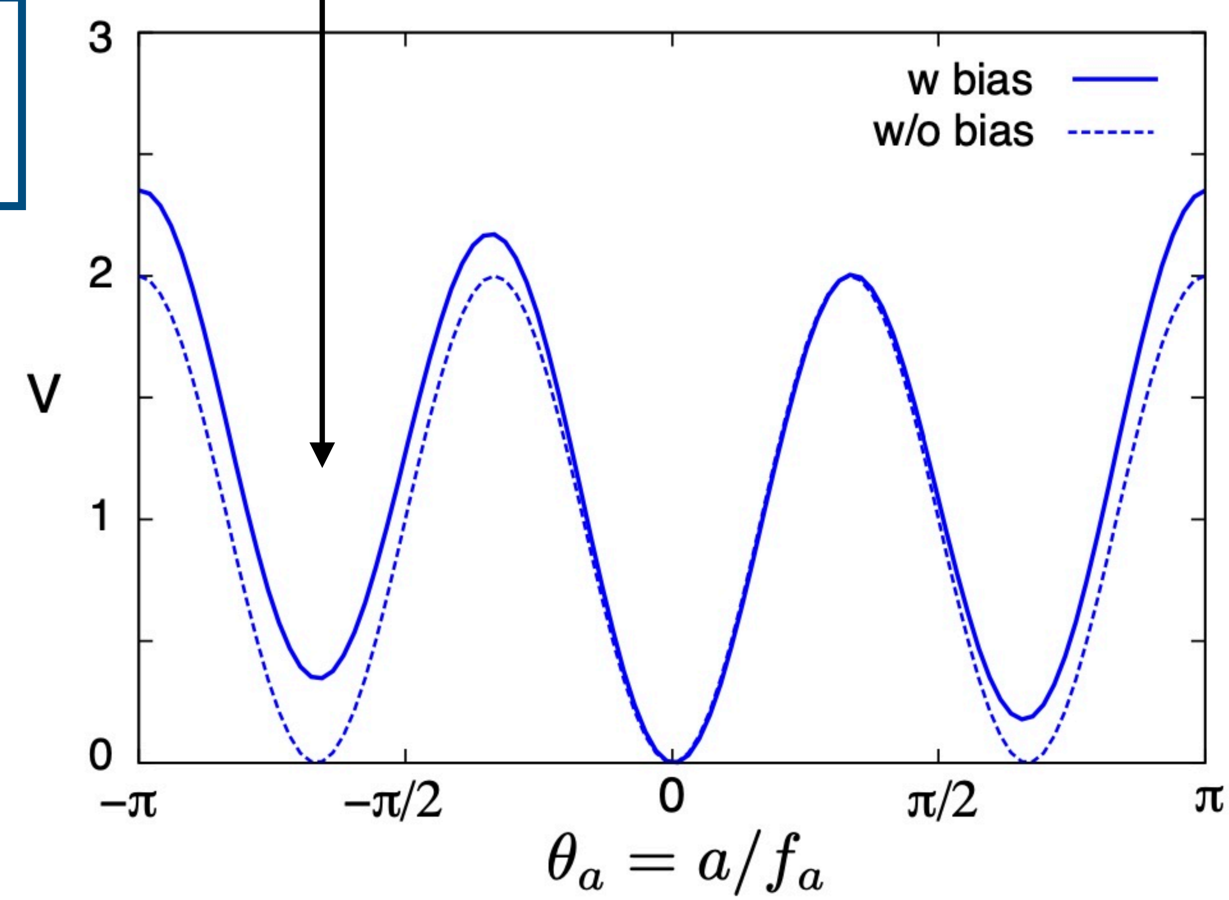
Bias Term

A potential solution to the domain wall problem is to introduce
a small explicit breaking of the $U(1)_{PQ}$ symmetry (**Bias term**).

Bias term lifts the vacuum degeneracy !

➡ **Domain walls collapse before they dominate the Universe.**

★ Axion DM abundance is changed.



However …

★ **Ad hoc introduction** of the bias term is unsatisfactory.

Potential Solutions

- **Special embedding** Hor, YN, Suzuki, Xu (2025)

  The SM gauge group is identified as a **special subgroup** of a UV group to obtain **small instanton effects** resolving the vacuum degeneracy of the axion potential.

- **Bias with a timer** Hao, Nakagawa, YN, Suzuki (2025)

  New scalar field interaction with the axion induces **a time-dependent bias term** only effective in the early Universe.

- **PQ symmetry non-restoration** Nakagawa, YN, Qiu, Wang, Wang (2025)

  PQ symmetry remains broken in the entire history of the Universe, avoiding the formation of axion strings and domain walls.

  …

Potential Solutions

- **Special embedding** Hor, YN, Suzuki, Xu (2025)

  The SM gauge group is identified as a **special subgroup** of a UV group to obtain **small instanton effects** resolving the vacuum degeneracy of the axion potential.

Our focus

- **Bias with a timer** Hao, Nakagawa, YN, Suzuki (2025)

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  …

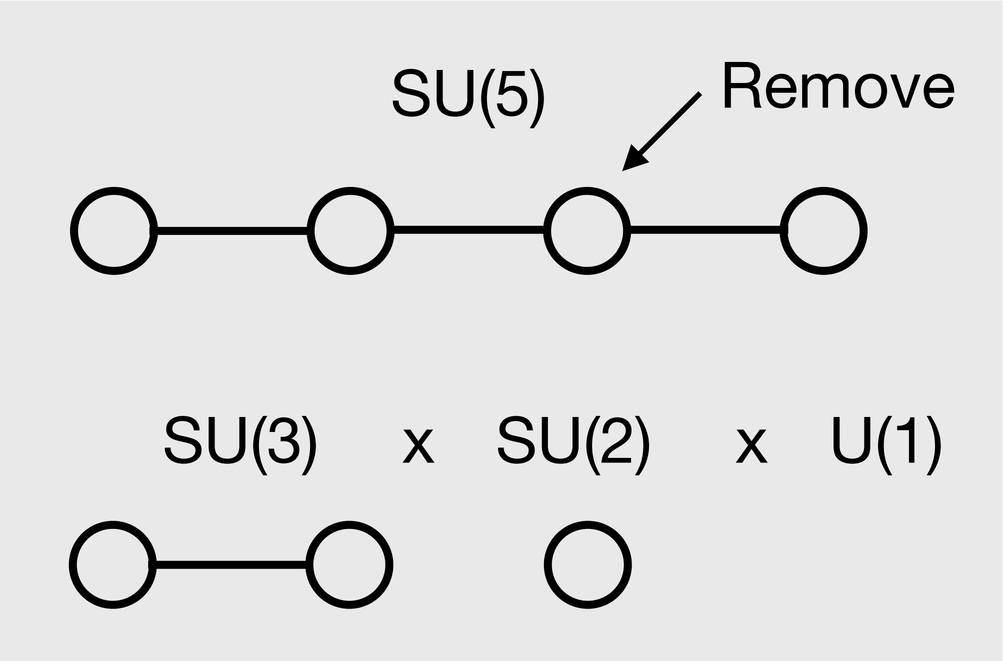
Special Subalgebra

Simple Lie algebras possess not only **regular** subalgebras but also **special** subalgebras.

Regular subalgebras : systematically obtained by removing nodes from Dynkin diagrams.

Special subalgebras :

do not follow this scheme !



→ Representation contents and group-theoretic indices are **reorganized** in a non-trivial manner.

Special Embedding

We focus on a gauge symmetry breaking : $SU(MN) \to SU(N)$

$T^m_{\rm UV}\ (m = 1, \ldots, (MN)^2 - 1)$: SU(MN) generators

$T^a_{\rm IR}\ (a = 1, ..., N^2 - 1)$: SU(N) generators

$$T^a_{\rm IR} = \underline{\mathcal{O}^{am}} T^m_{\rm UV}$$

Coefficients

r rep. of SU(N) is embedded into the fundamental rep. of SU(MN)

$$\mathrm{tr}(T^m_{\rm UV} T^n_{\rm UV}) = \frac{1}{2}\delta^{mn} \qquad \mathrm{tr}(T^a_{\rm IR} T^b_{\rm IR}) = \underline{T_{\rm IR}(\mathbf{r})}\delta^{ab}$$

Dynkin index

$$\Rightarrow \mathcal{O}^{am}\mathcal{O}^{bn}\delta_{mn} = c\delta^{ab} \qquad c \equiv \frac{T_{\rm IR}(\mathbf{r})}{1/2}$$

Special embedding corresponds to c > 1.

Special Embedding

Consider **a Weyl fermion** that transforms as the fundamental rep. of SU(MN) but behaves as the **r** rep. of the SU(N) subgroup :

$$\mathcal{D}_\mu \psi = \partial_\mu \psi - i g_{\mathrm{UV}} A^m_{\mathrm{UV},\mu} (T^m_{\mathrm{UV}}) \psi$$

$$\supset \partial_\mu \psi - i g_{\mathrm{IR}} A^a_{\mathrm{IR},\mu} (T^a_{\mathrm{IR}}) \psi$$

A part of SU(MN) gauge field is expressed in terms of SU(N) gauge field :

$$A^l_{\mathrm{UV},\mu} = \frac{g_{\mathrm{IR}}}{g_{\mathrm{UV}}} \underline{A^a_{\mathrm{IR},\mu}} (\mathcal{O})^{al}$$

Canonically normalized kinetic term ➡ $g_{\mathrm{IR}} = g_{\mathrm{UV}} / \sqrt{c}$

Theta term : $\displaystyle \int \frac{g^2_{\mathrm{UV}}}{8\pi^2} \mathrm{tr}(F_{\mathrm{UV}} \wedge F_{\mathrm{UV}}) = \int \frac{c g^2_{\mathrm{IR}}}{8\pi^2} \mathrm{tr}(F_{\mathrm{IR}} \wedge F_{\mathrm{IR}})$

UV Completion

Hor, YN, Suzuki, Xu, in preparation

Gauge symmetry breaking :

$$\underline{SU(4N)} \times SU(3)_2 \times SU(N)_2 \times U(1)_X \to \underline{SU(N) \times SU(3)_c} \times U(1)_Y$$

$\supset SU(4)_1 \times SU(N)_1$: **Special embedding** **Diagonal subgroup**

- All SM matter fields are charged under $SU(3)_2 \times U(1)_X$

- **A vector-like pair of PQ-charged fermions** transform as (anti-)fundamental reps. under SU(4N), so that **N_{DW} = 1**.

  After gauge symmetry breaking …

$$\Psi : (\mathbf{N}, \mathbf{3}) + (\mathbf{N}, \mathbf{1}) \ , \quad \bar{\Psi} : (\overline{\mathbf{N}}, \overline{\mathbf{3}}) + (\overline{\mathbf{N}}, \mathbf{1})$$

- The apparent vacuum degeneracy is lifted by **small instanton effects** on the axion potential that operates as <u>a PQ-violating bias term</u>.

Small Instanton

QCD θ-vacuum : superposition of *n*-vacua (energy degenerate but topologically distinct)

$$|\theta\rangle = \sum_{n=-\infty}^{\infty} e^{-in\theta}|n\rangle = \cdots + |0\rangle + e^{-i\theta}|1\rangle + \cdots$$

Instanton describes the tunneling effect between degenerate *n*-vacua

Instanton : localized object in Euclidean spacetime, satisfying Euclidean EOM with non-trivial topology and minimize Euclidean action

SU(2) BPST instanton solution with $Q = 1$:

$$\frac{g^2}{32\pi^2}\int d^4x\, F^{a\mu\nu}\widetilde{F}^a_{\mu\nu}\Big|_{\text{inst.}} = Q \quad (Q \in \mathbb{Z})$$

$$A^a_\mu(x)\Big|_{1-\text{inst.}} = 2\eta_{a\mu\nu}\frac{(x-x_0)_\nu}{(x-x_0)^2+\rho^2}$$

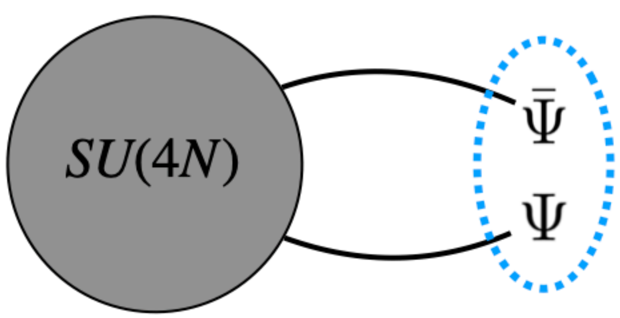
Position (x_0)　　Instanton size (ρ)

Small Instanton

Instantons contribute to the axion potential $\propto \exp\left(-\frac{8\pi^2}{g^2(1/\rho)}\right)$

In QCD, large-size instantons dominate the axion potential due to asymptotic freedom.

UV gauge interaction beyond QCD ➡ **Small instanton effects**



$$\sim \exp\left(\frac{2i}{f_a}\begin{pmatrix} \pi_8(x) + \frac{a(x)}{\sqrt{6}}\mathbf{1}_{3\times 3} & K(x) \\ K^\dagger(x) & -\frac{3a(x)}{\sqrt{6}} \end{pmatrix}\right)$$

’t Hooft vertex

$$\Rightarrow \quad V_{4N} \sim \Lambda_{\rm cut}\Psi\bar{\Psi}e^{-8\pi^2/(g_{SU(4N)}(\Lambda_{\rm cut}))^2}$$

$\Lambda_{\rm cut}$: cut-off scale

Post-Inflation Axion

PQ symmetry is spontaneously broken after reheating.

We focus on the scenario where the axion field starts to oscillate due to the axion mass originated from **the bias term** :

$$m_{\rm bias}^2 = \frac{1}{f_{\rm PQ}^2}\frac{\partial^2 V_{\rm bias}}{\partial \theta_a^2}$$

Temperature when the oscillation starts :

$$m_a(T_1) \approx m_{\rm bias} = 3H(T_1) \qquad H(T_1)^2 \approx \frac{\pi^2}{90 M_{\rm Pl}^2} g_* T_1^4$$

$$T_1 \;>\; T_{1,\rm QCD} \equiv 0.98~{\rm GeV}\left(\frac{f_a}{10^{12}~{\rm GeV}}\right)^{-0.19}$$

Temperature when the axion would start to oscillate with non-perturbative QCD effects if there was no bias term

Domain walls decay in a similar way as the standard $N_{\rm DW} = 1$ case.

Post-Inflation Axion

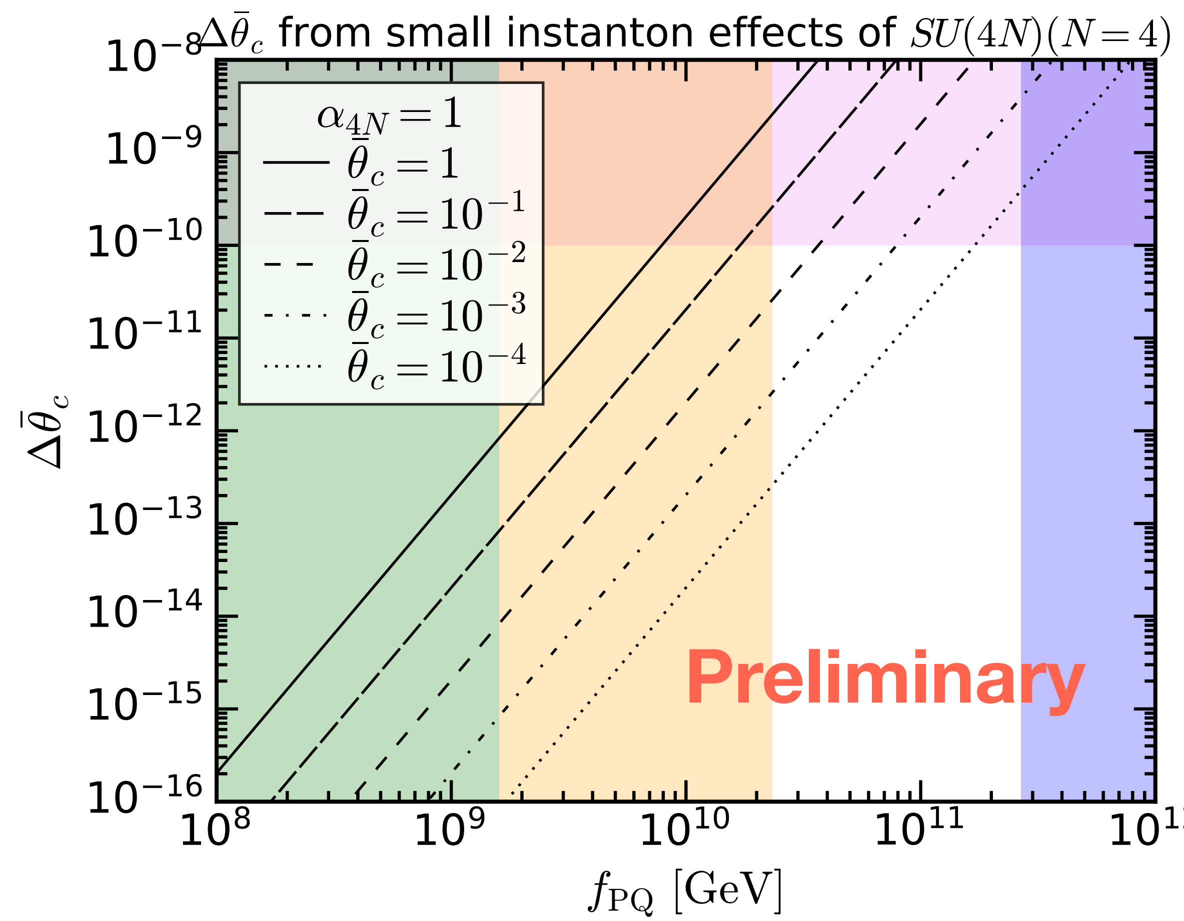
Axion abundance :

$$\Omega_a h^2 \approx 2 \times 10^{-12} \frac{f_{\rm PQ}}{T_1}$$

A large bias term shifts the axion potential minimum :

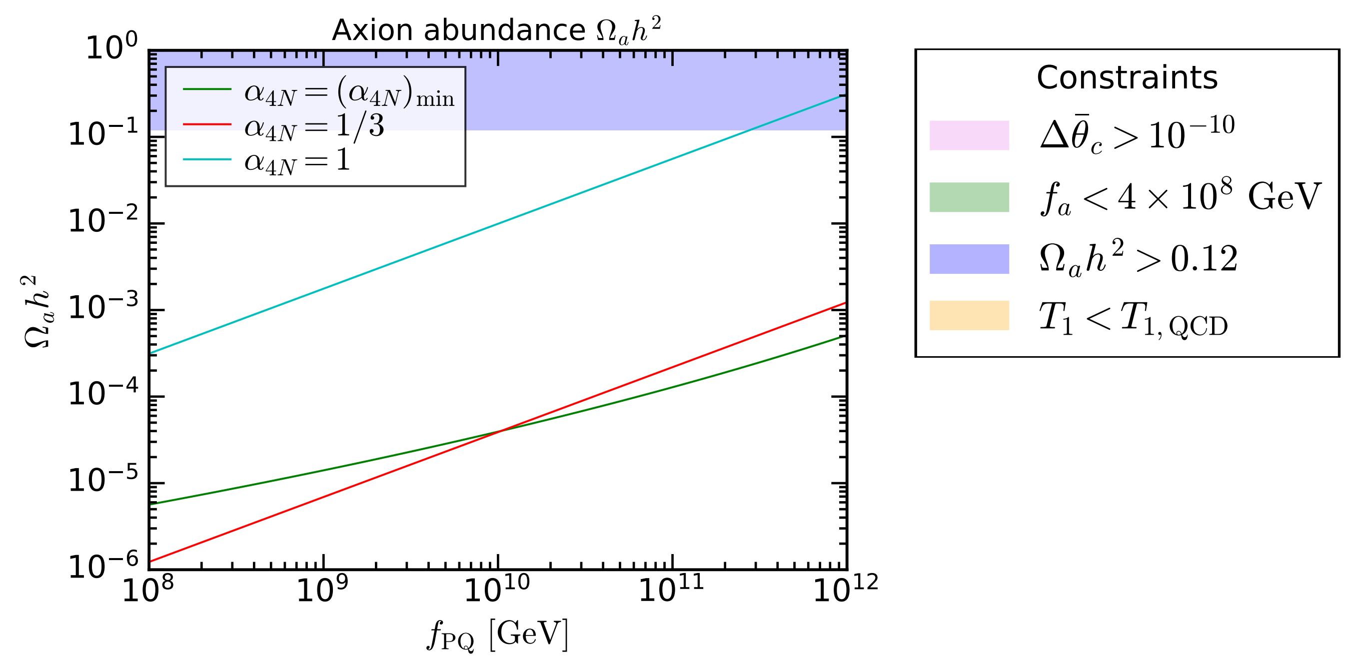
$$\Delta\bar{\theta}_c \approx \frac{V_{\rm bias}}{V_{\rm QCD}} \bar{\theta}_c$$

$\bar{\theta}_c$: Relative phase



Neutron EDM

➡ $\Delta\bar{\theta}_c \lesssim 10^{-10}$



Axion does not appear to explain the correct DM abundance…

Potential Solutions

- **Massless up quark**

  Existing global U(1) axial symmetry can rotate the θ term to zero.

  Ruled out by the lattice result.

- **Axion**

  Axion dynamically cancels the θ term at the minimum of its potential.

- **Spontaneous CP (or P) violation**

  CP is an exact symmetry of the Lagrangian but broken spontaneously at the vacuum.

  CKM phase is generated without reintroducing the θ term.

…

Spontaneous CPV

- Spontaneous CP violation (SCPV) provides **an axionless solution** to the strong CP problem.

- CP is an exact symmetry of the Lagrangian but broken spontaneously at the vacuum

→ Generation of the CKM phase without reintroducing a nonzero strong CP phase

Nelson-Barr mechanism

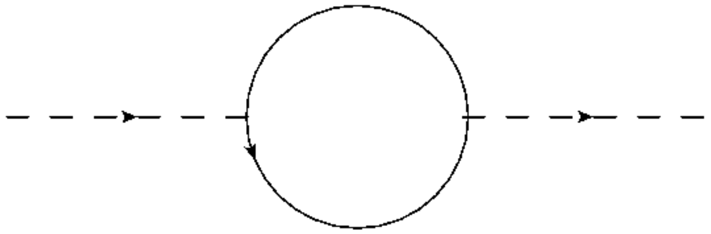
A vector-like pair of heavy quarks is introduced, so that the extended quark mass matrix transmits SCPV into the CKM matrix.

$$\mathcal{L} = \mu \underline{\bar{q} q} + a_{a\bar{f}} \underline{\eta_a} \bar{d}_{\bar{f}} q + y_{f\bar{f}} H Q_f \bar{d}_{\bar{f}} \quad \Rightarrow \quad \mathcal{M} = \begin{pmatrix} \mu & a_{a\bar{f}}\eta_a \\ \boxed{0} & m_d \end{pmatrix}$$

Heavy quarks — **CP breaking field**

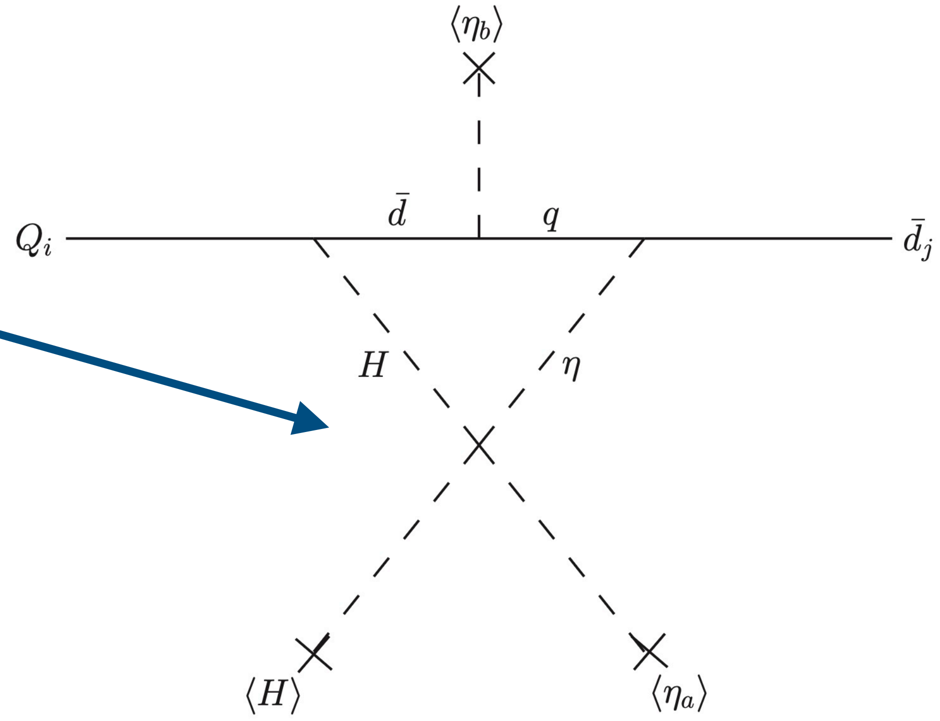
Challenges for SCPV

- **Naturalness problem**: the mechanism requires new scalar fields whose VEVs break CP much below the Planck scale.



- Sensitivity to **higher-dim. operators** and **radiative corrections**, regenerating a strong CP phase.

$$\mathcal{L} \supset \frac{\eta_b^*}{\Lambda}\eta_a \bar{q} q + \frac{\eta_a^*}{\Lambda} H Q \bar{q} + \underline{\gamma_{ab}\eta_a^*\eta_b H^\dagger H}$$



Dine, Draper (2015)

Challenges for SCPV

- **Naturalness problem**: the mechanism requires new scal… …ds whose VEVs break CP much below the Planck …

Supersymmetry (SUSY) offers a natural framework to address these difficulties !

- Sensi… …dim. **operators** and **radiative corrections**, regene… …g a strong CP phase.

✓ SUSY stabilizes the SCPV scale in much the same way it stabilizes the EW scale.

✓ SUSY can forbid or strongly suppress dangerous higher-dim. operators.

It's reasonable to consider SCPV in SUSY.

For non-SUSY approach, see *e.g.* Girmohanta, Lee, YN, Suzuki (2022).

SCPV in SUSY

The physical strong CP phase :

$$\bar{\theta} \equiv \theta - \arg\det\left(M_u M_d\right) - 3\arg\left(\underline{m_{\tilde{g}}}\right)$$

Gluino mass

To set the cosmological constant to zero

$$\Rightarrow \underline{\langle W \rangle} \sim \underline{m_{3/2}} M_{\rm Pl}^2$$

Gravitino mass

Complex phase generates a gluino mass phase via 1-loop anomaly med.

Constraint on $\bar{\boldsymbol{\theta}}$ $\Rightarrow$ $$\frac{\alpha_s}{4\pi}\frac{m_{3/2}}{m_{\tilde{g}}} < 10^{-10}$$

Gravitino mass must be sufficiently small.

SCPV in SUSY

- **Flat directions** are ubiquitous in SUSY.

  Valleys in field space along which the potential is exactly flat.

- Flat directions naturally contain a point to spontaneously break CP.

How to stabilize such a point ?

Giving positive masses for all scalar fields around the minimum.

- Two qualitatively different ways for the stabilization:

  ✓ Purely in a **supersymmetric** way ⬅ No low-energy mode …

  ✓ Through **SUSY breaking effects**

SCPV in SUSY

- **<u>Flat directions</u>** are ubiquitous in SUSY.

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>
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  ✓ Purely in a **supersymmetric** way

  ✓ Through **SUSY breaking effects** ← **Let's focus !**

SCPV via ~~SUSY~~

Liu, Nakagawa, YN, Wang (2025)

- **Supersymmetric potential**

$$V_F = \lambda^2|\phi_1\phi_2 - v^2|^2 + \underline{\lambda^2}|X|^2(|\phi_1|^2 + |\phi_2|^2)$$

Real parameter

$$\Rightarrow \quad \langle X\rangle = 0, \quad \langle\phi_1\rangle = v_1 e^{i\theta}, \quad \langle\phi_2\rangle = v_2 e^{-i\theta} \quad v_1 v_2 \equiv v^2$$

Not uniquely determined (v_1, θ)

SCPV scale (v^2)

$$\phi_1(x) = \left(v_1 + \frac{\sigma_1(x)}{\sqrt{2}}\right)\exp\left[i\left(\theta + \frac{\pi_1(x)}{\sqrt{2}v_1}\right)\right], \quad \phi_2(x) = \left(v_2 + \frac{\sigma_2(x)}{\sqrt{2}}\right)\exp\left[i\left(-\theta + \frac{\pi_2(x)}{\sqrt{2}v_2}\right)\right]$$

Massless modes :

$$s(x) = \frac{1}{f_a}(v_1\sigma_1(x) - v_2\sigma_2(x)), \quad a(x) = \frac{1}{f_a}(v_1\pi_1(x) - v_2\pi_2(x))$$

$$f_a \equiv \sqrt{v_1^2 + v_2^2}$$

SCPV via SUSY

Two terms with different periodicity are needed, *e.g.* $V = \mathcal{C}\cos\theta + \mathcal{D}\cos(2\theta)$

- **SUSY breaking**

$$V_{\rm soft} = \left(\frac{1}{2}b_1\phi_1^2 + \frac{1}{2}b_2\phi_2^2 + {\rm h.c.}\right) + m_1^2\phi_1^*\phi_1 + m_2^2\phi_2^*\phi_2$$

The potential is bounded from below. ➡ $|b_1| \le m_1^2 \;, \quad |b_2| \le m_2^2$

- **Non-perturbative effect**

SU(N) dark QCD with quarks coupled to SCPV fields.

$$\Rightarrow \quad V_{\rm dyn} = \frac{(\kappa_1^2 + \kappa_2^2)\,\Lambda^{6-\frac{2}{N}}}{|\kappa_1\phi_1 + \kappa_2\phi_2|^{2-\frac{2}{N}}}$$

Real coupling constants (→ κ_1, κ_2)

Λ : dark QCD dynamical scale

SCPV via ~~SUSY~~

$$V_{\rm tot} = V_F + V_{\rm soft} + V_{\rm dyn}$$

$$= b_1 \left(v_1^2 + \frac{b_2}{b_1}\frac{v^4}{v_1^2} \right) \cos(2\theta) + m_1^2 \left(v_1^2 + \frac{m_2^2}{m_1^2}\frac{v^4}{v_1^2} \right) + \frac{(\kappa_1^2+\kappa_2^2)\Lambda^{6-\frac{2}{N}}}{[\kappa_1^2 v_1^2 + \kappa_2^2 v^4/v_1^2 + 2\kappa_1\kappa_2 v^2 \cos(2\theta)]^{1-\frac{1}{N}}}$$

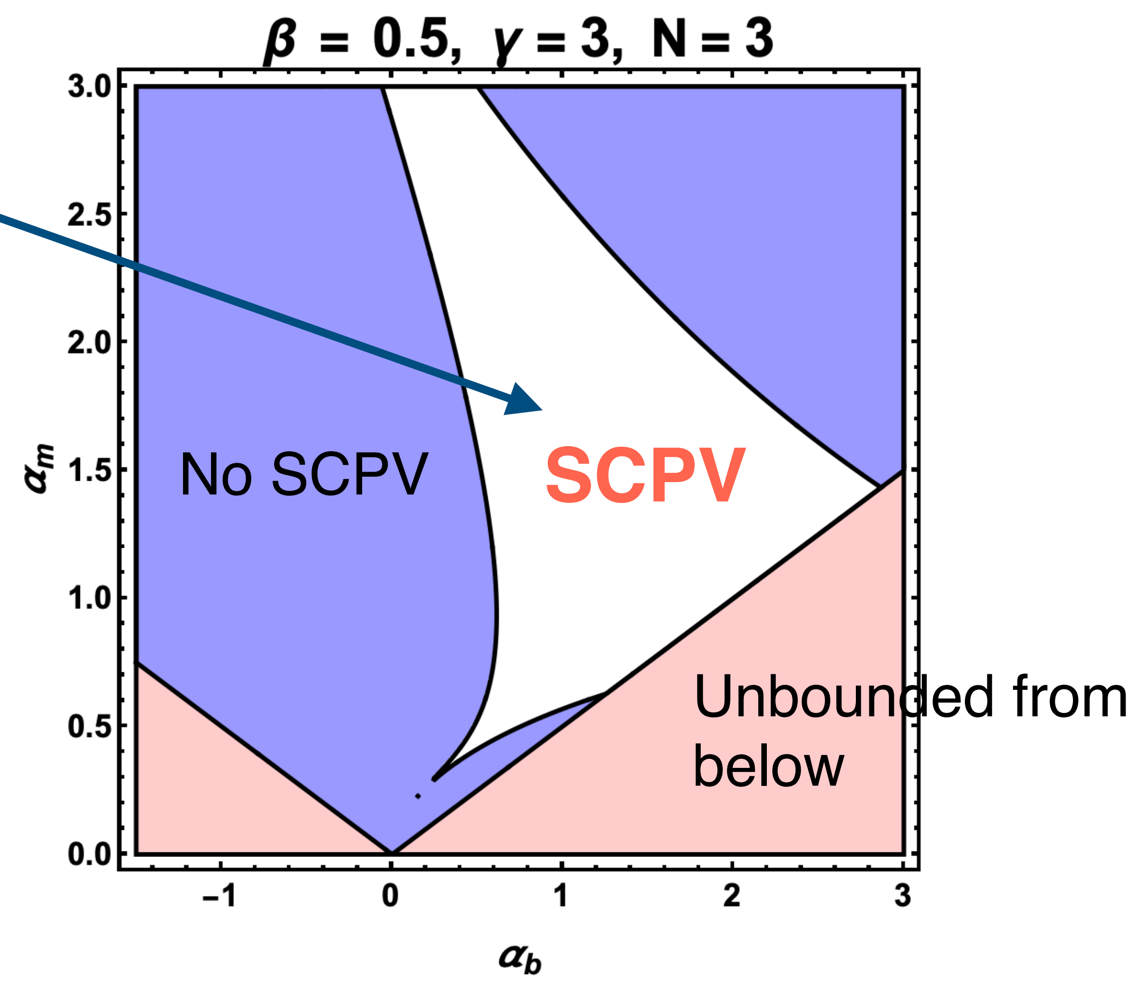
Stabilized at a non-trivial value $(\neq 0, \pi)$

All directions properly stabilized.

Existence of light modes :

$$m_a^2 \sim m_s^2 \lesssim m_{\rm soft}^2$$

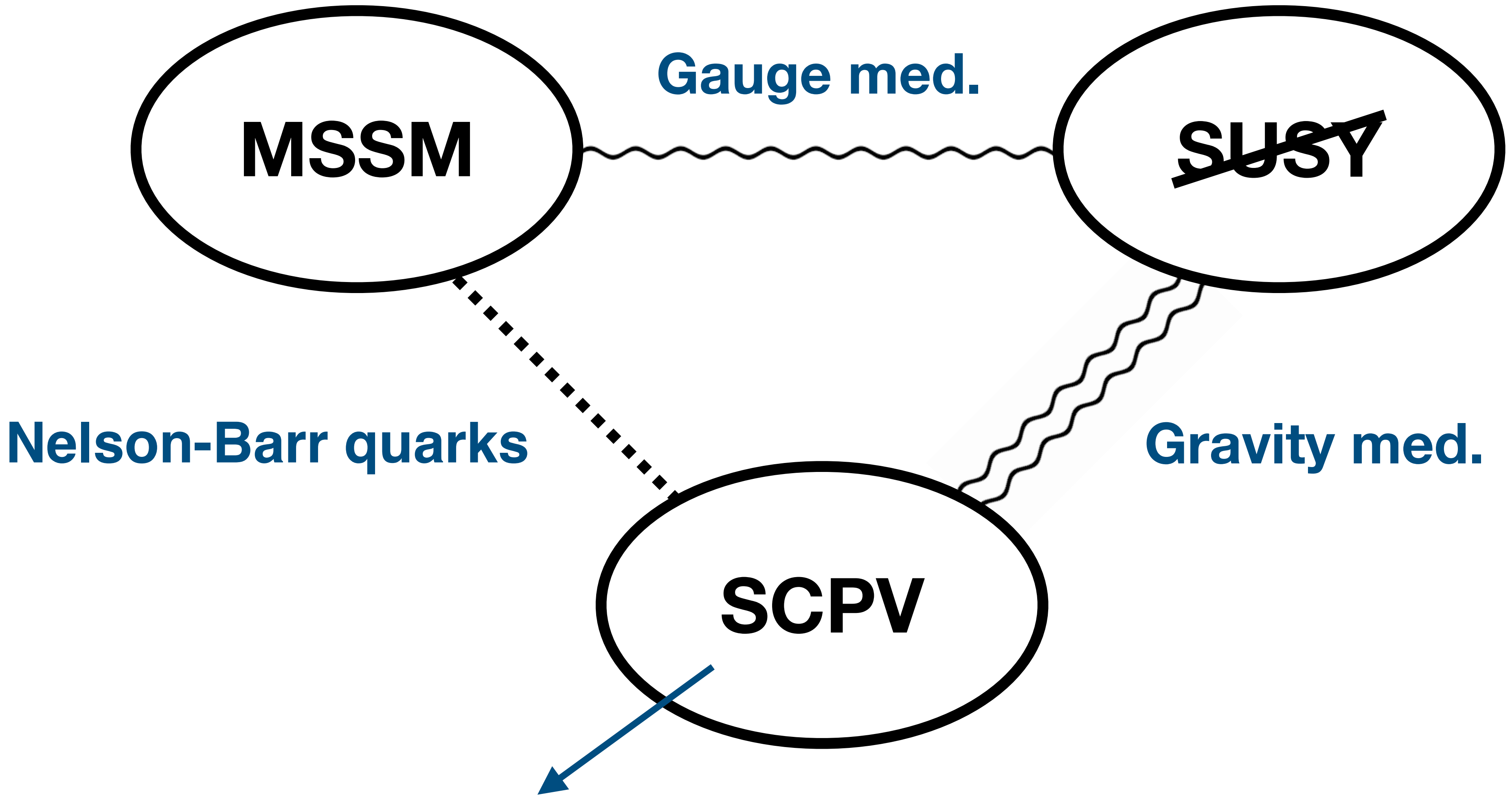
Much smaller than SCPV scale !



$\alpha_b \equiv b_2/b_1, \alpha_m \equiv m_2^2/m_1^2, \beta \equiv b_1/m_1^2, \gamma \equiv \Lambda^{6-\frac{2}{N}}/(m_1^2 v^{4-\frac{2}{N}})$

SCPV via ~~SUSY~~

Solving the strong CP problem via SCPV



$$m_a^2 \sim m_s^2 \sim m_{3/2}^2 < (10 - 100\,\mathrm{keV})^2$$

Light particles feebly interacting with SM are predicted !

Detailed phenomenology & cosmology will be explored in a future study.

Baryogenesis

Considering SCPV …

Fujikura, YN, Sato, Yamada (2022)

- What is **the source of CPV** for baryogenesis?
- High reheating temperature for thermal leptogenesis causes **the overproduction of gravitinos**.

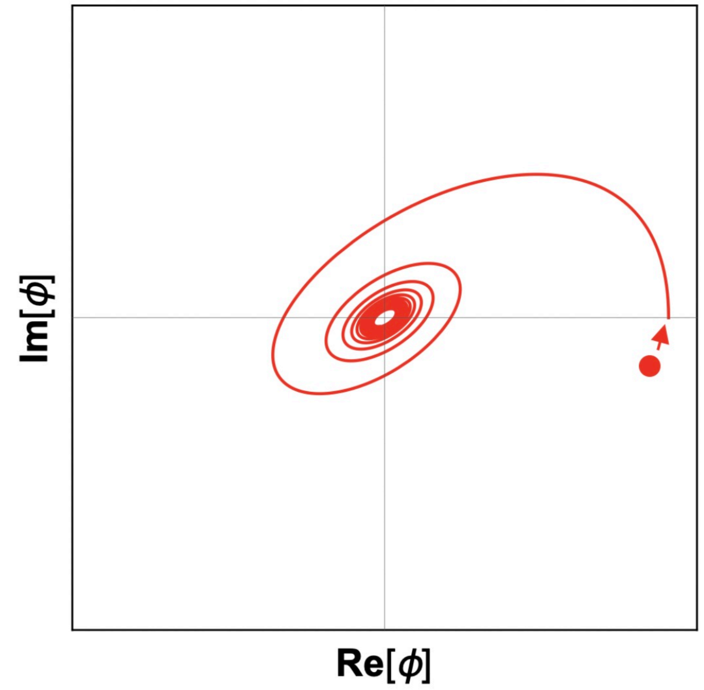
Affleck-Dine (AD) mechanism

realized in SUSY and compatible with a low reheating temperature.

✓ **AD field** (ϕ) coherent rotational motion leads to baryon asymmetry.

$$\phi^3 \approx Q\bar{q}L$$

$$\phi = \frac{\varphi}{\sqrt{2}} e^{i\theta}, \quad n_{B-L} = q_{B-L}\varphi^2\dot{\theta}$$



✓ Explicit CPV is not needed.

Baryogenesis

Fujikura, YN, Sato, Yamada (2022)

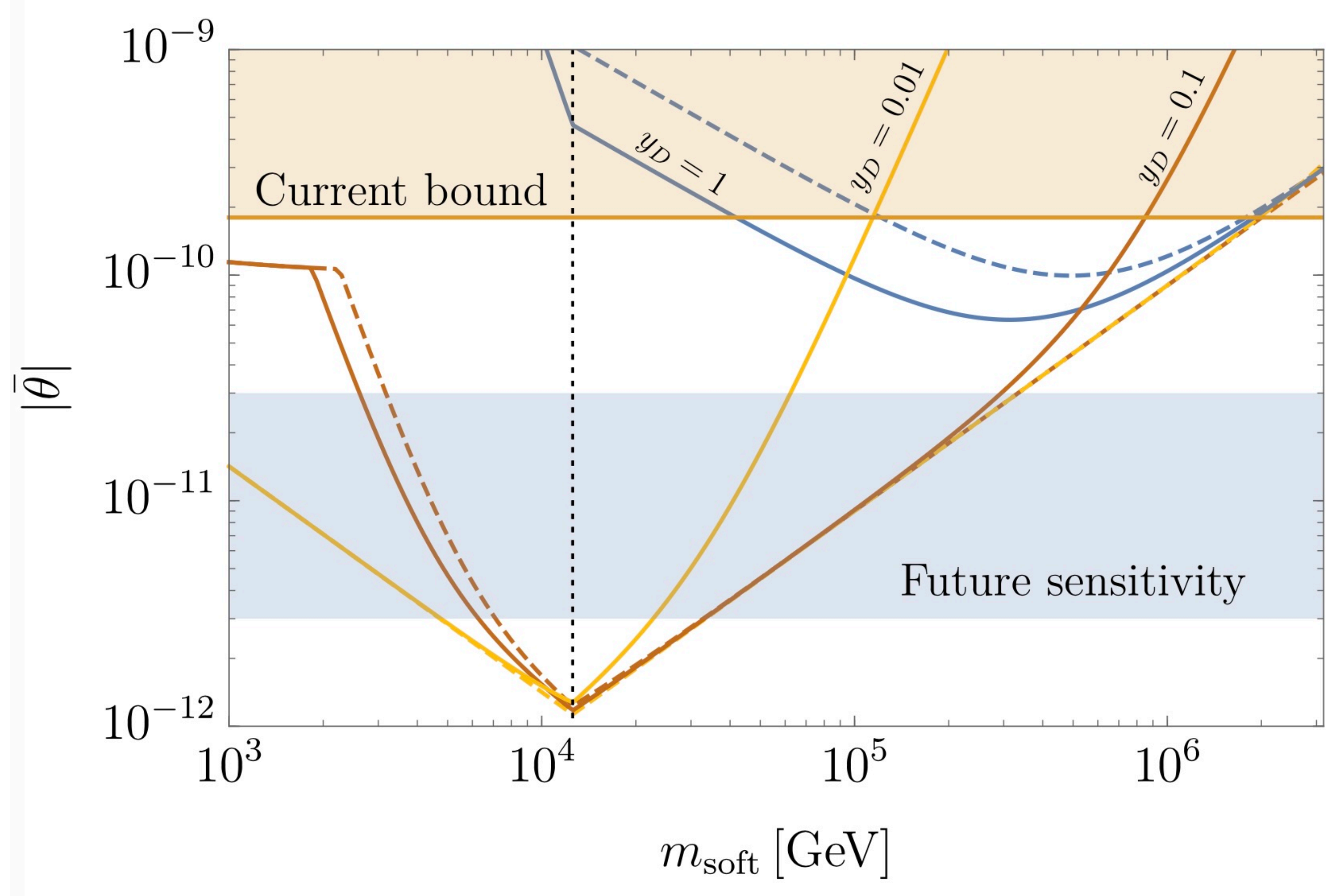
If **gravitino** gives DM …

Lyman-α constraint ➡ $m_{3/2} \gtrsim 5.3\,\mathrm{keV}$

Smallest value of $\bar{\theta}$

SCPV scale and reheating temperature are chosen to obtain the observed asymmetry and DM.

Consistency with SCPV via ~~SUSY~~ will be explored in a future study.



Neutron EDM is within the reach of near-future experiments !

Summary

Composite axion

- UV completion via **special embedding** leads to domain wall # = 1.
- **Small instanton effects** provide a bias term for the domain wall decay.

Spontaneous CP violation (SCPV) in SUSY

- SCPV vacuum stabilization via **SUSY breaking** predicts **light particles feebly interacting with SM**.
- **Neutron EDM** is within the reach of near-future experiments.

Thank you.