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Axion-light hadron interaction in chiral effective field theory

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Introduction

Strong CP problem

$$\mathcal{L}_{\rm QCD} = \sum_q \bar{q}(i\not{D} - m_q e^{i\theta_q})q - \frac{1}{4}G_{\mu\nu}G^{\mu\nu} + \theta\frac{\alpha_s}{8\pi}G_{\mu\nu}\tilde{G}^{\mu\nu}$$

$$q \to e^{i\gamma_5 \alpha} q \quad \longrightarrow \quad \theta_q \to \theta_q + 2\alpha\,, \qquad \theta \to \theta - 2\alpha$$

implying the invariant quantity: $\bar{\theta} = \theta + \theta_q$

$$\Rightarrow \quad \mathcal{L}_{\rm QCD} = \sum_q \bar{q}(i\not{D} - m_q)q - \frac{1}{4}G_{\mu\nu}G^{\mu\nu} + \boxed{\bar{\theta}\frac{\alpha_s}{8\pi}G_{\mu\nu}\tilde{G}^{\mu\nu}}$$

- $G_{\mu\nu}\tilde{G}^{\mu\nu} \sim \partial_\mu K^\mu$: **this total derivative is relevant, due to the nonperturbative QCD vacumm**
- **Naive guess from CKM:** $\theta_{\rm CPV} \sim O(1)$
- **Experimental constraints from neutron EDM:** $\bar{\theta} \leq 10^{-10}$

➢ **Strong CP problem: why $\bar{\theta}$ unnaturally tiny ?**

Peccei-Quinn mechanism to address strong CP problem

[Peccei,Quinn,PRL'77]

[Weinberg,PRL'78] [Wilzeck,PRL'78]

- **Promote constant $\bar{\theta}$ as a dynamical spin-0 field $a(x)$**
- **Impose new global U(1) PQ symmetry (anomalous under QCD)**

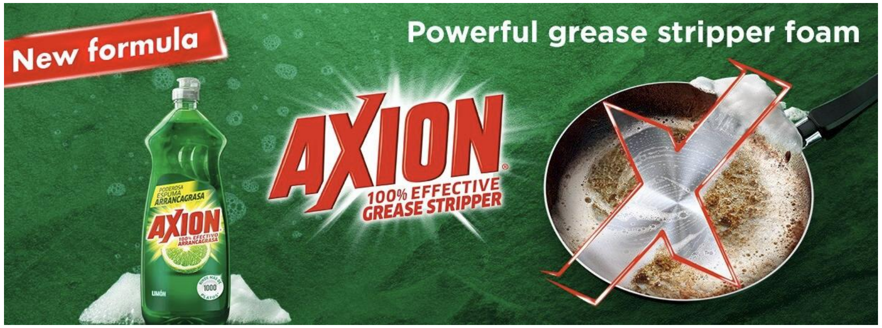
$$\mathcal{L}_a = \frac{1}{2}(\partial_\mu a)^2 + \frac{a}{f_a}\frac{g_s^2}{32\pi^2}G\tilde{G}$$

$$a(x) \to a(x) + \kappa f_a \Longrightarrow S \to S + \frac{g_s^2 \kappa}{32\pi^2}\int d^4x\, G\tilde{G} \quad (\textbf{cancel}\,\bar{\theta}\,\textbf{term})$$

- **Vafa-Witten theorem: VEV of $\langle a \rangle$ =0 in the vector-like theory, such as QCD**
- **Weinberg and Wilczek: PQ mechanism indicates a pseudo-Nambu-Goldstone boson**
- **This pNGB strips off the unwanted strong CP phase: Wilczek names it as Axion**

中文：轴子 (轴矢流耦合)

$$\frac{\partial_\mu a}{2f_a}\bar{q}\gamma^\mu\gamma_5 Q_a q$$



- **Original PQWW axion: $f_a \sim v_{EW} \approx 246$ GeV (visible axion)**

quickly ruled out by experiments: K→π a, J/ψ → γ a , ϒ → γ a, ...

astrophysical constraints: Supernovae, Red giant, ... (NN →NNa)

Generic effective axion Lagrangian for light-flavor quarks

$$\mathcal{L}_{\rm QCD}^{\rm axion} = \bar{q}(i\not{D} - M_q)q - \frac{1}{4}G_{\mu\nu}G^{\mu\nu} + \frac{1}{2}\partial_\mu a \partial^\mu a - \frac{1}{2}m_{a,0}^2 a^2 + \frac{a}{f_a}\frac{\alpha_s}{8\pi}G_{\mu\nu}\tilde{G}^{\mu\nu}$$
$$+ \frac{1}{4}g_{a\gamma}^0 aF\tilde{F} + \frac{\partial_\mu a}{2f_a}\bar{q}c_q^0\gamma^\mu\gamma_5 q$$

Diverse viable axion models

- **PQWW**: Y_{PQ} (SM fermion) $\neq 0$, $f_a \sim v_{EW}$ (ruled out)
- **KSVZ**: Y_{PQ} (SM fermion) $=0$, singlet Higgs and extra BSM fermions,

  $f_a >> v_{EW}$ (invisible axion)

  model-dependent terms vanish: $g_{a\gamma}^0=0$, $c_q^0=0$
- **DFSZ**: Y_{PQ} (SM fermion) $\neq 0$, extra singlet and doublet Higgs, $f_a >> v_{EW}$ (invisible)

  model-dependent terms retain: $g_{a\gamma}^0 \neq 0$, $c_q^0 \neq 0$
- **QCD axion / ALP (axion-like particle)**: bare axion mass term $m_{a,0}=0$ / $m_{a,0}\neq 0$

  ALP case: m_a and f_a are independent

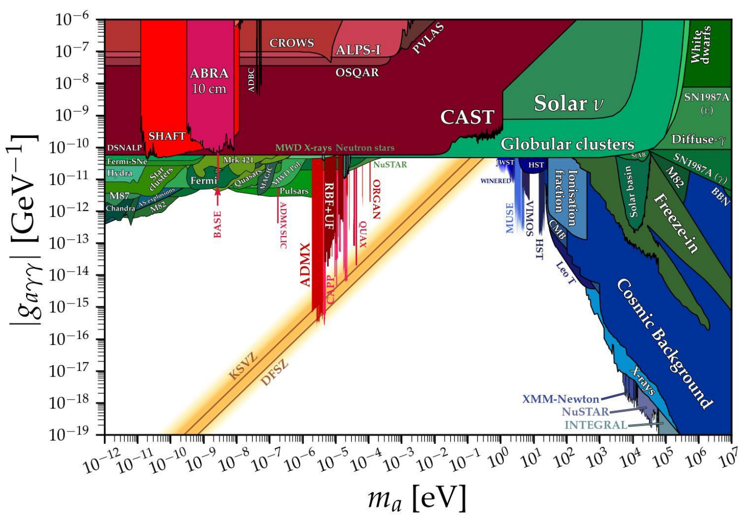
  QCD axion case: $m_a^2 \cong \frac{m_\pi^2 f_\pi^2}{4f_a^2}$ ($f_a >> f_\pi$, axion: a very light BSM particle)

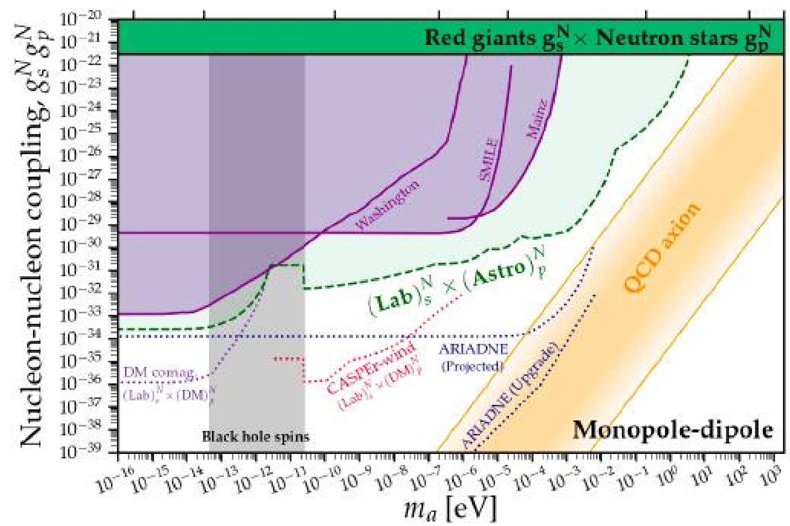
- **Various constraints from rather different experiments**

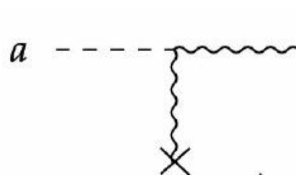
[Di Luzio, et al., Phy.Rep'20] [Sikivie, RMP'21] [Irastorza, Redondo,PPNP'18] ... ...

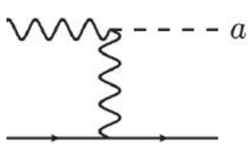
Cosmology, Astronomy, Colliders, Quantum precision measurements , Cavity Haloscope, ...

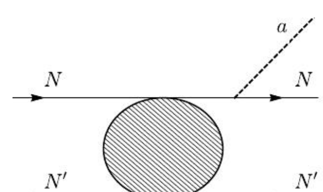
[O'Hare,Github, https://cajohare.github.io/AxionLimits/]

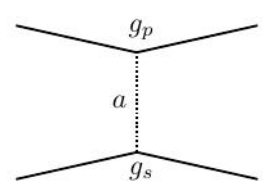












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Axion chiral perturbation theory

Axion chiral perturbation theory (AχPT)

- **We will focus on the QCD-like axion: $m_{a,0}\,(\neq 0) \ll f_a$ with model-independent $aG\tilde{G}$ interaction, i.e., the MODEL INDEPENDENT QCD axion interactions.**

- **Axion-hadron interactions are relevant at low energies.**

$$\mathcal{L}_{\rm QCD}^{\rm axion} = \bar{q}(i\not{D} - M_q)q - \frac{1}{4}G_{\mu\nu}G^{\mu\nu} + \frac{1}{2}\partial_\mu a \partial^\mu a - \frac{1}{2}m_{a,0}^2 a^2 + \boxed{\frac{a}{f_a}\frac{\alpha_s}{8\pi}G_{\mu\nu}\tilde{G}^{\mu\nu}}$$

Two ways to proceed:

(1) Remove the $aG\tilde{G}$ term via the quark axial transformation

$$q \to e^{i\frac{a}{2f_a}\gamma_5 Q_a} q$$

$\mathrm{Tr}(Q_a) = 1$

$$-\frac{a\alpha_s}{8\pi f_a}G\tilde{G} - \frac{\partial_\mu a}{2f_a}\bar{q}\gamma^\mu\gamma_5 Q_a q \qquad M_q \to M_q(a) = e^{-i\frac{a}{2f_a}Q_a} M_q e^{-i\frac{a}{2f_a}Q_a}$$

Mapping to χPT

$$\mathcal{L}_2 = \frac{F^2}{4}\langle D_\mu U D^\mu U^\dagger + \chi_a U^\dagger + U\chi_a^\dagger\rangle + \frac{\partial_\mu a}{2f_a}J_A^\mu|_{\rm LO}$$

$$\chi_a = 2B_0 e^{-i\frac{a}{2f_a}Q_a} M_q e^{-i\frac{a}{2f_a}Q_a} \qquad J_A^\mu|_{\rm LO} = -i\frac{F^2}{2}\langle Q_a(\partial^\mu U U^\dagger + U^\dagger \partial^\mu U)\rangle$$

- $Q_a = M_q^{-1}/\mathrm{Tr}(M_q^{-1})$ [Georgi,Kaplan,Randall, PLB'86]

- $J_A^\mu \partial_\mu a$ [Bauer, et al., PRL'21]

(2) Explicitly keep the $a G\widetilde{G}$ term and match it to χPT

Reminiscent:

QCD $U(1)_A$ anomaly that is caused by topological charge density $\omega(x) = \alpha_s G_{\mu\nu}\tilde{G}^{\mu\nu}/(8\pi)$ **is responsible for the massive singlet η_0.**

Axion could be similarly included as the η_0 via the U(3) χPT:

$$\mathcal{L}^{\rm LO} = \frac{F^2}{4}\langle u_\mu u^\mu \rangle + \frac{F^2}{4}\langle \chi_+ \rangle + \frac{F^2}{12} M_0^2 X^2$$

$$\mathcal{L}^{\rm NLO} = L_5 \langle u^\mu u_\mu \chi_+ \rangle + \frac{L_8}{2}\langle \chi_+\chi_+ + \chi_-\chi_- \rangle - \frac{F^2\,\Lambda_1}{12} D^\mu X D_\mu X - \frac{F^2\,\Lambda_2}{12} X \langle \chi_- \rangle\,,$$

$$U = u^2 = e^{i\frac{\sqrt{2}\Phi}{F}}\,, \qquad \chi = 2B(s+ip)\,, \qquad \chi_\pm = u^\dagger \chi u^\dagger \pm u \chi^\dagger u$$

$$u_\mu = i u^\dagger D_\mu U u^\dagger\,, \quad D_\mu U = \partial_\mu U - i(v_\mu + a_\mu)U + iU(v_\mu - a_\mu)$$

$$X = \log(\det U) - i\,\frac{a}{f_a} \qquad \Phi = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 + \frac{1}{\sqrt{3}}\eta_0 & \pi^+ & K^+ \\ \pi^- & \frac{-1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 + \frac{1}{\sqrt{3}}\eta_0 & K^0 \\ K^- & \overline{K}^0 & \frac{-2}{\sqrt{6}}\eta_8 + \frac{1}{\sqrt{3}}\eta_0 \end{pmatrix}$$

- **Q_a is not needed in U(3) χPT.**
- **$M_0^2 = 6\tau/F^2$, with τ the topological susceptibility. Note that $M_0^2 \sim O(1/N_c)$.**
- **δ expansion scheme: $\delta \sim O(p^2) \sim O(m_q) \sim O(1/N_c)$.**
- **Axion interactions enter via the axion-meson mixing terms.**

π-η-η'-a mixing in U(3) AχPT

[Gao,ZHG,Oller,Zhou,JHEP'23] [Gao,Hao,ZHG,et al.,EPJC'25]

LO

(mass mixing only)

$$\begin{pmatrix} \overline{\pi}^0 \\ \overline{\eta} \\ \overline{\eta}' \\ \overline{a} \end{pmatrix} = \begin{pmatrix} 1+v_{11} & -v_{12} & -v_{13} & -v_{14} \\ v_{12} & 1+v_{22} & -v_{23} & -v_{24} \\ v_{13} & v_{23} & 1+v_{33} & -v_{34} \\ v_{41} & v_{42} & v_{43} & 1+v_{44} \end{pmatrix} \begin{pmatrix} \pi^0 \\ \mathring{\eta} \\ \mathring{\eta}' \\ a \end{pmatrix}$$

$$v_{12} = -\frac{\epsilon}{\sqrt{3}} \frac{c_\theta - \sqrt{2} s_\theta}{m_\pi^2 - m_{\mathring{\eta}}^2}, \qquad v_{13} = -\frac{\epsilon}{\sqrt{3}} \frac{\sqrt{2} c_\theta + s_\theta}{m_\pi^2 - m_{\mathring{\eta}'}^2}, \qquad v_{23} = \frac{\sqrt{2} s_\theta^2 + c_\theta s_\theta - \sqrt{2} c_\theta^2}{3(m_{\mathring{\eta}'}^2 - m_{\mathring{\eta}}^2)} \epsilon, \qquad v_{41} = -\frac{M_0^2 \epsilon}{6(m_a^2 - m_\pi^2)} \frac{F}{f_a} \left[-\frac{(\sqrt{2} c_\theta - 2 s_\theta) s_\theta}{m_a^2 - m_{\mathring{\eta}}^2} + \frac{c_\theta (2 c_\theta + \sqrt{2} s_\theta)}{m_a^2 - m_{\mathring{\eta}'}^2} \right]$$

$$v_{42} = \frac{M_0^2 s_\theta}{\sqrt{6}(m_a^2 - m_{\mathring{\eta}}^2)} \frac{F}{f_a} - \frac{M_0^2 \epsilon}{3\sqrt{6}(m_a^2 - m_{\mathring{\eta}}^2)} \frac{F}{f_a} \left[\frac{c_\theta(-\sqrt{2} c_\theta^2 + c_\theta s_\theta + \sqrt{2} s_\theta^2)}{m_a^2 - m_{\mathring{\eta}'}^2} - \frac{s_\theta (2 c_\theta^2 + 2\sqrt{2} c_\theta s_\theta + s_\theta^2)}{m_a^2 - m_{\mathring{\eta}}^2} \right]$$

$$v_{43} = -\frac{M_0^2 c_\theta}{\sqrt{6}(m_a^2 - m_{\mathring{\eta}'}^2)} \frac{F}{f_a} - \frac{M_0^2 \epsilon}{3\sqrt{6}(m_a^2 - m_{\mathring{\eta}}^2)} \frac{F}{f_a} \left[\frac{c_\theta (c_\theta^2 - 2\sqrt{2} c_\theta s_\theta + 2 s_\theta^2)}{m_a^2 - m_{\mathring{\eta}'}^2} - \frac{s_\theta(-\sqrt{2} c_\theta^2 + c_\theta s_\theta + \sqrt{2} s_\theta^2)}{m_a^2 - m_{\mathring{\eta}}^2} \right]$$

... ...

with

$$m_{\mathring{\eta}}^2 = \frac{M_0^2}{2} + m_K^2 - \frac{\sqrt{M_0^4 - \frac{4 M_0^2 \Delta^2}{3} + 4\Delta^4}}{2}, \qquad m_{\mathring{\eta}'}^2 = \frac{M_0^2}{2} + m_K^2 + \frac{\sqrt{M_0^4 - \frac{4 M_0^2 \Delta^2}{3} + 4\Delta^4}}{2}, \qquad \sin\theta = -\left(\sqrt{1 + \frac{(3M_0^2 - 2\Delta^2 + \sqrt{9 M_0^4 - 12 M_0^2 \Delta^2 + 36 \Delta^4})^2}{32 \Delta^4}} \right)^{-1}$$

Physical masses after diagnolization

$$m_\eta^2 = m_{\mathring{\eta}}^2 + \frac{\epsilon}{3}(\sqrt{2} c_\theta + s_\theta)^2 + O(\epsilon^2) \qquad m_{\eta'}^2 = m_{\mathring{\eta}'}^2 + \frac{\epsilon}{3}(c_\theta - \sqrt{2} s_\theta)^2 + O(\epsilon^2)$$

$$\begin{aligned} m_a^2 = m_{a,0}^2 &+ \frac{M_0^2 F^2}{6 f_a^2} \left[1 + \frac{c_\theta^2 M_0^2}{m_{a,0}^2 - m_{\mathring{\eta}'}^2} + \frac{s_\theta^2 M_0^2}{m_{a,0}^2 - m_{\mathring{\eta}}^2} \right] \\ &+ \frac{M_0^4 F^2 \epsilon}{9 f_a^2} \left[\frac{s_\theta^2 (\sqrt{2} c_\theta + s_\theta)^2}{2(m_{a,0}^2 - m_{\mathring{\eta}}^2)^2} + \frac{c_\theta^2 (c_\theta - \sqrt{2} s_\theta)^2}{2(m_{a,0}^2 - m_{\mathring{\eta}'}^2)^2} \right. \\ &\left. + \frac{c_\theta s_\theta (\sqrt{2} c_\theta^2 - c_\theta s_\theta - \sqrt{2} s_\theta^2)}{(m_{a,0}^2 - m_{\mathring{\eta}}^2)(m_{a,0}^2 - m_{\mathring{\eta}'}^2)} \right] + O(\epsilon^2), \end{aligned}$$

$$\boldsymbol{m_a^2 = \frac{m_\pi^2 F^2}{4 f_a^2}}$$

[Weinberg,PRL'78]

(keep LO terms in m_π/m_K & m_π/M_0 & ε expansions)

NLO: (kinetic & mass mixing)

$$\mathcal{L} = \frac{1+\delta_k^{\eta}}{2}\partial_\mu\overline{\eta}\partial^\mu\overline{\eta} + \frac{1+\delta_k^{\eta'}}{2}\partial_\mu\overline{\eta}'\partial^\mu\overline{\eta}' + \delta_k^{\eta\eta'}\partial_\mu\overline{\eta}\partial^\mu\overline{\eta}' - \frac{m_{\overline{\eta}}^2+\delta_{m_\eta^2}}{2}\overline{\eta}\,\overline{\eta} - \frac{m_{\overline{\eta}'}^2+\delta_{m_{\eta'}^2}}{2}\overline{\eta}'\,\overline{\eta}' - \delta_{m^2}^{\eta\eta'}\overline{\eta}\,\overline{\eta}'$$
$$+ \frac{1+\delta_k^{\pi}}{2}\partial_\mu\overline{\pi}^0\partial^\mu\overline{\pi}^0 + \delta_k^{\pi\eta}\partial_\mu\overline{\pi}^0\partial^\mu\overline{\eta} + \delta_k^{\pi\eta'}\partial_\mu\overline{\pi}^0\partial^\mu\overline{\eta}' - \frac{m_{\overline{\pi}}^2+\delta_{m_\pi^2}}{2}\overline{\pi}^0\,\overline{\pi}^0 - \delta_{m^2}^{\pi\eta}\overline{\pi}^0\overline{\eta} - \delta_{m^2}^{\pi\eta'}\overline{\pi}^0\overline{\eta}'$$
$$+ \frac{1+\delta_k^{a}}{2}\partial_\mu\overline{a}\partial^\mu\overline{a} + \delta_k^{a\pi}\partial_\mu\overline{a}\partial^\mu\overline{\pi}^0 + \delta_k^{a\eta}\partial_\mu\overline{a}\partial^\mu\overline{\eta} + \delta_k^{a\eta'}\partial_\mu\overline{a}\partial^\mu\overline{\eta}' - \frac{m_{\overline{a}}^2+\delta_{m_a^2}}{2}\overline{a}\,\overline{a} - \delta_{m^2}^{a\pi}\overline{a}\,\overline{\pi}^0$$
$$- \delta_{m^2}^{a\eta}\overline{a}\,\overline{\eta} - \delta_{m^2}^{a\eta'}\overline{a}\,\overline{\eta}'$$

Separately handle the kinetic (x_{ij}) and mass (y_{ij}) mixing terms

$$\begin{pmatrix}\hat{\pi}^0\\ \hat{\eta}\\ \hat{\eta}'\\ \hat{a}\end{pmatrix} = \begin{pmatrix}1 & -y_{12} & -y_{13} & -y_{14}\\ y_{12} & 1 & -y_{23} & -y_{24}\\ y_{13} & y_{23} & 1 & -y_{34}\\ y_{14} & y_{24} & y_{34} & 1\end{pmatrix} \times \begin{pmatrix}1-x_{11} & -x_{12} & -x_{13} & -x_{14}\\ -x_{12} & 1-x_{22} & -x_{23} & -x_{24}\\ -x_{13} & -x_{23} & 1-x_{33} & -x_{34}\\ -x_{14} & -x_{24} & -x_{34} & 1-x_{44}\end{pmatrix}\begin{pmatrix}\overline{\pi}^0\\ \overline{\eta}\\ \overline{\eta}'\\ \overline{a}\end{pmatrix}$$

$$x_{11} = -\frac{\delta_k^{\pi}}{2},\quad x_{12} = -\frac{\delta_k^{\pi\eta}}{2},\quad x_{13} = -\frac{\delta_k^{\pi\eta'}}{2},\quad x_{14} = -\frac{\delta_k^{a\pi}}{2},\quad x_{22} = -\frac{\delta_k^{\eta}}{2},$$
$$x_{23} = -\frac{\delta_k^{\eta\eta'}}{2},\quad x_{24} = -\frac{\delta_k^{a\eta}}{2},\quad x_{33} = -\frac{\delta_k^{\eta'}}{2},\quad x_{34} = -\frac{\delta_k^{a\eta'}}{2},\quad x_{44} = -\frac{\delta_k^{a}}{2},$$

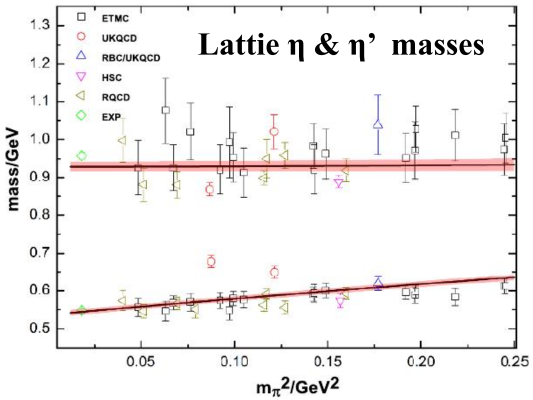
$\delta_X \sim L_5, L_8, \Lambda_1, \Lambda_2$

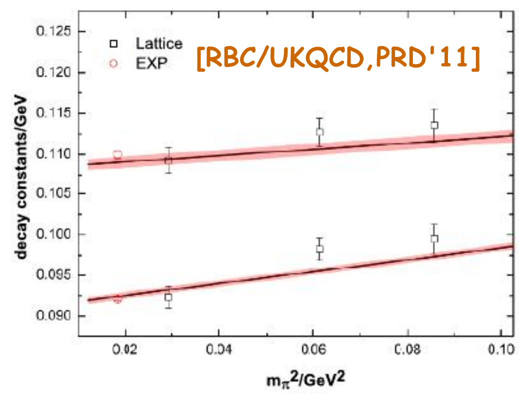
$$y_{12} = \frac{\delta_{m^2}^{\pi\eta} + x_{12}(m_{\overline{\eta}}^2 + m_{\overline{\pi}}^2)}{m_{\overline{\eta}}^2 - m_{\overline{\pi}}^2},\quad y_{13} = \frac{\delta_{m^2}^{\pi\eta'} + x_{13}(m_{\overline{\eta}'}^2 + m_{\overline{\pi}}^2)}{m_{\overline{\eta}'}^2 - m_{\overline{\pi}}^2},\quad y_{14} = \frac{\delta_{m^2}^{a\pi} + x_{14}(m_{a,0}^2 + m_{\overline{\pi}}^2)}{m_{a,0}^2 - m_{\overline{\pi}}^2},$$
$$y_{23} = \frac{\delta_{m^2}^{\eta\eta'} + x_{23}(m_{\overline{\eta}}^2 + m_{\overline{\eta}'}^2)}{m_{\overline{\eta}'}^2 - m_{\overline{\eta}}^2},\quad y_{24} = \frac{\delta_{m^2}^{a\eta} + x_{24}(m_{\overline{\eta}}^2 + m_{a,0}^2)}{m_{a,0}^2 - m_{\overline{\eta}}^2},\quad y_{34} = \frac{\delta_{m^2}^{a\eta'} + x_{34}(m_{\overline{\eta}'}^2 + m_{a,0}^2)}{m_{a,0}^2 - m_{\overline{\eta}'}^2}.$$

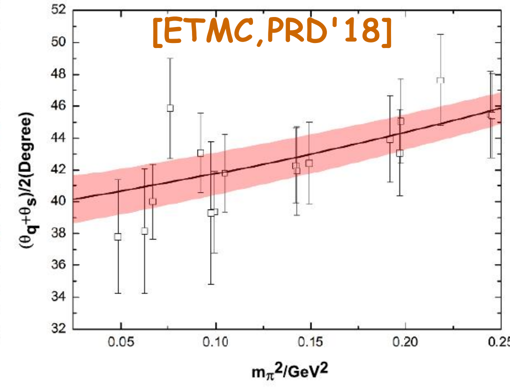
Fit to lattice data

[Gao,ZHG,Oller,Zhou,JHEP'23] [Gao,Hao,ZHG,et al.,EPJC'25]

Parameters	NLO Fit
F(MeV)	$91.05^{+0.42}_{-0.44}$
$10^3 \times L_5$	$1.68^{+0.05}_{-0.06}$
$10^3 \times L_8$	$0.88^{+0.04}_{-0.04}$
Λ_1	$-0.17^{+0.05}_{-0.05}$
Λ_2	$0.06^{+0.08}_{-0.09}$
$\chi^2/(\text{d.o.f.})$	$219.9/(111\text{–}5)$







Mixing pattern@NLO

$$\begin{pmatrix} \hat{\pi}^0 \\ \hat{\eta} \\ \hat{\eta}' \\ \hat{a} \end{pmatrix} = M^{\rm LO+NLO} \begin{pmatrix} \pi^0 \\ \eta_8 \\ \eta_0 \\ a \end{pmatrix}$$

$$M^{\rm LO+NLO} = \begin{pmatrix} 1+(0.015\pm0.001) & 0.017+(-0.007\pm0.001) & 0.009+(-0.011\pm0.001) & \frac{-12.8+(-0.13\pm0.02)}{f_a} \\ -0.019+(0.005\pm0.001) & 0.94+(0.21\pm0.01) & 0.33+(-0.21\pm0.03) & \frac{-34.3+(1.7^{+0.8}_{-0.7})}{f_a} \\ -0.003+(-0.001\pm0.000) & -0.33+(-0.18\pm0.02) & 0.94+(0.13^{+0.01}_{-0.02}) & \frac{-25.9+(0.2^{+0.4}_{-0.3})}{f_a} \\ \frac{12.1+(0.5\pm0.1)}{f_a} & \frac{23.8+(1.0^{+0.2}_{-0.1})}{f_a} & \frac{35.7+(1.7^{+0.2}_{-0.1})}{f_a} & 1+\frac{-921.5+(-56.6^{+7.9}_{-9.6})}{f_a^2} \end{pmatrix}$$

Mass decomposition@NLO

$$\begin{aligned} m_{\hat{\pi}} &= \left[134.9+(0.1\pm0.07)\right]\text{MeV}, \\ m_{\hat{K}} &= \left[492.1+(5.1^{+3.4}_{-3.3})\right]\text{MeV}, \\ m_{\hat{\eta}} &= \left[490.4+(61.1^{+10.0}_{-8.7})\right]\text{MeV}, \\ m_{\hat{\eta}'} &= \left[954.5+(-28.5^{+11.9}_{-10.9})\right]\text{MeV}, \\ m_{\hat{a}} &= \left[5.96+(0.12\pm0.02)\right]\mu\,\text{eV}\,\frac{10^{12}\,\text{GeV}}{f_a}, \end{aligned}$$

Two-photon couplings

$$\mathcal{L}_{WZW}^{\rm LO} = -\frac{3\sqrt{2}}{8\pi^2 F}\varepsilon_{\mu\nu\rho\sigma}\partial^\mu A^\nu \partial^\rho A^\sigma \langle Q^2\Phi\rangle, \qquad Q = {\rm Diag}(\tfrac{2e}{3}, -\tfrac{e}{3}, -\tfrac{e}{3})$$

$$\mathcal{L}_{WZW}^{\rm NLO} = t_1\frac{32\sqrt{2}B}{F}\varepsilon_{\mu\nu\rho\sigma}\partial^\mu A^\nu \partial^\rho A^\sigma \langle\left(M_q\Phi + \Phi M_q\right)Q^2\rangle + 16k_3\varepsilon_{\mu\nu\rho\sigma}\partial^\mu A^\nu \partial^\rho A^\sigma \langle Q^2\rangle\left(\frac{\sqrt{2}}{F}\langle\Phi\rangle - \frac{a}{f_a}\right)$$

* **Note: one needs the π-η-η'-*a* mixing as input to calculate $g_{a\gamma\gamma}$**

$$\begin{aligned} F_{\pi^0\gamma\gamma}^{\rm Exp} &= 0.274 \pm 0.002{\rm GeV}^{-1}, \\ F_{\eta\gamma\gamma}^{\rm Exp} &= 0.274 \pm 0.006{\rm GeV}^{-1}, \\ F_{\eta'\gamma\gamma}^{\rm Exp} &= 0.344 \pm 0.008{\rm GeV}^{-1}, \end{aligned}$$

→

$$\begin{aligned} t_1 &= -(3.8 \pm 2.4)\times 10^{-4}{\rm GeV}^{-2}, \\ k_3 &= (1.21 \pm 0.23)\times 10^{-4} \end{aligned}$$

isospin limit(LO) isospin breaking(LO) NLO

→

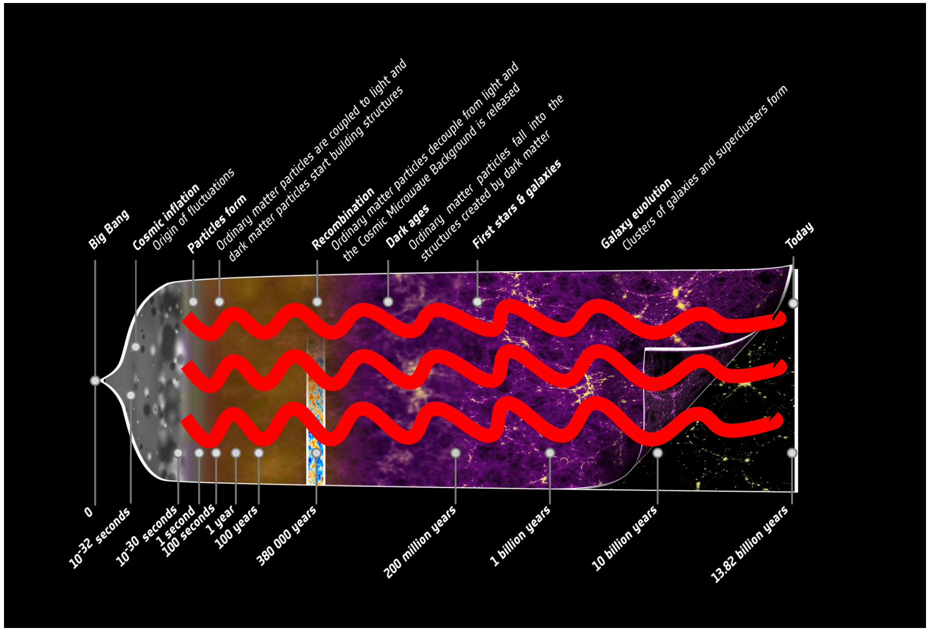
$$F_{a\gamma\gamma} = \frac{20.1 + 3.4 + (0.5 \pm 0.2)}{f_a}\times 10^{-3},$$

(IB corrections amount to be around 15%!)

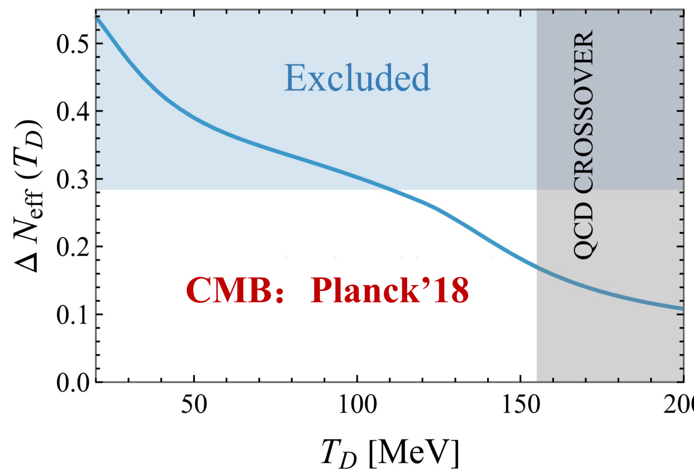
$$g_{a\gamma\gamma} = 4\pi\alpha_{em}F_{a\gamma\gamma} = \frac{\alpha_{em}}{2\pi f_a}\left(1.89 \pm 0.02\right).$$

which can be compared to: 1.92±0.04 [Grilli de Cortona, et al., JHEP'16] **and 2.05±0.03** [Lu, et al., JHEP'20]

Cosmology constraints on axion thermalization rate



- **Axions can be copiously produced from thermal bath in the early Universe.**
- **After decoupling, its thermal relics will leave imprints today.**



- Axion thermalization rate from reaction: $\underbrace{a + \cdots}_{i} \leftrightarrow \underbrace{1 + 2 + 3 + \cdots}_{j}$

$$\Gamma_a(T) = \frac{1}{n_a^{eq}} \int \left[\prod_i \frac{\mathrm{d}^3 p_i}{(2\pi)^3 2E_i}\right] \left[\prod_j \frac{\mathrm{d}^3 p_j}{(2\pi)^3 2E_j}\right] (2\pi)^4 \delta^4 \left(\sum_i p_i - \sum_j p_j\right) |\mathcal{M}_{\rm reaction}|^2 \prod_i f_i(x_i) \prod_j [1 \pm f_j(x_j)]$$

$$f_i(x_i) = \begin{cases} \frac{1}{e^{x_i} - 1}, & \text{bosonic,} \\ \frac{1}{e^{x_i} + 1}, & \text{fermionic,} \end{cases} \qquad x_i = E_i/T$$

"+" for Bose enhancement
"−" for Pauli blocking

Cosmology constraints on axion thermalization rate

Axion thermal production in the early Universe : Extra radiation (ΔN_{eff})

Extra effective number of relativisitc d.o.f :

$$\Delta N_{\rm eff} \simeq \frac{4}{7}\left(\frac{43}{4g_{\star s}(T_D)}\right)^{\frac{4}{3}}$$

$g_{\star s}(T)$: effective number of entropy d.o.f at temperature T

T_D: axion decoupling temperature from the thermal medium

- **CMB constraint (Plank'18)** [Aghanim et al., 2020] **:** $\Delta N_{eff} \leq 0.28$
- **T_D : Instantaneous decoupling approximation**

$$\Gamma_a(T_D) = H(T_D)$$

Axion thermalization rate

Hubble expansion parameter

$$\Gamma_a(T) = \frac{1}{n_a^{\rm eq}} \int {\rm d}\widetilde{\Gamma}\, |\mathcal{M}_{a-{\rm SM}}|^2 \prod_i f_i(x_i) \prod_j [1 \pm f_j(x_j)]$$

$$H(T) = T^2 \sqrt{4\pi^3 g_*(T)/45}/m_{\rm Pl}$$

Axion-SM particle scattering amplitudes

Key thermal channels of axion-SM scatterings at different temperatures

- ☞ $T_D \gtrsim 1$ GeV: $ag \leftrightarrow gg$.
  [Masso et al., 2002, Graf and Steffen, 2011]
- ☞ $T_D \lesssim 1$ GeV: Hadrons need to be included.
- ☞ $T_D \lesssim 200$ MeV: $a\pi \leftrightarrow \pi\pi$.
  [Chang and Choi, 1993, Hannestad et al., 2005, Giusarma et al., 2014, D'Eramo et al., 2022]

❑ **Reliable *aπ* interaction is crucial to determine Γ_a for $T_D < T_c \approx 155$ MeV**

- ➢ For a long time, only the LO $a\pi \leftrightarrow \pi\pi$ amplitude is employed to calculate Γ_a , e.g.,
  [Chang, Choi, PLB'93] [Hannestad, et al., JCAP'05] [Hannestad, et al., JCAP'05] [D'Eramo, et al., PRL'22] ... ...
- ➢ Recent NLO calculation of Γ_a : χPT invalid for $T_\chi > 70$ MeV **[Di Luzio, et al., PRL'21]**
- ➢ Chiral unitarization approach for $a\pi \leftrightarrow \pi\pi$: **[Di Luzio, et al., PRD'23]**
- ➢ **However, all the previous works have ignored thermal corrections to the *aπ* ↔*ππ* amplitudes. Fhe first estimation of such effect is given: [Wang, ZHG, Zhou, PRD'24]**

$$\Gamma_a(T) = \frac{1}{n_a^{\rm eq}} \int d\tilde{\Gamma} \boxed{|\mathcal{M}_{a\pi;\pi\pi}|^2} \prod_i f_i(x_i) \prod_j [1 + f_j(x_j)]$$

- ➢ **First realistic calculation of *aK* ↔*πK* shows significant contribution to axion thermalization rate: [Wang, ZHG, Zhou, PRD-Letter'25]**

Calculation of THERMAL *aπ ↔ππ* amplitudes at one-loop level

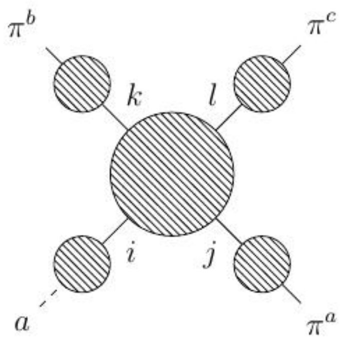
- Finite-temperature effects are included by imaginary time formalism (ITF), where
  [Kapusta and Gale, 2011, Bellac, 2011, Laine and Vuorinen, 2016]

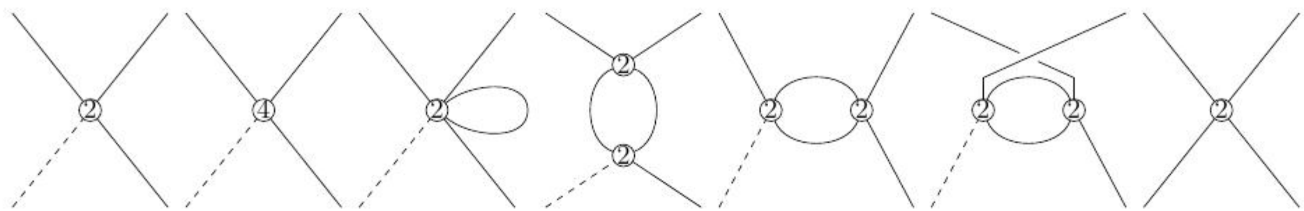
$$p^0 \to i\omega_n , \quad \text{with } \omega_n = 2\pi n T \, , n \in \mathbb{Z} \, ,$$

$$-i \int \frac{\mathrm{d}^d q}{(2\pi)^d} \to -i \int_\beta \frac{\mathrm{d}^d q}{(2\pi)^d} \equiv T \sum_n \int \frac{\mathrm{d}^{d-1} q}{(2\pi)^{d-1}} \, .$$

- Compute the thermal Green functions in ITF

$$G^T_{a\pi^a;\pi^b\pi^c}(p_1,p_2;p_3,p_4) = \sum_{i,j,k,l} G_{ai}(p_1^2) G_{\pi^a j}(p_2^2) G_{k\pi^b}(p_3^2) G_{l\pi^c}(p_4^2) A_{ij;kl}(p_1,p_2;p_3,p_4) \, .$$





Feynman diagrams for amputated functions up to NLO.

- The effective Lagrangian at $\mathcal{O}(p^4)$

$$\begin{aligned}
\mathcal{L}_4 \supset & \frac{l_3 + l_4}{16} \left\langle \chi_a U^\dagger + U\chi_a^\dagger \right\rangle \left\langle \chi_a U^\dagger + U\chi_a^\dagger \right\rangle + \frac{l_4}{8} \left\langle \partial_\mu U \partial^\mu U^\dagger \right\rangle \left\langle \chi_a U^\dagger + U\chi_a^\dagger \right\rangle \\
& - \frac{l_7}{16} \left\langle \chi_a U^\dagger - U\chi_a^\dagger \right\rangle \left\langle \chi_a U^\dagger - U\chi_a^\dagger \right\rangle + \frac{h_1 - h_3 - l_4}{16} \left[\left(\left\langle \chi_a U^\dagger + U\chi_a^\dagger \right\rangle \right)^2 \right. \\
& \left. + \left(\left\langle \chi_a U^\dagger - U\chi_a^\dagger \right\rangle \right)^2 - 2 \left\langle \chi_a U^\dagger \chi_a U^\dagger + U\chi_a^\dagger U \chi_a^\dagger \right\rangle \right] + \frac{\partial_\mu a}{2 f_a} J_A^\mu |_{\mathrm{NLO}} \, ,
\end{aligned}$$

$$\begin{aligned}
J_A^\mu|_{\mathrm{NLO}} \supset & - i l_1 \left\langle Q_a \left\{ \partial^\mu U, U^\dagger \right\} \right\rangle \left\langle \partial_\nu U \partial^\nu U^\dagger \right\rangle \\
& - i \frac{l_2}{2} \left\langle Q_a \left\{ \partial_\nu U, U^\dagger \right\} \right\rangle \left\langle \partial^\mu U \partial^\nu U^\dagger + \partial^\nu U \partial^\mu U^\dagger \right\rangle \\
& - i \frac{l_4}{4} \left\langle Q_a \left\{ \partial^\mu U, U^\dagger \right\} \right\rangle \left\langle \chi_a U^\dagger + U\chi_a^\dagger \right\rangle \, .
\end{aligned}$$

Unitarization of the partial-wave $a\pi \leftrightarrow \pi\pi$ amplitude

Inverse amplitude method (IAM)

$$\mathcal{M}^{\rm IAM}_{a\pi;IJ} = \frac{\left(\mathcal{M}^{(2)}_{a\pi;IJ}\right)^2}{\mathcal{M}^{(2)}_{a\pi;IJ} - \mathcal{M}^{(4)}_{a\pi;IJ}}$$

$$\mathcal{M}_{a\pi;IJ}(E_{cm}) = \frac{1}{2}\int_{-1}^{+1} {\rm d}\cos\theta\, \mathcal{M}_{a\pi;I}(E_{cm},\cos\theta) P_J(\cos\theta)$$

$${\rm Im}\mathcal{M}_{a\pi;IJ}(E_{cm}) \overset{s}{=} \frac{1}{2}\rho^T_{\pi\pi}(E_{cm})\mathcal{M}^{IJ^*}_{\pi\pi;\pi\pi}\mathcal{M}_{a\pi;IJ}\,, \quad (E_{cm} > 2m_\pi)$$

$$\rho^T_{\pi\pi}(E_{cm}) = \frac{\sigma_\pi(E^2_{cm})}{16\pi}\left[1 + 2n_B(\frac{E_{cm}}{2})\right]\,, \quad \sigma_\pi(s) = \sqrt{1 - \frac{4m_\pi^2}{s}}\,, \quad n_B(E) = \frac{1}{e^{E/T} - 1}$$

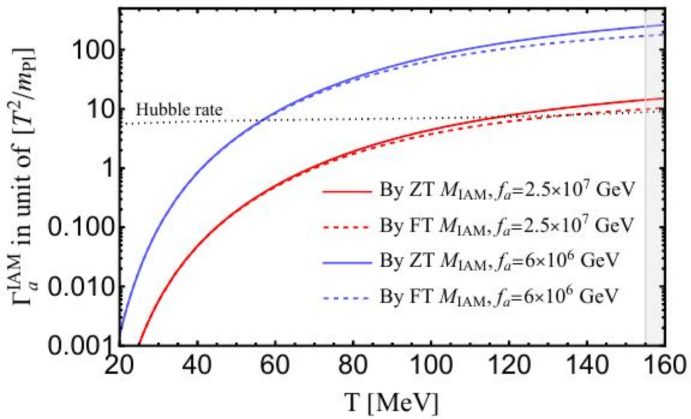
- Resonances poles on the second Riemann sheet

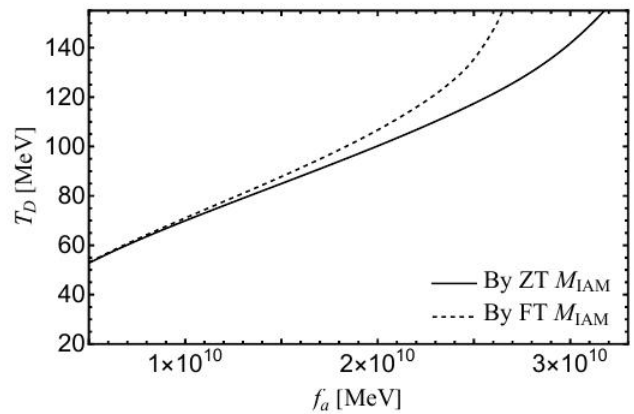
	$f_0(500)/\sigma$		$\rho(770)$	
	$M_\sigma \pm i\frac{\Gamma_\sigma}{2}$	$\lvert f_a g_{\sigma a\pi}\rvert$	$M_\rho \pm i\frac{\Gamma_\rho}{2}$	$\lvert f_a g_{\rho a\pi}\rvert$
$T = 0$ MeV	$422 \pm i240$ MeV	0.032 GeV^2	$739 \pm i72$ MeV	0.035 GeV^2
$T = 100$ MeV*	$368 \pm i310$ MeV	0.037 GeV^2	$744 \pm i77$ MeV	0.036 GeV^2

*Only include s-channel unitary thermal correction.

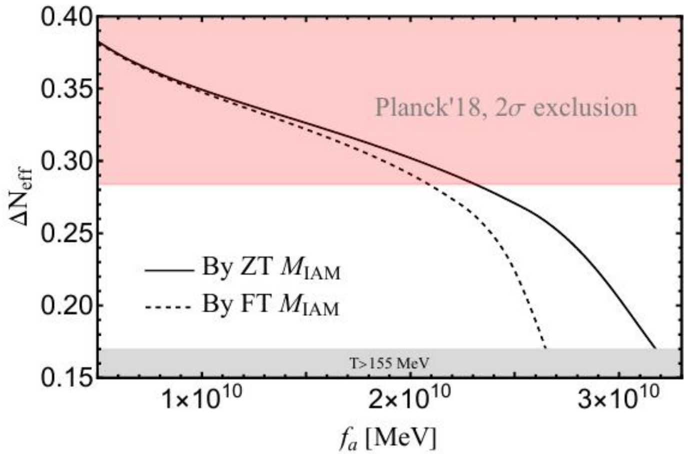
Resulst from the thermal-IAM improved axion rates

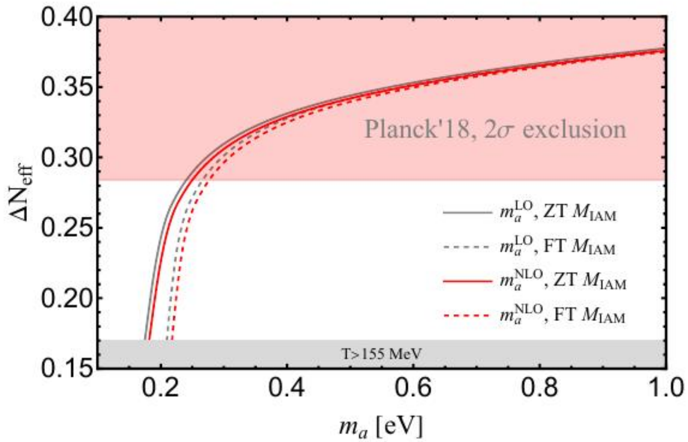
[Wang,ZHG,Zhou,PRD'24]





Updated bounds on the axion parameters





- The constrians

10% corrections are observed

	lower limit of f_a	upper limit of m_a by $m_a^{\rm LO}$	upper limit of m_a by $m_a^{\rm NLO}$
ZT	2.3×10^7 GeV	0.24 eV	0.25 eV
FT	2.1×10^7 GeV	0.27 eV	0.28 eV

Combined analyses with $a\pi \leftrightarrow \pi\pi$ & $aK \leftrightarrow \pi K$ channels

SU(3) Axion ChPT@LO **[Wang, ZHG, Zhou, PRD-Letter'25]**

$$\mathcal{L}_2 = \frac{F_\pi^2}{4}\langle \partial_\mu U \partial^\mu U^\dagger + \chi(a) U^\dagger + U \chi^\dagger(a)\rangle - \frac{\partial_\mu a}{2f_a}\sum_{i=1}^{8} C_i J^\mu_{A,i}$$

$$\chi(a) = 2B_0 e^{-i\frac{a}{2f_a}Q_a} M_q e^{-i\frac{a}{2f_a}Q_a} \qquad Q_a = M_q^{-1}/\langle M_q^{-1}\rangle \qquad J^\mu_{A,i} = i\frac{F_\pi^2}{4}\langle \lambda_i\{\partial^\mu U, U^\dagger\}\rangle$$

(singlet component of axial currents neglected)

$$C_3 = \frac{z(1-r^2)}{2r + z(1+r)^2}, \qquad C_8 = \frac{z(1+r)^2 - 4r}{\sqrt{3}\,[2r + z(1+r)^2]} \qquad z = \frac{m_s}{\hat{m}}, r = \frac{m_u}{m_d}, \hat{m} = \frac{m_u + m_d}{2}$$

Unitarized partial-wave axion-meson/meson-meson amplitudes

Unitarized meson-meson Amp:

$$T^{\rm uni}_{IJ} = T^{(2)}_{IJ}\cdot\left[T^{(2)}_{IJ} - T^{(4)\rm LECs}_{IJ} - T^{(2)}_{IJ}\cdot\mathcal{G}\cdot T^{(2)}_{IJ}\right]^{-1}\cdot T^{(2)}_{IJ}$$

$$\mathrm{Im}T = T^\dagger \cdot q/(8\pi\sqrt{s})\cdot T$$

Unitarized axion-meson Amp:

$$\vec{M}^{\rm uni}_{IJ} = T^{(2)}_{IJ}\cdot\left[T^{(2)}_{IJ} - T^{(4)\rm LECs}_{IJ} - T^{(2)}_{IJ}\cdot\mathcal{G}\cdot T^{(2)}_{IJ}\right]^{-1}\cdot \vec{M}^{(2)}_{IJ}$$

$$\mathrm{Im}\vec{M} = T^\dagger \cdot q/(8\pi\sqrt{s})\cdot \vec{M}$$

$$\mathcal{G} = \mathrm{diag}(G_n, G_m, \cdots) \qquad G_n(s) = G(a^n_{\rm sc}, s, m_{n_1}, m_{n_2}) = -\frac{1}{(4\pi)^2}\left[a^n_{\rm sc} - 1 + \log\frac{m^2_{n_2}}{\mu^2} + \frac{m_{i}^{2}}{} - \frac{\sqrt{\lambda(s, m^2_{n_1}, m^2_{n_2})}}{s}\log\frac{m^2_{n_1} + m^2_{n_2} - s + \sqrt{\lambda(s, m^2_{n_1}, m^2_{n_2})}}{2m_{n_1}m_{n_2}}\right]$$

Example:

$$\begin{pmatrix} M^{uni}_{00,\pi\pi} \\ M^{uni}_{00,KK}\end{pmatrix} = \begin{pmatrix} \widehat{T}^{\pi\pi\to\pi\pi}_{00} & \widehat{T}^{\pi\pi\to KK}_{00} \\ \widehat{T}^{\pi\pi\to KK}_{00} & \widehat{T}^{KK\to KK}_{00}\end{pmatrix}\begin{pmatrix} M^{(2)}_{00,\pi\pi} \\ M^{(2)}_{00,KK}\end{pmatrix}$$

Relevant channels: $S + P$ waves

(1) $a\,\pi^0 \to \pi^+ \pi^-$, $\pi^0 \pi^0$

IJ=00: $f_0(500), f_0(980)$ [K-Kbar coupled-channel included]

IJ=20: nonresonant case [single ππ channel]

(2) $a\,\pi^+ \to \pi^+ \pi^0$

IJ=11: $\rho(770)$ [K-Kbar coupled-channel included]

IJ=20: nonresonant case [single ππ channel, same as $a\pi^0$ case]

(3) $a\,K^+ \to \pi^+ K^0$, $\pi^0 K^+$

IJ=1/2 1: $K^*(892)$ [Kη coupled-channel included]

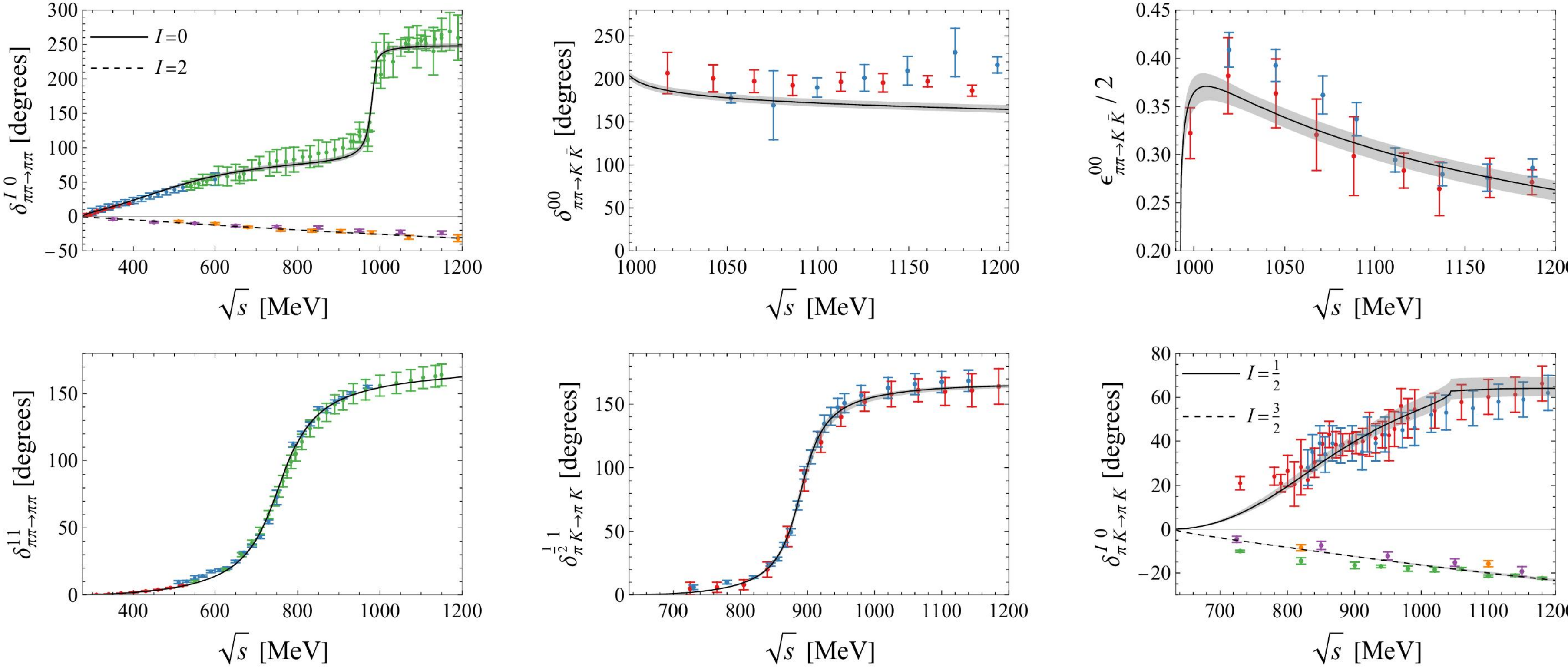
IJ=1/2 0: $K^*_0(700)$ [Kη coupled-channel included]

IJ=3/2 0: nonresonant case [single Kπ channel]

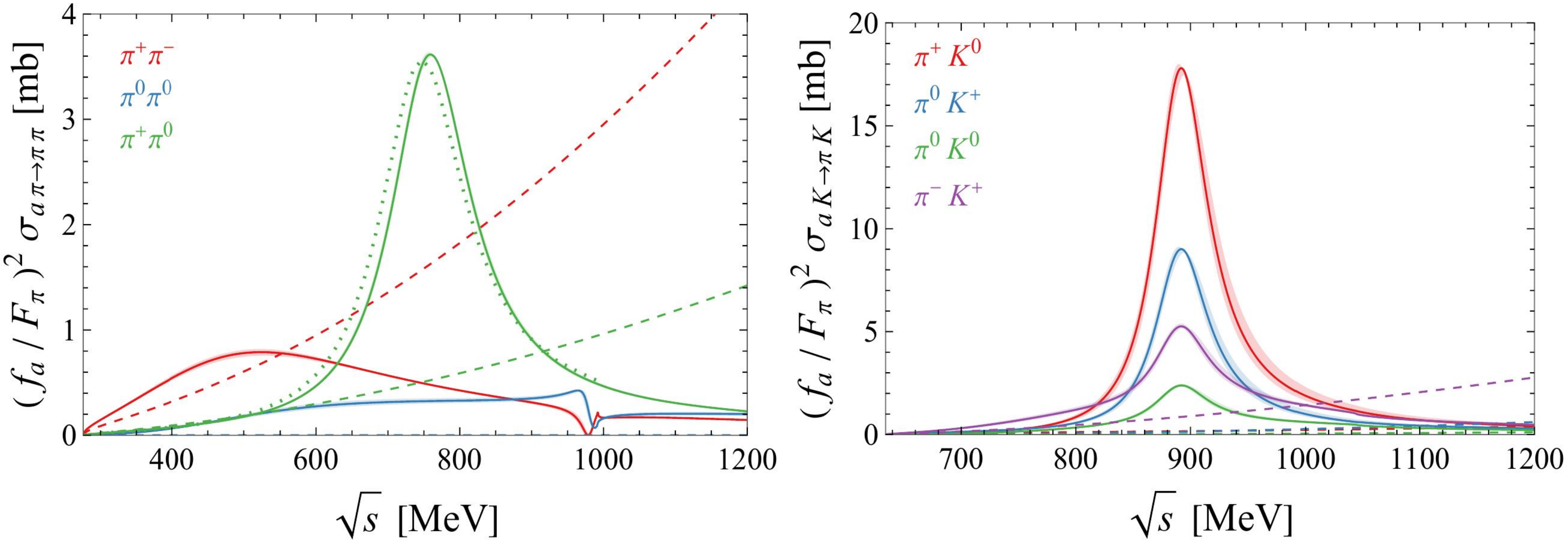
IJ=3/2 1: nonresonant case [neglected]

(4) $a\,K^0 \to \pi^- K^+$, $\pi^0 K^0$ [similar as aK^+ case]

(5) Other channels can be obtained via the charge-conjugation symmetry.



Subst. const.		Low energy constants	
$a_{\rm sc}^{\pi\pi,00}$	$-0.49^{+0.24}_{-0.23}$	$\hat{L}_1\times10^3$	$0.33^{+0.02}_{-0.02}$
$a_{\rm sc}^{K\bar{K},00}$	$-1.51^{+0.20}_{-0.19}$	$\hat{L}_2\times10^3$	$0.97^{+0.05}_{-0.05}$
$a_{\rm sc}^{11}$	$-1.38^{+0.33}_{-0.26}$	$\hat{L}_3\times10^3$	$-2.71^{+0.10}_{-0.11}$
$a_{\rm sc}^{\frac{1}{2}0}$	$0.15^{+0.18}_{-0.21}$	$\hat{L}_4\times10^3$	$-0.77^{+0.09}_{-0.11}$
$a_{\rm sc}^{\frac{1}{2}1}$	$1.53^{+0.76}_{-0.80}$	$\hat{L}_5\times10^3$	$3.51^{+1.39}_{-1.62}$
		$\hat{L}_6\times10^3$	$-1.47^{+0.20}_{-0.24}$
		$\hat{L}_7\times10^3$	$-0.77^{+0.24}_{-0.18}$
		$\hat{L}_8\times10^3$	$4.05^{+0.37}_{-0.45}$

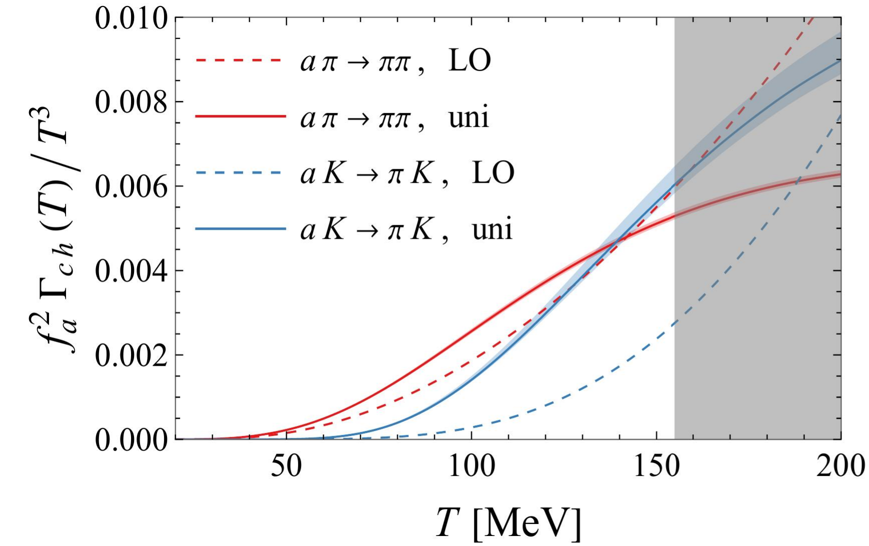


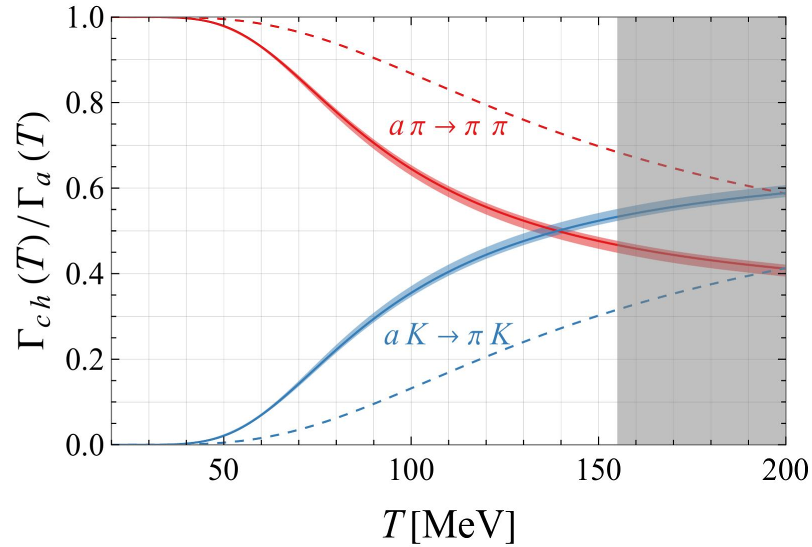
Resonance poles

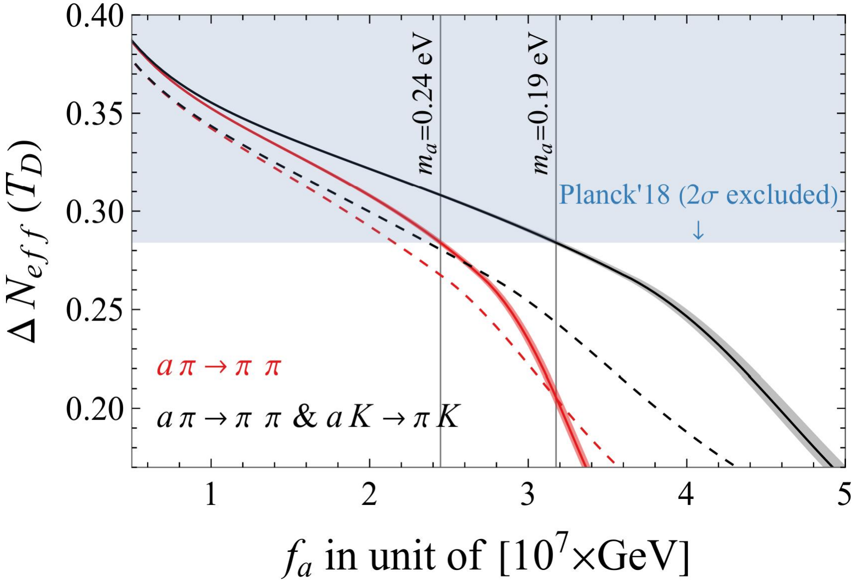
ρ: (754.3 - i67.9) MeV ; K^*: (889.5 - i28.0) MeV;

$f_0(500)$: 435.4- i238.0; $f_0(980)$: 981.1- i11.4; $K^*_0(700)$: 801.9- i195.2;

- **Clear enhancement from the unitarized amplitudes (solid lines)**
- **ρ(770) & K^*(892) lead to the most prominent effects**
- **Scalar resonances mostly give mild contributions**
- **aη related processes are much less important than the aK ones. (working in progress)**
- **axion-baryon is expected to be much suppressed, due to the heavy thresholds.**







[Wang, ZHG, Zhou, PRD-Letter'25]

$f_a \geq 2.45^{+0.03}_{-0.02} \times 10^7$ GeV **($a\pi \to \pi\pi$)**

$f_a \geq 3.18^{+0.04}_{-0.03} \times 10^7$ GeV **($a\pi \to \pi\pi$ + $aK \to \pi K$)**

- **Enhancement in $\tau \to \nu_\tau K a$ is also seen. However this belongs to a Cabibbo suppressed reaction.** **[Hao, Duan, ZHG, 2507.00383]**

Axion production in $\eta \to \pi\pi a$ decay

Axion production from $\eta \to \pi\pi a$ decay in SU(3) χPT

Why focus on axion in η decay:

- ✓ Valuable channel to search axion @colliders: many available experiments with large data samples of η/η' [BESIII, STCF, JLab, REDTOP, ...]
- ✓ $\eta \to \pi\pi\pi$ (IB suppressed), $\eta \to \pi\pi a$ (no IB suppression)
- ✓ $\eta \to \pi\pi a$: theoretically easier to handel than $\eta' \to \pi\pi a$ (next step)

Previous works:

- ❖ Most of them rely on leading-order χPT
- ❖ Possible issue: bulk contributions@LO χPT are constant terms, and potential large corrections from higher orders may result.
- ❖ Hadron resonance effects may lead to enhancements.

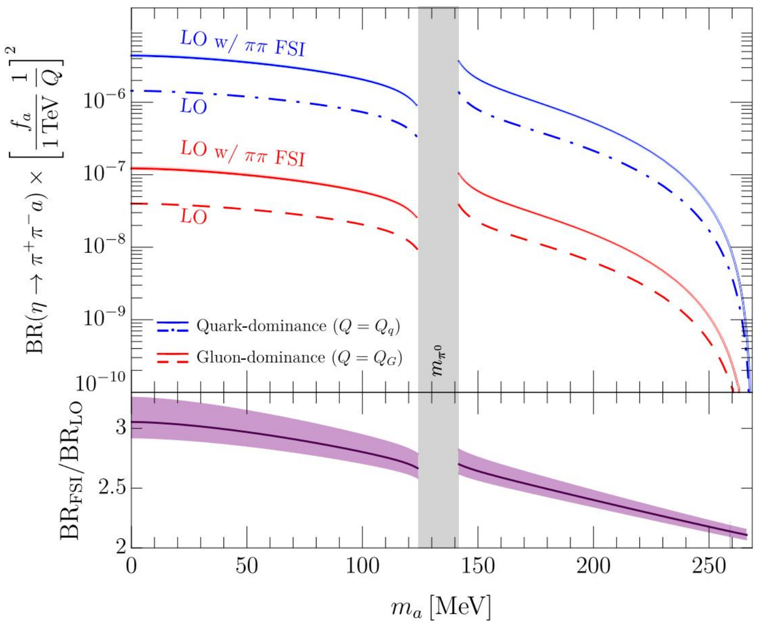
Advances in our work :

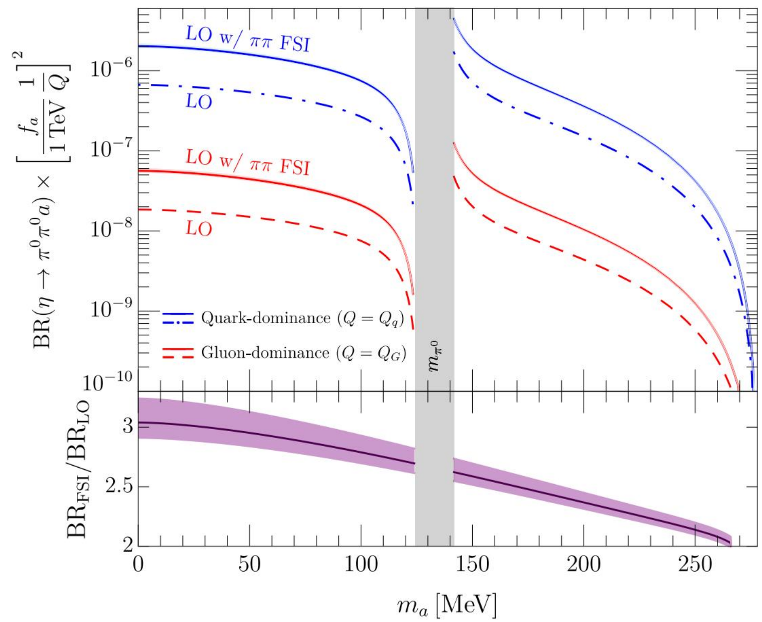
- ➢ Study of renormalization of $\eta \to \pi\pi a$ @1-loop level in SU(3) χPT
- ➢ To implement unitarization to the $\eta \to \pi\pi a$ χPT amplitude
- ➢ Uncertainty analyes in the phenomenological discussions

[Alves,Gonzalez-Solis,JHEP'24]

$$M_0(s) = P(s)\Omega_0^0(s)$$

η→ππa LO amplitude **Omnes function: ππ FSI**





Our improvements:

- **NLO perturbative decay amplitude include s- and $t(u)$-channel interactions perturbatively.**
- **The unitarized decay amplitude will be constructed to account for the s-channel $\pi\pi$ final state interaction (FSI) effect that respect the chiral symmetry.**
- **Dalitz plots will be explored to decode the dynamics in η→ππa.**

LO χPT Lagrangian

$$\mathcal{L}_2 = \frac{F^2}{4}\langle \partial_\mu U \partial^\mu U^\dagger + \chi_a U^\dagger + U \chi_a^\dagger \rangle + \frac{\partial_\mu a}{2 f_a} J_A^\mu\big|_{\mathrm{LO}} + \frac{1}{2}\partial_\mu a \partial^\mu a - \frac{1}{2} m_{a,0}^2 a^2$$

$$\chi_a = 2B_0 M(a) \qquad M(a) \equiv \exp\left(-i\frac{a}{2f_a}Q_a\right) M \exp\left(-i\frac{a}{2f_a}Q_a\right) \qquad J_A^\mu\big|_{\mathrm{LO}} = -i\frac{F^2}{2}\langle Q_a \left\{\partial^\mu U, U^\dagger\right\}\rangle$$

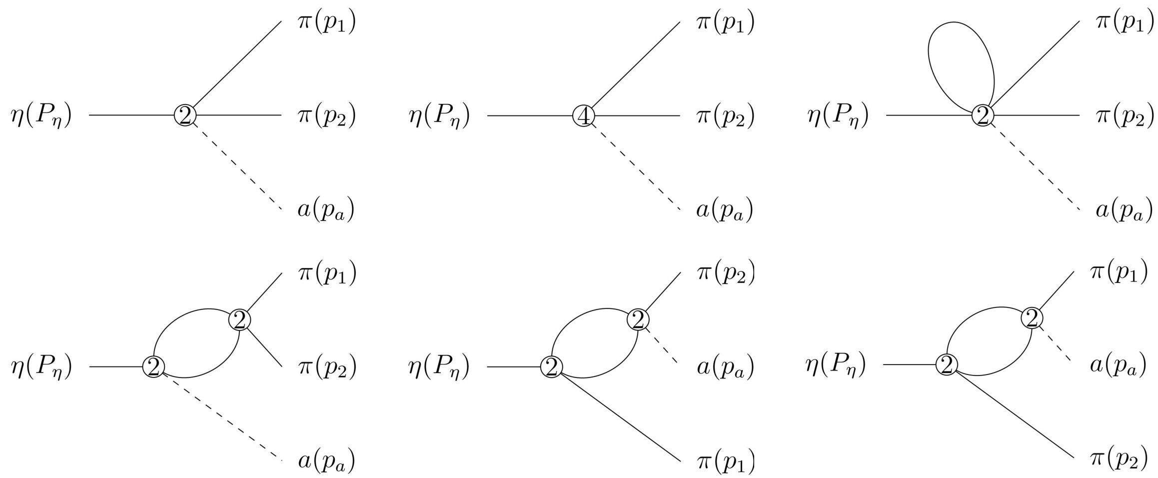
Note: we consider the octet part ($\bar{Q}_a$) of Q_a in SU(3) χPT

NLO χPT Lagrangian

$$\mathcal{L}_4 = L_1\langle \partial_\mu U \partial^\mu U^\dagger \rangle \langle \partial_\nu U \partial^\nu U^\dagger \rangle + \cdots + \frac{\partial_\mu a}{2 f_a} J_A^\mu\big|_{\mathrm{NLO}},$$

$$J_A^\mu\big|_{\mathrm{NLO}} = -4iL_1 \langle \bar{Q}_a \{U^\dagger, \partial^\mu U\}\rangle \langle \partial_\nu U \partial^\nu U^\dagger \rangle + \cdots .$$

Feynman diagrams up to NLO



Parameters

Masses and F_π [MeV]				LECs $L_i^r(\mu)$ at $\mu = 770$ MeV (in unit of 10^{-3})							
m_π	m_K	m_η	F_π	L_1^r	L_2^r	L_3^r	L_4^r	L_5^r	L_6^r	L_7^r	L_8^r
137	496	548	92.1	1.0(1)	1.6(2)	−3.8(3)	0.0(3)	1.2(1)	0.0(4)	−0.3(2)	0.5(2)

[J. Bijnens and G. Ecker, Ann. Rev. Nucl. Part. Sci. 64, 149 (2014)]

✓ **Renomarlization condition is verified to be consistent with conventional ChPT.**

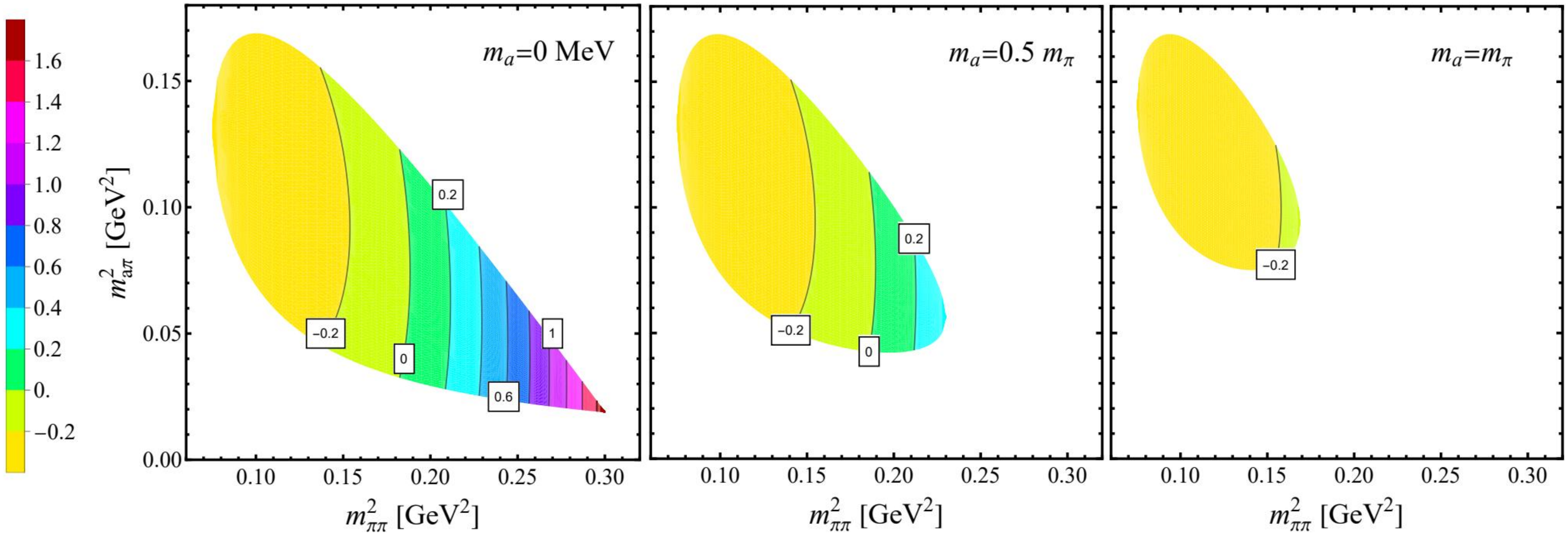
Observations:

- Strong isospin breaking effects enter the $\eta\to\pi\pi a$ amplitudes at the order of $(m_u-m_d)^2$
- In the isospin limit $(m_u=m_d)$, the amplitudes with $\pi^+\pi^-$ and $\pi^0\pi^0$ in $\eta\to\pi\pi a$ processes are identical.

- **Dalitz plots to show the NLO/LO convergence**

$$\left(2\mathcal{M}^{(2)}_{\eta;\pi\pi a}\mathrm{Re}\left(\mathcal{M}^{(4)}_{\eta;\pi\pi a}\right)+\left|\mathcal{M}^{(4)}_{\eta;\pi\pi a}\right|^2\right)/\left|\mathcal{M}^{(2)}_{\eta;\pi\pi a}\right|^2$$

[Wang,ZHG,Lu,Zhou, JHEP'24]



Important lessons:

- Non-perturbative effect in the $\pi\pi$ subsystem can be important.
- Perturbative treatment of the $a\pi$ subsystem is justified.

- **Unitarization of the partial-wave η→ππa amplitude**

$$\mathcal{M}^{00,\mathrm{Uni}}_{\eta;\pi\pi a}(s) = \frac{\mathcal{M}^{00,\mathrm{L}}_{\eta;\pi\pi a}(s)}{1 - G_{\pi\pi}(s) T^{00,(2)}_{\pi\pi\to\pi\pi}(s)} ,$$

$$G_{\pi\pi}(s) = -\frac{1}{(4\pi)^2}\left(\log\frac{m_\pi^2}{\mu^2} - \sigma_\pi(s)\log\frac{\sigma_\pi(s)-1}{\sigma_\pi(s)+1} - 1\right) ,$$

$$\mathcal{M}^{00,\mathrm{L}}_{\eta;\pi\pi a}(s) = \mathcal{M}^{00,(2)}_{\eta;\pi\pi a}(s) + \mathcal{M}^{00,(4)}_{\eta;\pi\pi a}(s) - G_{\pi\pi}(s)\,\mathcal{M}^{00,(2)}_{\eta;\pi\pi a}(s)\,T^{00,(2)}_{\pi\pi\to\pi\pi}(s) .$$

The unitarized amplitude satisfies the relation

$$\mathrm{Im}\mathcal{M}^{00,\mathrm{Uni}}_{\eta;\pi\pi a}(s) = \rho_{\pi\pi}(s)\mathcal{M}^{00,\mathrm{Uni}}_{\eta;\pi\pi a}(s)\left(T^{00,\mathrm{Uni}}_{\pi\pi\to\pi\pi}(s)\right)^* , \qquad (2m_\pi < \sqrt{s} < 2m_K)$$

with the unitarized PW ππ amplitude $T^{00,\mathrm{Uni}}_{\pi\pi\to\pi\pi}(s) = \dfrac{T^{00,(2)}_{\pi\pi\to\pi\pi}(s)}{1 - G_{\pi\pi}(s) T^{00,(2)}_{\pi\pi\to\pi\pi}(s)}$

- **Unitarized PW amplitude based on LO η→ππa amplitude**

$$\mathcal{M}^{00,\mathrm{Uni-LO}}_{\eta;\pi\pi a}(s) = \frac{\mathcal{M}^{00,(2)}_{\eta;\pi\pi a}(s)}{1 - G_{\pi\pi}(s) T^{00,(2)}_{\pi\pi\to\pi\pi}(s)} .$$

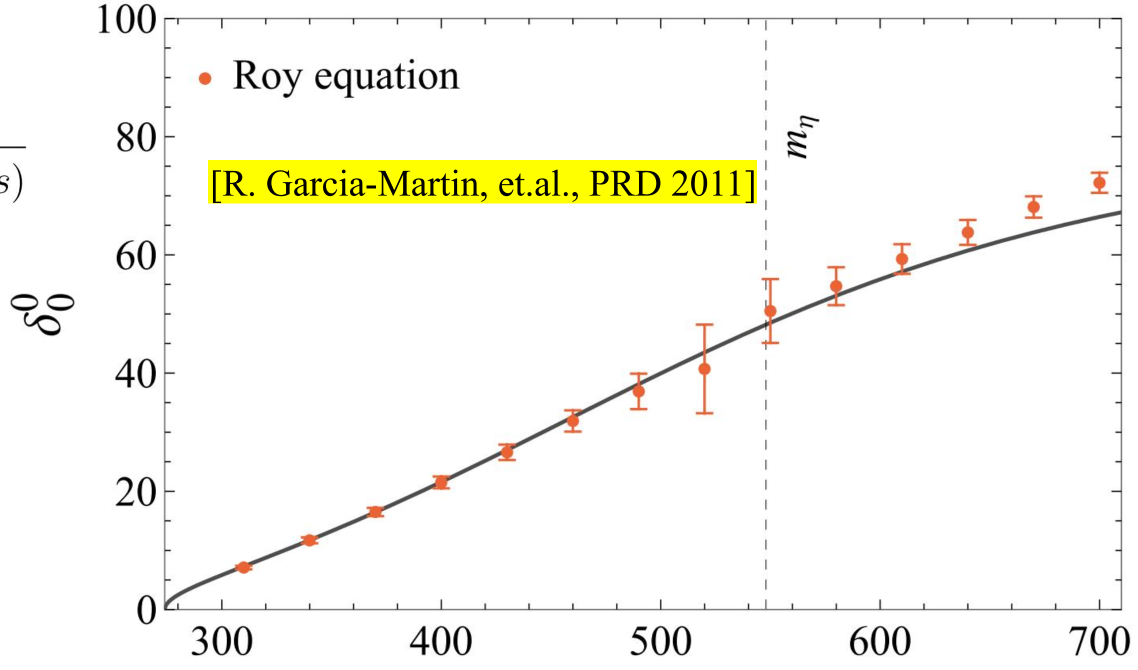
Resemble the method:

[Alves,Gonzalez-Solis,JHEP'24]

$$M_0(s) = P(s)\Omega^0_0(s)$$

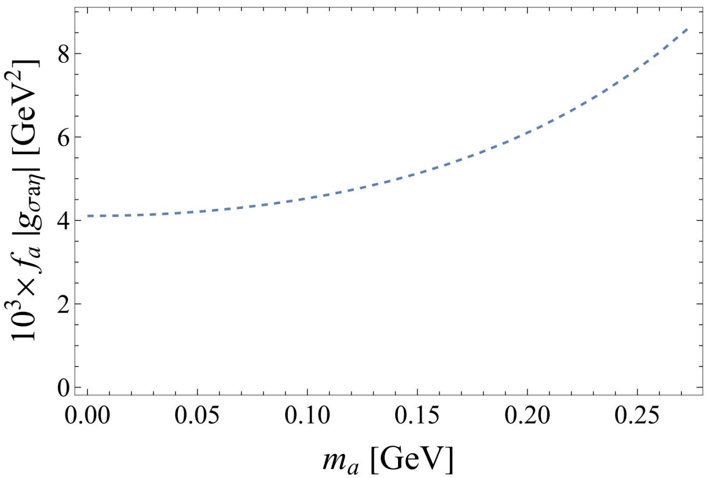
Phase shifts from the unitarized PW $\pi\pi$ amplitude

$$T^{00,\mathrm{Uni}}_{\pi\pi\to\pi\pi}(s) = \frac{T^{00,(2)}_{\pi\pi\to\pi\pi}(s)}{1 - G_{\pi\pi}(s) T^{00,(2)}_{\pi\pi\to\pi\pi}(s)}$$



- **Pole position of $f_0(500)/\sigma$:** $\sqrt{s_\sigma} = 457 \pm i251$ MeV

$$\mathcal{M}^{00,\mathrm{Uni,II}}_{\eta;\pi\pi a}(s)\big|_{s\to s_\sigma} \sim -\frac{g_{\sigma\pi\pi} g_{\sigma a\eta}}{s - s_\sigma}$$



Predictions of the $\eta\to\pi\pi a$ branching ratios by varying m_a

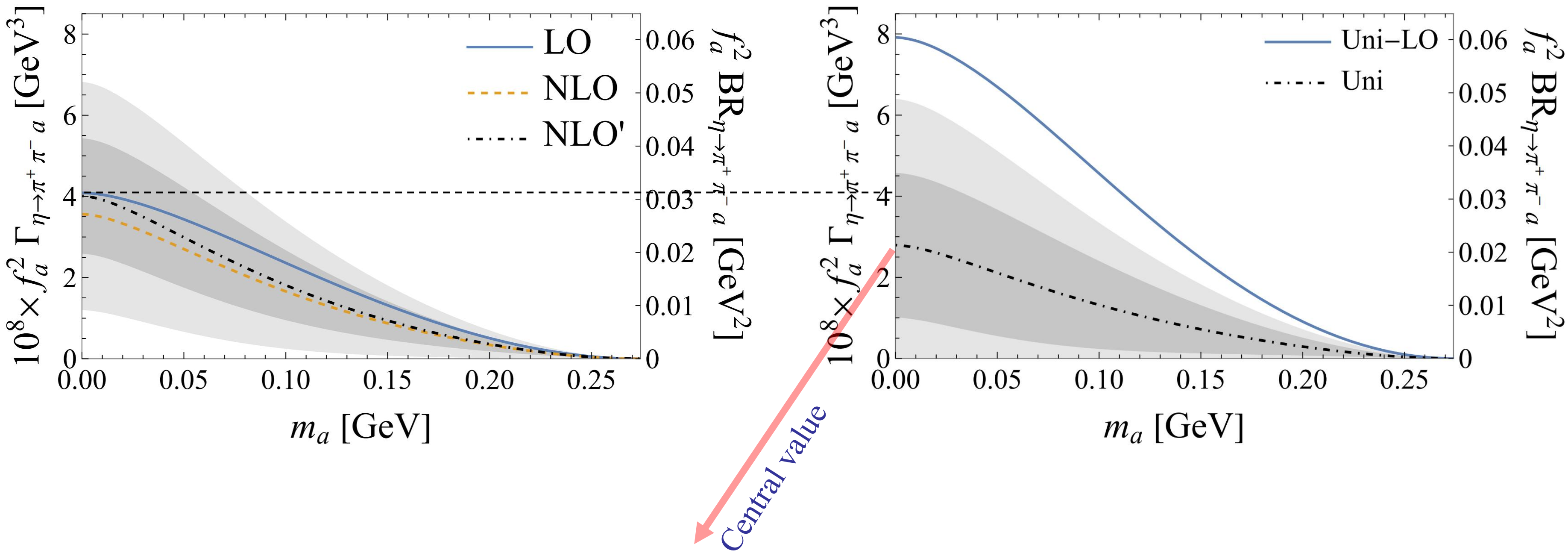
Uncertainty bands:

- ➢ **Lighter regions:**

L_1^r	L_2^r	L_3^r	L_4^r	L_5^r	L_6^r	L_7^r	L_8^r
1.0(1)	1.6(2)	−3.8(3)	0.0(3)	1.2(1)	0.0(4)	−0.3(2)	0.5(2)

- ➢ **Darker regions: freeze the 1/Nc suppressed ones (L_4,L_6,L_7)**

[Wang,ZHG,Lu,Zhou, JHEP'24]



$$\mathrm{BR}_{\eta\to\pi^+\pi^- a}\Big|_{m_a\to 0} = 2.1\times 10^{-2}\left(\frac{\mathrm{GeV}^2}{f_a^2}\right)$$

Possible detection channels: $a\to\gamma\gamma$, $a\to e^+e^-$, $a\to\mu^+\mu^-$

Summary

- **Chiral effecitive field theory provides a systematical and useful framework to study the axion-hadron reactions.**

- **Synergies of Lattice QCD, hadron phenomenologies and chiral EFT are demonstrated to be powerful to build axion amplitudes.**

- **Futher involvements with the experiments, cosmology, astronomy are needed to set up stronger constraints on axion parameters!**

谢谢！