Nuclear Science Computing Center at CCNU

Shear and Bulk Viscosities of Gluon Plasma across the Transition Temperature

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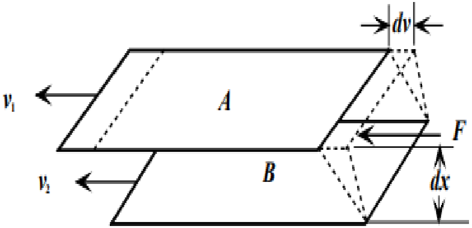
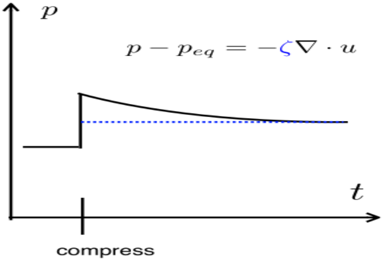
Viscosity of Gluon Plasma

- Sketch of η (shear viscosity) and ζ (bulk viscosity):





courtesy of Dabis S. Viswanath et al. courtesy of Hai-Tao Shu

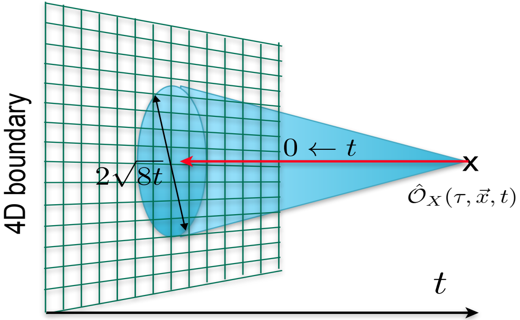
- Method: gradient flow: **extendable to full QCD**.

Noise Reduction Techniques: gradient flow

$$\dot{B}_\mu = (\partial_\mu + [B_\mu, \cdot])(\partial_\mu B_\nu - \partial_\nu B_\mu + [B_\mu, B_\nu]). \quad (1)$$

$$B_\nu^{\mathrm{LO}}(x, \tau_F) =$$

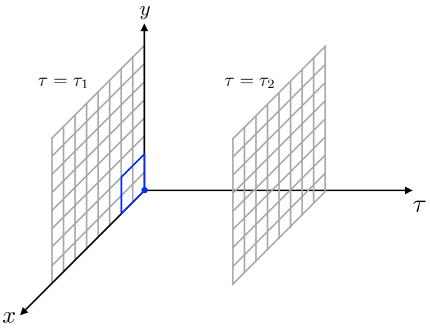
$$\int \mathrm{d}y \left(\frac{\sqrt{2\pi(8\tau_F)}}{2}\right)^{-4} \exp\left(\frac{-(x-y)^2}{\sqrt{8\tau_F}^2/2}\right) B_\nu(y).$$



courtesy of Hai-Tao Shu

Noise Reduction Techniques: blocking method

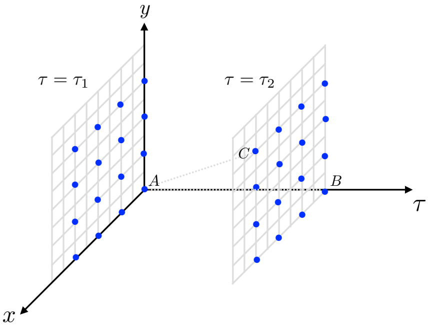
$$G(\tau_1-\tau_2)=\frac{a^3}{V}\sum_{\vec{x}\in V}\mathcal{O}(\tau_1,\vec{x})\sum_{\vec{y}\in V}\mathcal{O}(\tau_2,\vec{y})\ ,$$



L. Altenkort et al. PRD 105, 094505 (2022)

Noise Reduction Techniques: blocking method

$$G(\tau) = \frac{a^3}{V} \sum_{v_1} \Big[\sum_{\vec{m}\in v_1} \mathcal{O}(\tau_1, \vec{m}) \Big] \sum_{v_2} \Big[\sum_{\vec{n}\in v_2} \mathcal{O}(\tau_2, \vec{n}) \Big],$$



L. Altenkort et al. PRD 105, 094505 (2022)

Correlated Fit

$$\chi^2 = \sum_{i,j=1}^{n} [\bar{y}_i - f(\theta, x_i)] C_{ij}^{-1} [\bar{y}_j - f(\theta, x_j)], \tag{2}$$

$$X_{ij} = \frac{C_{ij}}{\sigma_i \sigma_j}, \tag{3}$$

$$\chi^2 = \sum_{i,j=1}^{n} \left(\frac{\bar{y}_i - f(\theta, x_i)}{\sigma_i}\right) (X^{-1})_{ij} \left(\frac{\bar{y}_j - f(\theta, x_j)}{\sigma_j}\right). \tag{4}$$

where X is called the correlation matrix.

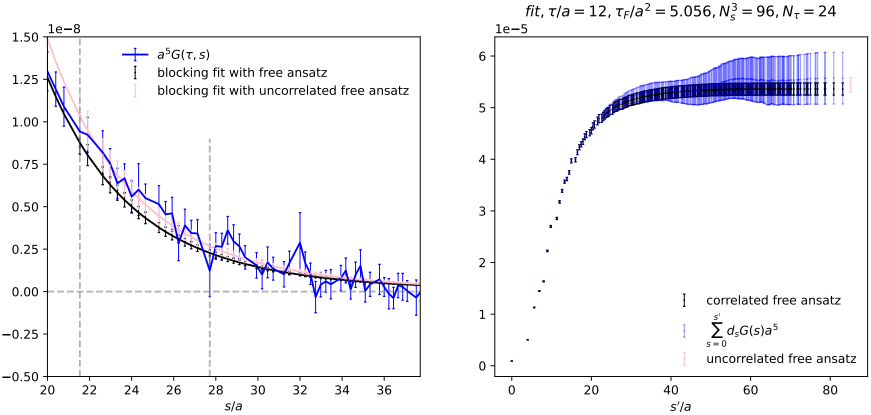
Free Ansatz

$$\langle E(\vec{r},\tau)E(0,0)\rangle_{\tau_F} \propto \sum_{n_1,n_2\in\mathcal{Z}} \frac{A(r_1)\,A(r_2)}{r_1^4 r_2^4} + \frac{A(r_1)\,B(r_2) + A(r_2)\,B(r_1)}{2r_1^4 r_2^4} + \frac{B(r_1)\,B(r_2)}{6r_1^6 r_2^6}\left(2\,(r_1\cdot r_2)^2 + r_1^2 r_2^2\right).$$

$$A(r,\tau_F) = 1 - \left(1 + \frac{r^2}{8\tau_F}\right) e^{-r^2/8\tau_F}, \tag{5}$$

$$B(r,\tau_F) = -2 + \left[2 - 2\frac{r^2}{8\tau_F} + \left(\frac{r^2}{8\tau_F}\right)^2\right] e^{-r^2/8\tau_F}. \tag{6}$$

Correlated Fit



Blocking fit of the bulk correlators at $1.5\,T_c$.

Correlated Fit

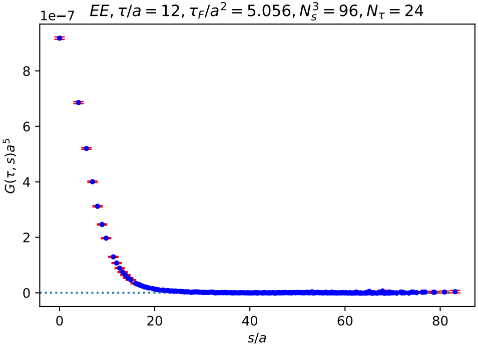
$$G(\tau) = G_{\text{dom}}(\tau) + G_{\text{mid}}(\tau) + G_{\text{tail}}(\tau) \tag{7}$$

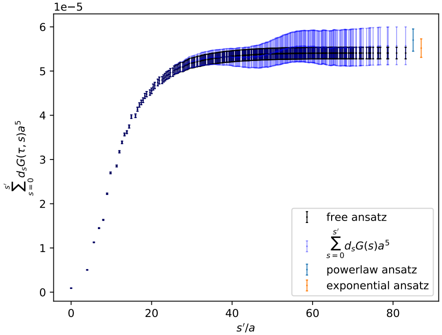
$$G_{\text{dom}}(\tau) \equiv \frac{a^3}{V_b} \sum_{s=0}^{s_0-1} d_s G(\tau, s), \tag{8}$$

$$G_{\text{mid}}(\tau) \equiv \frac{a^3}{V_b} \sum_{s=s_0}^{s_{\text{cut}}} d_s \left(x G_{\text{fit}}(\tau, s) + (1-x) G(\tau, s) \right), \tag{9}$$

$$G_{\text{tail}}(\tau) \equiv \frac{a^3}{V_b} \sum_{s>s_{\text{cut}}} d_s G_{\text{fit}}(\tau, s). \tag{10}$$

Bulk Correlator





Left: E in the energy momentum tensor as a function of distance at $T = 1.5T_c$; Right: sum of the correlator($s \leq s'$) as a function of spatial distance s'/a.

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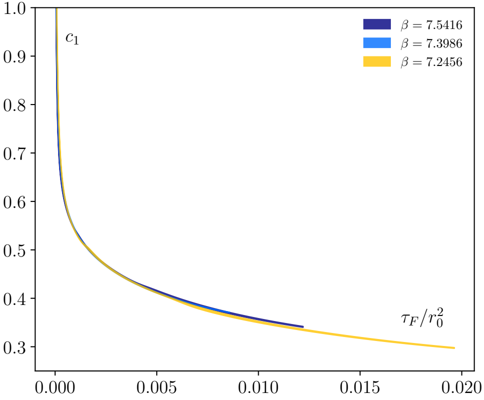
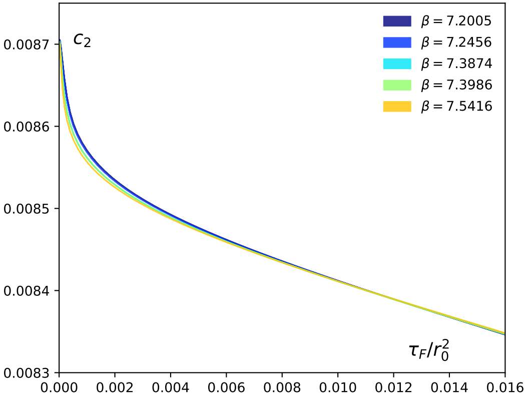
c_1 and c_2

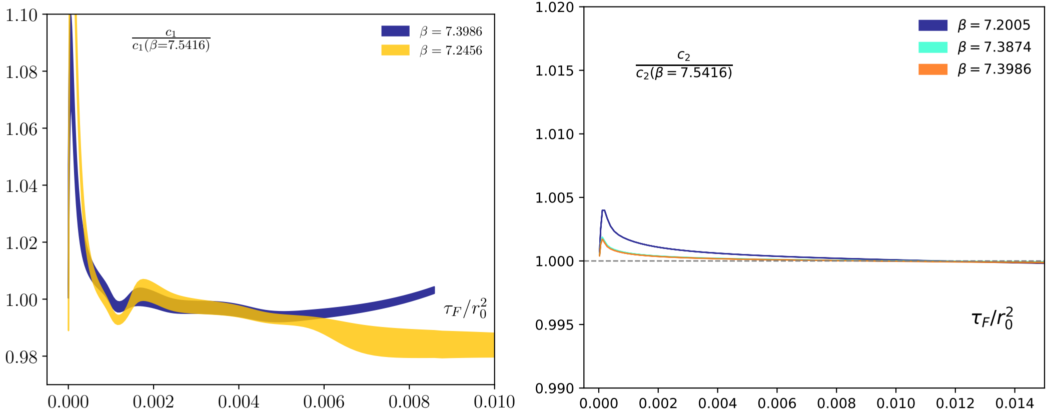
$$T_{\mu\nu}(\tau_{\rm F}, x) = c_1(\tau_{\rm F}) U_{\mu\nu}(\tau_{\rm F}, x) + 4c_2(\tau_{\rm F})\delta_{\mu\nu} E(\tau_{\rm F}, x).$$





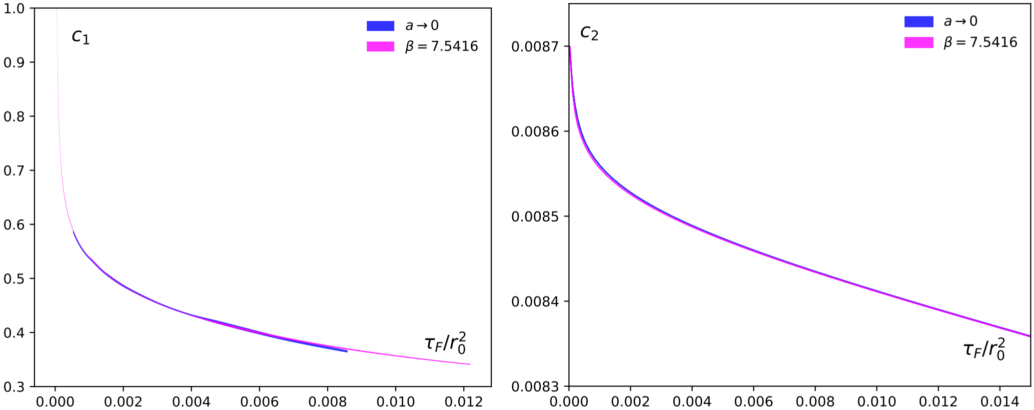
Left: Mixed c_1 at different lattice spacing; Right: c_2 measured at $T < T_c$ at different lattice spacing. For all the errors are quite smaller than 1%.

c_1 and c_2



Left: $c_1(\beta = 7.5416)$ is used as a normalization and so its error is not included in the ratio. Right: Cutoff effect analysis for c_2.

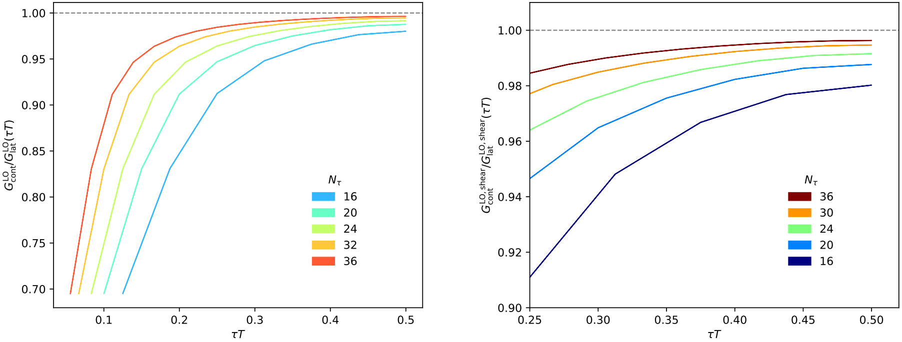
c_1 and c_2



Left: Based on the c_1^2's $a \to 0$ extrapolation. Right: Based on the c_2^2's $a \to 0$ extrapolation.

Lowest-perturbative Correlation Function

$$G^{\text{t.l.}}(\tau T) = G_{\text{lat}}(\tau T) \cdot \frac{G^{\text{LO}}_{\text{cont}}(\tau T)}{G^{\text{LO}}_{\text{lat}}(\tau T)}. \tag{11}$$



Tree-level improvement ratio for bulk (left) and shear (right) channel.

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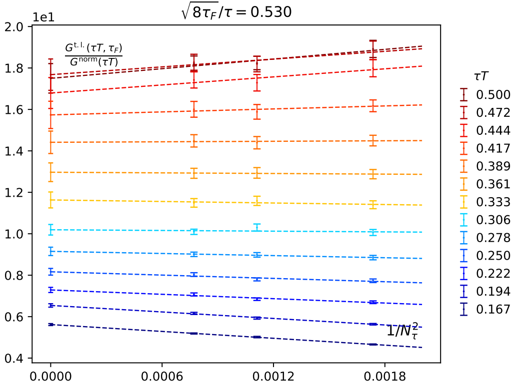
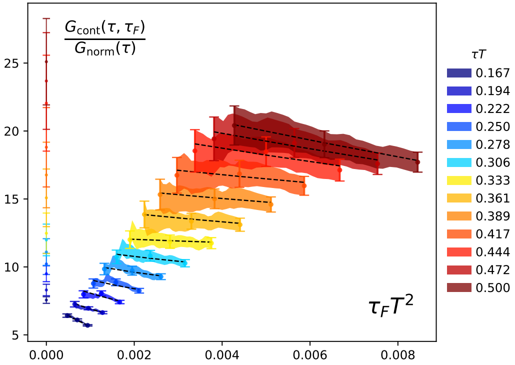
$a \to 0$ & $\tau_F \to 0$ extrapolation

$$\frac{G^{\text{t.l.}}(N_\tau)}{G_{\text{norm}}(N_\tau)} = m \cdot N_\tau^{-2} + b.$$



$$\theta(\tau_F) = \left(1 - c\left(\frac{g^2(\mu(\tau_F))}{(4\pi)}\right)^3\right)\theta(\tau_F = 0).$$



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Spectral Analysis

$$\text{M1}: \frac{\rho(\omega)}{\omega T^3} = \frac{A}{T^3} + B\frac{\rho_{\text{pert}}(\omega)}{\omega T^3},$$

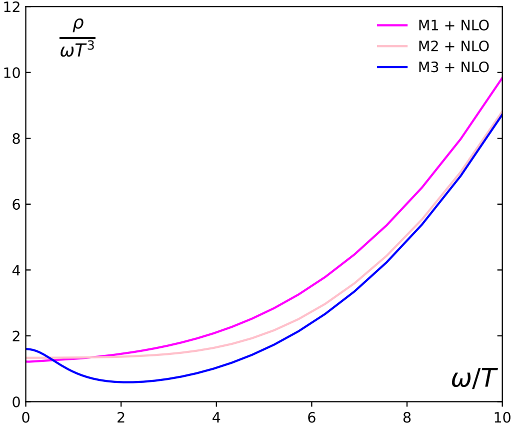
$$\text{M2}: \frac{\rho(\omega)}{\omega T^3} = \sqrt{\left(\frac{A}{T^3}\right)^2 + \left(B\frac{\rho_{\text{pert}}(\omega)}{\omega T^3}\right)^2},$$

$$\text{M3}: \frac{\rho(\omega)}{\omega T^3} = \frac{A}{T^3}\frac{C^2}{C^2 + (\omega/T)^2} + B\frac{\rho_{\text{pert}}(\omega)}{\omega T^3}.$$

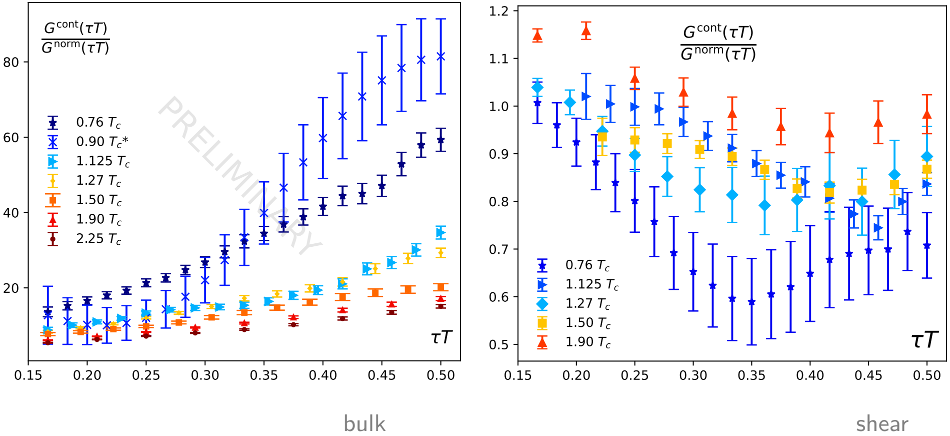
$$G(\tau) = \int_0^\infty \frac{dw}{\pi}\frac{\cosh[w(1/2T - \tau]}{\sinh(w/2T)}\rho(w, T).$$

$$\eta(T) = \lim_{\omega\to 0}\frac{\rho_{shear}(\omega, T)}{\omega},$$

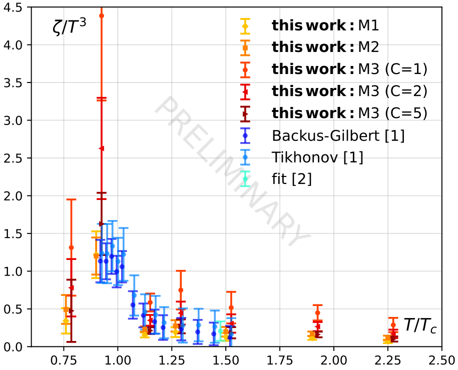
$$\zeta(T) = \frac{1}{9}\lim_{\omega\to 0}\frac{\rho_{bulk}(\omega, T)}{\omega}.$$



Results: temperature dependence of EMT correlators



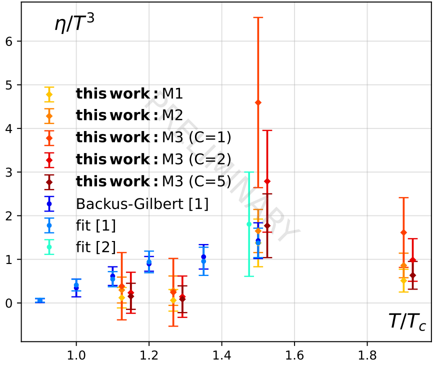
Temperature Dependence of The Bulk Viscosities



[1] N. Astrakhantsev et al. PRD 98, 054515 (2018)

[2] L. Altenkort, H.-T. Shu et al. PRD 108, 014503 (2023)

Temperature Dependence of The Shear Viscosities



[1] N. Astrakhantsev et al. JHEP 04, 101 (2017)

[2] L. Altenkort, H.-T. Shu et al. PRD 108, 014503 (2023)

Summary

- **Gradient flow strongly suppresses the UV fluctuations of the EMT correlators.**
- **Blocking method: an additional factor of 2 improvement for the STN ratio of the correlator.**
- ζ/T^3 **decreases with increasing** T **at** $T > T_c$ **in the quenched limit.**
- **There should be critical behavior around Tc.**

$\Rightarrow$ Bulk viscosity at 0.9 Tc is checking.

$\Rightarrow$ Shear viscosities are checking.