



一类分段光滑不连续映象中的边界碰撞分岔和余维分岔

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2025年11月27日

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研究背景与模型

分段光滑映象是不光滑动力系统的经典模型。其中，一维分段光滑映象形式相对简洁，并常常展现丰富的动力学。

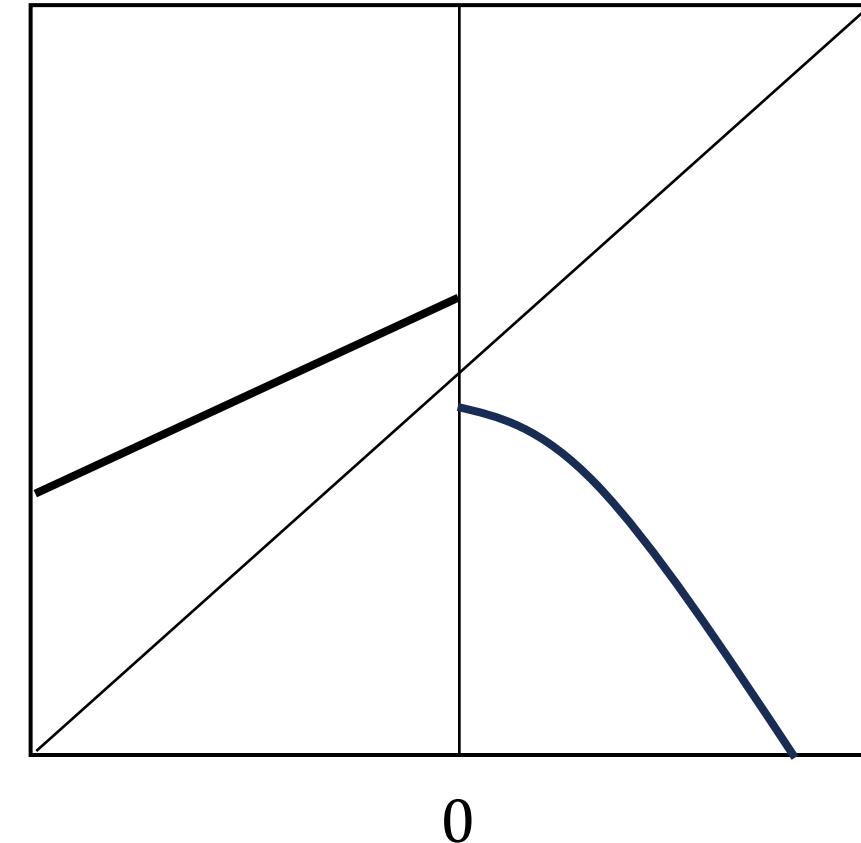
$$x_{n+1} = \begin{cases} f_L(x_n) = \alpha x_n + \mu & x_n < 0 \\ f_R(x_n) = \beta x_n^z + \mu + \delta & x_n > 0 \end{cases}$$

给定一个初值 x_0 代入上述方程迭代会得到一系列轨道点

$$x_1, x_2 \dots x_n \dots$$

轨道点下标表示迭代次数。

对于周期轨道满足： $x_{n+p} = x_n$





周期轨道的稳定性和边界碰撞分岔机制

稳定性分析

考虑在左支函数上迭代 $n - 1$ 次，在右支函数上迭代1次的周期模式，得到周期轨道稳定性条件如下

$$-1 \leq \alpha^{n-1} z \beta x_n^{z-1} \leq 1$$

$$\alpha^{n-1} z \beta x_n^{z-1} = -1 : \text{flip 分岔}$$



$$\mu_{n,\text{flip}} = \frac{1-\alpha}{1-\alpha^n} \left[\left(1 + \frac{1}{z}\right) (-\alpha^{n-1} \beta z)^{\frac{1}{1-z}} - \alpha^{n-1} \delta \right]$$

$$\alpha^{n-1} z \beta x_n^{z-1} = 1 : \text{fold 分岔}$$

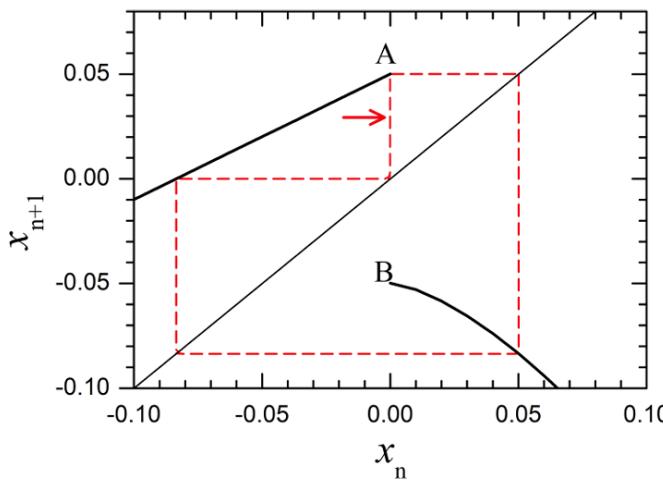


$$\mu_{n,\text{fold}} = \frac{1-\alpha}{1-\alpha^n} \left[\left(1 - \frac{1}{z}\right) (\alpha^{n-1} \beta z)^{\frac{1}{1-z}} - \alpha^{n-1} \delta \right]$$

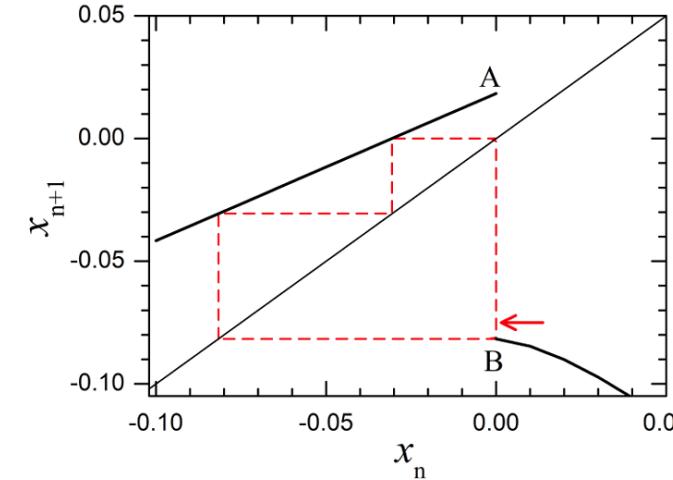


周期轨道的稳定性和边界碰撞分岔机制

边界碰撞及其临界条件



(a)



(b)

周期轨道:

$$x_1 < x_2 < x_3 < \dots < x_{n-1} < 0 < x_n \quad \Rightarrow$$

$$x_1 = x_{n+1}$$

$$\mu + \frac{\beta(1-\alpha)\alpha^{n-2}}{1-\alpha^{n-1}}\mu^z + \frac{(1-\alpha)\alpha^{n-2}}{1-\alpha^{n-1}}\delta \leq 0$$

$$\mu_{n,BC^-}$$

$$\alpha^{n-1}(\mu + \delta) + \frac{1 - \alpha^{n-1}}{1 - \alpha}\mu \geq 0$$

$$\mu_{n,BC^+}$$

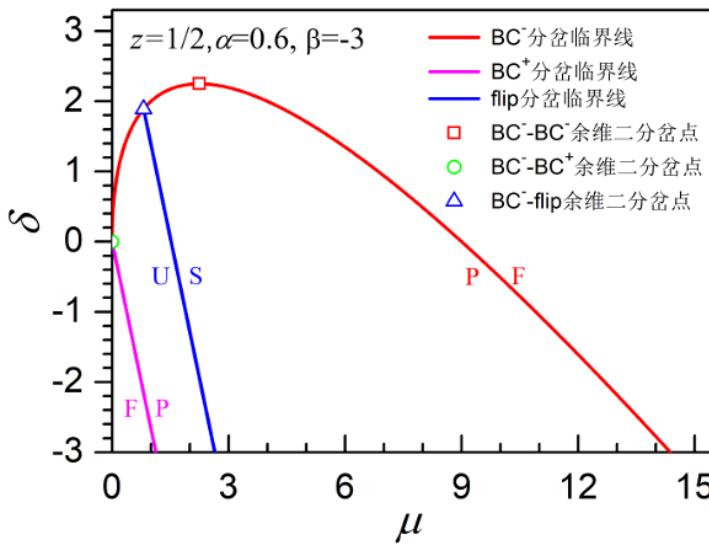
$$(a) \quad x_1 \geq \beta\mu^z + \mu + \delta$$

$$(b) \quad x_1 \leq \mu + \delta$$

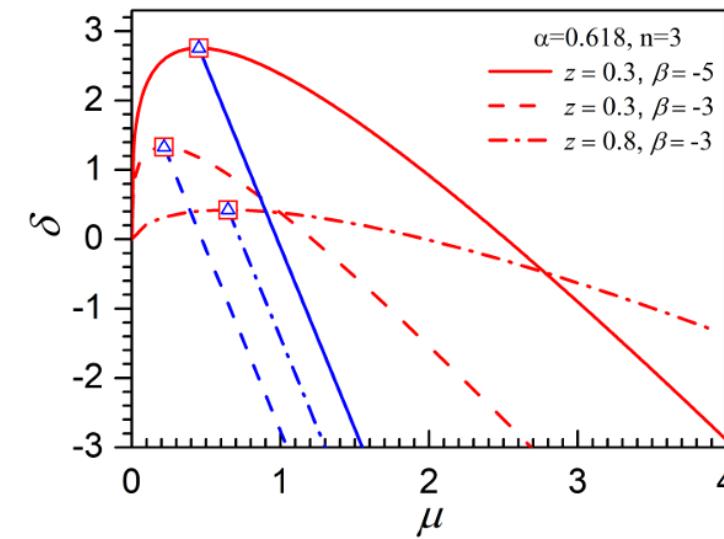


二维相平面中动力学行为的相图和余维分岔

- $0 < z < 1$



(a)



(b)

$$\alpha^n - 2\alpha + 1 = 0$$

$$\mu_{BC^- - BC^-} = \mu_0 z^{\frac{1}{1-z}}$$

$$\delta_{BC^- - BC^-} = \beta \mu_0^z (z-1) z^{\frac{z}{1-z}}$$

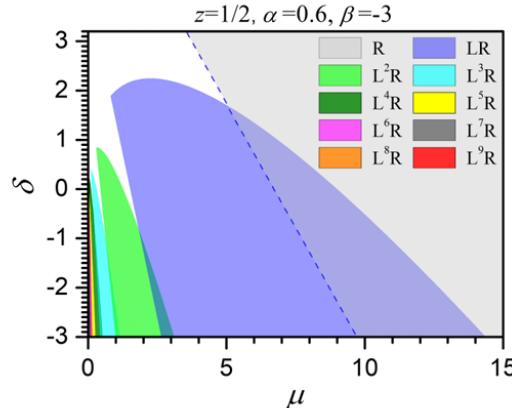
$$\mu_{BC^- - \text{flip}} = (-\alpha^{n-1} \beta z)^{\frac{1}{1-z}}$$

$$\delta_{BC^- - \text{flip}} = \beta \left(-\alpha^{n-1} \beta z \right)^{\frac{1}{1-z}} \left[\mu_0^{z-1} + (\alpha^{n-1} \beta z)^{-1} \right]$$

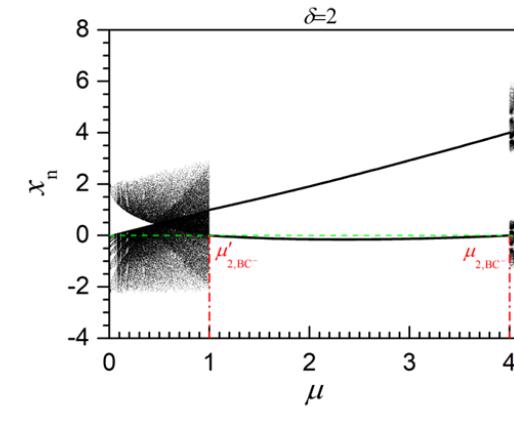


二维相平面中动力学行为的相图和余维分岔

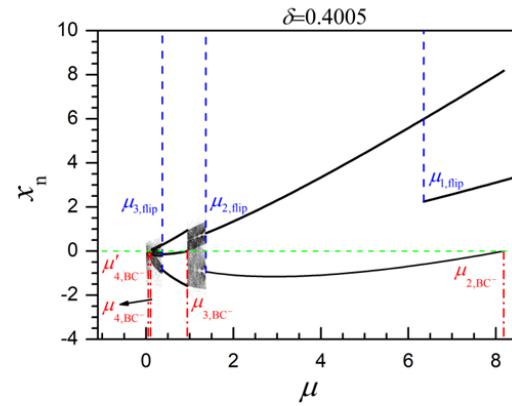
● $0 < z < 1$



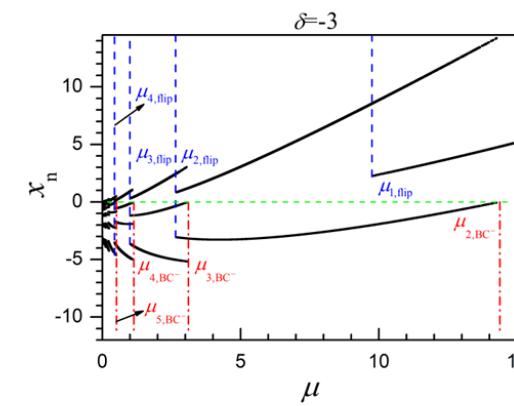
(a)



(b)



(c)



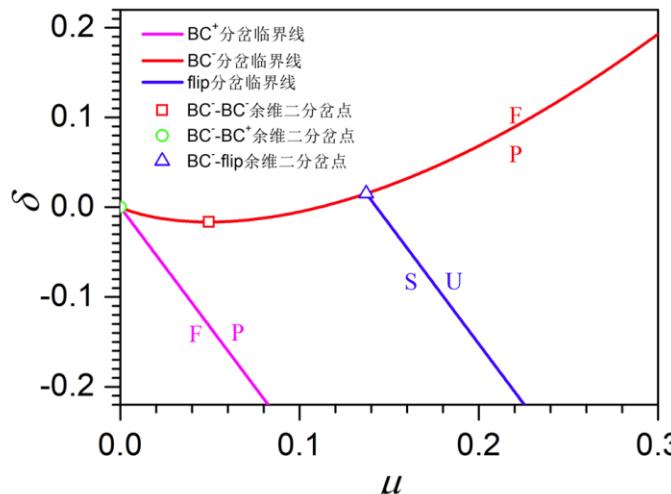
(d)

| δ | n | μ_{n,BC^+} | μ'_{n,BC^-} | μ_{n,BC^-} | $\mu_{n,flip}$ |
|----------|-----|----------------|-----------------|----------------|----------------|
| 2 | 1 | — | — | — | 4.75000 |
| 2 | 2 | — | 1.00000 | 4.00000 | — |
| 0.4005 | 1 | — | — | — | 6.3495 |
| 0.4005 | 2 | — | 0.01961 | 8.17939 | 1.36856 |
| 0.4005 | 3 | — | 0.02396 | 0.94129 | 0.37277 |
| 0.4005 | 4 | — | 0.05157 | 0.10493 | — |
| -3 | 1 | 3.00000 | — | — | 9.75000 |
| -3 | 2 | 1.12500 | — | 14.3739 | 2.64375 |
| -3 | 3 | 0.551020 | — | 3.10847 | 0.997347 |
| -3 | 4 | 0.297794 | — | 1.13912 | 0.442522 |
| -3 | 5 | 0.168633 | — | 0.510583 | 0.217806 |
| -3 | 6 | 0.0978786 | — | 0.253545 | 0.115003 |

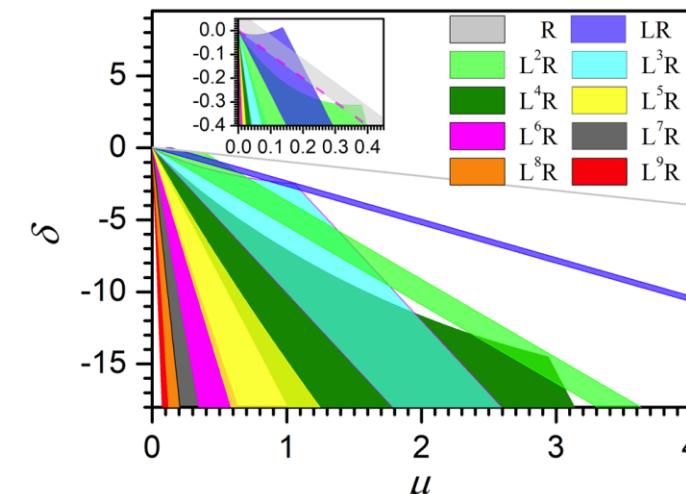


二维相平面中动力学行为的相图和余维分岔

● $z > 1$



(a)



(b)

$$\mu_{BC^- - BC^-} = \mu_0 z^{\frac{1}{1-z}}$$

$$\delta_{BC^- - BC^-} = \beta \mu_0^z (z-1) z^{\frac{z}{1-z}}$$

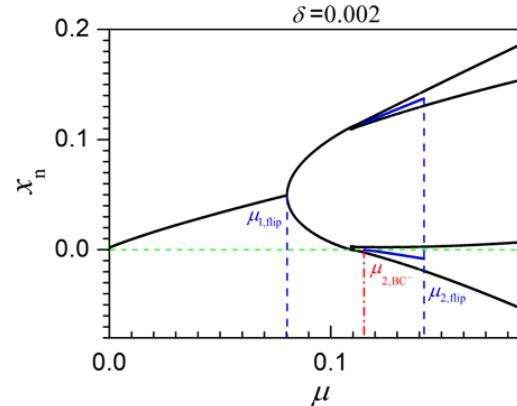
$$\mu_{BC^- - \text{flip}} = (-\alpha^{n-1} \beta z)^{\frac{1}{1-z}}$$

$$\delta_{BC^- - \text{flip}} = \beta \left(-\alpha^{n-1} \beta z \right)^{\frac{1}{1-z}} \left[\mu_0^{z-1} + \left(\alpha^{n-1} \beta z \right)^{-1} \right]$$

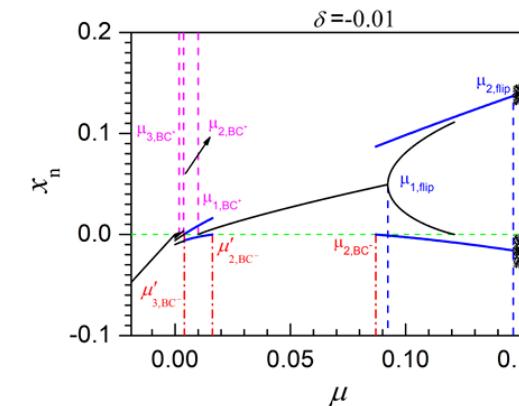


二维相平面中动力学行为的相图和余维分岔

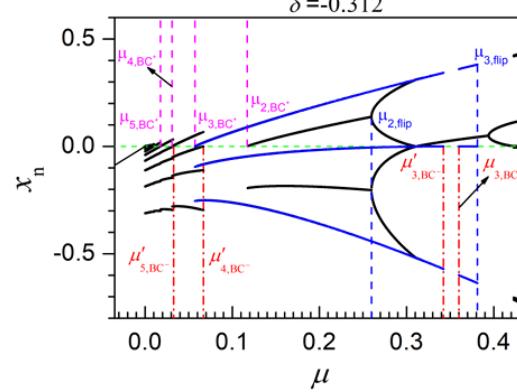
$z > 1$



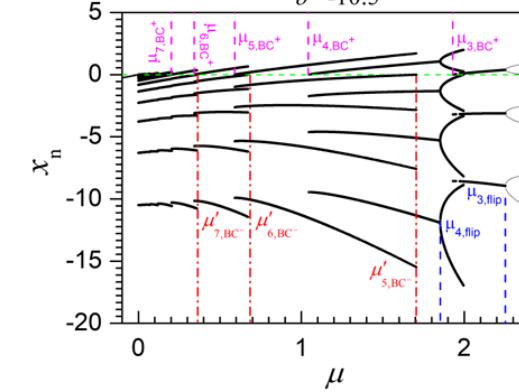
(a)



(b)



(c)



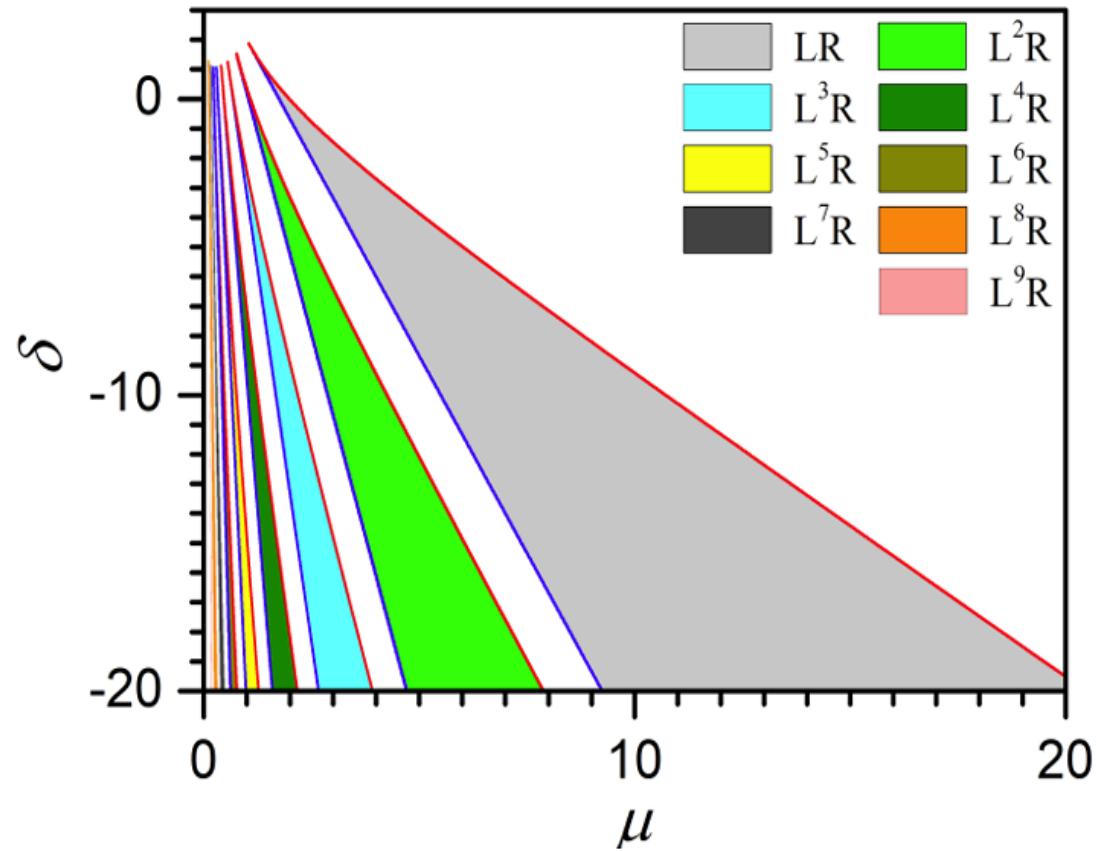
(d)

| δ | n | μ_{n,BC^+} | μ'_{n,BC^-} | μ_{n,BC^-} | $\mu_{n,flip}$ |
|----------|-----|----------------|-----------------|----------------|----------------|
| 0.002 | 1 | — | — | — | 0.08031 |
| 0.002 | 2 | — | — | 0.11501 | 0.14214 |
| -0.01 | 1 | 0.01 | — | — | 0.0923 |
| -0.01 | 2 | 0.00375 | 0.01617 | 0.08705 | 0.14664 |
| -0.01 | 3 | 0.00184 | 0.00404 | — | — |
| -0.01 | 4 | 0.000993 | 0.001882 | — | — |
| -0.01 | ... | ... | ... | ... | ... |
| -0.312 | 1 | 0.312 | — | — | 0.39431 |
| -0.312 | 2 | 0.117 | — | — | 0.25989 |
| -0.312 | 3 | 0.05731 | 0.34237 | 0.36 | 0.38132 |
| -0.312 | 4 | 0.03097 | 0.06682 | — | — |
| -0.312 | 5 | 0.01754 | 0.03273 | — | — |
| -0.312 | ... | ... | ... | ... | ... |
| -10.5 | 3 | 1.92857 | — | — | 2.25258 |
| -10.5 | 4 | 1.04228 | — | — | 1.85297 |
| -10.5 | 5 | 0.59022 | 1.70563 | — | — |
| -10.5 | 6 | 0.34258 | 0.68604 | — | — |
| -10.5 | 7 | 0.2016 | 0.36408 | — | — |
| -10.5 | ... | ... | ... | ... | ... |



二维相平面中动力学行为的相图和余维分岔

● $z < 0$



$$\mu_{BC--\text{fold}} = (\alpha^{n-1} \beta z)^{\frac{1}{1-z}}$$

$$\delta_{BC--\text{fold}} = \beta (\alpha^{n-1} \beta z)^{\frac{1}{1-z}} \left[\mu_0^{z-1} - (\alpha^{n-1} \beta z)^{-1} \right]$$

结论

- 给出了一类两分段不连续映象在特定周期模式失稳的临界点解析表达式；
- 得到了映象的双参数相平面，据此确定了不同控制参数下映象的长期动力学行为；
- 研究了相平面中的余维分岔行为，发现了一类特殊的余维二分岔点；



谢 谢