

# Probing Dark Matter with Space-based Gravitational-Wave Laser Interferometers

TANG, Yong 汤勇

University of Chinese Academy of Sciences



味物理讲座, 2025.11.13

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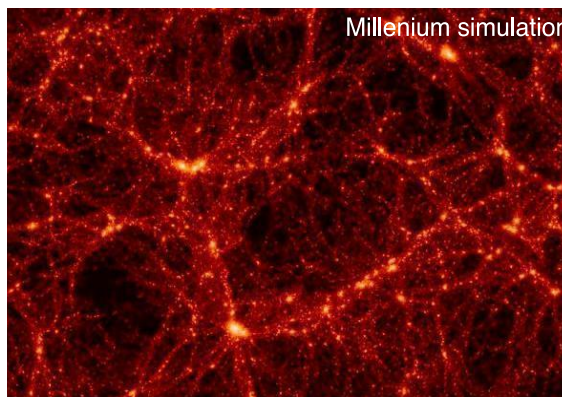
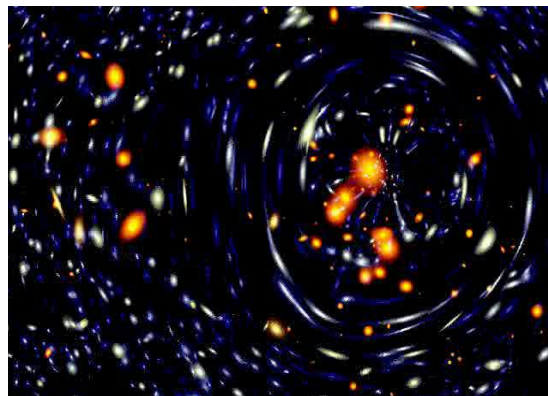
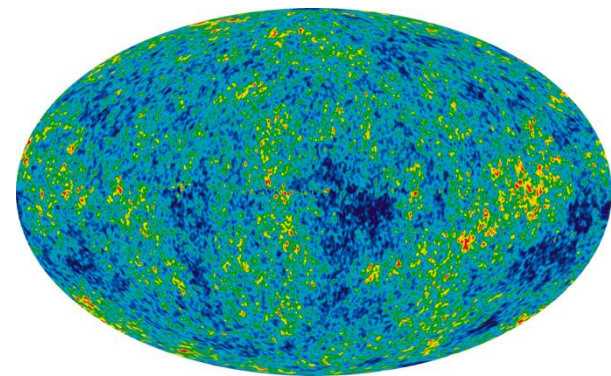
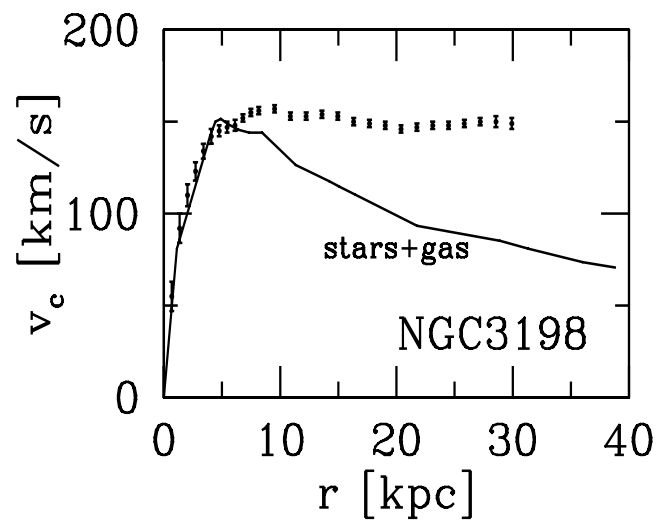
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Summary

姚跃辉 *et al*, arXiv: 2508.14655, 2506.09744, 2410.22072(PRD), 2404.01494(PRD), 2307.09197(PRD)

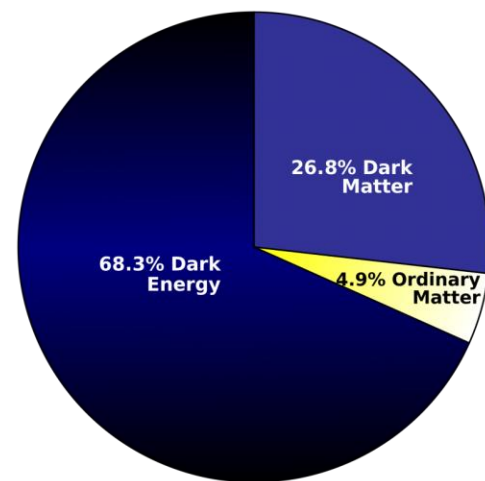
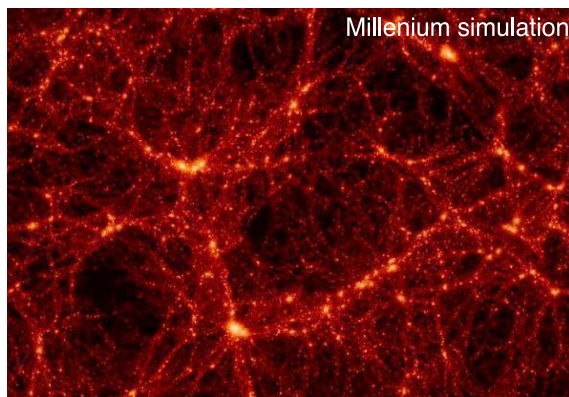
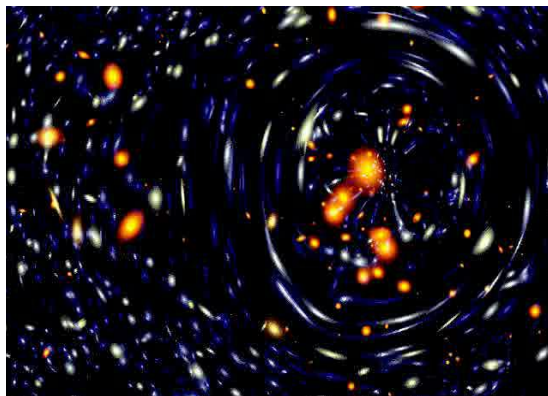
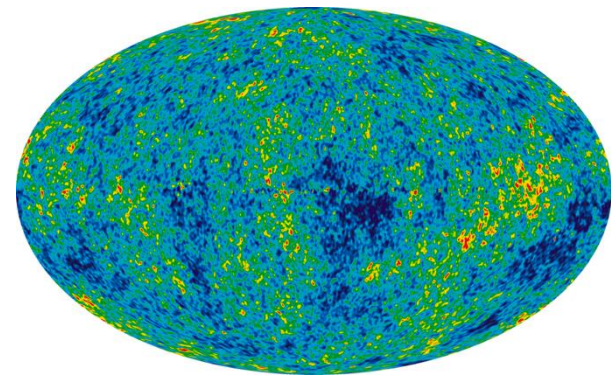
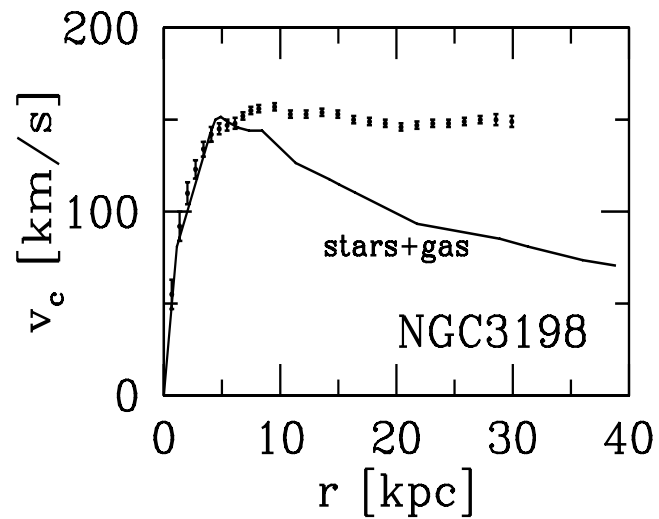
章子昌 *et al*, arXiv: 2505.09219(PRD), 2503.02573(PRD), 2403.18529(PRD), 2112.14041(SCMPA)

# Evidence for Dark Matter



- Rotation curve
- Gravitational lensing
- Bullet cluster
- Large-scale structure
- Anisotropy of CMB

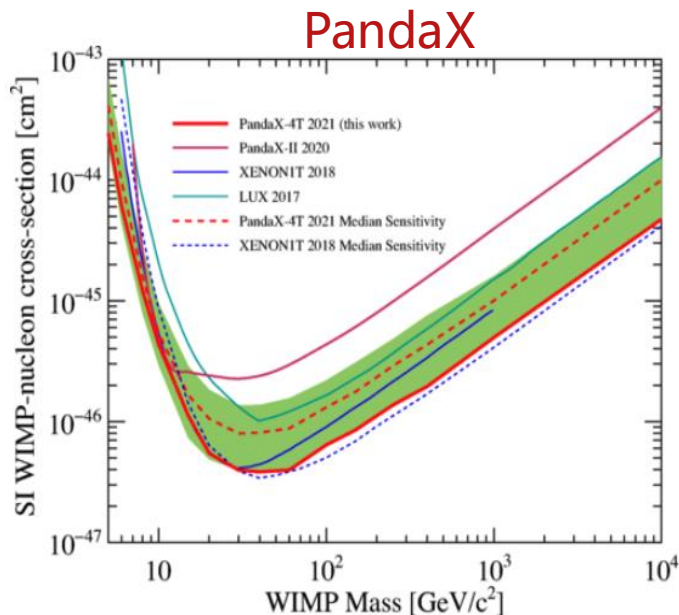
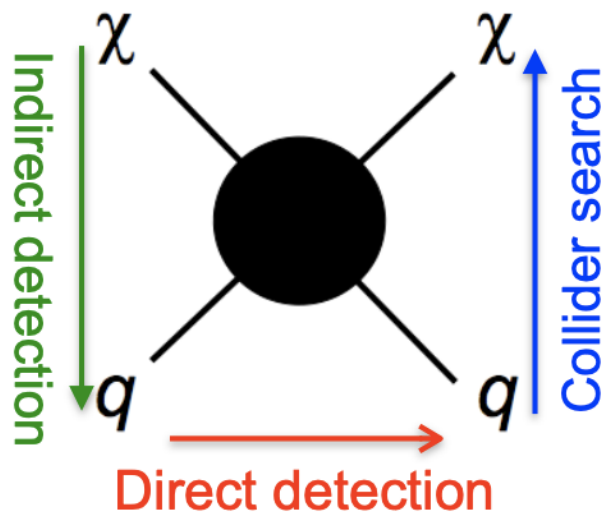
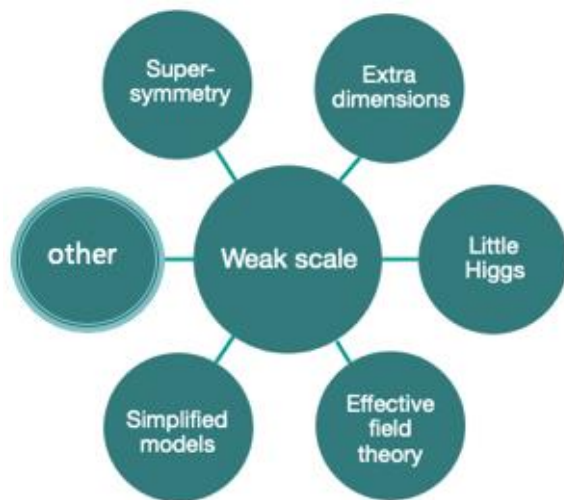
# Evidence for Dark Matter





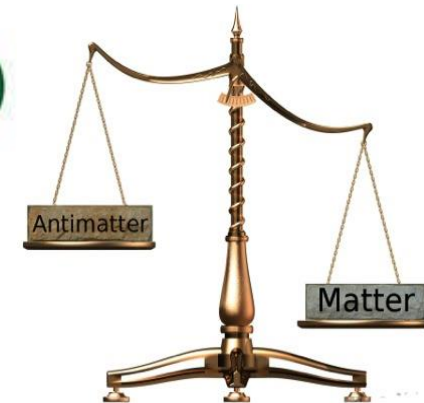
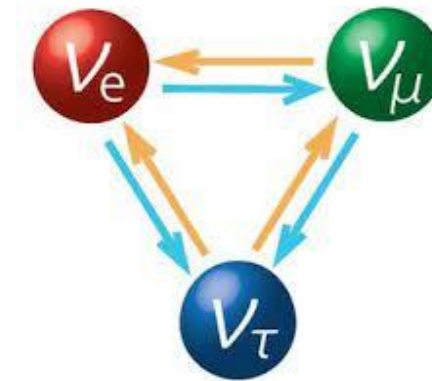
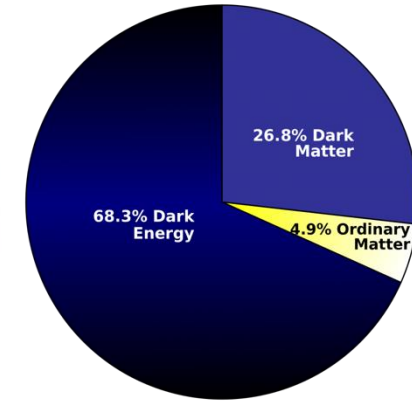
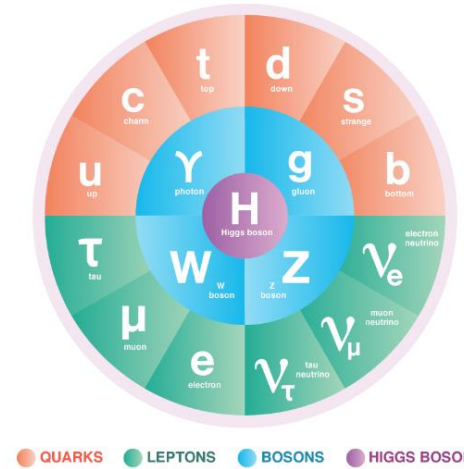
# Weakly Interacting Massive Particle (WIMP)

- Theories: supersymmetry, extra dimensions, .....
- Direct, indirect and collider searches
- Impressive progress



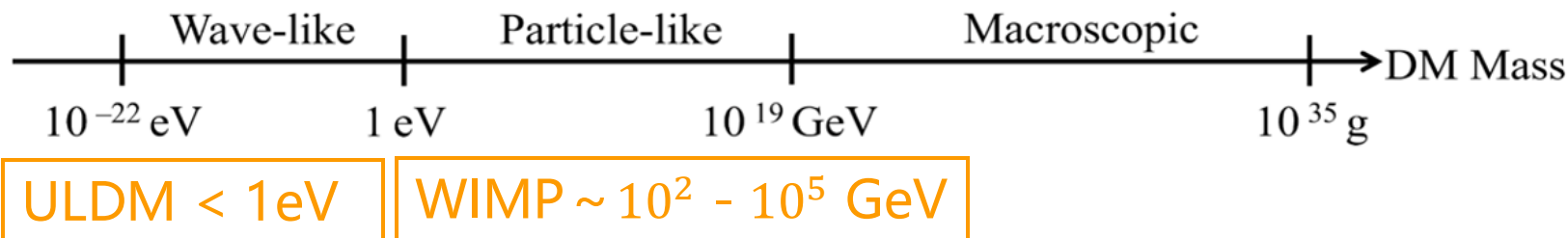
# More Motivations

- Standard model is not complete
- Dark Matter and Dark Energy
- Neutrino mass
- Matter-Antimatter asymmetry
- Theoretical Problems
  - Strong CP problem
  - Hierarchy problem
  - Fermion mass hierarchy
  - Unification of forces
  - .....



# Dark Matter Candidates

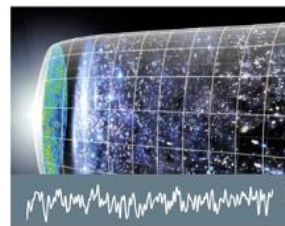
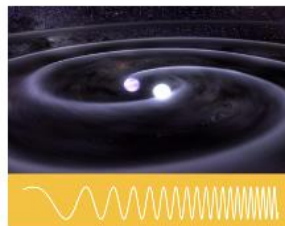
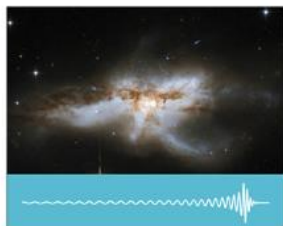
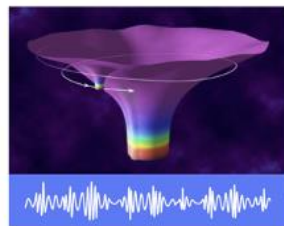
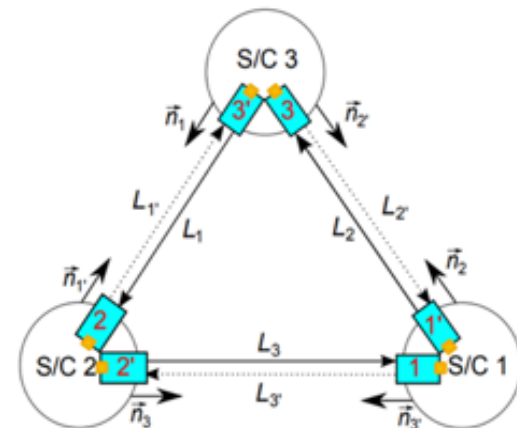
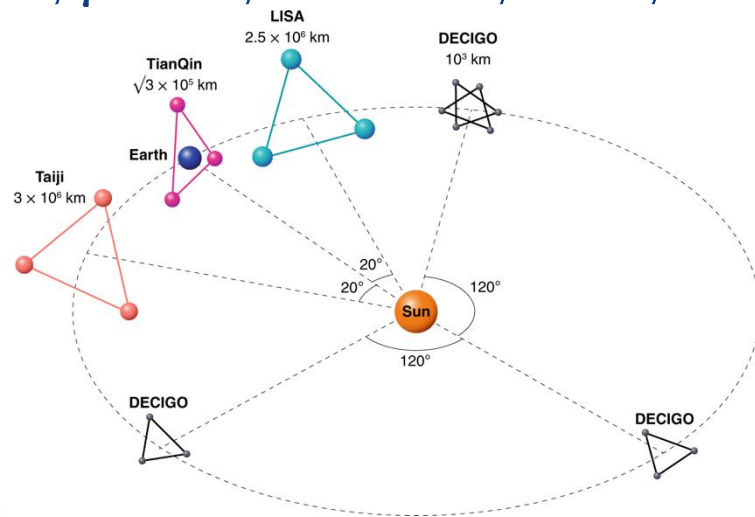
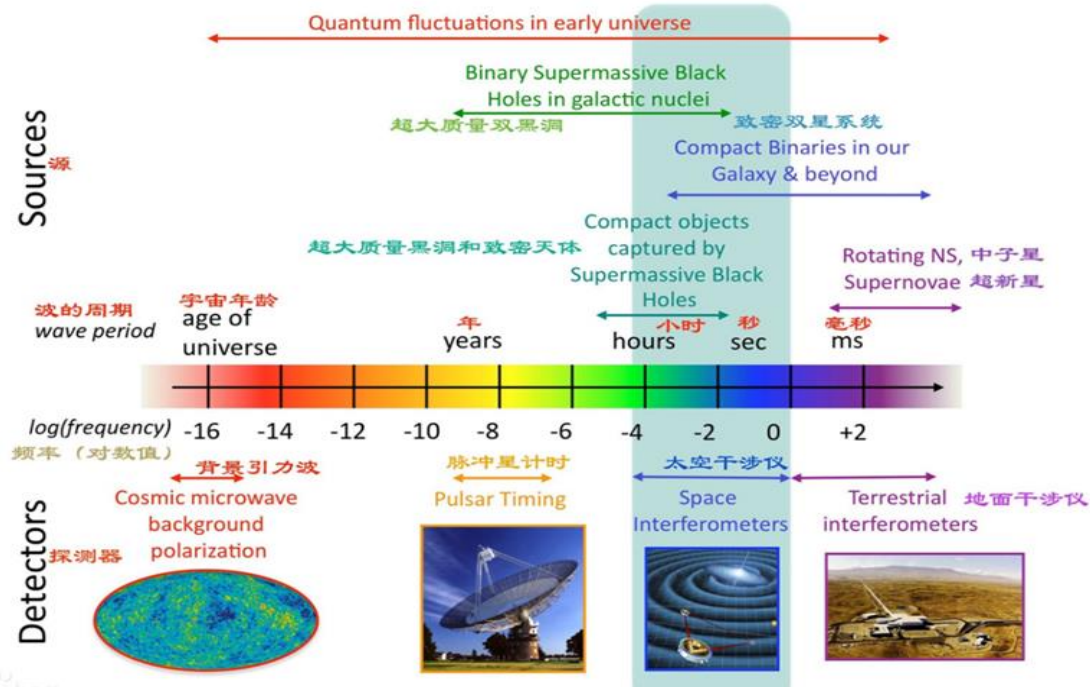
- Primordial black holes
- Super heavy particles
- Asymmetric DM
- Hidden sector DM
- .....
- **Weakly-interacting (WIMP)**
- Strongly-interacting (SIMP)
- Sterile neutrino
- **Ultralight DM, Axion (ALP), Dark photon, dilaton etc**



Since we do not know what DM is, we may explore as much as we can!

# GW Laser Interferometers in Space

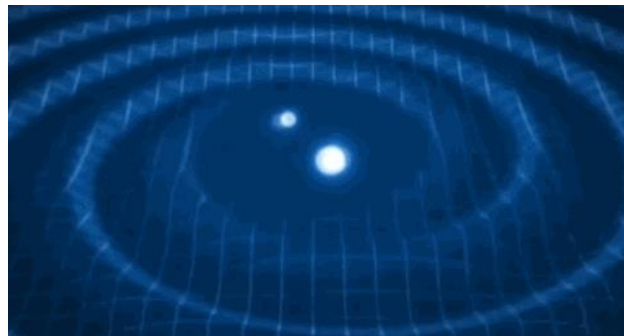
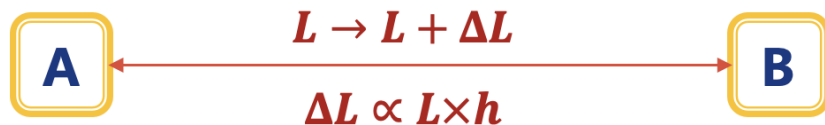
➤ LISA, Taiji, TianQin, LISAmass, Astrod-GW,  $\mu$ Ares, DECIGO, BBO,...



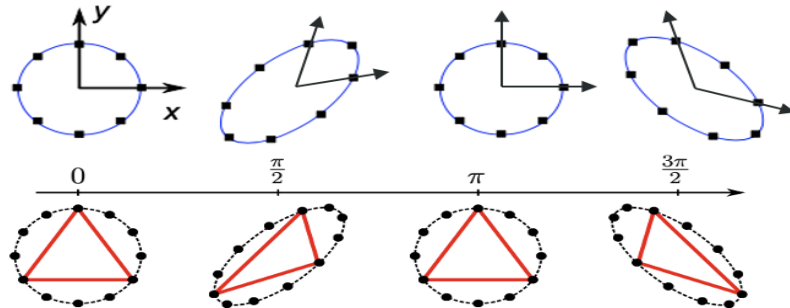
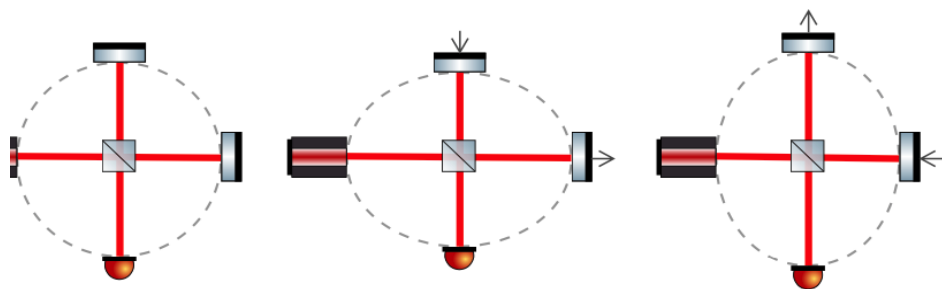


# Signal Response

- Gravitational wave can change the structure of spacetime, and the physical distance between objects



- $h \sim \Delta L / L \sim 10^{-12} \text{m} / 10^9 \text{m} \sim 10^{-21}$
- One can measure the length/phase by laser



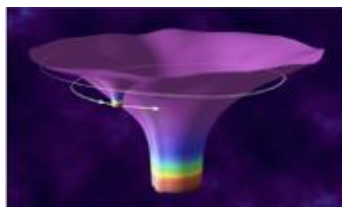
- Doppler shift  $\frac{\delta v(t)}{v_0} \equiv y_{BA} = -\frac{1}{2} \frac{n_i n_j}{1 + \vec{k} \cdot \vec{n}} \left[ h_{ij} \left( t - \frac{\vec{k} \cdot \vec{x}_B}{c} \right) - h_{ij} \left( t - \frac{\vec{k} \cdot \vec{x}_{A+L}}{c} \right) \right]$

# GW sources and waveforms

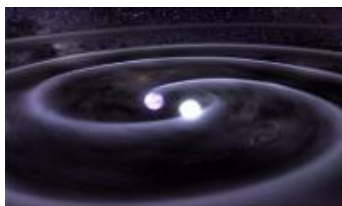
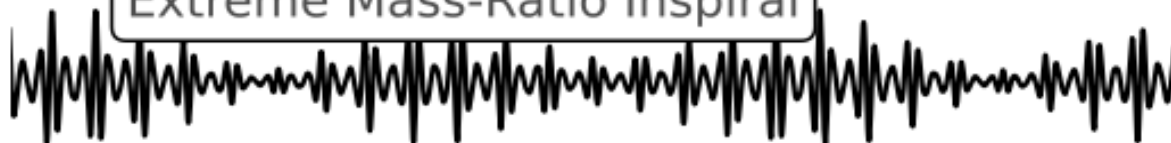
LISA, 2402.07571



Massive BH Binary Merger



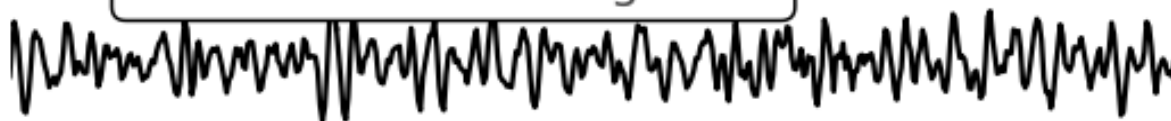
Extreme Mass-Ratio Inspiral



Galactic Binary

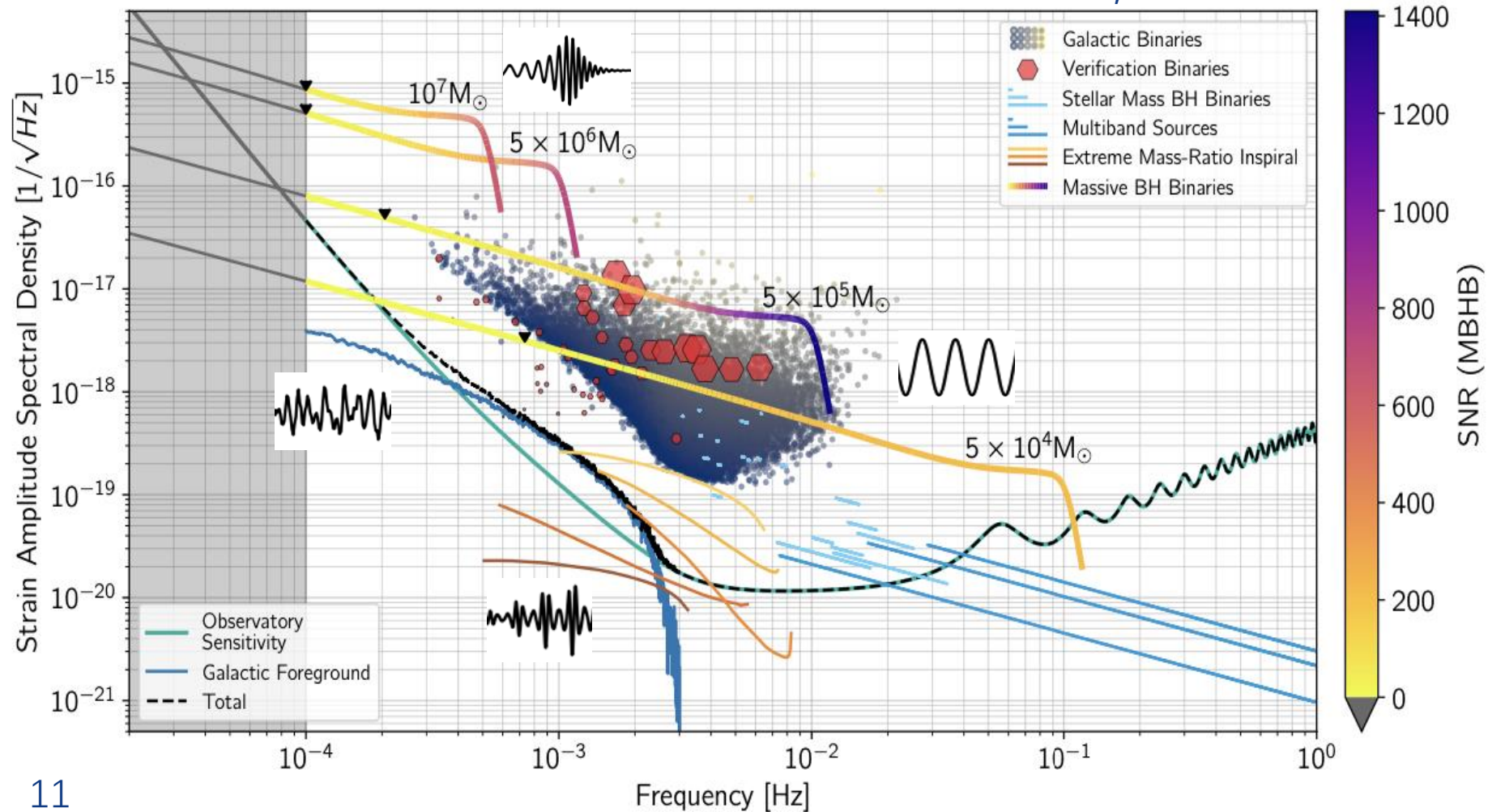


Stochastic GW Background



# GW sources and waveforms

LISA, 2402.07571



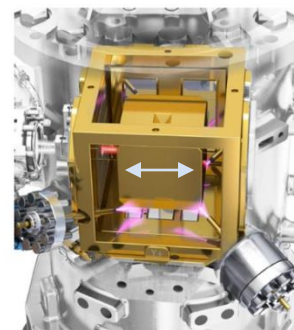
# Main Noises

## ➤ Laser frequency noise

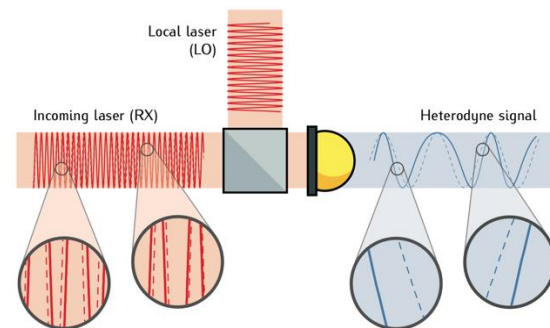
- The laser is not perfectly monochromatic, its frequency fluctuates.
- Need dedicated data analysis - Time-Delay Interferometry

## ➤ Secondary noises

- Acceleration noise of test mass,  $s_{acc} \sim 10^{-15} \text{ m/s}^2$ 
  - Residual charges, self-gravity, ...
- Optical metrology noise,  $s_{oms} \sim 10^{-12} \text{ m}$ 
  - Shot noise, relative intensity noise, ...



LISA, 2402.07571



Barke 2015

|                                                            | LISA | Taiji | Tianqin | BBO                  | DECIGO             |
|------------------------------------------------------------|------|-------|---------|----------------------|--------------------|
| $L$ ( $10^9$ m)                                            | 2.5  | 3     | 0.17    | 0.05                 | $1 \times 10^{-3}$ |
| $s_{acc}$ ( $10^{-15} \frac{\text{m}}{\sqrt{\text{Hz}}}$ ) | 3    | 3     | 1       | $3 \times 10^{-2}$   | $4 \times 10^{-4}$ |
| $s_{oms}$ ( $10^{-12} \frac{\text{m}}{\sqrt{\text{Hz}}}$ ) | 15   | 8     | 1       | $1.4 \times 10^{-5}$ | $2 \times 10^{-6}$ |

# Time-Delay Interferometry

Review by Tinto & Dhurandhar 2021

➤ There are multiple combinations

➤ Michelson channels

$$X(t) = (\eta_{2':322'} + \eta_{1:22'} + \eta_{3:2'} + \eta_{1'}) - (\eta_{3:2'3'3} + \eta_{1':3'3} + \eta_{2':3} + \eta_1),$$

$$Y(t) = (\eta_{3':133'} + \eta_{2:33'} + \eta_{1:3'} + \eta_{2'}) - (\eta_{1:3'1'1} + \eta_{2':1'1} + \eta_{3':1} + \eta_2),$$

$$Z(t) = (\eta_{1':211'} + \eta_{3:11'} + \eta_{2:1'} + \eta_{3'}) - (\eta_{2:1'2'2} + \eta_{3':2'2} + \eta_{1':2} + \eta_3).$$

➤ Sagnac channels

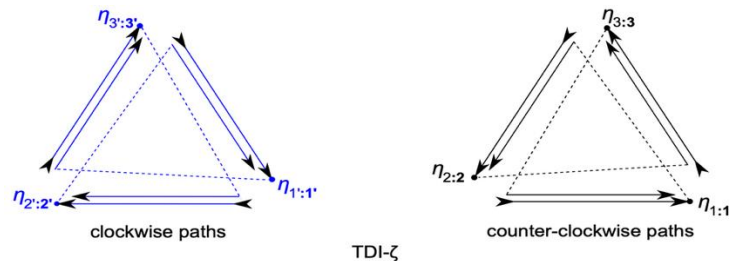
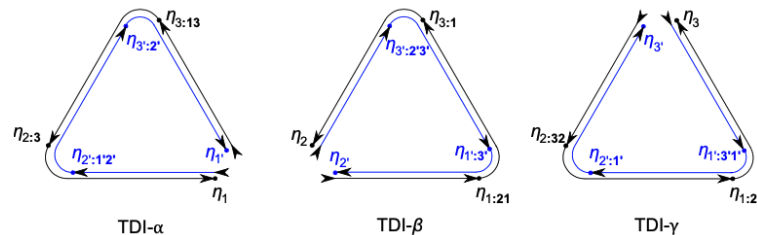
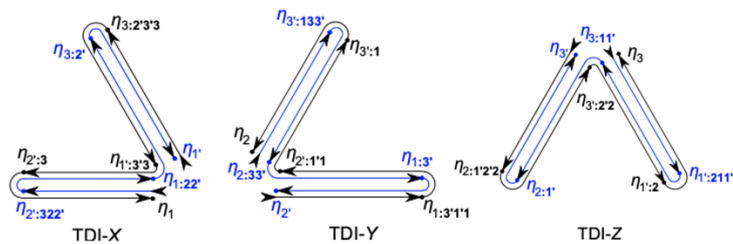
$$\alpha(t) = (\eta_{2':1'2'} + \eta_{3':2'} + \eta_{1'}) - (\eta_{3:13} + \eta_{2:3} + \eta_1),$$

$$\beta(t) = (\eta_{3':2'3'} + \eta_{1':3'} + \eta_{2'}) - (\eta_{1:21} + \eta_{3:1} + \eta_2),$$

$$\gamma(t) = (\eta_{1':3'1'} + \eta_{2':1'} + \eta_{3'}) - (\eta_{2:32} + \eta_{1:2} + \eta_3).$$

➤  $\zeta$  channel

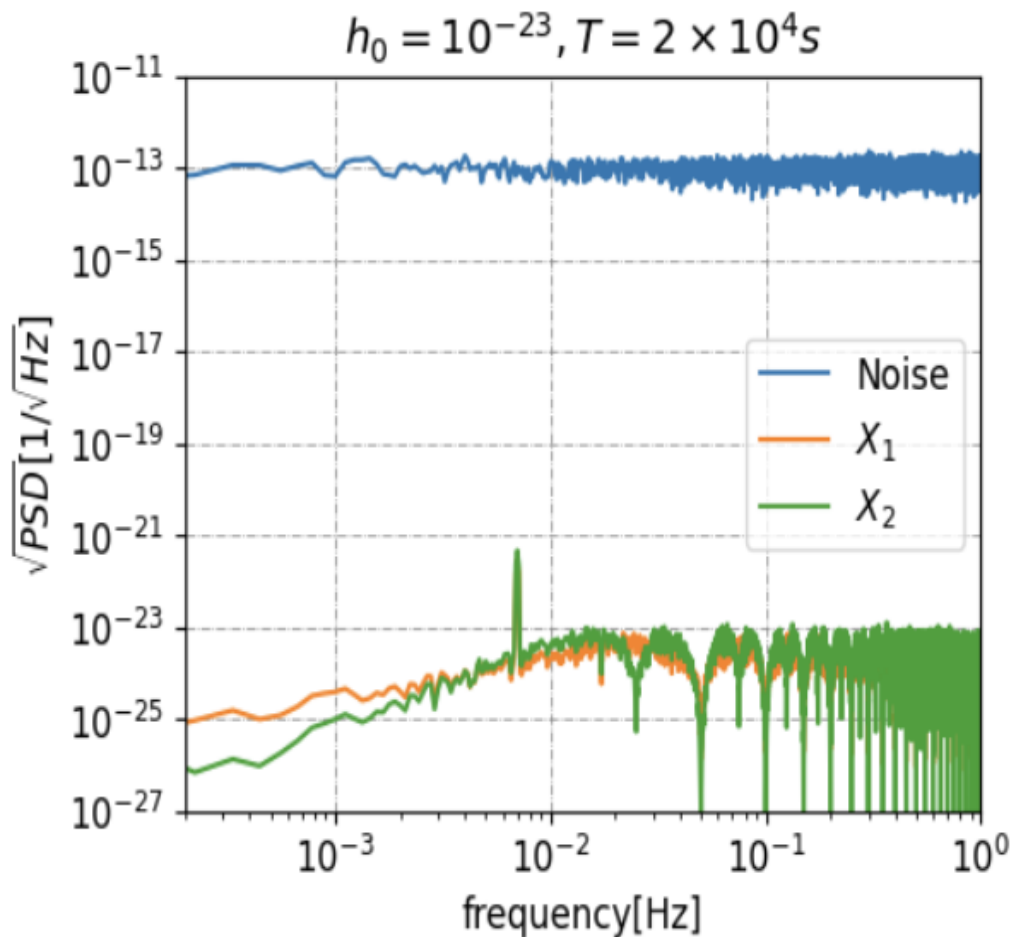
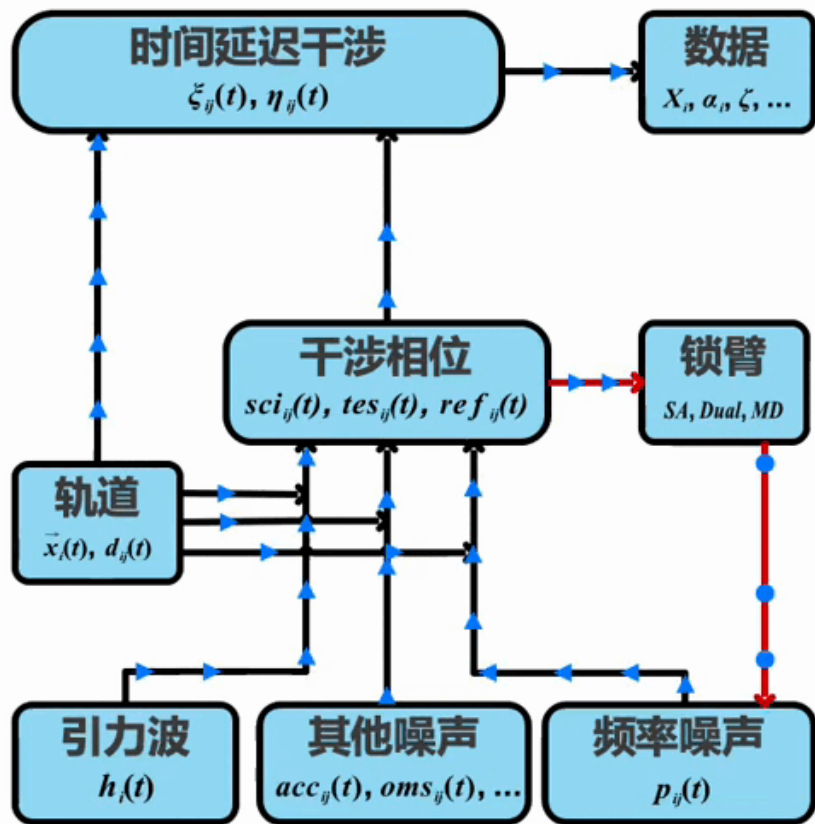
$$\zeta(t) = (\eta_{1':1'} + \eta_{2':2'} + \eta_{3':3'}) - (\eta_{1:1} + \eta_{2:2} + \eta_{3:3}).$$



Credit: Otto 2015



# Time-Delay Interferometry



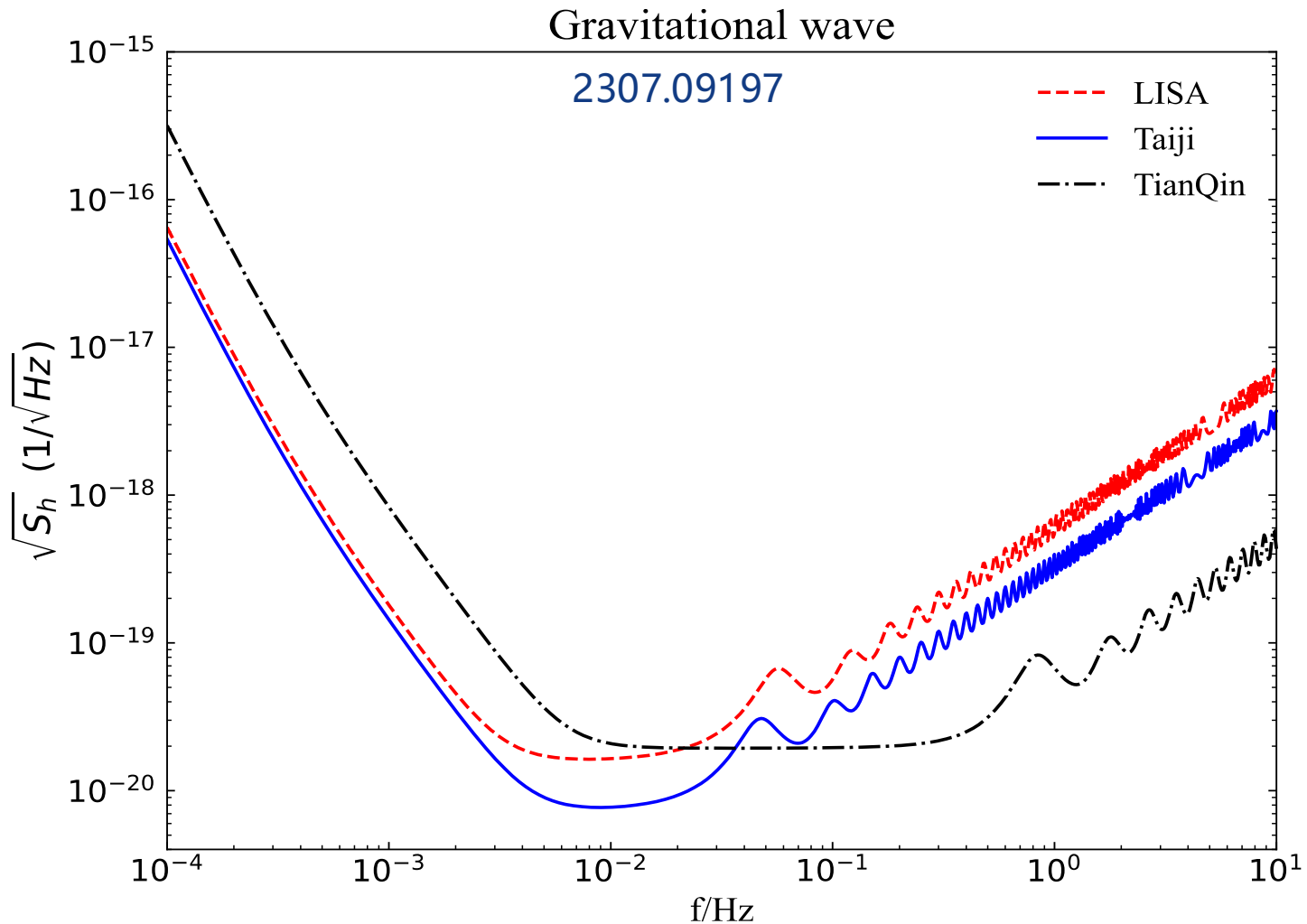
# Sensitivity on GW

## ➤ General

- Arm length
- Noise level
- Duration
- TDI

## ➤ Particular

- Direction
- Frequency
- Waveform



# Possible Effects from Dark Matter

## ➤ Direct - Interact with the detector directly

- Induce additional motion of test mass
- Change the size of test mass, etc
- Modify the propagation of laser light

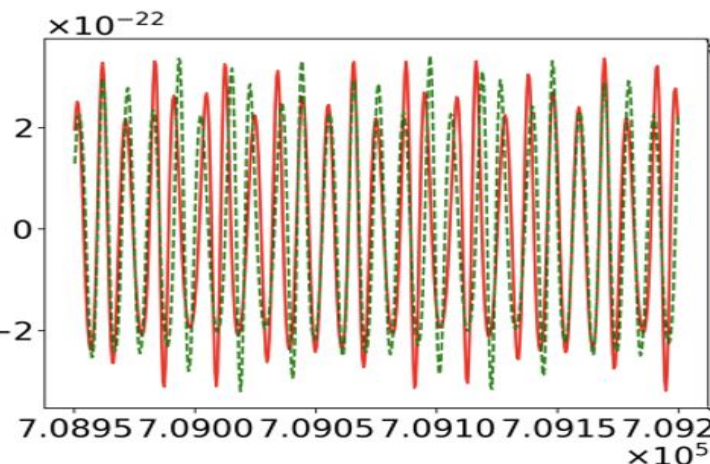


Refs. Pierce, Riles & Zhao 2018, Grote & Stadnik 2019, Morisaki & Suyama 2019, .....

## ➤ Indirect - Affect the messengers that interact with the detector

- Modify the GW from astrophysical sources
- Cosmological phase transition
- As new GW sources, PBHs, ...

Refs. Eda, Itoh, Kuroyanagi & Silk 2013, Yue, Han, Chen 2017, Bertone et al 2020, Cardoso et al 2022, Gong et al, .....



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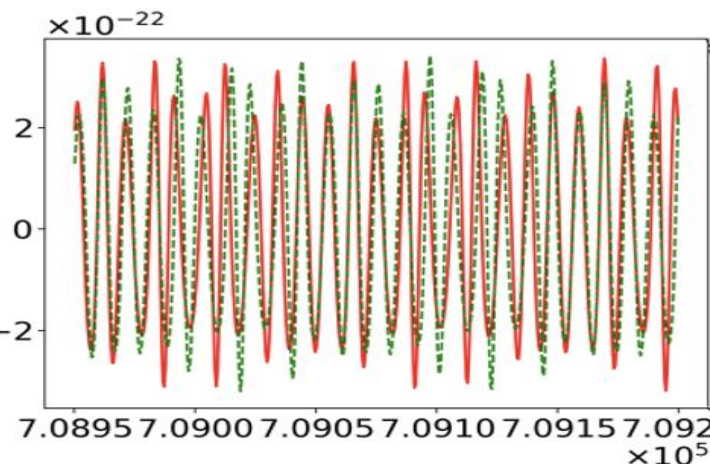


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# Ultralight/Wave Dark Matter

➤ Mass < 1 eV, QCD axion, ALP, Dark Photon, ...

➤ A phenomenological approach

➤ Scalar  $\mathcal{L} = \frac{1}{2}\partial^\mu\phi\partial_\mu\phi - \frac{1}{2}m_\phi^2\phi^2 - C\frac{\phi}{M_P}\mathcal{O}_{\text{SM}},$

➤ Light dilaton,  $\delta\mathcal{L} = \frac{\phi}{M_P} \left[ -\frac{d_g\beta_3}{2g_3}F_{\mu\nu}^A F^{A\mu\nu} - \sum_{i=u,d} (d_{m_i} + \gamma_{m_i}d_g) m_i \bar{\psi}_i \psi_i \right]$  Damour & Donoghue 2010

➤ ALP,  $-\frac{g_{a\gamma}}{4}aF_{\mu\nu}\tilde{F}^{\mu\nu},$

➤ Vector  $\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{1}{2}m_A^2 A^\nu A_\nu - \epsilon_D e J_D^\nu A_\nu,$

➤ Baryon number, B-L, Dark  $U(1)$ , .....

➤ Production Mechanism – viable DM candidate

scalar: misalignment, ...

vector: Graham, Mardon & Rajendran 2015, Ema, Nakayama & Tang 2019, Kolb & Long 2024

王清扬, Tang & Wu 2023, .....



# Wave Dark Matter Background

Foster, Rodd, Safdi 2018

- Number density is very large, behaves as classical wave

$$\Phi(x) = \sum_{\mathbf{v}} \frac{\sqrt{2\rho/N}}{m} e^{i(\omega t - \mathbf{k} \cdot \mathbf{x} + \theta_{\mathbf{v}})},$$

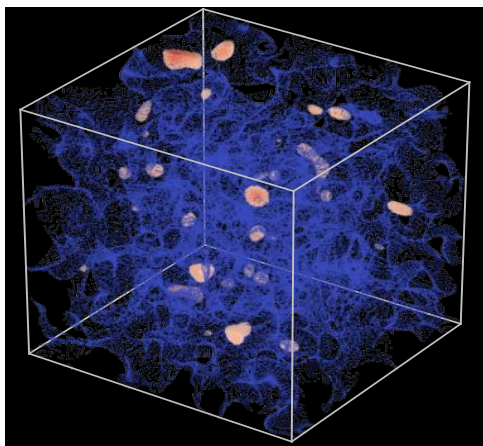
$$\vec{k} = m\vec{v}, \omega = 2\pi f \simeq m, v \sim 10^{-3},$$

$$f \approx 2.4 \times \left( \frac{m}{10^{-17} \text{eV}} \right) \text{mHz}$$

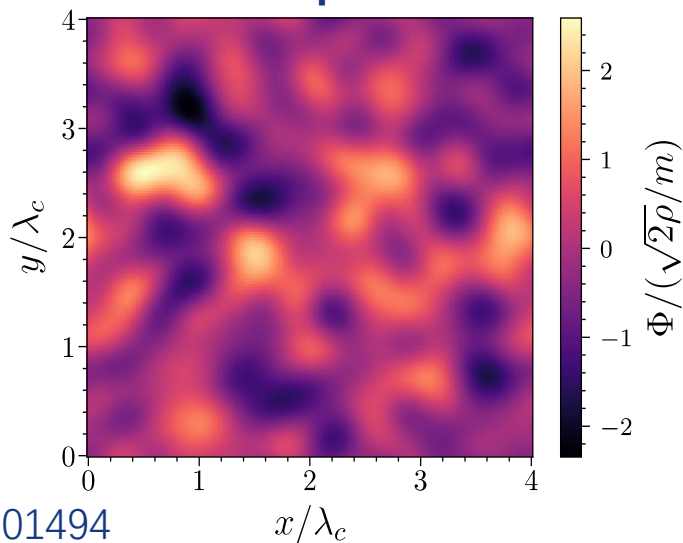
at a specific point  $\sim$  monochromatic within  $\tau_c$ .

$$\lambda_c = \frac{2\pi}{mv} \approx 1.24 \times 10^{11} \times \left( \frac{10^{-17} \text{eV}}{m} \right) \text{km}, \quad \tau_c = \frac{\lambda_c}{v} \approx 4 \times 10^8 \times \left( \frac{10^{-17} \text{eV}}{m} \right) \text{s},$$

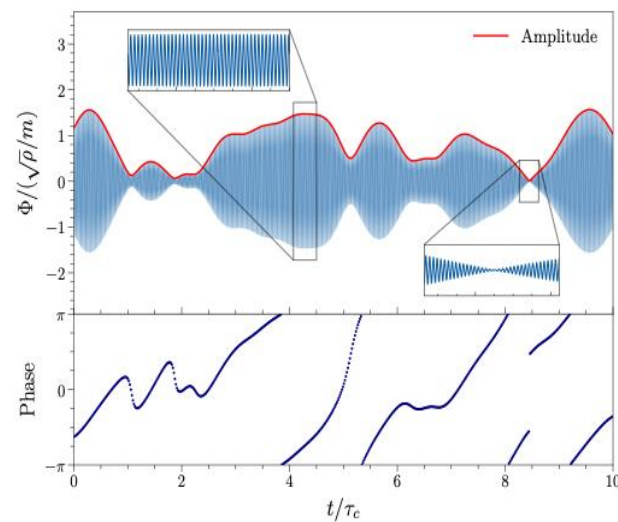
3D snapshot



2D snapshot



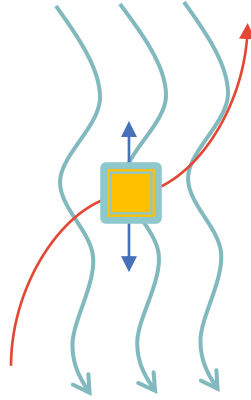
at some point



# Physical Effects

- Plane wave within  $\tau_c$ ,  $\phi(t, \vec{x}) = \phi_{\vec{k}} e^{i(\omega t - \vec{k} \cdot \vec{x} + \theta_0)}$ ,
- Scalar  $\phi$ ,  $\mathcal{L} = \frac{1}{2} \partial^\mu \phi \partial_\mu \phi - \frac{1}{2} m_\phi^2 \phi^2 - C \frac{\phi}{M_P} \mathcal{O}_{\text{SM}}$ ,
- Interaction depending on the underlying theory, e.g.  

$$C \frac{\phi}{M_P} m_\psi \bar{\psi} \psi \Rightarrow m_\psi \rightarrow \left(1 + C \frac{\phi}{M_P}\right) m_\psi, \quad S = - \int m(\phi) \sqrt{-\eta_{\mu\nu}} dx^\mu dx^\nu.$$
- Additional gradient or force



$$\delta x^i(t, \vec{x}) = \mathcal{M}_s \hat{k}^i e^{im_\phi(t - v \hat{k} \cdot \vec{x})}, \quad \mathcal{M}_s \propto \phi_{\vec{k}} |\vec{k}| / m_\phi^2$$

- Vector  $\mathcal{L} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \frac{1}{2} m_A^2 A^\nu A_\nu - \epsilon_D e J_D^\nu A_\nu$ ,  $\vec{A}(t, \vec{x}) = |\vec{A}| \hat{e}_A e^{i(\omega t - \vec{k} \cdot \vec{x})}$ ,  

$$\delta x^i(t, \vec{x}) = \mathcal{M}_v \hat{e}_A^i e^{im_A(t - v \hat{k} \cdot \vec{x})}, \quad \mathcal{M}_v \propto \epsilon_D e q_{D,j} |\vec{A}| / m_A M_j$$

- Axion – laser propagation

# Ultralight/Wave DM - Signal Response

- DM couples to SM particles, inducing oscillations of test mass, effectively changing the length
- One-way Doppler shift



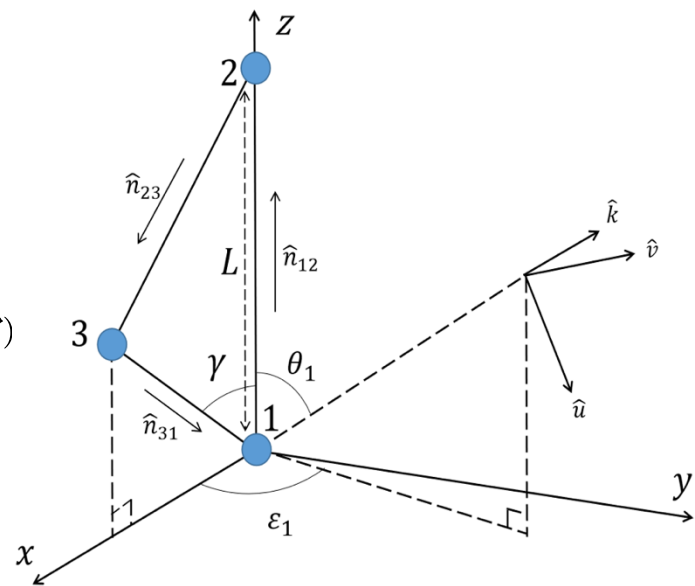
$$\delta t_{rs} = -\hat{n}_{rs} \cdot [\delta \vec{x}(t, \vec{x}_r) - \delta \vec{x}(t - L, \vec{x}_s)],$$

$$\frac{\delta \nu_{rs}}{\nu_0} = \frac{\nu_{rs} - \nu_0}{\nu_0} = -\frac{d\delta t_{rs}}{dt}.$$

- Fractional frequency change

$$y_{rs}(t) \equiv \frac{\delta \nu_{rs}}{\nu_0} = \mu_{rs} [h(t, \vec{x}_r) - h(t - L, \vec{x}_s)], \quad h(t, \vec{x}) \propto e^{im(t - v\hat{k} \cdot \vec{x})}$$

$$\mu_{rs} = \begin{cases} \hat{k} \cdot \hat{n}_{rs} & \text{for scalar field,} \\ \hat{e}_A \cdot \hat{n}_{rs} & \text{for vector field,} \\ \frac{\hat{n}_{rs}^i \hat{n}_{rs}^j e_{ij}(\hat{k}, \psi)}{2(1 + \hat{n}_{rs} \cdot \hat{k})} & \text{for gravitational wave,} \end{cases}$$



# Transfer Functions

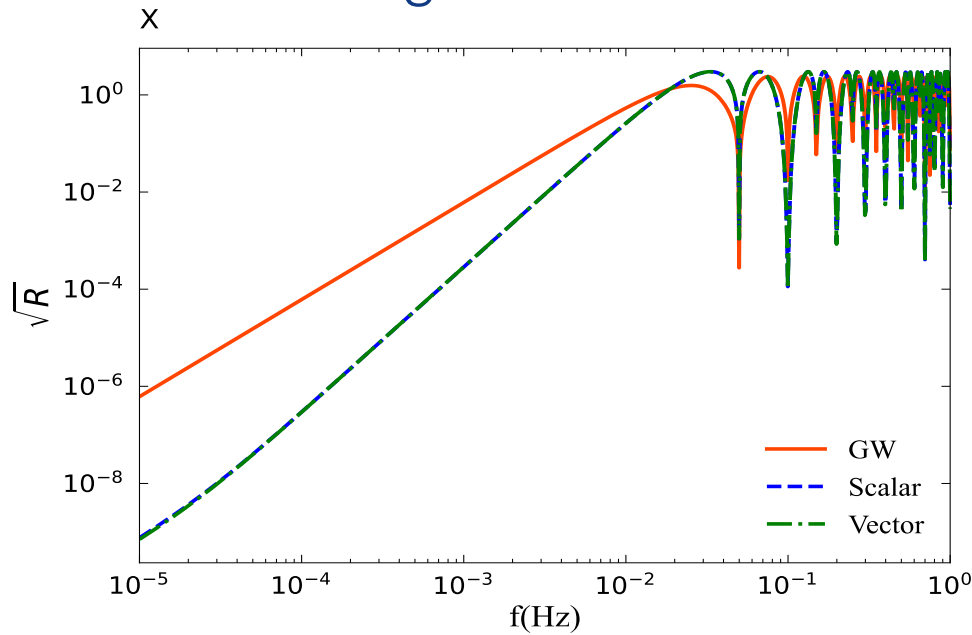
- Signal response in frequency domain  $h(t) = \frac{\sqrt{T}}{2\pi} \int_0^\infty \tilde{h}(\omega) e^{i\omega t} d\omega$
- One-way single link  $y_{rs}(t) = \mu_{rs} \frac{\sqrt{T}}{2\pi} \int_0^\infty d\omega \tilde{h}(\omega) e^{i\omega t} \left[ e^{-i\vec{k} \cdot \vec{x}_r} - e^{-i(\tau + \vec{k} \cdot \vec{x}_s)} \right],$   
 $\tilde{y}_{rs}(\omega) = \mu_{rs} \tilde{h}(\omega) \left[ e^{-i(\vec{k} \cdot \vec{x}_r)} - e^{-i(\tau + \vec{k} \cdot \vec{x}_s)} \right]. \quad \tau = 2\pi f L$
- Transfer function, sky and polarization averaged

$$R(\omega) = \left| \frac{\tilde{y}_{rs}(\omega)}{\tilde{h}(\omega)} \right|^2, \quad \text{normalized}$$

$$I_s \equiv \frac{1}{4\pi} \int_{-1}^1 d\cos\theta_1 \int_0^{2\pi} d\epsilon_1 \dots,$$

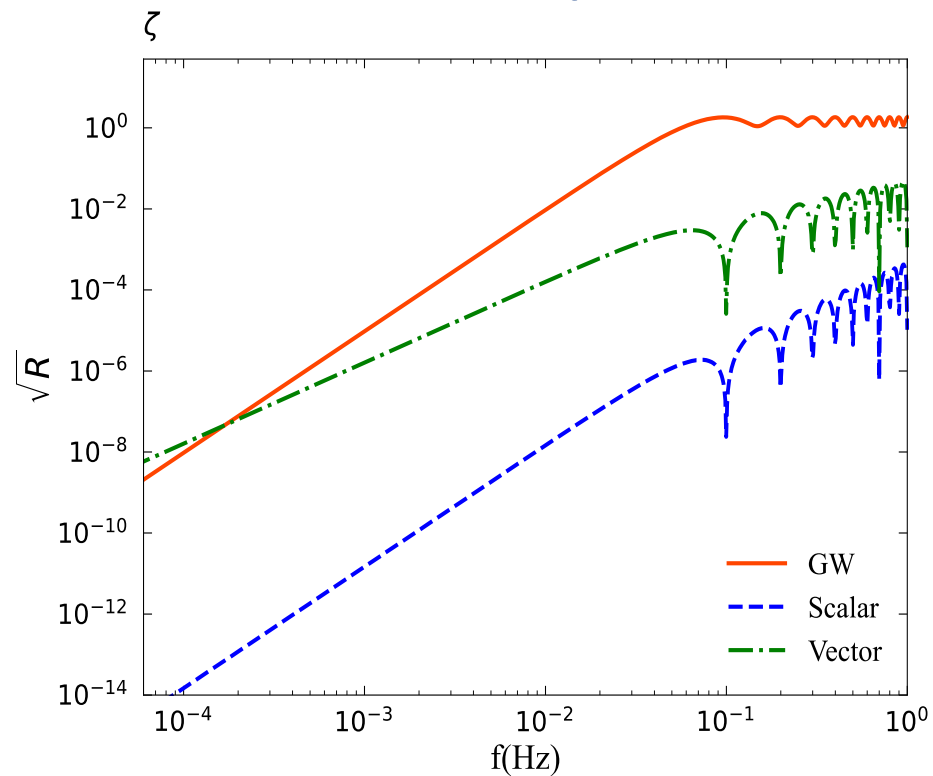
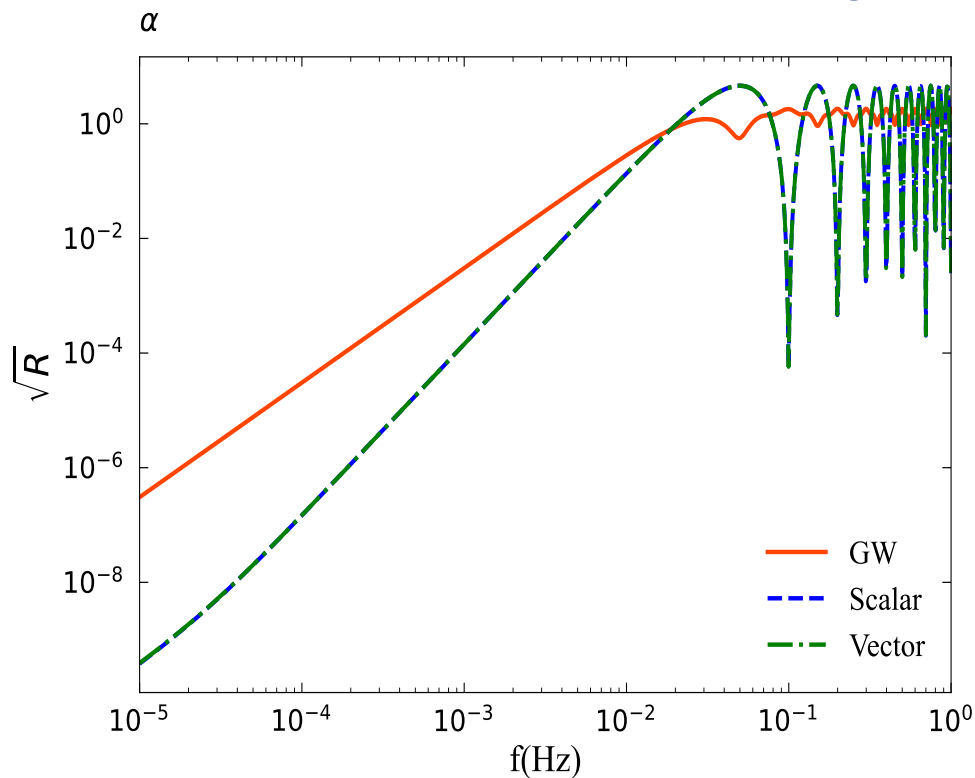
$$I_v \equiv \frac{1}{16\pi^2} \int_{-1}^1 d\cos\theta_1 \int_0^{2\pi} d\epsilon_1 \int_{-1}^1 d\cos\theta_2 \int_0^{2\pi} d\epsilon_2 \dots,$$

$$I_{GW} \equiv \frac{1}{8\pi^2} \int_{-1}^1 d\cos\theta_1 \int_0^{2\pi} d\epsilon_1 \int_0^{2\pi} d\psi \dots$$



# Transfer Functions

- Different channels have different transfer functions
- DM is also different from gravitational wave, velocity effect, ...



# Sensitivity – Michelson Interferometer

➤ Defined by  $S_O(f) = \frac{N_O(f)}{R_O(f)}$ ,  $N_X = 16 \sin^2(\tau) \{ [3 + \cos(2\tau)] S_{acc} + S_{oms} \}$ ,  $\tau = 2\pi f L$

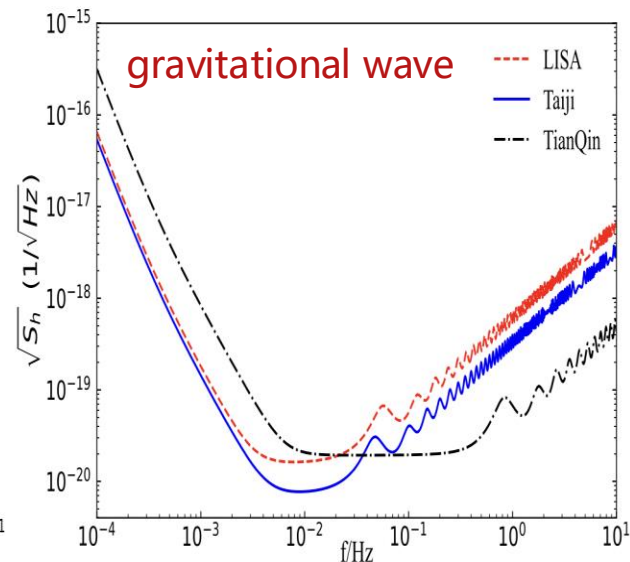
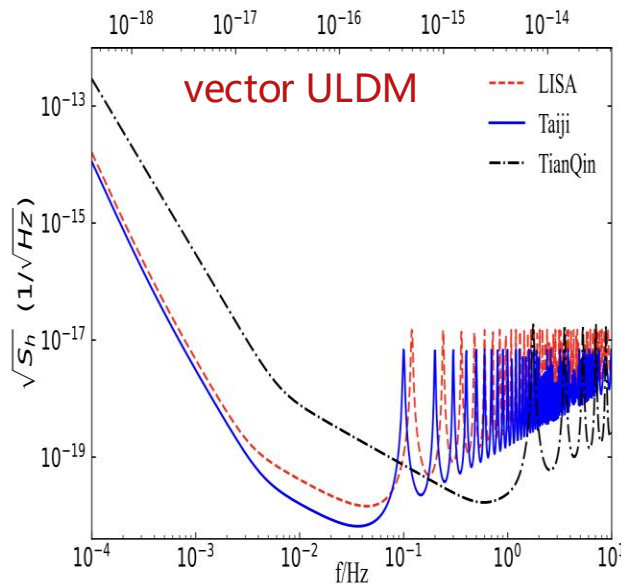
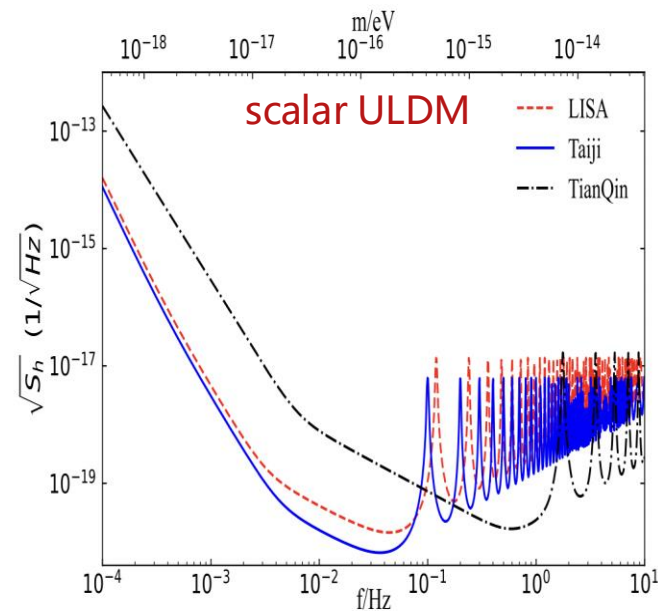
$$S_{oms}(f) = \left( s_{oms} \frac{2\pi f}{c} \right)^2 \left[ 1 + \left( \frac{2 \times 10^{-3} \text{ Hz}}{f} \right)^4 \right] \frac{1}{\text{Hz}},$$

$$S_{acc}(f) = \left( \frac{s_{acc}}{2\pi f c} \right)^2 \left[ 1 + \left( \frac{0.4 \times 10^{-3} \text{ Hz}}{f} \right)^2 \right] \left[ 1 + \left( \frac{f}{8 \times 10^{-3} \text{ Hz}} \right)^4 \right] \frac{1}{\text{Hz}},$$

$$\text{LISA : } s_{oms} = 15 \times 10^{-12} \text{ m}, s_{acc} = 3 \times 10^{-15} \text{ m/s}^2,$$

$$\text{Taiji : } s_{oms} = 8 \times 10^{-12} \text{ m}, s_{acc} = 3 \times 10^{-15} \text{ m/s}^2,$$

$$\text{TianQin : } s_{oms} = 1 \times 10^{-12} \text{ m}, s_{acc} = 1 \times 10^{-15} \text{ m/s}^2.$$



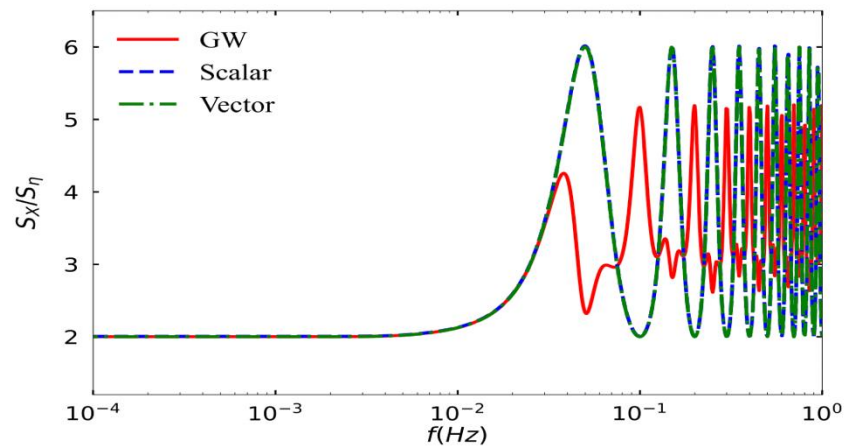
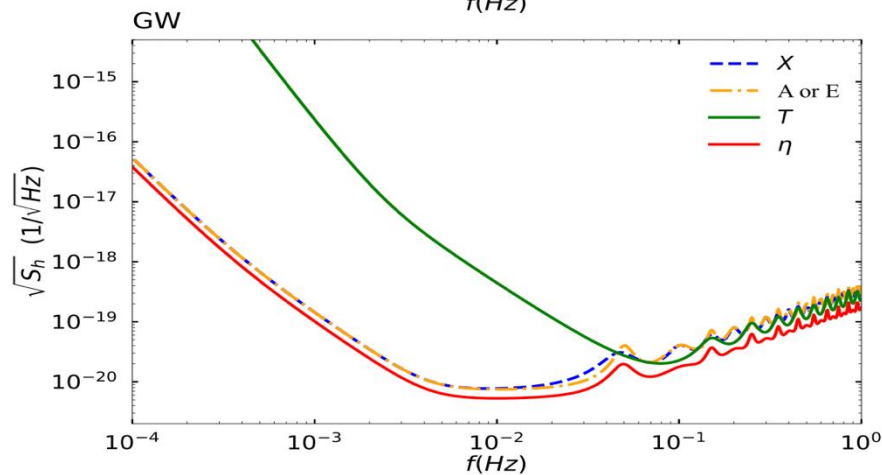
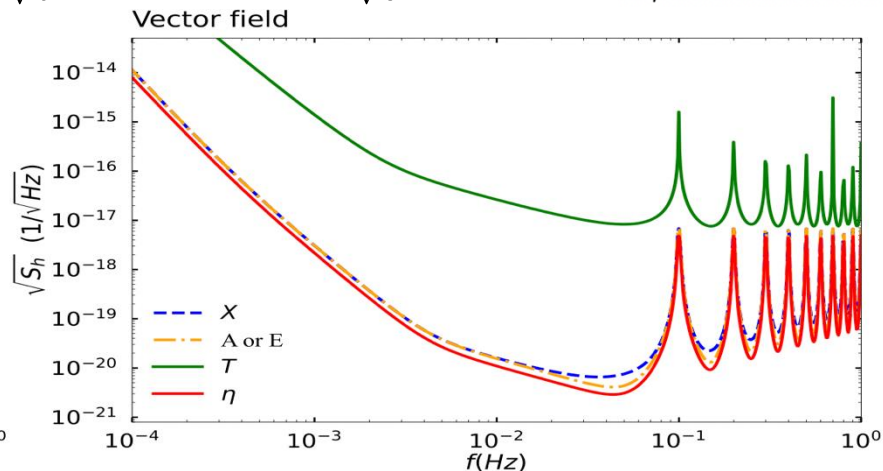
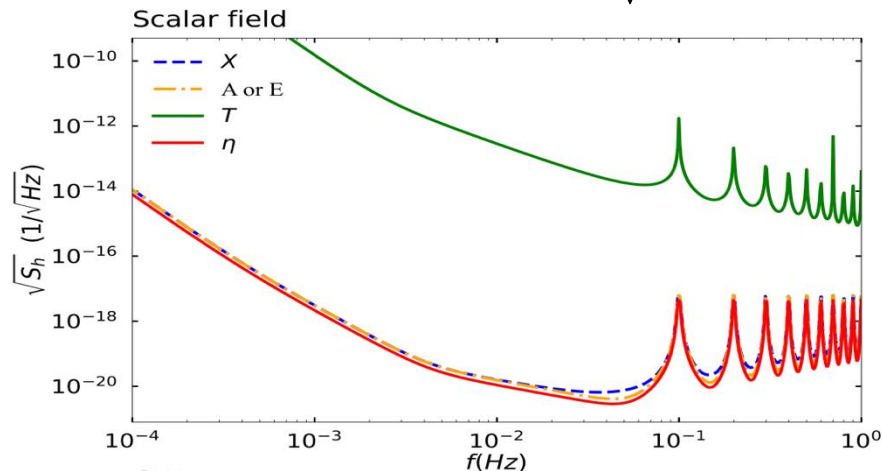


# Sensitivity – Other Interferometers

Prince, Tinto, Larson & Armstrong

## ➤ Optimal channels

$$A = \frac{1}{\sqrt{2}} [Z - X], E = \frac{1}{\sqrt{6}} [X - 2Y + Z], T = \frac{1}{\sqrt{3}} [X + Y + Z]. \quad \frac{1}{S_\eta} = \frac{1}{S_A} + \frac{1}{S_E} + \frac{1}{S_T}$$



# Sensitivity on scalar DM

➤ Strong sector

$$\delta\mathcal{L} = \frac{\phi}{M_P} \left[ -\frac{d_g\beta_3}{2g_3} F_{\mu\nu}^A F^{A\mu\nu} - \sum_{i=u,d} (d_{m_i} + \gamma_{m_i} d_g) m_i \bar{\psi}_i \psi_i \right]$$

Damour & Donoghue

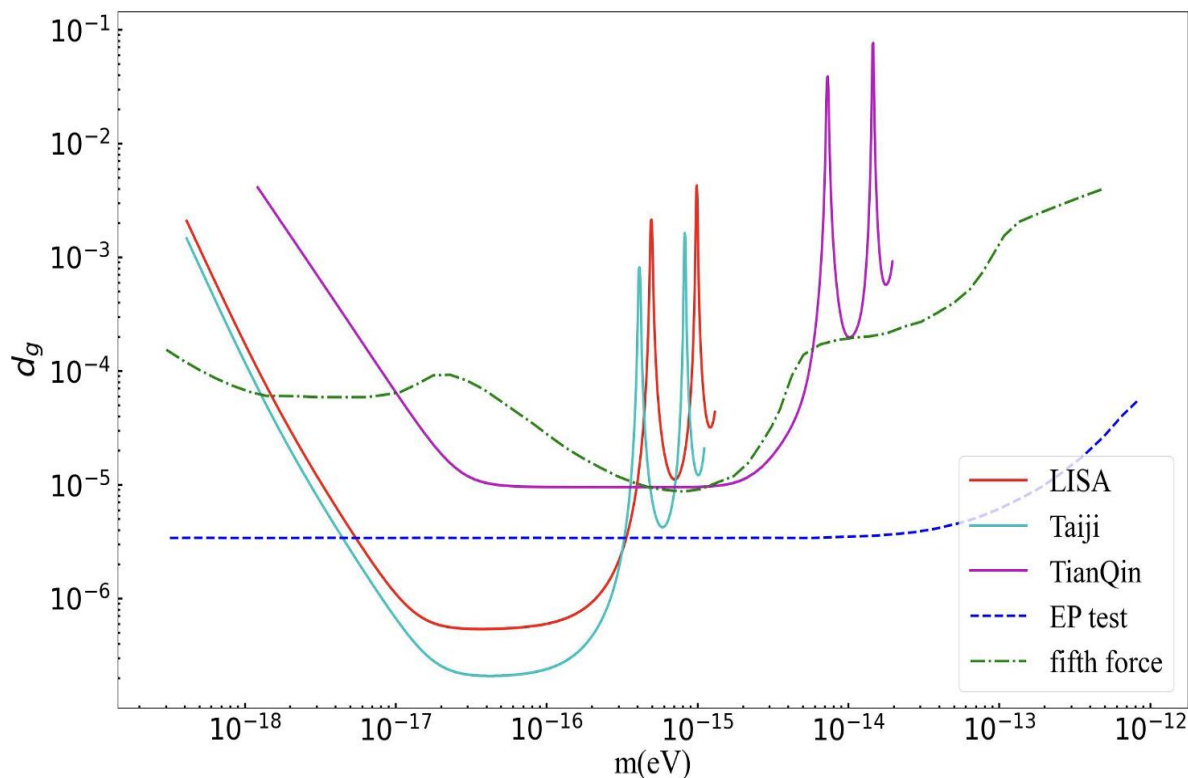
➤ At nucleus level,  
coupling is not  $\propto$  mass

➤ Equivalence principle  
is violated.

➤ MICROSCOPE

➤ Pulsars

Shao, Wex, Kramer 2019



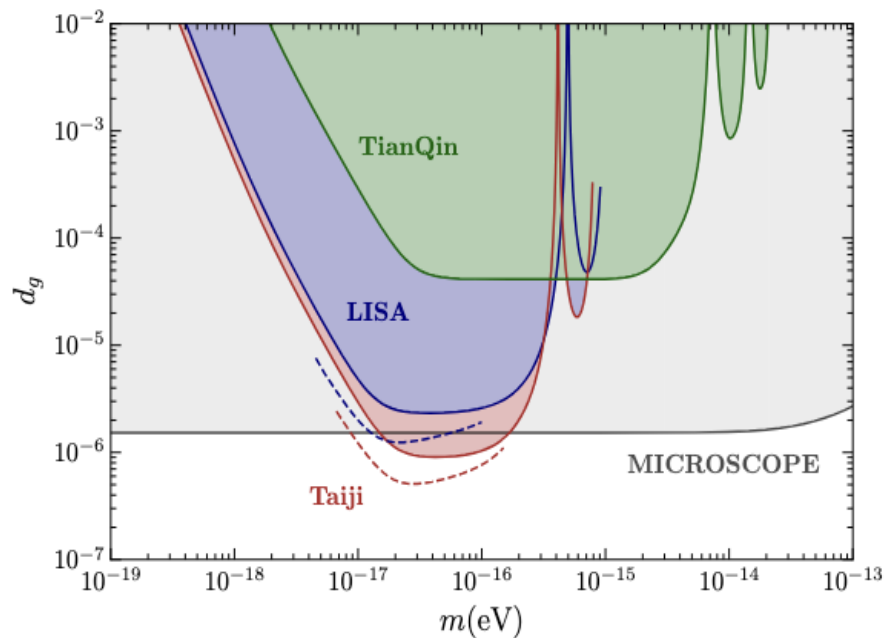
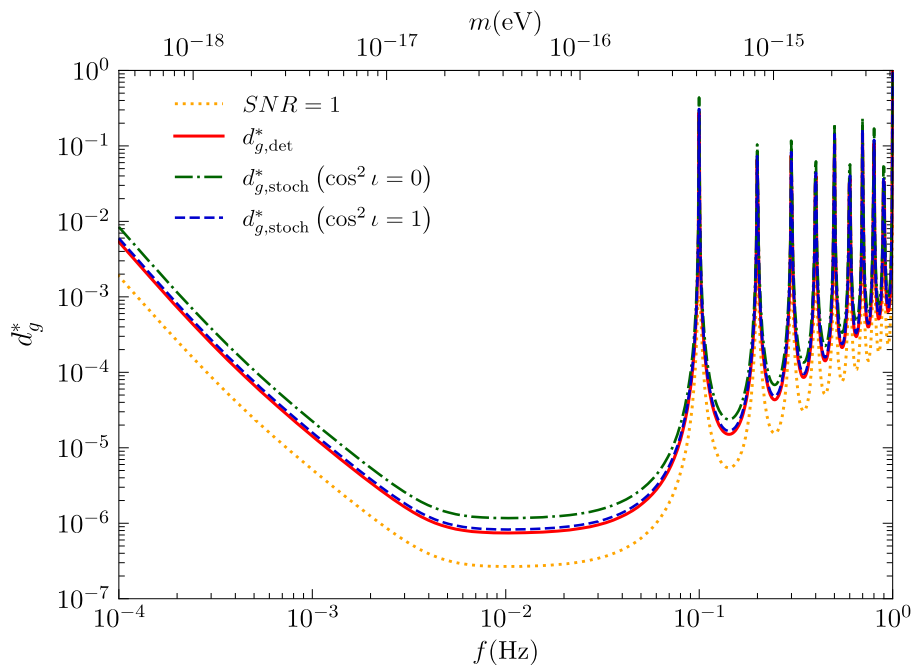
# Statistical Effects

## ➤ Velocity distributions, likelihood analysis

$$S_O(\lambda_{\min}) = \left( \frac{\ln \alpha}{\ln \gamma} - 1 \right) N_O, \quad \lambda_{\min}^2 = \frac{N_O}{\Gamma_O S_\Phi} \left( \frac{\ln \alpha}{\ln \gamma} - 1 \right).$$

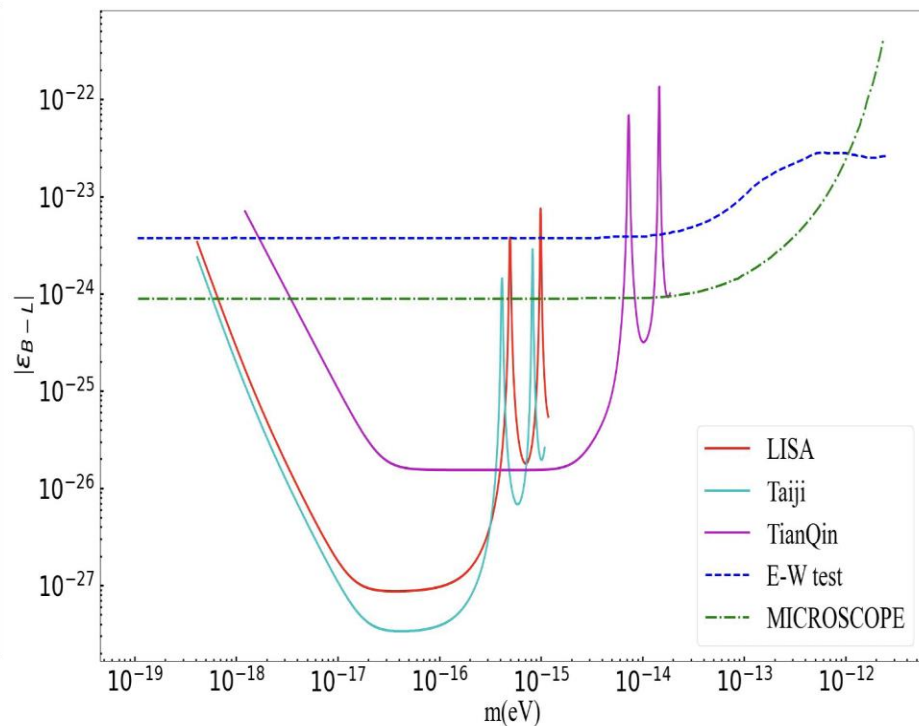
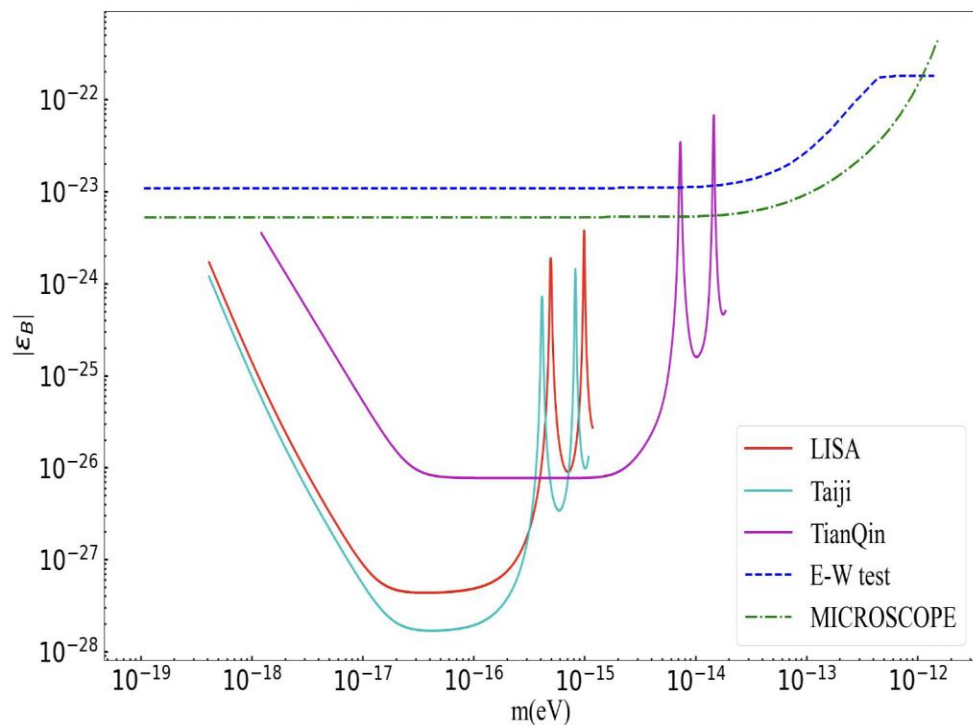
$$\gamma = \int_{P_*(\alpha)}^{\infty} dP \mathcal{L}_{\text{stoch}} \left( \tilde{d} \middle| S_O(\lambda_{\min}), N_O \right), \quad \text{detection probability}$$

$$\alpha = \int_{P_*}^{\infty} dP \mathcal{L}_{\text{stoch}} \left( P \middle| \lambda = 0, N_O \right). \quad \text{false alarm rate}$$



# Sensitivity on vector DM

- For example, vector fields couple to baryon number  $B$ , or  $B-L$ , effectively neutron number. Sensitivity on ratio  $\epsilon_D = e_D/e$

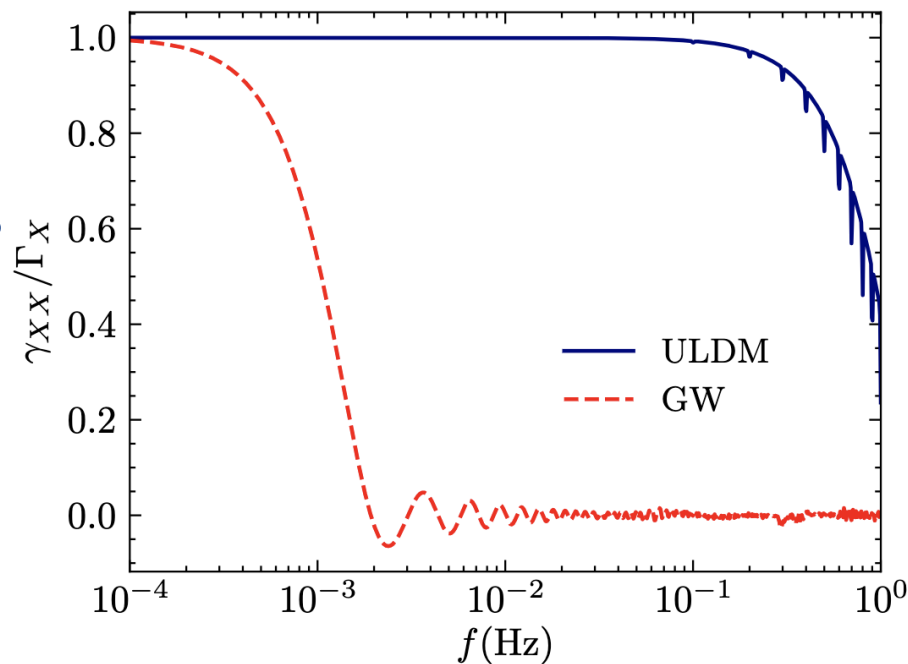


# Correlation

- Network of detectors

$$\gamma_{ab}(f, v) := \int d\hat{k} \chi_{ab}(f, \hat{k}, v), \quad \lambda_{\min}^2 = \frac{N}{2\Gamma_X S_\Phi} \left( \frac{\ln \alpha}{\ln \gamma} - 1 \right).$$

- Correlation length is larger
- Sensitivity is enhanced by  $\sqrt{n}$
- Different from SGWB searches which has  $\sqrt{T}$  enhancement.
- How to distinguish them.
- Anisotropy and TDI



# Sensitivity on Axion-Photon Coupling

- Axion-photon coupling

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}\partial_\mu a\partial^\mu a - \frac{1}{2}m^2 a^2 - \frac{g_{a\gamma}}{4}aF_{\mu\nu}\tilde{F}^{\mu\nu},$$

- Modifies Maxwell's eqs

$$\nabla \cdot \mathbf{E} = -g_{a\gamma}\nabla a \cdot \mathbf{B},$$

$$\frac{\partial \mathbf{E}}{\partial t} - \nabla \times \mathbf{B} = -g_{a\gamma}\left(\frac{\partial a}{\partial t} \cdot \mathbf{B} + \nabla a \times \mathbf{E}\right), \quad \left(\frac{\partial^2}{\partial t^2} - \nabla^2\right)\mathbf{B} = g_{a\gamma}\dot{a}\nabla \times \mathbf{B}.$$

- Dispersion relation for two polarizations,  $\omega^2 - k^2 = \pm g_{a\gamma}\dot{a}k$  Birefringence

- Axion dark matter background affects the phase speed of laser with different polarization,  $v_{\pm} = \frac{\omega}{k} \simeq 1 \pm \frac{g_{a\gamma}\dot{a}}{2\omega}$ .

- Equivalent phase changes

- Might be detected by LISA and Taiji detectors, etc

- Need modifications

Melissinos 2009,  
DeRocco & Hook 2018  
Nagano+2019  
DarkGEO 2024  
LIDA 2024, ...



# Sensitivity on Axion-Photon Coupling

- Signal response  $L = \int_{t-\Delta T_{\pm}}^t dt v_{\pm} = \Delta T_{\pm} \pm \frac{g_{a\gamma}}{2\omega} [a(t) - a(t - \Delta T_{\pm})]$ ,
- Equivalent phase changes

$$\Delta T_{\pm} \simeq L \mp \frac{g_{a\gamma}}{2\omega} [a(t) - a(t - L)] .$$

$$\eta_{rs}(t) = -\frac{d(\Delta T_{\pm})}{dt} = \pm \frac{img_{a\gamma}}{2\omega} [a(t) - a(t - L)] ,$$

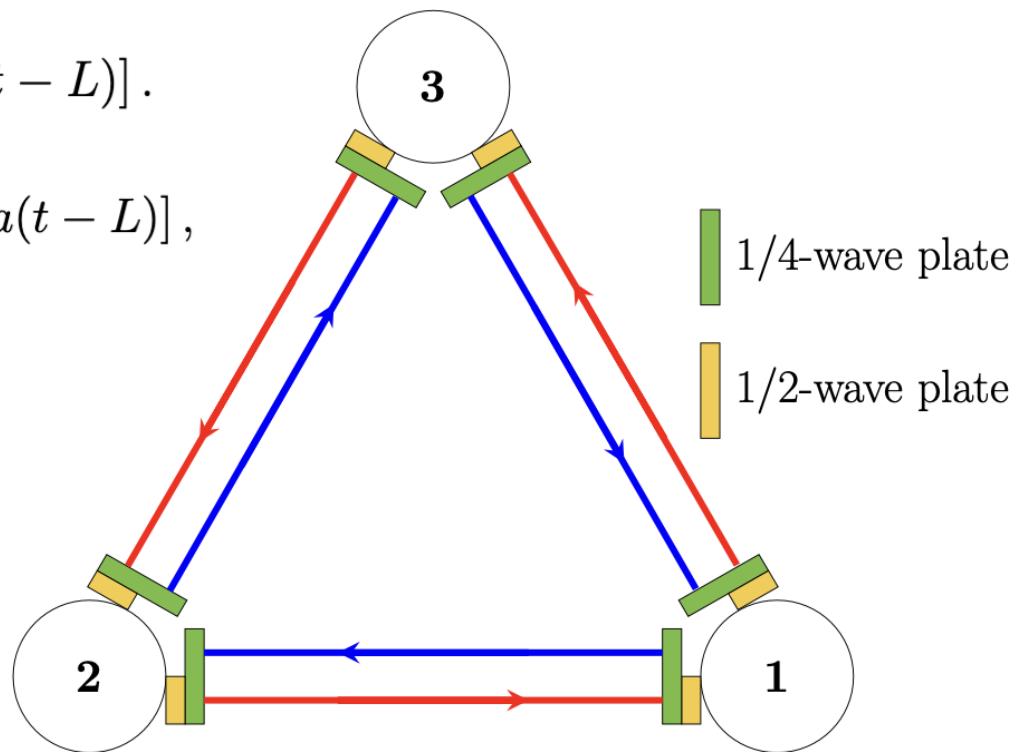
- Linearly polarized – pol angle

$$\mathbf{E}_r(t) = \begin{bmatrix} 1 \\ i \end{bmatrix} \frac{e^{i\omega(t-\Delta T_+)}}{2} + \begin{bmatrix} 1 \\ -i \end{bmatrix} \frac{e^{i\omega(t-\Delta T_-)}}{2} .$$

$$E_x = +\cos \left[ g_{a\gamma} \frac{a(t) - a(t - L)}{2} \right] \cos[\omega(t - L)]$$

$$E_y = -\sin \left[ g_{a\gamma} \frac{a(t) - a(t - L)}{2} \right] \cos[\omega(t - L)]$$

- Need modifications



姚跃辉, Jiang, YT, 2410.22072

# Sensitivity on Axion-Photon Coupling

- Signal response
- Equivalent phase changes

$$\Delta T_{\pm} \simeq L \mp \frac{g_{a\gamma}}{2\omega} [a(t) - a(t - L)] .$$

$$\eta_{rs}(t) = -\frac{d(\Delta T_{\pm})}{dt} = \pm \frac{img_{a\gamma}}{2\omega} [a(t) - a(t - L)] ,$$

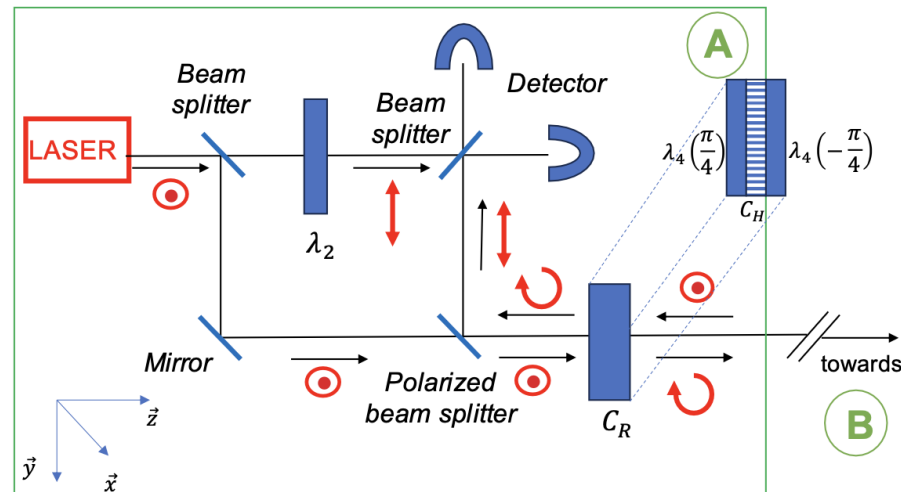
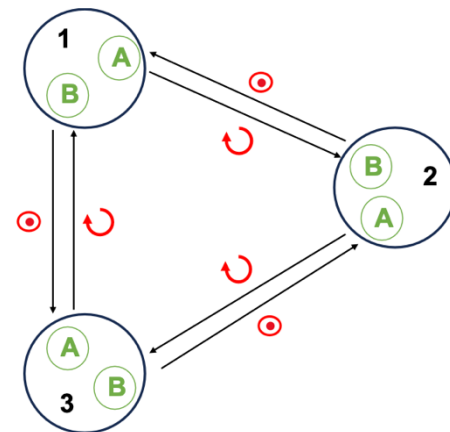
- Linearly polarized – pol angle

$$\mathbf{E}_r(t) = \begin{bmatrix} 1 \\ i \end{bmatrix} \frac{e^{i\omega(t-\Delta T_+)}}{2} + \begin{bmatrix} 1 \\ -i \end{bmatrix} \frac{e^{i\omega(t-\Delta T_-)}}{2} .$$

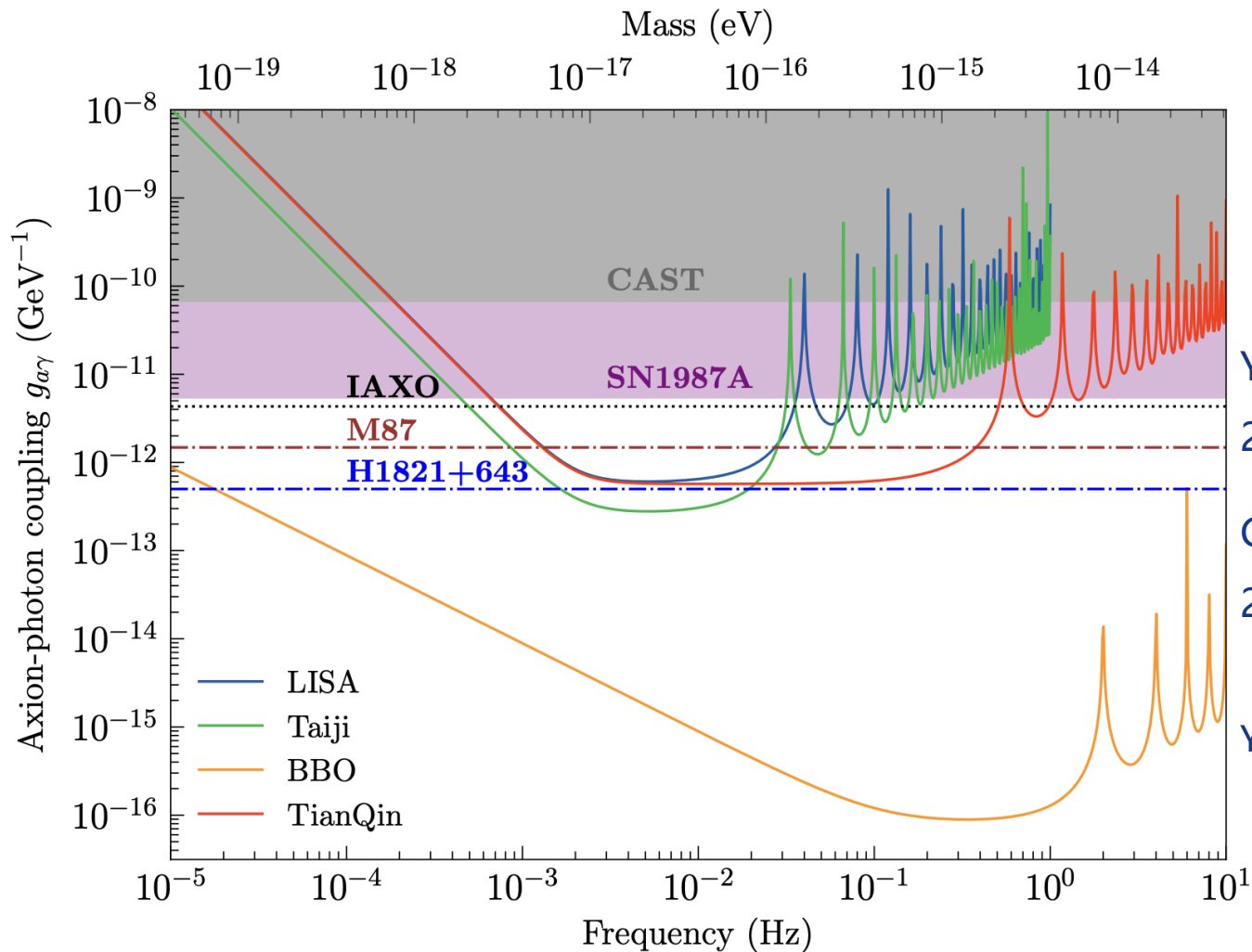
$$E_x = + \cos \left[ g_{a\gamma} \frac{a(t) - a(t - L)}{2} \right] \cos[\omega(t - L)]$$

$$E_y = - \sin \left[ g_{a\gamma} \frac{a(t) - a(t - L)}{2} \right] \cos[\omega(t - L)]$$

- Need modifications



# Sensitivity on Axion-Photon Coupling



Yao, Jiang, YT,  
2410.22072, Oct 29, 2024

Gue, Hees, Wolf,  
2410.17763, Oct 23, 2024

Yao, Bi, Yin, Huang, 2504.10083

# Orbital Modulation

## ➤ Spectral split

$$y_{rs}(t) \supset \hat{n}_{rs} : \frac{\mathbf{h}(t_s - \hat{k} \cdot \mathbf{x}_s) - \mathbf{h}(t - \hat{k} \cdot \mathbf{x}_r)}{2(1 - \hat{k} \cdot \hat{n}_{rs})} : \hat{n}_{rs},$$

$$y_{rs}(t) \supset g \hat{n}_{rs} \cdot [\mathbf{A}(t, \mathbf{x}_r) - \mathbf{A}(t_s, \mathbf{x}_s)],$$

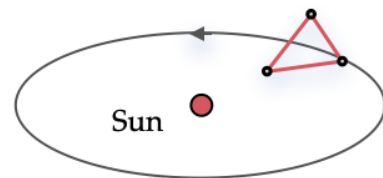
binary inspiral



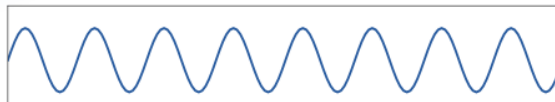
induced test mass motion



heliocentric orbit



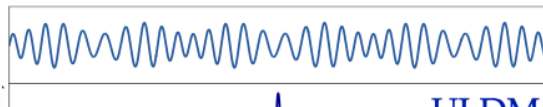
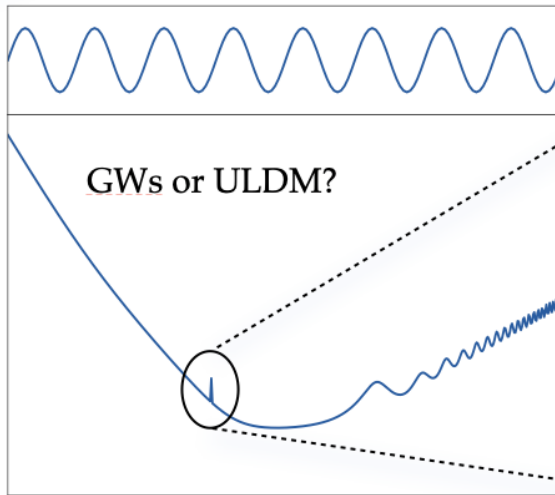
time series



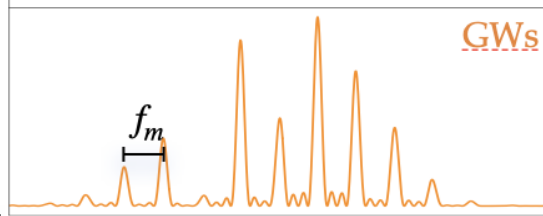
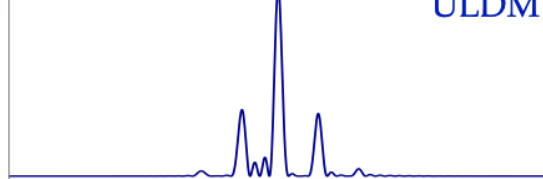
power spectrum

GWs or ULDM?

frequency

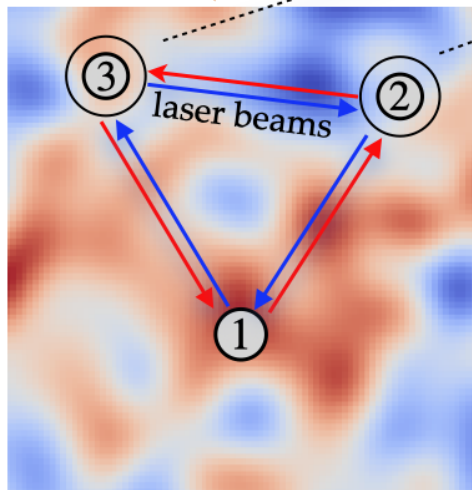


ULDM



GWs

spectral harmonics



ULDM background

# Orbital Modulation

- Spectral split  $y_{rs}(t) \supset \hat{n}_{rs} : \frac{\mathbf{h}(t_s - \hat{\mathbf{k}} \cdot \mathbf{x}_s) - \mathbf{h}(t - \hat{\mathbf{k}} \cdot \mathbf{x}_r)}{2(1 - \hat{\mathbf{k}} \cdot \hat{n}_{rs})} : \hat{n}_{rs},$   $y_{rs}(t) \supset g\hat{n}_{rs} \cdot [\mathbf{A}(t, \mathbf{x}_r) - \mathbf{A}(t_s, \mathbf{x}_s)],$

$$X(t) = (y_{13} + y_{31,2} + y_{12,22} + y_{21,322}) - (y_{12} + y_{21,3} + y_{13,33} + y_{31,233}),$$

- We adopt the Keplerian orbit

$$\begin{aligned} x_i(t) &= r_{\odot} \cos \alpha + \frac{1}{2} e r_{\odot} [\cos(2\alpha - \beta_i) - 3 \cos \beta_i] & e \text{ is the orbital eccentricity,} \\ &+ \frac{1}{8} e^2 r_{\odot} [3 \cos(3\alpha - 2\beta_i) - 10 \cos \alpha - 5 \cos(\alpha - 2\beta_i)], & \alpha = 2\pi f_m t + \kappa \\ y_i(t) &= r_{\odot} \sin \alpha + \frac{1}{2} e r_{\odot} [\sin(2\alpha - \beta_i) - 3 \sin \beta_i] & f_m = 1/\text{yr}, \beta_i = 2\pi(i-1)/3 + \lambda \\ &+ \frac{1}{8} e^2 r_{\odot} [3 \sin(3\alpha - 2\beta_i) - 10 \sin \alpha + 5 \sin(\alpha - 2\beta_i)], \\ z_i(t) &= -\sqrt{3} e r_{\odot} \cos(\alpha - \beta_i) \\ &+ \sqrt{3} e^2 r_{\odot} [\cos^2(\alpha - \beta_i) + 2 \sin^2(\alpha - \beta_i)], \end{aligned} \quad (\text{A1})$$

# Orbital Modulation

- Spectral split  $y_{rs}(t) \supset \hat{n}_{rs} : \frac{\mathbf{h}(t_s - \hat{\mathbf{k}} \cdot \mathbf{x}_s) - \mathbf{h}(t - \hat{\mathbf{k}} \cdot \mathbf{x}_r)}{2(1 - \hat{\mathbf{k}} \cdot \hat{n}_{rs})} : \hat{n}_{rs},$   $y_{rs}(t) \supset g\hat{n}_{rs} \cdot [\mathbf{A}(t, \mathbf{x}_r) - \mathbf{A}(t_s, \mathbf{x}_s)],$

$$X(t) = (y_{13} + y_{31,2} + y_{12,22} + y_{21,322}) - (y_{12} + y_{21,3} + y_{13,33} + y_{31,233}),$$

- For GWs, it can be rewritten as

$$X(t) \simeq 16\pi^2 f^2 L^2 \sum_{n=-\infty}^{+\infty} e^{i2\pi(f+nf_m)t} \times \sum_{l=-2j}^{2j} \mathcal{D}^{(n-l)}(f, \hat{\mathbf{k}}) \sum_{p=+, \times} G_p^{(l)}(\hat{\mathbf{k}}) H_p,$$

- For ULDM, it is

$$X(t) \simeq -2im^3 L^3 \sum_{n=-j}^j e^{i2\pi(f_c+nf_m)t} \sum_p \mathcal{G}_p^{(n)} a_p(\mathbf{x}_1),$$



# Orbital Modulation

## ➤ Spectral split

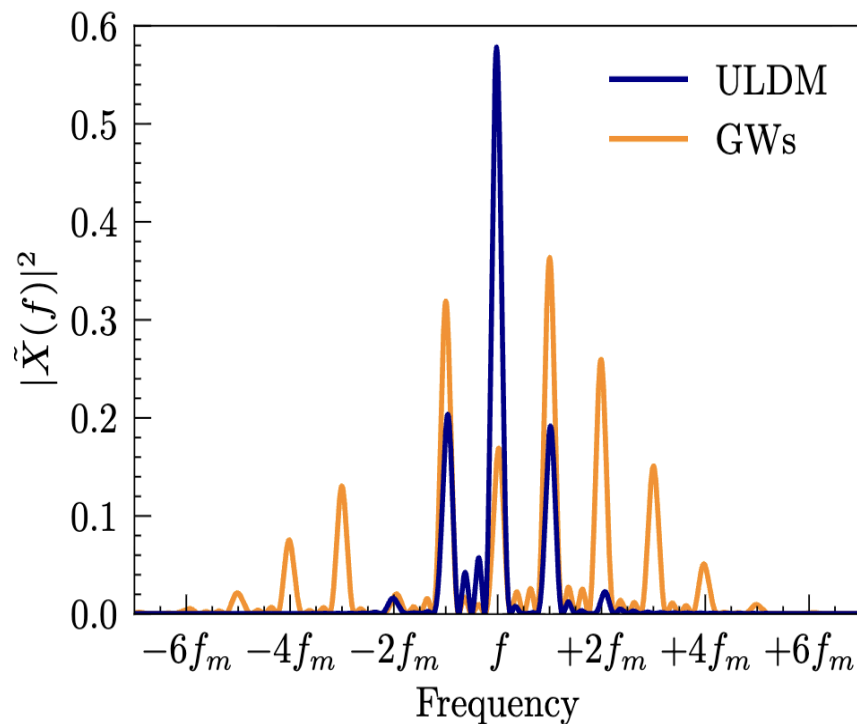
$$y_{rs}(t) \supset \hat{n}_{rs} : \frac{\mathbf{h}(t_s - \hat{\mathbf{k}} \cdot \mathbf{x}_s) - \mathbf{h}(t - \hat{\mathbf{k}} \cdot \mathbf{x}_r)}{2(1 - \hat{\mathbf{k}} \cdot \hat{n}_{rs})} : \hat{n}_{rs},$$

$$y_{rs}(t) \supset g \hat{n}_{rs} \cdot [\mathbf{A}(t, \mathbf{x}_r) - \mathbf{A}(t_s, \mathbf{x}_s)],$$

$$X(t) = (y_{13} + y_{31,2} + y_{12,22} + y_{21,322}) - (y_{12} + y_{21,3} + y_{13,33} + y_{31,233}),$$

| $n$ | $G_+^{(n)}$                                                                  | $G_\times^{(n)}$                                                    |
|-----|------------------------------------------------------------------------------|---------------------------------------------------------------------|
| 0   | $-\frac{9\sqrt{3}}{128}[3 + \cos(2\theta)] \sin(2(\phi - \lambda))$          | $\frac{9\sqrt{3}}{32} \cos \theta \cos(2(\phi - \lambda))$          |
| 1   | $-\frac{9i}{64} \sin(2\theta) e^{i(\kappa + \phi - 2\lambda)}$               | $-\frac{9}{32} \sin \theta e^{i(\kappa + \phi - 2\lambda)}$         |
| 2   | $\frac{9\sqrt{3}i}{128} [1 - \cos(2\theta)] e^{i2(\kappa - \lambda)}$        | 0                                                                   |
| 3   | $\frac{3i}{64} \sin(2\theta) e^{i(3\kappa - 2\lambda - \phi)}$               | $-\frac{3}{32} \sin \theta e^{i(3\kappa - 2\lambda - \phi)}$        |
| 4   | $\frac{\sqrt{3}i}{256} [3 + \cos(2\theta)] e^{2i(2\kappa - \lambda - \phi)}$ | $-\frac{\sqrt{3}}{64} \cos \theta e^{2i(2\kappa - \lambda - \phi)}$ |

| $n$ | $\mathcal{G}_x^{(n)}$                  | $\mathcal{G}_y^{(n)}$                  | $\mathcal{G}_z^{(n)}$                          |
|-----|----------------------------------------|----------------------------------------|------------------------------------------------|
| 0   | $-\frac{3}{4} \sin \lambda$            | $\frac{3}{4} \cos \lambda$             | 0                                              |
| 1   | 0                                      | 0                                      | $-\frac{\sqrt{3}i}{4} e^{i(\kappa - \lambda)}$ |
| 2   | $\frac{i}{8} e^{i(2\kappa - \lambda)}$ | $\frac{1}{8} e^{i(2\kappa - \lambda)}$ | 0                                              |



# Null-Response Channel

- For a source from some direction, we construct a null-response channel that is insensitive to ULDM or GWs,

$$\tilde{\eta} = a_1 \tilde{\alpha} + a_2 \tilde{\beta} + a_3 \tilde{\gamma}.$$

- The Signal-to-Noise Ratio (SNR)

$$\text{SNR}_{\eta}^2 = \int df \frac{\left| a_1 \tilde{\alpha}_s + a_2 \tilde{\beta}_s + a_3 \tilde{\gamma}_s \right|^2}{\mathbf{a}^\dagger \mathbf{S}_{\alpha}(f) \mathbf{a}},$$

- SNR = 0 gives

$$a_1 \tilde{\alpha}_s + a_2 \tilde{\beta}_s + a_3 \tilde{\gamma}_s = 0.$$

# Null-Response Channel

- In GR, two polarizations of GW, the response can be written as

$$\tilde{\alpha}_s = \alpha_+^{\text{gw}}(f, \hat{k}) \tilde{h}_+(f) + \alpha_\times^{\text{gw}}(f, \hat{k}) \tilde{h}_\times(f),$$

- The condition  $a_1 \tilde{\alpha}_s + a_2 \tilde{\beta}_s + a_3 \tilde{\gamma}_s = 0$  can be satisfied by

$$\left( \mathbf{a} \cdot \mathbf{x}_+^{\text{gw}}(\hat{k}) \right) \tilde{h}_+ + \left( \mathbf{a} \cdot \mathbf{x}_\times^{\text{gw}}(\hat{k}) \right) \tilde{h}_\times = 0,$$

- Where

$$\mathbf{x}_p^{\text{gw}}(\hat{k}) = [\alpha_p^{\text{gw}}(\hat{k}), \beta_p^{\text{gw}}(\hat{k}), \gamma_p^{\text{gw}}(\hat{k})]^T$$

- This can be solved by

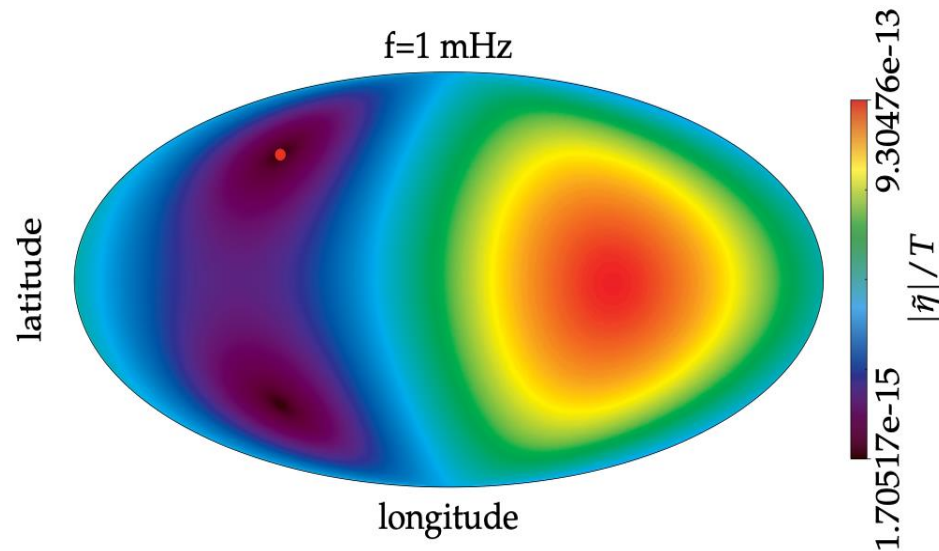
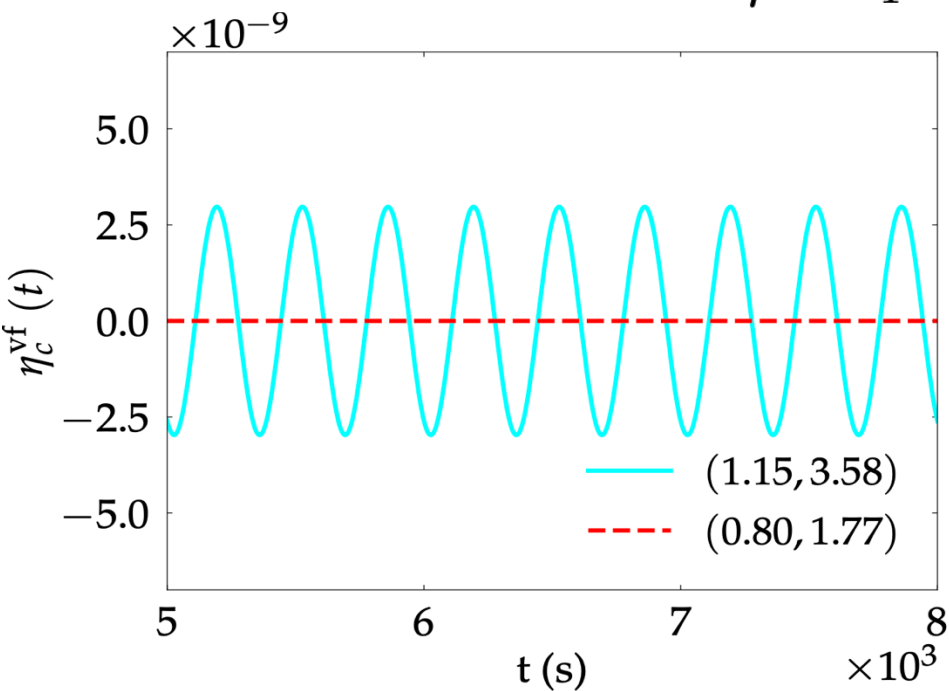
$$\mathbf{a}_c^{\text{gw}}(\hat{k}) = \mathbf{x}_+^{\text{gw}}(\hat{k}) \times \mathbf{x}_\times^{\text{gw}}(\hat{k}).$$

- So NRC is given by  $\tilde{\eta}_c^{\text{gw}}(f, \hat{k}; \hat{k}_c) = \mathbf{a}_c^{\text{gw}}(\hat{k}_c) \cdot [\tilde{\alpha}, \tilde{\beta}, \tilde{\gamma}]$

# Null-Response Channel

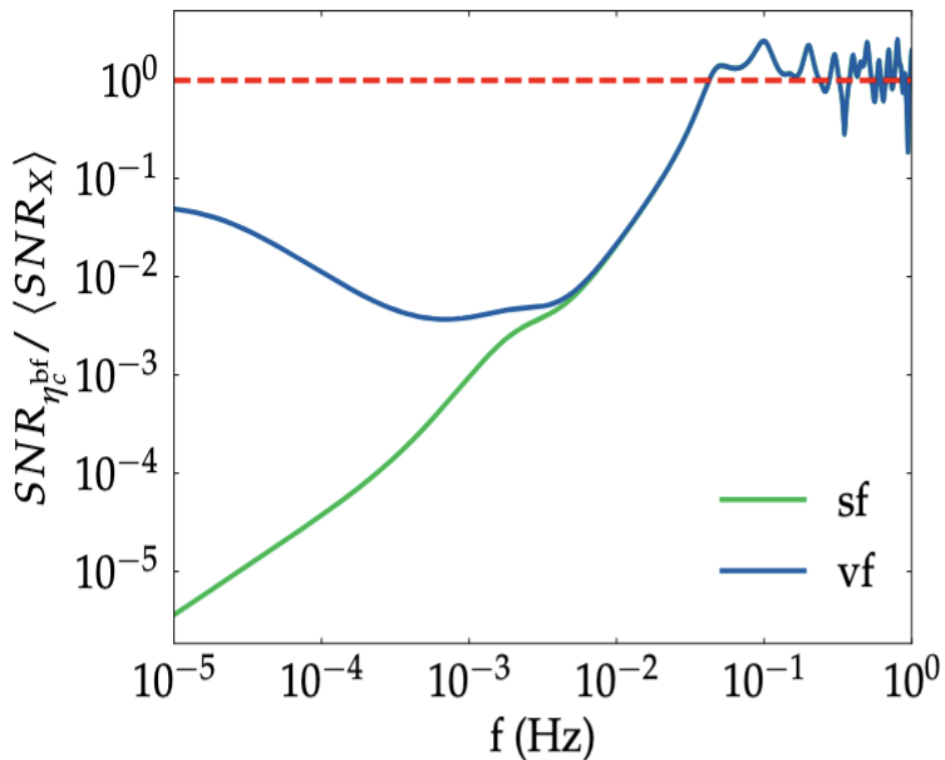
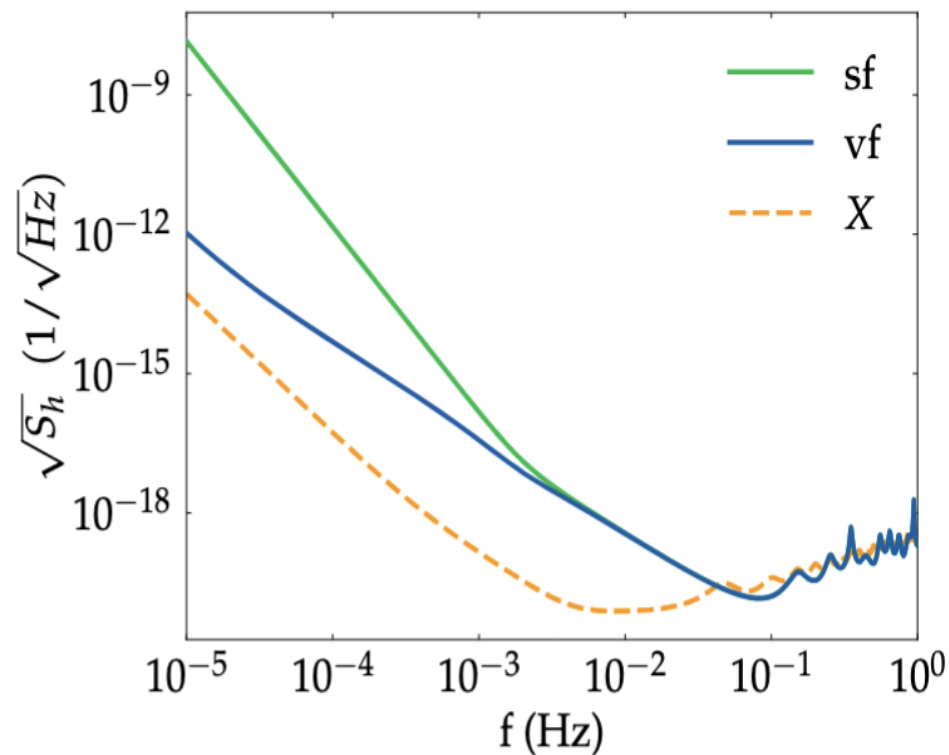
- For a source from some direction, we construct a null-response channel that is insensitive to ULDM or GWs,

$$\tilde{\eta} = a_1 \tilde{\alpha} + a_2 \tilde{\beta} + a_3 \tilde{\gamma}.$$



# Null-Response Channel

- For a source from some direction, we construct a null-response channel that is insensitive to ULDM or GWs



# Detecting Ultralight Dark Matter Gravitationally

- Can we probe DM with gravitational interaction in solar system
- What if dark matter has only gravitational interaction?
- Motivations from theories of gravity, particle physics, ...
- Viable scenarios in cosmology
  - Axion or axion-like particle, misalignment
  - Dark photon, vacuum fluctuation

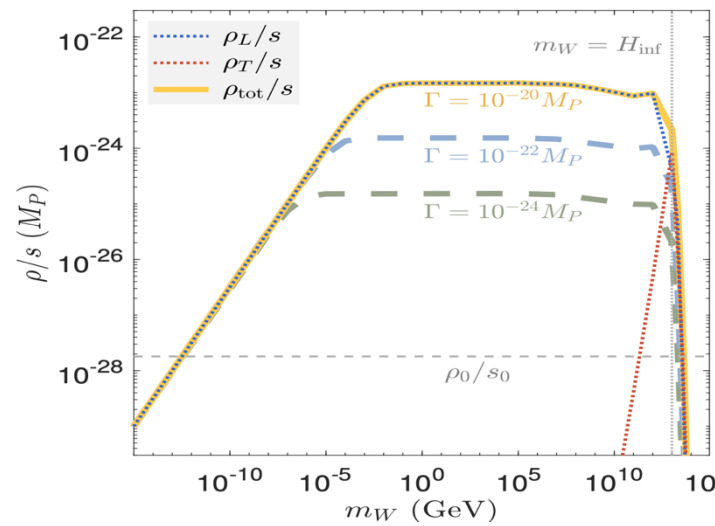
Graham, Mardon, Rajendran, 1504.02102(PRD)

Ema, Nakayama, Tang, 1903.10973(JHEP),

Ahmed, Grzadkowski, Socha, 2005.01766(JHEP)

Kolb, Long, 2009.03828(JHEP), ...

王清扬, Tang, Wu, 2203.15452(PRD)



# Detecting Ultralight Dark Matter Gravitationally

- Metric perturbation in solar system

$$ds^2 = -(1 + 2\Psi)dt^2 + (1 - 2\Phi)\delta_{ij}dx^i dx^j + h_{ij}dx^i dx^j$$

- Einstein equations

$$\partial_i \partial^i \Phi = 4\pi G T_{00},$$

$$3\ddot{\Phi} + \partial_i \partial^i (\Psi - \Phi) = 4\pi G T_k^k,$$

$$\ddot{h}_{ij} = 16\pi G \left( T_{ij} - \frac{1}{3} \delta_{ij} T_k^k \right),$$

$$\Psi^j \simeq \Phi^j \simeq \pi G \frac{\rho}{m^2} = \frac{7 \times 10^{-26} \rho}{0.4 \text{ GeV/cm}^3} \left( \frac{10^{-18} \text{ eV}}{m} \right)^2,$$

$$h_{ij}^v \propto h_0 \simeq \frac{8}{3} \pi G \frac{\rho}{m^2} = \frac{2 \times 10^{-25} \rho}{0.4 \text{ GeV/cm}^3} \left( \frac{10^{-18} \text{ eV}}{m} \right)^2,$$

$$h_{ij}^s \simeq h_0 v^2 / 2$$

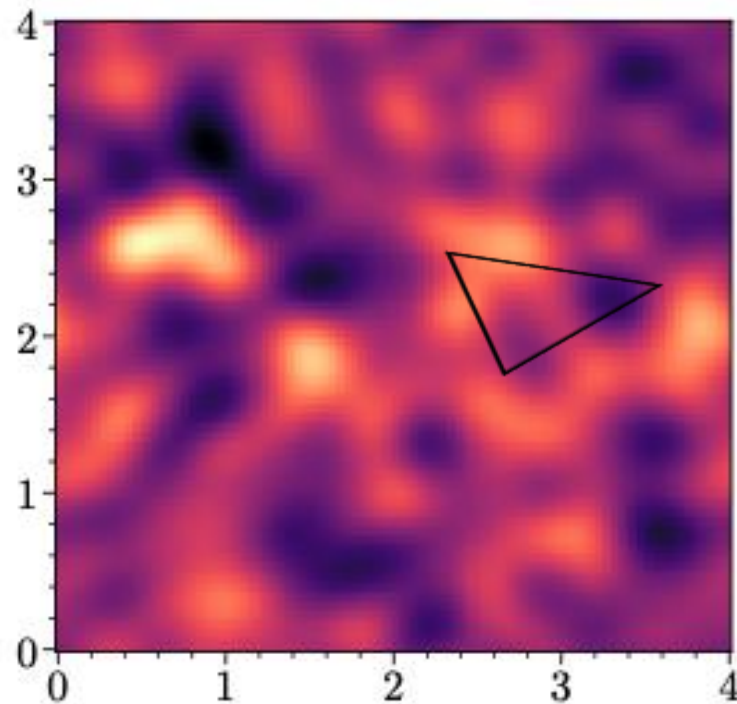
- Perturbation for scalar is suppressed.

H. Kim 2023

- Not for  $h_{ij}$  from vector

Nomura, Ito & Soda 2020

PTA, e.g. Khmelnitsky & Rubakov 2014,...





# Detecting Ultralight Dark Matter Gravitationally

➤ Tensor perturbation

$$S_{\Phi}^s \simeq \frac{64}{15} \kappa^4 \rho^2 v^2 L^4 T \left[ v^2 \sin^2 \gamma + 5 m^2 L^2 \sin^2 \frac{\gamma}{2} \right],$$

**scalar**

$$S_h^v(\epsilon_{ij}) \simeq \frac{256}{9} \kappa^4 \rho^2 L^4 T [(\hat{n}_{12}^i \hat{n}_{12}^j - \hat{n}_{13}^i \hat{n}_{13}^j) \epsilon_{ij}]^2$$

**vector**

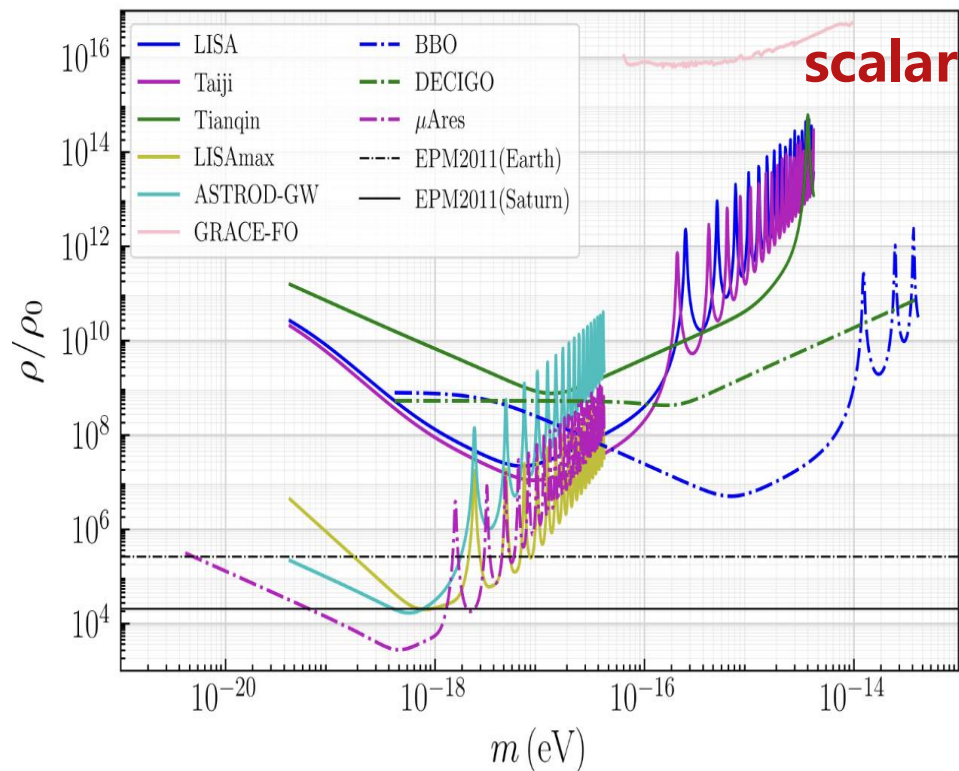
TABLE I. Arm lengths and instrumental noises of several planned laser interferometers in space. In the last row we give the sensitivities on vector ULDM with mass  $5.0 \times 10^{-19}$  eV. Here  $L$  is the nominal arm length of triangle constellation, while  $s_{\text{acc}}$  and  $s_{\text{oms}}$  are the acceleration noise of test mass and noise from optical metrology system, respectively. Note that LISA/LISAmx/Taiji/Tianqin adopt frequency-dependent noise power spectra [101],  $s_{\text{acc}}^2 \propto [1 + (0.4 \times 10^{-3} \text{ Hz}/f)^2][1 + (f/8 \times 10^{-3} \text{ Hz})^4]$  and  $s_{\text{oms}}^2 \propto 1 + (2 \times 10^{-3} \text{ Hz}/f)^4$ .

|                                                                       | LISA               | Taiji              | Tianqin            | BBO                  | DECIGO             | $\mu$ Ares | LISAmx | ASTROD-GW |
|-----------------------------------------------------------------------|--------------------|--------------------|--------------------|----------------------|--------------------|------------|--------|-----------|
| $L$ ( $10^9$ m)                                                       | 2.5                | 3                  | 0.17               | 0.05                 | $1 \times 10^{-3}$ | 395        | 260    | 260       |
| $s_{\text{acc}}$ ( $10^{-15} \frac{\text{m/s}^2}{\sqrt{\text{Hz}}}$ ) | 3                  | 3                  | 1                  | $3 \times 10^{-2}$   | $4 \times 10^{-4}$ | 1          | 3      | 3         |
| $s_{\text{oms}}$ ( $10^{-12} \frac{\text{m}}{\sqrt{\text{Hz}}}$ )     | 15                 | 8                  | 1                  | $1.4 \times 10^{-5}$ | $2 \times 10^{-6}$ | 50         | 15     | 100       |
| $\frac{\rho}{\rho_0}$ ( $5.0 \times 10^{-19}$ eV)                     | $7.95 \times 10^2$ | $6.53 \times 10^2$ | $3.82 \times 10^3$ | $2.00 \times 10^2$   | $1.34 \times 10^2$ | 0.44       | 7.67   | 4.32      |

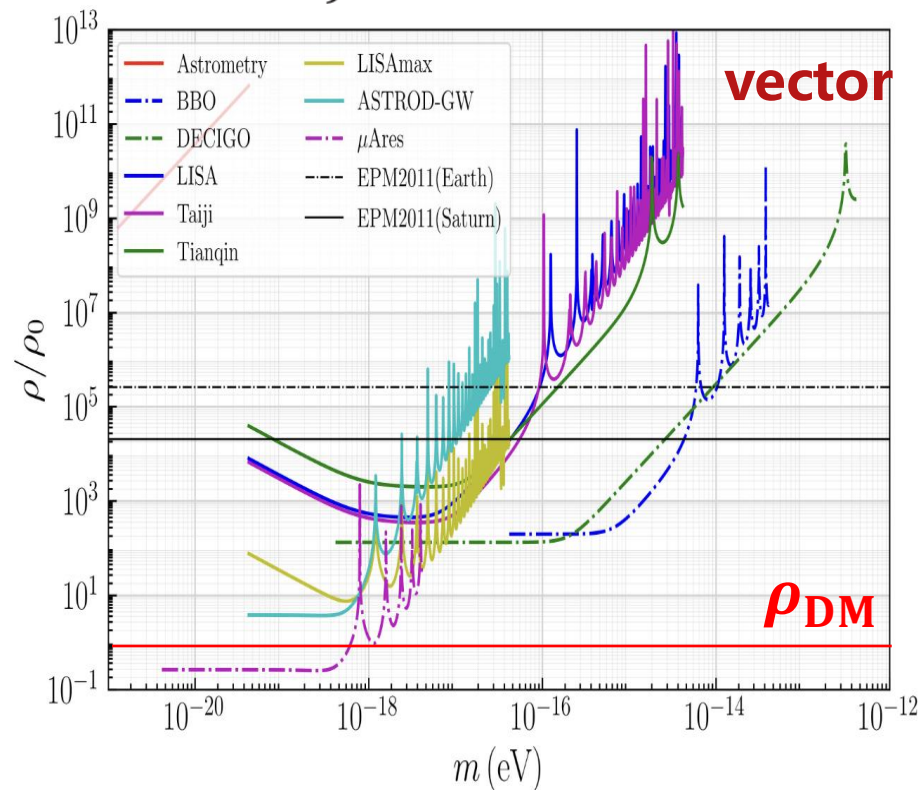
# Detecting Ultralight Dark Matter Gravitationally

## ➤ Tensor perturbation

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$$S_h^v(\epsilon_{ij}) \simeq \frac{256}{9} \kappa^4 \rho^2 L^4 T [(\hat{n}_{12}^i \hat{n}_{12}^j - \hat{n}_{13}^i \hat{n}_{13}^j) \epsilon_{ij}]^2$$



# WIMP DM Spikes around Black Holes

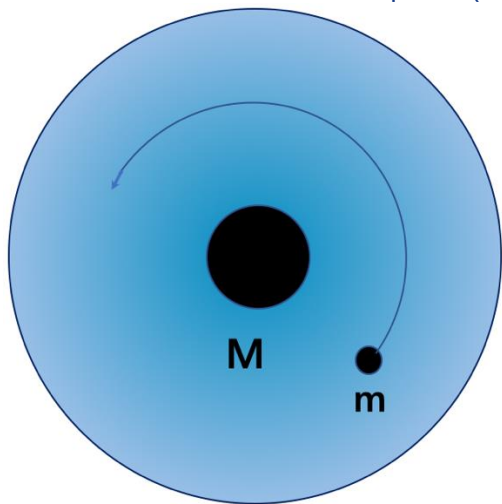
- WIMP DM particles accretion around BH → DM spike

- NFW profile → spiky density profile Gondolo & Silk (1999)

$$\rho(r) \propto r^{-\gamma}, 0 \leq \gamma \leq 2 \quad \Rightarrow \quad \rho_{\text{spike}}(r) = \rho_{\text{sp}} \left( \frac{r_{\text{sp}}}{r} \right)^{\alpha}, \quad \alpha = \frac{9 - 2\gamma}{4 - \gamma}$$

- Dynamical friction → Gravitational wave

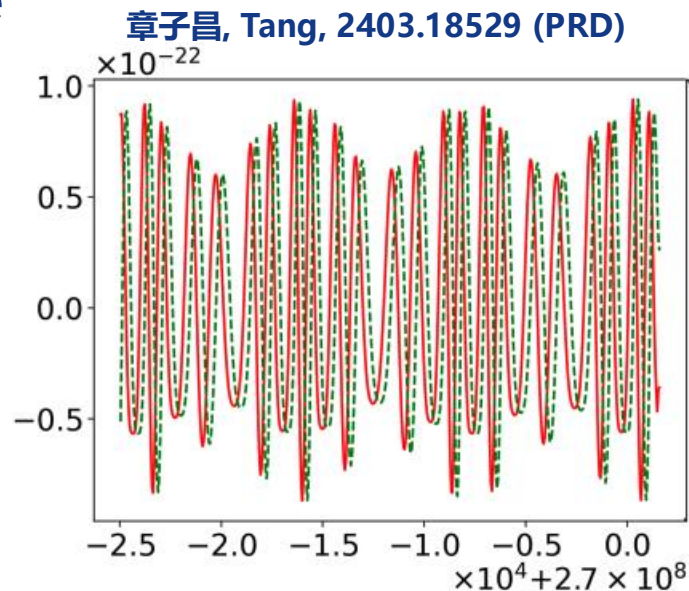
Extreme-mass-ratio Inspiral (EMRI)



$$m \frac{d^2 \mathbf{x}}{dt^2} = \mathbf{F}_G + \mathbf{F}_{\text{DF}}$$

$$h_{ij} \sim \frac{G}{d} \frac{d^2 Q_{ij}}{dt^2},$$

$$Q_{ij} \sim m \left( x_i x_j - \frac{1}{3} x^2 \delta_{ij} \right)$$



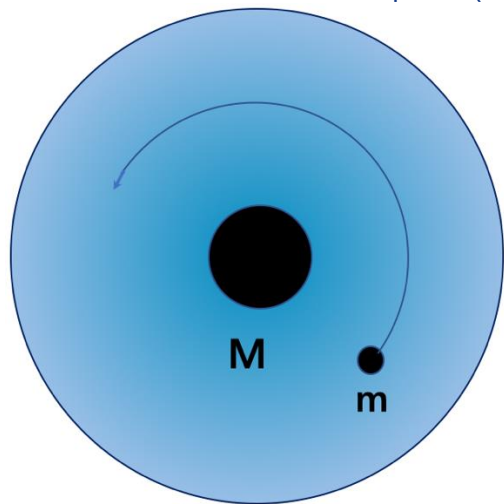
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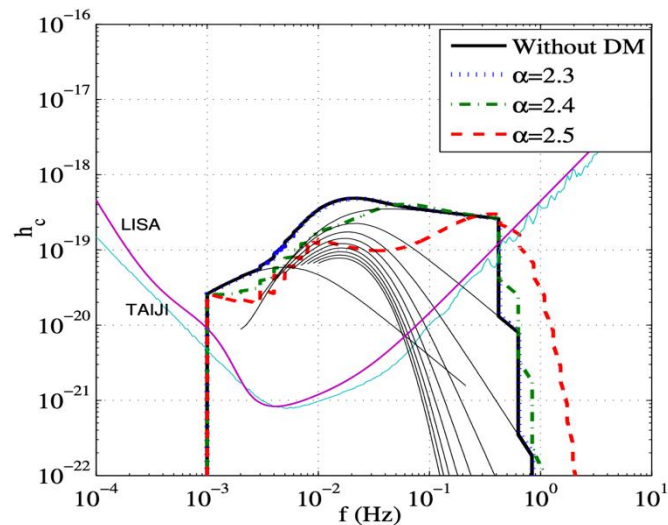


$$m \frac{d^2 \mathbf{x}}{dt^2} = \mathbf{F}_G + \mathbf{F}_{\text{DF}}$$

$$h_{ij} \sim \frac{G}{d} \frac{d^2 Q_{ij}}{dt^2},$$

$$Q_{ij} \sim m \left( x_i x_j - \frac{1}{3} x^2 \delta_{ij} \right)$$

Li, Tang, Wu, 2112.14041



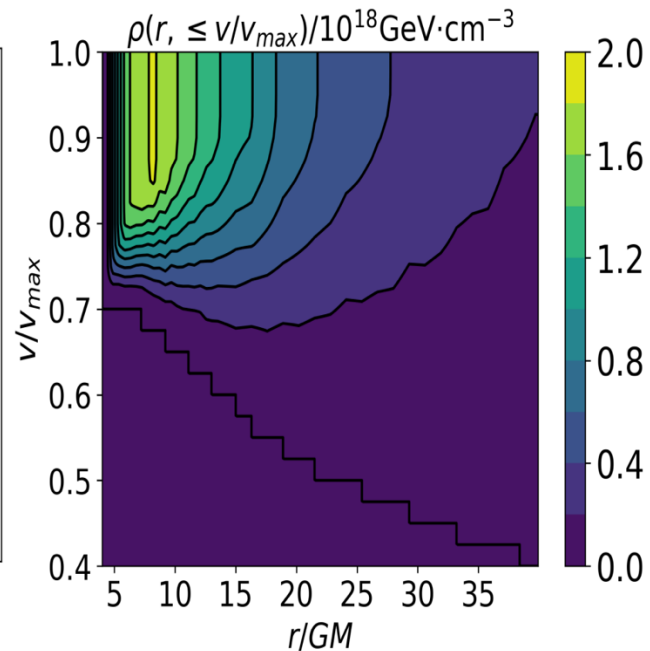
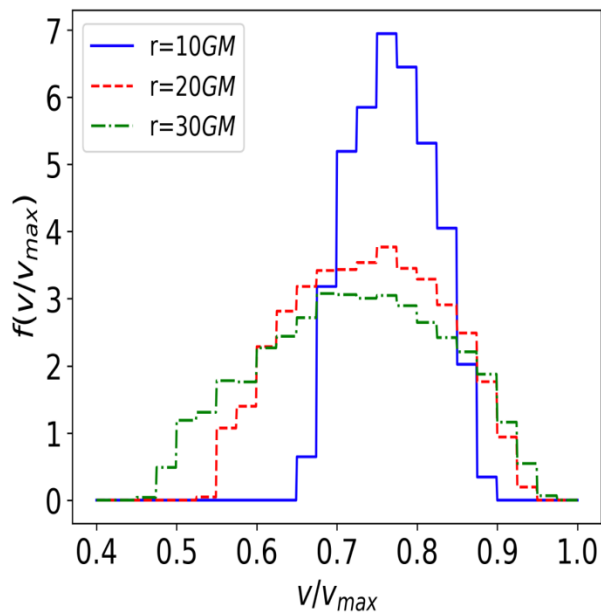
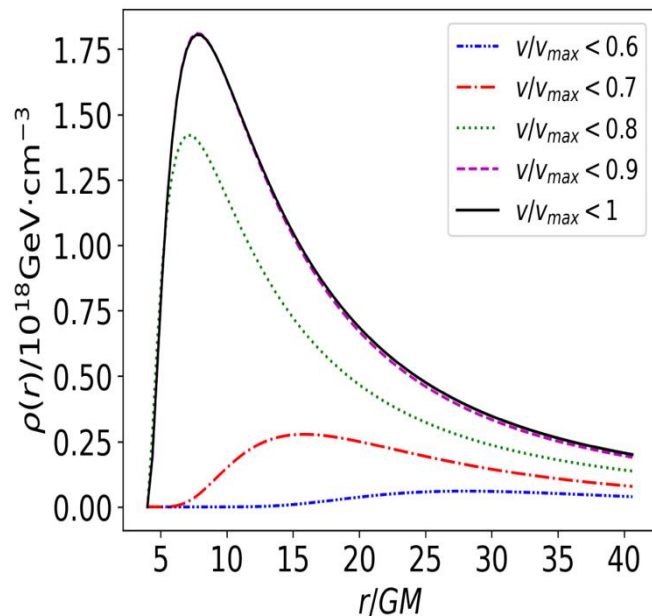
# Density and Velocity Distribution

## ➤ Relativistic treatment (Will et al)

$$\rho(r, \alpha = 1) = \frac{\kappa}{(r/GM)^\omega} \left(1 - \frac{4GM}{r}\right)^\eta.$$

| Halo parameters                               | Index      | Fitting parameters                                   |
|-----------------------------------------------|------------|------------------------------------------------------|
| $(M_{halo}, r_s)$                             | $(\gamma)$ | $(\kappa, \eta, \omega)$                             |
| $(10^{12} M_\odot, 20 \text{ kpc})$           | -          | $(5.33 \times 10^{20} \text{ GeV/cm}^3, 1.99, 2.07)$ |
| $(4.5 \times 10^8 M_\odot, 1.85 \text{ kpc})$ | -          | $(6.15 \times 10^{24} \text{ GeV/cm}^3, 2.03, 2.11)$ |
| $(7.3 \times 10^8 M_\odot, 1.85 \text{ kpc})$ | 7/4        | $(5.83 \times 10^{26} \text{ GeV/cm}^3, 2.04, 2.16)$ |

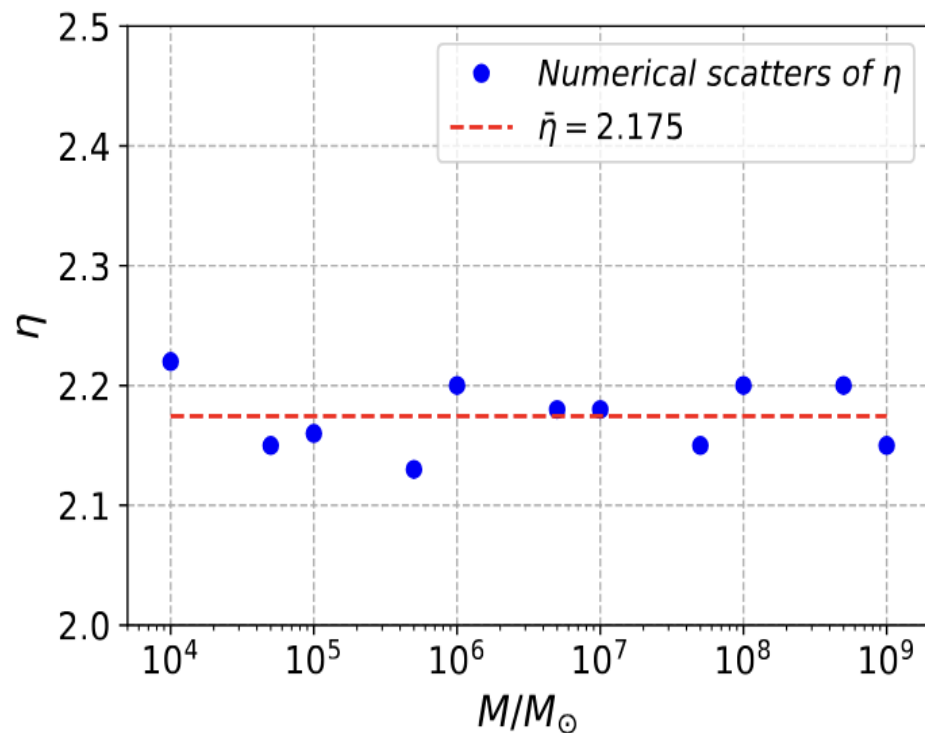
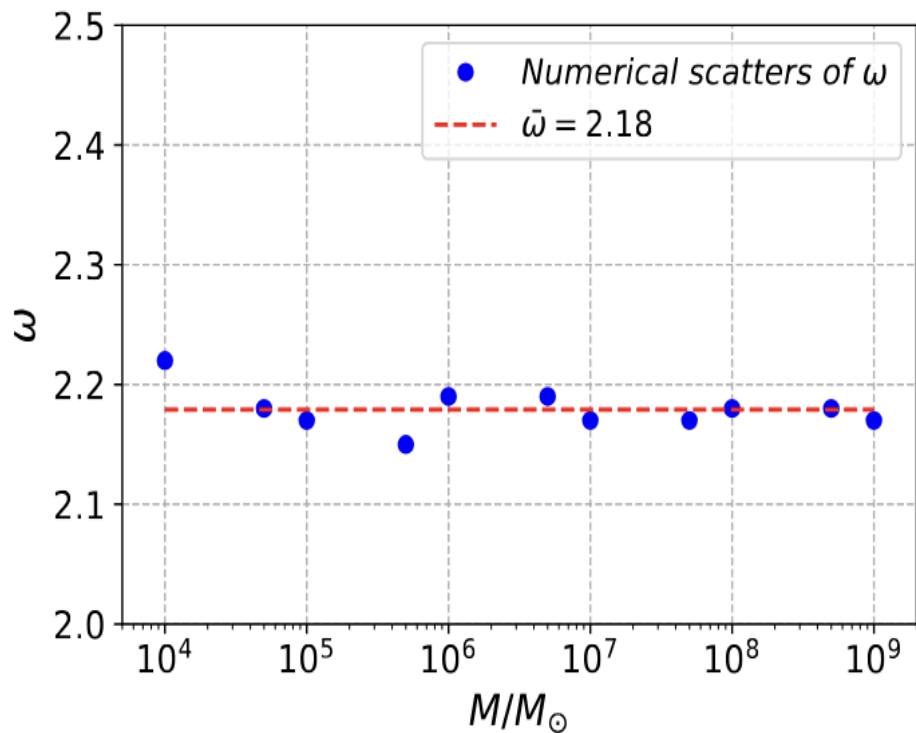
章子昌, Tang, 2403.18529 (PRD)



# Dark Matter Spike

➤ Universal profile  $\rho(r) = \frac{\kappa(\text{GeV} \cdot \text{cm}^{-3})}{(r/GM)^\omega} \left(1 - \frac{4GM}{r}\right)^\eta$

$$\log(\kappa/(\text{GeV} \cdot \text{cm}^{-3})) = a \cdot \log(M/M_\odot) + b \quad (a, b) = (-1.62, 31.5).$$





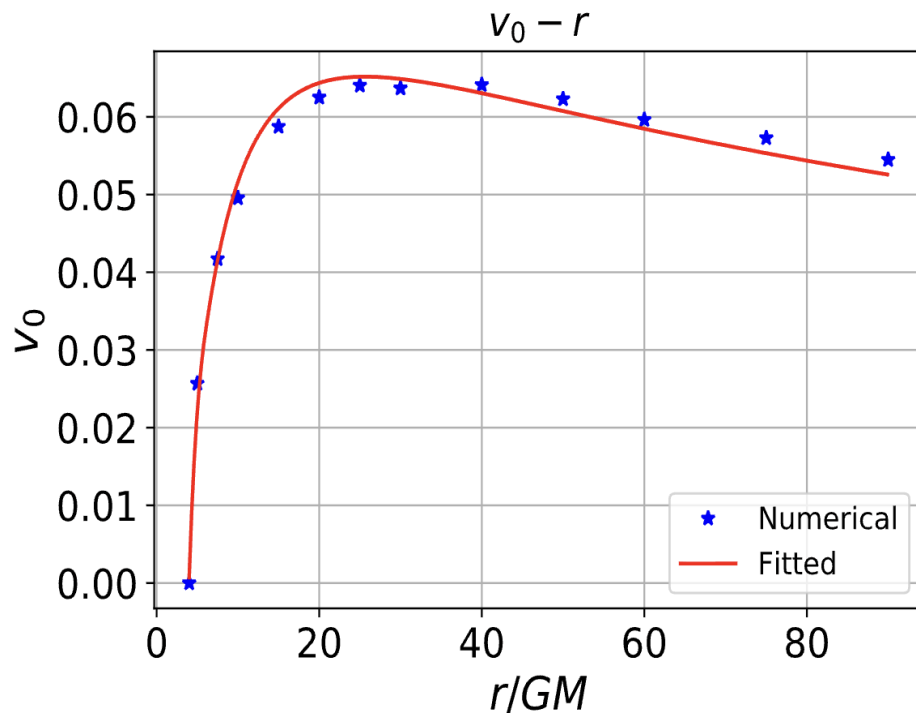
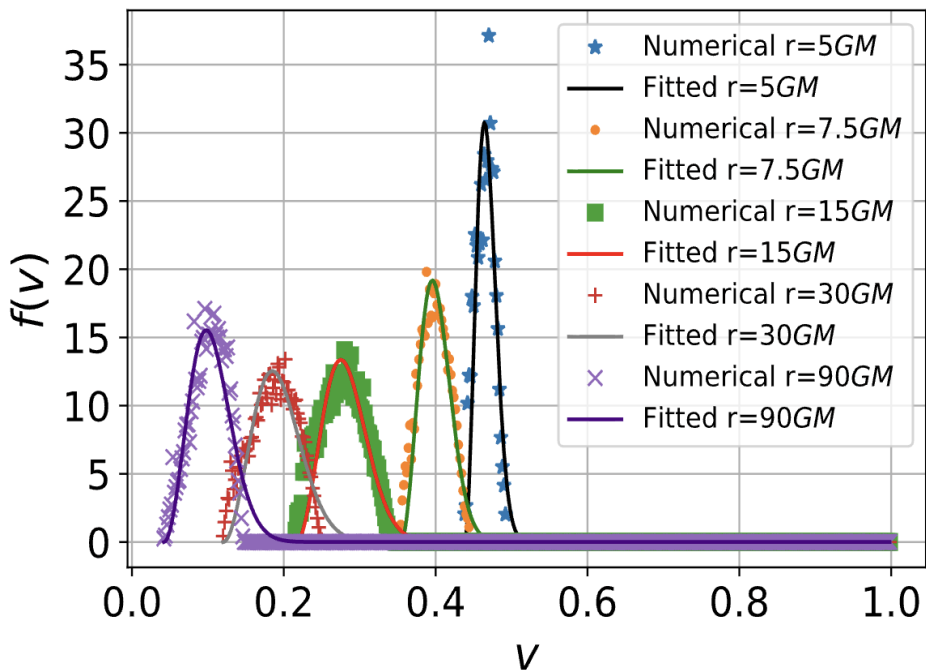
# Velocity Distribution

## ➤ Fitted with Maxwell-Boltzmann distribution

$$f_r(v) = \frac{4\pi [v - v_{\min}(r)]^2}{[\pi v_0^2(r)]^{\frac{3}{2}}} e^{-[v - v_{\min}(r)]^2 / v_0^2(r)},$$

章子昌, Yuan, Tang, 2503.02573(PRD)

Fitted curves

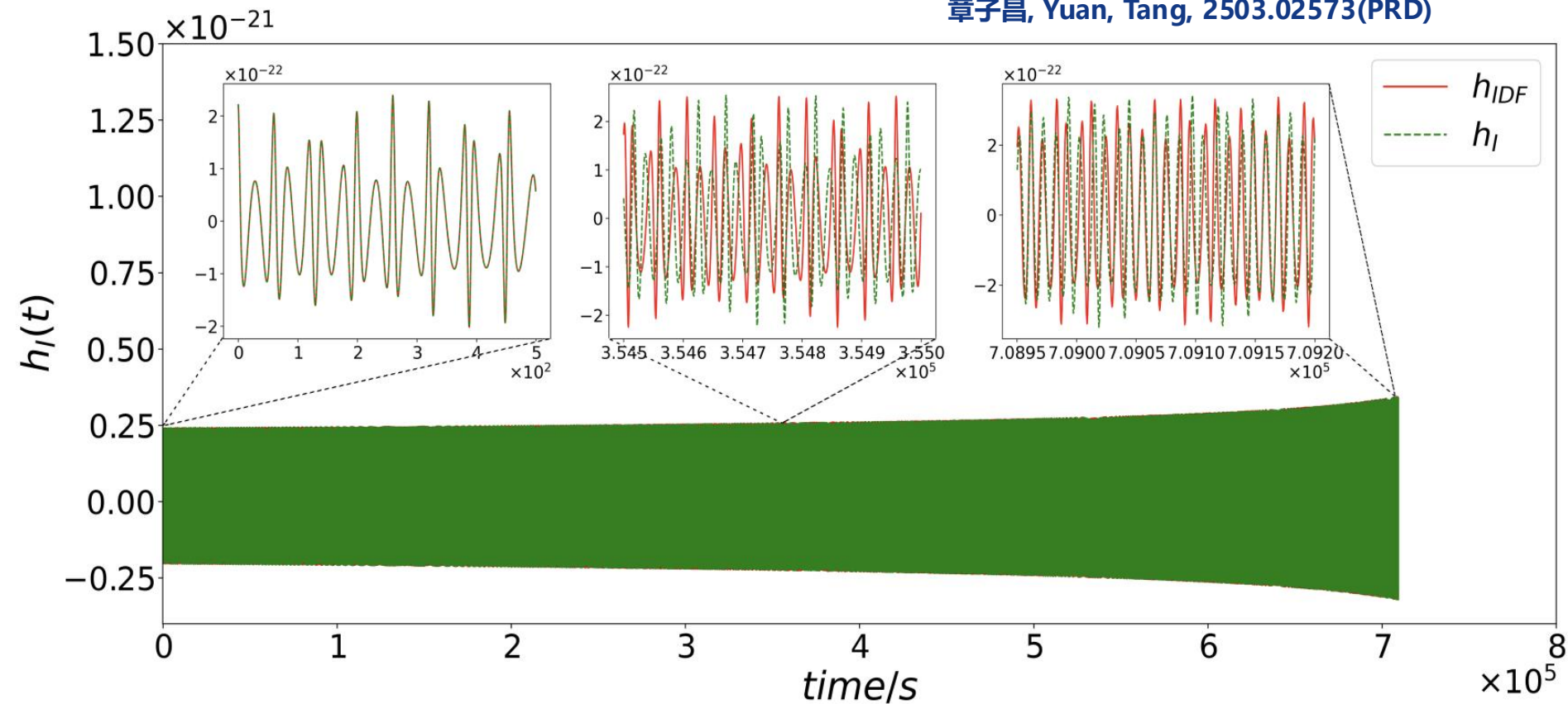




# Effects on EMRI

## ➤ Phase shift of the waveform of GW

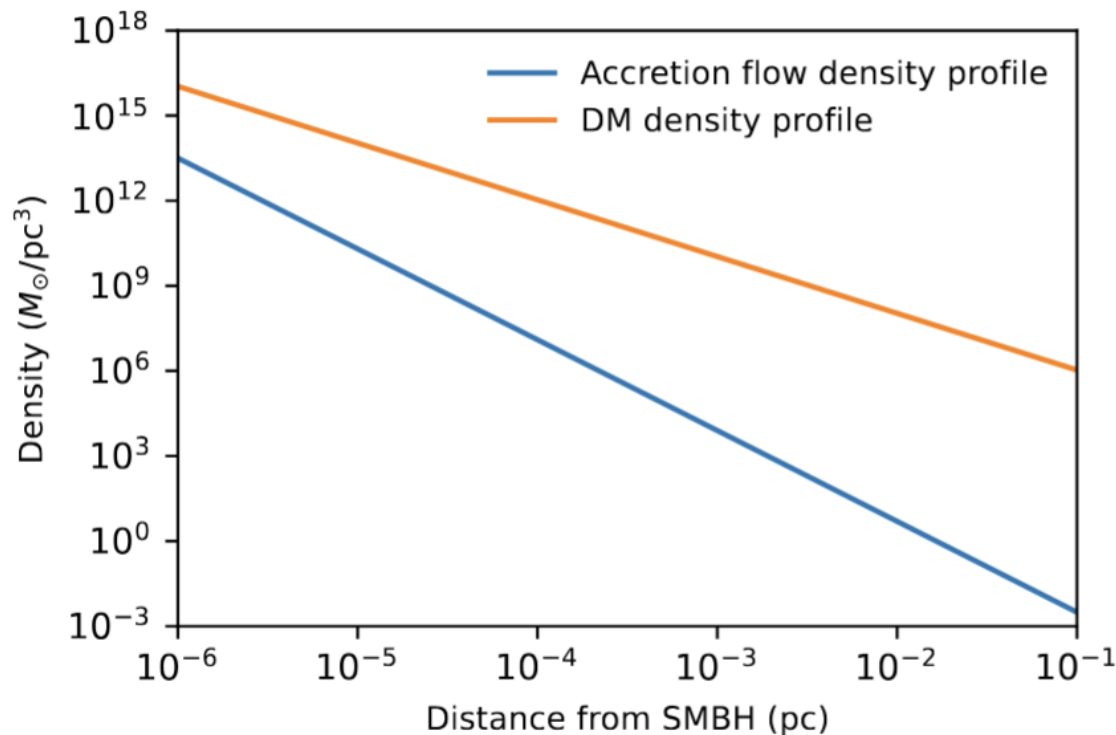
章子昌, Yuan, Tang, 2503.02573(PRD)



# Effects on Early EMRIs

- In Milky Way center, SMBH with mass  $4 \times 10^6 M_{\odot}$ , possible many EMRIs at early stages, GW background

Feng, Tang, Wu, 2506.02937



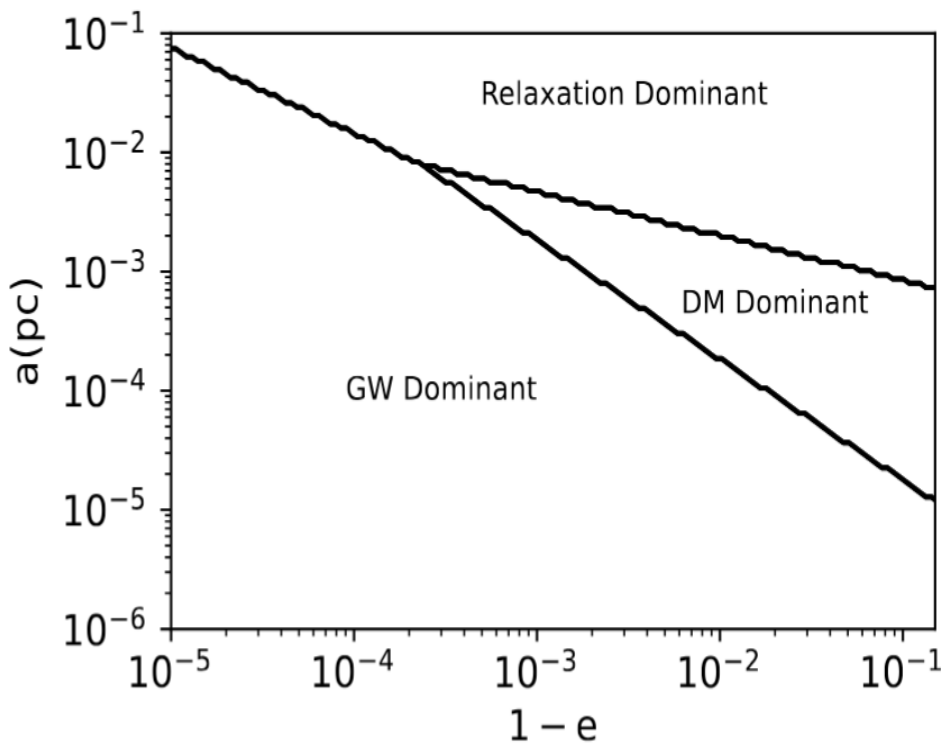
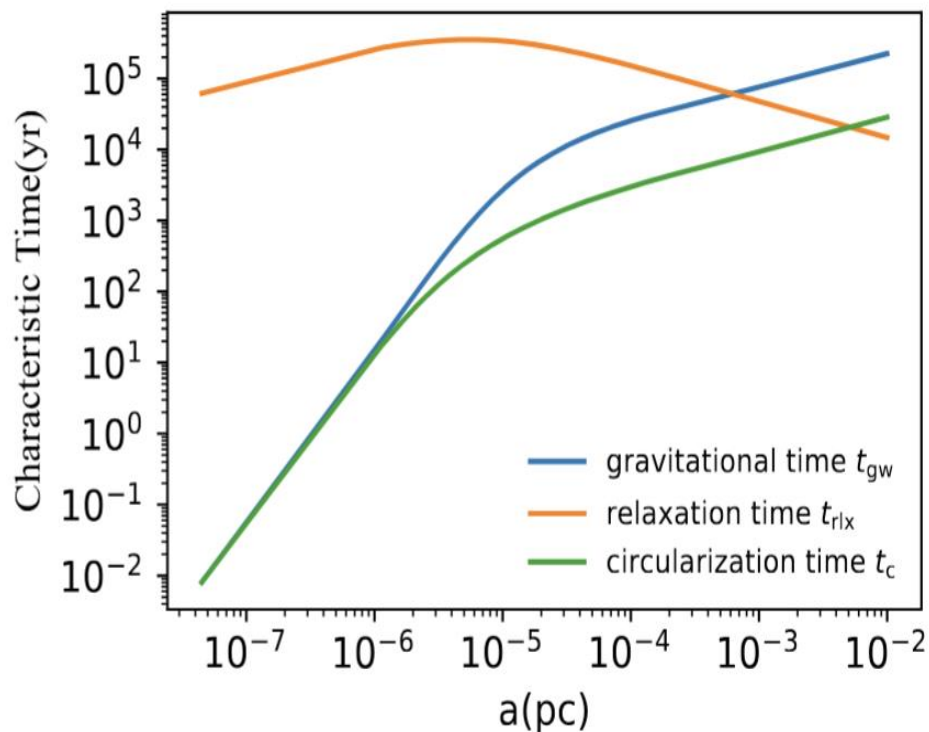
$$\rho(r) = \rho_c \left( \frac{r_c}{r} \right)^{\gamma}, \quad \gamma = \frac{9 - 2\beta}{4 - \beta},$$

$$\left\langle \frac{da}{dt} \right\rangle = \left\langle \frac{da}{dt} \right\rangle_{\text{df}} + \left\langle \frac{da}{dt} \right\rangle_{\text{gw}},$$
$$\left\langle \frac{de}{dt} \right\rangle = \left\langle \frac{de}{dt} \right\rangle_{\text{df}} + \left\langle \frac{de}{dt} \right\rangle_{\text{gw}}.$$

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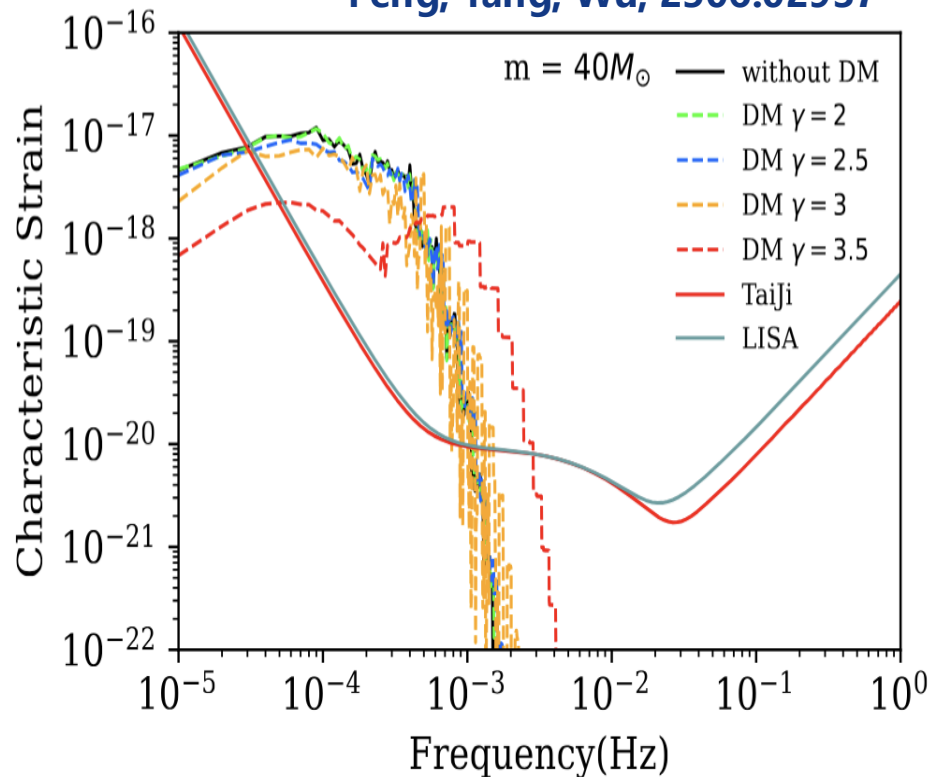
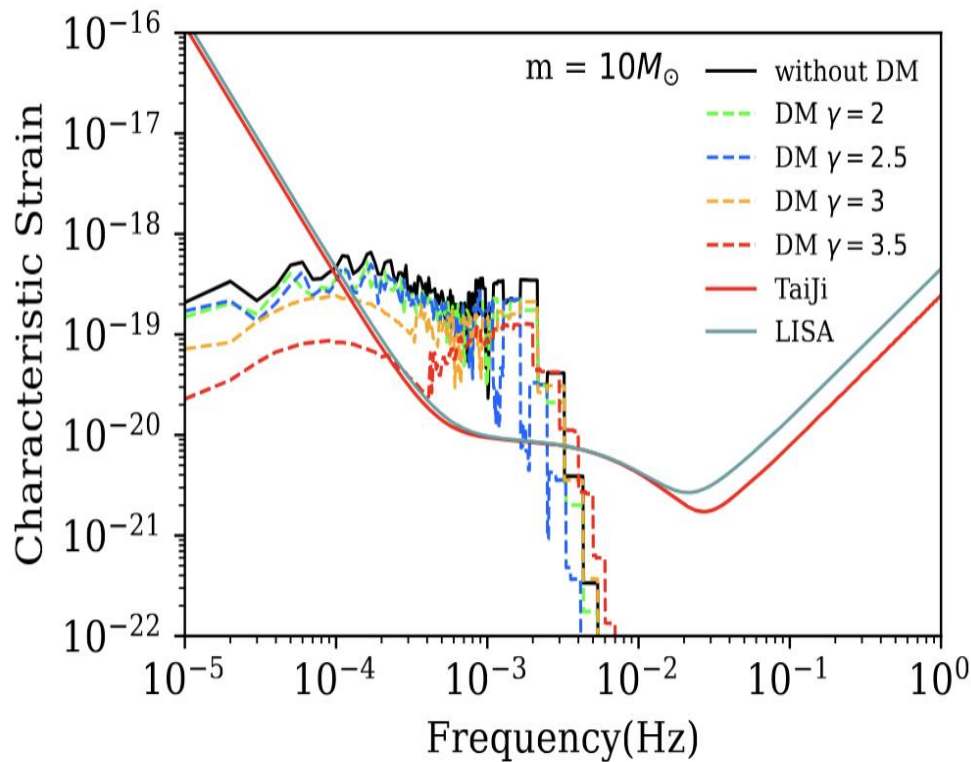
Feng, Tang, Wu, 2506.02937



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Feng, Tang, Wu, 2506.02937



# Summary

- We discuss how GW laser interferometers in space may help to understand the nature of dark matter.
- Wave/Ultralight Dark Matter
  - induce the oscillation of test masses, leading to signals in detectors. Also sensitive to axion-photon coupling
  - Metric perturbation by vector ULDM may be detectable in next-generation interferometers.
- Weakly-interacting massive particles
  - can form spikes around black holes and affect the orbiting compact objects, imprinting on the waveform of GW.

**Thanks!**