Lattice QCD Constraints on Hadron Physics

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CONTENTS

Masses and widths of baryon resonances & Hamiltonian Effective Field Theory

Masses and widths of baryon resonances

- Naive quark model predicts wrong mass order for baryon resonances:
  eg. $N^0(1535) \sim udd$ vs $\Lambda^0(1405) \sim uds$.
- It is still not clear what is dominant in these excited baryons:
  the triquark core, meson-baryon structure, or ...?
- Usually masses and widths are extracted from the scattering experiments:
  tangled with different quantum numbers.
- Nowadays Lattice QCD provides abundant of finite-volume mass spectra:
  starting from the first principle of QCD;
  easier to extract results for one quantum number independently.

Scattering Data and Lattice QCD data are both important.

- Lüscher formalism
  - model independent:
    the hadron interactions and structures are not needed;
  - single-channel efficient:

$$\text{lattice QCD energies} \xrightarrow{\text{Lüscher formalism}} \text{scattering phaseshifts} \Longrightarrow \text{masses and widths.}$$

- Hamiltonian Effective Field Theory (HEFT)
  - multi-channel friendly;
  - Scattering Data and Lattice QCD data can be simultaneously analyzed with HEFT

$$\left.\begin{array}{l}\text{scattering data}\\ \\ \text{lattice QCD data}\end{array}\right\} \xrightarrow{\text{HEFT}} \text{effective potentials} \Longrightarrow \text{masses and widths;}$$

    Hadron interactions and structures can be studied.

- Other approaches

- $N(1/2^-)$

- $\Lambda(1/2^-)$

- $\Sigma(1/2^-)$

$N^*(1535)$ with πN Scattering

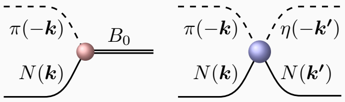
$N^*(1535)$ is the lowest resonance with $I(J^P) = \frac{1}{2}(\frac{1}{2}^-)$.

- One needs to consider the interactions

  among the bare baryon N_0^*, πN channel, and ηN channel.

$$G^2_{\pi N; N_0^*}(k) = \frac{3g^2_{\pi N; N_0^*}}{4\pi^2 f^2}\omega_\pi(k)$$

$$V^S_{\pi N, \pi N}(k, k') = \frac{3g^S_{\pi N}}{4\pi^2 f^2}\frac{m_\pi + \omega_\pi(k)}{\omega_\pi(k)}\frac{m_\pi + \omega_\pi(k')}{\omega_\pi(k')}$$

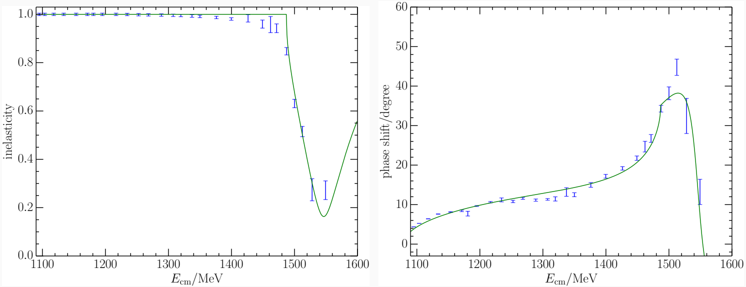


- Phase shifts and inelasticities

  are obtained by solving Bethe-Salpeter equation with the interactions.

$$T_{\alpha,\beta}(k, k'; E) = V_{\alpha,\beta}(k, k') + \sum_\gamma \int q^2 dq V_{\alpha,\gamma}(k, q) \frac{1}{E - \sqrt{m^2_{\gamma_1} + q^2} - \sqrt{m^2_{\gamma_2} + q^2} + i\epsilon} T_{\gamma,\beta}(q, k'; E)$$

$N^*(1535)$ with πN scattering at infinite volume



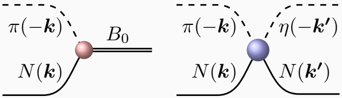
πN Scattering with $I(J^P) = \frac{1}{2}(\frac{1}{2}^-)$.

Our Pole: $1531 \pm 29 - $ i 88 ± 2 MeV.

Particle Data Group: $1510 \pm 20 - $ i 85 ± 40 MeV.

Z. W. Liu, W. Kamleh, D. B. Leinweber, F. M. Stokes, A. W. Thomas and J. J. Wu, Phys. Rev. Lett. 116 (2016) no.8, 082004

Discretization in finite volume



$$\tilde{G}_i(k_n) = \sqrt{\frac{C_3(n)}{4\pi}}\left(\frac{2\pi}{L}\right)^{3/2} G_i(k_n),$$

$$\tilde{V}^S_{i,j}(k_n, k_m) = \frac{\sqrt{C_3(n)C_3(m)}}{4\pi}\left(\frac{2\pi}{L}\right)^3 V^S_{i,j}(k_n, k_m).$$

$C_3(n)$ represents the number of summing the squares of three integers to equal n.

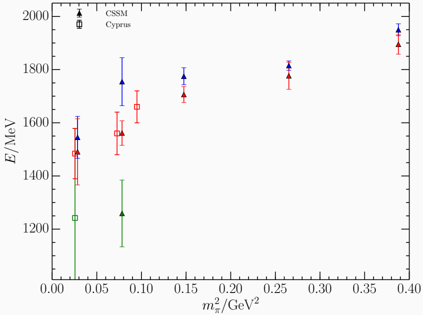
With the eigen-solution of the discretized Hamiltonian, one can obtain the mass spectrum and the components.

$$H_0 = \mathrm{diag}\{m^0_{N_1}, \omega_{\pi N}(k_0),\ \omega_{\eta N}(k_0), \omega_{\pi N}(k_1),\ \omega_{\eta N}(k_1), ...\},$$

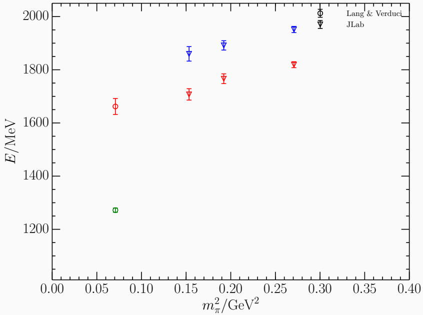
$$H_I = \begin{pmatrix} 0 & \tilde{G}_{\pi N}(k_0) & \tilde{G}_{\eta N}(k_0) & \tilde{G}_{\pi N}(k_1) & \tilde{G}_{\eta N}(k_1) & \cdots \\ \tilde{G}_{\pi N}(k_0) & \tilde{V}^S_{\pi N,\pi N}(k_0, k_0) & 0 & \tilde{V}^S_{\pi N,\pi N}(k_0, k_1) & 0 & \cdots \\ \tilde{G}_{\eta N}(k_0) & 0 & 0 & 0 & 0 & \cdots \\ \tilde{G}_{\pi N}(k_1) & \tilde{V}^S_{\pi N,\pi N}(k_1, k_0) & 0 & \tilde{V}^S_{\pi N,\pi N}(k_1, k_1) & 0 & \cdots \\ \tilde{G}_{\eta N}(k_1) & 0 & 0 & 0 & 0 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix},$$

Spectra at Finite Volumes

3 sets of lattice QCD data at different pion masses and finite volumes



$L \approx 3$ fm



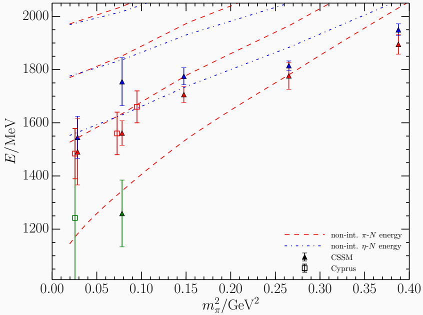
$L \approx 2$ fm

N^* Spectra with $I(J^P) = \frac{1}{2}(\frac{1}{2}^-)$ at finite volumes

Spectra at Finite Volumes

3 sets of lattice QCD data at different pion masses and finite volumes

Non-interacting energies of the two-particle channels



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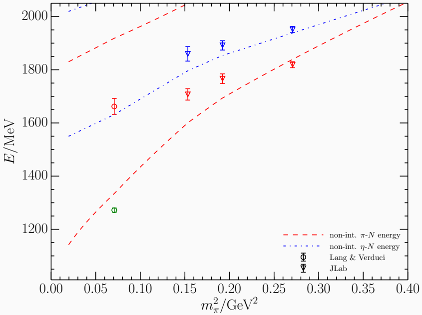

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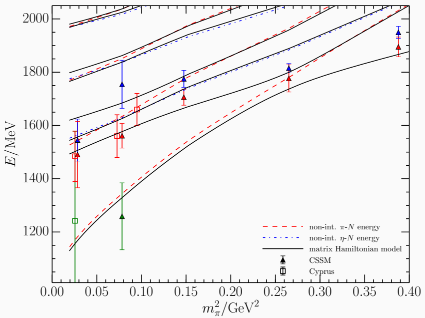
Non-interacting energies of the two-particle channels

Eigenenergies of Hamiltonian effective field theory



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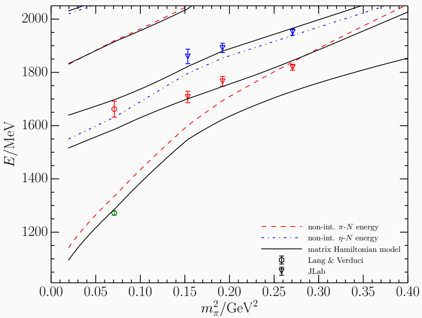

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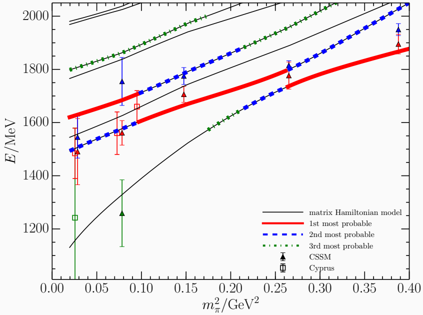
Spectra at Finite Volumes

3 sets of lattice QCD data at different pion masses and finite volumes

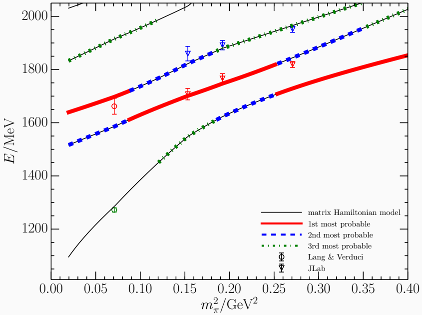
Eigenenergies of Hamiltonian effective field theory

Coloured lines indicating most probable states observed in LQCD

We not only provide the mass but also analyze why some states are observed on the lattice



$L \approx 3$ fm



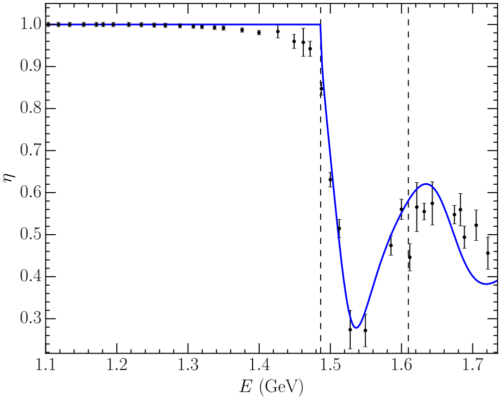
$L \approx 2$ fm

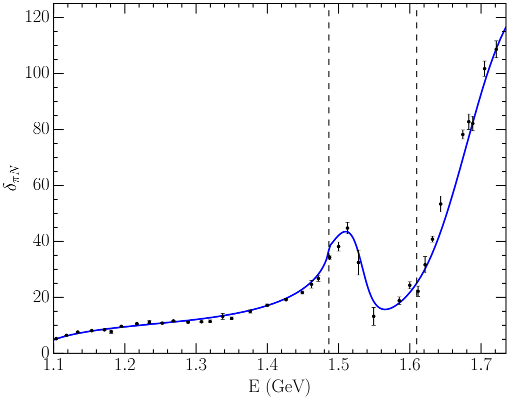
N^* Spectra with $I(J^P) = \frac{1}{2}(\frac{1}{2}^-)$ at finite volumes

Explicitly including $N^*(1650)$ as well as $N^*(1535)$

In Phys. Rev. D 108 (2023) 9, 094519, we consider

- two bare baryon states N_1 and N_2;
- πN, ηN, and $K\Lambda$;
- more experimental data with larger energies (1.60, 1.75) GeV.





Pole positions for $N^*(1535)$ and $N^*(1650)$

In the Particle Data Group (PDG) tables, the poles for the two low-lying odd-parity nucleon resonances are given as

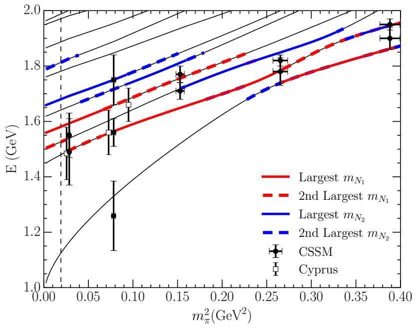
$$E_{N^*(1535)} = 1510 \pm 10 - (65 \pm 10)i \text{ MeV},$$
$$E_{N^*(1650)} = 1655 \pm 15 - (67 \pm 18)i \text{ MeV}.$$

Using HEFT, two poles for $N^*(1535)$ and $N^*(1650)$ in the second Riemann sheet are found at energies

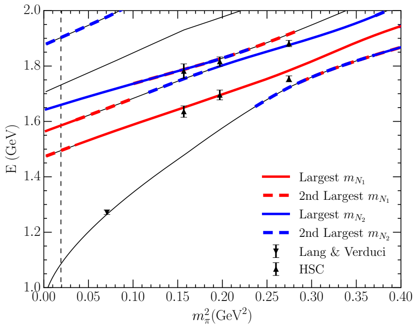
$$E_1 = 1500 - 50i \text{ MeV},$$
$$E_2 = 1658 - 56i \text{ MeV}.$$

Our results are in excellent agreement with the PDG pole positions.

Finite-volume spectrum



$L \sim 3$ fm



$L \sim 2$ fm

C. D. Abell, D. B. Leinweber, Z.-W. Liu, A. W. Thomas, J.-J. Wu, PRD 108 (2023) 9,094519

- $N(1/2^-)$

- $\Lambda(1/2^-)$

- $\Sigma(1/2^-)$

$\Lambda(1/2^-)$ resonances

- $\Lambda(1405)$ is close to the $\bar{K}N$ threshold.
- Its molecular interpretation can easily explain why $m_{\Lambda(1405)} > m_{N(1535)}$.

- If so, where can one find the lightest P-wave $|uds\rangle$ baryon expected in the conventional quark model?
- We will show this P-wave triquark core goes in $\Lambda(1670)$.

$\Lambda(1405)$ and $\Lambda(1670)$ with $K^{-}p$ scattering

- The well-known Weinberg-Tomozawa potentials are used.

  momentum-dependent, non-separable

$$v^{I} = \sum_{\alpha,\beta} \int d^3\vec{k}\, d^3\vec{k}' \, |\alpha(\vec{k})\rangle \, V^{I}_{\alpha,\beta}(k,k') \, \langle\beta(\vec{k}')| \,,$$

$$V_{\alpha,\beta}(k,k') = g_{\alpha,\beta} \frac{\omega_{\alpha_M}(k) + \omega_{\beta_M}(k')}{8\pi^2 f^2 \sqrt{2\omega_{\alpha_M}(k)}\sqrt{\omega_{\beta_M}(k')}}$$

$|\alpha\rangle$=$|\pi\Sigma\rangle$, $|\bar{K}N\rangle$, $|\eta\Lambda\rangle$, $|K\Xi\rangle$, $|\pi\Lambda\rangle$

- two scenarios: with or without a bare baryon

$$g^{I} = \sum_{\alpha,B_0} \int d^3\vec{k} \left\{ |\alpha(\vec{k})\rangle \, G^{I\dagger}_{\alpha,B_0}(k) \, \langle B_0| + |B_0\rangle \, G^{I}_{\alpha,B_0}(k) \, \langle\alpha(\vec{k})| \right\},$$

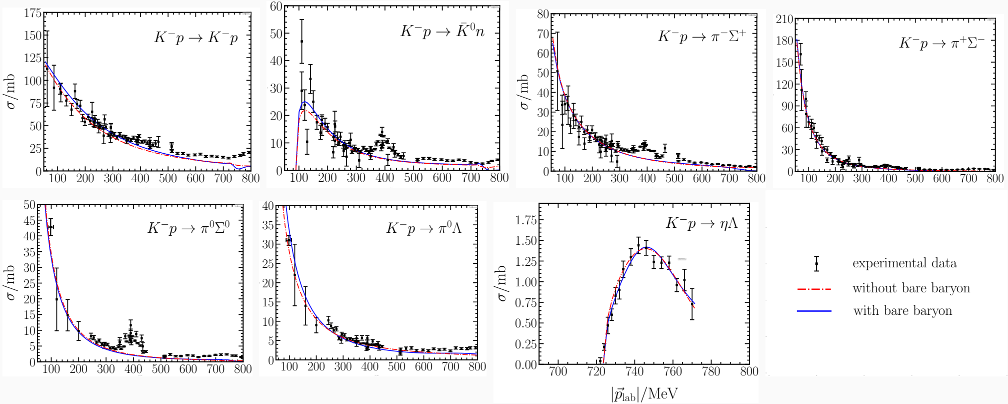
where

$$G^{I}_{\alpha,B_0}(k) = \frac{\sqrt{3}\, g^{I}_{\alpha,B_0}}{2\pi f} \sqrt{\omega_{\pi}(k)}\, u(k).$$

$$H^{I}_{\mathrm{int}} = g^{I} + v^{I}.$$

$\Lambda(1405)$ and $\Lambda(1670)$ with $K^- p$ scattering

We can fit the cross sections of $K^- p$ well both with and without a bare baryon.

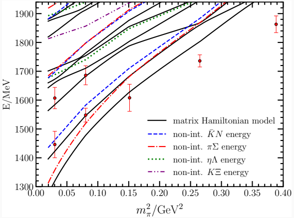


Two-pole structure of $\Lambda(1405)$: $1424 - i67$ MeV, $1428 - i24$ MeV ;

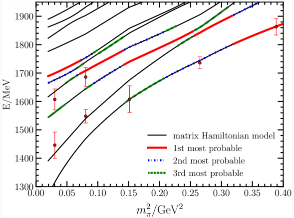
Pole for $\Lambda(1670)$: $1674 - i11$ MeV.

J.-J. Liu, Z.-W. Liu, K. Chen, D. Guo, D. B. Leinweber, X. Liu, A. W. Thomas, Phys. Rev. D 109 (2024) 5, 054025.

Spectrum of $\Lambda(1/2^-)$ on the Lattice



without a bare baryon



with a bare baryon

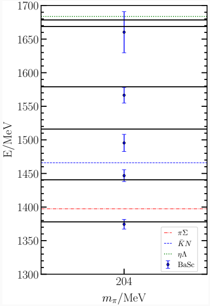
Λ Spectra with $S = -1, I(J^P) = 0(\frac{1}{2}^-)$ in the finite volume.

- The bare baryon is important for interpreting lattice QCD data at large pion masses.
- The bare triquark core is important for $\Lambda(1670)$.
- $\Lambda(1405)$ is mainly a $\bar{K}N$ molecular state.
  containing very little of bare baryon at physical pion mass.

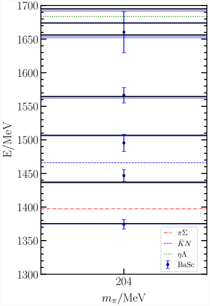
J.-J. Liu, Z.-W. Liu, K. Chen, D. Guo, D. B. Leinweber, X. Liu, A. W. Thomas, Phys. Rev. D 109 (2024) 5, 054025.

Comparison with recent BaSc lattice simulations

Λ Spectra with $S = -1, I(J^P) = 0(\frac{1}{2}^-)$ in the finite volume



without a bare baryon



with a bare baryon

- The BaSc lattice collaboration observed all HEFT states with multiquark interpolating operators;
- The right HEFT results with bare Λ fit the lattice simulations better;
- The left HEFT results without bare triquark core lose the 1σ consistence with the lattice simulations.

Baryon Scattering (BaSc) Collaboration, Phys.Rev.Lett. 132 (2024) 5, 051901; Phys.Rev.D 109 (2024) 1, 014511

J.-J. Liu, Z.-W. Liu, K. Chen, D. Guo, D. B. Leinweber, X. Liu, A. W. Thomas, Phys. Rev. D 109 (2024) 5, 054025.

- $N(1/2^-)$
- $\Lambda(1/2^-)$
- $\Sigma(1/2^-)$

$\Sigma(1/2^-)$ resonances

- In naive quark model, $\Sigma(1/2^-)$ resonances should have been similar to $\Lambda(1/2^-)$ ones except their isospin.
  However, meson-baryon coupled channel effects would make them very different.

- It is even debating how many there are $\Sigma(1/2^-)$ low-lying resonances.

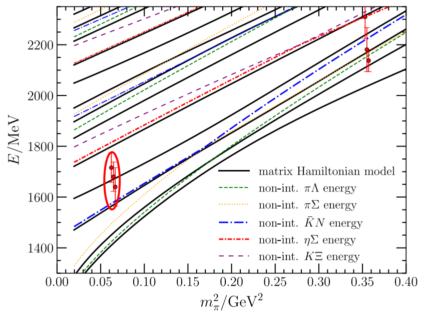
  $\Sigma(1/2^-)$ in PDG2019: $\Sigma(1480)$* $\Sigma(1620)$* $\Sigma(1750)$*** $\cdots$

  $\Sigma(1/2^-)$ in PDG2024: $\Sigma(1620)$* $\Sigma(1750)$*** $\cdots$

- The contributions of $\Sigma\left(\frac{1}{2}^-\right)$ family are not dominant in most processes.
  e.g. in the $\Lambda_c^+ \to \Lambda\pi^+\eta$ decay, $3/2^+$ $\Sigma(1385)$ dominates rather than $\Sigma\left(\frac{1}{2}^-\right)$ around 1.4 GeV *[BESIII:2024mbf]*.
- Analyzing the lattice QCD mass spectra of $\Sigma\left(\frac{1}{2}^-\right)$ thus becomes especially important.

Spectrum of $\Sigma(1/2^-)$ on the Lattice

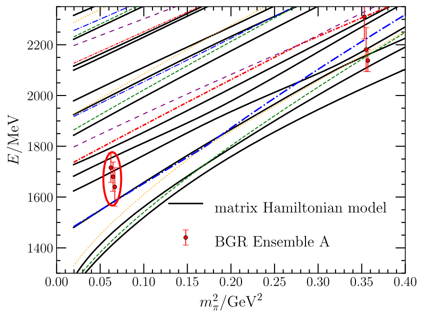
without a bare baryon Σ_0



In the red circle:

3 lattice QCD data vs 2 HEFT eigenlevels.

with a bare baryon Σ_0



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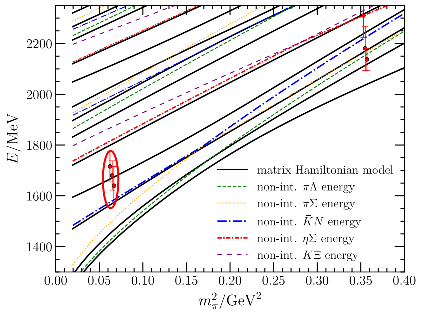
3 lattice QCD data vs 3 HEFT eigenlevels.

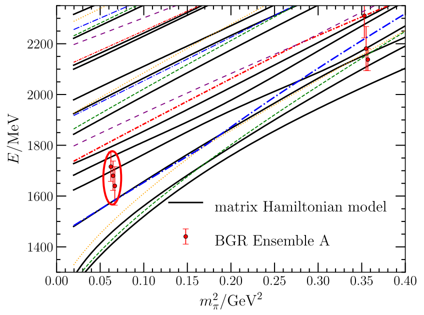
BGR Collaboration, PRD 87 (2013) 074504.
Z.-L. Ma, Z.-W. Liu, J.-J. Liu, PRD 113 (2026) 014037.

Spectrum of $\Sigma(1/2^-)$ on the Lattice

without a bare baryon Σ_0



with a bare baryon Σ_0



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3 lattice QCD data vs 2 HEFT eigenlevels.

HEFT pole: $1666 - 63\,i$ MeV

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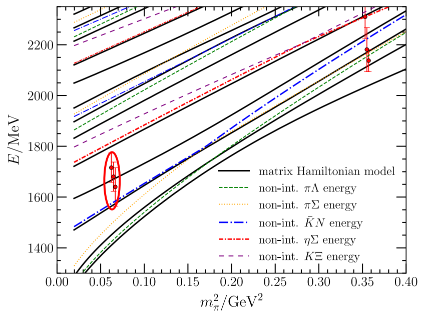
3 lattice QCD data vs 3 HEFT eigenlevels.

HEFT poles: $1687 - 110\,i$, $1714 - 14\,i$

BGR Collaboration, PRD 87 (2013) 074504.
Z.-L. Ma, Z.-W. Liu, J.-J. Liu, PRD 113 (2026) 014037.

Spectrum of $\Sigma(1/2^-)$ on the Lattice

without a bare baryon Σ_0



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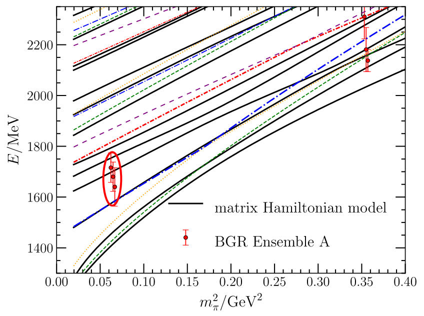

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HEFT poles: $1687 - 110\,i$, $1714 - 14\,i$

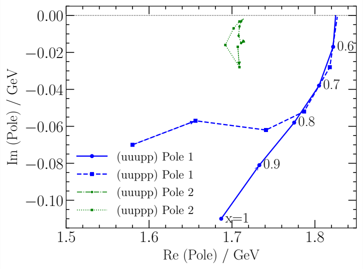
Lattice QCD data support this scenario!

BGR Collaboration, PRD 87 (2013) 074504.
Z.-L. Ma, Z.-W. Liu, J.-J. Liu, PRD 113 (2026) 014037.

$\Sigma(1/2^-)$ poles on different Riemann sheets with the bare baryon Σ_0

Riemann sheets are labeled with "u"/"p" referred to unphysical/physical corresponding to channel sequence ($\pi\Lambda$, $\pi\Sigma$, $\bar{K}N$, $\eta\Sigma$, $K\Xi$).

This work	Pole 1 (MeV)	Pole 2 (MeV)	Other work	Pole 1 (MeV)	Pole 2 (MeV)
Sheet (uuupp)	$1687 - 110\,i$	$1714 - 14\,i$	*[Sarantsev:2019xxm]*	$1689 - 103\,i$	$1680 - 19\,i$
Sheet (uuppp)	$1580 - 70\,i$	$1707 - 17\,i$	*[Zhang:2013sva]*	$1501 - 86\,i$	$1708 - 79\,i$

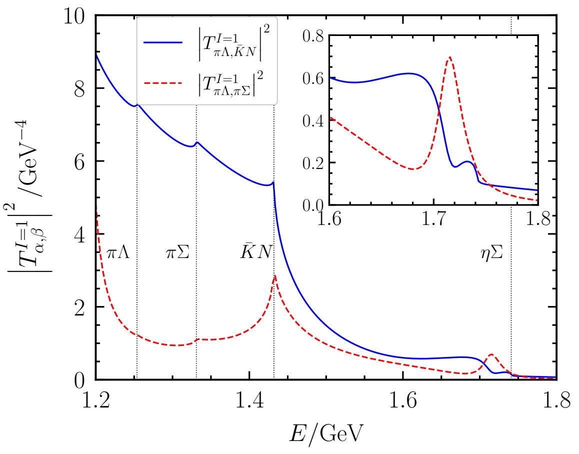


There are two poles on each Riemann sheet.

1st pole deviates $\sim$100 MeV on different Riemann sheets.

If scaling the relevant couplings, the poles on different Riemann sheets would approach each other.

Peaks/dips/cusps in the T matrices



Z.-L. Ma, Z.-W. Liu, J.-J. Liu, Phys. Rev. D 113 (2026) 1, 014037.

For $\left|T^{I=1}_{\pi\Lambda,\bar{K}N}\right|^2$ (blue lines)

- a wide peak around 1.68 GeV
- a narrow one around 1.73 GeV

corresponding to the two poles found on the (uuupp) sheet.

A clear cusp-like structures at the $\bar{K}N$ threshold in the $\pi\Sigma \to \pi\Lambda$ scattering.

Pion electroproduction off a nucleon & Extended Lellouch–Lüscher Formalism

Currently, we have shown the analyses of $N^*(1650)$ and $N^*(1535)$ based on

- the $\pi N \to \pi N$ scattering data,
- lattice QCD spectrum of N^*.

Next, we will present their roles in the electromagnetic processes:

- $\gamma + N \to \pi + N$
- lattice QCD simulation of electric dipole magnitudes.

Pion Photoproduction off Nucleon with Hamiltonian EFT

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- combining
  - $\pi N \to \pi N$
  - lattice QCD spectrum of N^*
  - $\gamma + N \to \pi + N$

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- $\gamma + N \to \pi + N$
  - γNN etc. couplings are not adjusted

Pion Photoproduction off Nucleon with Hamiltonian EFT

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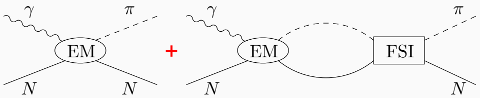

Pion Photoproduction off Nucleon with Hamiltonian EFT

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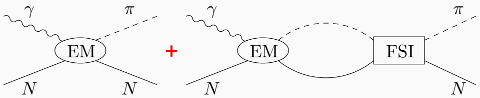

$$\begin{aligned}\mathcal{M}(\gamma N \to \pi N) \quad \sim \quad & \mathcal{M}^{\text{EM}}(\gamma N \to \pi N) \\ & + \mathcal{M}^{\text{EM}}(\gamma N \to \pi N) \otimes \mathcal{M}^{\text{FSI}}(\pi N \to \pi N) \\ & + \mathcal{M}^{\text{EM}}(\gamma N \to \eta N) \otimes \mathcal{M}^{\text{FSI}}(\eta N \to \pi N)\end{aligned}$$

Pion Photoproduction off Nucleon with Hamiltonian EFT

- combining
  - $\pi N \rightarrow \pi N$
  - lattice QCD spectrum of N^*
  - $\gamma + N \rightarrow \pi + N$

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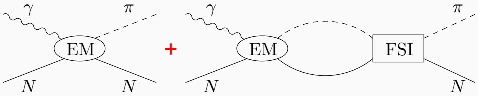

$$\mathcal{M}(\gamma N \rightarrow \pi N) \sim \mathcal{M}^{\text{EM}}(\gamma N \rightarrow \pi N) + \mathcal{M}^{\text{EM}}(\gamma N \rightarrow \pi N) \otimes \mathcal{M}^{\text{FSI}}(\pi N \rightarrow \pi N) + \mathcal{M}^{\text{EM}}(\gamma N \rightarrow \eta N) \otimes \mathcal{M}^{\text{FSI}}(\eta N \rightarrow \pi N)$$

  - Finite State Interaction (FSI) part can be determined independently

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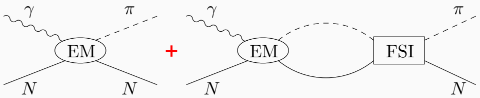

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  - Finite State Interaction (FSI) part can be determined independently
  - understand the structure of $N(1535)$ and the interactions of $\pi N/\eta N$ at low energies and near the resonance

Pion Photoproduction off Nucleon with Hamiltonian EFT

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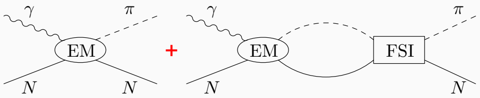


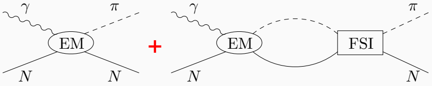
$$\begin{aligned}\mathcal{M}(\gamma N \to \pi N) \quad \sim \quad & \mathcal{M}^{\mathrm{EM}}(\gamma N \to \pi N) \\ & + \mathcal{M}^{\mathrm{EM}}(\gamma N \to \pi N) \otimes \mathcal{M}^{\mathrm{FSI}}(\pi N \to \pi N) \\ & + \mathcal{M}^{\mathrm{EM}}(\gamma N \to \eta N) \otimes \mathcal{M}^{\mathrm{FSI}}(\eta N \to \pi N)\end{aligned}$$

- Finite State Interaction (FSI) part can be determined independently
- understand the structure of $N(1535)$ and the interactions of $\pi N/\eta N$ at low energies and near the resonance
- necessities for the photon-nucleus investigation

Electromagnetic Multipoles

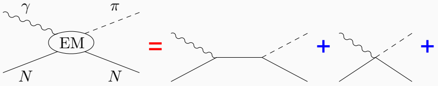
- $|\gamma N\rangle \rightarrow |\phi\,(\vec{k}),\ N(-\vec{k}, s_z'^N)\rangle,$
- $|\gamma N\rangle \rightarrow |\phi\, N; k, J, J_z, L\rangle\ ,$
- $|\gamma N\rangle \rightarrow |\phi\, N; k, J, J_z, \lambda_N'\rangle\ ,$

$k_x, k_y, k_z, s_z'^N$
$k,\ \ J,\ \ J_z,\ \ L$
$k,\ \ J,\ \ J_z, \lambda_N'$

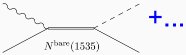


Partial wave decomposition:

$$V_{\alpha,\gamma N}(J, \lambda_N', \lambda_\gamma, \lambda_N; k, q) = 2\pi \int_{-1}^{1} \mathrm{d}(\cos\theta) \sum_{s_z'^N} d^J_{\lambda_\gamma - \lambda_N, -\lambda_N'}(\theta) d^{1/2}_{s_z'^N, -\lambda_N'}(\theta)^* \mathcal{M}_{\alpha,\gamma N}(s_z'^N, \lambda_N, \lambda_\gamma; \vec{k}, \vec{q}),$$



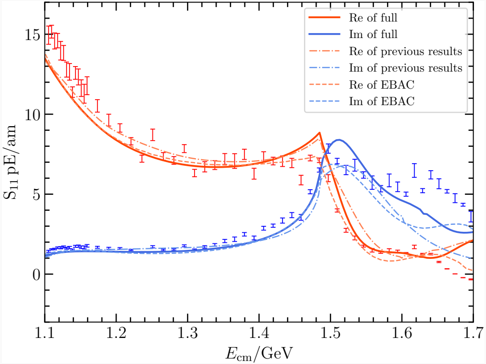
$$V^{JLS;\lambda_\gamma\lambda_N}_{\alpha,\gamma N}(k, q) = \sqrt{\frac{2L+1}{2J+1}} \sum_{\lambda_N'} \langle L, S, 0, -\lambda_N' | J, -\lambda_N'\rangle \times V_{\alpha,\gamma N}(J, \lambda_N', \lambda_\gamma, \lambda_N; k, q).$$



D. Guo and Z. W. Liu, Phys. Rev. D **105** (2022) no.11, 11

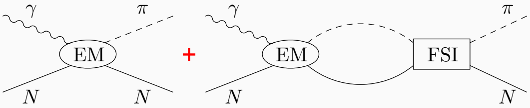
Electric dipole amplitudes E_{0+} with two bare states

Our results can describe the electric dipole amplitudes E_{0+} extracted from experiments.

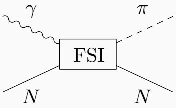


Yu Zhuge, Zhan-Wei Liu, Derek B. Leinweber, Anthony W. Thomas, Phys.Rev.D 110 (2024) 9, 094015.

The bare core in $N^*(1535)$



- If $N^*(1535)$ has no bare core, it would play roles ONLY in finite state interaction



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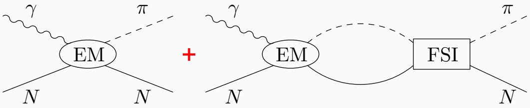

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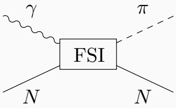
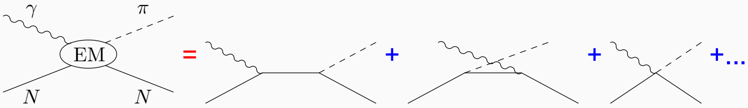
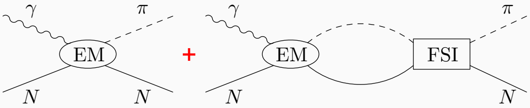


- If with bare core, $N^*(1535)$ also plays roles in electromagnetic potential



The bare core in $N^*(1535)$



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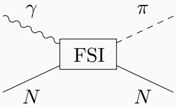

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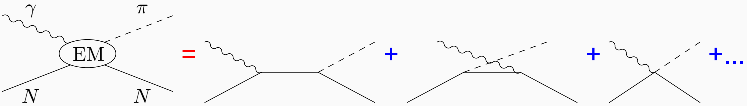
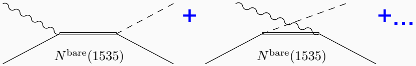
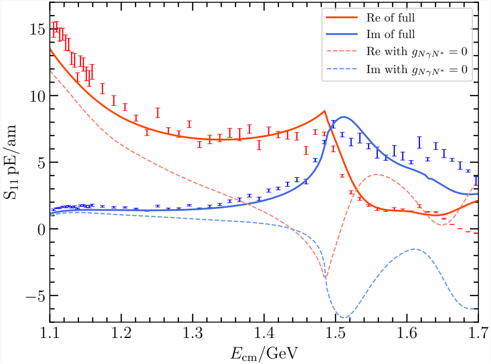




The bare core in $N^*(1535)$ cannot be absent in pion photoproduction

If without the bare core in $N^*(1535)$,
E_0^+ would change much!



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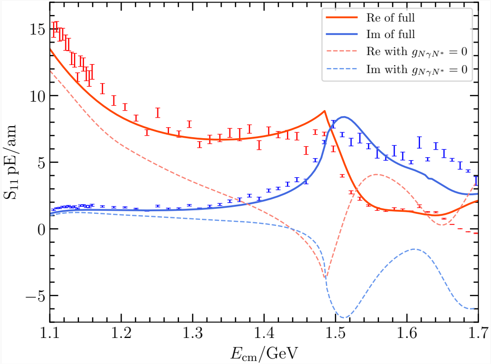
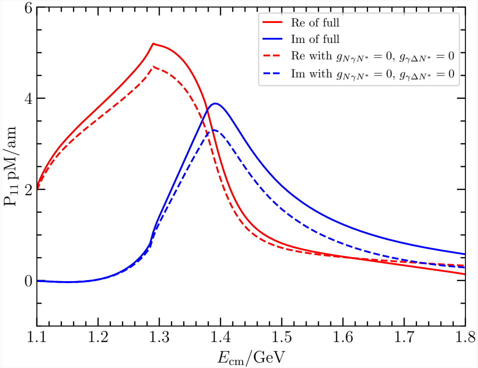


V.S.

If without the bare core in $N^*(1440)$,
M_1^- would change little!



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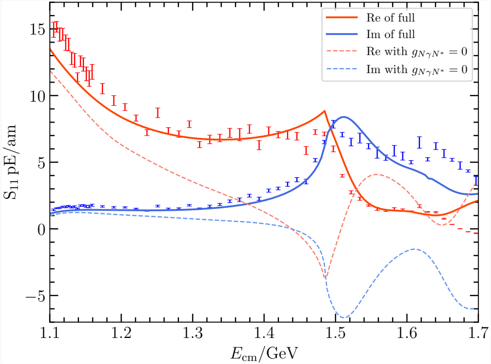

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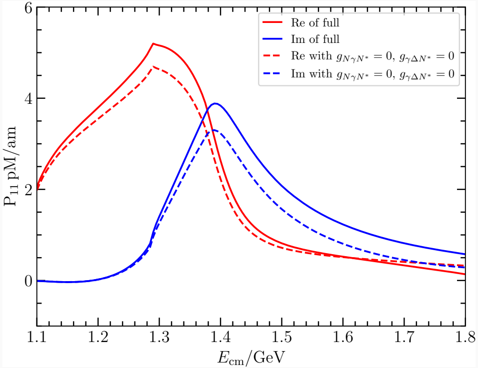
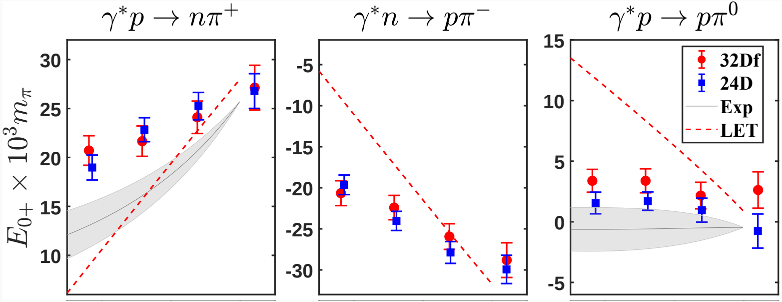


Yu Zhuge, Zhan-Wei Liu, Derek B. Leinweber, Anthony W. Thomas, Phys. Rev. D 110 (2024) 9, 094015.

Latest lattice QCD data on E_0^+

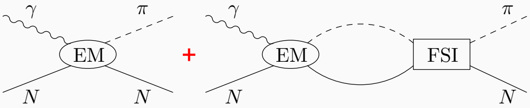
The lattice QCD results is very close to the partial wave analysis from the Jülich-Bonn-Washington collaboration.



First lattice QCD simulation of pionproduction at threshold!

Gao, Yu-Sheng and Zhang, Zhao-Long and Feng, Xu and Jin, Lu-Chang and Liu, Chuan and Meißner, Ulf-G., Lattice QCD Study of Pion Electroproduction and Weak Production from a Nucleon, Phys. Rev. Lett. 134 (2025) 17, 171904

Direct extension of our previous work

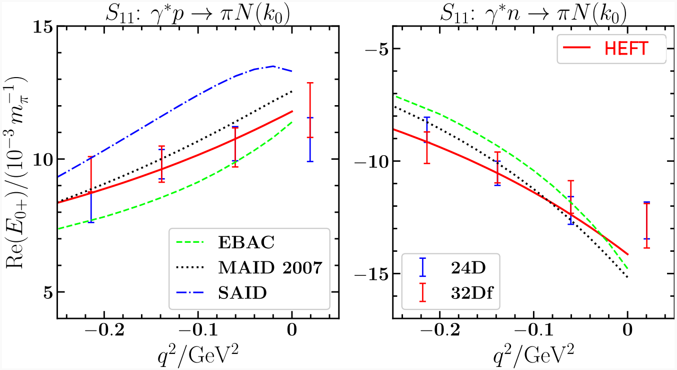


From the real photon ($q^2 = 0$) to the virtual spacelike photon ($q^2 < 0$), we

- do not adjust the previous parameters,
- add the form factors of neutrons and pion:
  - $F(q^2 = 0) = 1$,
  - $F(q^2 < 0) < 1$,
  - $F(q^2)$ is well determined by the experiment.

Latest lattice QCD data and our HEFT results

Our HEFT results can describe the lattice QCD E_{0+} very well.



Y. Zhuge, Z.-W. Liu, D. B. Leinweber, A. W. Thomas, arXiv: 2603.06055.

The Lellouch–Lüscher formula

- The original lattice QCD E^L_{0+} are not strictly equal to the physical E_{0+}.
- Lellouch & Lüscher proved
  a corrected factor is needed for general electroweak processes *[Lellouch:2000pv]* :

$$F_{\mathrm{LL}} = \left| \frac{E_{0+}|_{E_{G(n)}}}{E^L_{0+}|_{E_{G(n)}}} \right| = \sqrt{\frac{2\pi\, C_3(n)}{(kL)^3} \left(\tilde{k}\frac{d\phi}{d\tilde{k}} + k\frac{d\delta}{dk} \right)},$$

where $\tilde{k} \equiv \frac{kL}{2\pi}$, k is πN 3-momentum, $C_3(n)$ represents the number of summing the squares of three integers to equal n.

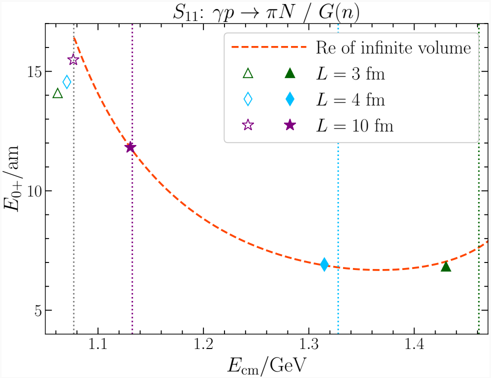
The function $\phi(\tilde{k})$ is related to the ζ function, and $\delta(k)$ is the phase shift.

- This factor only depends on the final state interactions.
- The published lattice QCD data has been transformed with Lellouch–Lüscher factor.

HEFT can study both infinite volume E_{0+} and finite volume E^L_{0+}

Infinite volume $\quad E_{0+} \sim \quad V_{\pi N,\gamma^* N}(k,q) + \sum_{\alpha=\pi N,\eta N,\dots} \int \mathrm{d}k'\, k'^2 V_{\alpha,\gamma^* N}(k',q) \frac{1}{E_{\rm cm} - \omega_\alpha(k') + i\epsilon} T_{\pi N,\alpha}(k,k';E_{\rm cm})\,,$

Finite volume $\quad E^L_{0+} \sim \quad \sum_{\alpha,n_i} c^{(i)}_{n_i} V_{\alpha(n_i),\gamma^* N}, \qquad c^{(i)}_{n_i}$ is the eigenvector in the box.



- With L increasing, E^L_{0+} gets closer to E_{0+}.
- The finite volume effects are much smaller for the first excited states compared to the ground states.

Y. Zhuge, Z.-W. Liu, D. B. Leinweber, A. W. Thomas, arXiv: 2603.06055.

We have proposed an extended Lellouch–Lüscher formalism

- The Lellouch–Lüscher formula can only extrapolate the absolute value $|E_{0+}|$

$$F_{\rm LL} = \left| \frac{E_{0+}|_{E_{G(1)}}}{E^L_{0+}|_{E_{G(1)}}} \right| = \sqrt{\frac{12\pi}{(kL)^3} \left(\tilde{k}\frac{d\phi}{d\tilde{k}} + k\frac{d\delta}{dk} \right)} \, .$$

- If separable potentials $V_{\alpha,\alpha'}(k,k') = h_\alpha(k)\, h_{\alpha'}(k')$ are employed, $\alpha^{(\prime)} = \gamma N, \pi N, \eta N, ...$,

$$F_{\rm sep} = \frac{E_{0+}|_{E_{G(1)}}}{E^L_{0+}|_{E_{G(1)}}} = \frac{2\sqrt{6}\pi^2\, T_{\pi N,\pi N}\left(E_{G(1)}, k_{G(1)}, k_1\right)}{L^3 U_1^{\rm bind} c_1^{\pi N}} \, ,$$

where $c_1^{\pi N} = \langle \pi N(k_1)|G(1)\rangle = 1 + O(L^{-6}) \approx 1$, $U_1^{\rm bind} = E_{\pi N(1)} - E_{G(1)}$ denotes its finite-volume binding energy and $T_{\pi N,\pi N}$ is corresponding T matrix in infinite volume.

- This simple factor also depends only on the final state interactions.
- It can provide extrapolations to both the real and imaginary parts.

L/fm	$\mathrm{Re}F_{\rm HEFT}$	$\mathrm{Re}F_{\rm sep}$	$\mathrm{Im}F_{\rm HEFT}$	$\mathrm{Im}F_{\rm sep}$	$\lvert F_{\rm HEFT}\rvert$	$F_{\rm LL}$
3	1.029	1.028	0.376	0.375	1.096	1.095
4	0.995	0.990	0.222	0.221	1.019	1.016
10	0.999	0.999	0.122	0.122	1.007	1.006

Summary

Summary

- We have studied the $N(1535)$ and $N(1650)$ based on analyzing
  (1) $\pi N \to \pi N$, (2) lattice QCD spectrum of N^*,
  (3) $\gamma + N \to \pi + N$, and (4) lattice QCD simulation of pionproduction.
- Both the conventional quark model states and coupled baryon-meson channels are important for $N(1/2^-)$ resonances $N(1535)$ and $N(1650)$,
- The Roper results are insensitive to whether including the bare basis state or not.

- $\Lambda(1405)$ is mainly a $\bar{K}N$ molecule, but the 1P triquark core $\Lambda(1P)$ plays an important role in $\Lambda(1670)$.
- More precise lattice QCD data will be very essential to solve whether the $\Sigma(1620)$ should exist or not.

- We propose a novel extension of the well-known Lellouch-Luscher formalism that extrapolates both real and imaginary parts of amplitudes for general electroweak processes.

Thank you for your attention!