Nucleon sigma term in BChPT at two-loop order

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Outline

- Introduction
- The tension between lattice QCD and phenomenology
- Two-loop extraction of pion-nucleon sigma term
- Summary and outlook

Introduction

Why nucleon sigma term?

- Anatomy of the nucleon mass
    - Trace of the energy-momentum tensor

$$T^{\mu}_{\mu} = \left[\frac{\beta_{\rm QCD}}{2g} G^{a}_{\mu\nu} G^{\mu\nu}_{a} + \sum_{q} \gamma_m m_q \bar{q} q \right] + \sum_{q} m_q \bar{q} q$$

    - Explicit chiral symmetry breaking term (classical)
    - γ_m anomalous dimension of the mass operator.
    - Trace anomaly owing to quantum effects

- Nucleon mass budget

$$m_N = \langle N | \underbrace{\frac{\beta_{QCD}}{2g} G^{a}_{\mu\nu} G^{\mu\nu}_{a} + \gamma_m \sum_{q} m_q \bar{q} q}_{\text{Trace anomaly}} + (\underbrace{m_u \bar{u} u + m_d \bar{d} d}_{\text{sigma term}} + \cdots) | N \rangle$$

Why nucleon sigma term?

- Dark matter detection [Hill & Solon PLB(2012)]
  - Scalar coupling of the nucleon

$$\langle N | m_q \bar{q} q | N \rangle = m_N f_q$$

  - Dark matter of scalar nature

$$\mathscr{L}_{\chi q} = C \frac{m_q}{\Lambda^3} \bar{\chi}\chi\bar{q}q \quad \Longrightarrow \quad \sigma_{\chi N} = \frac{m_{\chi N}^2}{\pi \Lambda^6} \left| \sum_q C_q f_q \right|^2$$

- Condensate in nuclear matter [Kaiser & Weise PLB(2008)]

$$\frac{\langle \Psi | \bar{q}q | \Psi \rangle}{\langle 0 | \bar{q}q | 0 \rangle} = 1 - \frac{\rho}{F_\pi^2} \left\{ \frac{\sigma_{\pi N}}{M_\pi^2} \left[1 - \frac{3k_f^2}{10m_N^2} + \frac{9k_f^4}{56m_N^4} \right] + \cdots \right\}$$

Basics of pion-nucleon sigma term

- **Definition**
  - Scalar form factor of the nucleon $\sigma(t) = \frac{1}{2m_N}\langle N(p')\,|\,\hat{m}(\bar{u}u + \bar{d}d)\,|\,N(p)\rangle\ , \quad t = (p' - p)^2$
  - Nucleon sigma term (**Direct approach**) $\sigma_{\pi N} = \sigma(t=0) = \frac{1}{2m_N}\langle N(p)\,|\,\hat{m}(\bar{u}u + \bar{d}d)\,|\,N(p)\rangle$
  - Feynman-Hellman theorem $\frac{\partial m_N^2}{\partial \hat{m}} = \langle N(p)\,|\,\bar{u}u + \bar{d}d\,|\,N(p)\rangle$
- **Therefore, the nucleon sigma term via FH theorem. (FH approach)**
  - Gell-Mann-Oakes-Renner relation: $M^2 = 2B_0\hat{m}$

$$\sigma_{\pi N} = \hat{m}\frac{\partial m_N}{\partial \hat{m}} = M^2\frac{\partial m_N}{\partial M^2}$$

Nucleon mass as input

Status of sigma term from lattice QCD

- **FH appoach**
  - χQCD 15 (2+1): $\sigma_{\pi N} = 45.9(7.4)(2.8)$ MeV [Yang, et al., [χQCD], PRD(2016)]
  - BMW 15 (2+1): $\sigma_{\pi N} = 38(8)(8)$ MeV [Durr, et al., PRL(2016)]
  - RQCD 22 (2+1): $\sigma_{\pi N} = 43.9(4.7)$ MeV [Bali, et al., [RQCD], JHEP(2023)]
  - …

- **Direct approach**
  - NME 21(2+1+1): $\sigma_{\pi N} = 59.6(7.4)$ MeV [Gupta, et al., PRL(2021)]
  - Mainz 23 (2+1): $\sigma_{\pi N} = 43.7(3.6)$ MeV [Agadjanov, et al., PRL(2023)]
  - ETMC 24 (2+1+1): $\sigma_{\pi N} = 41.9(8.1)$ MeV [Alexandrou, et al., PRD(2025)]
  - …..
  - In the direct approach, the $N\sigma$ or $N\pi\pi$ effects might be important [Barca, et al., arXiv:2508.09006 [hep-lat].

The tension between lattice QCD and phenomenology

Roy-Steiner equation determination

- Basic formula:

$$\sigma_{\pi N} = \underbrace{F_\pi^2 (d_{00}^+ + 2M_\pi^2 d_{01}^+)}_{\text{RS equation}} + \underbrace{(\Delta_D - \Delta_\sigma)}_{\text{Dis. Relation}} - \underbrace{\Delta_R}_{\text{B}\chi\text{PT}}$$

- Subthreshold parameters output of the RS equations

$$d_{00}^+ = -1.36(3) M_\pi^{-1} \qquad [\text{KH}: \ -1.46(10) M_\pi^{-1}]$$
$$d_{01}^+ = \ 1.16(3) M_\pi^{-3} \qquad [\text{KH}: \ 1.14(2) \ M_\pi^{-3}]$$

- $\Delta_D - \Delta_\sigma = (1.8 \pm 0.2)$ MeV [Hoferichter, Ditsche, Kubis and Meissner, JHEP(2012)]
- $\Delta_R \lesssim 2$ MeV [Bernard, Kaiser and Meissner, PLB(1996)]
- Isospin breaking in the Cheng-Dashen theorem shifts $\sigma_{\pi N}$ by $+3$ MeV

$$\sigma_{\pi N} = (59.1 \pm 1.9_{\text{RS}} \pm 3.0_{\text{LET}}) \text{ MeV} = (59.1 \pm 3.5) \text{ MeV}$$

[NB: recover $\sigma_{\pi N} = 45$ MeV if KH80 scattering lengths are used] [Courtesy of Ulf-G. Meissner]

The long-standing tension

- Phenomenological (RS equation)

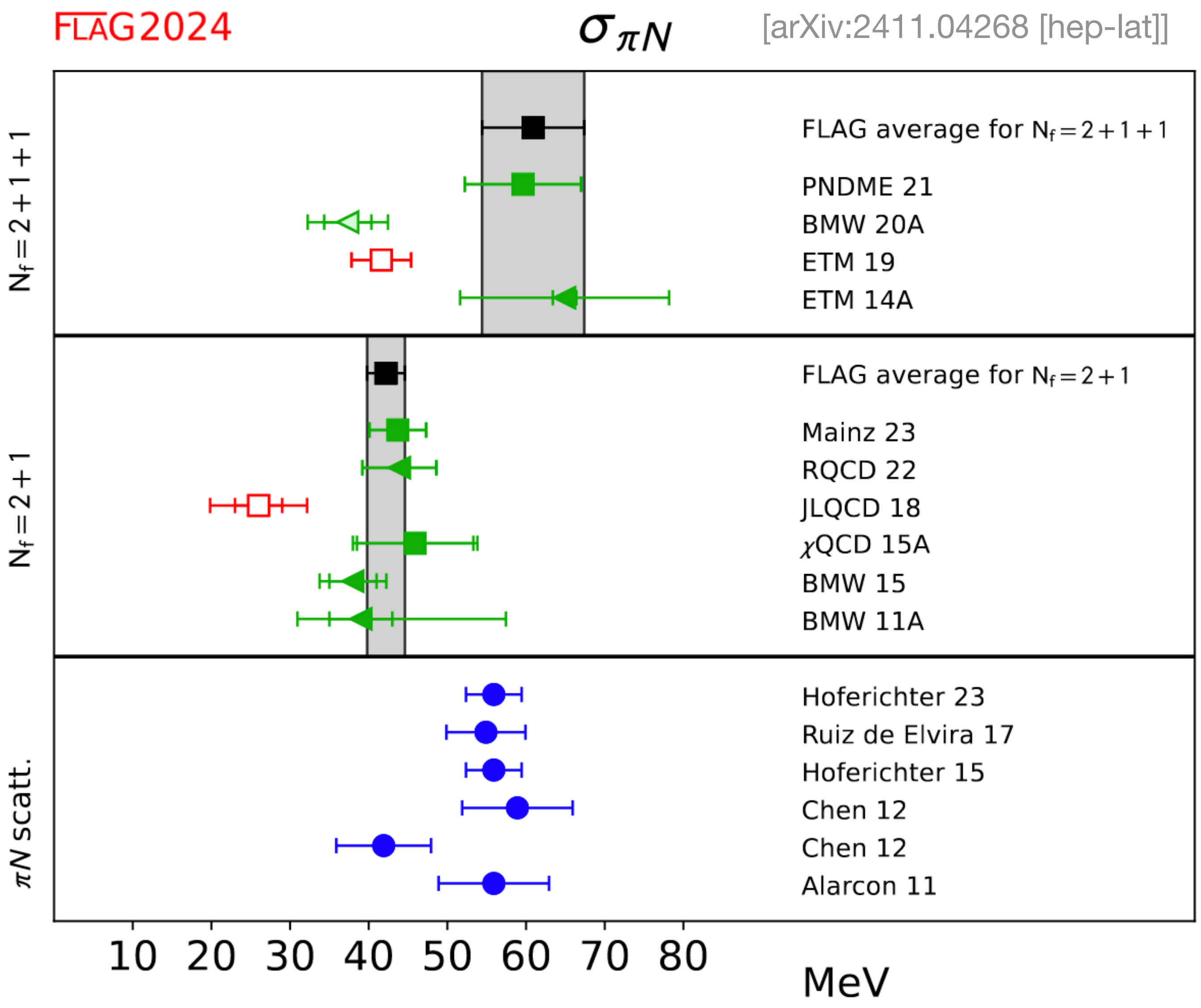
  $\sigma_{\pi N} = 59.1(3.5)$ MeV

- FLAG Average $N_f = 2+1+1$

  $\sigma_{\pi N} = 60.9(6.5)$ MeV

- FLAG Average $N_f = 2+1$

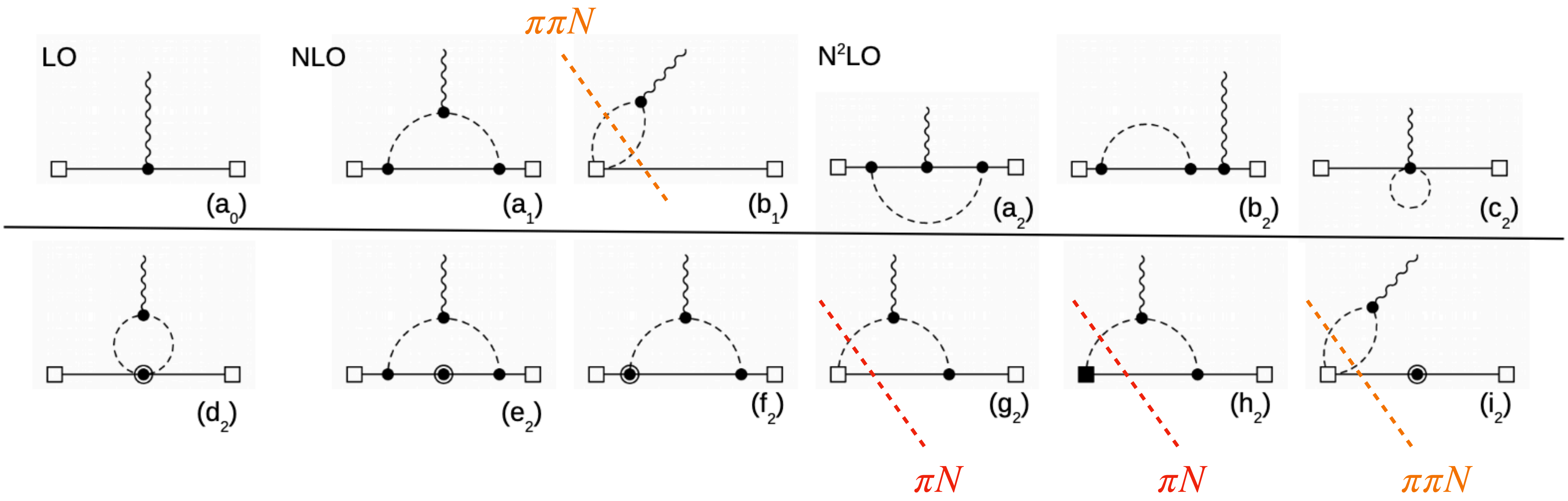
  $\sigma_{\pi N} = 42.2(2.4)$ MeV



Tension between phenomenological and $N_f = 2+1$ lattice QCD determinations.

Excited-state contamination?

- ESC from an EFT point of view [Gupta, et al., PRL(2021)]



- Spin-1/2, positive parity: N, πN, $\pi\pi N$, $\cdots$

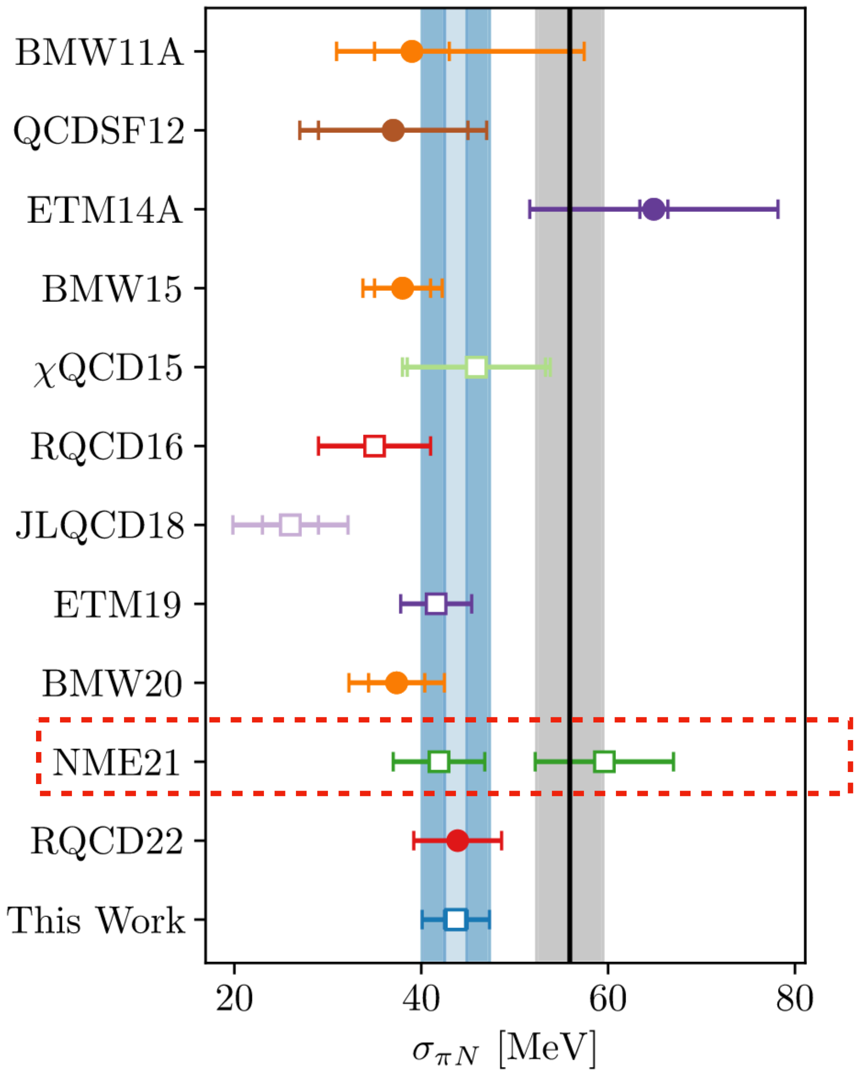
Excited-state contamination?

- **NME 21** [Gupta, et al., PRL(2021)]
  - πN & $\pi\pi N$ excited states (e.s.)

$$\sigma_{\pi N} = \begin{cases} 41.9(4.9)\,\text{MeV} & \text{w/o e. s.} \\ 59.6(7.4)\,\text{MeV} & \text{w e. s.} \end{cases}$$

- **Mainz 23** [Agadjanov, et al., PRL(2023)]
  - An upward trend for $\sigma_{\pi N}$, when using priors similar to NME21, albeit not as pronounced.

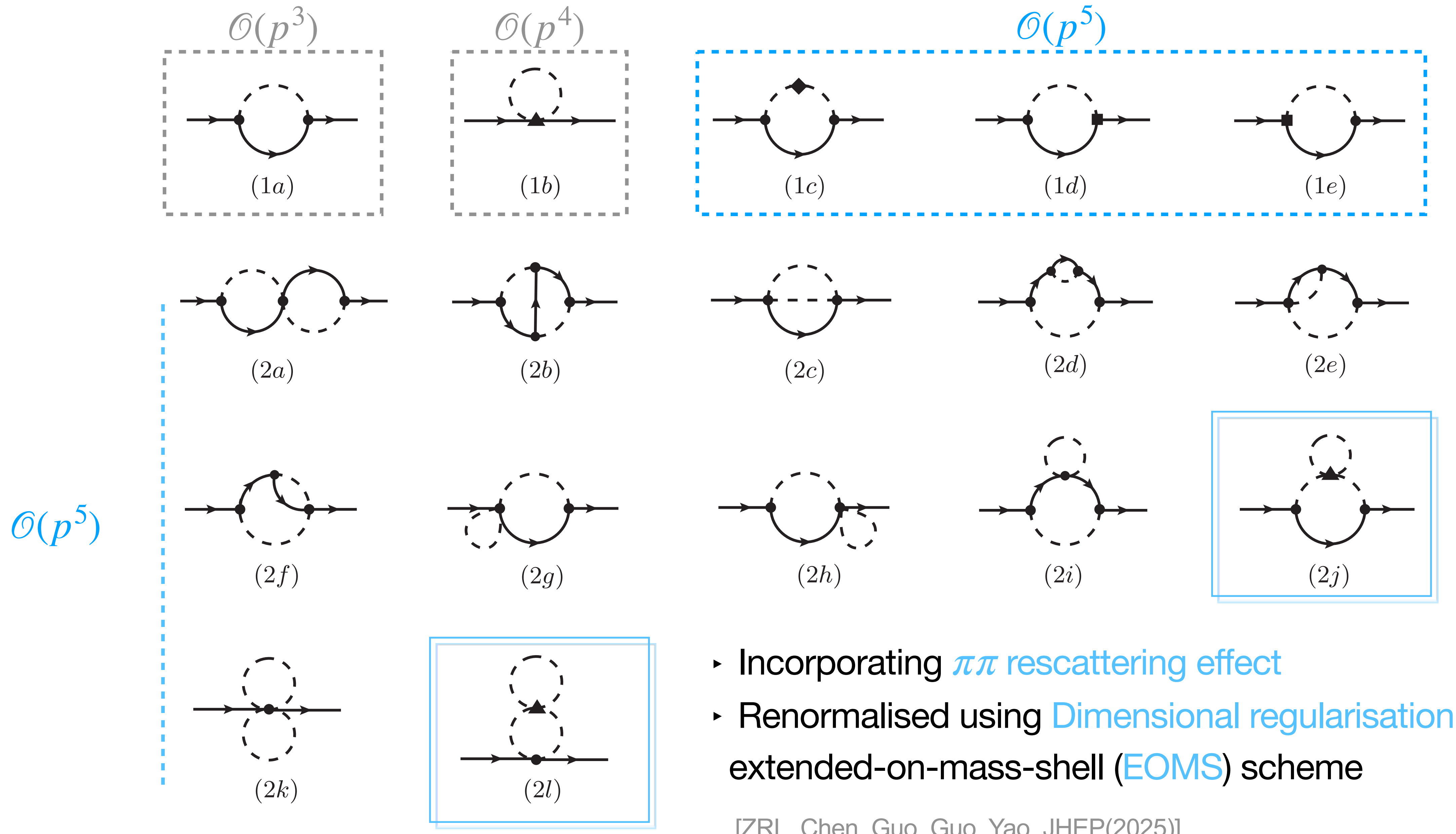
$$\sigma_{\pi N} = 43.7(3.6)\ \text{MeV}$$



The subtraction of ESC does not play the sole role in alleviating the tension!

Two-loop extraction of pion-nucleon sigma term

The nucleon mass at two-loop order



- Incorporating $\pi\pi$ rescattering effect
- Renormalised using Dimensional regularisation and extended-on-mass-shell (EOMS) scheme

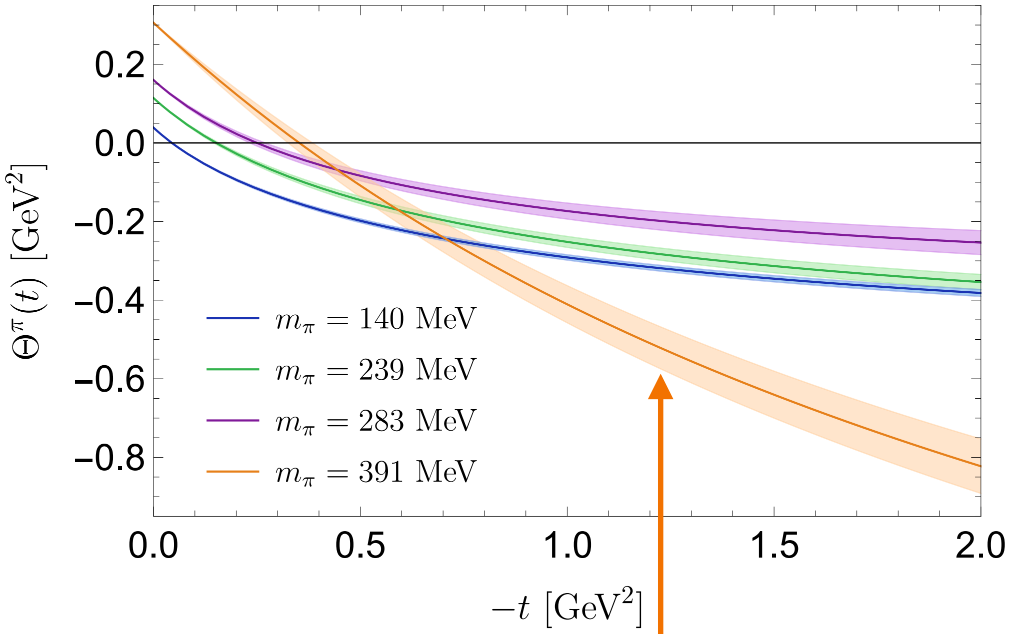
[ZRL, Chen, Guo, Guo, Yao, JHEP(2025)]

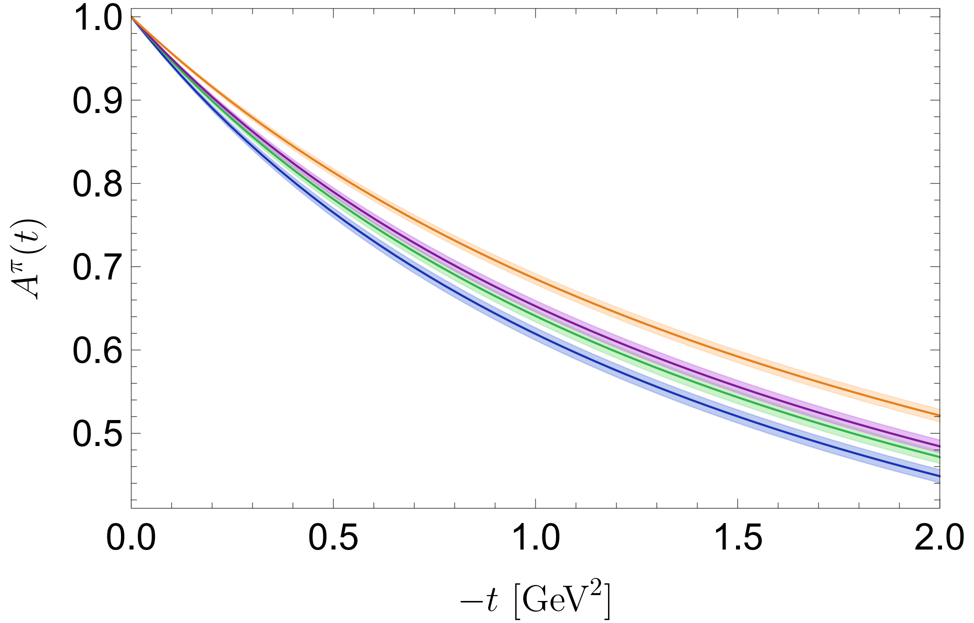
$\pi\pi$ rescattering effect

- Using the $\pi\pi$ scattering phase shifts at unphysical pion masses (239 MeV, 283 MeV, 391 MeV) obtained from Roy equation analyses

[Cao, Guo, Li, Yao, Nature Com.(2025)]

[Cao, Guo, Li, Wu, Yao, arXiv:2507.05375 [hep-ph]]





Fast change before $m_\pi = 391$ MeV:
σ meson becomes a $\pi\pi$ bound state!

Isoscalar $\pi\pi$ contribution is significant at unphysical pion masses!

[R.A. Briceno et al. [HadSpec], PRL 118 (2017) 022002]

Two-loop sigma term in BχPT

- **Chiral expression up to $\mathcal{O}(p^5)$**
  - Tree + one loop + two loop $\sigma_{\pi N} = -4c_1 M^2 - 4e_m M^4 + \Delta\sigma_{\pi N}^{(1)} + \Delta\sigma_{\pi N}^{(2)}$
  - One-loop: $\Delta\sigma_{\pi N}^{(1)} = \sigma_{\pi N}^{(1a)} + \sigma_{\pi N}^{(1b)} + \sigma_{\pi N}^{(1c)} + \sigma_{\pi N}^{(1d)} + \sigma_{\pi N}^{(1e)}\ ,$
  - Two-loop: $\Delta\sigma_{\pi N}^{(2)} = \sigma_{\pi N}^{(2a)} + \sigma_{\pi N}^{(2b)} + \cdots + \sigma_{\pi N}^{(2l)} + \sigma_{\pi N}^{(2')} + \sigma_{\pi N}^{\text{sub.}}\ .$

$$\sigma_{\pi N}^{(1a)} = -\frac{3ig_A^2 M^2 m (J_{11} + M^2 J_{21})}{2F^2}$$

$$\sigma_{\pi N}^{(2a)} = \frac{3g_A^2 M^2 m}{2F^4}(M^4 I_{12011} + M^4 I_{21011} + M^2 I_{02011} + 2M^2 I_{11011} + M^2 I_{20011} + I_{10011} + I_{01011})$$

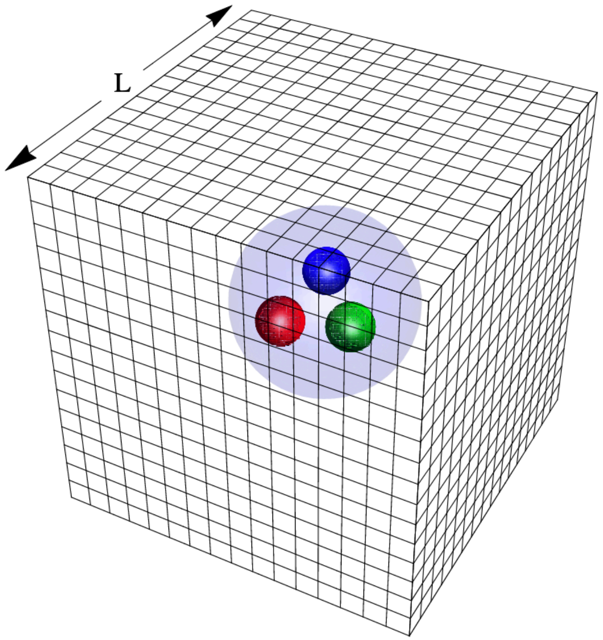
✓ $J_{\nu_1\nu_2}$ and $I_{\nu_1\nu_2\nu_3\nu_4\nu_5}$ are one- and two-loop Feynman integrals.

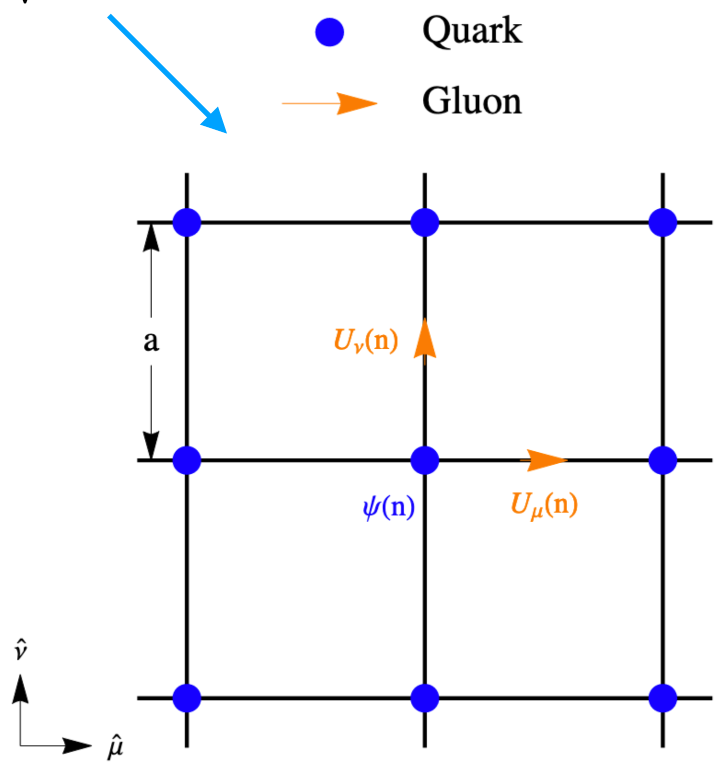
Sigma term on the lattice

- Lattice artifacts: finite-volume & lattice spacing corrections

Continuous Infinite volume *Discrete Finite volume*

$$\sigma_{\pi N} \quad \Longrightarrow \quad \sigma_{\pi N} + \Delta_L \sigma_{\pi N} + b_\pi \frac{a}{\sqrt{t_0}} M_\pi^2,$$





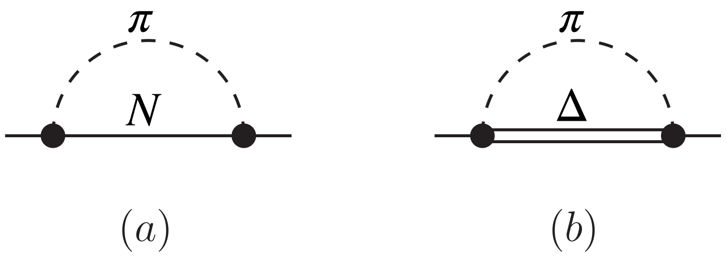
Sigma term on the lattice

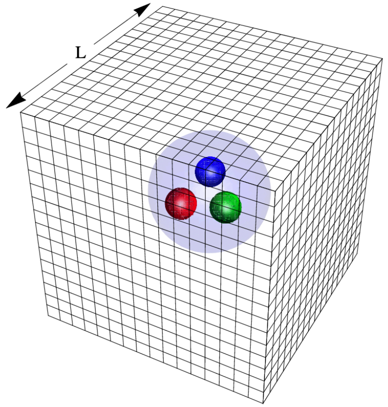
- Our treatment of FV corrections

✓ ChPT at finite volume:

[Liang, Yao, JHEP(2022)]

$$\Delta_L \sigma_{\pi N} = \Delta_L \sigma_{\pi N}^{(N)}(M_\pi; L) + \Delta_L \sigma_{\pi N}^{(\Delta)}(M_\pi; L)$$





✓ Our full expression (N for example):

$$\Delta_L \sigma_{\pi N}^{(N)}(M_\pi; L) = \sum_{n_s} \vartheta(n_s) \int_0^1 \mathrm{d}x_1 \frac{3 g_A^2 m M_\pi^2}{32 F^2 \pi^2} \left[2K_0\left(\sqrt{\mathcal{M}_N^2}\right) + \frac{L^2 M_\pi^2 n_s (x_1 - 1)}{\sqrt{\mathcal{M}_N^2}} K_1\left(\sqrt{\mathcal{M}_N^2}\right) \right]$$

Ensemble	H102	N101	H105	C101	S400	N451	D450	D452	N203	S201	N200	D200	E250	N302	J303	E300
BChPT w/o Δ	−4.72	−0.94	−6.17	−1.49	−8.22	−1.70	−0.68	−1.11	−2.86	−16.3	−3.90	−1.73	−0.52	−9.07	−3.80	−1.06
BChPT w Δ	−9.17	−1.62	−11.7	−2.52	−16.5	−3.03	−1.11	−1.75	−5.37	−33.3	−7.25	−2.89	−0.76	−18.3	−6.94	−1.69

✓ Mainz 23 uses the expression at infinite L and treat b_L free:

$$\Delta_L \sigma_{\pi N}^{(N)}(M_\pi; L) \xrightarrow[L\to\infty]{} b_L \left(\frac{M_\pi^3}{M_\pi L} - \frac{M_\pi^3}{2} \right) \exp(-M_\pi L) \ , \quad b_L = 9 g_A^2 / (8\pi F^2) \ .$$

Sigma term on the lattice

- **Direct approach: Mainz 23** [Agadjanov, et al., PRL(2023)]
  - Effective scalar form factor (ESFF)

$$G_S^{\mathrm{eff}}(t, t_s) \equiv \mathrm{Re}\frac{C_3(t, t_s; \mathbf{0})}{C_2(t_s; \mathbf{0})}$$

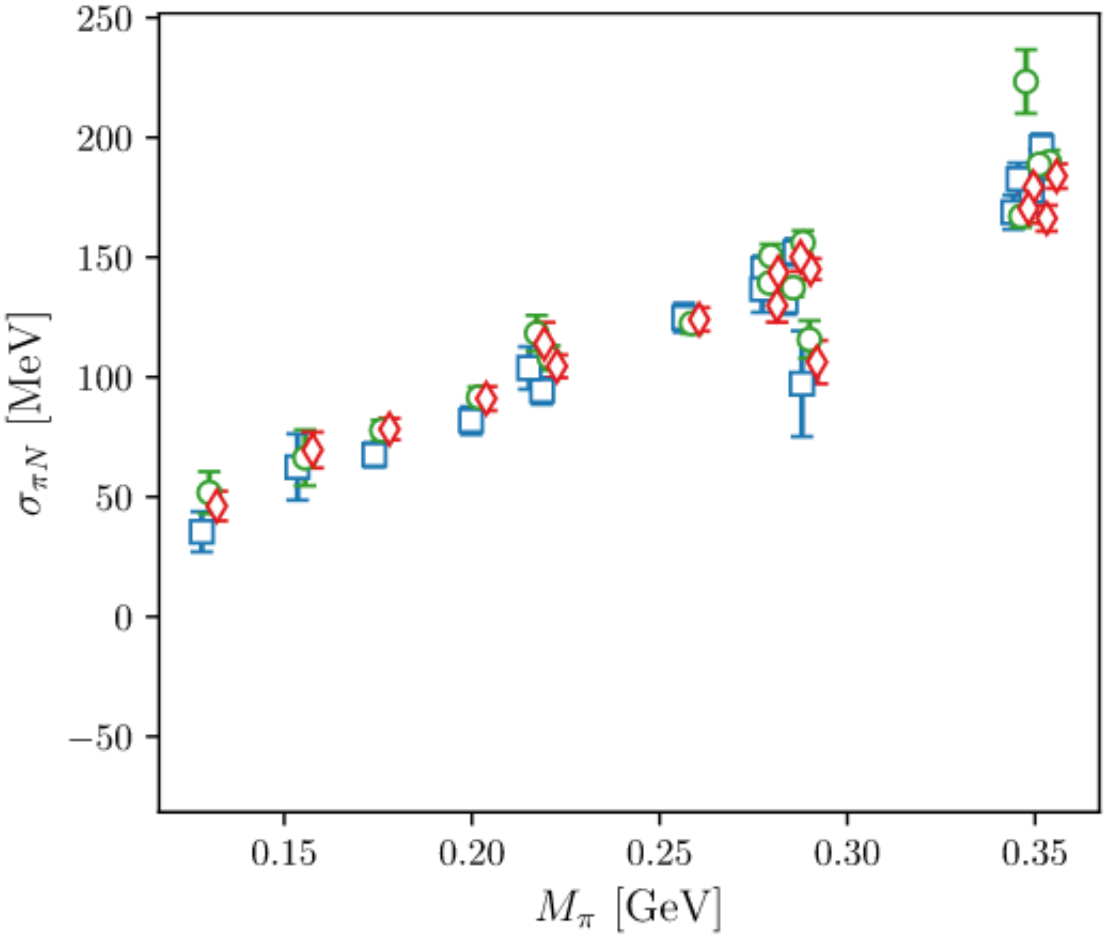
  - Summation method: summed correlator (SC)

$$S(t_s) = a \sum_{t=a}^{t_s - a} G_S^{\mathrm{eff}}(t, t_s) \xrightarrow{t_s \gg \Delta^{-1}} b_1 + (t_s - a) G_S$$

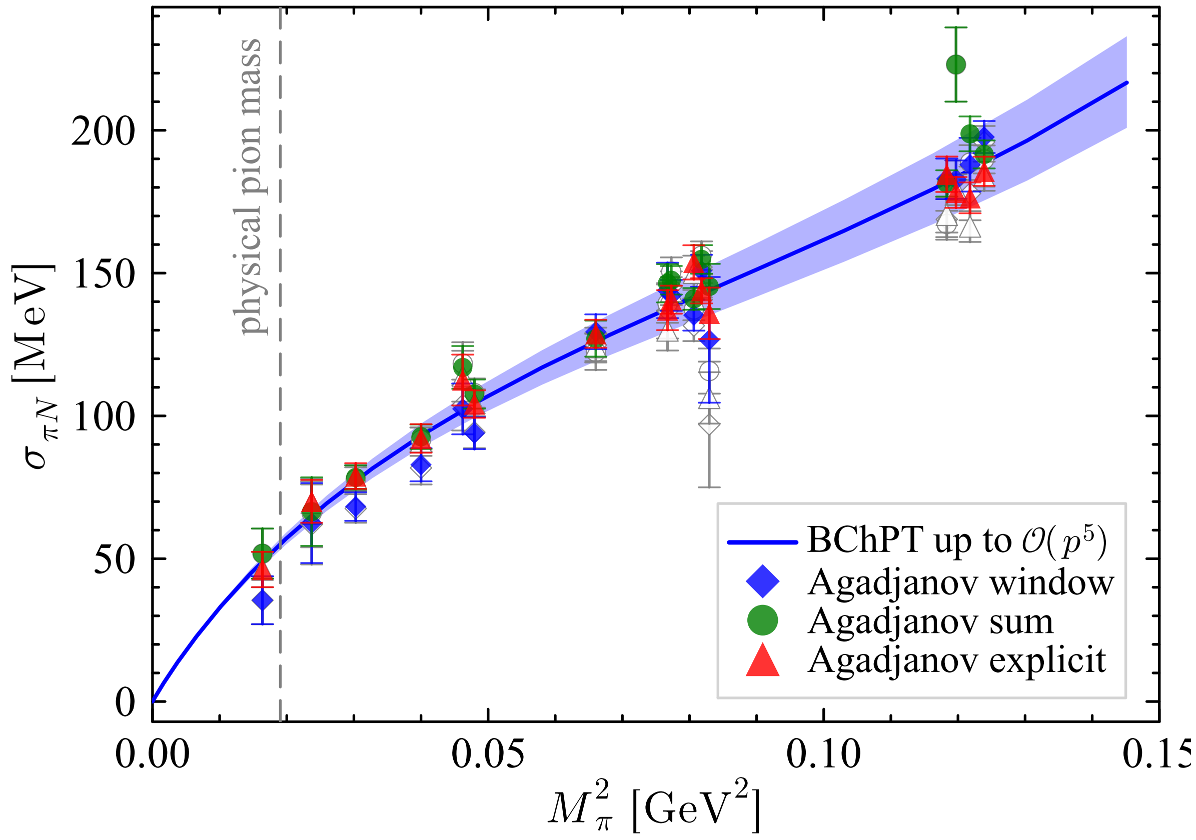
Δ is the energy gap between the g.s. and 1st e.s

- **Excited-state analysis**

✓ "window": window average of the SC
✓ "sum": explicit two-state fit to the SC
✓ "Explicit": explicit two-state fit to the ESFF



Chiral extrapolation



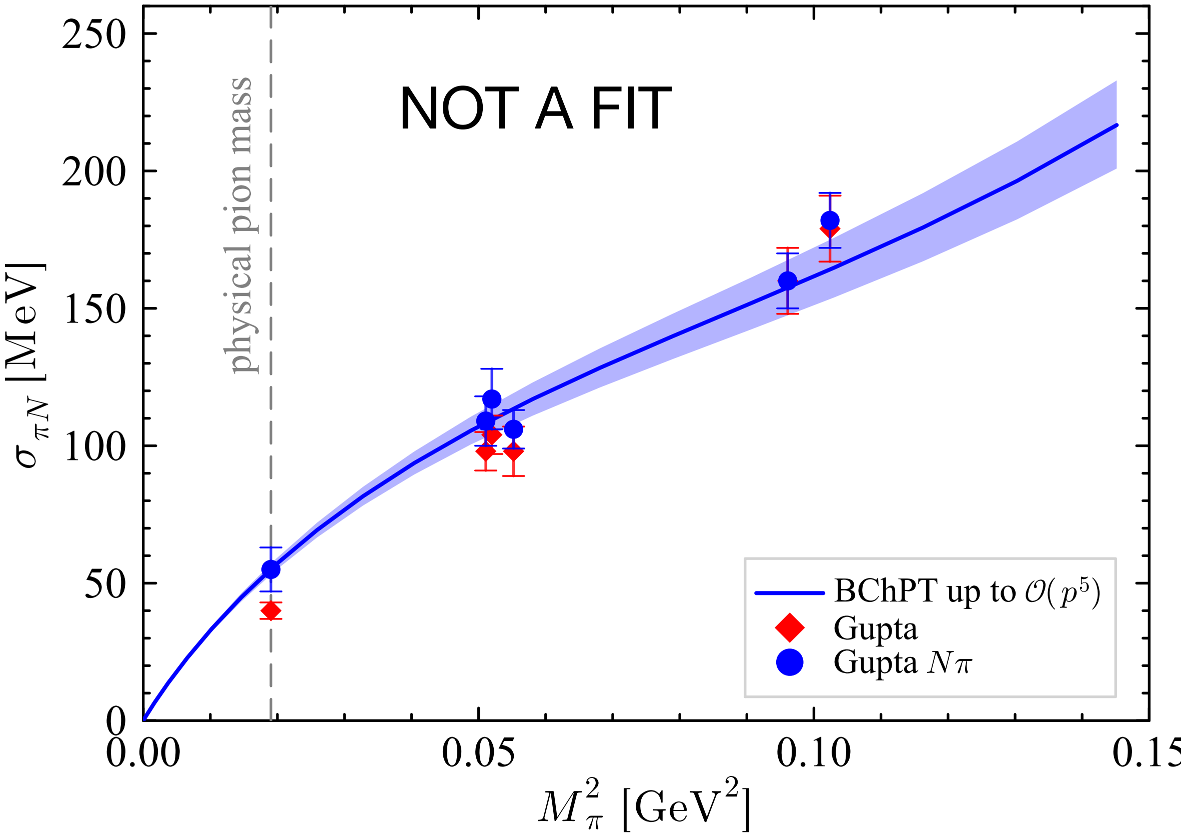
Fit procedure

$$\chi^2 = \chi^2_{m_N} + \omega_i \cdot \{\chi^2_{\text{win}}, \chi^2_{\text{sum}}, \chi^2_{\text{exp}}\}$$

- Constrained by physical m_N
- Three ESC approaches with weighting $\omega_i = \{1/2, 0, 1/2\}$
- Subtracting Finite volume corrections and lattice spacing effects

LECs	Values	Correlation matrix $\tilde{m}$	Correlation matrix $\tilde{c}_1$	Correlation matrix $\tilde{e}_m$
$\tilde{m}$ [MeV]	863.7 ± 2.2	1.000	0.948	−0.640
$\tilde{c}_1$ [GeV^{-1}]	-1.07 ± 0.02		1.000	−0.703
$\tilde{e}_m$ [GeV^{-3}]	-5.64 ± 0.22			1.000
χ^2/d.o.f.	$16.49/(16+1-4) \simeq 1.27$			

Chiral extrapolation

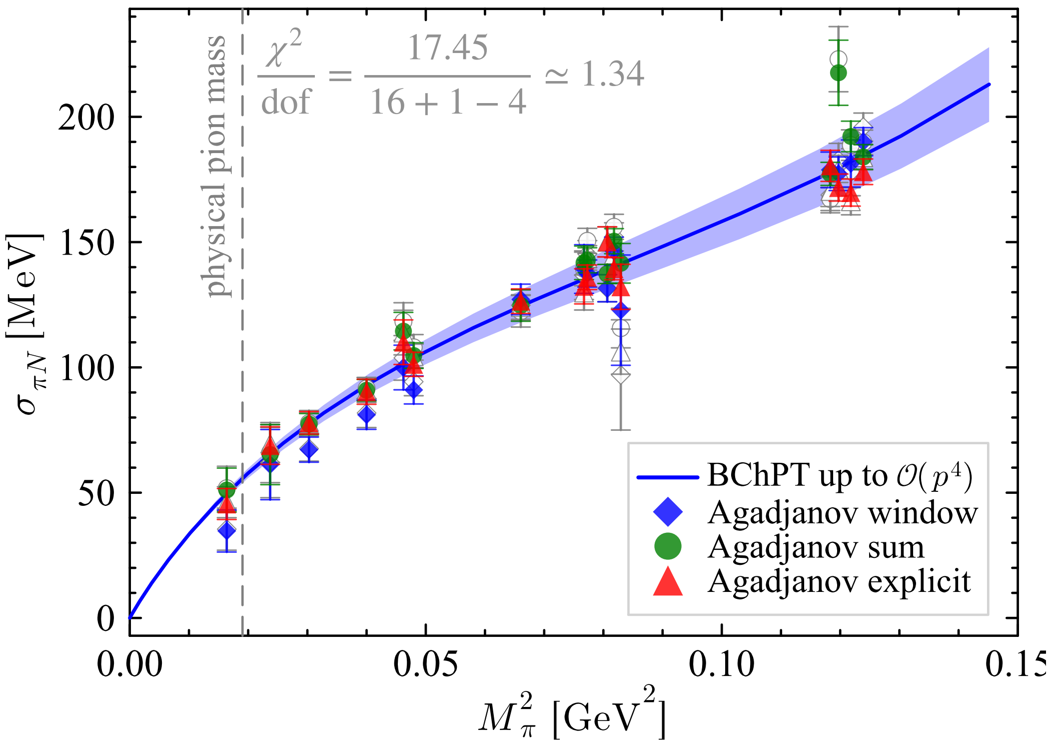


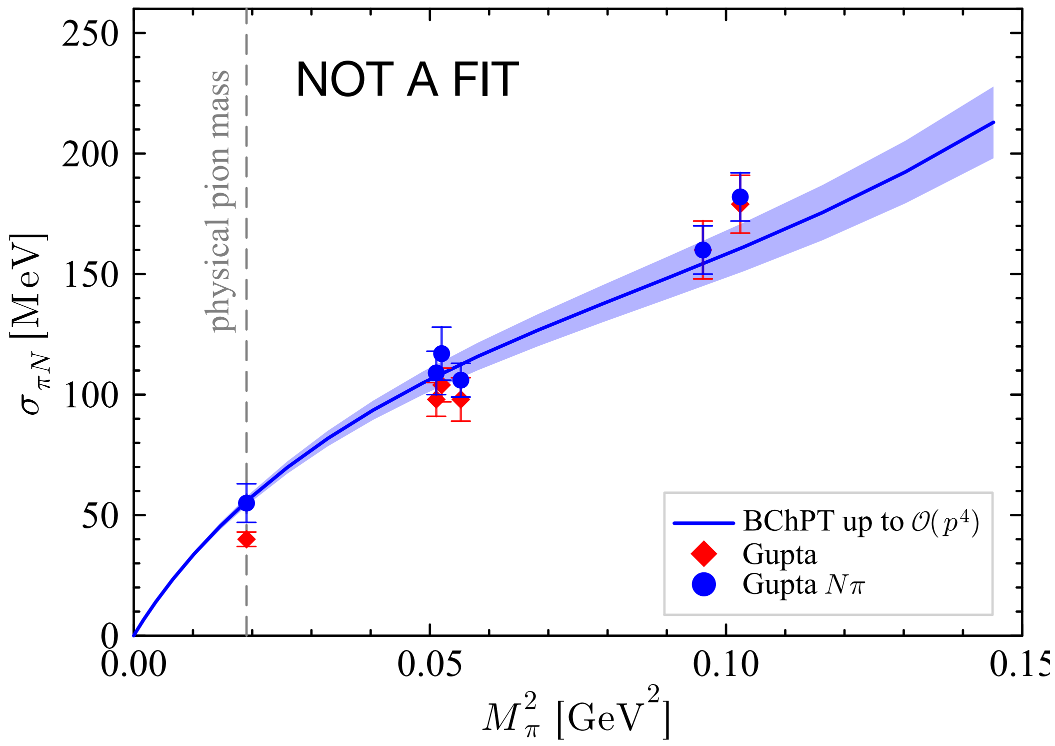
- Sigma term (M_{π^+}): $\sigma_{\pi N} = 56.1(2.6)$ MeV
- Sigma term (M_{π^0}): $\bar{\sigma}_{\pi N} = 53.3(2.4)$ MeV

In remarkable consistency with the dataset in which the ESC is properly accounted for.

Chiral extrapolation

- Comparative fit: up to $\mathcal{O}(p^4)$



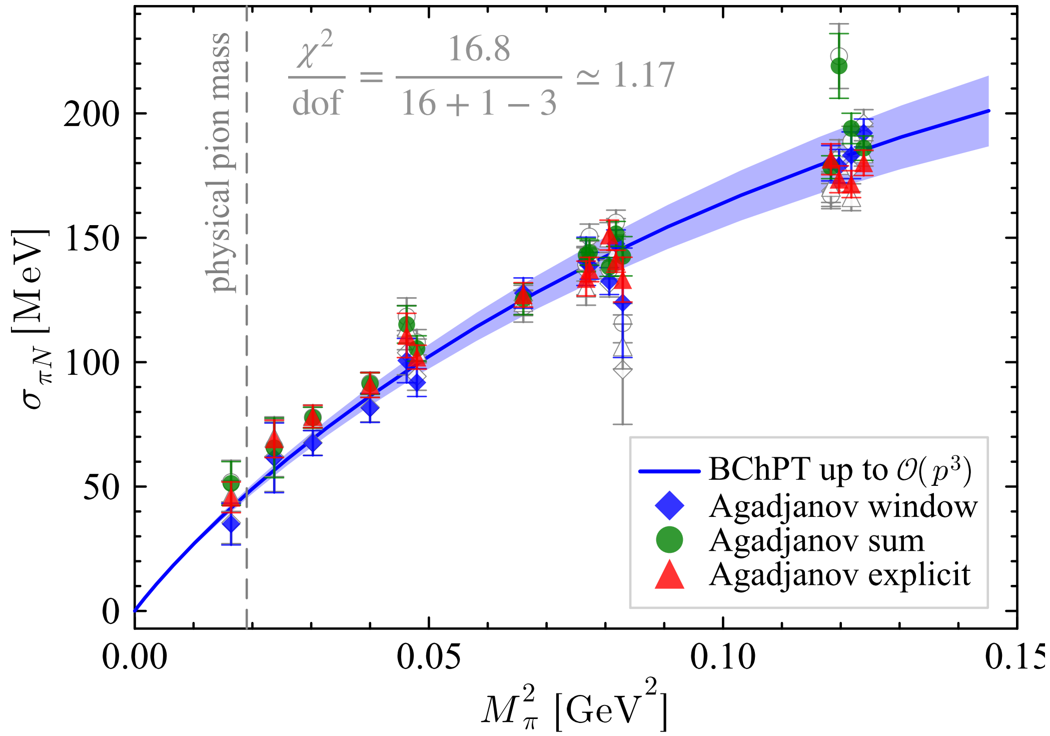


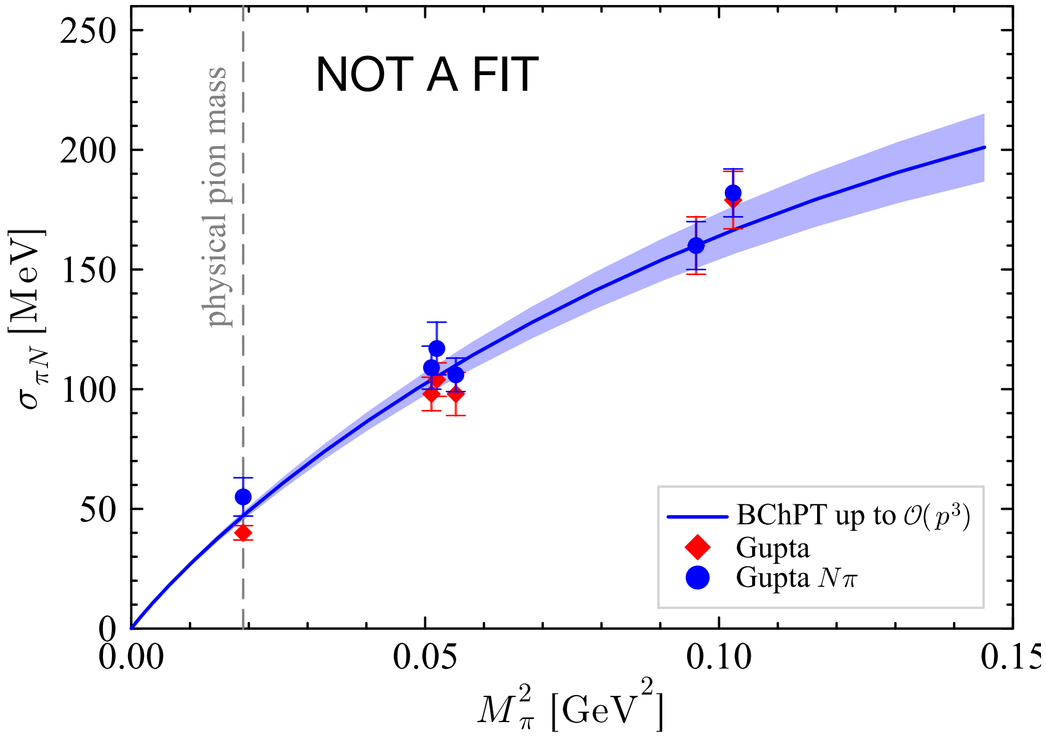
- Parameters: $\tilde{m} = 872.7(2.0)$ MeV, $\tilde{c}_1 = -1.09(2)$ GeV^{-1}, $\tilde{e}_m = -1.39(21)$ GeV^{-3}
- Sigma term (M_{π^+}): $\sigma_{\pi N} = 56.9(2.0)$ MeV
- Sigma term (M_{π^0}): $\bar{\sigma}_{\pi N} = 54.1(1.9)$ MeV

The e_m term mimics part of the two-loop contribution through parameter adjustment

Chiral extrapolation

- Comparative fit: up to $\mathcal{O}(p^3)$





  - Parameters: $\tilde{m} = 886.1(2.1)$ MeV, $\tilde{c}_1 = -0.84(3)$ GeV^{-1}
  - Sigma term (M_{π^+}): $\sigma_{\pi N} = 48.7(2.1)$ MeV
  - Sigma term (M_{π^0}): $\bar{\sigma}_{\pi N} = 46.0(1.9)$ MeV

The $\mathcal{O}(p^3)$ fit indeed yields small nucleon sigma term!

Two-loop extraction of $\sigma_{\pi N}$

- Precise determination of $\sigma_{\pi N}$ in the physical world ($M_\pi = M_\pi^+$)

$$\sigma_{\pi N} = 56.1 \pm (2.0)_{\rm stat} \pm (1.5)_{\rm sys_1} \pm (0.6)_{\rm sys_2} {\rm MeV} = 56.1(2.6)\ {\rm MeV}$$

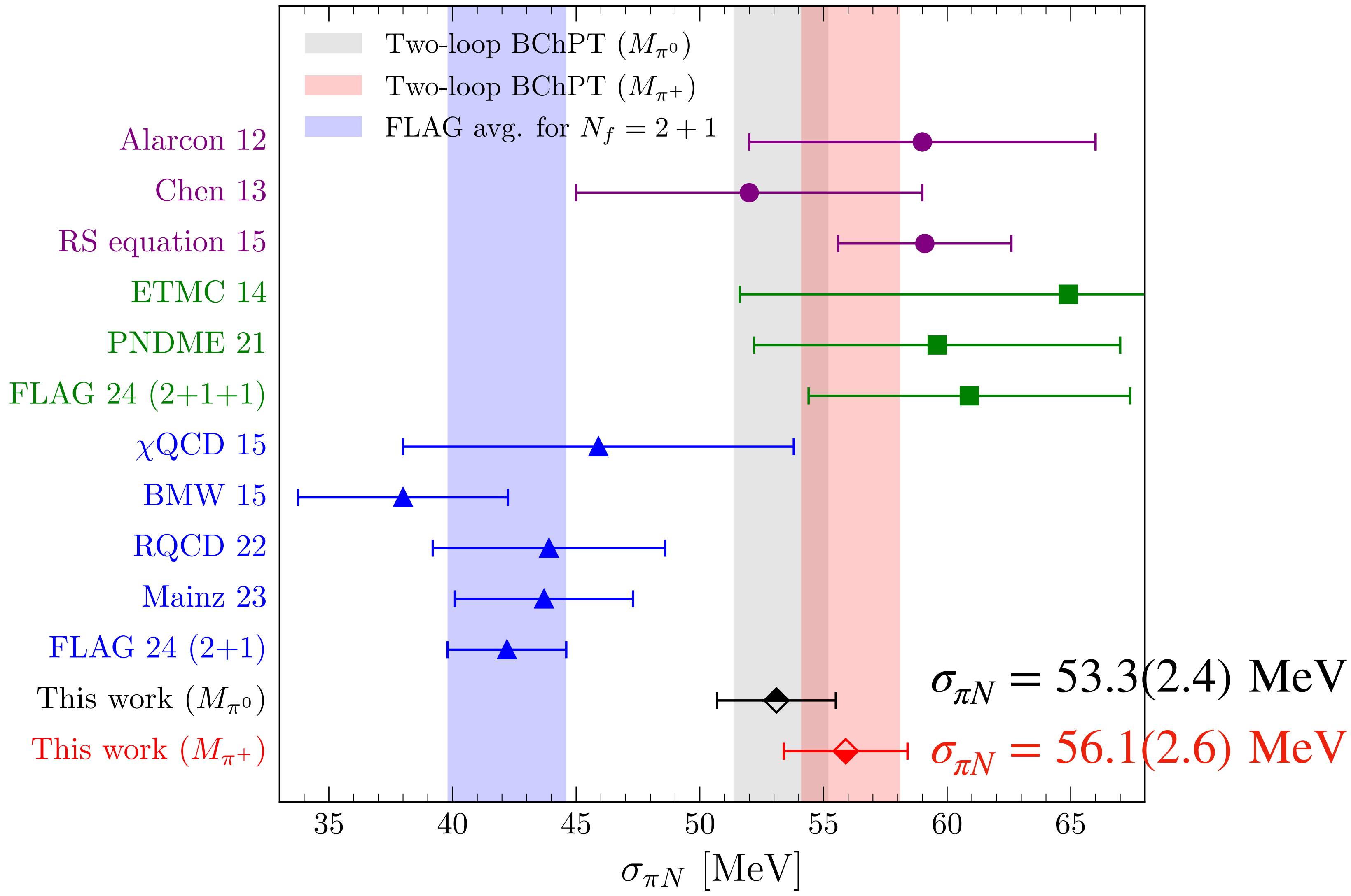
- Error budget

(i) stat: propagated from the 1σ errors of the fitted parameters $(\tilde{m}, \tilde{c}_1, \tilde{e}_m)$;

(ii) sys_1: arising from the errors in the one-loop renormalized LECs (c_2, c_3) involved in the $\mathcal{O}(p^4)$ chiral loop contribution;

(iii) sys_2: due to truncation of the chiral expansion at $\mathcal{O}(p^5)$.

Comparison



Resolving the tension between lattice QCD and phenomenology!

Summary and outlook

Summary and outlook

- The two-loop representation of the sigma term is established in EOMS BChPT for the first time.
  - Method 1: from the nucleon mass via the Feynman-Hellmann theorem.
  - Method 2: a direct calculation of the forward isoscalar-scalar nucleon matrix element.
  - Chiral extrapolation: $N_f = 2 + 1$ lattice QCD data at unphysical quark masses.
  - Our result of the sigma term:

$$\sigma_{\pi N} = 56.1(2.6)\ \mathrm{MeV}$$

- The long-standing tension between lattice QCD and dispersive determinations is naturally resolved.
  - Owing to the incorporation of intermediate ππ rescattering effects that begin to contribute at two-loop order.
  - The general importance of ππ rescattering effects in observables coupled to isoscalar scalar currents.

Thanks!