γW-exchange contributions in neutron β decay

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Outline

1. neutron β decay and radiative corrections

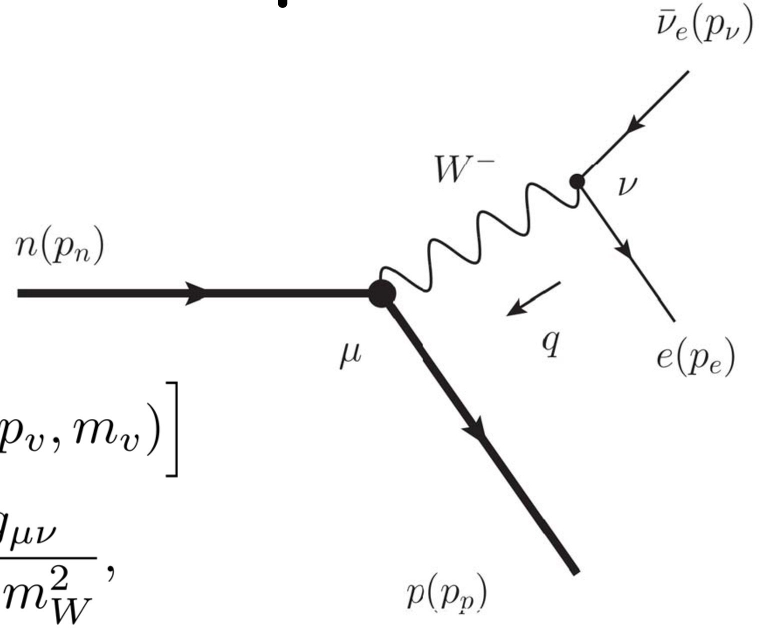
2. γW-exchange in forward limit

3. γW-exchange beyond forward limit

4. a short summary

neutron β decay at tree level

At tree level, taking neutron and proton as an effective particles



$$\mathcal{M}^W = -\frac{ig^2 V_{ud}}{8}\left[\bar{u}(p_e, m_e)\gamma^\nu(1-\gamma_5)u(p_v, m_v)\right]$$
$$\left[\bar{u}(p_p, m_p)\Gamma^\mu_{Wnp}(q)u(p_n, m_n)\right]\frac{-ig_{\mu\nu}}{q^2 - m_W^2},$$

one can extract V_{ud} from the lift time of neutron

$$|V_{ud}|^2 \propto \frac{1}{\tau_n(1+3\lambda^2)}\frac{1}{1+\mathrm{RC}}$$

$$\Gamma^\mu_{Wnp}(l) = \left(f_1(l^2)\gamma^\mu + i\frac{f_2(l^2)}{2m_N}\sigma^{\mu\rho}l_\rho + \frac{f_3(l^2)}{2m_N}l^\mu\right) + \left(f_4(l^2)\gamma^\mu + i\frac{f_5(l^2)}{2m_N}\sigma^{\mu\rho}l_\rho + \frac{f_6(l^2)}{2m_N}l^\mu\right)\gamma_5$$

radiative corrections in neutron βdecay

RCs in life time of neutron

$$\mathrm{RC} = \frac{\alpha}{2\pi}\bar{g}(E_m) + \Delta_R^V$$

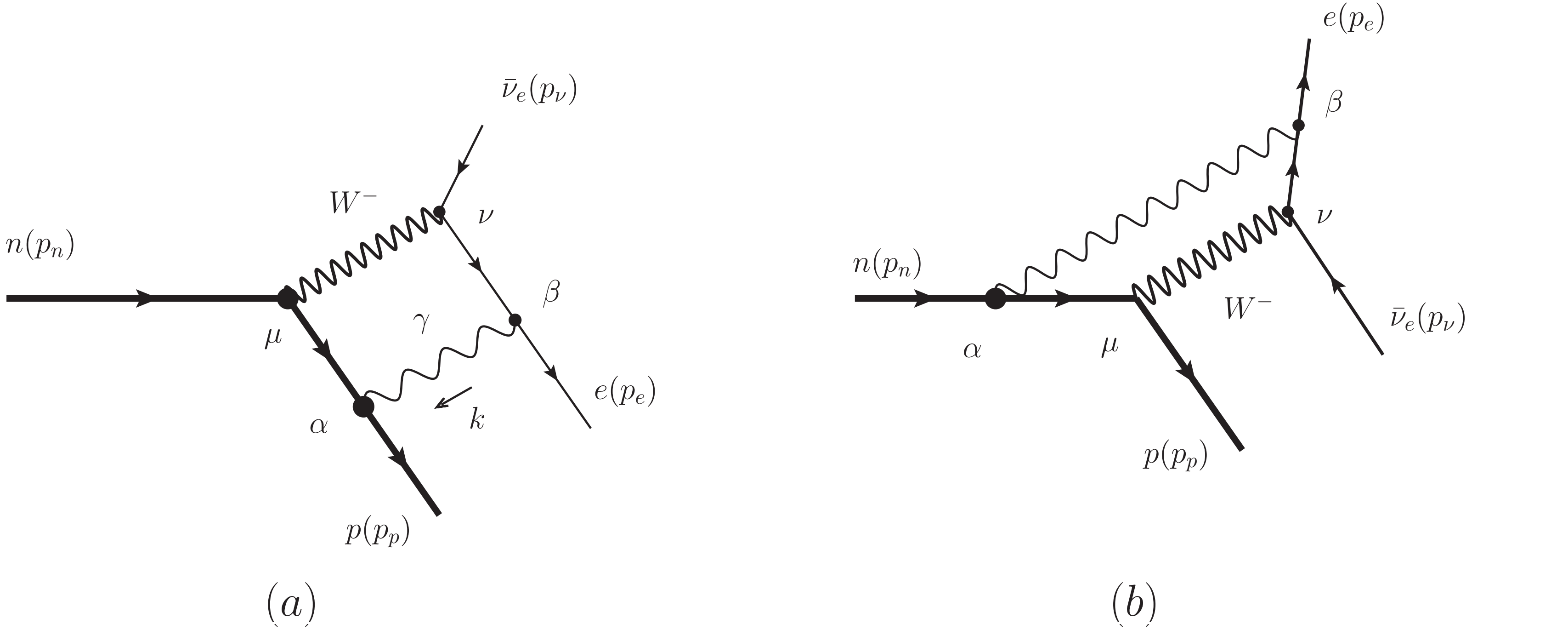
$$\Delta_R^V = \frac{\alpha}{2\pi}\left[3\ln\frac{m_Z}{m_\rho} + \ln\frac{m_Z}{m_W} + \tilde{a}_g\right] + \delta_{\mathrm{HD}}^{\mathrm{QED}} + 2\,\Box_{\gamma W}^V$$

$$\Box_{\gamma W}^V = \frac{\alpha}{2\pi}\left[\frac{1}{2}\ln\frac{m_W}{m_A} + C_{\mathrm{Born}} + \frac{1}{2}\mathrm{Ag}\right]$$

Among all the EW RCs, γW-exchange contribution plays special role.

A. Sirlin, PR164 (1967) 1767, RMP50 (1978) 573 , W. J. Marciano and A. Sirlin, PRL 96 (2006) 032002

γW-exchange contributions



These contributions
(1) change the angle dependence, not only ratios.
(2) non-zero imaginary part.
(3) deependent on hadronic structure

$$\Gamma^\mu_{\gamma pp}(l) = ie\left[F_1^p(l^2)\gamma^\mu + i\frac{F_2^p(l^2)}{2m_p}\sigma^{\mu\nu}l_\nu\right], \quad \Gamma^\mu_{\gamma nn}(l) = ie\left[F_1^n(l^2)\gamma^\mu + i\frac{F_2^n(l^2)}{2m_n}\sigma^{\mu\nu}l_\nu\right]$$

γW-exchange in literatures

Theoretical calculations in literatures

(1) current algebra

(2) model form factors (FFs) as input

(3) low energy effective theory (HBχPT)

(4) dispersion relations (DRs)

(5) Lattice QCD

used approximations or limits:

(1) forward limit

(2) current conserve

(3) only correction on lifetime

C_{Born}:γW with elastic intermediate state

We use hadronic model as example (which goes back to low energy effective interactions, also obeys some special DRs)

When only consider the elastic intermediate state, the following quantities are used

$$\Gamma^{\mu}_{\gamma pp}(l) = ie\Big[F_1^p(l^2)\gamma^\mu + i\frac{F_2^p(l^2)}{2m_p}\sigma^{\mu\nu}l_\nu\Big],$$

$$\Gamma^{\mu}_{\gamma nn}(l) = ie\Big[F_1^n(l^2)\gamma^\mu + i\frac{F_2^n(l^2)}{2m_n}\sigma^{\mu\nu}l_\nu\Big],$$

$$\Gamma^{\mu}_{Wnp}(l) = \Big(f_1(l^2)\gamma^\mu + i\frac{f_2(l^2)}{2m_N}\sigma^{\mu\rho}l_\rho + \frac{f_3(l^2)}{2m_N}l^\mu\Big) + \Big(g_1(l^2)\gamma^\mu + i\frac{g_2(l^2)}{2m_N}\sigma^{\mu\rho}l_\rho + \frac{g_3(l^2)}{2m_N}l^\mu\Big)\gamma_5,$$

C_{Born} in literatures

1st approximation: neglect some interactions

$$f_1 = g_V\,, f_2 = g_M\,, f_3 = g_S = 0,$$
$$f_4 = g_A, f_5 = g_T = 0, f_6 = g_P.$$

2nd approximation: forward limit (FWL) before loop

$$\bar{u}(p_e, m_e)\Gamma_L^{\omega\nu}(p_e, p_\nu, k)u(p_\nu, m_\nu) \approx \bar{u}(0,0)\Gamma_L^{\omega\nu}(0,0,k)u(0,0),$$
$$\bar{u}(p_p, m_p)\Gamma_H^{\rho\mu}(p_p, p_n, k)u(p_n, m_n) \approx \bar{u}(p, m_N)\Gamma_H^{\rho\mu}(p,p,k)u(p, m_N)$$

which means

$$\mathcal{M}^{(a)} \approx -i\int \frac{1}{(2\pi)^4}\bar{u}(0,0)\Gamma_L^{\omega\nu}(0,0,k)u(0,0)\bar{u}(p,m_N)\Gamma_H^{\rho\mu}(p,p,k)u(p,m_N)$$
$$\times \frac{-ig_{\mu\nu}}{k^2 - m_W^2}\frac{-ig_{\rho\omega}}{k^2}\frac{1}{k^2 + 2p\cdot k}\frac{1}{k^2 - m_e^2},$$

C_{Born} in literatures

3rd inner contributions and outer contributions, leptonic part

$$
\begin{aligned}
& \bar{u}(p_e, m_e)(-ie\gamma^\mu) S_F(p_e + k, m_e)(-i\gamma^\nu)(g_e^V - g_e^A\gamma_5) u(p_v, m_v) \\
= \quad & \frac{e}{(p_e+k)^2 - m_e^2} \bar{u}(p_e, m_e)\left[...... + i\epsilon^{\mu\nu\lambda\alpha} k_\lambda \gamma_\alpha \right](g_e^V - g_e^A\gamma_5) u(p_\nu, m_\nu) \\
= \quad & \frac{e}{(p_e+k)^2 - m_e^2} \bar{u}(p_e, m_e)[i\epsilon^{\mu\nu\lambda\alpha} k_\lambda \gamma_\alpha (g_e^V - g_e^A\gamma_5)] u(p_v, m_v) + \text{outer} \\
\equiv \quad & \frac{e}{(p_e+k)^2 - m_e^2} i\epsilon^{\lambda\beta\nu\rho} k_\rho L_\lambda + \text{outer}
\end{aligned}
$$

$$
\gamma^\mu\gamma^\lambda\gamma^\nu = g^{\mu\lambda}\gamma^\nu - g^{\mu\nu}\gamma^\lambda + g^{\lambda\nu}\gamma^\mu - i\epsilon^{\mu\lambda\nu\alpha}\gamma_\alpha\gamma^5
$$

C_{Born} in literatures

4th FCC approximation for hadronic parts:
after applying the Dirac equation etc., only keep $\varepsilon_{\mu\nu\rho\sigma}$

$$
\begin{aligned}
&\bar{u}(p,m_N)\left[\gamma^\mu(\not{p}-\not{k}+m_N)\gamma^\nu\right]u(p,m_N)\\
&=\bar{u}(p,m_N)\left[i\epsilon^{\mu\nu\rho\sigma}k_\rho\gamma_\sigma\gamma^5+......\right]u(p,m_N)\\
&\approx\ \bar{u}(p,m_N)\left[i\epsilon^{\mu\nu\rho\sigma}k_\rho\gamma_\sigma\gamma^5\right]u(p,m_N),
\end{aligned}
$$

And also some other similar relations.

beyond FW limit and at amplitude level

our aim: γW-exchange contributions beyond FW at amplitude level -- 16 helicity amplitudes

$$\mathcal{M} \equiv \sum_{i=1}^{16} c_i O_i, \qquad c_i^{\gamma W} = ?$$

γW-exchange at amplitude level

In the practical calculation, when choosing the covariant form for O_i, it is very difficult to calculate the corresponding c_i, so we choose Pauli spinor form for O_i as

$$O_1 \equiv [\xi_p^\dagger \xi_n][\xi_e^\dagger \boldsymbol{\sigma} \cdot \boldsymbol{n}_e \eta_\nu]$$

$$O_2 \equiv [\xi_p^\dagger \xi_n][\xi_e^\dagger \boldsymbol{\sigma} \cdot \boldsymbol{n}_\nu \eta_\nu]$$

$$O_3 \equiv [\xi_p^\dagger \xi_n][\xi_e^\dagger \eta_\nu]$$

$$O_4 \equiv [\xi_p^\dagger \xi_n][\xi_e^\dagger i\boldsymbol{\sigma} \cdot (\boldsymbol{n}_e \times \boldsymbol{n}_\nu) \eta_\nu]$$

$$O_5 \equiv [\xi_p^\dagger \boldsymbol{\sigma} \cdot \boldsymbol{n}_e \xi_n][\xi_e^\dagger \eta_\nu]$$

$$O_6 \equiv [\xi_p^\dagger \boldsymbol{\sigma} \cdot \boldsymbol{n}_\nu \xi_n][\xi_e^\dagger \eta_\nu]$$

$$O_7 \equiv [\xi_p^\dagger i\boldsymbol{\sigma} \cdot (\boldsymbol{n}_e \times \boldsymbol{n}_\nu) \xi_n][\xi_e^\dagger \boldsymbol{\sigma} \cdot \boldsymbol{n}_e \eta_\nu]$$

$$O_8 \equiv [\xi_p^\dagger i\boldsymbol{\sigma} \cdot (\boldsymbol{n}_e \times \boldsymbol{n}_\nu) \xi_n][\xi_e^\dagger \boldsymbol{\sigma} \cdot \boldsymbol{n}_\nu \eta_\nu]$$

$$O_9 \equiv [i\xi_p^\dagger \boldsymbol{\sigma} \xi_n] \times [\xi_e^\dagger \boldsymbol{\sigma} \eta_\nu] \cdot \boldsymbol{n}_e,$$

$$O_{10} \equiv [i\xi_p^\dagger \boldsymbol{\sigma} \xi_n] \times [\xi_e^\dagger \boldsymbol{\sigma} \eta_\nu] \cdot \boldsymbol{n}_\nu,$$

$$O_{11} \equiv [\xi_p^\dagger \boldsymbol{\sigma}_i \xi_n][\xi_e^\dagger \boldsymbol{\sigma}_i \eta_\nu]$$

$$O_{12} \equiv [\xi_p^\dagger \boldsymbol{\sigma} \cdot \boldsymbol{n}_e \xi_n][\xi_e^\dagger \boldsymbol{\sigma} \cdot \boldsymbol{n}_e \eta_\nu]$$

$$O_{13} \equiv [\xi_p^\dagger \boldsymbol{\sigma} \cdot \boldsymbol{n}_e \xi_n][\xi_e^\dagger \boldsymbol{\sigma} \cdot \boldsymbol{n}_\nu \eta_\nu]$$

$$O_{14} \equiv [\xi_p^\dagger \boldsymbol{\sigma} \cdot \boldsymbol{n}_\nu \xi_n][\xi_e^\dagger \boldsymbol{\sigma} \cdot \boldsymbol{n}_e \eta_\nu]$$

$$O_{15} \equiv [\xi_p^\dagger \boldsymbol{\sigma} \cdot \boldsymbol{n}_\nu \xi_n][\xi_e^\dagger \boldsymbol{\sigma} \cdot \boldsymbol{n}_\nu \eta_\nu]$$

$$O_{16} \equiv [\xi_p^\dagger i\boldsymbol{\sigma} \cdot (\boldsymbol{n}_e \times \boldsymbol{n}_\nu) \xi_n][\xi_e^\dagger \eta_\nu]$$

γW-exchange at amplitude level

To compare the results in literature, we separate the amplitude as

$$\mathcal{M} \equiv \mathcal{M}^{\mathrm{Fermi}} + \mathcal{M}^{\mathrm{GT}}$$

$$\mathcal{M}^{\mathrm{Fermi}} \equiv \sum_{i=1}^{4} c_i^{\mathrm{Fermi}} O_i, \mathcal{M}^{\mathrm{GT}} \equiv \sum_{i=5}^{16} c_i^{\mathrm{GT}} O_i$$

coefficients c_i at tree level beyond FWL

Beyond FW limit, but taking E_e, E_ν, m_e, E_0 as samll quantities comparing with m_n. Finally one has

$$c_{1,\mathrm{LO}}^{\mathrm{OBE}} = g_V \eta \qquad c_{9,\mathrm{LO}}^{\mathrm{OBE}} = g_A \beta$$
$$c_{2,\mathrm{LO}}^{\mathrm{OBE}} = g_V \qquad c_{10,\mathrm{LO}}^{\mathrm{OBE}} = -g_A$$
$$c_{3,\mathrm{LO}}^{\mathrm{OBE}} = -g_V[1+\eta\beta] \qquad c_{11,\mathrm{LO}}^{\mathrm{OBE}} = -g_A(1-\eta\beta)$$
$$c_{4,\mathrm{LO}}^{\mathrm{OBE}} = -g_V \eta \qquad c_{12,\mathrm{LO}}^{\mathrm{OBE}} = 0$$
$$c_{5,\mathrm{LO}}^{\mathrm{OBE}} = g_A \eta \qquad c_{13,\mathrm{LO}}^{\mathrm{OBE}} = -g_A \eta$$
$$c_{6,\mathrm{LO}}^{\mathrm{OBE}} = g_A \qquad c_{14,\mathrm{LO}}^{\mathrm{OBE}} = -g_A \eta$$
$$c_{7,\mathrm{LO}}^{\mathrm{OBE}} = 0 \qquad c_{15,\mathrm{LO}}^{\mathrm{OBE}} = 0$$
$$c_{8,\mathrm{LO}}^{\mathrm{OBE}} = 0 \qquad c_{16,\mathrm{LO}}^{\mathrm{OBE}} = g_A \eta$$

When recoil contributions are neglected, one has

$$\eta = 0, \beta = 0$$

$$N \equiv \frac{2m_n}{m_W^2}\sqrt{E_\nu(E_e+m_e)}, \qquad \eta \equiv \sqrt{\frac{E_e - m_e}{E_e + m_e}}, \qquad \beta = \boldsymbol{n}_e \cdot \boldsymbol{n}_\nu$$

γW-exchange at amplitude level

To calculate $c_i^{\gamma W}$, the following parameters are needed:

$$m_n, m_p, m_e, m_\nu, \alpha_e, F_{1,2}^{p,n}, f_i; \quad g, V_{ud}$$

assumed form factors

For EM FFs, we take a very general form as

$$F_1^p(l^2) = F_{10}^p \sum_{j=1}^{N_1} a_{1j} G(l^2, \Lambda_{1j}^2, n_{1j}), \qquad F_2^p(l^2) = F_{20}^p \sum_{j=1}^{N_2} a_{2j} G(l^2, \Lambda_{2j}^2, n_{2j}),$$

$$F_1^n(l^2) = F_{10}^n \sum_{j=1}^{N_3} a_{3j} G(l^2, \Lambda_{3j}^2, n_{3j}), \qquad F_2^n(l^2) = F_{20}^n \sum_{j=1}^{N_4} a_{4j} G(l^2, \Lambda_{4j}^2, n_{4j}),$$

with $$G(l^2, \Lambda^2, n) \equiv \frac{(-1)^n}{(l^2 - \Lambda^2)^n}$$

For weak FFs, we take

$$f_i(l^2) = f_{i0} \sum_{j=1}^{\bar{N}_i} b_{ij} G(l^2, \bar{\Lambda}_{ij}^2, \bar{n}_{ij}), \qquad f_{1,2}(Q^2) = F_{1,2}^p(Q^2) - F_{1,2}^n(Q^2),$$

γW-exchange in the FW limit

For tree diagram, one has

$$c_{2,\mathrm{LO}}^{W} = -c_{3,\mathrm{LO}}^{W} = g_V\,, \quad c_{6,\mathrm{LO}}^{W} = ic_{10,\mathrm{LO}}^{W} = -c_{11,\mathrm{LO}}^{W} = g_A$$

For γW, one has

$$\begin{aligned} c_{2,\mathrm{LO}}^{\gamma W} =& g_A[d_{2,1}F_{10}^{p} + d_{2,2}F_{20}^{p} + d_{2,3}F_{10}^{n} + d_{2,4}F_{20}^{n}], \\ c_{6,\mathrm{LO}}^{\gamma W} =& g_V[d_{6,1}^{V}F_{10}^{p} + d_{6,2}^{V}F_{20}^{p} + d_{6,3}^{V}F_{10}^{n} + d_{6,4}^{V}F_{20}^{n}] \\ &+ g_M[d_{6,1}^{M}F_{10}^{p} + d_{6,2}^{M}F_{20}^{p} + d_{6,3}^{M}F_{10}^{n} + d_{6,4}^{M}F_{20}^{n}] \end{aligned}$$

$g_V \equiv f_{10}, g_M \equiv f_{20}, g_A \equiv f_{40}$

$d_{3,j} = -d_{2,j}\,, \quad d_{10,j} = -id_{6,j}\,, \quad d_{11,j} = -d_{6,j}$

γW-exchange in the FW limit

$$d_{2,i} = \sum_{j,k} \hat{\mathcal{F}}_{ij,4k} \Big[\frac{X_1(\Lambda_{ij}, \Lambda_{4k})}{2m_N^2(\Lambda_{ij}^2 - \Lambda_{4k}^2)} - \frac{\Lambda_{ij} Z_1(\Lambda_{4k}) - \Lambda_{4k} Z_1(\Lambda_{ij})}{m_N^4 \Lambda_{ij}\Lambda_{4k}(\Lambda_{ij}^2 - \Lambda_{4k}^2)} \Big]$$

$$d_{6,1}^V = \sum_{j,k} \hat{\mathcal{F}}_{1j,1k} \Big[\frac{X_2(\Lambda_{1j}, \Lambda_{1k})}{6m_N^2(\Lambda_{1j}^2 - \Lambda_{1k}^2)} - \frac{2[\Lambda_{1j} Z_2(\Lambda_{1k}) - \Lambda_{1k} Z_2(\Lambda_{1j})]}{3m_N^4 \Lambda_{1j}\Lambda_{1k}(\Lambda_{1j}^2 - \Lambda_{1k}^2)} \Big],$$

$$d_{6,2}^V = \sum_{j,k} \hat{\mathcal{F}}_{2j,1k} \Big[\frac{X_3(\Lambda_{2j}, \Lambda_{1k})}{6m_N^2(\Lambda_{2j}^2 - \Lambda_{1k}^2)} - \frac{2[\Lambda_{2j} Z_3(\Lambda_{1k}) - \Lambda_{1k} Z_3(\Lambda_{2j})]}{3m_N^4 \Lambda_{2j}\Lambda_{1k}(\Lambda_{2j}^2 - \Lambda_{1k}^2)} \Big],$$

$$d_{6,3}^V = [d_{6,1}^V \text{ replacing the index } 1j, 1k \text{ by } 3j, 3k],$$

$$d_{6,4}^V = [d_{6,2}^V \text{ replacing the index } 2j, 2k \text{ by } 4j, 4k]$$

$$d_{6,1}^M = [d_{6,2}^V \text{ replacing the indexes } 2j \text{ and } 1k \text{ to } 1j \text{ and } 2k\text{, respectively}],$$

$$d_{6,2}^M = \sum_{j,k} \hat{\mathcal{F}}_{2j,2k} \Big[\frac{X_4(\Lambda_{2j}, \Lambda_{2k})}{6m_N^2(\Lambda_{2j}^2 - \Lambda_{2k}^2)} - \frac{\Lambda_{2j} Z_4(\Lambda_{2k}) - \Lambda_{2k} Z_4(\Lambda_{2j})}{m_N^4 (\Lambda_{2j}^2 - \Lambda_{2k}^2)} \Big],$$

$$d_{6,3}^M = [d_{6,1}^M \text{ replacing the index } 1j \text{ by } 3j],$$

$$d_{6,4}^M = [d_{6,2}^M \text{ replacing the index } 2j \text{ by } 4j]$$

$$\hat{\mathcal{F}}_{ij,mk} \equiv a_{ij} b_{mk} \frac{(-1)^{n_{ij}+\bar{n}_{mk}}}{(n_{ij}-1)!\,(\bar{n}_{mk}-1)!} \frac{d^{n_{ij}-1}}{d(\Lambda_{ij}^2)^{n_{ij}-1}} \frac{d^{\bar{n}_{mk}-1}}{d(\bar{\Lambda}_{mk}^2)^{\bar{n}_{mk}-1}}$$

C_{Born}

then one has

$$\frac{\alpha_e}{2\pi}\delta_i \equiv \frac{\mathcal{C}_{i,\text{LO}}^{\gamma W}}{\mathcal{C}_{i,\text{LO}}^{W}}$$

$$C_{\text{Born}}^{\text{F}} = \delta_2 = \delta_3$$

$$C_{\text{Born}}^{\text{GT}} = \delta_6 = \delta_{10} = \delta_{11}$$

For comparison, we separate the corrections as

$$C_{\text{Born}}^{\text{F}} \equiv C_{\text{Born}}^{\text{F},g_A} + C_{\text{Born}}^{\text{F},g_M},$$

$$C_{\text{Born}}^{\text{GT}} \equiv C_{\text{Born}}^{\text{GT},g_V} + C_{\text{Born}}^{\text{GT},g_M}$$

FFs used in the practical numerical results

For f_4, we take the simple form as

$$\bar{N}_4 = 1, \bar{n}_{41} = 1, b_{41} = \Lambda_W^4, \bar{\Lambda}_{i1} = \Lambda_W = 1.09 \pm 0.05 \text{ GeV}$$

For EM FFs, we take three forms as examples

(I)

$$\begin{aligned}
&N_1 = 2, n_{1j} = 2, a_{11} = 0.152, \Lambda_{11} = 0.726, a_{12} = 1.270, \Lambda_{12} = 1.294, \\
&N_2 = 2, n_{2j} = 3, a_{21} = 0.359, \Lambda_{21} = 1.000, a_{22} = 0.656, \Lambda_{22} = 1.004, \\
&N_3 = 2, n_{3j} = 2, a_{31} = \Lambda_{31}^4, \Lambda_{31} = 1.288, a_{32} = -\Lambda_{32}^4, \Lambda_{32} = 1.378, F_{10}^n = 1, \\
&N_4 = 2, n_{4j} = 3, a_{41} = 0.041, \Lambda_{41} = 0.699, a_{42} = 2.087, \Lambda_{42} = 1.214, \quad \text{(typeI)}
\end{aligned}$$

FFs used in the practical numerical results

(II)

$$N_1 = 1, n_{11} = 2, a_{11} = \Lambda_{11}^4, \Lambda_{11} = 0.960,$$
$$N_2 = 1, n_{21} = 3, a_{21} = \Lambda_{21}^6, \Lambda_{21} = 1.003,$$
$$N_3 = 2, n_{3j} = 1, a_{31} = \Lambda_{31}^2, \Lambda_{31} = 0.847, a_{32} = -\Lambda_{32}^2, \Lambda_{32} = 0.914, F_{10}^n = 1,$$
$$N_4 = 1, n_{41} = 3, a_{41} = \Lambda_{41}^6, \Lambda_{41} = 1.038, \qquad \text{(typeII)}$$

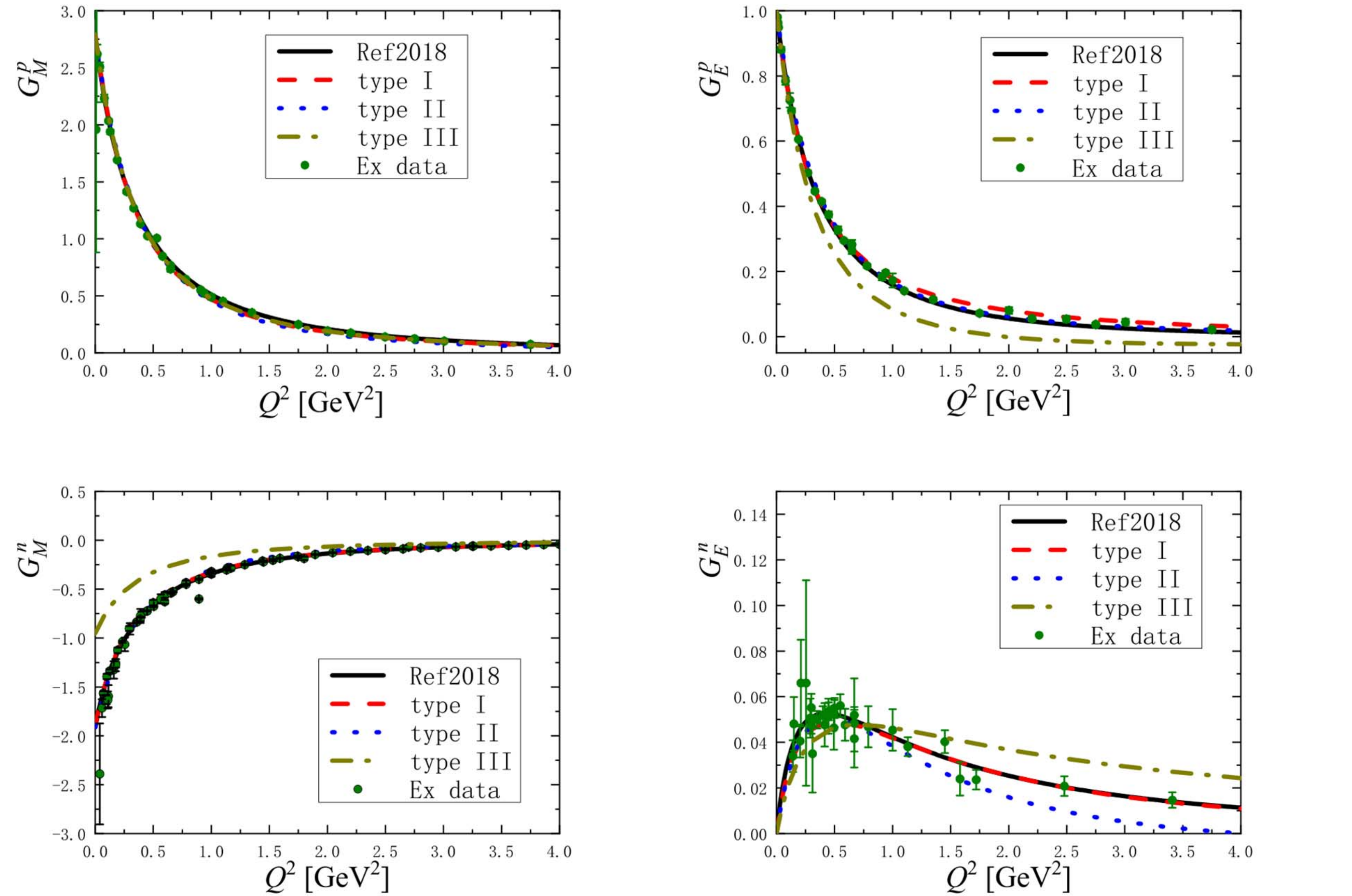
(III)

$$N_i = 1, n_{i1} = 2, a_{i1} = \Lambda_{i1}^4, \Lambda_{i1} = \Lambda_\gamma = 0.84, F_{10}^n = 0 \qquad \text{(typeIII)}$$

other used parameters

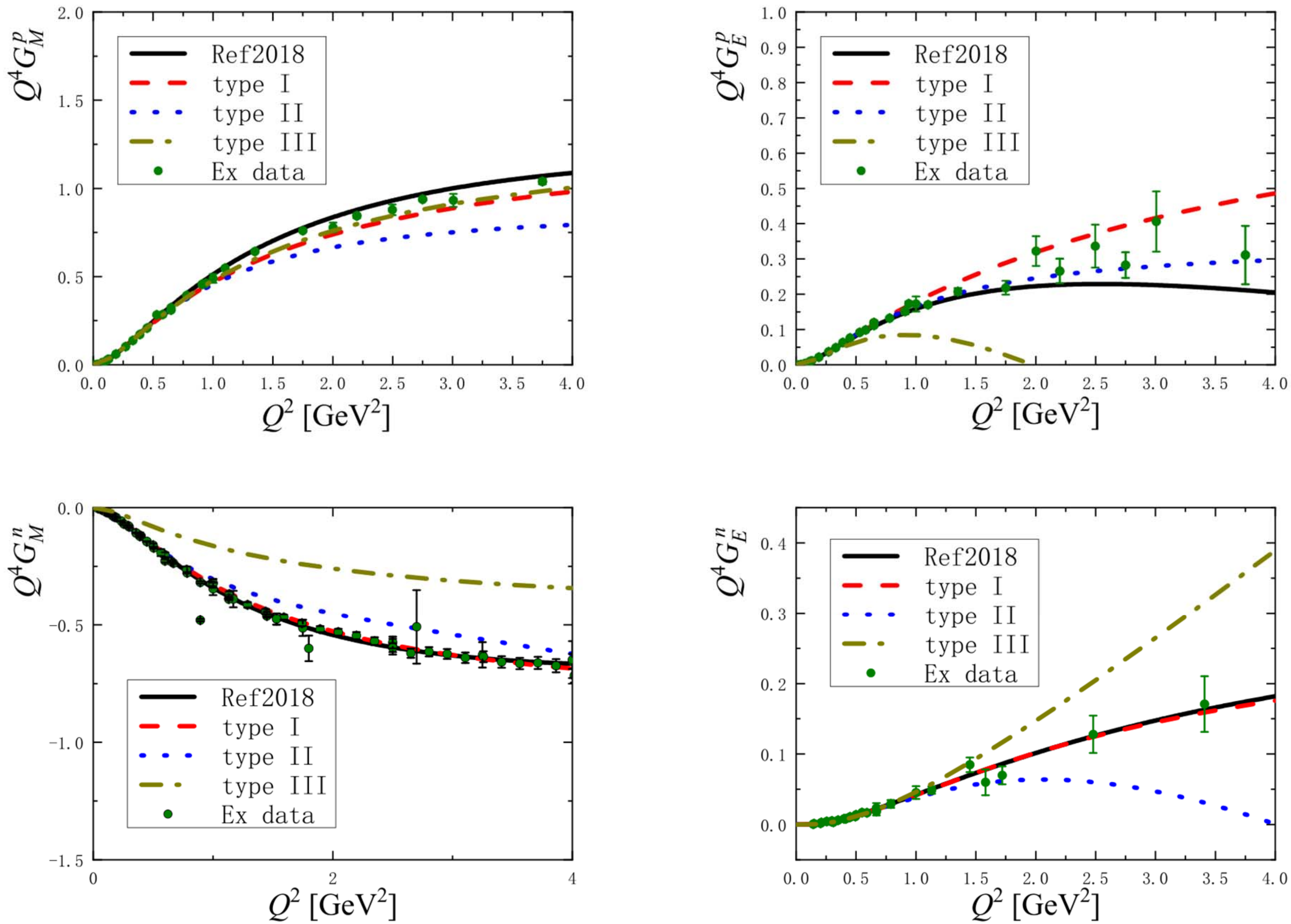
$$m_n = 939.56542 \text{ MeV}, m_p = 938.27209 \text{ MeV}, m_e = 0.51100 \text{ MeV},$$
$$F_{10}^p = 1, F_{20}^p = 1.793, F_{20}^n = -1.913,$$
$$g_V = 1, g_A = -1.26, g_M = F_{20}^p - F_{20}^n = 3.706$$

EM FFs vs. Ex-data

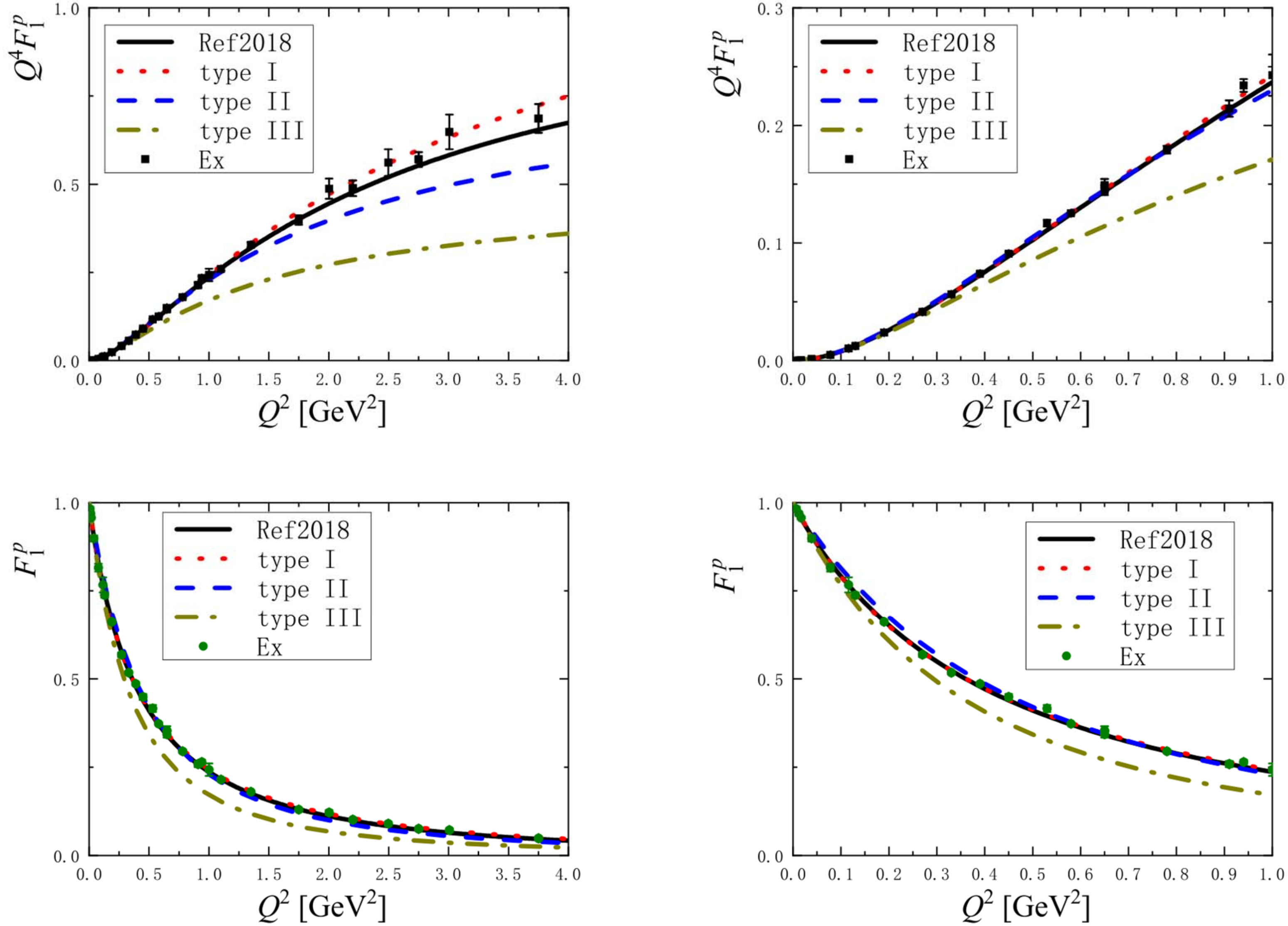


Zhihong Ye, etc. PLB777(2018)8

EM FFs vs. Ex-data



F_1^p vs. Ex-data



Numerical results with different FFs

$$\text{type I}: \begin{cases} C_{\text{Born}}^{\text{F},g_A} = 1.048F_{10}^p + 0.967F_{20}^p - 0.027F_{10}^n + 0.968F_{20}^n = 0.906, \\ C_{\text{Born}}^{\text{GT},g_V} = 0.465F_{10}^p + 0.231F_{20}^p - 0.013F_{10}^n + 0.231F_{20}^n = 0.423, \\ C_{\text{Born}}^{\text{GT},g_M} = [0.226F_{10}^p - 0.004F_{20}^p - 0.0058F_{10}^n - 0.005F_{20}^n]g_M = 0.825 \end{cases}$$

$$\text{type II}: \begin{cases} C_{\text{Born}}^{\text{F},g_A} = 1.062F_{10}^p + 0.968F_{20}^p - 0.028F_{10}^n + 0.985F_{20}^n = 0.887, \\ C_{\text{Born}}^{\text{GT},g_V} = 0.478F_{10}^p + 0.235F_{20}^p - 0.014F_{10}^n + 0.239F_{20}^n = 0.428, \\ C_{\text{Born}}^{\text{GT},g_M} = [0.232F_{10}^p - 0.004F_{20}^p - 0.005F_{10}^n - 0.005F_{20}^n]g_M = 0.844 \end{cases}$$

$$\text{type III}: \begin{cases} C_{\text{Born}}^{\text{F},g_A} = 0.999F_{10}^p + 0.999F_{20}^p + 0.999F_{20}^n = 0.882, \\ C_{\text{Born}}^{\text{GT},g_V} = 0.414F_{10}^p + 0.223F_{20}^p + 0.223F_{20}^n = 0.388, \\ C_{\text{Born}}^{\text{GT},g_M} = [0.223F_{10}^p - 0.007F_{20}^p - 0.007F_{20}^n]g_M = 0.832 \end{cases}$$

comparison with results in literatures

	$C_{\text{Born}}^{\text{F},g_A}$	$C_{\text{Born}}^{\text{GT},g_V}$	$C_{\text{Born}}^{\text{GT},g_M}$
Ref. [Towner1992]	0.881 ± 0.014	not calculated	not calculated
Ref. [Hayen2021]	0.91(5)	0.39(1)	0.78(2)
type I	0.951	0.442	0.835
type II	1.009	0.475	0.876
type III	0.882	0.388	0.832

The results for $C_{\text{Born}}^{\text{F},g_A}$ with type I, II, III are consistent with those given in Refs within the error.

Different from the case $C_{\text{Born}}^{\text{F},g_A}$, our results (type I) for $C_{\text{Born}}^{\text{GT},g_V}$ and $C_{\text{Born}}^{\text{GT},g_M}$ are about 8% and 6% larger than those given by Ref, respectively.

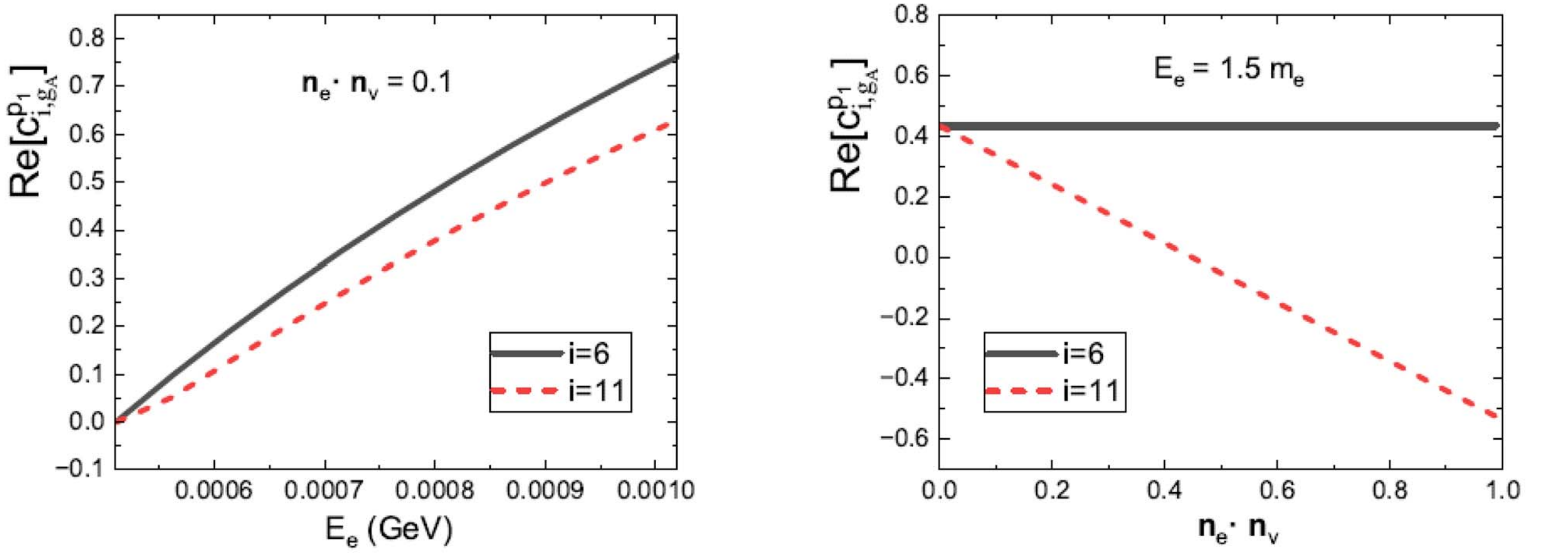
[10]NPAA 540 (1992) 478, [11]PRD103 (2021) 113001

contributions from different parts

contributions/All	$C_{\text{Born}}^{\text{F},g_A}(f_4)$	$C_{\text{Born}}^{\text{GT},g_V}(f_1)$	$C_{\text{Born}}^{\text{GT},g_M}(f_2)$
F_1^p	115%	110%	102%
$F_2^{p,n}$	-13%	-7%	1%
F_1^n	-3%	-3%	-3%

γW-exchange beyond the FW limit

when go beyond FW limit, our numerical calculations show that only the contribution from proton F_1 and g_A, give significant E_e dependent contribution as



γW-exchange beyond the FW limit

corresponding contributions to C_{Born}

	$C_{\mathrm{Born}}^{\mathrm{F},g_A}$	$C_{\mathrm{Born}}^{\mathrm{GT},g_V}$	$C_{\mathrm{Born}}^{\mathrm{GT},g_M}$	$C_{\mathrm{Born}}^{\mathrm{GT},g_A}$
Ref.[Hayen2021]	0.91(5)	0.39(1)	0.78(2)	not calculated
Ref.[Seng2023]	0.91(5)	Total 1.22(1)		not calculated
Ref.[Cao2025] FWL before loop	0.951	0.442	0.835	0
FWL after loop	0.984	0.452	0.841	0
beyond FWL	1.007	0.436	0.807	0.191

a new non-zero contribution appears when beyong FWL

Next step

1. dispersion relations beyond the forward limit
2. other processes such as weak decay of meson

Short Summary

1. The inner γW-exchange contributions with Born intermediate in the forward limit are calculated at the amplitude level.

2. The numerical result for $C_{\rm Born}^{\rm F,g_A}$ is consistent with the previous results, while the results for $C_{\rm Born}^{\rm GT,g_V}$ and $C_{\rm Born}^{\rm GT,g_M}$ are about 8% and 6% larger than the precious results.

3. Beyond FWL, new contribution is found for $C_{\rm Born}^{\rm GT,g_A}$.

Thanks!

any comments, suggestions, and discussions are **Welcome!**

请大家批评指正！

Expressions for some functions

$$X_1(x,y) \equiv x^2 \log\frac{m_N^2}{x^2} - y^2 \log\frac{m_N^2}{y^2} + 6m_N^2 \log\frac{x^2}{y^2},$$

$$X_2(x,y) \equiv x^2 \log\frac{m_N^2}{x^2} - y^2 \log\frac{m_N^2}{y^2} - 6m_N^2 \log\frac{x^2}{y^2},$$

$$X_3(x,y) \equiv x^2 \log\frac{m_N^2}{x^2} - y^2 \log\frac{m_N^2}{y^2},$$

$$X_4(x,y) \equiv 2x^2 \log\frac{m_N^2}{x^2} - 2y^2 \log\frac{m_N^2}{y^2} + m_N^2 \log\frac{x^2}{y^2},$$

$$Y(x) \equiv \log[\frac{x+\sqrt{-4m_N^2+x^2}}{2m_N}],$$

$$Z_1(x) \equiv (-4m_N^2+x^2)^{3/2} Y(x),$$

$$Z_2(x) \equiv (-4m_N^2+x^2)^{1/2}(8m_N^2+x^2) Y(x),$$

$$Z_3(x) \equiv (-4m_N^2+x^2)^{1/2}(2m_N^2+x^2) Y(x),$$

$$Z_4(x) \equiv (-4m_N^2+x^2)^{1/2} x Y(x).$$