Fragmentation functions for production of Bc mesons

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Outline

1. Background

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4. Summary

1. Background

- **Only meson state with two different heavy flavors**
  - Only weak decay is possible => weak interaction
- **Its production can be described by NRQCD factorization**
  - A lot of the dynamics can be calculated perturbatively
  - The production mechanism of Bc is simpler than that of heavy quarkonium
- **It was first observed by CDF collaboration in 1998**

(u,d,s-1963; c-1974; b-1977; t-1995)

- Two Bc(2S) excited states were observed in 2019

PHYSICAL REVIEW LETTERS **122,** 132001 (2019)

Editors' Suggestion | Featured in Physics

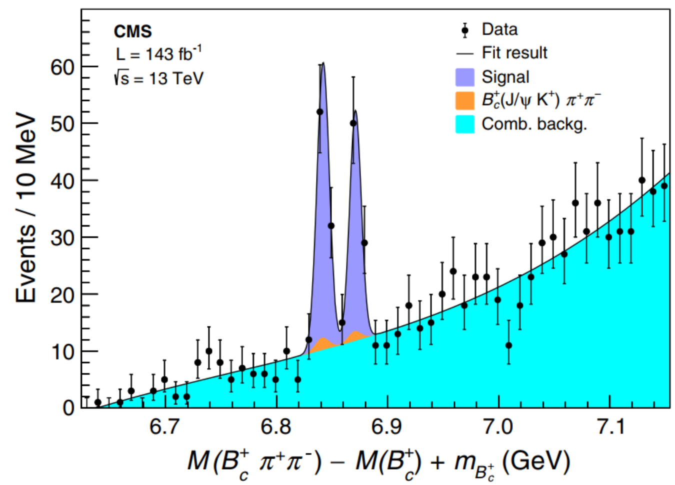
Observation of Two Excited B_c^+ States and Measurement of the $B_c^+(2S)$ Mass in pp Collisions at $\sqrt{s} = 13$ TeV

A. M. Sirunyan *et al.*[*]
(CMS Collaboration)

(Received 1 February 2019; revised manuscript received 18 February 2019; published 2 April 2019)

Signals consistent with the $B_c^+(2S)$ and $B_c^{*+}(2S)$ states are observed in proton-proton collisions at $\sqrt{s} = 13$ TeV, in an event sample corresponding to an integrated luminosity of 143 fb^{-1}, collected by the CMS experiment during the 2015–2018 LHC running periods. These excited $\bar{b}c$ states are observed in the $B_c^+\pi^+\pi^-$ invariant mass spectrum, with the ground state B_c^+ reconstructed through its decay to $J/\psi\pi^+$. The two states are reconstructed as two well-resolved peaks, separated in mass by $29.1 \pm 1.5(\mathrm{stat}) \pm 0.7(\mathrm{syst})$ MeV. The observation of two peaks, rather than one, is established with a significance exceeding five standard deviations. The mass of the $B_c^+(2S)$ meson is measured to be $6871.0 \pm 1.2(\mathrm{stat}) \pm 0.8(\mathrm{syst}) \pm 0.8(B_c^+)$ MeV, where the last term corresponds to the uncertainty in the world-average B_c^+ mass.

DOI: 10.1103/PhysRevLett.122.132001



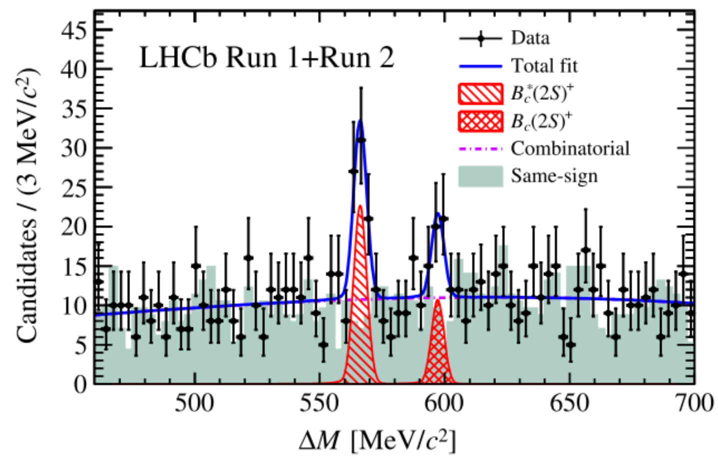
PHYSICAL REVIEW LETTERS **122,** 232001 (2019)

Observation of an Excited B_c^+ State

R. Aaij *et al.*[*]
(LHCb Collaboration)

(Received 29 March 2019; revised manuscript received 23 April 2019; published 11 June 2019)

Using pp collision data corresponding to an integrated luminosity of 8.5 fb^{-1} recorded by the LHCb experiment at center-of-mass energies of $\sqrt{s} = 7$, 8, and 13 TeV, the observation of an excited B_c^+ state in the $B_c^+\pi^+\pi^-$ invariant-mass spectrum is reported. The observed peak has a mass of $6841.2 \pm 0.6(\mathrm{stat}) \pm 0.1(\mathrm{syst}) \pm 0.8(B_c^+)$ MeV$/c^2$, where the last uncertainty is due to the limited knowledge of the B_c^+ mass. It is consistent with expectations of the $B_c^*(2^3S_1)^+$ state reconstructed without the low-energy photon from the $B_c^*(1^3S_1)^+ \to B_c^+\gamma$ decay following $B_c^*(2^3S_1)^+ \to B_c^*(1^3S_1)^+\pi^+\pi^-$. A second state is seen with a global (local) statistical significance of 2.2σ (3.2σ) and a mass of $6872.1 \pm 1.3(\mathrm{stat}) \pm 0.1(\mathrm{syst}) \pm 0.8(B_c^+)$ MeV$/c^2$, and is consistent with the $B_c(2^1S_0)^+$ state. These mass measurements are the most precise to date.



- Two P-wave Bc excited states were observed for the first time

PHYSICAL REVIEW LETTERS **135,** 231902 (2025)

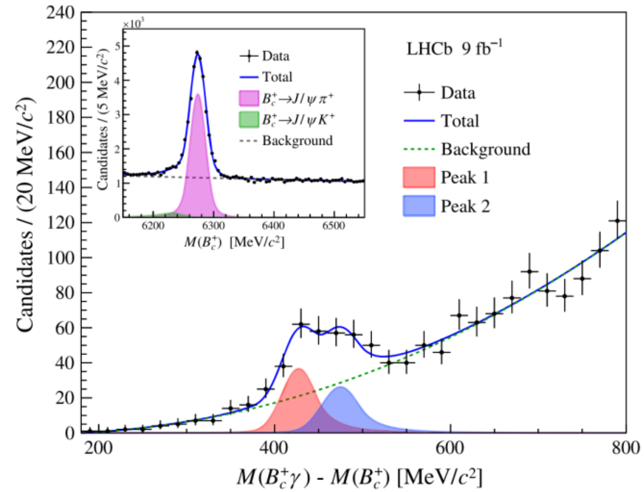
Editors' Suggestion

Observation of Orbitally Excited B_c^+ States

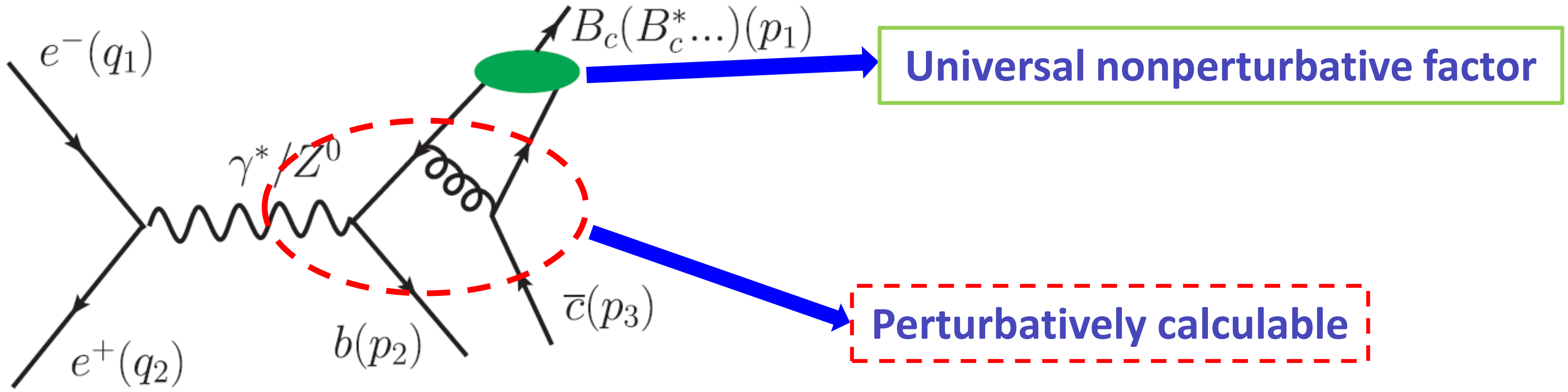
R. Aaij *et al.*[*]
(LHCb Collaboration)

(Received 9 July 2025; accepted 3 October 2025; published 3 December 2025)

The observation of a wide peaking structure in the $B_c^+\gamma$ mass spectrum is reported using proton-proton collision data collected by the LHCb detector at center-of-mass energies of 7, 8, and 13 TeV, corresponding to a total integrated luminosity of 9 fb^{-1}. The statistical significance over the background-only hypothesis exceeds seven standard deviations. The width of the observed structure is larger than the expectation from a single-peak hypothesis, and is well described by an effective minimal model consisting of two narrow peaks located at $6704.8 \pm 5.5 \pm 2.8 \pm 0.3\ \mathrm{MeV}/c^2$ and $6752.4 \pm 9.5 \pm 3.1 \pm 0.3\ \mathrm{MeV}/c^2$. The uncertainty terms are statistical, systematic, and associated to the knowledge of the B_c^+ mass, respectively. The measured peak locations are in line with theoretical predictions for lowest excited *P*-wave B_c^+ states, marking the first observation of orbitally excited beauty-charm mesons and providing important insights into the internal dynamics of hadrons containing two heavy quarks.



- NRQCD factorization



$$d\sigma(e^+ + e^- \to Bc + b + \bar{c})$$
$$= \sum_n d\hat{\sigma}(e^+ + e^- \to c\bar{b}[n] + b + \bar{c}) \langle O^{Bc}(n) \rangle$$

NRQCD factorization

Short-distance coefficients

Long-distance matrix elements

- NRQCD factorization

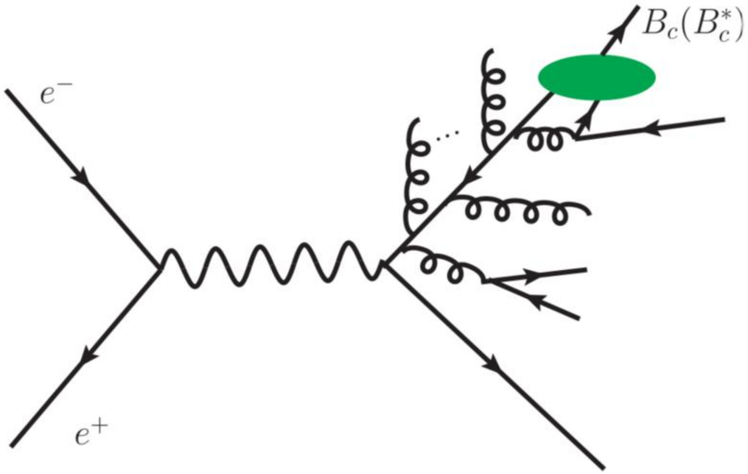
$$d\sigma(e^+ + e^- \to Bc + b + \bar{c})$$
$$= \sum_n d\sigma(e^+ + e^- \to (c\bar{b})[n] + b + \bar{c})\langle O^{\mathrm{Bc}}(n)\rangle$$

Energy scales:
$\sqrt{s}, m_Q$

Log-terms appear in short-distance coefficients:

$$\boxed{\alpha_s^m \sum_{n=0}^{\infty} \alpha_s^n \ln^n(s/m_Q^2)}$$

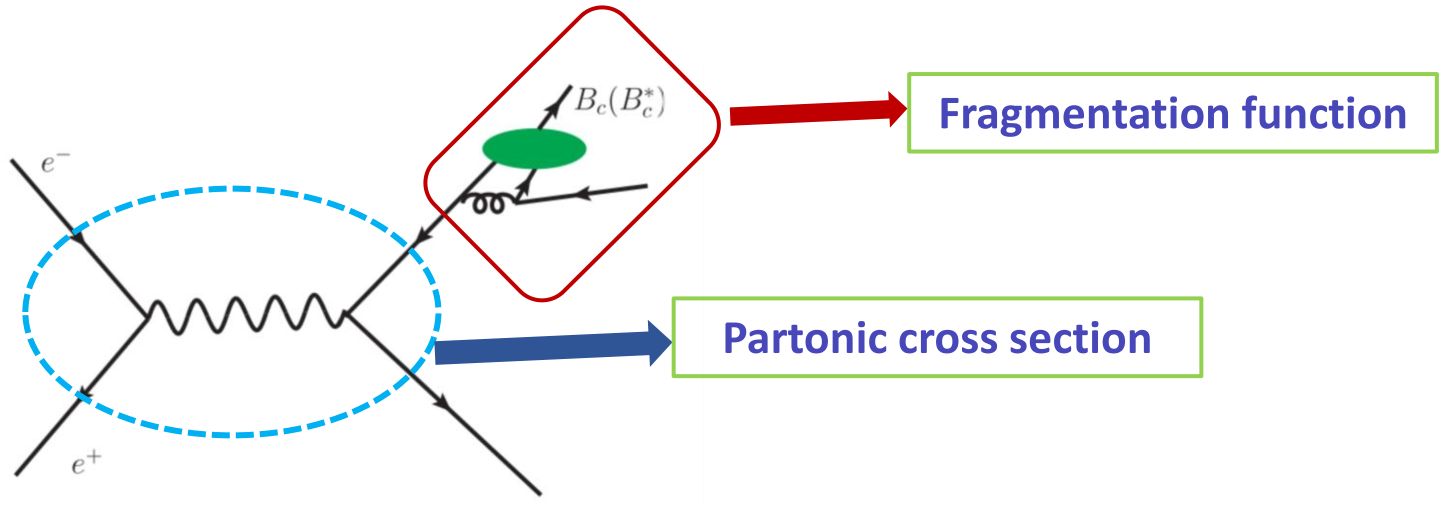
Collinear emission



Spoil or weak the convergence of the series

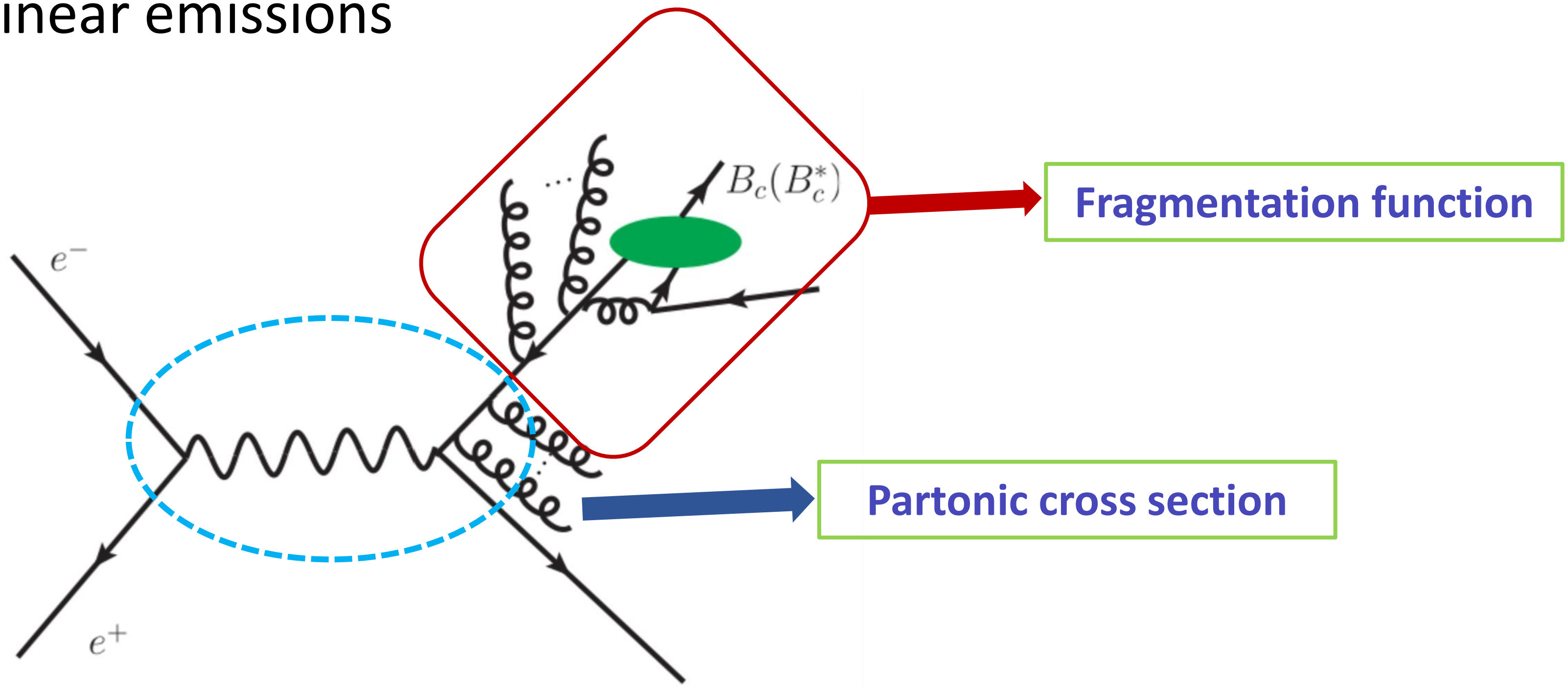
$\ln(p_t^2/m_Q^2)$ appearing in the production at a hadron collider

- Fragmentation-function approach



$$d\sigma(e^+ + e^- \to Bc(p) + b + \bar{c})$$
$$= \sum_i d\hat{\sigma}(e^+ + e^- \to i + X)(p/z, \mu_F) \otimes D_{i\to Bc}(z, \mu_F) + \mathrm{O}(m_Q^2/s)$$

Collinear emissions



Large Logrithms of s/m_Q^2?

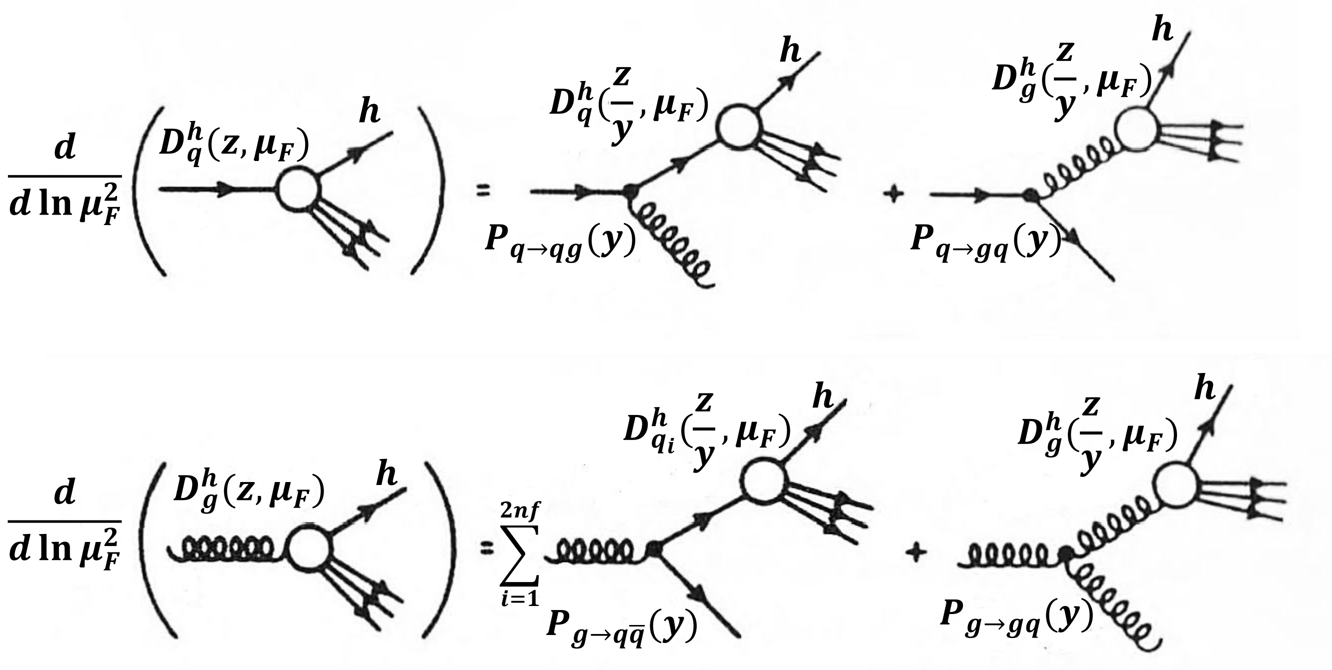
Involving $\ln(s/\mu_F^2)$

$\mu_F = O(\sqrt{s})$

$$d\sigma(e^+ + e^- \to Bc(p) + b + \bar{c})$$
$$= \sum_i \boxed{d\hat{\sigma}(e^+ + e^- \to i + X)(p/z, \mu_F)} \otimes \boxed{D_{i\to Bc}(z, \mu_F)} + O(m_Q^2/s)$$

Involving $\ln(\mu_F^2/m_Q^2)$

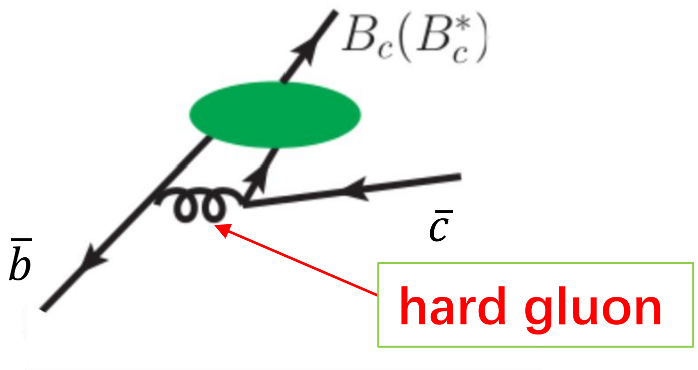
DGLAP evolution



Collinear log-terms can be resummed through the DGLAP evolution.

Boundary condition: $D_i^h(z,\mu_{F0})$ with $\mu_{F0} = O(m_Q)$

NRQCD factorization for FFs:



The fragmentation process contains perturbatively calculable information

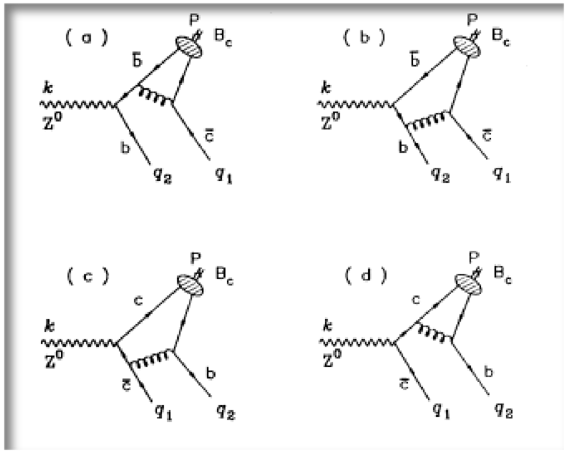
$$D_{i\to Bc}(z,\mu_{F0}) = \sum_n d_{i\to c\bar{b}[n]}(z,\mu_{F0})\langle O^{Bc}(n)\rangle$$

Involving $\ln(\mu_{F0}^2/m_Q^2)$ $\mu_{F0} = O(m_Q)$

The FFs at a higher scale can be obtained by solving the DGLAP equations

➢ LO fragmentation functions for the Bc production

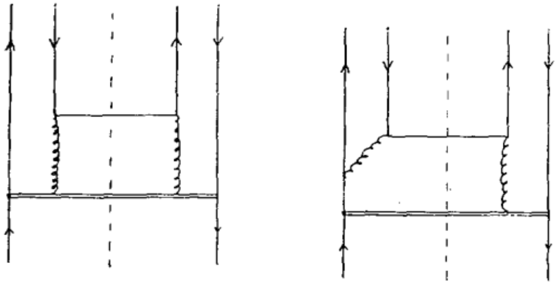
- Extracting from the LO calculation of process $Z^0 \to Bc + b + \bar{c}$



C.-H. Chang, Y.-Q. Chen, PRD 46, 3845, (1992);

E. Braaten et al, PRD 48, R5049, (1993).

- Calculating from the definition:



J.-P. Ma, PLB 332, 398, (1994).

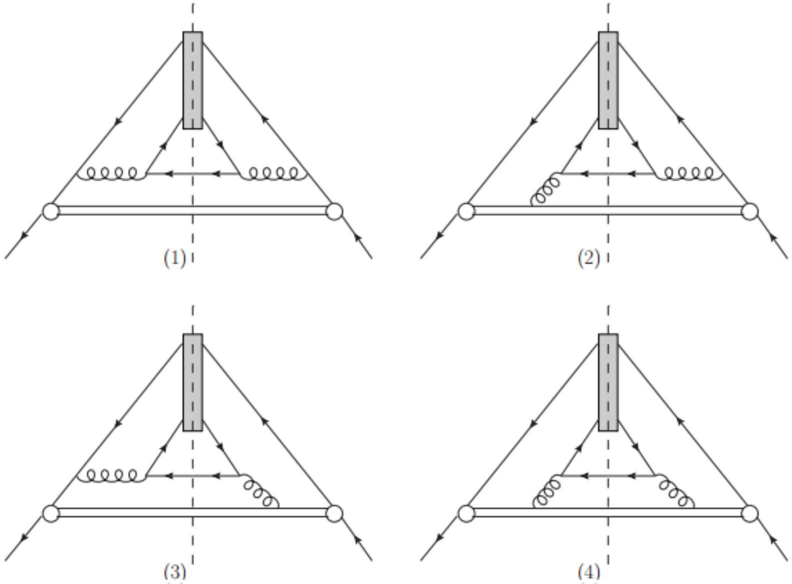
2. FFs at NLO

Based on the definition of FFs by **Collins and Soper.**

Nucl. Phys. B 194, 445, (1982).

Process independent approach

LO cut diagrams:



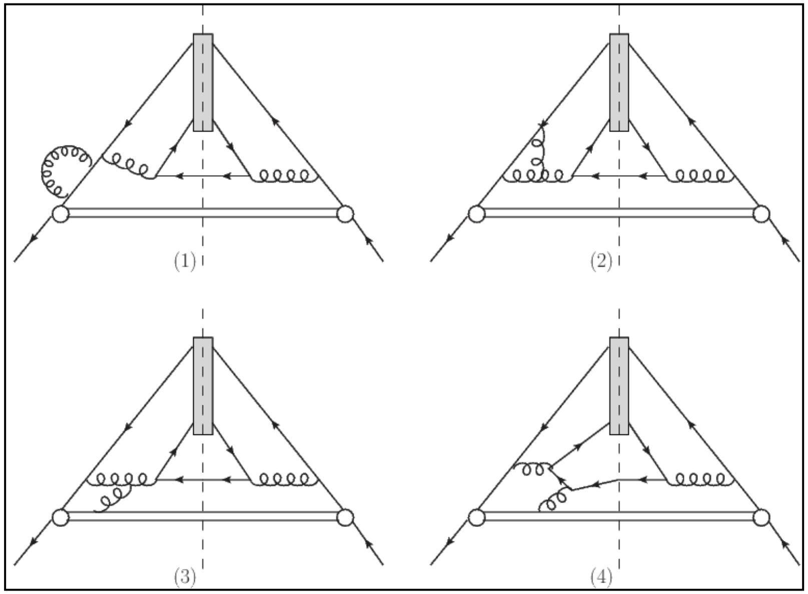
LO fragmentation functions:

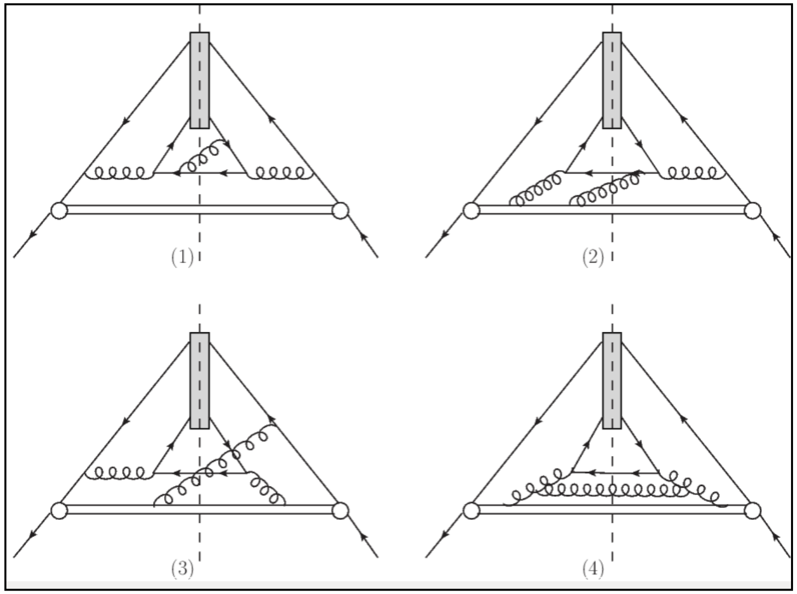
$$D^{\rm LO}_{\bar{b}\to B_c}(z) = \frac{2\alpha_s^2 z(1-z)^2|R_S(0)|^2}{81\pi r_c^2(1-r_b z)^6 M^3}[6-18(1-2r_c)z+(21-74r_c+68r_c^2)z^2 -2r_b(6-19r_c+18r_c^2)z^3+3r_b^2(1-2r_c+2r_c^2)z^4],$$

$$D^{\rm LO}_{\bar{b}\to B_c^*}(z) = \frac{2\alpha_s^2 z(1-z)^2|R_S(0)|^2}{27\pi r_c^2(1-r_b z)^6 M^3}[2-2(3-2r_c)z+3(3-2r_c+4r_c^2)z^2 -2r_b(4-r_c+2r_c^2)z^3+r_b^2(3-2r_c+2r_c^2)z^4].$$

NLO corrections

Sample NLO cut diagrams





54 virtual cut diagrams， 72 real cut diagrams.

- **Virtual corrections**

Loop-integral reduction

Many integrals containing an eikonal line, e.g,

$$\int \frac{d^D l}{[(l-p_1)^2 - m_1^2 + i\varepsilon][(l-p_2)^2 - m_2^2 + i\varepsilon][(l-p_3)^2 - m_3^2 + i\varepsilon](l\cdot n + i\varepsilon)}$$

- **Real corrections**

UV and IR divergences!

$$\mathrm{D}^{real}_{\bar{b}\to Bc}(z) = \int N_{CS} d\phi_{real}(A_{real} - A_S) + \int N_{CS} d\phi_{real} A_S$$

Calculated in 4 dimensions

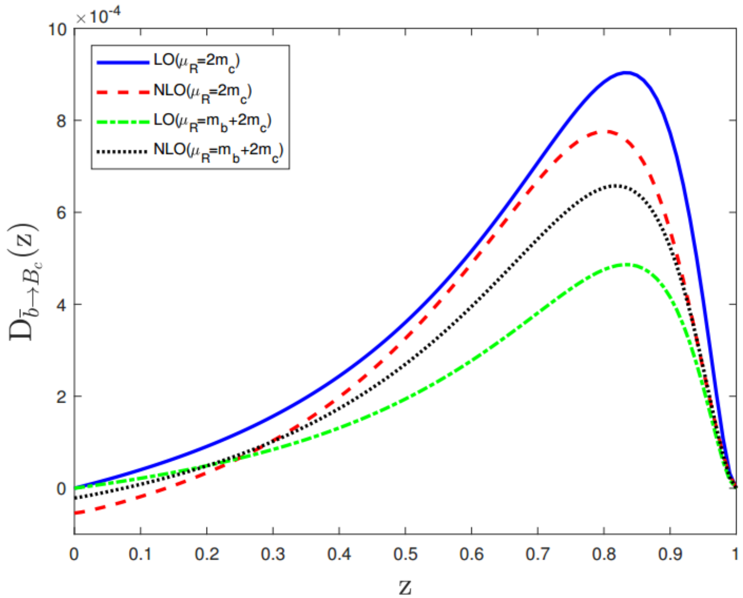
Calculated in d dimensions

Various types of subtraction terms need to be integrated!

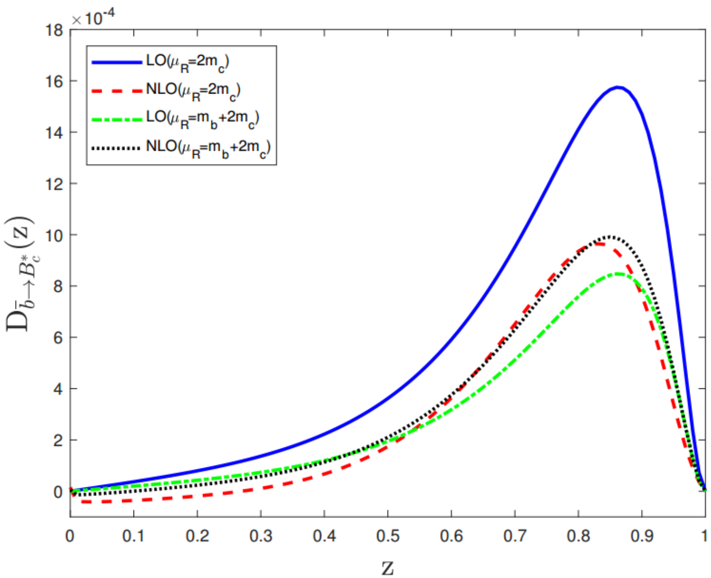
NLO results

PRD 100, 034004, (2019),
X.-C. Zheng, C.-H. Chang, T.-F Feng, X.-G. Wu.

NLO fragmentation functions for $\bar{b} \to B_c$ and $\bar{b} \to B_c^*$

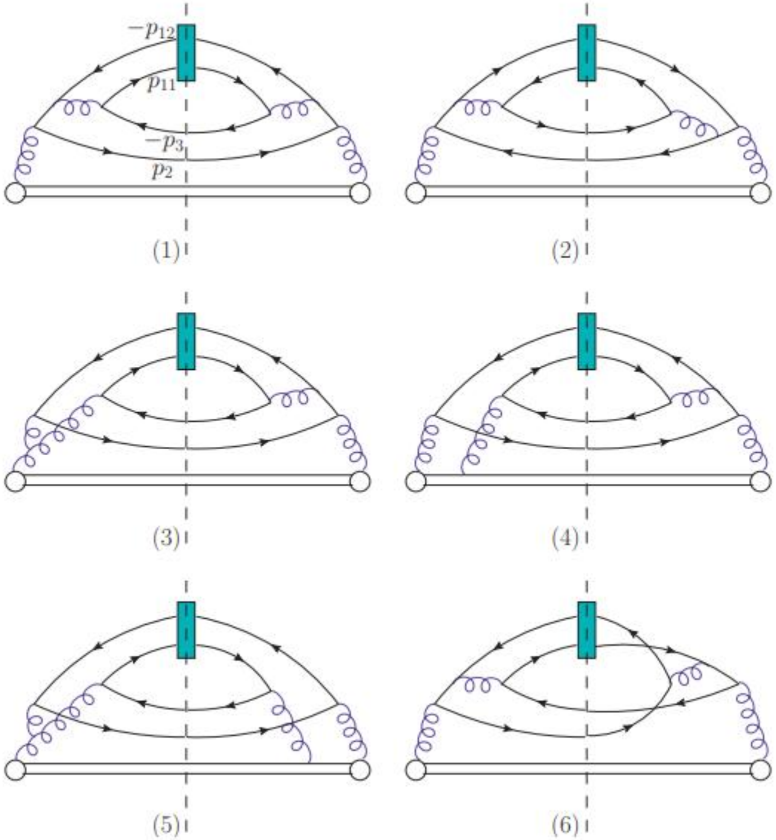


$$D_{\bar{b}\to Bc}(z, \mu_F = m_b + 2m_c)$$



$$D_{\bar{b}\to B_c^*}(z, \mu_F = m_b + 2m_c)$$

g->Bc(Bc*) FFs

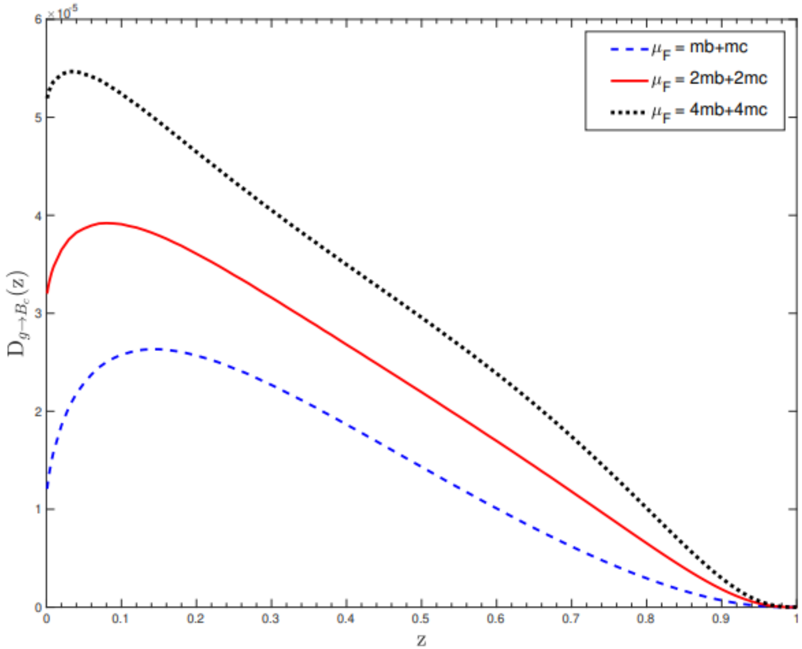


末态有三个粒子，且相空间积分存在紫外发散。

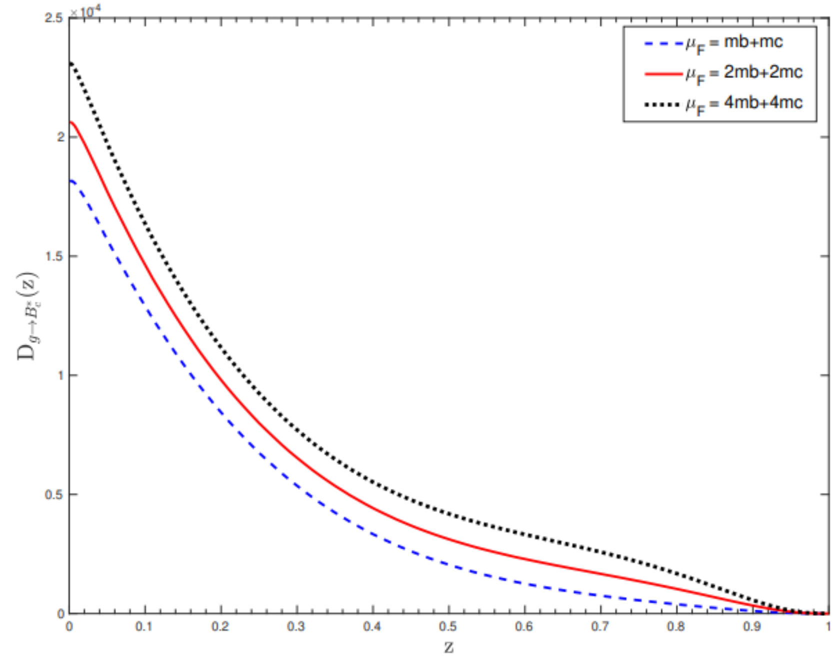
Six of the 49 cut diagrams

g->Bc(Bc*) FFs

JHEP 05, 036, (2022),
X.-C. Zheng, C.-H. Chang, X.-G. Wu.



g->Bc FFs



g->Bc* FFs

PHYSICAL REVIEW D **106**, 054030 (2022)

Gluon fragmentation into $B_c^{(*)}$ in NRQCD factorization

Feng Feng,[1,2,*] Yu Jia,[2,3,†] and Deshan Yang[3,2,‡]
[1]China University of Mining and Technology, Beijing 100083, China
[2]Institute of High Energy Physics, Chinese Academy of Sciences, Beijing 100049, China
[3]School of Physical Sciences, University of Chinese Academy of Sciences, Beijing 100049, China

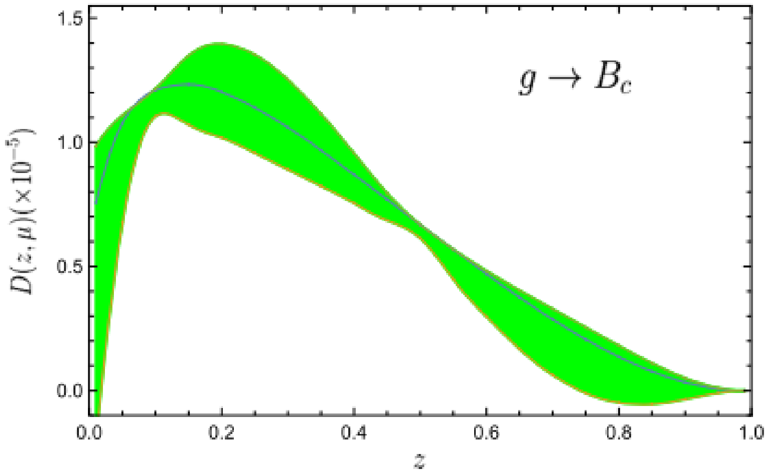
(Received 17 January 2022; accepted 8 September 2022; published 26 September 2022)

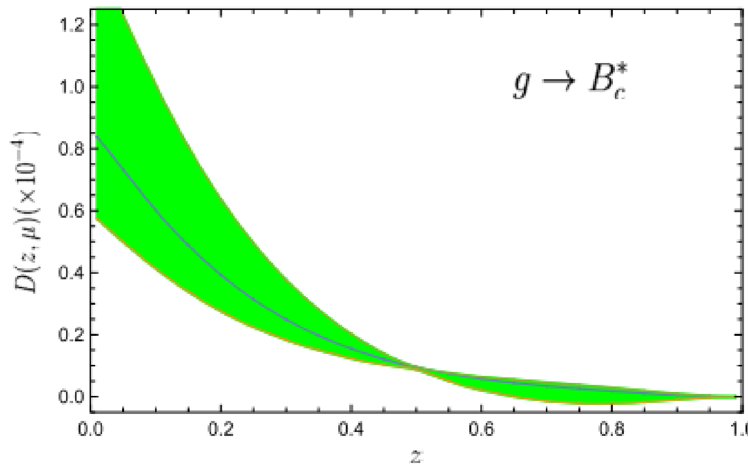
The universal fragmentation functions of gluon into the flavored quarkonia B_c and (polarized) B_c^* are computed within NRQCD factorization framework at the lowest order in velocity expansion and strong coupling constant. It is mandatory to invoke the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi renormalization program to render the NRQCD short-distance coefficients UV finite in a pointwise manner. The calculation is facilitated with the sector decomposition method, with the final results presented with high numerical accuracy. This knowledge is useful to enrich our understanding toward the large-p_T behavior of $B_c^{(*)}$ production at LHC experiment.

DOI: 10.1103/PhysRevD.106.054030

$c_1^{Bc}(z)$	z=0.1	Z=0.2	Z=0.5	Z=0.8	Z=0.9
Zheng et al	0.2323	0.2311	0.1282	0.02612	0.006144
Feng et al	0.2324	0.2311	0.1282	0.02612	0.006143

$c_1^{Bc^*}(z)$	z=0.1	Z=0.2	Z=0.5	Z=0.8	Z=0.9
Zheng et al	1.155	0.7554	0.1822	0.03412	0.009589
Feng et al	1.155	0.7550	0.1822	0.03411	0.009586

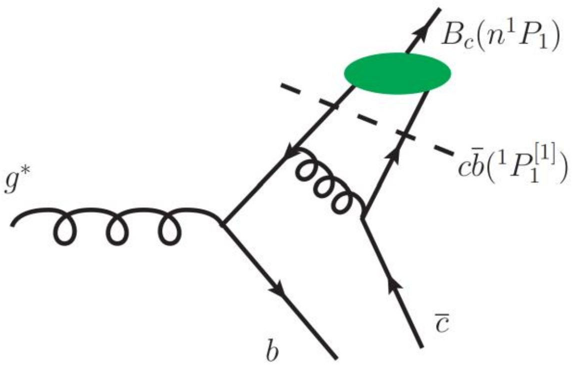




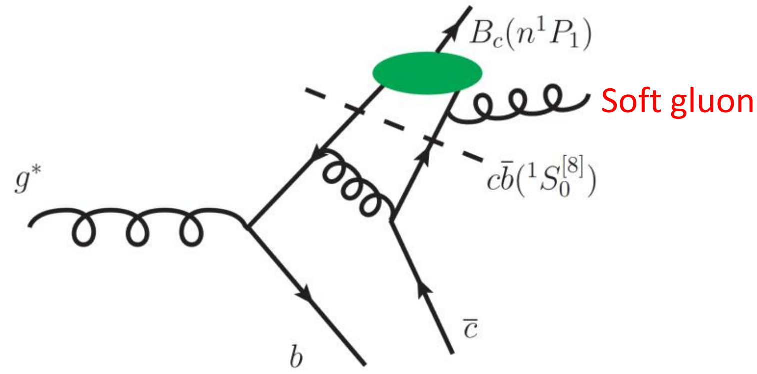
g->Bc(nP) FFs

At LO in *v*, the FFs can be factorized as

$$\begin{aligned}
D_{g\to B_c(n^1P_1)}(z,\mu_F) = & \; d_{g\to(c\bar{b})[^1P_1^{[1]}]}(z,\mu_F)\langle\mathcal{O}^{B_c(n^1P_1)}(^1P_1^{[1]})\rangle \\
& + d_{g\to(c\bar{b})[^1S_0^{[8]}]}(z,\mu_F)\langle\mathcal{O}^{B_c(n^1P_1)}(^1S_0^{[8]})\rangle, \\
D_{g\to B_c(n^3P_J)}(z,\mu_F) = & \; d_{g\to(c\bar{b})[^3P_J^{[1]}]}(z,\mu_F)\langle\mathcal{O}^{B_c(n^3P_J)}(^3P_J^{[1]})\rangle \\
& + d_{g\to(c\bar{b})[^3S_1^{[8]}]}(z,\mu_F)\langle\mathcal{O}^{B_c(n^3P_J)}(^3S_1^{[8]})\rangle,
\end{aligned}$$



Color-singlet mechanism



Color-octet mechanism

The heavy quark spin symmetry (HQSS) relation:

$$\langle \mathcal{O}^{B_c(n^3P_J)}(^3P_J^{[1]})\rangle \approx \frac{(2J+1)}{3}\langle \mathcal{O}^{B_c(n^1P_1)}(^1P_1^{[1]})\rangle,$$
$$\langle \mathcal{O}^{B_c(n^3P_J)}(^3S_1^{[8]})\rangle \approx \frac{(2J+1)}{3}\langle \mathcal{O}^{B_c(n^1P_1)}(^1S_0^{[8]})\rangle.$$

The color-singlet LDMEs can be estimated by wave function:

$$\langle \mathcal{O}^{B_c(n^1P_1)}(^1P_1^{[1]})\rangle \approx \frac{9N_c}{2\pi}|R'_{nP}(0)|^2.$$

The RGE of the color-octet LDMEs:

$$\mu_\Lambda \frac{d}{d\mu_\Lambda}\langle \mathcal{O}^{B_c(n^1P_1)}(^1S_0^{[8]})\rangle = \frac{4\alpha_s(\mu_\Lambda)}{27\pi m_{\rm red}^2}\langle \mathcal{O}^{B_c(n^1P_1)}(^1P_1^{[1]})\rangle,$$

$$\langle \mathcal{O}^{B_c(n^1P_1)}(^1S_0^{[8]})\rangle_{\mu_\Lambda} = \langle \mathcal{O}^{B_c(n^1P_1)}(^1S_0^{[8]})\rangle_{\mu_{\Lambda 0}} + \frac{8}{27\beta_0 m_{\rm red}^2}\ln\left(\frac{\alpha_s(\mu_{\Lambda 0})}{\alpha_s(\mu_\Lambda)}\right)\langle \mathcal{O}^{B_c(n^1P_1)}(^1P_1^{[1]})\rangle,$$

The color-octet LDMEs can be estimated through this equation.

Calculation of SDCs

Applying NRQCD factorization to an on-shell quark pair:

$$D_{g\to(c\bar{b})[n]}(z,\mu_F)=\sum_{n'} d_{g\to(c\bar{b})[n']}(z,\mu_F)\langle\mathcal{O}^{(c\bar{b})[n]}(n')\rangle.$$

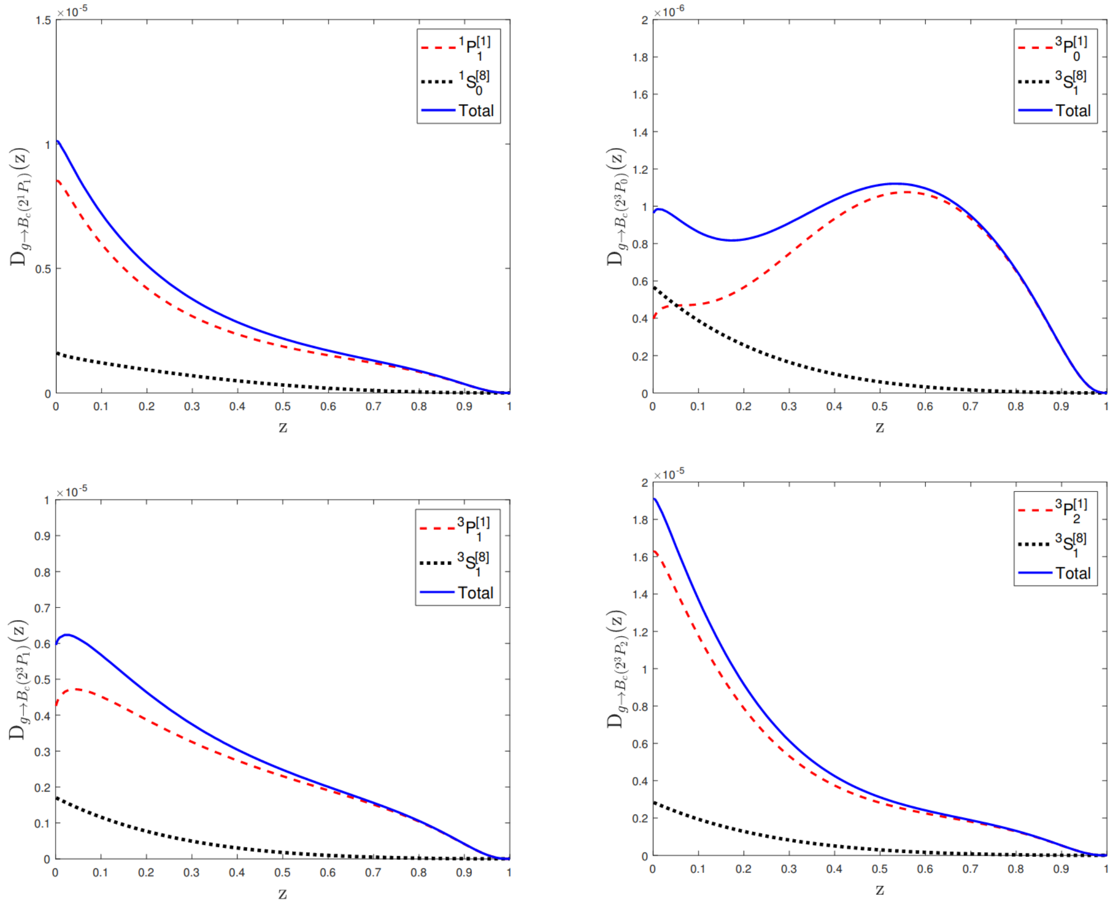
Calculated through Feynman diagrams

Calculated from the operator definition

$$\begin{aligned}
\langle\mathcal{O}^{(c\bar{b})[^1S_0^{[8]}]}(^1S_0^{[8]})\rangle &= N_c^2-1,\\
\langle\mathcal{O}^{(c\bar{b})[^3S_1^{[8]}]}(^3S_1^{[8]})\rangle &= (d-1)(N_c^2-1),\\
\langle\mathcal{O}^{(c\bar{b})[^1P_1^{[1]}]}(^1P_1^{[1]})\rangle &= 2(d-1)N_c\,\boldsymbol{q}^2,\\
\langle\mathcal{O}^{(c\bar{b})[^3P_J^{[1]}]}(^3P_J^{[1]})\rangle &= 2\,(2J+1)N_c\boldsymbol{q}^2,
\end{aligned}$$

The SDCs can be determined through **matching**.

FFs for 2P Bc excited states: **arXiv: 2602.01211, X.-C. Zheng, X.-G. Wu.**



Fitting functions for FFs:

arXiv: 2602.01211, X.-C. Zheng, X.-G. Wu.

$$d_{g\to(c\bar{b})[^{2S+1}L_J^{[1(8)]}]}(z,\mu_F) = \frac{\alpha_s}{2\pi}\ln\frac{\mu_F^2}{4M^2}\int_z^1\left[\sum_{Q=\bar{b},c}P_{Qg}(y)d^{\rm LO}_{Q\to(c\bar{b})[^{2S+1}L_J^{[1(8)]}]}(z/y)\right]\frac{dy}{y} + \frac{\alpha_s^3}{M^{(2L+3)}}f_{[^{2S+1}L_J^{[1(8)]}]}(z),$$

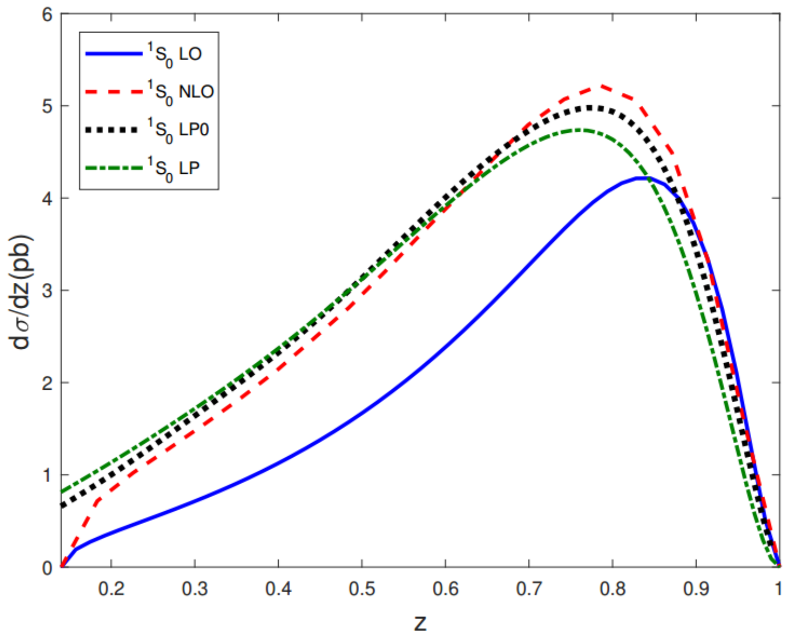
$$f_{[^{2S+1}L_J^{[1(8)]}]}(z) = N\, z^\alpha(1-z)^\beta\left(1+\sum_{i=1}^{6}a_i\, z^i\right).$$

State	N	α	β	a_1	a_2	a_3	a_4	a_5	a_6
$^1P_1^{[1]}$	6.705	0.01418	2.172	-2.293	7.423	-18.93	47.56	-65.72	41.25
$^3P_0^{[1]}$	1.297	0.06646	2.269	0.7862	4.369	34.07	140.1	-381.9	336.1
$^3P_1^{[1]}$	3.753	0.03236	2.138	2.702	-15.01	51.22	-66.48	26.77	18.66
$^3P_2^{[1]}$	7.580	0.01234	2.114	-1.318	-2.371	14.21	-17.72	4.866	8.175
$^1S_0^{[8]}$	1.420	-0.01096	3.101	0.3302	7.717	-33.55	86.58	-108.5	56.07
$^3S_1^{[8]}$	1.619	0.001004	1.675	-2.113	2.244	-2.408	2.907	-2.043	0.4914

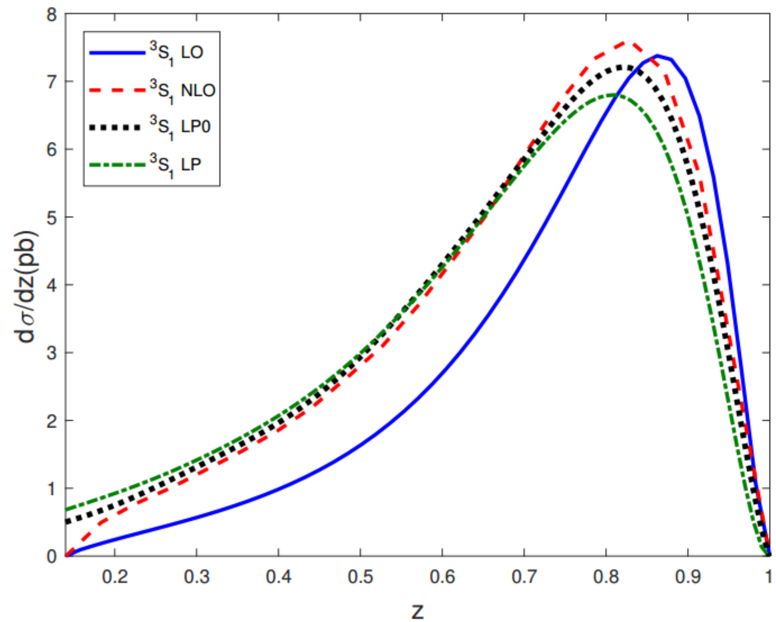
3. Applications

PRD 100, 034004, (2019),
X.-C. Zheng, C.-H. Chang, X.-G. Wu.

- Bc production at the Z pole



$d\sigma / dz(Bc)$



$d\sigma / dz(Bc^*)$

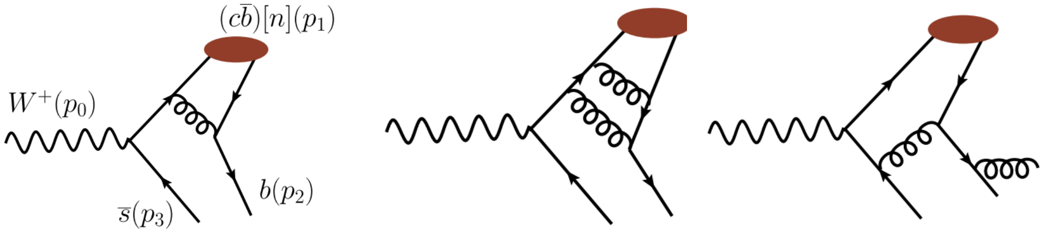
LO,NLO: direct NRQCD approach

LP0： fragmentation approach, no DGLAP evolution.

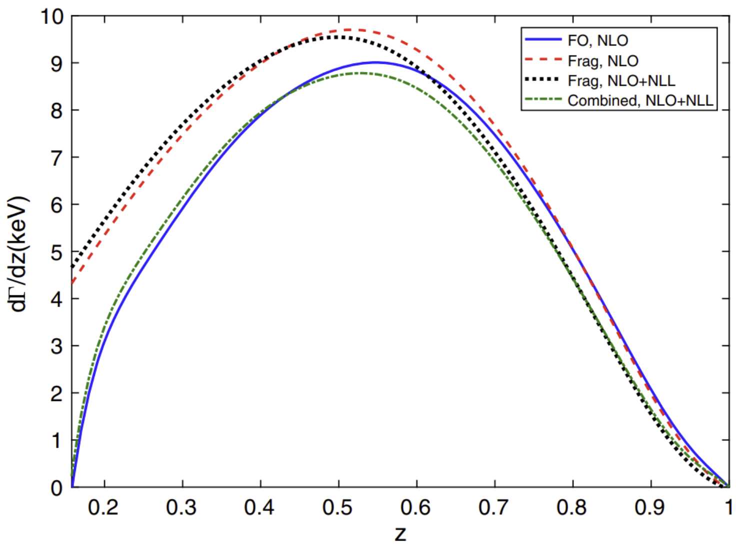
LP： fragmentation approach, evolved with DGLAP equation.

- Bc production via W^+-boson decay

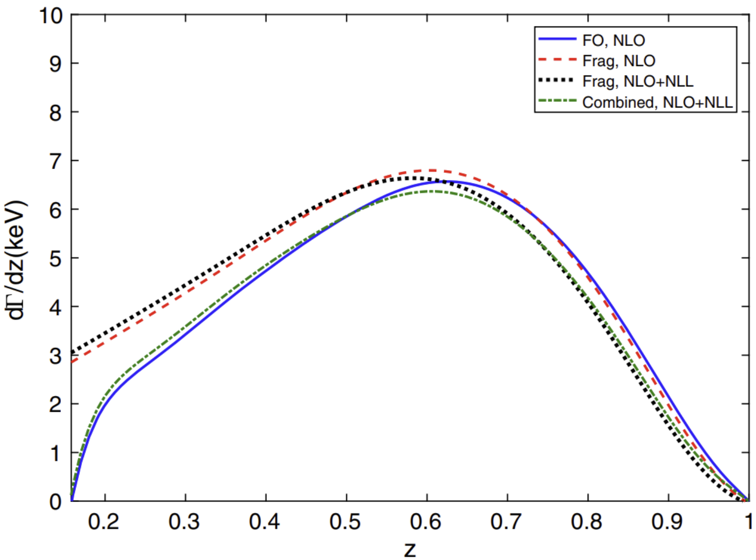
PRD 101, 034029, (2020),
X.-C. Zheng, C.-H. Chang, X.-G. Wu, et al.



Typical Feynman diagrams under the Fixed-order approach



$d\Gamma / dz(Bc)$



$d\Gamma / dz(Bc^*)$

- Bc production via Higgs-boson decay

PRD 107, 074005, (2023), X.-C. Zheng, X.-G. Wu et al.

Two sources of large logarithms:

- Renormalization of the Yukawa couplings;
- Collinear emission of gluons and quarks.

Running quark masses

DGLAP evolution

Contributions	Direct NRQCD	FF approach
$\bar{b}$-fragmentation	1.20	1.22
c-fragmentation	4.13×10^{-3}	4.26×10^{-3}
Interference	1.25×10^{-2}	
Total	1.22	1.22

Decay width under the fixed-order calculation

Contributions	B_c
$\bar{b}$-fragmentation	0.673
c-fragmentation	1.47×10^{-3}
g-fragmentation	-1.80×10^{-3}
Triangle top-loop	4.59×10^{-2}
Total	0.719

Decay width after resuming the large log terms

Summary

- Fragmentation-function approach can be used to **resum the large logarithms** arising from the collinear emissions;
- The **NLO fragmentation functions** for a quark or gluon into S- and P-wave Bc mesons have been obtained;
- These **fragmentation functions** can be studied at the high energy colliders, such as CEPC, FCC-ee, etc.

Thank you！

第八届全国重味物理与量子色动力学研讨会

Apr 24 – 28, 2026
Asia/Shanghai timezone

Enter your search term

- Overview
- Timetable
- Registration
- Participant List

Chongqing University
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会议通知

作为检验标准模型和寻找新物理的重要场所，以及解释物质反物质不对称性的关键方向，重味物理的研究对于理解量子色动力学基本理论和揭示强子内部结构均有重要的科学意义。B介子工厂已经累积了可观的实验数据，正在运行的LHCb和BELLE-II，以及对撞能量亮度持续提高的BESIII实验都将为我们提供更多更精确的测量。实验家们近年来在重味物理领域取得了诸多重要成果。这就迫切要求理论同行从强相互作用的基本理论出发，提高理论计算的精度，为精确检验标准模型以及寻找新物理做好准备。当前，重味强子弱衰变的高精度理论研究在高阶辐射修正和高阶幂次修正两个方向均取得了一系列令人振奋的新进展，在CP破坏的方向上也有一些可喜的成果。同时，核子部分子结构研究也即将进入EIC时代，这必将为强相互作用的研究注入新的活力。

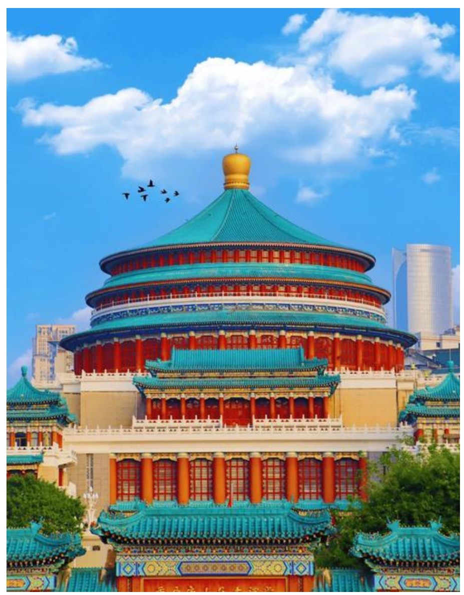
为此，我们将在重庆举办"第八届全国重味物理与量子色动力学研讨会"，诚邀本领域的实验和理论同行，共同探讨重味物理与强相互作用物理领域的最新研究进展，开展广泛而深入的学术讨论与思想碰撞，促进理论物理与实验物理的融合，以期开启更多富有成效的科学合作，取得更多有意义的研究成果。

本次会议由重庆大学、重庆科技大学、南开大学、中国科学院高能物理研究所、南京师范大学和重庆物理学会共同举办，重庆大学承办。以下是会议相关事项：

（一） 会议时间：2026年4月24日–4月28日，其中4月24日报到。

（二） 会议地点：重庆山城国际会议中心。位于重庆市沙坪坝区文广大道18号。

（三） 会议网站：https://indico.ihep.ac.cn/event/28363/



会议网站：https://indico.ihep.ac.cn/event/28363/