$f_0(980)$ 结构的能标敏感性研究

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Overview

I: The controversy of $f_0(980)$ structure

II: 2πDAs and H_{l4} decays

i: From π-LCDAs to 2πDAs

ii: Energy dependent structure of $f_0(980)/\,[\pi\pi]_{\rm S}$ in B_s/D_s decays

III: Summary and Prospect

The controversy of $f_0(980)$ structure

- Experimental identification is particularly challenging (large width)
- From the quantum theory, it is a superposition of all possible Fock states

$$|f_0(980)\rangle, \quad |[\pi\pi]_{\mathrm{S}}\rangle = \psi_{q\bar{q}}|q\bar{q}\rangle + \psi_{q\bar{q}g}|q\bar{q}g\rangle + \psi_{q\bar{q}q\bar{q}}|q\bar{q}q\bar{q}\rangle + \cdots$$

- **Hadron spectroscopy** provides clear evidence for the complex config

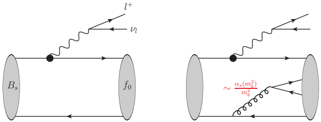
○ conventional resonant with possible gluonball component
[J.D. Weinstein and N. Isgur, PRL 48. 659 (1982)]

○ exotic multiparticle state (compact tetraquark state, molecular state)
[Y.R. Liu, H.X. Chen, W. Chen, X. Liu and S.L. Zhu, PPNP 107. 237-320 (2019)]
[F.K. Guo, C. Hanhart, U.G. Meissner, Q. Wang, Q. Zhao and B.S. Zou, RMP 90.015004 (2018)]

- **The underlying partonic dynamics** can not be extracted directly from spectral analysis, even though it reveals these exotic configurations

- **Semileptonic** B, D **decays** are powerful probe of the underlying structure
- In terms of LCDAs, scale-dependent functions

- **Semileptonic** B, D **decays** are powerful probe of the underlying structure
- **color transparency mechanism in $B_{(s)} \to f_0 l^+ \nu_l$ decays**



† high Fock states' contribution is doubly suppressed by α_s and $\mathcal{O}(1/Q^2)$, FSI is weak

- **the mechanism fails in $D_s \to f_0 l^+ \nu_l$ decays**

- the cascade decay analyses of $D_s \to (f_0 \to \pi\pi)e\nu$ under $q\bar{q}$ ansatz consists well with data
- $D_s \to f_0$ FFs & Flatté resonant model & fine tuning of the mixing angle

  [**SC** and S. L. Zhang, EPJC 84. 379 (2024)]
  [D.D. Hu, X.G. Wu, H.J. Tian, T. Zhong, and H.B. Fu, PRD 112. 056023 (2025)]

  - the scale dependence have revealed in $D_s \to f_0$ form factor $\Downarrow$
  - the seemingly agreement in D_{l4} decays is attributed to the cascade framework
  - the Flatté parameterization disrupts the assessment of color transparency $\Uparrow$

- a model-independent study directly from the $\pi\pi$ signal state

$H \to \pi\pi$ form factors in H_{l4} decays

- **QCD dynamics of H_{l4} decays is incorporated in $H \to \pi\pi$ form factors** instead of the $H \to f_0$ ffs followed by $f_0 \to \pi\pi$ in the cascade decay

$$i\langle \pi^+(k_1)\pi^-(k_2)|\bar{s}\gamma_\nu(1-\gamma_5)c|D_s(p)\rangle = F_\perp(q^2,k^2,\zeta)\,\frac{2}{\sqrt{k^2}\sqrt{\lambda_B}}\,i\epsilon_{\nu\alpha\beta\gamma}\,q^\alpha\,k^\beta\,\bar{k}^\gamma$$
$$+F_t(q^2,k^2,\zeta)\,\frac{q_\nu}{\sqrt{q^2}} + F_0(q^2,k^2,\zeta)\,\frac{2\sqrt{q^2}}{\sqrt{\lambda_B}}\left(k_\nu - \frac{k\cdot q}{q^2}q_\nu\right)$$
$$+F_\parallel(q^2,k^2,\zeta)\,\frac{1}{\sqrt{k^2}}\left(\bar{k}_\nu - \frac{4(q\cdot k)(q\cdot\bar{k})}{\lambda_B}\,k_\nu + \frac{4k^2(q\cdot\bar{k})}{\lambda_B}\,q_\nu\right)$$

  - $\lambda = \lambda(m_B^2, k^2, q^2)$ is the Källén function, $q\cdot k = (m_B^2 - q^2 - k^2)/2$,
    $q\cdot\bar{k} = \sqrt{\lambda}\beta_\pi(k^2)\cos\theta_\pi/2 = \sqrt{\lambda}\,(2\zeta-1)$, $\beta_\pi(k^2) = \sqrt{1-4m_\pi^2/k^2}$

- **QCDF** (QCD factorization) in the large two-pion mass
  - [P. Böer, T. Feldmann and D. van Dyk, JHEP 1702. 133] $T_I \propto F_{B\to\pi}\otimes\phi_\pi$,

- **SU(3) flavor symmetry/breaking** with the intermediate resonance
  - [R.M. Wang, Y.G. Xu, J.H. Sheng, X.D. Cheng and et.al., 2301.00090, PRD 112. 033002 (2025)]

$H \to \pi\pi$ form factors

- **LQCD** (Lattice QCD) in the ρ resonance region with a simple BW model
  - [L. Leskovec and et.al, PRL 134.161901 (2025), **Editors' Suggestion**]

- **HChPT** (Heavy-meson Chiral Perturbative Theory) in the large q^2 by taking dispersive methods in terms of Omnés functions
  - [X.-W. Kang, B. Kubis, C. Hanhart, and U.-G. Meißner, PRD 89. 053015 (2014)]

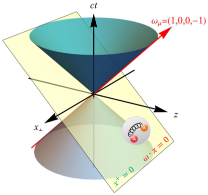
  in the full phase-space by a novel parameterization with unitarity
  - [F. Herren, B. Kubis and R. van Tonder, PRD 112, 014037 (2025), **Editors' Suggestion**]

- **LCSRs** (Light-cone sum rules) in the small and intermediate q^2
  - [**SC**, A. Khodjamirian and J. Virto, JHEP 05(2017)157] B-meson LCSRs , [S. Descotes-Genon, A. Khodjamirian, J. Virto and K.K. Vos, JHEP 12(2019)083, 06(2023)034] $B \to K\pi$
  - [C. Hambrock and A. Khodjamirian, NPB 905. 379-390(2016)] 2πDAs LCSRS of $F_{\parallel,\perp}$
  - [**SC**, A. Khodjamirian and J. Virto, PRD(R) 96. 051901(2017)] timelike-helicity FF F_t and F_0
  - [**SC**, PRD 99. 053005(2019)] 2πDAs updates and $B \to [\pi\pi]_{\mathrm{S,P}}$ FFs
  - [**SC** and J.M Shen, EPJC 6:554(2020), **SC** and S.L Zhang, EPJC 84:379(2024)] Pheno
  - [**SC**, PRD 112. L111301(2025)] first study of twist-three 2πDAs and $|V_{ub}|$ extraction
  - [**SC**, L.Y. Dai, J.M. Shen and S.L. Zhang, PRD 113. L031901(2026)] $D_s \to [\pi\pi]_{\mathrm{S}}\, e\nu$ this talk

- From π-LCDAs to 2πDAs
  [**SC**, PRD 112. L111301(2025)]

- Energy dependent structure of $f_0(980)/[\pi\pi]_{\rm S}$ in B_s/D_s decays [**SC**, L.Y. Dai, J.M. Shen and S.L. Zhang, PRD 113. L031901(2026)]

LCDAs Exclusive QCD processes with larger momentum transfers



- The Lorentz and gauge invariant ME

$$\langle 0|\bar{u}(x)\gamma_\mu\gamma_5 d(-x)|\pi^-(p)\rangle = f_\pi \int_0^1 du\, e^{i(2u-1)p\cdot x}\left[ip_\mu\left(\phi(u,\mu)+\frac{x^2}{4}\phi_1^4(u,\mu)\right) -\right.$$

$$\langle 0|\bar{u}(x)\gamma_\mu\gamma_5 d(-x)|\rho^-(p)\rangle = f_\rho m_\rho \int_0^1 du\, e^{i(2u-1)p\cdot x}\left[p_\mu \frac{\epsilon^{(\lambda)}\cdot x}{p\cdot x}\left(\phi_\parallel(u,\mu)-\phi_\perp^3\right.\right.$$

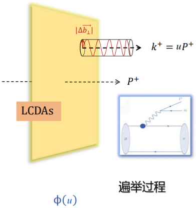
$$\langle 0|\bar{q}(x)\gamma_\mu\gamma_5 q(-x)|f_0(p)\rangle = p_\mu \int_0^1 du\, e^{i(2u-1)p\cdot x}\left[\phi(u,\mu)+\cdots\right]$$

- LCDAs are dimensionless functions of u and renormalization scale μ
- the probability amplitudes to **find the meson** in a state with minimal number of constitutes and have small transversal separation of order $1/\mu$
- the current accuracy up to three-particle ($q\bar{q}g$) current

- **longitudinal** $\otimes$ *transversal* dofs
- the TMD (μ dependence) is governed by the *RGE*
- the LMD is described in terms of irreducible representations of the corresponding symmetry group

collinear subgroup of conformal group $SL(2,R)\cong SU(1,1)\cong SO(2,1)$

$\Rightarrow$ collinear twist $\Rightarrow$ **the Gegenbauer polynomials**



- LCDAs of pion achieved **great success** in describing large Q^2 processes.

  △ the establishment and development of the pQCD factorization $F_\pi(Q^2)$

$$\int_0^1 du_i \phi(u_1, \tilde{Q}_1)\, T_H(u_i, Q) \phi(u_2, \tilde{Q}_2)$$ 1980s — [Lepage & Brodsky 1980, Efremov & Radyushkin 1980]

$$\Downarrow$$

$$\int_b^{\bar{b}} du_1 \int_{a/u_1}^{1-a/\bar{u}_1} \phi(u_1, k_{1T})\, T_H(u_i, Q) \phi(u_2, k_{2T})$$ 1990s — [Huang, Shen & Kroll 1991, Huang, Shen & Ma 1994]

$$\Downarrow$$

$$\psi(u_1, \mu_{r_1})\, T_H(u_i, b_i, Q)\, S_t(u_i)\, e^{-S(u_i, b_i, Q)}\, \psi(u_2, \mu_{r_2})$$ 2000s — [Botts & Sterman 1989, Li & Sterman 1992, Li 1999]

$$\Downarrow$$

$$\sum_{t_i} \psi^{t_1}(u_1, \mu_{r_1})\, T_H^{t_i,\mathrm{LO+NLO}}(u_i, b_i, Q) S_t(u_i)\, e^{-S(u_i, b, Q)}\, \psi^{t_2}(u_2, \mu_{r_2})$$ 2010s — [Li, Shen, Wang & Zou 2011, **SC**, Fan & Xiao 2014]

$$\sum_{t_i} \psi^{t_1}(u_1, b_1, \mu_{r_1})\, T_H^{t_i,\mathrm{LO+NLO}}(u_i, b_i, Q)\, S_t(u_i)\, e^{-S(u_i, b, Q)}\, \psi^{t_2}(u_2, b_2, \mu_{r_2})$$ 2020s — [Chai & **SC** 2025] [Chen^2, Feng & Jia 2024], [Ji, Shi, Wang^3 & Yu 2025]

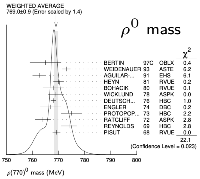
- LCDAs of proton serves as the fundamental input to explain *ep* scattering
  [Chen^2, Feng, Hu, Jia 2025], [Huang, Shi, Wang, Zhao 2025], [Yu, **SC**, Han, Li, Yu 2025]

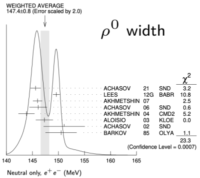
- The non-perturbative input for HFP theoretical studies that determines the precision and accuracy   predicted the CPVs in the $B \to \pi\pi, K\pi$ decays and et.al.,

- $\cdots$

The limitations of the single particle picture

- V, S meson LCDAs reveal certain limitations in the precision testing era
  - △ the probability amplitudes to **find the light meson** in a state with $\cdots$
  - △ the $\pi\pi$ mass in $[0.554, 0.996]$GeV is selected to identify candidates for the $\rho(770)$





- A second-best approach frequently employed in phenomenology is the **cascade decay framework**

$$\mathcal{M}(B^0 \to \pi^0\pi^- l^+\nu_l) = \mathcal{M}(B^0 \to \rho^- l^+\nu_l)\,\mathrm{BW}(s)\,\mathcal{M}(\rho \to \pi\pi)$$

- **model dependence, large uncertainties in f_0 involved processes**

- How to accurately describe the **width effects** of unstable intermediate particles, the contributions and **interference effects** of different partial waves, and the **QCD backgrounds** from non-resonant states

2πDAs provide a most general description of $\pi\pi$ spectral

- Chiral-even LC expansion with gauge factor $[x, 0]$ [Polyakov 1999, Diehl 1998]

$$\langle \pi^a(k_1)\pi^b(k_2)|\bar{q}_f(zn)\gamma_\mu\tau q_{f'}(0)|0\rangle = \kappa_{ab}\, k_\mu \int du\, e^{iuz(k\cdot n)}\, \Phi_{\parallel}^{ab,ff'}(u,\zeta,k^2,\mu)$$

$$\langle 0|\bar{q}(x)\gamma_\mu\gamma_5 q(-x)|f_0(p)\rangle = p_\mu \int_0^1 du\, e^{i(2u-1)p\cdot x}\phi(\mu,u)$$

- 2πDAs is decomposed in terms of $C_n^{3/2}(2u-1)$ and $C_\ell^{1/2}(2\zeta-1)$

$$\Phi^{I=1}(u,\zeta,k^2,\mu) = 6u(1-u)\sum_{n=0,\mathrm{even}}^{\infty}\sum_{l=1,\mathrm{odd}}^{n+1} B_{n\ell}^{I=1}(k^2,\mu)C_n^{3/2}(2u-1)C_\ell^{1/2}(2\zeta-1)$$

$$\Phi^{I=0}(u,\zeta,k^2,\mu) = 6u(1-u)\sum_{n=1,\mathrm{odd}}^{\infty}\sum_{l=0,\mathrm{even}}^{n+1} B_{n\ell}^{I=0}(k^2,\mu)C_n^{3/2}(2u-1)C_\ell^{1/2}(2\zeta-1)$$

- **Evolution from $4m_\pi^2$ to large k^2** via the Watson theorem of $\pi\pi$ scattering amplitudes

$$B_{n\ell}^I(k^2) = B_{n\ell}^I(0)\,\mathrm{Exp}\left[\sum_{m=1}^{N-1}\frac{k^{2m}}{m!}\frac{d^m}{dk^{2m}}\ln B_{n\ell}^I(0) + \frac{k^{2N}}{\pi}\int_{4m_\pi^2}^{\infty} ds\,\frac{\delta_\ell^I(s)}{s^N(s-k^2-i0)}\right]$$

△ **2πDAs in a wide range of energies is given by δ_ℓ^I and a few subtraction constants**

2πDAs

- The subtraction constants of $B_{n\ell}(k^2)$ at low k^2 (around the threshold)

(nl)	$B^{\parallel}_{n\ell}(0)$	$c_1^{\parallel,(nl)}$	$\frac{d}{dk^2}\ln B^{\parallel}_{n\ell}(0)$	$B^{\perp}_{n\ell}(0)$	$c_1^{\perp,(nl)}$	$\frac{d}{dk^2}\ln B^{\perp}_{n\ell}(0)$
(01)	1	0	1.46 $\to$ 1.80	1	0	0.68 $\to$ 0.60
(21)	-0.113 $\to$ 0.218	-0.340	0.481	0.113 $\to$ 0.185	-0.538	-0.153
(23)	0.147 $\to$ -0.038	0	0.368	0.113 $\to$ 0.185	0	0.153
(10)	-0.556	-	0.413	-	-	-
(12)	0.556	-	0.413	-	-	-

△ firstly studied in the effective low-energy theory based on instanton vacuum [Polyakov 1999]

△ updated with the kinematical constraints and the new a_2^{π}, a_2^{ρ} [**SC** 2019, 2023]

- 2πDAs were introduced at leading twist [Polyakov 1999, Diehl 1998]

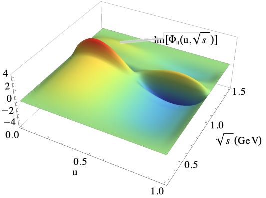
$$\langle \pi^a(k_1)\pi^b(k_2)|\bar{q}_f(zn)\gamma_\mu \tau q_{f'}(0)|0\rangle = \kappa_{ab}\, k_\mu \int dx\, e^{iuz(k\cdot n)}\, \Phi_{\parallel}^{ab,ff'}(u,\zeta,k^2)$$

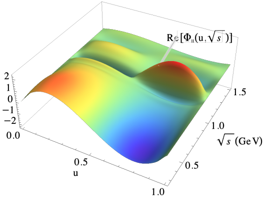
- improved to twist-three level recently [**SC** 2025]

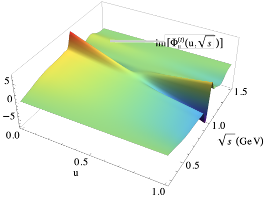
$$\langle\pi(k_1)\pi(k_2)|\bar{q}(0)q(x)|0\rangle = \int du e^{i\bar{u}k\cdot x}\frac{ik^2(k\cdot x)}{2f_{2\pi}^{\perp}}\Phi_{\parallel}^{(s)},$$

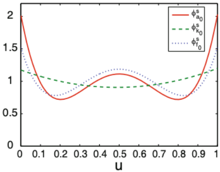
$$\langle\pi(k_1)\pi(k_2)|\bar{q}(0)\sigma^{\mu\nu}q(x)|0\rangle = -\frac{i}{f_{2\pi}^{\perp}}\int du e^{i\bar{u}k\cdot x}\Big[\frac{k_\mu \bar{k}_\nu - k_\nu \bar{k}_\mu}{2\zeta-1}\Phi_\perp - k^2\frac{k_\mu x_\nu - k_\nu x_\mu}{k\cdot x}\Phi_{\parallel}^{(t)}\Big].$$

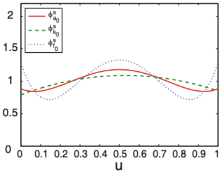
2πDAs of isospin scalar two-pion system

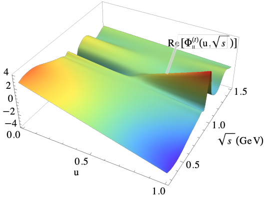












[Han, et.al., Eur. Phys. J. A (2013) 49: 78]   [Lü, et.al., Phys. Rev. D 75. 056001(2007)]

- asymmetry of the twist-3 2πDAs to the parton momentum fraction u
- symmetric for f_0 obtained under the single particle approximation
- where QCD sum rules dictate that the asymmetric component vanishes

$D_s \to [\pi\pi]_S \, e^+\nu_e$ decay

- $D_s \to f_0 e^+\nu$ [CLEO '09], $D_{(s)} \to a_0 e^+\nu$ [BESIII '18, '21], $D^+ \to f_0/\sigma e^+\nu$[BESIII '19]
- $D_s \to f_0(\to \pi^0\pi^0, K_sK_s)e^+\nu$ [BESIII 22], $D_s \to f_0(\to \pi^+\pi^-)e^+\nu$[BESIII 23]

$$\mathcal{B}(D_s \to f_0(\to \pi^0\pi^0)e^+\nu) = (7.9 \pm 1.4 \pm 0.3) \times 10^{-4}$$
$$\mathcal{B}(D_s \to f_0(\to \pi^+\pi^-)e^+\nu) = (17.2 \pm 1.3 \pm 1.0) \times 10^{-4}$$
$$f_+^{f_0}(0)|V_{cs}| = 0.504 \pm 0.017 \pm 0.035$$

- single particle (narrow width limit) $\quad D_s \to f_0 e^+\nu$

$$\frac{d\Gamma(D_s^+ \to f_0 l^+\nu)}{dq^2} = \frac{G_F^2|V_{cs}|^2\lambda^{3/2}(m_{D_s}^2, m_{f_0}^2, q^2)}{192\pi^3 m_{D_s}^3}|f_+(q^2)|^2, \ D_s \to f_0 \text{ FF}$$

- improvement with the width effect by resonant models $\quad D_s \to [f_0 \to] \, \pi\pi e^+\nu$

$$\frac{d\Gamma(D_s^+ \to [\pi\pi]_S \, l^+\nu)}{dsdq^2} = \frac{1}{\pi}\frac{G_F^2|V_{cs}|^2}{192\pi^3 m_{D_s}^3}|f_+(q^2)|^2 \frac{\lambda^{3/2}(m_{D_s}^2, s, q^2)\, g_1\beta_\pi(s)}{|m_S^2 - s + i(g_1\beta_\pi(s)) + g_2\beta_K(s))|^2}, \text{ BESIII}$$

- calculate directly the signal channel $D_s \to [\pi\pi]_S \, e^+\nu$

$$\frac{d^2\Gamma(D_s^+ \to [\pi\pi]_S \, l^+\nu)}{dsdq^2} = \frac{G_F^2|V_{cs}|^2 \, \beta_{\pi\pi}(s)\sqrt{\lambda_{D_s}}q^2}{3(4\pi)^5 m_{D_s}^3}\sum_{\ell=0}^{\infty}|F_0^{(\ell)}(q^2, s)|^2, \ D_s \to \pi\pi \text{ FF}$$

$D_s \to f_0$ FFs and cascade decay $D_s \to (f_0 \to) \pi\pi e^+ \nu$

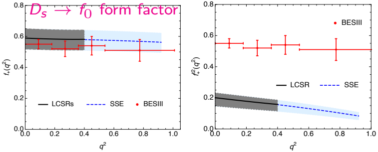
- $\{M^2, s_0\} = \{5.0 \pm 0.5, 6.0 \pm 0.5\}\mathrm{GeV}^2$

this work	3pSRs(07)	LFQM(09)	CLFD/DR(08)	LCSRs(10)
0.63 ± 0.04	0.96	0.87	$0.86/0.90$	0.30 ± 0.03

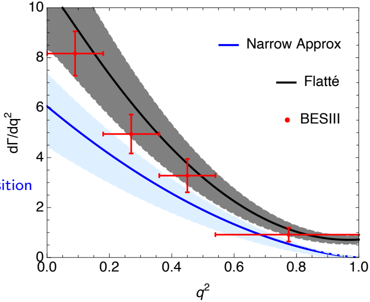
- the BESIII result in the $\pi^+\pi^-$ system $f_+(0) = 0.518 \pm 0.018 \pm 0.036$ [BESIII 23]

different input of the decay constant $\tilde{f}_{f_0} = 335$ MeV, much larger than 180 MeV in LCSRs(10)
we add the first gegenbauer expansion terms in the LCDAs, up-to-date parameters

$s\bar{s}$-$n\bar{n}$ mixing scenario of f_0 with $\theta = 20^\circ \pm 10^\circ$



- Twist-3 LCDAs give dominate contribution in $D_s \to f_0$ transition
  - the uncertainty estimation is conservative
  - without NLO correction
  - we need a model independent calculation
  - for the QCD understanding
  - and the future partial-wave measurement

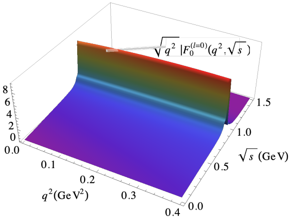


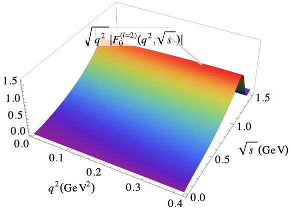
Differential decay width of $D_s^+ \to (f_0 \to) [\pi\pi]_S e^+ \nu_e$

$D_s \to [\pi\pi]_S$ FFs and $D_s \to [\pi\pi]_S e^+\nu$ decay

- The LCSRs ℓ'-wave $D_s \to [\pi\pi]_S$ form factors ($\ell' = \text{even}$ & $\ell' \leqslant n+1$)

$$\sqrt{q^2} F_0^{(\ell')}(q^2, k^2) = \frac{m_c(m_c+m_s)\sqrt{q^2}\sqrt{\lambda_{Ds}}}{m_{D_s}^2 f_{D_s}} \sum_{n=1,\text{odd}}^{\infty} \frac{\beta_\pi(k^2)}{\sqrt{2\ell'+1}} J_n^0(q^2, k^2, M^2, s_0)\, B_{n\ell,\parallel}^{l=0}(k^2)\, I_{\ell\ell'}$$





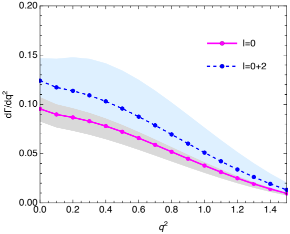
- Twist-2 and twist-3 contributions to $D_s \to \pi\pi, f_0$ form factors at $q^2 = 0$

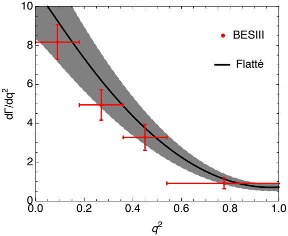
under the $q\bar{q}$ approximation

Form Factors	Twist-2	Twist-3	Total
$\sqrt{q^2}F_0^{(l=0)}(0) = \sqrt{q^2}F_t^{(l=0)}(0)$	$0.20^{+0.02}_{-0.02} - i0.24^{+0.02}_{-0.02}$	$-0.41^{+0.04}_{-0.05} + i0.51^{+0.02}_{-0.04}$	$-0.21^{+0.02}_{-0.01} + i0.27^{+0.03}_{-0.02}$
$\sqrt{q^2}F_0^{(l=2)}(0) = \sqrt{q^2}F_t^{(l=2)}(0)$	$0.27^{+0.03}_{-0.02} + i0.21^{+0.02}_{-0.01}$	$-0.55^{+0.02}_{-0.03} - i0.41^{+0.05}_{-0.04}$	$-0.28^{+0.02}_{-0.02} - i0.20^{+0.02}_{-0.01}$
$f_+(0) = f_0(0)$	$0.20^{+0.03}_{-0.05}$	$0.41^{+0.04}_{-0.06}$	$0.61^{+0.05}_{-0.07}$

$D_s \to [\pi\pi]_S$ FFs and $D_s \to [\pi\pi]_S\, e^+\nu$ decay

- The differential decay width on momentum transfers





$D_s \to [\pi\pi]_S\, e^+\nu_e$	$D_s \to [f_0 \to \pi\pi]\, e^+\nu_e$ [23]	Data [25]
$0.81^{+0.34}_{-0.14}$	$18.8^{+4.5}_{-3.8}$	17.2 ± 1.6

- Differential widths $d\Gamma/dq^2$ is two-order in magnitude smaller than the data
- non-$q\bar{q}$ Fock states are the dominate component of $[\pi\pi]_S$ in charm decays
- in consistent with the assessment of color transparency is severe disrupted by the Flatté model

- much different in B decays non-asymptotic leading twist dominated [**SC** 2025]

- go further to multi-particle DiPion LCDAs in CHARM ($q\bar{q}g$, $q\bar{q}q\bar{q}$)

Summary and Prospect

- 2πDAs provide a general description of the $\pi\pi$ mass spectrum
- 2πDAs are a crucial input for the study of H_{l4} decays
- 2πDAs have been studied at the three-twist level, explaining the multi-particle picture of scalar mesons
- wishlists related to the (future) colliders
  - possible anomalies in the FCNC processes of the $D \to \pi\pi l^+ l^-$ decay
  - the processes e^+e^- annihilation $\gamma^* \to \pi\pi\gamma$ and $\gamma^*\gamma \to \pi\pi$

Thank you for your patience.