奇异夸克偶素的格点 QCD 研究

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Outline

Strangeonium: A Key System in Light Hadron Physics

- Strangeonium ($s\bar{s}$) provides a clean laboratory for studying low-energy QCD dynamics.
- Experimentally, states like $\phi(1020)$, $\phi(1680)$, $h_1(1410)$,$f_2(1525)$ may have dominant $s\bar{s}$ components.
- The strangeonium spectrum is essential for understanding flavor mixing and for identifying glueballs via mixing with scalar mesons.
- **Open questions**: Precise spectrum, role of disconnected diagrams, mixing with glueballs.
- First-principles lattice QCD calculation including full disconnected contributions and mixing analysis.

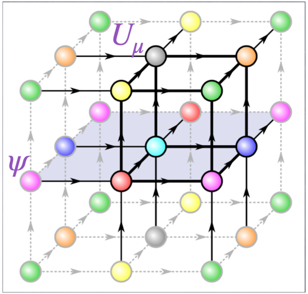
Lattice QCD Formalism

$$Z = \int DAD\psi D\bar{\psi} e^{iS[A,\psi,\bar{\psi}]}$$

$$\xrightarrow[t\to i\tau]{U_\mu = e^{iA_\mu}} \int DU \det M[U] e^{-S_g[U]}$$

$$\langle \widehat{O}[U,\psi,\bar{\psi}]\rangle = \frac{1}{Z}\int DU \det M[U] e^{-S_g[U]} \hat{O}[U]$$

$$\langle \widehat{\mathcal{O}}[U,\psi,\bar{\psi}]\rangle = \frac{1}{N}\sum_i \mathcal{O}[U_i] + O\left(\frac{1}{\sqrt{N}}\right)$$



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Simulation Details and Techniques

- 1-flavor asymmetric configurations with dynamical strange sea quarks.

β	ξ	a_t^{-1}(GeV)	$L^3 \times T$	m_π(MeV)	N_{conf}
2.4	5	6.66	$16^3 \times 128$	686	3931

- Distillation method: efficient construction of correlation matrices and disconnected diagrams.
- Operator set covering various J^{PC} channels (see next slide).

Operator Construction

J^{PC}	operators					
0^{--}	$\gamma^5\gamma^i\nabla^i$					
0^{++}	$\gamma^i\nabla^i$	$\gamma^4\gamma^i\nabla^i$	$\gamma^4\gamma^5\gamma^i\mathbb{B}^i$	1		
0^{+-}	$\gamma^5\gamma^i\mathbb{B}^i$					
0^{-+}	$\gamma^4\gamma^5\gamma^i\nabla^i$	$\gamma^i\mathbb{B}^i$	$\gamma^4\gamma^i\mathbb{B}^i$	γ^5		
1^{--}	∇^i	$\epsilon_{ijk}\gamma^5\gamma^j\nabla^k$	$\lvert\epsilon_{ijk}\rvert\gamma^j\mathbb{D}^k$	$\lvert\epsilon_{ijk}\rvert\gamma^4\gamma^j\mathbb{D}^k$	$\gamma^5\mathbb{B}^i$	γ^i
1^{++}	$\epsilon_{ijk}\gamma^j\nabla^k$	$\epsilon_{ijk}\gamma^4\gamma^j\nabla^k$	$\lvert\epsilon_{ijk}\rvert\gamma^5\gamma^j\mathbb{D}^k$	$\epsilon_{ijk}\gamma^4\gamma^5\gamma^j\mathbb{B}^k$	$\gamma^5\gamma^i$	
1^{-+}	$\gamma^4\nabla^i$	$\epsilon_{ijk}\gamma^4\gamma^5\gamma^j\nabla^k$	$\epsilon_{ijk}\gamma^j\mathbb{B}^k$	$\epsilon_{ijk}\gamma^4\gamma^j\mathbb{B}^k$		
1^{+-}	$\gamma^5\nabla^i$	$\gamma^4\gamma^5\nabla^i$	$\lvert\epsilon_{ijk}\rvert\gamma^4\gamma^5\gamma^j\mathbb{D}^k$	$\mathbb{B}^i$	$\epsilon_{ijk}\gamma^5\gamma^j\mathbb{B}^k$	$\gamma^i\gamma^j$
2^{++}	$\lvert\epsilon_{ijk}\rvert\gamma^j\nabla^k$	$\lvert\epsilon_{ijk}\rvert\gamma^4\gamma^j\nabla^k$	$\epsilon_{ijk}\gamma^5\gamma^j\mathbb{D}^k$	$\lvert\epsilon_{ijk}\rvert\gamma^4\gamma^5\gamma^j\mathbb{B}^k$		
2^{--}	$\mathbb{Q}_{ijk}\gamma^5\gamma^j\nabla^k$	$\epsilon_{ijk}\gamma^j\mathbb{D}^k$	$\mathbb{Q}_{ijk}\gamma^4\gamma^j\mathbb{D}^k$			
2^{-+}	$\mathbb{Q}_{ijk}\gamma^4\gamma^5\gamma^j\nabla^k$	$\mathbb{Q}_{ijk}\gamma^j\mathbb{B}^k$	$\mathbb{Q}_{ijk}\gamma^4\gamma^j\mathbb{B}^k$			
2^{+-}	$\mathbb{Q}_{ijk}\gamma^4\gamma^5\gamma^j\mathbb{D}^k$	$\mathbb{Q}_{ijk}\gamma^5\gamma^j\mathbb{B}^k$				

Two-Point Correlation Functions with Disconnected Diagrams

$$C^{(2)}_{ji}(\vec{p},t) = \sum_{\vec{x}} e^{-i\vec{p}\cdot\vec{x}} \left\langle O_j(\vec{x},t) O_i^\dagger(\vec{0},0) \right\rangle$$

$$= \underbrace{\sum_{\vec{x}} e^{-i\vec{p}\cdot\vec{x}} \left\langle \Phi_j G(\vec{x},t;\vec{0},0) \Phi_i G(\vec{0},0;\vec{x},t) \right\rangle}_{\text{Connected}}$$

$$+ \underbrace{\left\langle \Phi_j G(\vec{x},t;\vec{x},t) \Phi_i G(\vec{0},0;\vec{0},0) \right\rangle}_{\text{Disconnected}}$$

Variational Method

- Solve generalized eigenvalue problem:

$$C(t)v^n(t) = \lambda_n(t)C(t_0)v^n(t)$$

- Fit eigenvalues:

$$\lambda_n(t) = (1 - A_n)e^{-m_n(t-t_0)} + A_n e^{-m'_n(t-t_0)}$$

Level Identification

- Practical issue: using too many operators does not always yield stable extractions; we typically use 2–3 operators per channel.

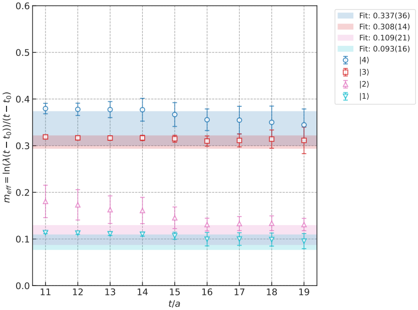

Figure 2: Example: pseudoscalar channel with four operators.

Disconnected Contributions: Pseudoscalar Channel

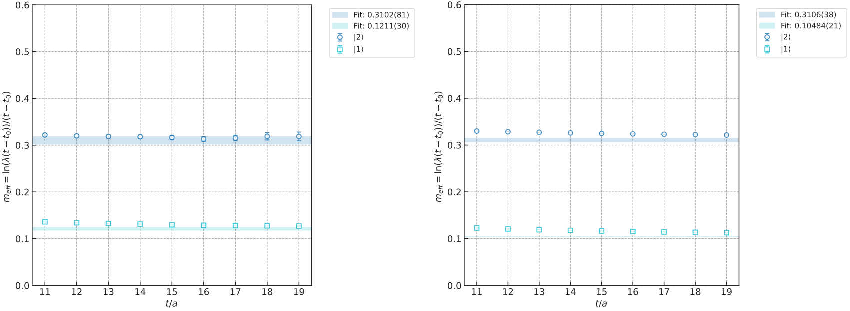

Figure 3: Pseudoscalar effective masses: left (connected+disconnected), right (connected only).

Disconnected Diagram Effects: Scalar Channel

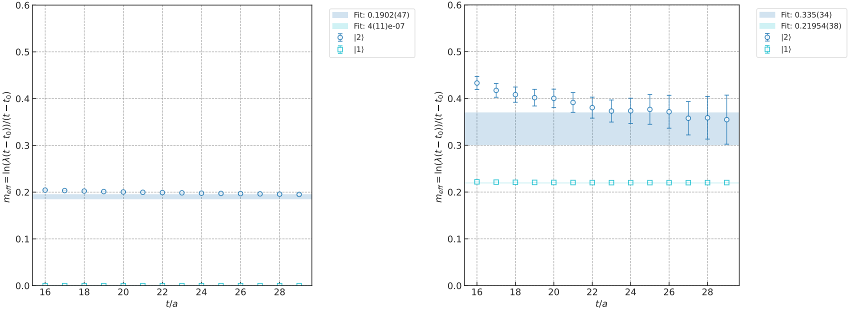

Figure 4: Scalar effective masses: left (connected+disconnected), right (connected only).

Mass Spectrum

Meson	E_1 (GeV)	E_2 (GeV)	E_1 (GeV)	E_2 (GeV)	PDG (GeV)	Width (MeV)
η_s	0.6983(54)	2.069(29)	0.806(21)	2.066(56)	–	–
ϕ	1.0355(79)	2.029(16)	1.035(11)	1.982(67)	1.020	4
f_0	1.462(11)	2.23(23)	1.267(33)	–	1.370	200
f_1	1.560(13)	2.49(14)	1.585(14)	2.56(29)	1.510	73
f_2	1.629(13)	2.382(65)	1.624(14)	2.64(14)	1.525	72
h_1	1.580(14)	2.45(16)	1.575(14)	2.643(93)	1.409	78
0^{--}	3.34(28)	–	3.65(22)	–	–	–
0^{+-}	2.546(60)	–	2.649(71)	–	–	–
1^{-+}	2.225(25)	2.92(19)	2.212(90)	2.63(35)	–	–
2^{--}	2.046(19)	2.70(36)	2.050(21)	3.14(23)	–	–
2^{-+}	2.041(23)	2.418(32)	2.040(28)	2.35(22)	–	–
2^{+-}	2.53(22)	–	2.20(32)	–	–	–

Table 1: Strangeonium mass spectrum obtained in this work. Ground states are stable; excited states require further study.

Strangeonium Spectrum

- Ground state masses are stable and reliable; excited states require further investigation.
- Discrepancies between ground state masses and experimental values may arise from decay channel effects, finite volume effects, and discretization errors.
- We also compute states with exotic quantum numbers. The 1^{-+} state has a mass of $2.212(90)$ GeV, which can be interpreted as a strangeonium-like candidate corresponding to $\eta_1(1855)$.
- Future work will involve larger volumes and smaller lattice spacings to control systematic uncertainties.

Mixing Mechanism

- Dynamical quarks allow glueballs to mix with mesons of the same J^{PC}.
- For 0^{++}, consider mixing between a pure glueball $|G\rangle$ and pure $|s\bar{s}\rangle$:

$$\begin{pmatrix} |g\rangle \\ |f_0\rangle \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} |G\rangle \\ |s\bar{s}\rangle \end{pmatrix}$$

- Hamiltonian:

$$\hat{H} = \begin{pmatrix} m_G & x \\ x & m_{s\bar{s}} \end{pmatrix}$$

- Mixing angle θ and mixing energy x related by:

$$m_\pm = \bar{m} \pm \frac{1}{2}\sqrt{\Delta^2 + 4x^2}, \quad \sin\theta = \mathrm{sgn}(x\Delta)\sqrt{\frac{\delta - 1}{2\delta}}$$

with $\Delta = m_G - m_{s\bar{s}}$, $\delta = \sqrt{1 + 4x^2/\Delta^2}$.

Extraction of Mixing Parameters

- Build correlation matrix:

$$\mathbf{C}(t) = \begin{pmatrix} C_{GG}(t) & C_{Gs}(t) \\ C_{sG}(t) & C_{ss}(t) \end{pmatrix}$$

- Variational method yields optimized operators:

$$\mathcal{O}^{(i)} = w_G^{(i)} \mathcal{O}_G + w_s^{(i)} \mathcal{O}_s$$

- Assuming $\langle 0|\mathcal{O}_s|G\rangle \approx 0$ and $\langle 0|\mathcal{O}_G|s\bar{s}\rangle \approx 0$, we get:

$$|\tan\theta| = \sqrt{-\frac{Z_s^2 Z_G^1}{Z_s^1 Z_G^2}} \approx \sqrt{-\frac{C_{s2}(t) C_{G1}(t)}{C_{s1}(t) C_{G2}(t)}}$$

Variational Results

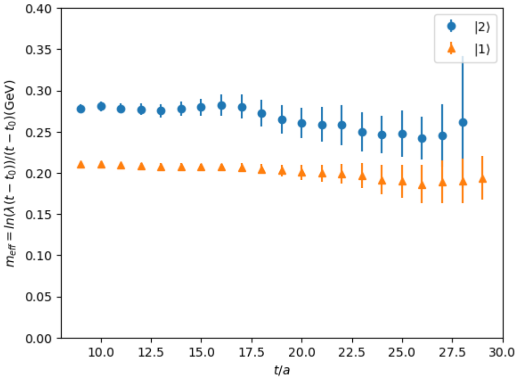

Figure 5: Variational results for the correlation matrix of scalar glueball and scalar meson.

Mixing Angle

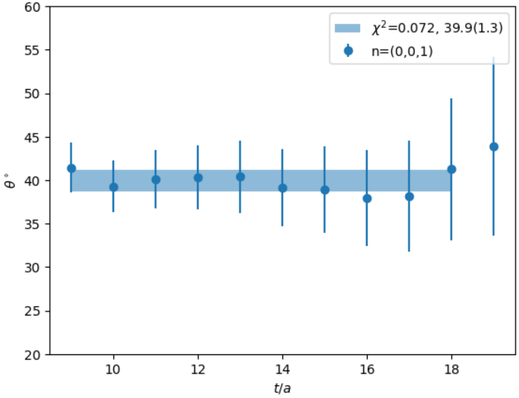

Figure 6: Extracted mixing angle.

Mixing Energy and Pure Glueball Mass

- Mixing angle: $\theta = 39.9^{\circ}(1.3)$.
- Mixing energy: $x = 237(21)$ MeV.
- Pure glueball mass: $m_G \approx 1.566(43)$ GeV.

Summary

- First lattice QCD calculation of strangeonium spectrum including disconnected diagrams on 1-flavor asymmetric configurations.
  - Ground state masses: η_s 0.806(21) GeV, f_0 1.462(11) GeV, f_1 1.585(14),f_2 1.624(14),h_1 1.575(14) etc.
  - Exotic 1^{-+} state at 2.212(90) GeV as a candidate for $\eta_1(1855)$.
  - mixing angle $\theta = 39.9(1.3)^\circ$, mixing energy $x = 237(21)$ MeV.
- Disconnected diagrams significantly affect pseudoscalar and scalar channels.
- Results provide first-principles inputs for phenomenology.
- **Outlook:** Larger volumes, smaller lattice spacings to control systematics; refine excited states and high-spin channels.

Thank you!

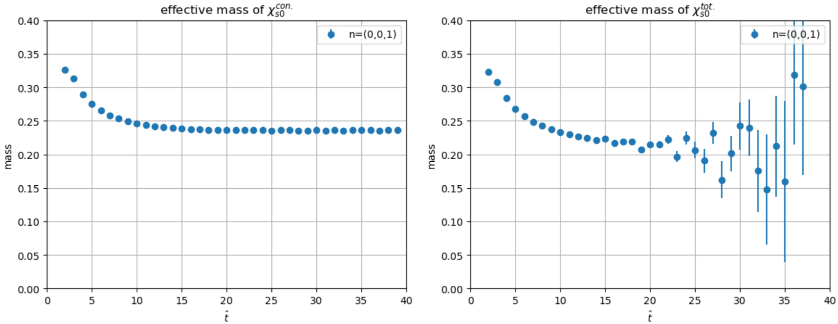

Figure 7: Effective mass of χ_{s0}

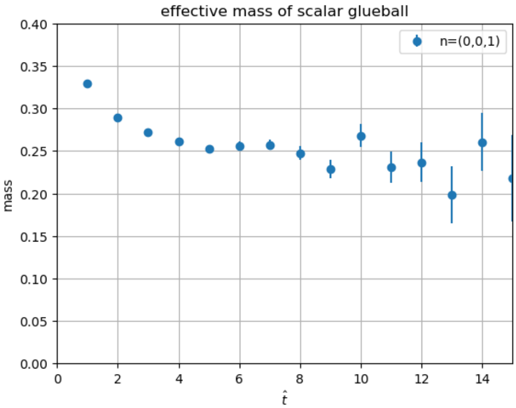

Figure 8: Effective mass of scalar glueball