The semi-leptonic decays $B \to D^{(*)}(1S, 2S)\ell\nu_\ell$ in LFQM

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Outline

Introduction

- The ground states $D_{(s)}$ and $D^*_{(s)}$ have been well determined in experiments, their produtions and decays have been extensively studied.
- While the first radially excited states $D_{(s)}(2S)$ and $D^*_{(s)}(2S)$ have not been well confirmed.
- For example both $D_0(2550)$ and $D_J(2580)$ are considered as the candidates of $D(2^1S_0)\frac{1}{2}(0^-)$ state.
- $D_{s0}(2590)$ observed by LHCb is suggested to a candidate of $D_s(2^1S_0)0(0^-)$ state. Some theoretical works suggest that it may not be a pure state but rather has D^*K component.

Introduction

- $D_1^*(2680)^0, D^*(2650)^0$ and $D_1^*(2600)^0$ observed by LHCb are considered as the candidated of netrual $D(2^3S_1)\frac{1}{2}(1^-)$. The $D^*(2640)^\pm$ discovered by Delphi have masses consistent with the predictions of charged $D(2^3S_1)\frac{1}{2}(1^-)$, while it has not been confirmed in any other experiments.
- Except for the assignment of $D_s(2^3S_1)0(1^-)$ to $D_{s1}^*(2700)^\pm$, $D_s(1^3D_1)$ and a mixture of $D_s(2^3S_1)$ and $D_s(1^3D_1)$ are also been proposed.
- We will assume these several particles discovered in experiments as the corresponding first excited D mesons in the calcualtions:
  $D_0(2550) \to D(2^1S_0),\ D_{s0}(2590)^\pm \to D_s(2^1S_0),$
  $D_1^*(2600)^0, D^*(2640)^\pm \to D(2^3S_1),\ D_{s1}^*(2700)^\pm \to D_s(2^3S_1).$

Introduction

At the end of the twentieth century, Jaus put forward the covariant light-front quark model (CLFQM). The CLFQM has some unique advantages:

- The light-front wave functions descrbing the hadron through quark and gluon degrees of freedom can preserve a Lorentz invariant formalism.
- The final state meson at $q^2 = 0$ is usually relativistic. The CLFQM with relativistic effects involved is suitable to study hadronic transition form factors.

The CLFQM general calculation procedure

The CLFQM general calculation procedure

The Bauer-Stech-Wirble (BSW) form factors for $B \to D$ and $B \to D^*$ transitions are defined as follows,

$$\begin{aligned}
\left\langle D\left(P''\right)\left|V_\mu\right|B\left(P'\right)\right\rangle &= \left(P_\mu - \frac{m_B^2 - m_D^2}{q^2} q_\mu\right) F_1^{BD}\left(q^2\right) + \frac{m_B^2 - m_D^2}{q^2} q_\mu F_0^{BD}\left(q^2\right), \\
\left\langle D^*\left(P'', \varepsilon''\right)\left|V_\mu\right|B\left(P'\right)\right\rangle &= -\frac{1}{m_{D^*} + m_B} \epsilon_{\mu\nu\alpha\beta} \varepsilon''^{*\nu} P^\alpha q^\beta V^{BD^*}\left(q^2\right), \\
\left\langle D^*\left(P'', \varepsilon''\right)\left|A_\mu\right|B\left(P'\right)\right\rangle &= i\left\{\left(m_{D^*} + m_B\right) \varepsilon''^*_\mu A_1^{BD^*}\left(q^2\right) - \frac{\varepsilon''^* \cdot P}{m_{D^*} + m_B} P_\mu A_2^{BD^*}\left(q^2\right)\right. \\
&\quad \left. -2 m_{D^*} \frac{\varepsilon''^* \cdot P}{q^2} q_\mu \left[A_3^{BD^*}\left(q^2\right) - A_0^{BD^*}\left(q^2\right)\right]\right\},
\end{aligned}$$

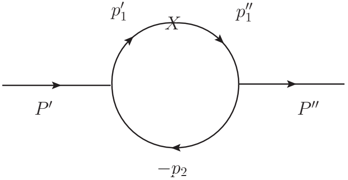
where $P = P' + P''$, $q = P' - P''$, ϵ is the polarization vector. The four-momentum of the

initial (final) meson is $P' = p_1' + p_2$ $(P'' = p_1'' + p_2)$.

The CLFQM general calculation procedure

The decay amplitude in the lowest order for the transitions $B \to D$ and $B \to D^*$

$$\begin{aligned} \mathcal{B}_\mu^{BD} &= -i^3 \frac{N_c}{(2\pi)^4} \int d^4 p_1' \frac{h_B' h_D''}{N_1' N_1'' N_2} S_\mu^{BD}, \\ \mathcal{B}_\mu^{BD^*} &= -i^3 \frac{N_c}{(2\pi)^4} \int d^4 p_1' \frac{h_B' \, (i h_{D^*}'')}{N_1' N_1'' N_2} S_{\mu\nu}^{BD^*} \varepsilon''^{*\nu}, \end{aligned} \tag{1}$$



- $N_1'^{(\prime\prime)} = p_1'^{(\prime\prime)2} - m_1'^{(\prime\prime)2}$, $N_2 = p_2^2 - m_2^2$ arise from the quark propagators.

The traces S_{μ}^{BD} and $S_{\mu\nu}^{BD^*}$ can be obtained directly using Lorentz contraction,

$$
\begin{aligned}
S_{\mu}^{BD} &= \mathrm{Tr}\left[\gamma_5\left(\not p_1'' + m_1''\right)\gamma_\mu\left(\not p_1' + m_1'\right)\gamma_5\left(-\not p_2 + m_2\right)\right], \\
S_{\mu\nu}^{BD^*} &= \left(S_V^{BD^*} - S_A^{BD^*}\right)_{\mu\nu} \\
&= \mathrm{Tr}\left[\left(\gamma_\nu - \frac{1}{W_V''}\left(p_1'' - p_2\right)_\nu\right)\left(\not p_1'' + m_1''\right)\left(\gamma_\mu - \gamma_\mu\gamma_5\right)\left(\not p_1' + m_1'\right)\gamma_5\left(-\not p_2 + m_2\right)\right],
\end{aligned}
\tag{2}
$$

The covariant vertex function h_M'' with $M = D(1S, 2S), D^*(1S, 2S)$ is defined as

$$
\begin{aligned}
h_M'' &= \left(M''^2 - M_0''^2\right)\sqrt{\frac{x_1 x_2}{N_c}}\frac{1}{\sqrt{2}\widetilde{M}_0''}\varphi, \\
M_0''^2 &= \left(e_1'' + e_2\right)^2 = \frac{p_\perp'^2 + m_1''^2}{x_1} + \frac{p_\perp^2 + m_2^2}{x_2}, \quad \widetilde{M}_0'' = \sqrt{M_0''^2 - \left(m_1'' - m_2\right)^2}.
\end{aligned}
\tag{3}
$$

The CLFQM general calculation procedure

The phenomenological Gaussian-type wave function φ depicts the light-front momentum distribution amplitude for the S-wave mesons,

$$\varphi(1S) = 4\left(\frac{\pi}{\beta^2}\right)^{\frac{3}{4}}\sqrt{\frac{dp_z}{dx_2}}\exp\left(-\frac{p_z^2+p_\perp^2}{2\beta^2}\right),$$
$$\varphi(2S) = 4\left(\frac{\pi}{\beta^2}\right)^{\frac{3}{4}}\sqrt{\frac{dp_z}{dx_2}}\exp\left(-\frac{p_z^2+p_\perp^2}{2\beta^2}\right)(3-2\frac{p_z^2+p_\perp^2}{\beta^2}),$$

- The shape parameter β can be fixed by fitting the corresponding decay constant.

Transition Form Factors

Input parameters:

- The constituent quark masses(GeV): $m_c = 1.4$, $m_s = 0.37$, $m_{u,d} = 0.25$;
- The masses of the initial and the final mesons(GeV): $m_D = 1.86966$,

  $m_{D(2S)} = 2.549$, $m_{D_s} = 1.96835$, $m_{D_s(2S)} = 2.591$,

  $m_{D_s^{\star\pm}} = 2.1122, m_{D^{\star 0}} = 2.0068, m_{D^{\star\pm}} = 2.0102$, $m_B = 5.279$,

  $m_{D_s^{\star\pm}(2S)} = 2.732$, $m_{D^{\star 0}(2S)} = 2.627, m_{D^{\star\pm}(2S)} = 2.637, m_{B_s^0} = 5.4154$.
- The CKM matrix element: $|V_{cb}| = (41.1 \pm 1.2) \times 10^{-3}$

Transition Form Factors

Input parameters:

- The shape parameters fitted by the decay constants:

$$\beta_{D_s^{\star\pm}(1S)} = 0.534^{+0.014}_{-0.014}, \beta_{D^{\star 0}(1S)} = 0.500^{+0.140}_{-0.187}, \beta_D = 0.466^{+0.022}_{-0.021},$$

$$\beta_{D_s^{\star\pm}(2S)} = 0.473^{+0.041}_{-0.041}, \beta_{D^{\star 0}(2S)} = 0.456^{+0.004}_{-0.003}, \beta_{D_s} = 0.600^{+0.026}_{-0.025},$$

$$\beta_{D^{\star\pm}(1S)} = 0.502^{+0.041}_{-0.041}, \beta_{D^{\star\pm}(2S)} = 0.453^{+0.004}_{-0.003},\ \beta_{\bar{B}_s^0} = 0.626^{+0.045}_{-0.045},$$

$$\beta_{D(2S)} = 0.297^{+0.041}_{-0.041}, \beta_{D_s(2S)} = 0.422^{+0.026}_{-0.025}, \beta_B = 0.555^{+0.060}_{-0.060}.$$

- mean life$(10^{-12}s)$: $\tau_{B^0} = (1.519 \pm 0.004)$, $\tau_{B_s^0} = (1.520 \pm 0.005)$,

  $\tau_{B^\pm} = (1.638 \pm 0.004)$.

Transition Form Factors

- All the calculations are carried out within the $q^+ = 0$ reference frame, where the form factors can only be obtained at spacelike momentum transfers $q^2 = -q_\perp^2 \leq 0$,
- The parameterized form factors are extrapolated from the space-like region to the time-like region by using

$$F\left(q^2\right) = \frac{F(0)}{\left(1 - q^2/m^2\right)\left[1 - a\left(q^2/m^2\right) + b\left(q^2/m^2\right)^2\right]}. \tag{4}$$

- $F(q^2)$ denotes different form factors, such as $F_1(q^2), F_0(q^2), V(q^2)$, $A_0(q^2), A_1(q^2),\ A_2(q^2)$.

Transition Form Factors

- The form factors of the transtions $B_{(s)} \to D_{(s)}(1S, 2S)$ in the CLFQM. The uncertainties are from the decay constants of $B_{(s)}$ and final state mesons.

	$F(0)$	$F(q^2_{max})$	a	b
F_1^{BD}	$0.66^{+0.00+0.01}_{-0.01-0.01}$	$0.81^{+0.01+0.01}_{-0.00-0.01}$	$0.80^{+0.01+0.04}_{-0.02-0.04}$	$0.86^{+0.01+0.02}_{-0.01-0.02}$
F_0^{BD}	$0.66^{+0.00+0.01}_{-0.01-0.01}$	$0.70^{+0.02+0.01}_{-0.03-0.01}$	$0.46^{+0.14+0.00}_{-0.12-0.01}$	$0.77^{+0.01+0.05}_{-0.01-0.05}$
$F_1^{BD(2S)}$	$0.26^{+0.01+0.01}_{-0.02-0.02}$	$0.34^{+0.02+0.01}_{-0.03-0.03}$	$0.99^{+0.04+0.16}_{-0.10-0.18}$	$0.66^{+0.07+0.17}_{-0.12-0.15}$
$F_0^{BD(2S)}$	$0.26^{+0.01+0.02}_{-0.01-0.02}$	$0.32^{+0.03+0.02}_{-0.00-0.04}$	$0.65^{+0.03+0.05}_{-0.01-0.04}$	$-0.24^{+0.02+0.01}_{-0.03-0.03}$
$F_1^{B_sD_s}$	$0.67^{+0.00+0.01}_{-0.00-0.01}$	$0.81^{+0.00+0.00}_{-0.01-0.01}$	$0.82^{+0.00+0.02}_{-0.01-0.02}$	$0.96^{+0.01+0.03}_{-0.02-0.03}$
$F_0^{B_sD_s}$	$0.67^{+0.00+0.01}_{-0.00-0.01}$	$0.71^{+0.01+0.01}_{-0.02-0.01}$	$0.48^{+0.00+0.08}_{-0.01-0.08}$	$0.85^{+0.02+0.06}_{-0.02-0.06}$
$F_1^{B_sD_s(2S)}$	$0.26^{+0.02+0.02}_{-0.02-0.02}$	$0.29^{+0.02+0.01}_{-0.02-0.02}$	$0.59^{+0.12+0.05}_{-0.16-0.01}$	$0.35^{+0.11+0.04}_{-0.13-0.02}$
$F_0^{B_sD_s(2S)}$	$0.26^{+0.01+0.01}_{-0.02-0.02}$	$0.25^{+0.02+0.01}_{-0.02-0.02}$	$-0.09^{+0.22+0.22}_{-0.14-0.26}$	$-0.07^{+0.13+0.18}_{-0.21-0.08}$

Transition Form Factors

- The form factors of the transtions $B \to D^*(1S, 2S)$ in the CLFQM. The uncertainties are from the decay constants of B and final state mesons.

F	F(0)	$F(q^2_{max})$	a	b
V^{BD^*}	$0.77^{+0.00+0.01}_{-0.00-0.01}$	$0.94^{+0.00+0.01}_{-0.00-0.01}$	$0.78^{+0.00+0.03}_{-0.01-0.03}$	$0.82^{+0.04+0.14}_{-0.08-0.16}$
$A_0^{BD^*}$	$0.75^{+0.00+0.06}_{-0.01-0.02}$	$0.79^{+0.00+0.05}_{-0.02-0.02}$	$0.17^{+0.01+0.01}_{-0.01-0.00}$	$0.12^{+0.07+0.02}_{-0.06-0.03}$
$A_1^{BD^*}$	$0.67^{+0.00+0.01}_{-0.01-0.11}$	$0.76^{+0.00+0.01}_{-0.01-0.01}$	$0.38^{+0.01+0.01}_{-0.01-0.01}$	$0.19^{+0.05+0.09}_{-0.05-0.09}$
$A_2^{BD^*}$	$0.58^{+0.00+0.00}_{-0.01-0.02}$	$0.69^{+0.01+0.01}_{-0.01-0.02}$	$0.68^{+0.02+0.00}_{-0.02-0.00}$	$0.66^{+0.25+0.10}_{-0.12-0.16}$
$V^{BD^*(2S)}$	$0.19^{+0.05+0.01}_{-0.06-0.01}$	$0.19^{+0.01+0.03}_{-0.04-0.01}$	$0.11^{+0.05+0.03}_{-0.06-0.05}$	$0.34^{+0.00+0.03}_{-0.04-0.06}$
$A_0^{BD^*(2S)}$	$0.27^{+0.04+0.01}_{-0.05-0.00}$	$0.28^{+0.00+0.01}_{-0.02-0.00}$	$0.24^{+0.05+0.00}_{-0.06-0.00}$	$-0.07^{+0.15+0.03}_{-0.29-0.06}$
$A_1^{BD^*(2S)}$	$0.16^{+0.04+0.01}_{-0.05-0.00}$	$0.15^{+0.05+0.02}_{-0.04-0.02}$	$-0.23^{+0.04+0.00}_{-0.04-0.00}$	$0.17^{+0.03+0.01}_{-0.01-0.01}$
$A_2^{BD^*(2S)}$	$-0.05^{+0.04+0.01}_{-0.04-0.00}$	$0.01^{+0.00+0.00}_{-0.00-0.00}$	$-1.10^{+0.02+0.01}_{-0.01-0.01}$	$-0.63^{+0.02+0.01}_{-0.01-0.00}$

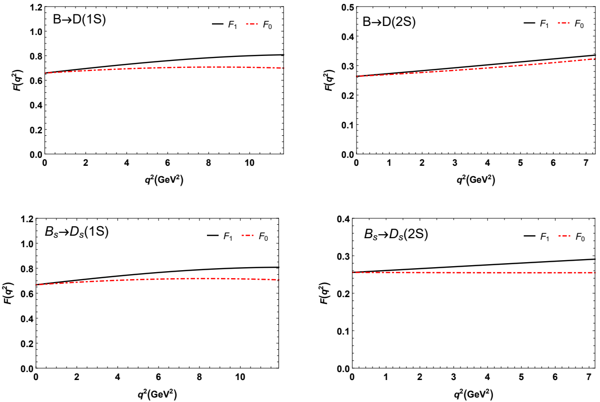
Transition Form Factors

- The form factors of the transtions $B_s \to D_s^*(1S, 2S)$ in the CLFQM. The uncertainties are from the decay constants of B_s and final state mesons.

F	F(0)	$F(q^2_{max})$	a	b
$V^{B_sD_s^*}$	$0.78^{+0.01+0.01}_{-0.01-0.01}$	$0.87^{+0.00+0.00}_{-0.00-0.01}$	$0.86^{+0.01+0.04}_{-0.01-0.04}$	$1.11^{+0.01+0.02}_{-0.01-0.02}$
$A_0^{B_sD_s^*}$	$0.74^{+0.01+0.01}_{-0.01-0.01}$	$0.69^{+0.01+0.01}_{-0.01-0.00}$	$0.23^{+0.01+0.01}_{-0.01-0.01}$	$0.21^{+0.00+0.01}_{-0.00-0.01}$
$A_1^{B_sD_s^*}$	$0.66^{+0.01+0.01}_{-0.02-0.02}$	$0.95^{+0.00+0.00}_{-0.00-0.01}$	$0.81^{+0.01+0.02}_{-0.01-0.02}$	$0.93^{+0.01+0.01}_{-0.01-0.01}$
$A_2^{B_sD_s^*}$	$0.57^{+0.00+0.00}_{-0.01-0.00}$	$0.68^{+0.01+0.00}_{-0.01-0.00}$	$0.80^{+0.00+0.03}_{-0.01-0.03}$	$0.96^{+0.01+0.03}_{-0.01-0.02}$
$V^{B_sD_s^*(2S)}$	$0.26^{+0.03+0.04}_{-0.03-0.04}$	$0.28^{+0.00+0.02}_{-0.09-0.10}$	$0.25^{+0.02+0.04}_{-0.01-0.04}$	$0.30^{+0.03+0.04}_{-0.01-0.00}$
$A_0^{B_sD_s^*(2S)}$	$0.31^{+0.02+0.01}_{-0.03-0.02}$	$0.33^{+0.00+0.08}_{-0.01-0.07}$	$0.21^{+0.01+0.03}_{-0.05-0.00}$	$-0.09^{+0.04+0.10}_{-0.02-0.14}$
$A_1^{B_sD_s^*(2S)}$	$0.21^{+0.02+0.03}_{-0.03-0.03}$	$0.20^{+0.00+0.01}_{-0.07-0.06}$	$-0.16^{+0.20+0.04}_{-0.17-0.02}$	$0.12^{+0.00+0.12}_{-0.01-0.07}$
$A_2^{B_sD_s^*(2S)}$	$-0.01^{+0.02+0.06}_{-0.02-0.06}$	$0.01^{+0.00+0.03}_{-0.01-0.01}$	$-4.03^{+0.51+0.75}_{-0.47-1.38}$	$4.31^{+0.00+2.13}_{-0.00-2.44}$

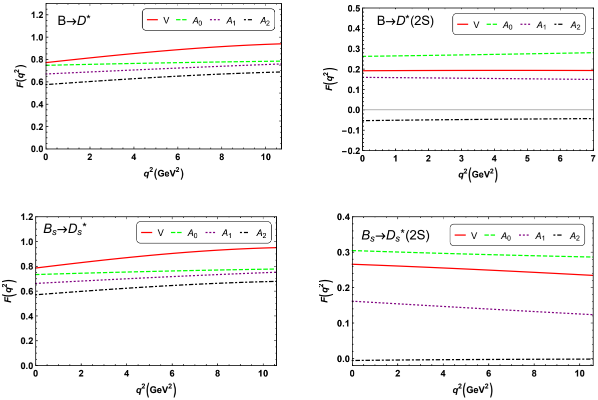
Transition Form Factors

图: The q^2-dependence of the $B_{(s)} \to D_{(s)}(1S, 2S)$ transtion form factors.



Transition Form Factors

图: The q^2-dependence of the $B_{(s)} \to D^*_{(s)}(1S, 2S)$ transtion form factors.



Semi-leptonic Decays

$$B \to D^{(*)}(1S, 2S)\ell\nu_\ell$$

The differential decay widths of the semileptonic $B_{(s)}$ decays can be obtained by the combinations of the helicity amplitudes via the form factors

$$\frac{d\Gamma(B_{(s)} \to D_{(s)}\ell\nu)}{dq^2} = (1-\hat{m}_\ell^2)^2 \frac{\sqrt{\lambda(q^2)}G_F^2|V_{cb}|^2}{384 m_{B_{(s)}}^3 \pi^3} \Big\{ (\hat{m}_\ell^2+2)\,\lambda(q^2)F_1^2(q^2) + 3\hat{m}_\ell^2(m_{B_{(s)}}^2 - m_{D_{(s)}}^2)^2 F_0^2(q^2) \Big\},$$

$$\frac{d\Gamma_L(B_{(s)} \to D^*_{(s)}\ell\nu)}{dq^2} = (1-\hat{m}_\ell^2)^2 \frac{\sqrt{\lambda(q^2)}G_F^2|V_{cb}|^2}{384 m_{B_{(s)}}^3 \pi^3} \Big\{ 3\hat{m}_\ell^2\lambda(q^2)A_0^2(q^2) + (\hat{m}_\ell^2+2) \times \Big| \frac{1}{2m_{D^*_{(s)}}} \Big[(m_{B_{(s)}}^2 - m_{D^*_{(s)}}^2 - q^2)(m_{B_{(s)}} + m_{D^*_{(s)}})A_1(q^2) - \frac{\lambda(q^2)}{m_{B_{(s)}} + m_{D^*_{(s)}}} A_2(q^2) \Big] \Big|^2 \Big\},$$

Semi-leptonic Decays $B \to D^{(*)}(1S, 2S)\ell\nu_\ell$

$$\frac{d\Gamma^{\pm}(B_{(s)} \to D^*_{(s)}\ell\nu)}{dq^2} = (1-\hat{m}_\ell^2)^2 \frac{\sqrt{\lambda(q^2)}\, G_F^2 |V_{cb}|^2}{384 m_{B_{(s)}}^3 \pi^3} \Bigg\{ (m_\ell^2 + 2q^2)\lambda(q^2) \times \left| \frac{V(q^2)}{m_{B_{(s)}} + m_{D^*_{(s)}}} \mp \frac{(m_{B_{(s)}} + m_{D^*_{(s)}})A_1(q^2)}{\sqrt{\lambda(q^2)}} \right|^2 \Bigg\},$$

where $\lambda(q^2) = (m_{B_{(s)}}^2 + m_{D^*_{(s)}}^2 - q^2)^2 - 4m_{B_{(s)}}^2 m_{D^*_{(s)}}^2$, $\hat{m}_\ell^2 = m_\ell^2/q^2$ and m_ℓ is the mass of the lepton ℓ with $\ell = e, \mu, \tau$. The combined transverse and total differential decay widths are defined as

$$\frac{d\Gamma_T}{dq^2} = \frac{d\Gamma_+}{dq^2} + \frac{d\Gamma_-}{dq^2}, \quad \frac{d\Gamma}{dq^2} = \frac{d\Gamma_L}{dq^2} + \frac{d\Gamma_T}{dq^2}. \tag{5}$$

For the $B_{(s)} \to D^*_{(s)}(1S, 2S)\ell\nu_\ell$ decays, defining the polarization fraction is important owing to the existence of different polarizations

$$f_L = \frac{\Gamma_L}{\Gamma_L + \Gamma_+ + \Gamma_-}. \quad (6)$$

The analytical expression of the forward-backward asymmetry is defined as

$$A_{FB} = \frac{\int_0^1 \frac{d\Gamma}{dcos\theta} dcos\theta - \int_{-1}^0 \frac{d\Gamma}{dcos\theta} dcos\theta}{\int_{-1}^1 \frac{d\Gamma}{dcos\theta} dcos\theta} = \frac{\int b_\theta(q^2) dq^2}{\Gamma_{B \to D^{(*)}\ell\nu_\ell}}. \quad (7)$$

Semi-leptonic Decays $B \to D^{(*)}(1S, 2S)\ell\nu_\ell$

- The branching ratios of the decays $B \to D(1S)\ell^+\nu_\ell(\%)$.

Modes	$\mathcal{B}(B^0 \to D^-(1S)e^+\nu_e)$	$\mathcal{B}(B^0 \to D^-(1S)\mu^+\nu_\mu)$	$\mathcal{B}(B^0 \to D^-(1S)\tau^+\nu_\tau)$
This work(CLFQM)	$1.96^{+0.06}_{-0.06}(\sim 1.5\sigma)$	$1.96^{+0.06}_{-0.06}(\sim 1.5\sigma)$	$0.51^{+0.02}_{-0.02}(\sim 2.2\sigma)$
PQCD	2.19	2.19	0.82
PQCD+Lattice	1.95	1.95	0.62
CQM	2.74	2.74	0.73
HQET	–	–	0.64
PQCD	2.03	2.03	0.87
CPQCD	–	1.65	0.554
PDG	2.10 ± 0.07	2.10 ± 0.07	0.98 ± 0.21
Modes	$\mathcal{B}(B^+ \to D^0(1S)e^+\nu_e)$	$\mathcal{B}(B^+ \to D^0(1S)\mu^+\nu_\mu)$	$\mathcal{B}(B^+ \to D^0(1S)\tau^+\nu_\tau)$
This work(CLFQM)	$2.11^{+0.06}_{-0.06}(\sim 1.6\sigma)$	$2.11^{+0.06}_{-0.06}(\sim 1.6\sigma)$	$0.55^{+0.02}_{-0.02}(\sim 0.9\sigma)$
RQM	2.53	2.53	0.68
PQCD	2.29	2.29	0.86
PQCD+Lattice	2.10	2.10	0.69
LQCD	–	–	0.65
HQET	–	–	0.66
PQCD	2.19	2.19	0.95
PDG	2.26 ± 0.07	2.26 ± 0.07	0.77 ± 0.25

Semi-leptonic Decays $B \to D^{(*)}(1S, 2S)\ell\nu_\ell$

- The branching ratios of the decays $B_s \to D_s(1S)\ell^+\nu_\ell(\%)$.

Modes	$\mathcal{B}(B_s^0 \to D_s^-(1S)e^+\nu_e)$	$\mathcal{B}(B_s^0 \to D_s^-(1S)\mu^+\nu_\mu)$	$\mathcal{B}(B_s^0 \to D_s^-(1S)\tau^+\nu_\tau)$
This work(CLFQM)	$2.05^{+0.12}_{-0.13}$	$2.04^{+0.11}_{-0.11}(\sim 1.1\sigma)$	$0.53^{+0.04}_{-0.07}$
NCQM	2.32	–	0.67
LCSR	1.817	1.817	0.606
QCDSR	2.03	2.03	–
LCSR	1.0	1.0	0.33
pQCD	2.13	2.13	0.84
RQM	2.1	2.1	0.62
CQM	$2.73 - 3.00$	$2.73 - 3.00$	–
pQCD	1.97	1.97	0.72
pQCD+Lattice	1.84	1.84	0.63
LQCD	$2.013 - 2.469$	$2.013 - 2.469$	$0.619 - 0.724$
BS	$1.4 - 1.7$	$1.4 - 1.7$	$0.47 - 0.55$
LQCD	2.31	2.31	0.69
CQM	2.89	2.88	0.78
PDG	–	2.29 ± 0.21	–

Semi-leptonic Decays $B \to D^{(*)}(1S, 2S)\ell\nu_\ell$

- The branching ratios of the decays $B \to D^*(1S)\ell^+\nu_\ell(\%)$.

Modes	$B^0 \to D^{*-}(1S)e^+\nu_e$	$B^0 \to D^{*-}(1S)\mu^+\nu_\mu$	$B^0 \to D^{*-}(1S)\tau^+\nu_\tau$
This work	$4.97^{+0.36}_{-0.16}(\sim 0.3\sigma)$	$4.95^{+0.36}_{-0.16}(\sim 0.3\sigma)$	$1.13^{+0.07}_{-0.04}(\sim 3.1\sigma)$
RQM	6.28	6.28	1.45
PQCD	5.32	5.32	1.53
PQCD+Lattice	4.63	4.63	1.25
PQCD	4.52	4.52	1.36
HQET	–	–	1.29
CQM	6.64	6.64	1.57
CPQCD	–	4.33	1.175
PDG	4.87 ± 0.09	4.87 ± 0.09	1.48 ± 0.09
Modes	$B^+ \to D^{*0}(1S)e^+\nu_e$	$B^+ \to D^{*0}(1S)\mu^+\nu_\mu$	$B^+ \to D^{*0}(1S)\tau^+\nu_\tau$
This work	$5.37^{+0.39}_{-0.17}(\sim 0.3\sigma)$	$5.35^{+0.38}_{-0.17}(\sim 0.3\sigma)$	$1.22^{+0.08}_{-0.04}(\sim 3.1\sigma)$
RQM	6.81	6.77	1.52
PQCD	5.53	5.53	1.60
PQCD+Lattice	4.89	4.89	1.34
PQCD	4.87	4.87	1.47
HQET	–	–	1.34
PDG	5.26 ± 0.10	5.26 ± 0.10	1.88 ± 0.20

Semi-leptonic Decays $B \to D^{(*)}(1S, 2S)\ell\nu_\ell$

- The branching ratios of the decays $B_s \to D_s^*(1S)\ell^+\nu_\ell(\%)$.

Modes	$\mathcal{B}(B_s^0 \to D_s^{*-}(1S)e^+\nu_e)$	$\mathcal{B}(B_s^0 \to D_s^{*-}(1S)\mu^+\nu_\mu)$	$\mathcal{B}(B_s^0 \to D_s^{*-}(1S)\tau^+\nu_\tau)$
This work(CLFQM)	$5.02^{+0.48}_{-0.54}$	$5.00^{+0.48}_{-0.53}(\sim 0.3\sigma)$	$1.13^{+0.09}_{-0.10}$
NCQM	6.26	$-$	1.53
PQCD	4.76	4.76	1.44
RQM	5.3	5.3	1.3
RQM	2.54	2.54	0.70
CQM	$7.49 - 7.66$	$7.49 - 7.66$	$-$
pQCD	5.04	5.04	1.45
pQCD+Lattice	4.42	4.42	1.20
BS	$5.1 - 5.8$	$5.1 - 5.8$	$1.2 - 1.3$
LQCD	5.25	5.25	1.31
CQM	$1.89 - 6.61$	$1.89 - 6.61$	$-$
CQM	6.42	6.39	1.53
PDG	$-$	5.2 ± 0.5	$-$

Semi-leptonic Decays $B \to D^{(*)}(1S, 2S)\ell\nu_\ell$

- The branching ratios of the decays $B \to D^{(*)}(2S)\ell\nu_\ell(10^{-3})$.

Modes	This work	BS	Modes	This work	BS
$\mathcal{B}(B^0 \to D^-(2S)e^+\nu_e)$	$1.44^{+0.00+0.18+0.15}_{-0.00-0.19-0.19}$	0.139	$\mathcal{B}(B^+ \to D^0(2S)e^+\nu_e)$	$1.55^{+0.00+0.19+0.17}_{-0.00-0.21-0.20}$	0.15
$\mathcal{B}(B^0 \to D^-(2S)\mu^+\nu_\mu)$	$1.43^{+0.00+0.18+0.15}_{-0.00-0.19-0.19}$	0.138	$\mathcal{B}(B^+ \to D^0(2S)\mu^+\nu_\mu)$	$1.54^{+0.00+0.19+0.17}_{-0.00-0.21-0.20}$	0.149
$\mathcal{B}(B^0 \to D^-(2S)\tau^+\nu_\tau)$	$0.18^{+0.00+0.03+0.03}_{-0.00-0.03-0.03}$	0.0135	$\mathcal{B}(B^+ \to D^0(2S)\tau^+\nu_\tau)$	$0.19^{+0.00+0.03+0.03}_{-0.00-0.04-0.04}$	0.0149
$\mathcal{B}(B^0 \to D^{*-}(2S)e^+\nu_e)$	$2.67^{+0.00+0.20+0.11}_{-0.00-0.24-0.14}$	0.1942	$\mathcal{B}(B^+ \to D^{*0}(2S)e^+\nu_e)$	$2.93^{+0.00+0.21+0.13}_{-0.00-0.26-0.16}$	0.2123
$\mathcal{B}(B^0 \to D^{*-}(2S)\mu^+\nu_\mu)$	$2.65^{+0.00+0.20+0.11}_{-0.00-0.24-0.14}$	0.1932	$\mathcal{B}(B^+ \to D^{*0}(2S)\mu^+\nu_\mu)$	$2.91^{+0.00+0.21+0.13}_{-0.00-0.26-0.16}$	0.2113
$\mathcal{B}(B^0 \to D^{*-}(2S)\tau^+\nu_\tau)$	$0.19^{+0.00+0.02+0.10}_{-0.00-0.02-0.11}$	0.0137	$\mathcal{B}(B^+ \to D^{*0}(2S)\tau^+\nu_\tau)$	$0.21^{+0.00+0.02+0.11}_{-0.00-0.03-0.11}$	0.0155

- The branching ratios of the decays $B_s \to D_s^{(*)}(2S)\ell\nu_\ell(10^{-3})$.

Modes	This work	BS	RQM	Modes	This work	BS	RQM
$\mathcal{B}(B_s^0 \to D_s^-(2S)e^+\nu_e)$	$1.35^{+0.00+0.25+0.14}_{-0.00-0.23-0.22}$	0.314	2.7	$\mathcal{B}(B_s^0 \to D_s^{*-}(2S)e^+\nu_e)$	$2.25^{+0.01+1.09+0.64}_{-0.01-0.70-0.80}$	0.5873	3.8
$\mathcal{B}(B_s^0 \to D_s^-(2S)\mu^+\nu_\mu)$	$1.34^{+0.00+0.25+0.14}_{-0.00-0.23-0.22}$	0.312	–	$\mathcal{B}(B_s^0 \to D_s^{*-}(2S)\mu^+\nu_\mu)$	$2.23^{+0.01+1.09+0.64}_{-0.01-0.70-0.80}$	0.5842	–
$\mathcal{B}(B_s^0 \to D_s^-(2S)\tau^+\nu_\tau)$	$0.14^{+0.00+0.03+0.02}_{-0.00-0.03-0.03}$	0.0244	0.11	$\mathcal{B}(B_s^0 \to D_s^{*-}(2S)\tau^+\nu_\tau)$	$0.15^{+0.00+0.05+0.06}_{-0.00-0.07-0.06}$	0.0405	0.15

Semi-leptonic Decays $B \to D^{(*)}(1S, 2S)\ell\nu_\ell$

- For the decays $B_{u,d} \to D\ell'\nu_{\ell'}$ and $B_s \to D_s\ell'\nu_{\ell'}$ with $\ell' = e, \mu$, their branching ratios deviate from the PDG data by about 1.5σ and 1.1σ. For the decays $B_{u,d} \to D^*\ell'\nu_{\ell'}$ and $B_s \to D_s^*\ell'\nu_{\ell'}$, the deviations are about 0.3σ.
- The branching ratios of all the considered decays with $\tau\nu_\tau$ involved exihibit systematically smaller than the experimental values. The deviations between our predictions and the data for the decays $B^0 \to D^{(*)-}\tau^+\nu_\tau$, $B^+ \to D^{*0}\tau^+\nu_\tau$ exceed 2σ or even 3σ.
- The branching ratios of the decays $B \to D^{(*)}(2S)\ell\nu_\ell$ lie in the range $10^{-4} \sim 10^{-3}$, which are about one order larger than those predicted by BS equation. For the decays $B_s \to D_s^{(*)}(2S)\ell\nu_\ell$, our predictions are $3 \sim 5$ times of those given by BS equation, while are consistent with RQM calculations.

Semi-leptonic Decays $B \to D^{(*)}(1S, 2S)\ell\nu_\ell$

- As one of the pillars of the SM, the ratio $R(D^{(*)})$ is a powerful test of the LFU,

$$\mathcal{R}\left(D^{(*)}\right) = \frac{Br\left(B \to D^{(*)}\tau\nu\right)}{Br\left(B \to D^{(*)}\ell'\nu_{\ell'}\right)}. \quad (8)$$

	$\mathcal{R}\left(D^{-}(1S)\right)$	$\mathcal{R}\left(D^{0}(1S)\right)$	$\mathcal{R}\left(D^{*-}(1S)\right)$	$\mathcal{R}\left(D^{*0}(1S)\right)$
This work	$0.260 \pm 0.014(\sim 3.1\sigma)$	0.261 ± 0.013	$0.228 \pm 0.026(\sim 2.1\sigma)$	0.228 ± 0.026
RQM	–	0.269	0.231	0.224
PQCD+Lattice	0.318	0.329	0.270	0.274
PQCD	0.429	0.434	0.301	0.302
Belle-II	0.418	–	0.306	–
LHCb	0.249	–	0.402	–
HFLAF	0.347 ± 0.025	-	0.288 ± 0.012	-
	$\mathcal{R}\left(D^{-}(2S)\right)$	$\mathcal{R}\left(D^{0}(2S)\right)$	$\mathcal{R}\left(D^{*-}(2S)\right)$	$\mathcal{R}\left(D^{*0}(2S)\right)$
This work	0.125 ± 0.037	0.123 ± 0.039	0.072 ± 0.0431	0.072 ± 0.043
BS	0.097	0.099	0.071	0.073
	$\mathcal{R}\left(D_s^{-}(1S)\right)$	$\mathcal{R}\left(D_s^{*-}(1S)\right)$	$\mathcal{R}\left(D_s^{-}(2S)\right)$	$\mathcal{R}\left(D_s^{*-}(2S)\right)$
This work	0.259 ± 0.033	0.226 ± 0.031	0.104 ± 0.038	0.067 ± 0.052
RQM	0.276	0.276	–	–
RQM	0.295	0.245	–	–
pQCD+Lattice	0.342	0.271	–	–
BS	–	–	0.078	0.069

Semi-leptonic Decays $B \to D^{(*)}(1S, 2S)\ell\nu_\ell$

- Forward-backward asymmetries A_{FB} for the decays $B \to D^{(*)}(1S)\ell\nu_\ell$.

Channels	$B^0 \to D^-(1S)e^+\nu_e$	$B^0 \to D^-(1S)\mu^+\nu_\mu$	$B^0 \to D^-(1S)\tau^+\nu_\tau$
A_{FB}	$4.4^{+0.0+0.0+0.1}_{-0.0-0.0-0.1} \times 10^{-7}$	$0.02^{+0.00+0.00+0.00}_{-0.00-0.00-0.00}$	$0.36^{+0.00+0.00+0.01}_{-0.00-0.01-0.01}$
CQM	-11.7×10^{-7}	–	-0.36
PQCD	–	–	0.35
PQCD+Lattice	–	–	0.36
Channels	$B^+ \to D^0(1S)e^+\nu_e$	$B^+ \to D^0(1S)\mu^+\nu_\mu$	$B^+ \to D^0(1S)\tau^+\nu_\tau$
A_{FB}	$4.4^{+0.00+0.00+0.01}_{-0.00-0.00-0.00} \times 10^{-7}$	$0.02^{+0.00+0.00+0.00}_{-0.00-0.00-0.00}$	$0.36^{+0.00+0.04+0.01}_{-0.00-0.05-0.01}$
RQM	-9.8×10^{-7}	-0.013	-0.37
Channels	$B_s^0 \to D_s^-(1S)e^+\nu_e$	$B_s^0 \to D_s^-(1S)\mu^+\nu_\mu$	$B_s^0 \to D_s^-(1S)\tau^+\nu_\tau$
A_{FB}	$4.4^{+0.0+0.0+0.1}_{-0.0-0.0-0.1} \times 10^{-7}$	$0.02^{+0.00+0.00+0.00}_{-0.00-0.00-0.00}$	$0.36^{+0.00+0.04+0.02}_{-0.00-0.02-0.01}$
RQM	-9.7×10^{-7}	-0.013	-0.36
PQCD	–	–	0.36
PQCD+Lattice	–	–	0.36

Channels	$B^0 \to D^{*-}(1S)e^+\nu_e$	$B^0 \to D^{*-}(1S)\mu^+\nu_\mu$	$B^0 \to D^{*-}(1S)\tau^+\nu_\tau$
A_{FB}	$-0.20^{+0.00+0.00+0.00}_{-0.00-0.01-0.01}$	$-0.20^{+0.00+0.00+0.00}_{-0.00-0.01-0.01}$	$-0.14^{+0.00+0.00+0.01}_{-0.00-0.00-0.04}$
CQM	0.19	–	0.027
PQCD	–	–	-0.085
PQCD+Lattice	–	–	-0.054
HQET	–	–	-0.084
Channels	$B^+ \to D^{*0}(1S)e^+\nu_e$	$B^+ \to D^{*0}(1S)\mu^+\nu_\mu$	$B^+ \to D^{*0}(1S)\tau^+\nu_\tau$
A_{FB}	$-0.20^{+0.00+0.01+0.00}_{-0.00-0.00-0.01}$	$-0.20^{+0.00+0.01+0.00}_{-0.00-0.00-0.01}$	$-0.14^{+0.00+0.00+0.01}_{-0.00-0.00-0.04}$
RQM	-0.22	-0.23	-0.32
Channels	$B_s^0 \to D_s^{*-}(1S)e^+\nu_e$	$B_s^0 \to D_s^{*-}(1S)\mu^+\nu_\mu$	$B_s^0 \to D_s^{*-}(1S)\tau^+\nu_\tau$
A_{FB}	$-0.20^{+0.00+0.01+0.02}_{-0.00-0.00-0.01}$	$-0.20^{+0.00+0.01+0.02}_{-0.00-0.00-0.01}$	$-0.14^{+0.00+0.00+0.01}_{-0.00-0.00-0.01}$
RQM	-0.26	-0.27	-0.32
PQCD	–	–	-0.083
PQCD+Lattice	–	–	-0.050

Semi-leptonic Decays $B \to D^{(*)}(1S, 2S)\ell\nu_\ell$

- Forward-backward asymmetries A_{FB} for the decays $B \to D^{(*)}(2S)\ell\nu_\ell$.

Channel	$B^0 \to D^-(2S)e^+\nu_e$	$B^0 \to D^-(2S)\mu^+\nu_\mu$	$B^0 \to D^-(2S)\tau^+\nu_\tau$
$\mathcal{A}_{\rm FB}$	$6.6^{+0.0+0.8+0.8}_{-0.0-0.9-0.7} \times 10^{-7}$	$0.02^{+0.00+0.00+0.00}_{-0.00-0.00-0.00}$	$0.35^{+0.00+0.05+0.05}_{-0.00-0.06-0.06}$
Channels	$B^+ \to D^0(2S)e^+\nu_e$	$B^+ \to D^0(2S)\mu^+\nu_\mu$	$B^+ \to D^0(2S)\tau^+\nu_\tau$
$\mathcal{A}_{\rm FB}$	$6.6^{+0.0+0.8+0.7}_{-0.0-0.9-0.8} \times 10^{-7}$	$0.02^{+0.00+0.00+0.00}_{-0.00-0.00-0.00}$	$0.35^{+0.00+0.05+0.05}_{-0.00-0.06-0.06}$
Channels	$B_s^0 \to D_s^-(2S)e^+\nu_e$	$B_s^0 \to D_s^-(2S)\mu^+\nu_\mu$	$B_s^0 \to D_s^-(2S)\tau^+\nu_\tau$
$\mathcal{A}_{\rm FB}$	$6.6^{+0.0+1.1+0.6}_{-0.0-1.0-1.0} \times 10^{-7}$	$0.02^{+0.00+0.00+0.00}_{-0.00-0.00-0.00}$	$0.37^{+0.00+0.07+0.06}_{-0.00-0.08-0.08}$
Channels	$B^0 \to D^{*-}(2S)e^+\nu_e$	$B^0 \to D^{*-}(2S)\mu^+\nu_\mu$	$B^0 \to D^{*-}(2S)\tau^+\nu_\tau$
$\mathcal{A}_{\rm FB}$	$-0.12^{+0.00+0.01+0.06}_{-0.00-0.02-0.05}$	$-0.11^{+0.00+0.01+0.06}_{-0.00-0.02-0.05}$	$-0.05^{+0.00+0.00+0.03}_{-0.00-0.02-0.03}$
Channels	$B^+ \to D^{*0}(2S)e^+\nu_e$	$B^+ \to D^{*0}(2S)\mu^+\nu_\mu$	$B^+ \to D^{*0}(2S)\tau^+\nu_\tau$
$\mathcal{A}_{\rm FB}$	$-0.12^{+0.00+0.01+0.06}_{-0.00-0.02-0.05}$	$-0.12^{+0.00+0.01+0.06}_{-0.00-0.02-0.05}$	$-0.05^{+0.00+0.00+0.03}_{-0.00-0.02-0.03}$
Channels	$B_s^0 \to D_s^{*-}(2S)e^+\nu_e$	$B_s^0 \to D_s^{*-}(2S)\mu^+\nu_\mu$	$B_s^0 \to D_s^{*-}(2S)\tau^+\nu_\tau$
$\mathcal{A}_{\rm FB}$	$-0.13^{+0.00+0.04+0.06}_{-0.00-0.06-0.05}$	$-0.13^{+0.00+0.04+0.06}_{-0.00-0.06-0.05}$	$-0.07^{+0.00+0.02+0.04}_{-0.00-0.06-0.03}$

Semi-leptonic Decays $B \to D^{(*)}(1S, 2S)\ell\nu_\ell$

- The values of $\mathcal{A}_{FB}(B_{(s)} \to D_{(s)}\ell\nu_\ell)$ exhibit a clear hierarchical relationship, While the difference among $\mathcal{A}_{FB}(B_{(s)} \to D^*_{(s)}\ell\nu_\ell)$ is not significant.
- $\mathcal{A}_{\text{FB}}(B_{(s)} \to D_{(s)}\ell\nu_\ell)$ are positive, while $\mathcal{A}_{\text{FB}}(B_{(s)} \to D^*_{(s)}\ell\nu_\ell)$ are negative.
- Our predictions are consistent with those given by the PQCD, PQCD+Lattice and HQET approaches, while have the opposite sign compared to the calculations in CQM and RQM except for those for the decays $B_{(s)} \to D^*_{(s)}\ell\nu_\ell$.
- The forward-backward asymmetries of the decays $B_{(s)} \to D^{(*)}_{(s)}(2S)\ell\nu_\ell$ are similar with those of the corresponding decays $B_{(s)} \to D^{(*)}_{(s)}(1S)\ell\nu_\ell$.

Semi-leptonic Decays $B \to D^{(*)}(1S, 2S)\ell\nu_\ell$

- The longitudinal polarization fractions f_L for the decays $B_{(s)} \to D^*_{(s)}(nS)\ell\nu_\ell$ in Region 1, Region 2 and total physical region.

Observables	Region 1	Region 2	Total	Observables	Region 1	Region 2	Total
$f_L(B^0 \to D^{*-}(1S)l'^+\nu_{l'})$	0.35	0.20	$0.55^{+0.00+0.00+0.03}_{-0.00-0.04-0.02}$	$f_L(B^0 \to D^{*-}(2S)\ell'^+\nu_{\ell'+})$	0.50	0.18	$0.68^{+0.00+0.06+0.03}_{-0.00-0.05-0.03}$
$f_L(B^0 \to D^{*-}(1S)\tau^+\nu_\tau)$	0.20	0.25	$0.45^{+0.00+0.00+0.01}_{-0.00-0.03-0.02}$	$f_L(B^0 \to D^{*-}(2S)\tau^+\nu_\tau)$	0.22	0.29	$0.51^{+0.00+0.03+0.01}_{-0.00-0.06-0.02}$
PQCD	–	–	0.42	–	–	–	–
PQCD+Lattice	–	–	0.43	–	–	–	–
SM	–	–	0.457	–	–	–	–
SM	–	–	0.441	–	–	–	–
Belle	–	–	$0.60 \pm 0.08 \pm 0.04$	–	–	–	–
LHCb	–	–	$0.41 \pm 0.06 \pm 0.03$	–	–	–	–
Observables	Region 1	Region 2	Total	Observables	Region 1	Region 2	Total
$f_L(B^+ \to D^{*0}(1S)\ell'^+\nu_{\ell'+})$	0.35	0.20	$0.55^{+0.00+0.00+0.03}_{-0.00-0.03-0.02}$	$f_L(B^+ \to D^{*0}(2S)\ell'^+\nu_{\ell'+})$	0.49	0.18	$0.68^{+0.00+0.05+0.03}_{-0.00-0.06-0.03}$
RQM	–	–	0.55	–	–	–	–
$f_L(B^+ \to D^{*0}(1S)\tau^+\nu_\tau)$	0.21	0.25	$0.46^{+0.00+0.00+0.01}_{-0.00-0.03-0.06}$	$f_L(B^+ \to D^{*0}(2S)\tau^+\nu_\tau)$	0.22	0.29	$0.51^{+0.00+0.4+0.03}_{-0.00-0.06-0.03}$
RQM	–	–	0.47	–	–	–	–
Observables	Region 1	Region 2	Total	Observables	Region 1	Region 2	Total
$f_L(B^0_s \to D^{*-}_s(1S)\ell'^+\nu_{\ell'+})$	0.34	0.20	$0.54^{+0.00+0.04+0.05}_{-0.00-0.03-0.07}$	$f_L(B^0_s \to D^{*-}_s(2S)\ell'^+\nu_{\ell'})$	0.51	0.17	$0.68^{+0.00+0.31+0.15}_{-0.00-0.20-0.22}$
RQM	–	–	0.49	–	–	–	–
$f_L(B^0_s \to D^{*-}_s(1S)\tau^+\nu_\tau)$	0.19	0.25	$0.45^{+0.00+0.00+0.05}_{-0.00-0.02-0.03}$	$f_L(B^0_s \to D^{*-}_s(2S)\tau^+\nu_\tau)$	0.23	0.28	$0.51^{+0.01+0.12+0.11}_{-0.01-0.18-0.11}$
PQCD	–	–	0.42	–	–	–	–
PQCD+Lattice	–	–	0.43	–	–	–	–
RQM	–	–	0.42	–	–	–	–

Semi-leptonic Decays $B \to D^{(*)}(1S, 2S)\ell\nu_\ell$

- Region 1 $m_\ell^2 < q^2 < \frac{(m_B - m_{D^*(nS)})^2 + m_\ell^2}{2}$
  Region 2 $\frac{(m_B - m_{D^*(nS)})^2 + m_\ell^2}{2} < q^2 < (m_B - m_{D^*(nS)})^2$ with $n = 1, 2$.
- For the decays $B_{(s)} \to D^*_{(s)}(nS)\ell'\nu_{\ell'}$, the longitudial polarization fractions from Region 1 are larger than those from Region 2, while it is contrary to the decays $B_{(s)} \to D^*_{(s)}(nS)\tau\nu_\tau$.
- Relations $f_L(B_{(s)} \to D^*_{(s)}(nS)\ell'\nu_{\ell'}) > f_L(B_{(s)} \to D^*_{(s)}(nS)\tau\nu_\tau), f_L(B_{(s)} \to D^*_{(s)}(2S)\ell\nu_\ell) > f_L(B_{(s)} \to D^*_{(s)}(1S)\ell\nu_\ell)$.
- Our predictions for the considered decays are consistent with other theoretical calculations. $f_L(B^0 \to D^{*-}(1S)\tau\nu_\tau)$ is in good agreement with the experimental data given by LHCb, but smaller than Belle measurement.

Summary

- The form factors of the transitions $B_{(s)} \to D_{(s)}(2S), D^*_{(s)}(2S)$ are much smaller than those of the corrresponding transtions $B_{(s)} \to D_{(s)}(1S), D^*_{(s)}(1S)$ because of the different structures of the wave functions between the ground and radilly excited charmed mesons.
- The branching ratios of the decays $B_{(s)} \to D_{(s)}\ell'\nu_{\ell'}$ and $B_{(s)} \to D^*_{(s)}\ell'\nu_{\ell'}$ are consistent well with the experimental data, while those of the decays $B_{(s)} \to D_{(s)}\tau\nu_\tau$ and $B_{(s)} \to D^*_{(s)}\tau\nu_\tau$ are systematically smaller than the data.
- The branching ratios of the decays $B_{(s)} \to D_{(s)}(2S)\ell\nu_\ell$ and $B_{(s)} \to D^*_{(s)}(2S)\ell\nu_\ell$ lie in the range $10^{-4} \sim 10^{-3}$, which can be observed by current LHCb and Belle II experiments.
- There exist similar forward-backward asymmetry (polarization) characteristics between the decays $B \to D^{(*)}(1S)\ell\nu_\ell$ and $B \to D^{(*)}(2S)\ell\nu_\ell$.

Thank you for your attention!

- $f_D = 0.2058 \pm 0.0089$ GeV, $f_{D_s} = 0.2499 \pm 0.0005$ Gev, $f_{D^*} = 0.245^{+0.023}_{-0.022}$ GeV, $f_{D_s^*} = 0.272^{+0.039}_{-0.038}$ GeV.