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The production of $D\bar{D}$ and $D_s\bar{D}_s$ bound states in the B decay within the Bethe–Salpeter framework

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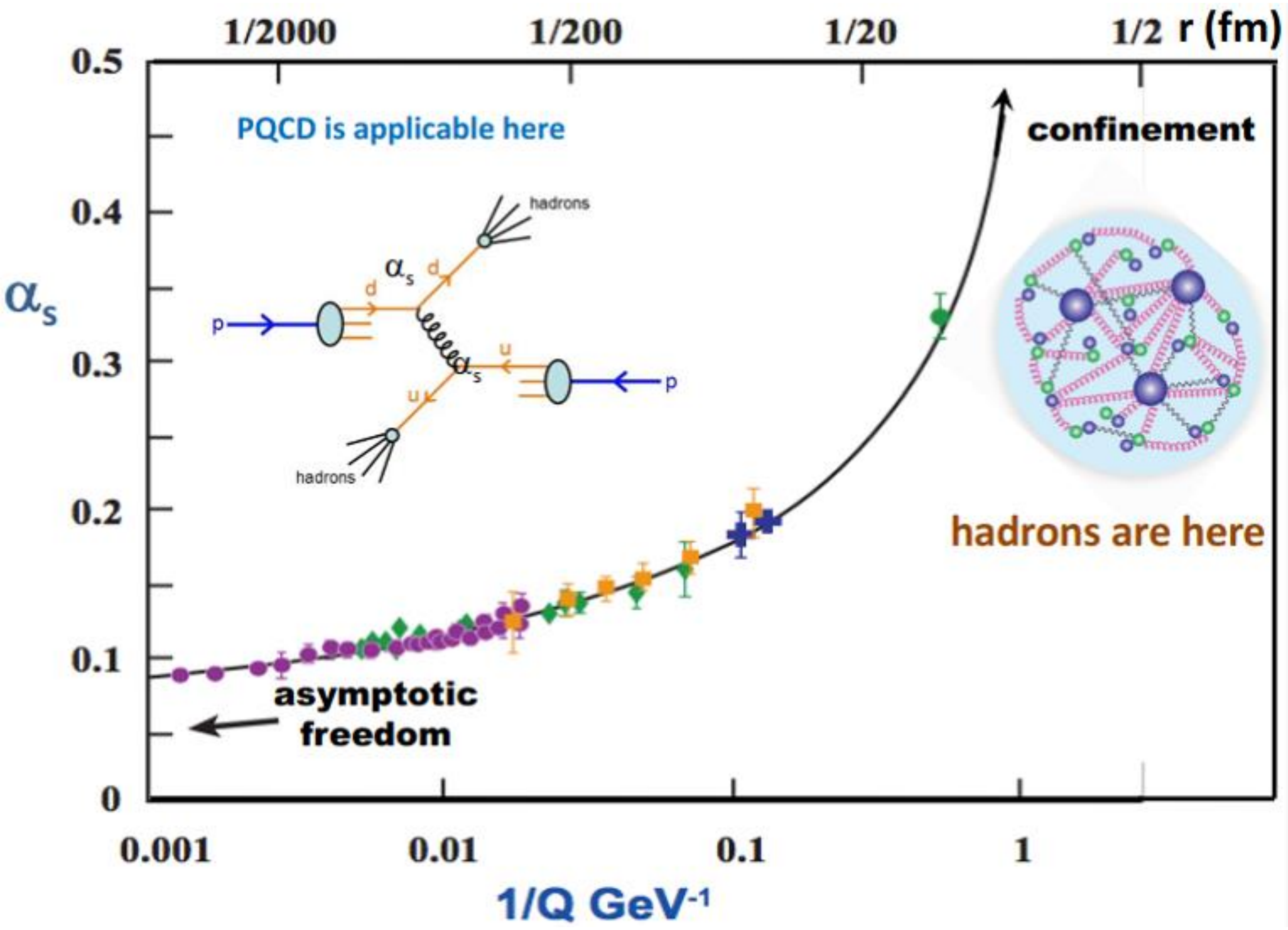
QCD



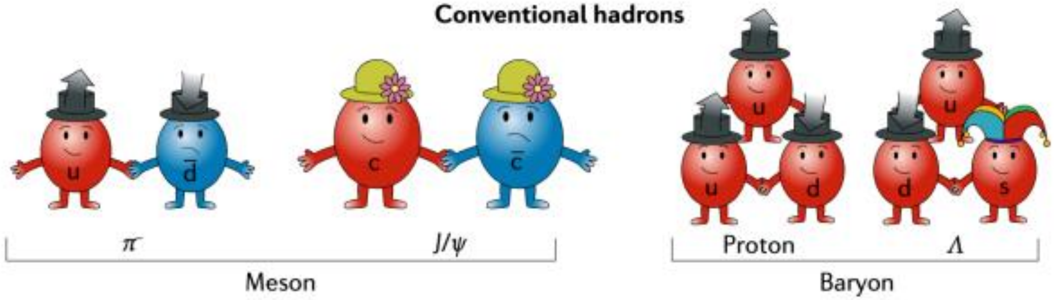
The behavior of the QCD coupling strength $\alpha_S(\mathrm{Q})$ vs. 1/Q (bottom axis) and distance (top axis) (Front. Phys. (Beijing) 19 (2024), 14701).

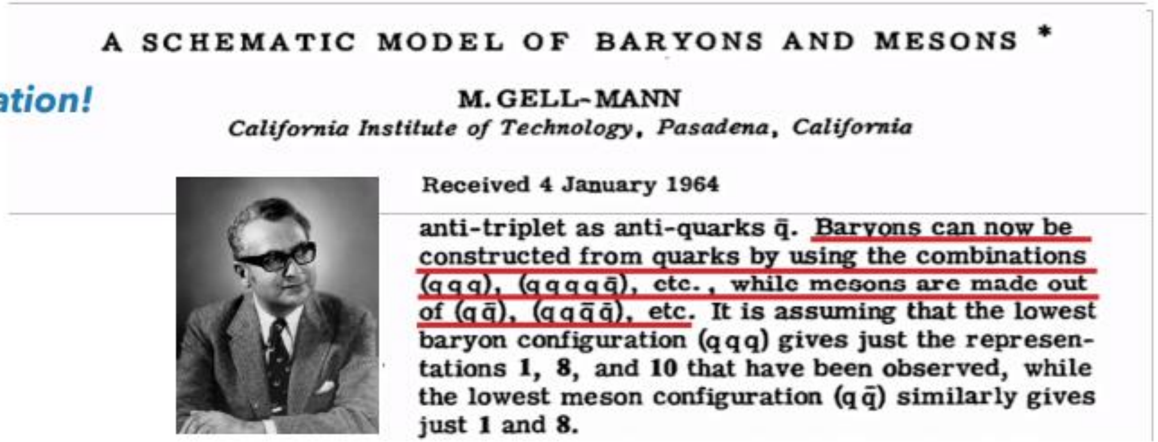
- The strong interaction is fundamentally described by QCD.
- In QCD, the basic constituents are quarks and gluons.
- Two essential features of QCD are color confinement and asymptotic freedom.
- Because of color confinement, quarks and gluons are not observed as free particles; instead, only hadrons are seen in experiments.

Hadrons

- **Quark Model** [1964 by Gell-Mann and Zweig]

Lowest Configuration!



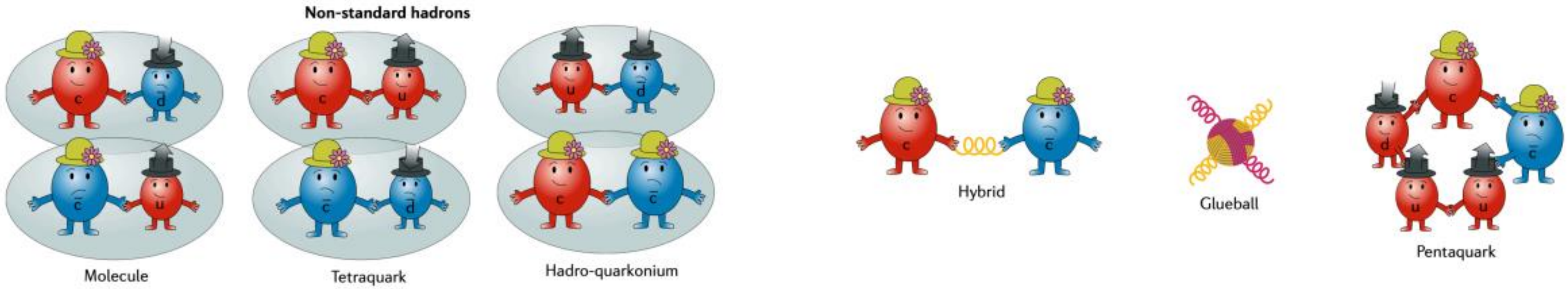

A SCHEMATIC MODEL OF BARYONS AND MESONS *

M. GELL-MANN
California Institute of Technology, Pasadena, California

Received 4 January 1964

anti-triplet as anti-quarks q̄. Baryons can now be constructed from quarks by using the combinations (q q q), (q q q q q̄), etc., while mesons are made out of (q q̄), (q q q̄ q̄), etc. It is assuming that the lowest baryon configuration (q q q) gives just the representations **1**, **8**, and **10** that have been observed, while the lowest meson configuration (q q̄) similarly gives just **1** and **8**.

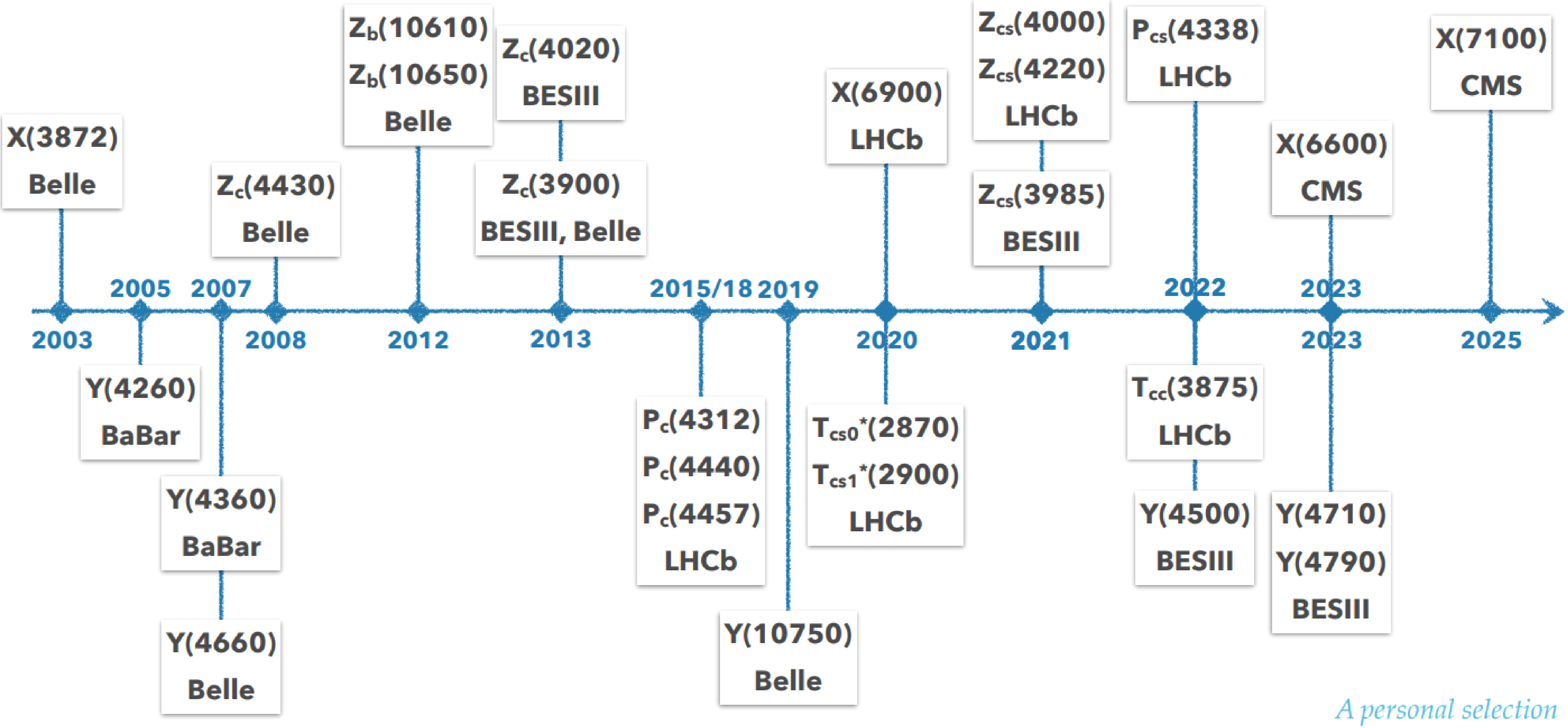
- **Exotic hadrons:**



C. Z. Yuan, S. L. Olsen, Nature Reviews Physics 1, 480 (2019)

Exotic Hadron Candidates

Yuping Guo @ 第二十届全国中高能核物理大会



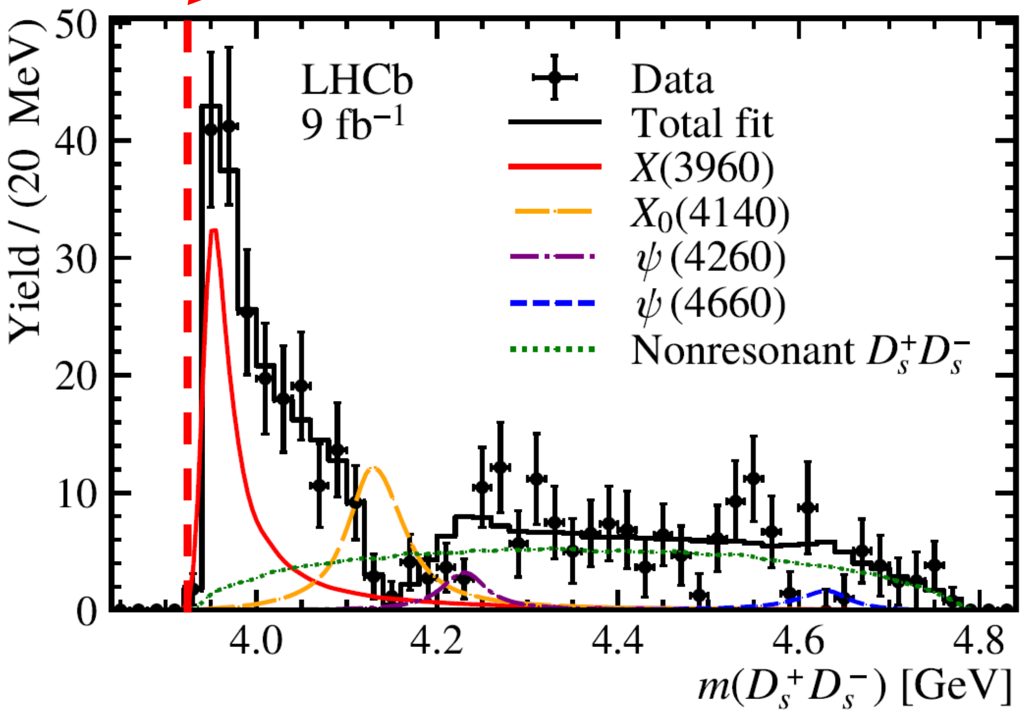
A personal selection

Experiment data

- Observed $X(3960)$ in $B^+ \to D_s^+ D_s^- K^+$

$T_{D_sD_s}$

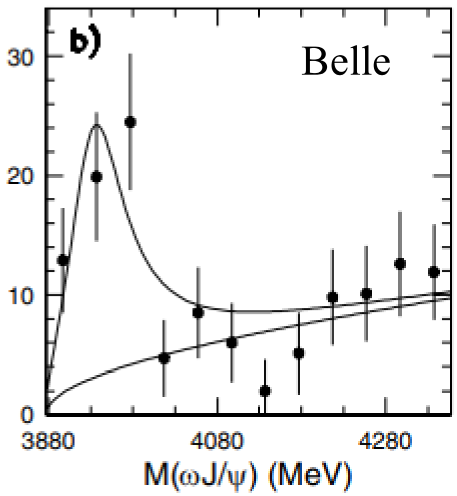
PRL131, 071901 (2023)



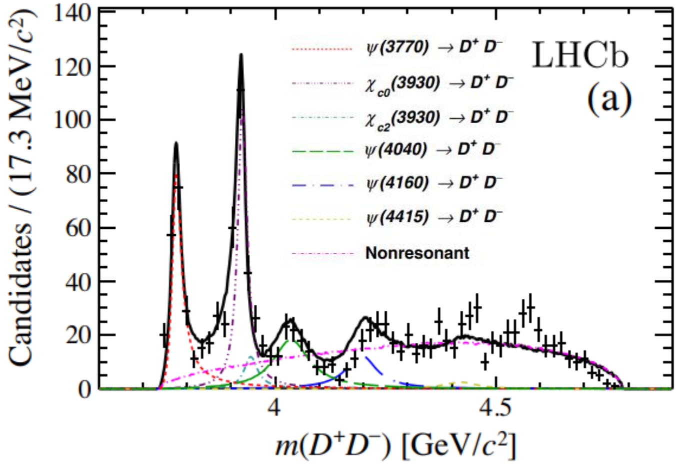
- $J^{PC} = 0^{++}$
- M = $3956 \pm 5 \pm 10$ MeV
- $\Gamma = 43 \pm 12 \pm 8$ MeV

- $X(3915)$ observed by BaBar, BES, Bell, LHCb, and other experiments.

PRL 94, 182002 (2005)



PRD102, 112003 (2020)



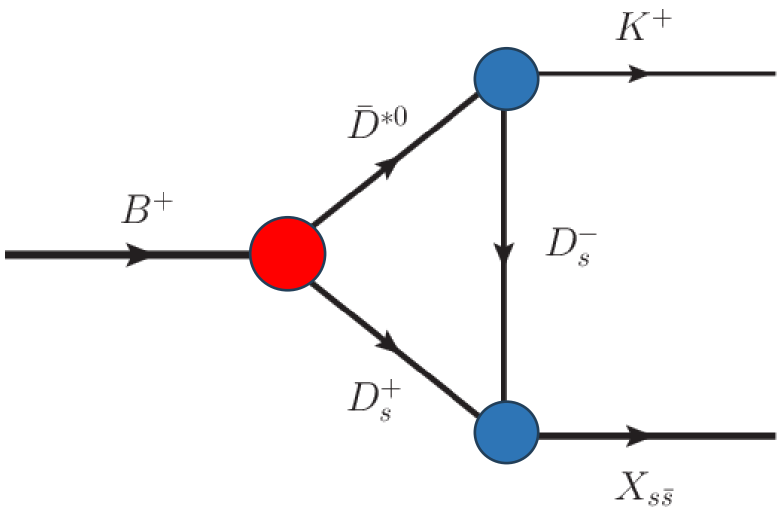
- $M = 3922.1 \pm 1.8$ MeV (PDG)
- $M_{\chi_{c0}} = 3923.8 \pm 1.5 \pm 0.4$ MeV (LHCb)
- $M_{\chi_{c2}} = 3926.8 \pm 2.4 \pm 0.8$ MeV (LHCb)

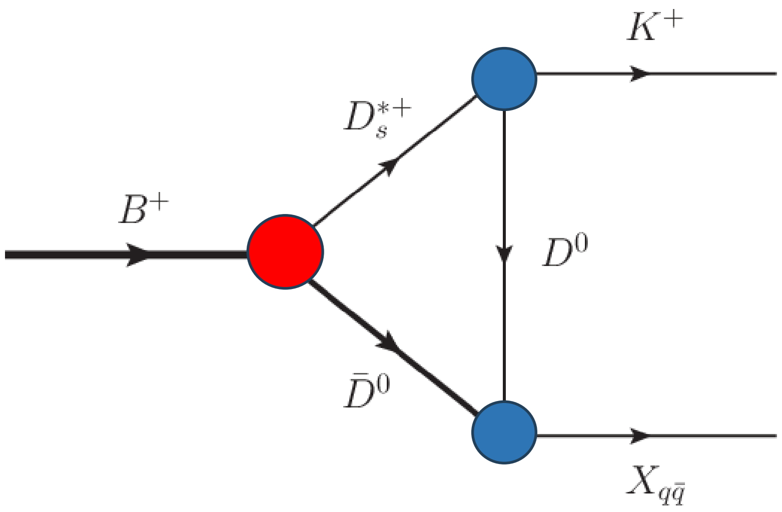
$T_{D_sD_s} = 3936.7$ MeV

Theoretical research

- The coupled-channel $D\bar{D}$ and $D_s\bar{D}_s$ interaction have been investigated by both the chiral unitary approach [PRD107, 034007 (2023)] and lattice QCD [JHEP06, 035 (2021)], where both methods consistently predict the existence of $D\bar{D}$ and $D_s\bar{D}_s$ bound states.
- The freshly measured $D_s^+D_s^-$ invariant mass distribution in the $B^+ \to D_s^+D_s^-K^+$ reaction can be well described by a $D_s\bar{D}_s$ bound or virtual state below threshold [PRD106, 094002 (2022)].
- The existence of $D\bar{D}$ and $D_s\bar{D}_s$ bound states have been studied in various approaches, like Bethe-Salpeter equation [Progr.Phys. 41, 65-93 (2021), EPJC81, 732 (2021), PRD105, 114019 (2022)], potential model [CPC41, 053105 (2017)], Lippmann-Schwinger equation [Sci.Bull. 66, 1288-1295 (2021)], coupled-channel Bethe-Salpeter equation [PRD112, 016003 (2025)], and unitarized coupled channel framework [Phys.Rev.D 76 (2007) 074016].
- The production of $D\bar{D}$ and $D_s\bar{D}_s$ bound states been studied in the $B \to XK$[PRD107, 016003 (2023) and EPJC76, 121(2016)], $\gamma\gamma \to D\bar{D}$ [PRD103, 054008 (2021) and PLB827, 136982 (2022)], $\psi(3770) \to \gamma D\bar{D}$ [EPhJC 80, 510 (2020)], $\Lambda_b \to \Lambda D\bar{D}$ [PRD103, 114013 (2021)], $B^- \to K^-\eta\eta_c$ [PRD109, 094014 (2024)], and $B^- \to K^-J/\psi\omega$ [EPJC83, 309 (2023)].
- The decay of $X(3915)$ as $D_s\bar{D}_s$ bound states also been study n Ref. [PRD 91, 114014 (2015) and CPC50, 022001(2026)].
- ……

$B \to XK$ in the Bethe-Salpeter equation





- The decay $B \to XK$ consists of a weak-interaction part and a strong-interaction part.
- The weak process $B \to \bar{D}^* D$ is treated within the naive factorization approach.
- The strong-interaction part is described by the effective Lagrangian and the Bethe–Salpeter equation.

$$\mathcal{M} = \int \frac{d^4p}{(2\pi)^4} \mathcal{A}_{weak}(B \to \bar{D}^* D)\Gamma(\bar{D}^* \to \bar{D}K)\chi_P(p) \qquad D = (D^0, D^+, D_s^+)$$

The weak interaction

- Within the naive factorization framework, the decay amplitude for $B \to \bar{D}^* D$ is given by

$$\mathcal{A}_{\text{weak}} = \frac{G_F}{\sqrt{2}} v_{cb} v_{cs} a_1 \langle D^{(*)} | s\bar{c} | 0 \rangle \langle \bar{D}^{(*)} | c\bar{b} | B \rangle$$

- The matrix elements of pseudoscalar and vector mesons with the vacuum can be parameterized as

$$\langle D_s^+ | c\bar{s} | 0 \rangle = f_{D_s^+} p^{\mu}_{D_s^+}$$
$$\langle D_s^{*+} | \bar{s}c | 0 \rangle = m_{D_s^{*+}} f_{D_s^{*+}} \epsilon^*_{\mu}$$

- The weak-current transition matrix elements for B $\to \bar{D}^{*0} / \bar{D}^0$

$$\langle \bar{D}^{*0} | c\bar{b} | B^+ \rangle$$
$$= \epsilon^*_{\beta} \{ i\epsilon^{\alpha\beta\mu\nu} P_{\mu} q_{\nu} \frac{V(q^2)}{m_{\bar{D}^{*0}} + m_{B^+}} - g^{\alpha\beta} (m_{\bar{D}^{*0}} + m_{B^+}) A_1(q^2) + P^{\alpha} P^{\beta} \frac{A_2(q^2)}{m_{\bar{D}^{*0}} + m_{B^+}} + q^{\alpha} P^{\beta} [\frac{m_{\bar{D}^{*0}} + m_{B^+}}{q^2} A_1(q^2)$$
$$- \frac{m_{B^+} - m_{\bar{D}^{*0}}}{q^2} A_2(q^2) - \frac{2m_{\bar{D}^{*0}}}{q^2} A_0(q^2)] \}$$

$$\langle \bar{D}^0 | c\bar{b} | B\text{^}+ \rangle = [(p_{B^+} + p_{\bar{D}^0})^{\mu} - \frac{m^2_{B^+} - m^2_{\bar{D}^0}}{q'^2} q'^{\mu}] F_{1D}(q'^2) + \frac{m^2_{B^+} - m^2_{\bar{D}^0}}{q'^2} q'^{\mu} F_{0D}(q'^2)$$

Factorization parameter

- The decay constants $f_{D_s^+}$ and $f_{D_s^{*+}}$ take from Ref. [EPJC80, 113 (2020)]

$$f_{D_s^+} = 250 \text{ MeV}$$
$$f_{D_s^{*+}} = 272 \text{ MeV}$$

- The form factors of $F_{1,0D}(t)$, $A_{0,1,2}(t)$, and $V(t)$ with $t \equiv q^{(')2}$ are parameterized as

$$F(t) = \frac{F(0)}{1 - a(t/m_B^2) + b(t/m_B^2)}$$

with $(F_{1D}(0), a, b) = (0.67, 1.22, 0.36)$, $(F_{0D}(0), a, b) = (0.67, 0.63, 0.01)$, $(A_0(0), a, b) = (0.68, 1.21, 0.36)$, (A_1(0),a,b)=(0.65,0.60,0.00), $(A_2(0), a, b) = (0.61, 1.12, 0.31)$, and $(V(0), a, b) = (0.77, 1.25, 0.38)$.

J. Phys. G 39, 025005 (2012).

Strong interaction

- $\Gamma(\bar{D}^* \to \bar{D}K)$ is described by the effective Lagrangian

$$\mathcal{L}_{\mathcal{D}^*\mathcal{D}\mathcal{P}} = -ig_{D^*D\mathcal{P}}(D_i^\dagger \partial_\mu \mathcal{P}_{ij} D_j^{*\mu} - D_i^{*\mu\dagger} \partial_\mu \mathcal{P}_{ij} D_j) + \text{H.c.}$$

- The $D\bar{D}$ and $D_s\bar{D}_s$ bound state are characterized by the Bethe-Salpeter wave function

$$\chi_P(x_1, x_2) = \langle 0 | T\phi(x_1)\bar{\phi}(x_2) | P \rangle = e^{-iPX} \int \frac{d^4p}{(2\pi)^4} e^{-ipx} \chi_P(p),$$

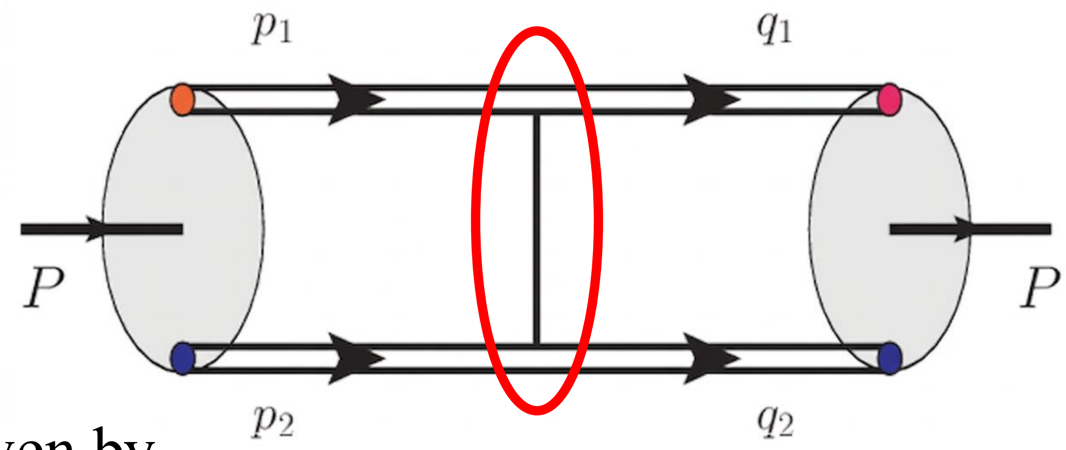
- In momentum space, the Bethe–Salpeter equation takes the form

$$\chi_P(p) = S(p_1) \int \frac{d^4q}{(2\pi)^4} K(P, p, q) \chi_P(q) S(p_2)$$

- The normalization condition for the Bethe–Salpeter wave function is

$$i \int \frac{d^4p d^4p'}{(2\pi)^8} \bar{\chi}_P(p) \frac{\partial}{\partial P^0} [I_P(p, p') + \bar{K}_P(p, p')] \chi_P(p') = 1$$

Bethe-Salpeter equation



- The effective Lagrangian for the $DD\mathcal{V}$ interaction

$$\mathcal{L}_{DD\mathcal{V}} = -ig_{DD\mathcal{V}} D_i^\dagger \overleftrightarrow{\partial}_\mu D_j (\mathcal{V})_{ij} + \text{H.c.}$$

- At tree level, the t-channel interaction kernel is given by

$$K_P(p,p') = (2\pi)^4 \delta^4(p_1' + p_2' - p_1 - p_2) g_{DDV}^2 (p_1 + p_1')^\mu (p_2 + p_2')^\nu \Delta_{\mu\nu}(k)$$

- A form factor is introduced at each vertex

$$F(k^2) = \frac{\Lambda^2 - m^2}{\Lambda^2 - k^2}$$

with $\Lambda = \alpha\Lambda_{\text{QCD}} + \text{m}$.

Bethe-Salpeter equation

- In the covariant instantaneous approximation

$$p_l = q_l = 0$$

$$p_l = p \cdot v$$
$$p_t = p - p_l v$$

- The interaction kernel is constructed from vector-meson exchanges:

  ρ and ω for the $D\bar{D}$ system

  ϕ for the $D_s\bar{D}_s$ system:

- The coupling constants

  1) hidden gauge symmetry approach $g_{DDV} = \frac{1}{\sqrt{2}}\beta g_V \approx 3.69$ PRD68, 114001(2003)

  with $\beta \approx 0.9$ and $g_V \approx 5.8$

  2) LQCD $g_{DD\rho} = 4.84 \pm 0.34$ PLB719, 103 (2013)

  3) LCSR $g_{DD\rho} = 2.64 \pm 0.58$, $g_{D_sD_s\phi} = 2.9 \pm 0.68$ EPJ.C 52 ,553 (2007)

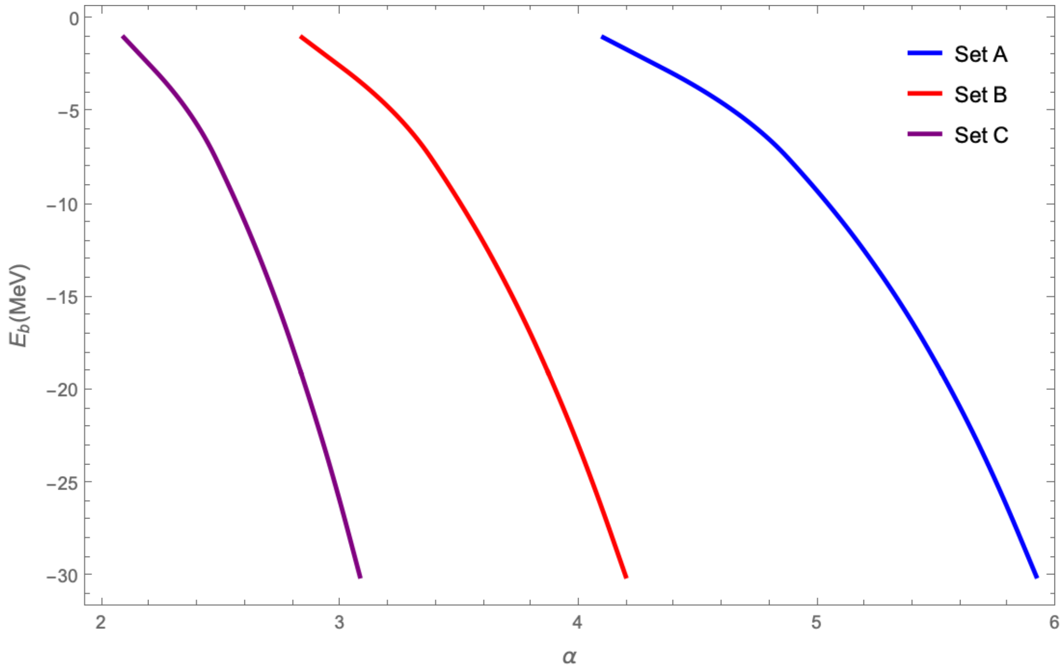
  4) LCSR(next-to-leading order) $g_{DD\rho} = 4.30^{+0.82}_{-0.72}$, JHEP03, 106(2025)

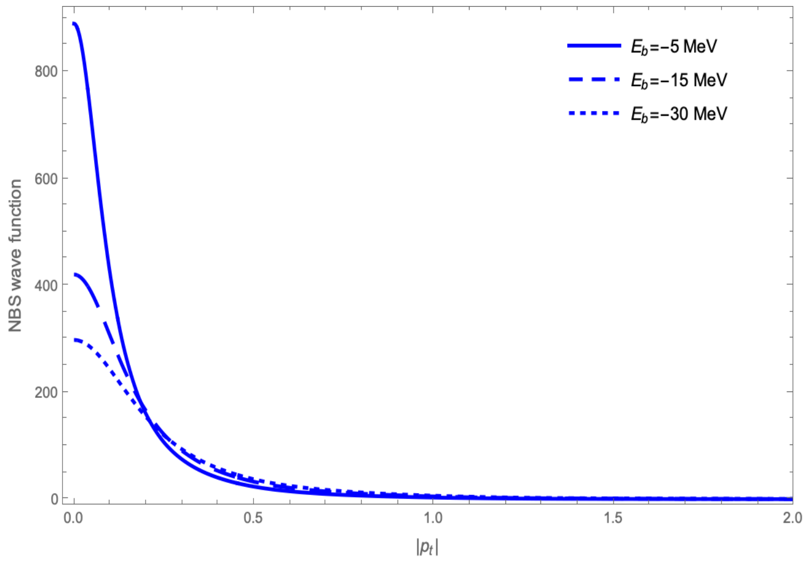
  $g_{DD\omega} = 2.80^{+0.54}_{-0.48}$, $g_{D_sD_s\phi} = 3.65^{+0.93}_{-0.59}$

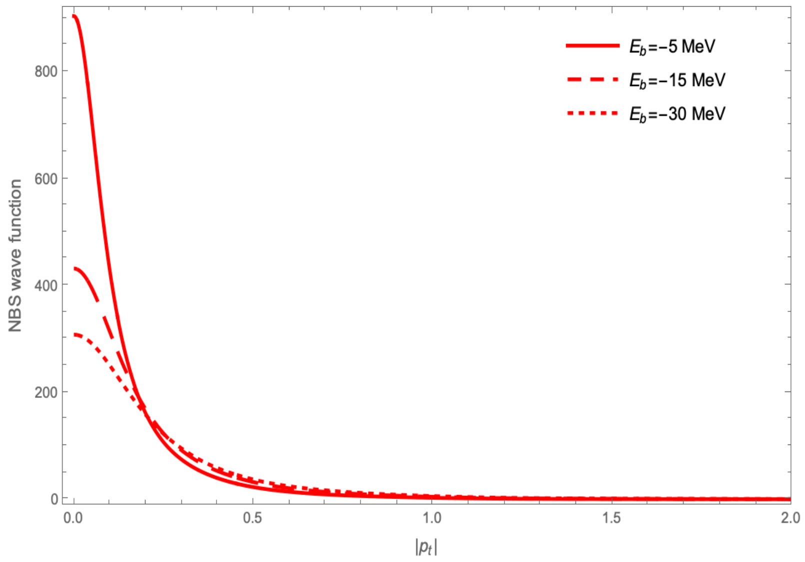
Set A: **Lower limit**, Set B: **Central value**, Set C: **Upper limit**

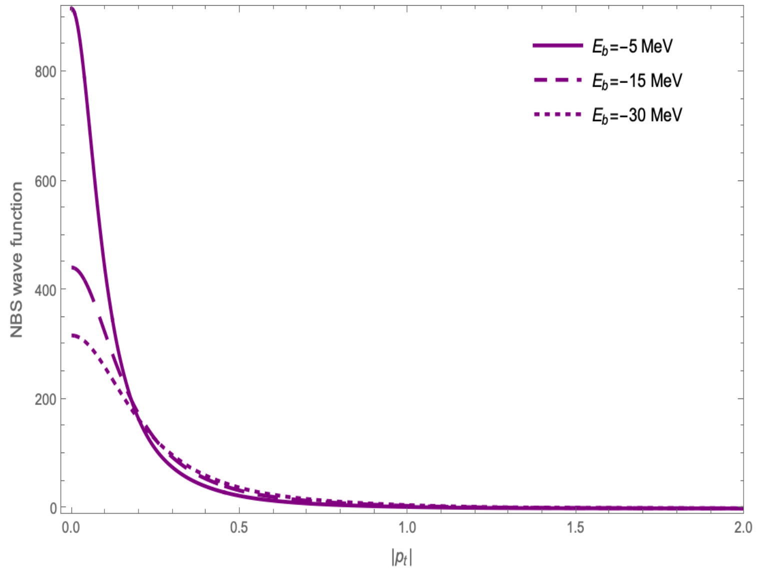
$D\bar{D}$ Bethe-Salpeter equation results (Preliminary)

	$g_{DD\rho}$	$g_{DD\omega}$
Set A:	3.58	2.32
Set B:	4.30	2.80
Set C:	5.12	3.34









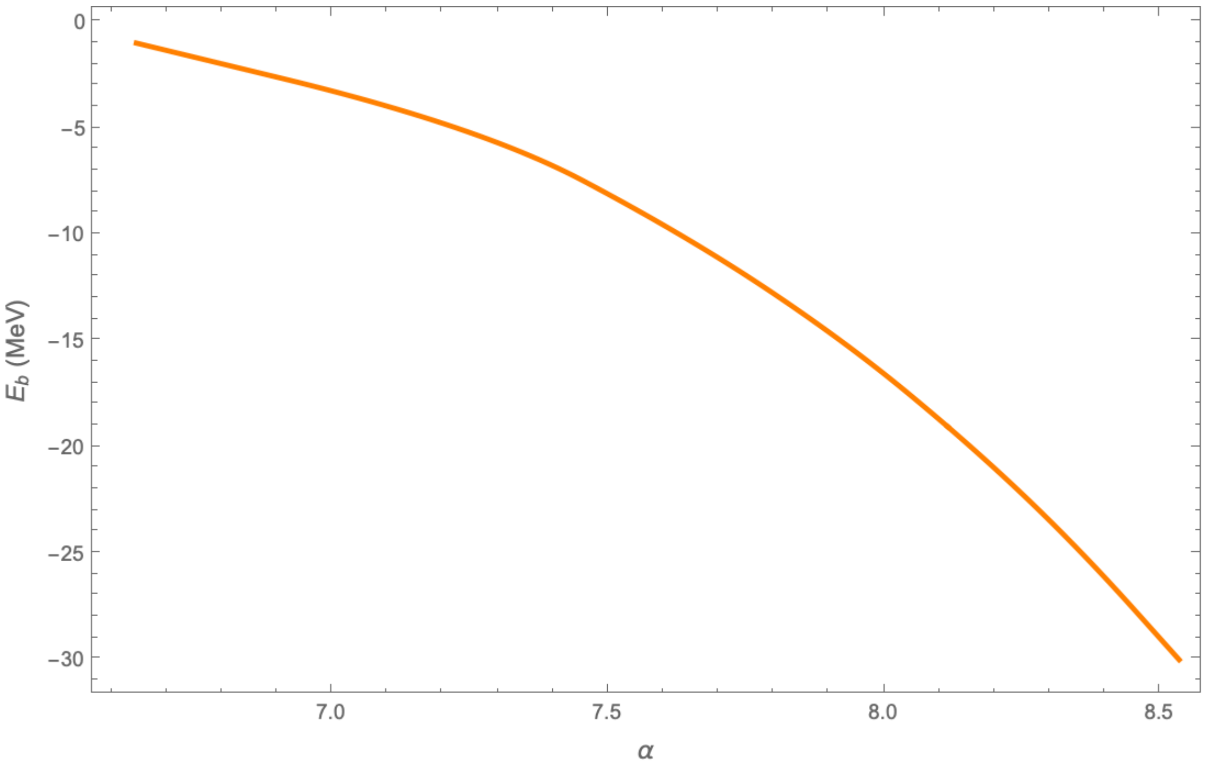
$D_s\bar{D}_s$ Bethe-Salpeter equation results (Preliminary)

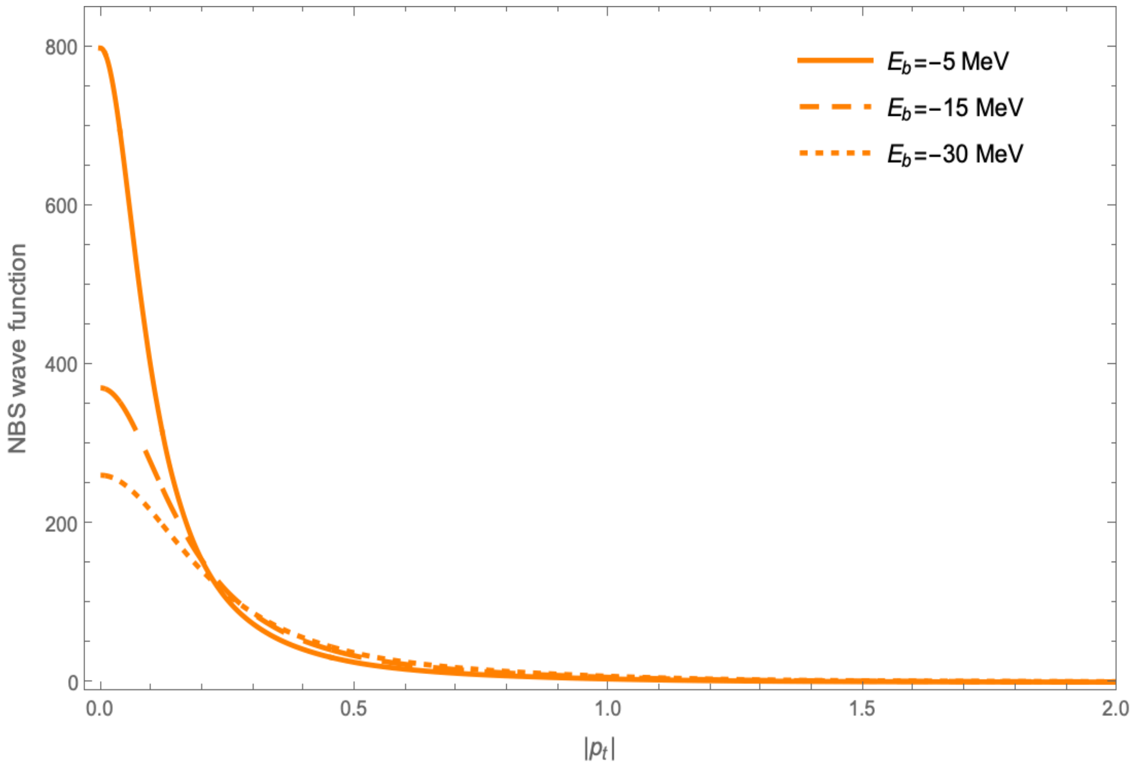
$g_{D_sD_s\phi}$

Set A: **3.06** ✗

Set B: **3.65** ✗

Set C: **4.58** ✓





The amplitudes for $B^+ \to X_{ss}K^+$ and $B^+ \to X_{qq}K^+$

- The amplitudes of $B^+ \to D_s^+ \overline{D}^{*0}$ and $B^+ \to D_s^{*+} \overline{D}^0$

$$\begin{aligned}
&\mathcal{A}(B^+ \to D_s^+ \overline{D}^{*0}) \\
&= \frac{G_F}{\sqrt{2}} V_{cb} V_{cs}^* a_1 f_{D_s} \frac{\epsilon_\alpha^*}{m_B + m_{D^*}} \{-q^\alpha (m_{B^+} + m_{\overline{D}^{*0}}) A_1(q^2) + q^\mu P_\mu P_\alpha A_2(q^2) \\
&+ P_\alpha \left[(m_{B^+} + m_{\overline{D}^{*0}})^2 A_1(q^2) - \left(m_{B^+}^2 - m_{\overline{D}^{*0}}^2\right) A_2(q^2) - 2m_{\overline{D}^{*0}} (m_{B^+} + m_{\overline{D}^{*0}}) A_0(q^2) \right] \} \\
&= \mathcal{M}^\alpha (B^+ \to D_s^+ \overline{D}^{*0}) \epsilon_\alpha^*
\end{aligned}$$

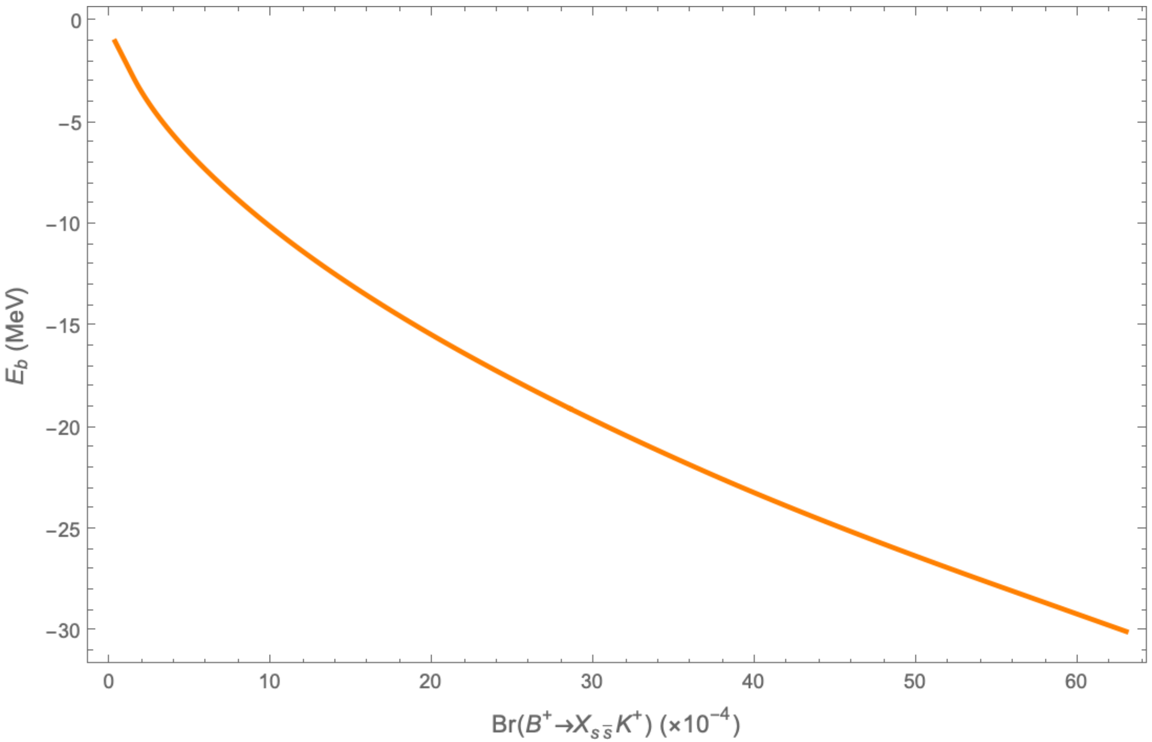
$$\mathcal{A}(B^+ \to D_s^{*+} \overline{D}^0) = \frac{2G_F}{\sqrt{2}} V_{cb} V_{cs} a_1' m_{D_s^{*+}} f_{D_s^{*+}} \epsilon_\alpha \, p_{\overline{D}^0}^\alpha F_{1D}(q'^2) = \mathcal{M}^\alpha (B^+ \to D_s^{*+} \overline{D}^0) \epsilon_\alpha$$

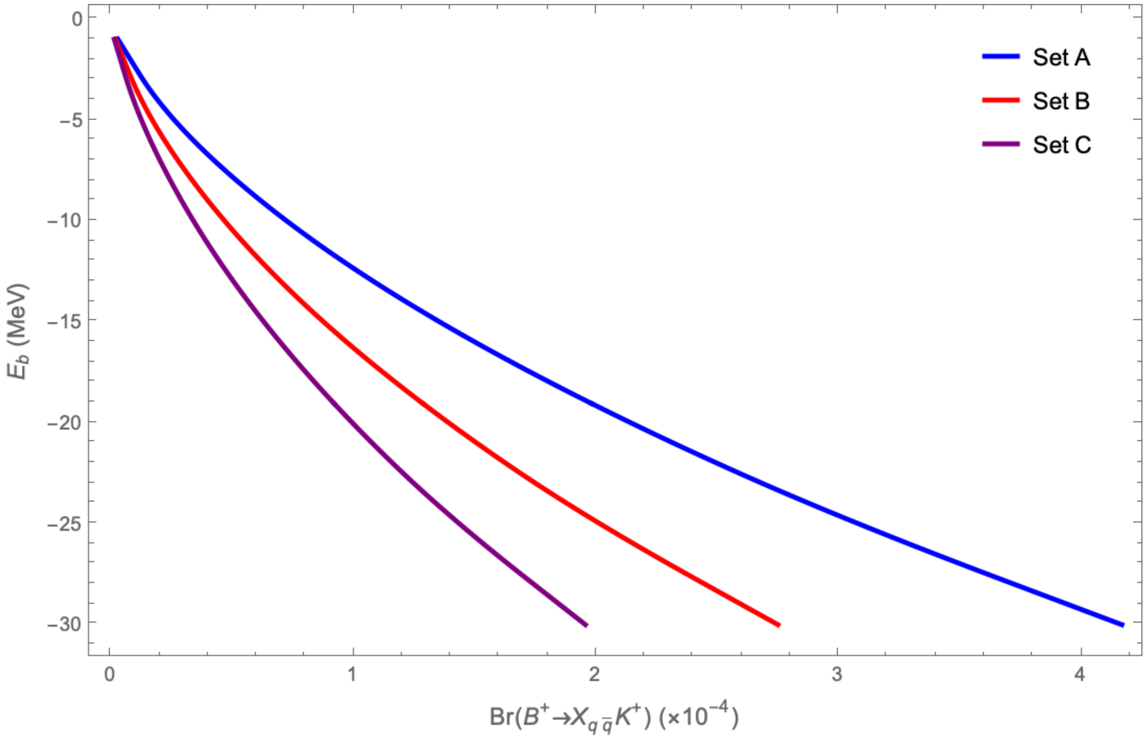
- The amplitudes for $B^+ \to X_{ss}K^+$ and $B^+ \to X_{qq}K^+$

$$\mathcal{A}_{B^+ \to X_{ss}K^+} = \int \frac{d^4 p}{(2\pi)^4} g_{\overline{D}^* \overline{D}_s K} \mathcal{M}^\alpha (B^+ \to D_s^+ \overline{D}^{*0}) p_{K^+}^\beta \frac{-i(g_{\alpha\beta} - p_{\overline{D}^{*0}\alpha} p_{\overline{D}^{*0}\beta} / m_{\overline{D}^{*0}}^2)}{p_{\overline{D}^{*0}}^2 - m_{\overline{D}^{*0}}^2} \chi_P(p)$$

$$\mathcal{A}_{B^+ \to X_{qq}K^+} = \int \frac{d^4 p}{(2\pi)^4} g_{\overline{D}_s^* D K} \mathcal{M}^\alpha (B^+ \to D_s^{*+} \overline{D}^0) p_{K^+}^\beta \frac{-i(g_{\alpha\beta} - p_{D_s^{*+}\alpha} p_{D_s^{*+}\beta} / m_{D_s^{*+}}^2)}{p_{D_s^{*+}}^2 - m_{D_s^{*+}}^2} \chi_P(p)$$

Branching ratios (Preliminary)





Decay modes	Our results	PRD107, 016003 (2023)	Exp[PDG]
$B^+ \to X_{s\bar{s}}K^+$	$(0.34 \sim 63.0) \times 10^{-4}$	$(2.1 \sim 17.0) \times 10^{-4}$	$\mathrm{Br}(B^+ \to \mathrm{X}(3915)K^+) < 2.8 \times 10^{-4}$
$B^+ \to X_{\mathrm{q}\bar{\mathrm{q}}}K^+$	$(0.0156 \sim 4.17) \times 10^{-4}$	$(0.9 \sim 6.7) \times 10^{-4}$	$\mathrm{Br}(B^+ \to \eta_c \eta K^+) < 2.2 \times 10^{-4}$

Summary

✓ The $D\bar{D}$ and $D_s\bar{D}_s$ systems have been studied within the Bethe–Salpeter framework.

- The $D\bar{D}$ system can form a bound state.
- Whether the $D_s\bar{D}_s$ system can form a bound state requires further investigation.
- The coupling constants significantly affect the existence of the bound state and the value of the binding energy.

✓ The branching ratios of $B^+ \to X_{s\bar{s}}K^+$ and $B^+ \to X_{\mathrm{q\bar{q}}}K^+$ have been studied.

- The binding energy significantly affects the decay branching ratio.

Thank you for your attention !