多强子散射问题的格点量子色动力学研究

燕浩波

Based on:

1. **Haobo Yan** et al., **Phys. Rev. D** 111 (2025) 014503 on $D\pi \to D_0^*(2300)$
2. **Haobo Yan** et al., arXiv:26xx.xxxx on $D\pi - D\eta - D_s\bar{K}$ scattering
3. **Haobo Yan** et al., **Phys. Rev. Lett** 133 (2024) 211906 on $\pi\pi\pi \to \omega(782)$ (**Editors' Suggestion**)
4. **Haobo Yan** et al., **Phys. Rev. Lett** xxx (2026) xxxxxx on $\pi\pi\pi \to \pi(1300)$
5. **Haobo Yan** et al., **JHEP** 10 (2025) 210 on OpTion package

北京大学
PEKING UNIVERSITY

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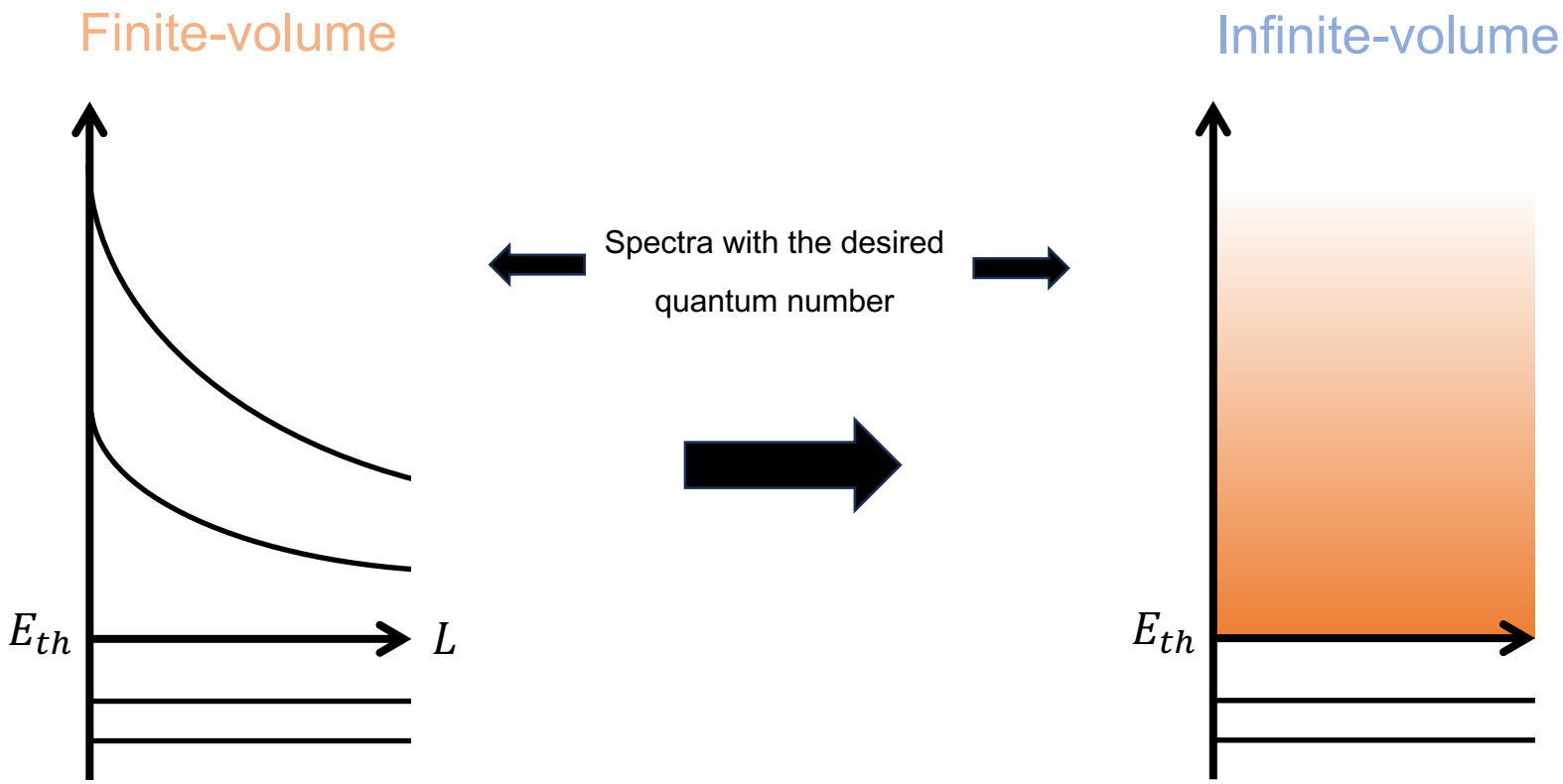
Milestones of hadron scattering

- Two-body
  - 1991: Lüscher – the famous formula
  - 1992-2014 – development of Lüscher formalism
  - 2012: energy-dependent phase shift of $I = 2$ $\pi\pi$
  - 2013: $\pi\pi \to \rho$
  - 2012-now: scattering with mesons, baryons…
- Three-body
  - 2014-2017: development of three-particle formalism
  - 2021: energy-dependent $I = 3$ $\pi\pi\pi$
  - 2021: $\pi\pi\pi \to a_1(1260)$
  - 2024: $\pi\pi\pi \to \omega(782)$
  - 2026: $\pi\pi\pi \to \pi(1300)$
  - ?

Routine

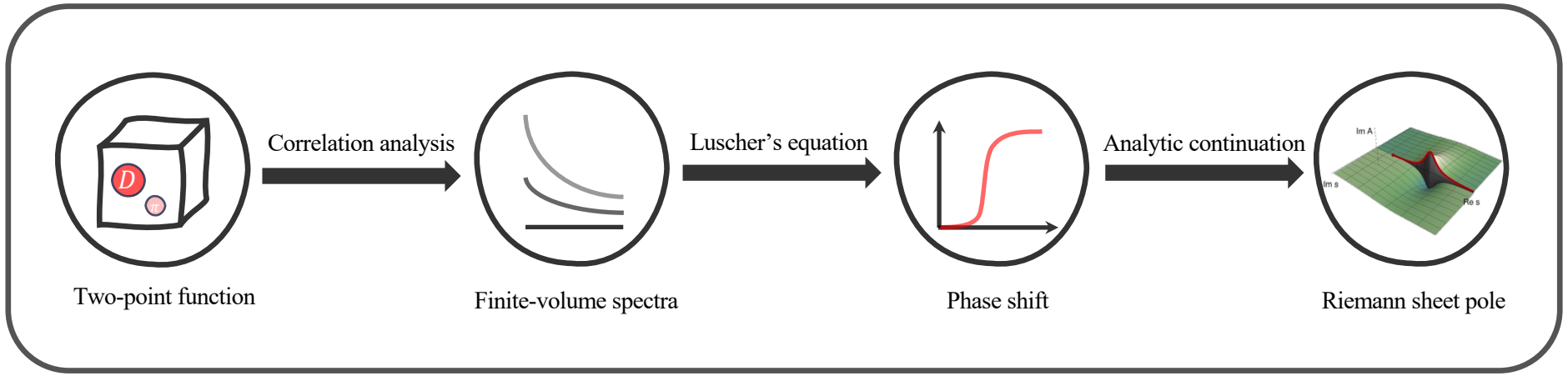


- How to extract the infinite-volume information from finite-volume spectra?

  → Quantization condition: $f\left(\mathcal{M}(E_n),\ \mathcal{F}(E_n, L)\right) = 0$

- More E_n and L give more constraints on $\mathcal{M}(E_n)$

Methodology – 两体



- Operators from OpTion
- Two-body spectra
- Apply the quantization conditions
- Analytic continuate and search for poles

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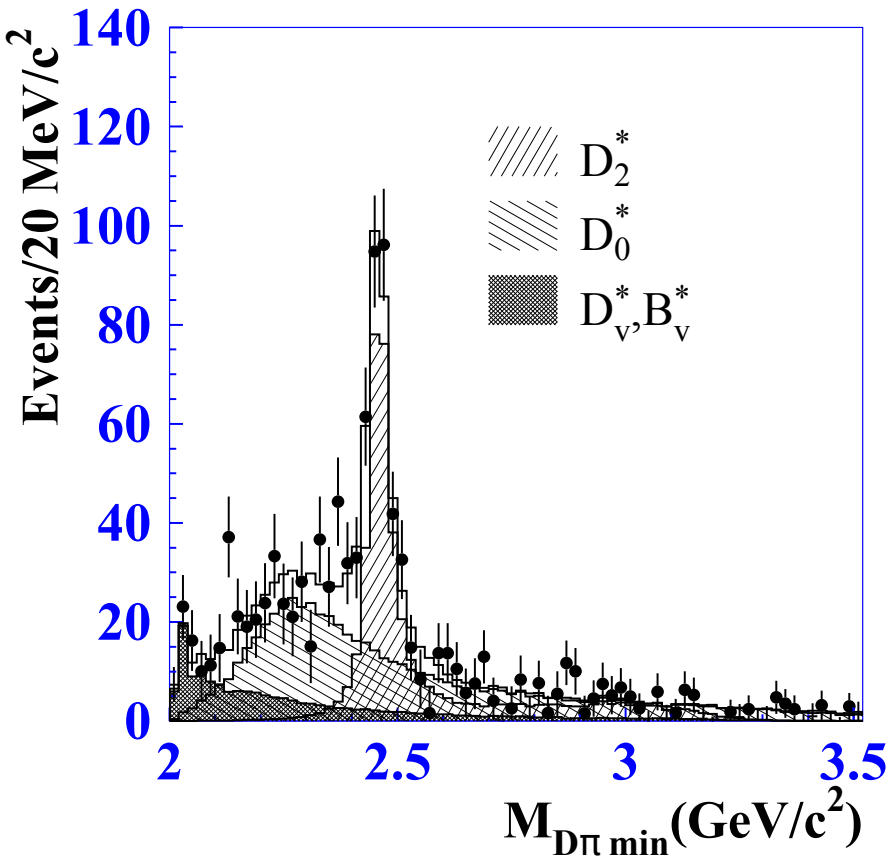
Puzzles on $D_0^*(2300)$

- A broad resonance $D_0^* \to D\pi$ was found by Belle collaboration in 2004, **Breit-Wigner is used**



Satpathy et al, PRB 159 (2003) 553.

- $m_{D_0^*} \approx m_{D_{s0}^*}$
- $m_{D_0^*}^{ChPT} \ll m_{D_0^*}^{exp}$
- could be explained by UChPT where a **two-pole structure** is proposed

Albaladejo et al, PLB 767 (2017) 465

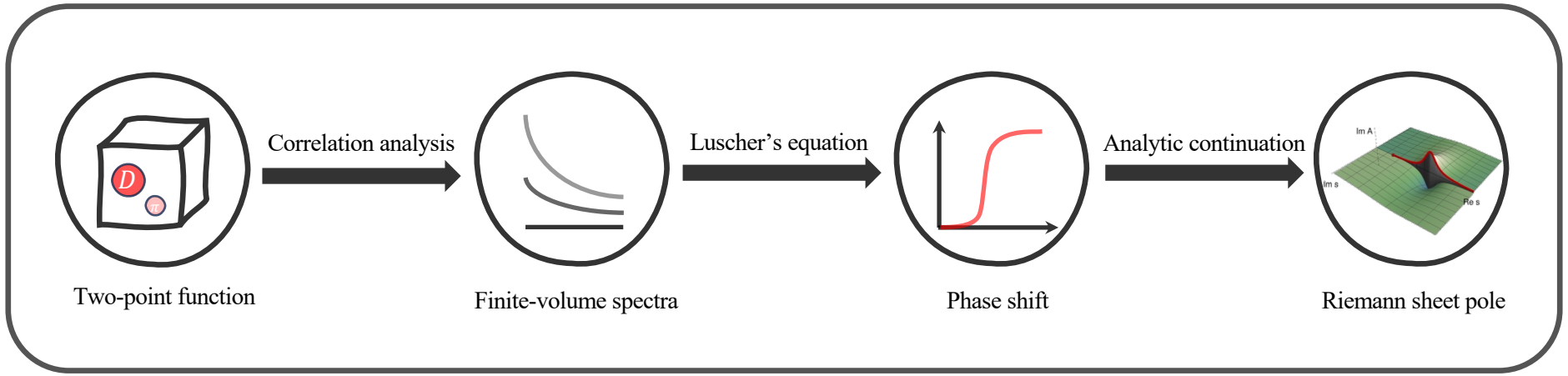
- Should be tested on the lattice at a series of m_π
- Input for T_{cc}

Lattice setup

configuration	volume	a/fm	m_π/MeV	N_{cfgs}
C48P14	$48^3 \times 96$	$0.10530(18)$	$135.5(1.6)$	259
F32P21	$32^3 \times 64$	$0.07746(18)$	$210.9(2.2)$	459
F48P21	$48^3 \times 96$	$0.07746(18)$	$207.2(1.1)$	222
F32P30	$32^3 \times 96$	$0.07746(18)$	$303.2(1.3)$	567
F48P30	$48^3 \times 96$	$0.07746(18)$	$303.4(9)$	201
H48P32	$48^3 \times 144$	$0.05187(26)$	$317.2(0.9)$	274

- CLQCD ensembles with $N_f = 2 + 1$ Clover fermions [CLQCD, 2024]
- 4 different pion masses to track the chiral behavior
- 3 lattice spacings to estimate the discretization error

Methodology



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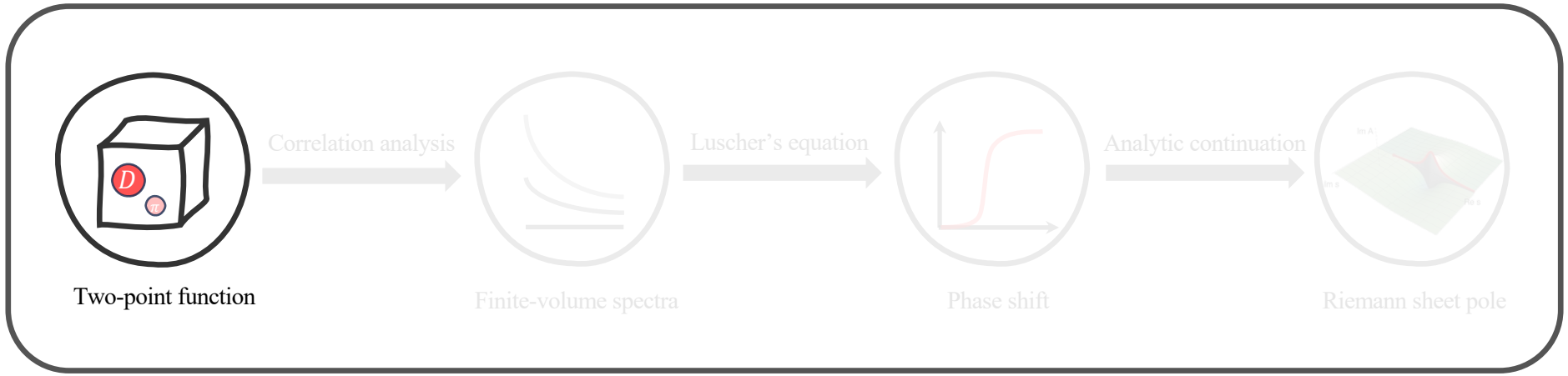
Methodology



- Operators from OpTion

Operator consTruction (OpTion)

- We need to build the operator to create the hadronic states ($D\pi, \pi\pi\pi, \ldots$) from the vacuum
- The rotation symmetry **broken** to $O(3) \to O_h$ or even more
- A Mathematica package Option has been developed to construct **general** N-hadron operators
- OpTion has been a routine tool in the CLQCD collaboration

README MIT license

OpTion

license MIT Mathematica 10.x 11.x 12.x 13.x

Releases 2

OpTion 1.1.0 Latest
on Jul 24

+ 1 release

Packages

```
(* I=3 πππ or I=1 πππ π(1300) *)
Print /@ NHadronOperatorAll[{0, 0, 0}, "A1-", 1, MaxMom, {"P", "P", "P"}];
打印

P1[0] P2[0] P3[0]

P1[ex] P2[0] P3[-ex] + P1[-ex] P2[0] P3[ex] + P1[ey] P2[0] P3[-ey] + P1[-ey] P2[0] P3[ey] + P1[ez] P2[0] P3[-ez] + P1[-ez] P2[0] P3[ez]

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```
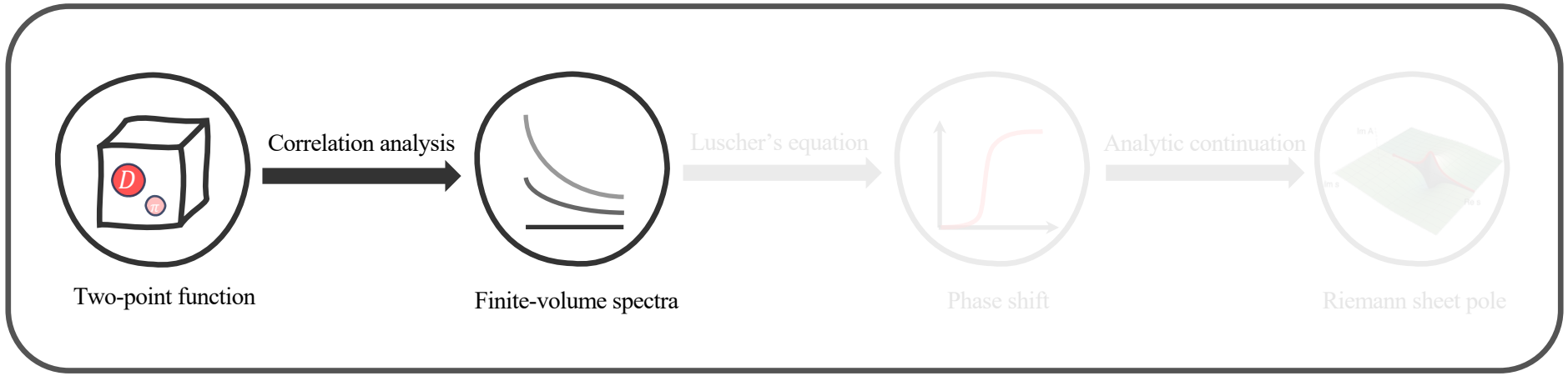
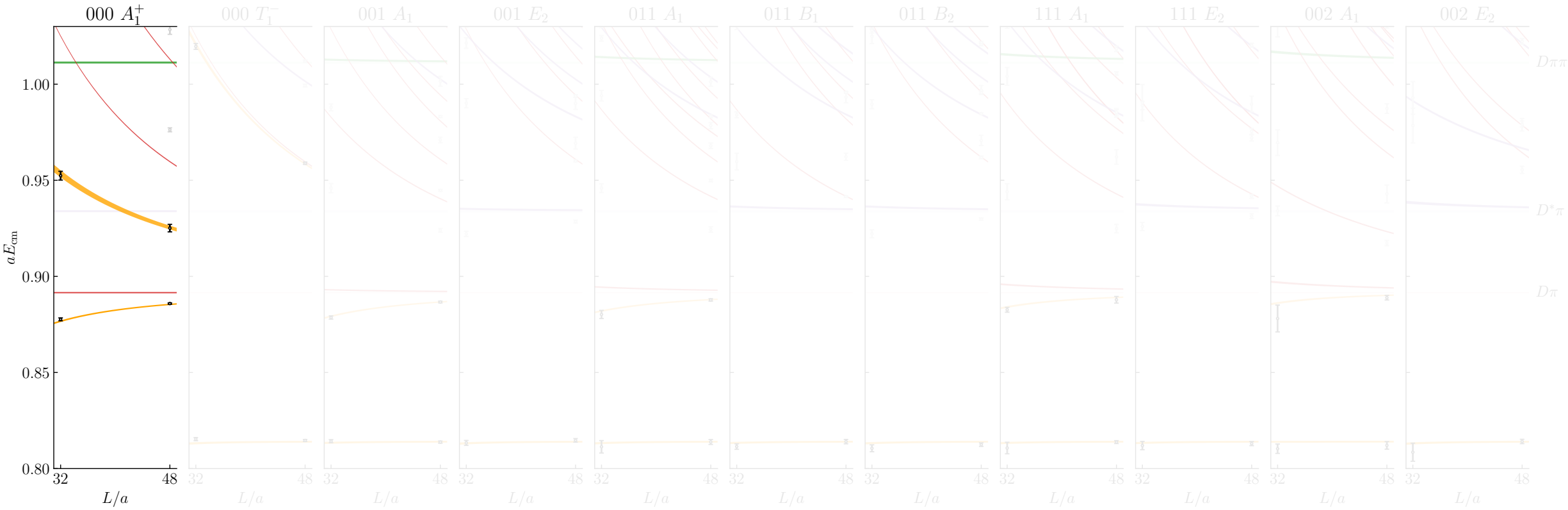

Haobo Yan (燕浩波), Chuan Liu, Liuming Liu, Yu Meng, **JHEP** 10 (2025) 210 https://github.com/wittscien/OpTion

Methodology



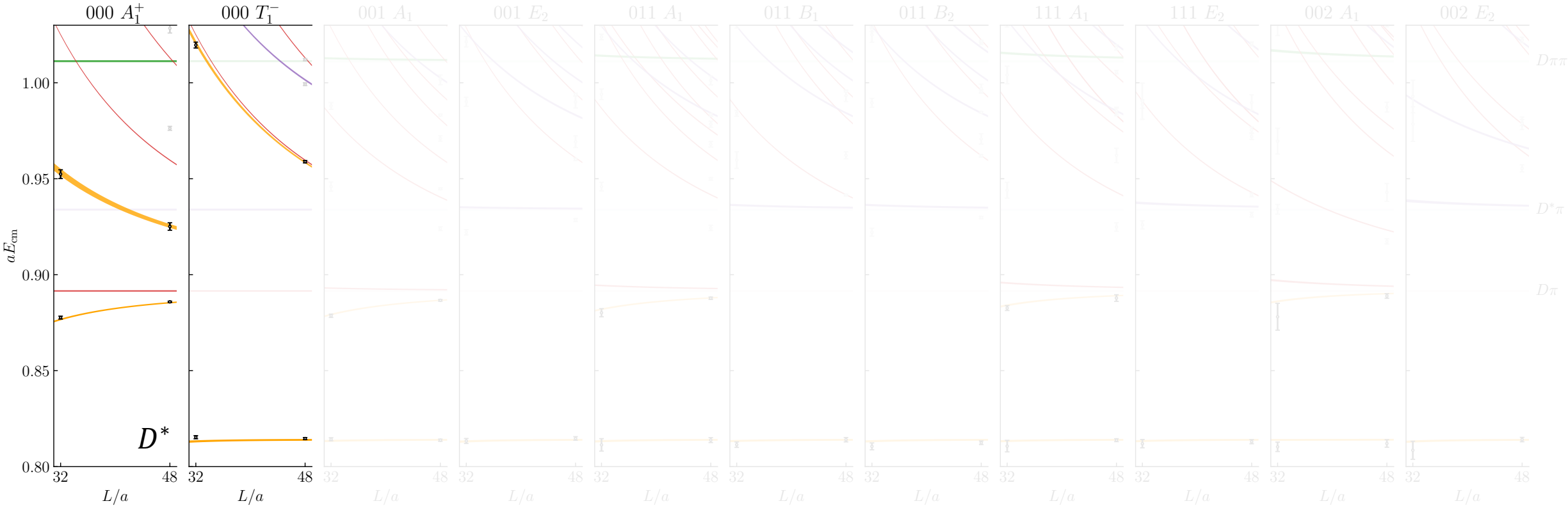
- Operators from OpTion
- Two-body spectra

Eg: Finite-volume spectra at $m_\pi \approx 305$ MeV



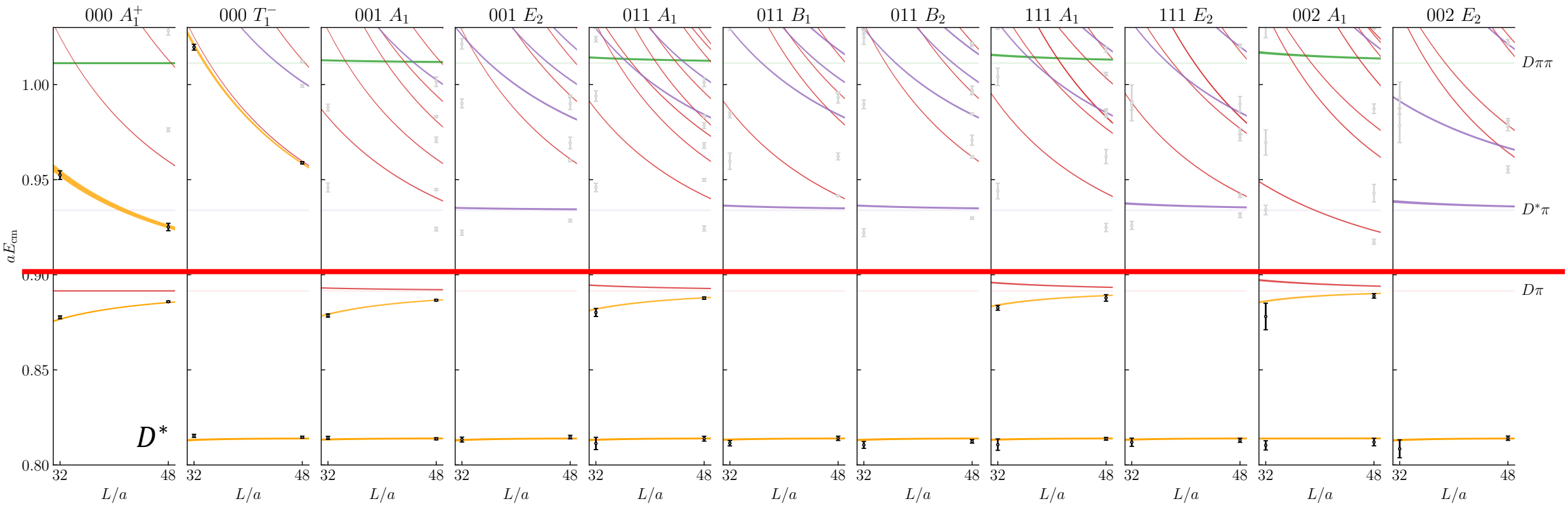
- Strong **attraction** in the S wave

Eg: Finite-volume spectra at $m_\pi \approx 305$ MeV



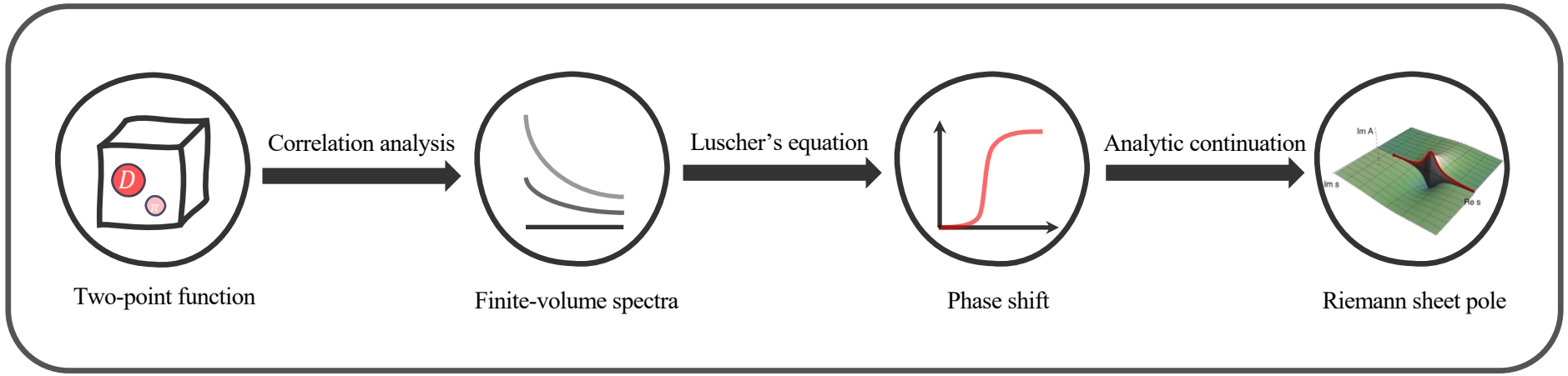
- Strong **attraction** in the S wave
- Negligible interaction in the P wave

Eg: Finite-volume spectra at $m_\pi \approx 305$ MeV



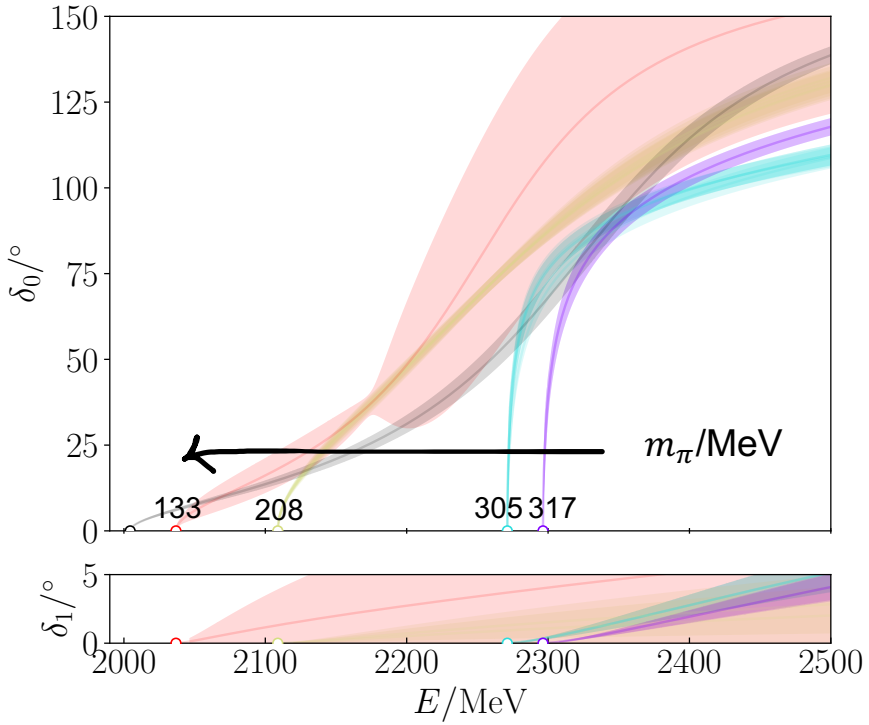
- Strong **attraction** in the S wave
- Negligible interaction in the P wave
- Many moving frames to constrain strongly on the amplitude
- Restricted below inelastic thresholds

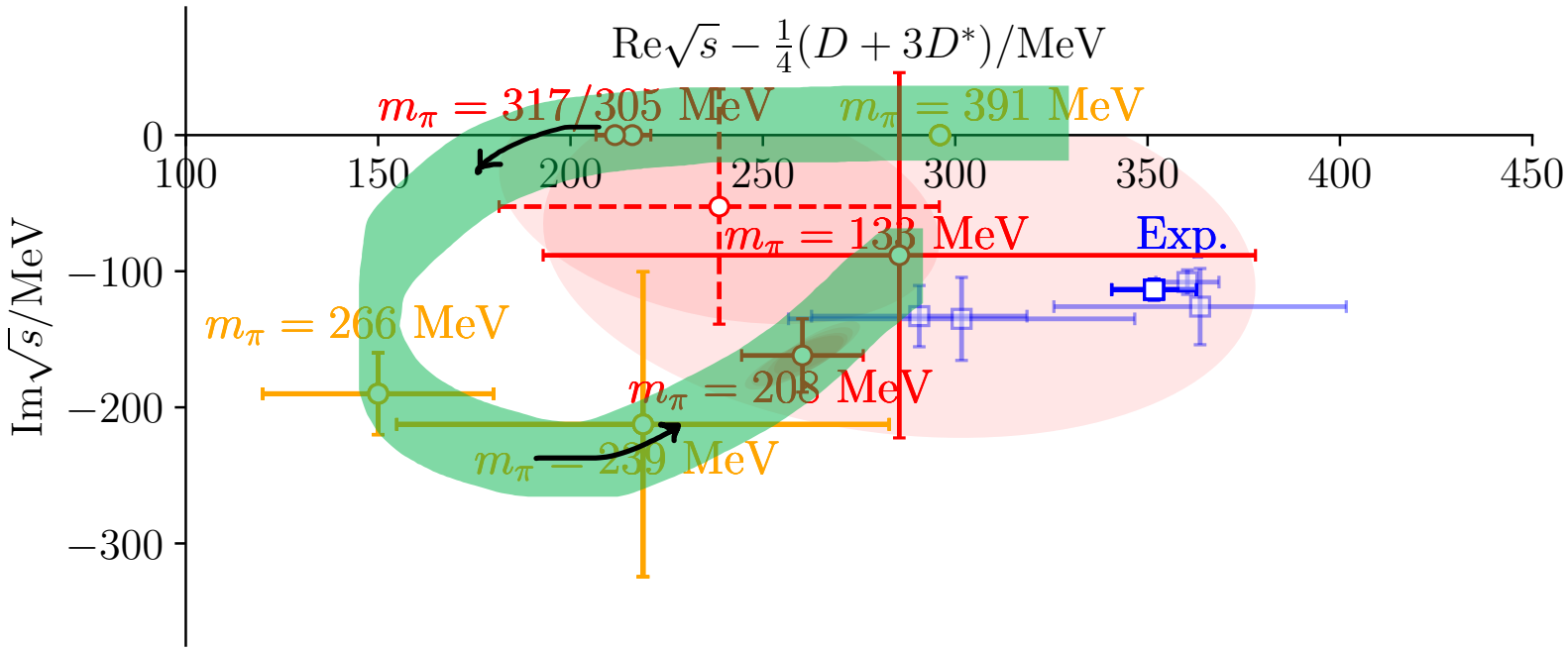
Methodology



- Operators from OpTion
- Two-body spectra
- Apply the quantization conditions
- Analytic continuate and search for poles

Phase shift & poles



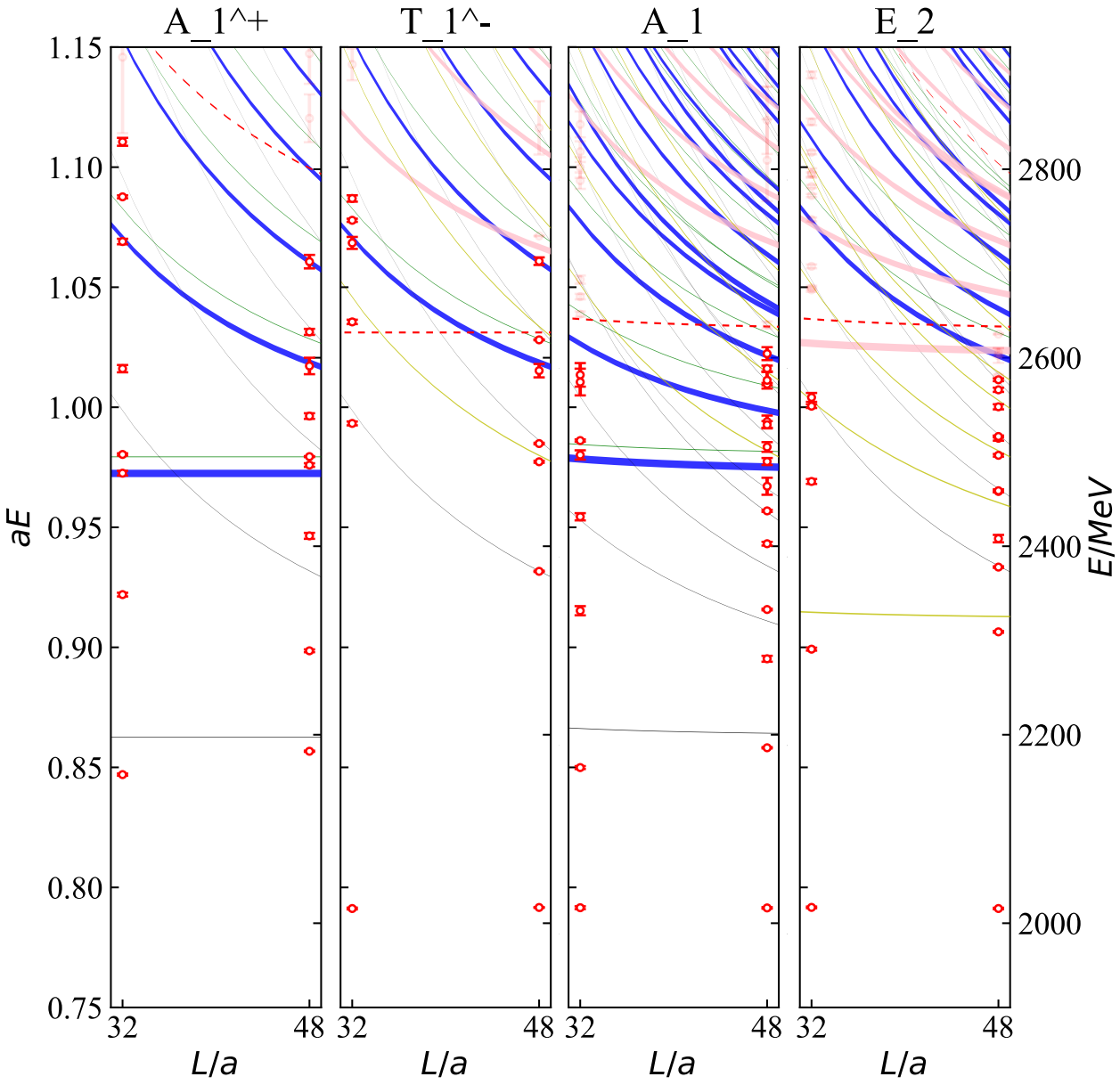


- Only **one pole** within the single-channel scattering picture
- An clear trend for the motion of the D_0^* pole is identified

bound state → virtual state → resonance

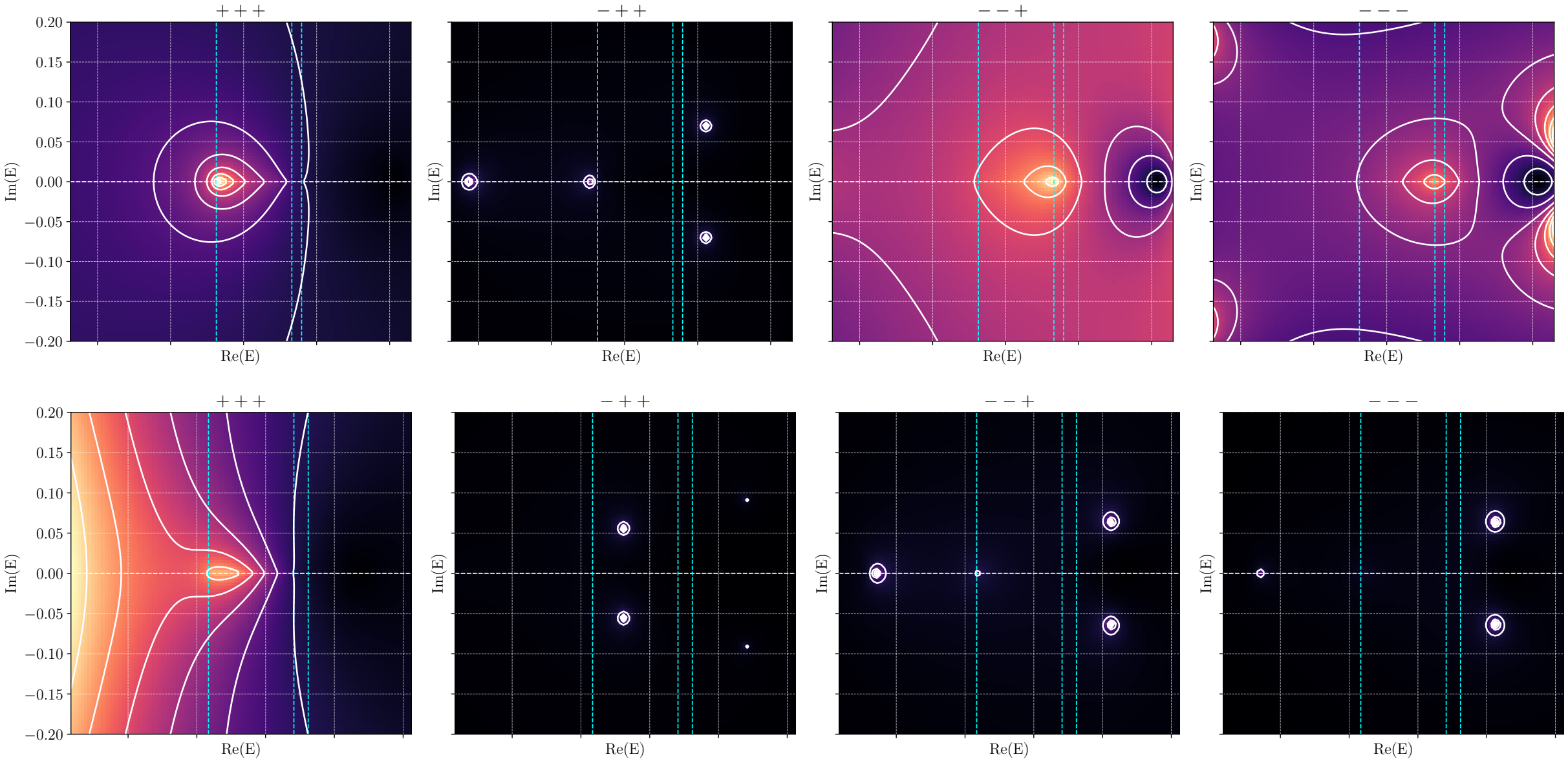
Haobo Yan (燕浩波), Chuan Liu, Liuming Liu, Yu Meng, Hanyang Xing, **Phys. Rev. D** 111 (2025) 014503

$D\pi - D\eta - D_s\bar{K} - D^*\pi$ coupled channel scattering

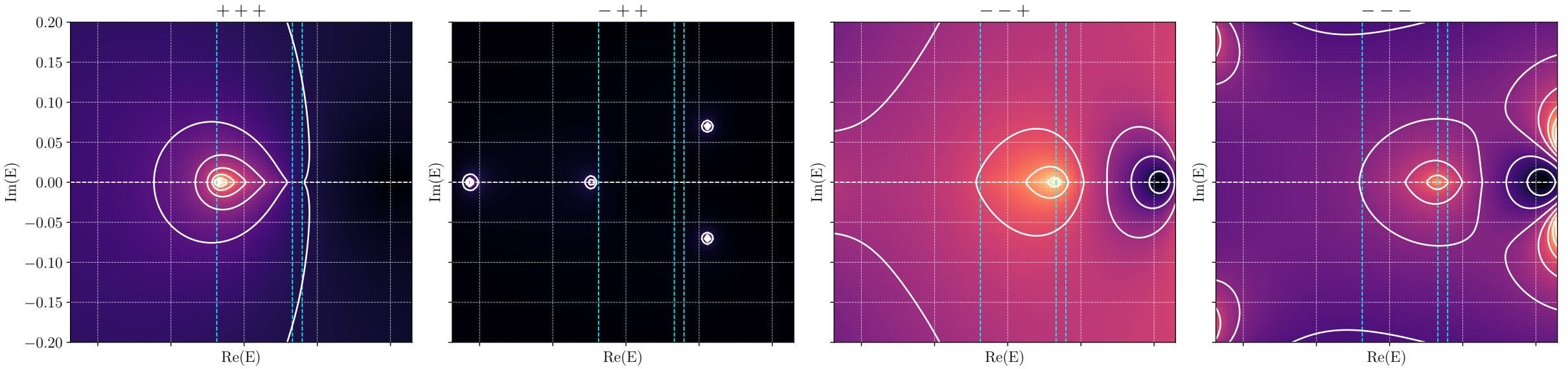


- $\mathcal{O}(1000)$ energy levels
- All irreps are useful
- $D\pi - D\eta - D_s\bar{K}$ S and D wave
- $D^*\pi$ ${}^3S_1$, ${}^3P_0$, ${}^3P_1$, ${}^3P_2$, ${}^3D_1$, ${}^3D_2$, ${}^3D_3$ wave

$D\pi - D\eta - D_s\bar{K} - D^*\pi$ coupled channel scattering – pole



$D\pi - D\eta - D_s\bar{K} - D^*\pi$ coupled channel scattering – pole



- **We do find 'two poles'**
- Combined picture
  - $M_\pi = 391$ MeV: bound + virtual + resonance
  - $M_\pi = 305$ MeV : virtual + virtual + resonance
  - $M_\pi = 208$ MeV : resonance + resonance

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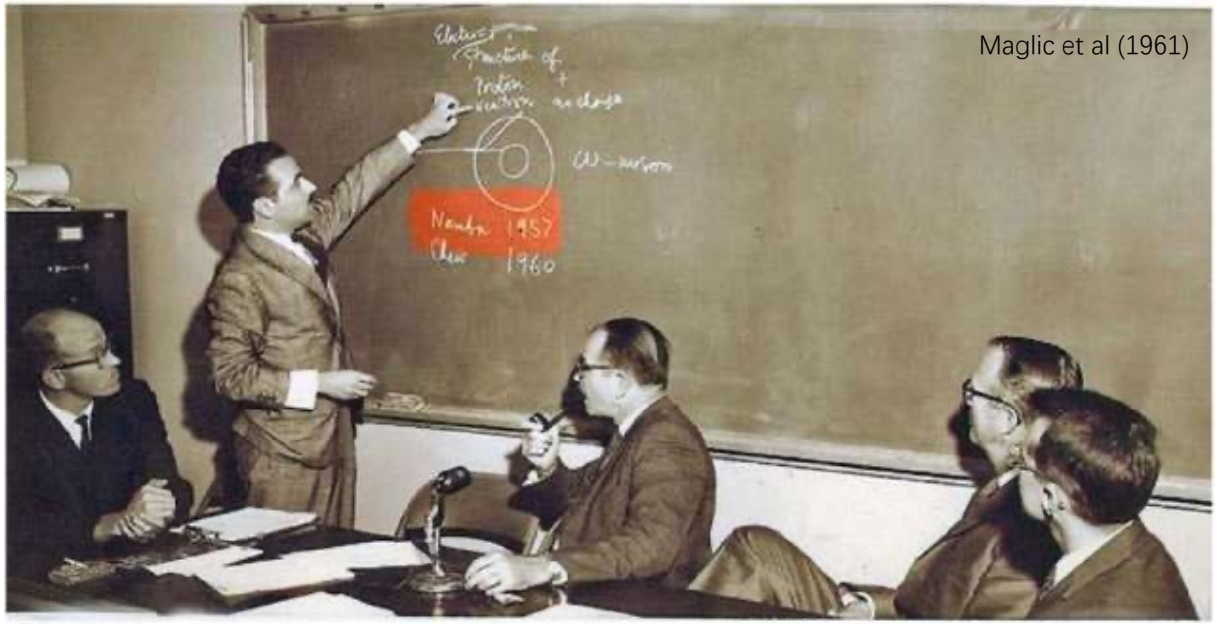
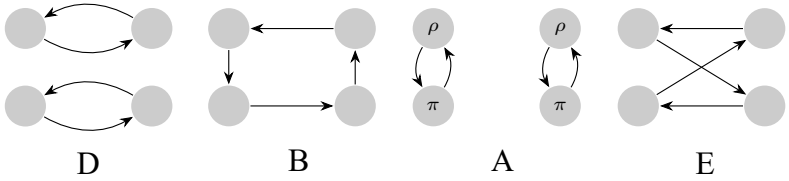
Towards the three-body problem

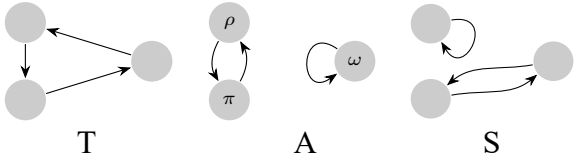


- Many nonperturbative puzzles are three-body problems
- Difficult: lattice and formalism
- In heavy flavor: $T_{cc}(3875) \to DD^* \to DD\pi$
- Could be studied from DD^* scattering at the unphysical world

Contractions



$\sim\mathcal{O}(10)$



$\sim\mathcal{O}(1)$

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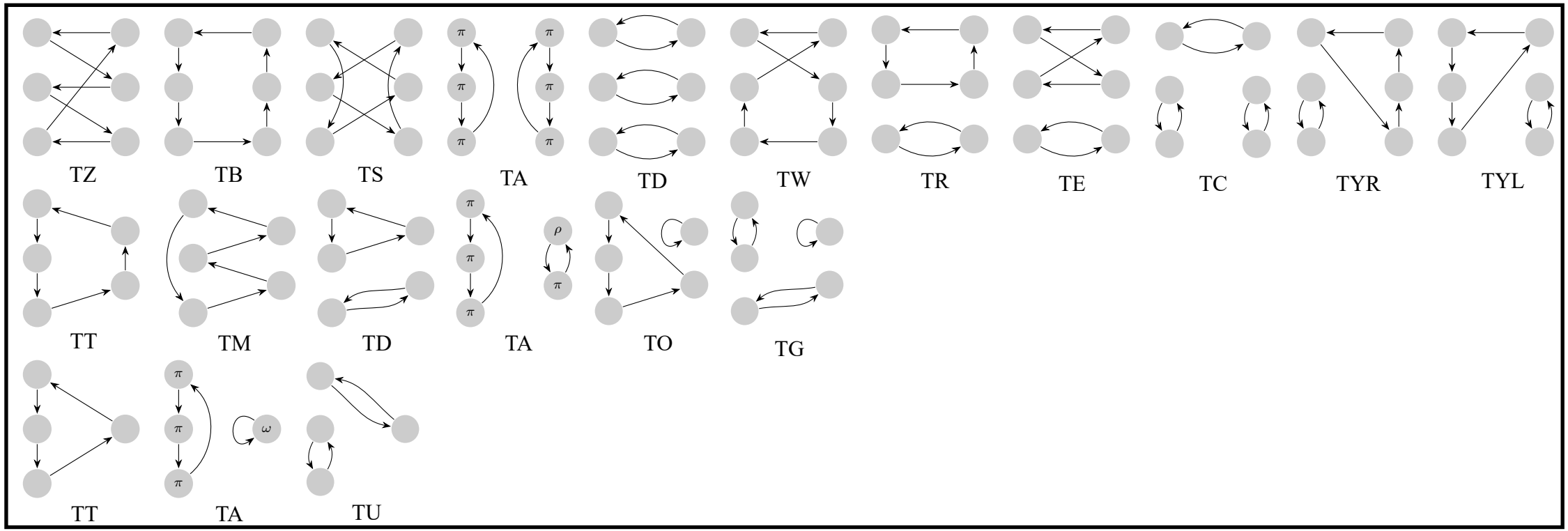
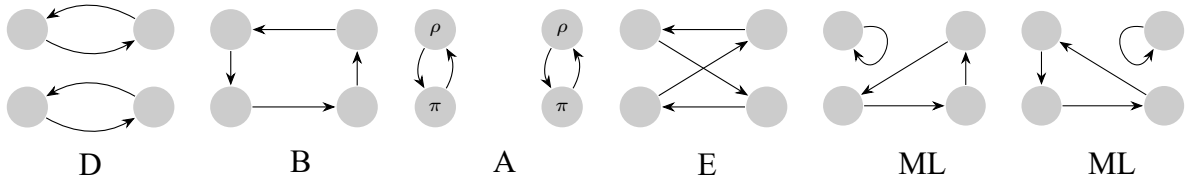
Contractions



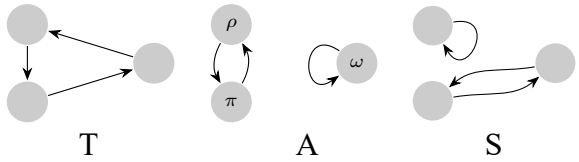
$\sim\mathcal{O}(300)$

$\sim\mathcal{O}(50)$

$\sim\mathcal{O}(10)$



$\sim\mathcal{O}(10)$



$\sim\mathcal{O}(1)$

$\sim\mathcal{O}(1)$

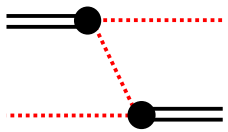
- A code is written to write code
- Diagrams can be assembled from smaller blocks

Quantization condition

State-of-the-art formalisms

- FVU [Mai and Döring, 2017]
- RFT [Hansen and Sharpe, 2014]
- NREFT [Hammer, Pang, and Rusetsky, 2017]

- Using FVU (**F**inite-**V**olume **U**nitarity)



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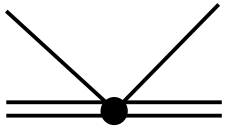

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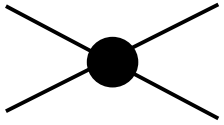
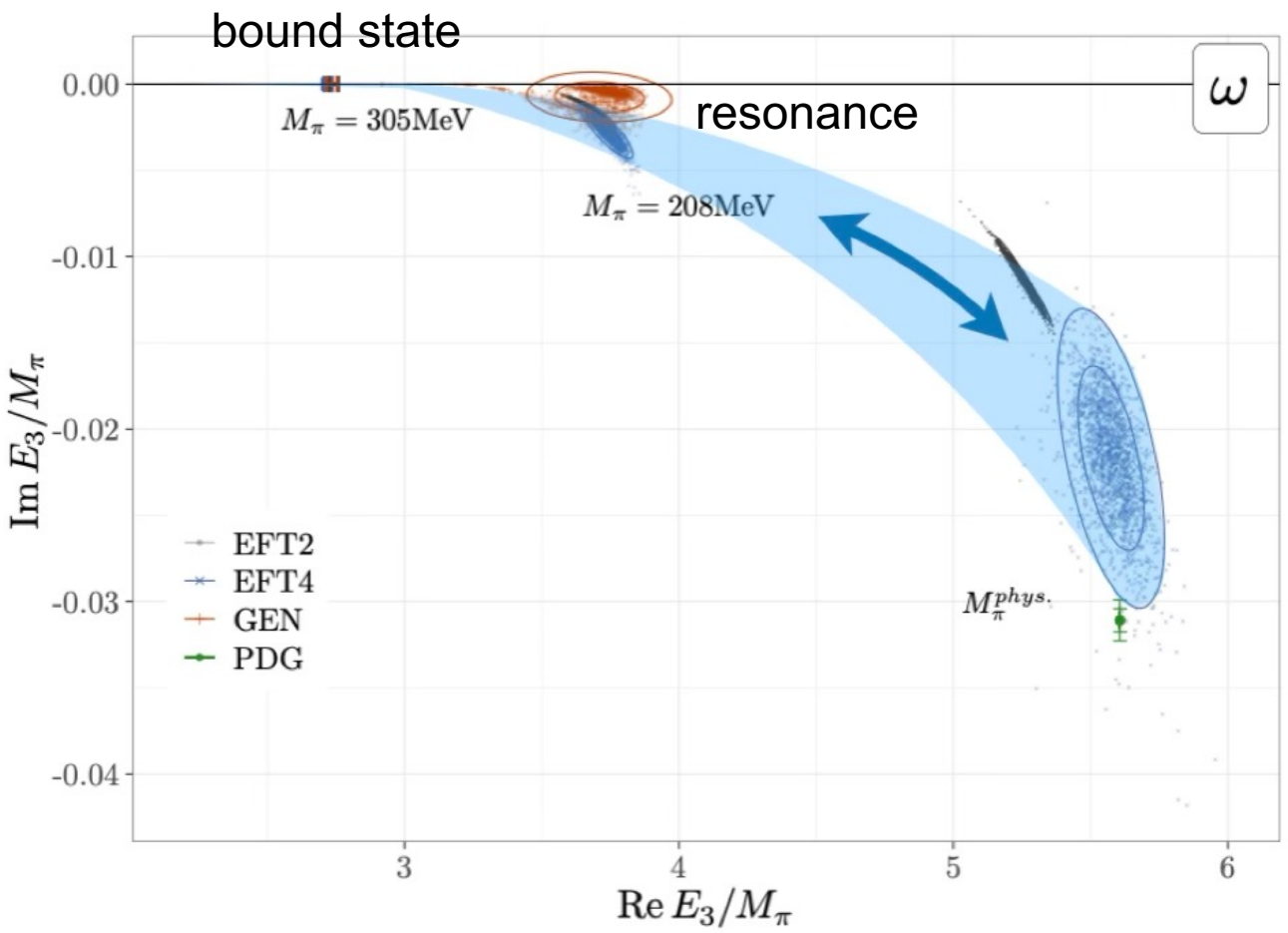


$\tilde{K}^{-1}$

- Two-body is input to three-body

For review of the EFT used, see Meißner (1988).

History: $\pi\pi\pi \to \omega(782)$



FVU [Mai and Döring, 2017]

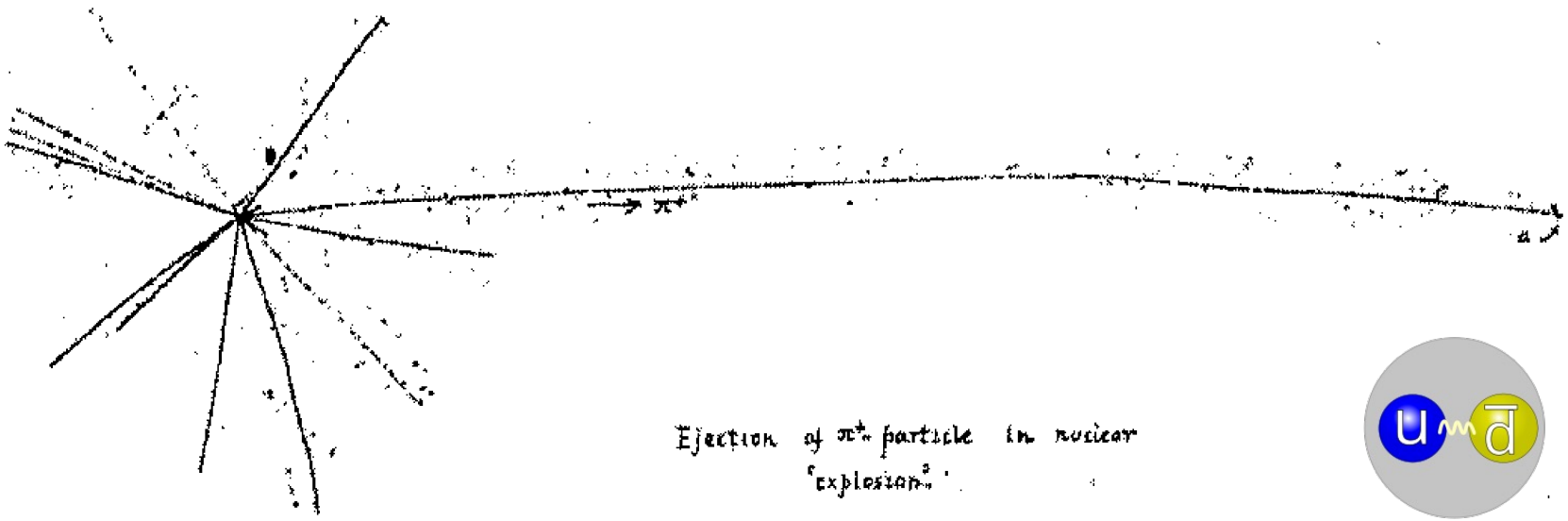
- Three-body quantization condition: FVU
- Solving an integral equation
- Extrapolate to the physical pion mass
- Different parametrizations are consistent

$$M_{\omega(782)} = (778.0 \pm 11.2) - i(3.0 \pm 5)\ \text{MeV}$$

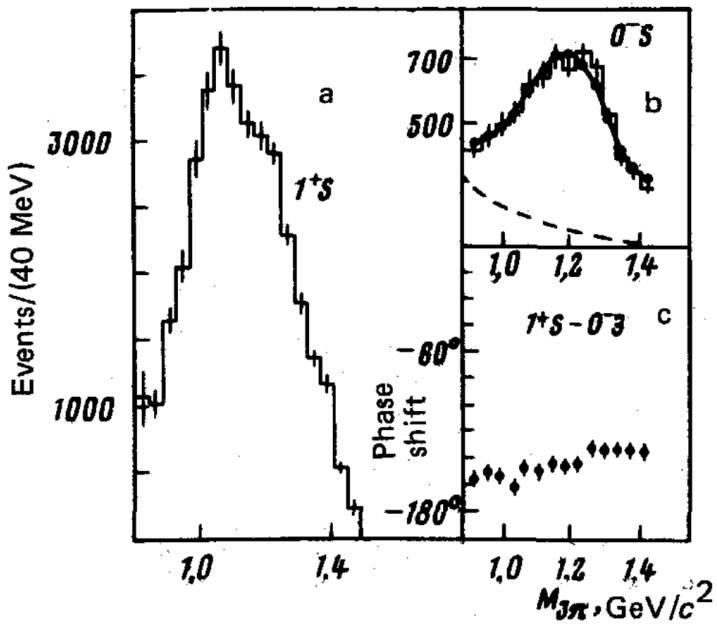
Haobo Yan (燕浩波), Maxim Mai, Marco Garofalo, Ulf-G. Meißner, Chuan Liu, Liuming Liu, Carsten Urbach, **Phys. Rev. Lett** 133 (2024) 211906, **Editors' Suggestion**

Excitation of pion

Bellini et al (1961)



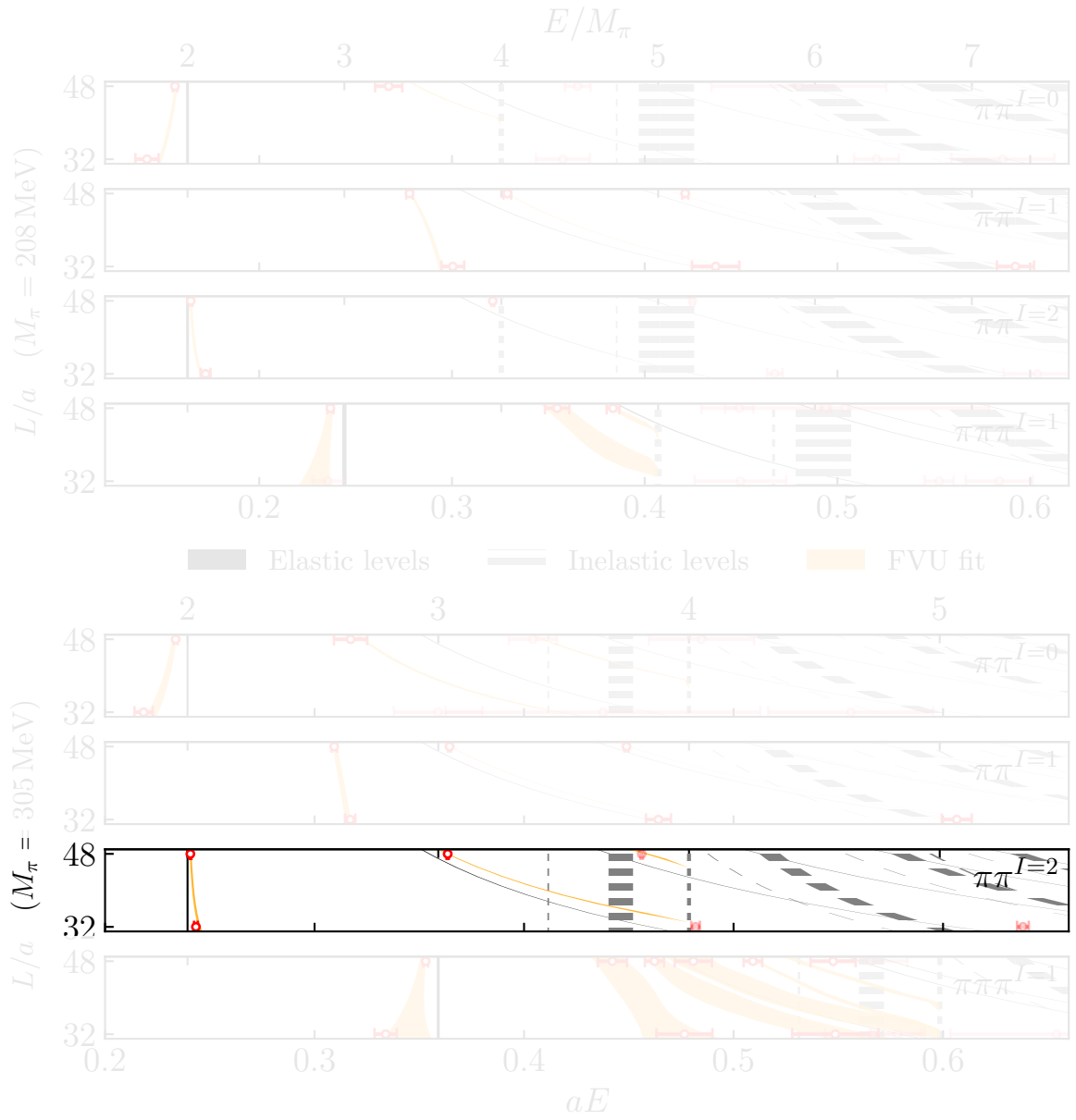
Discovery of π



Evidence of $\pi(1300)$

- π: the lightest hadron, Goldstone boson。The existence of $\pi(1300)$ is not solid, and 10x heavier
- A test of the soft-pion theorem: $m_{\pi(1300)} f_{\pi(1300)} \to 0$ in the chiral limit
- beyond post-prediction
- Possible hybrid content
- Decay to $\pi\pi\pi$ without centrifugal barrier: a second step towards Roper → $N\pi\pi$

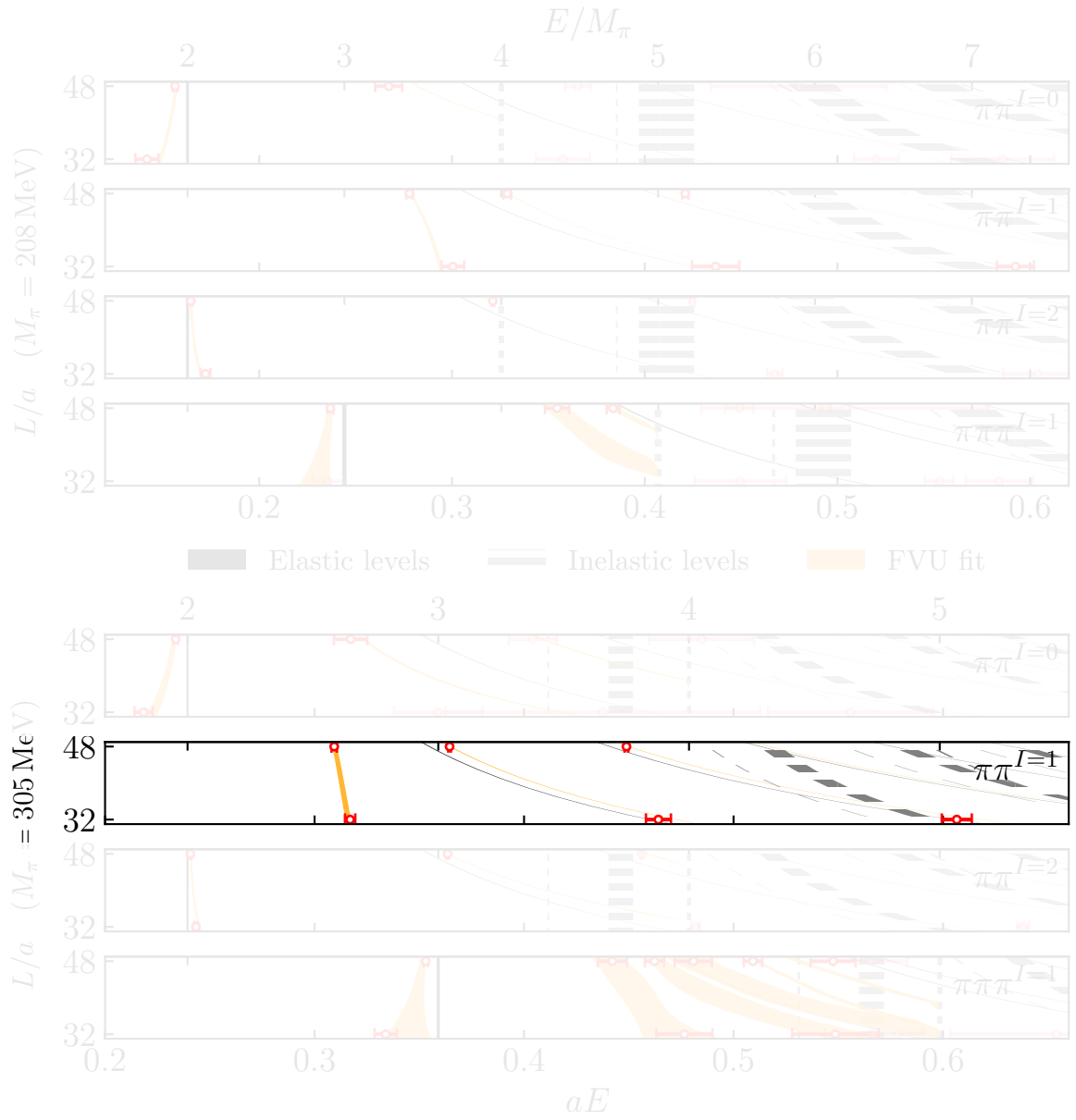
Finite-volume spectra and fit



- $I = 0, 1, 2$ $\pi\pi$ levels as expected
- Aggregation of additional states in $\pi\pi\pi$

- Indication of a resonance in 0^-

Finite-volume spectra and fit



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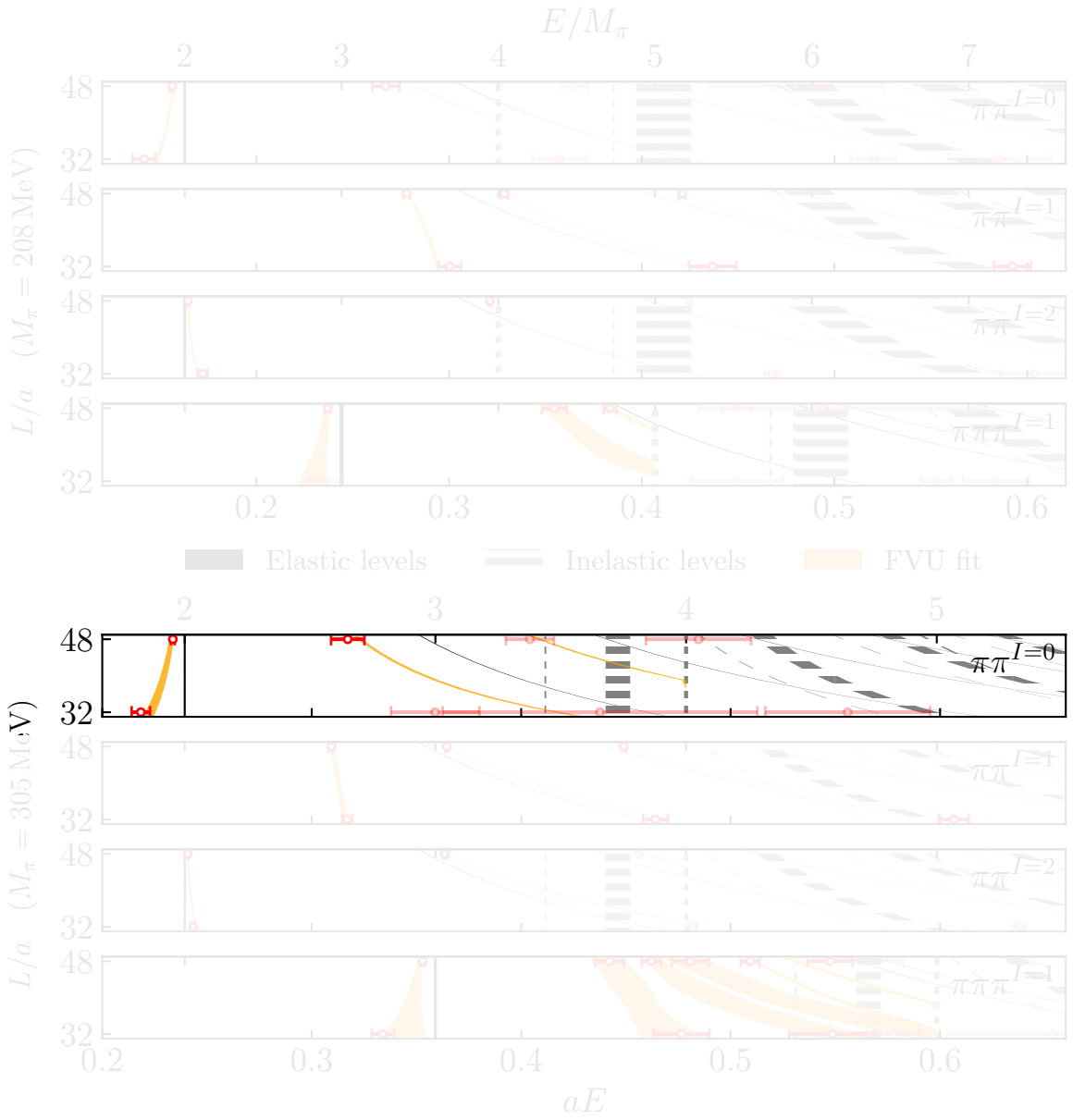

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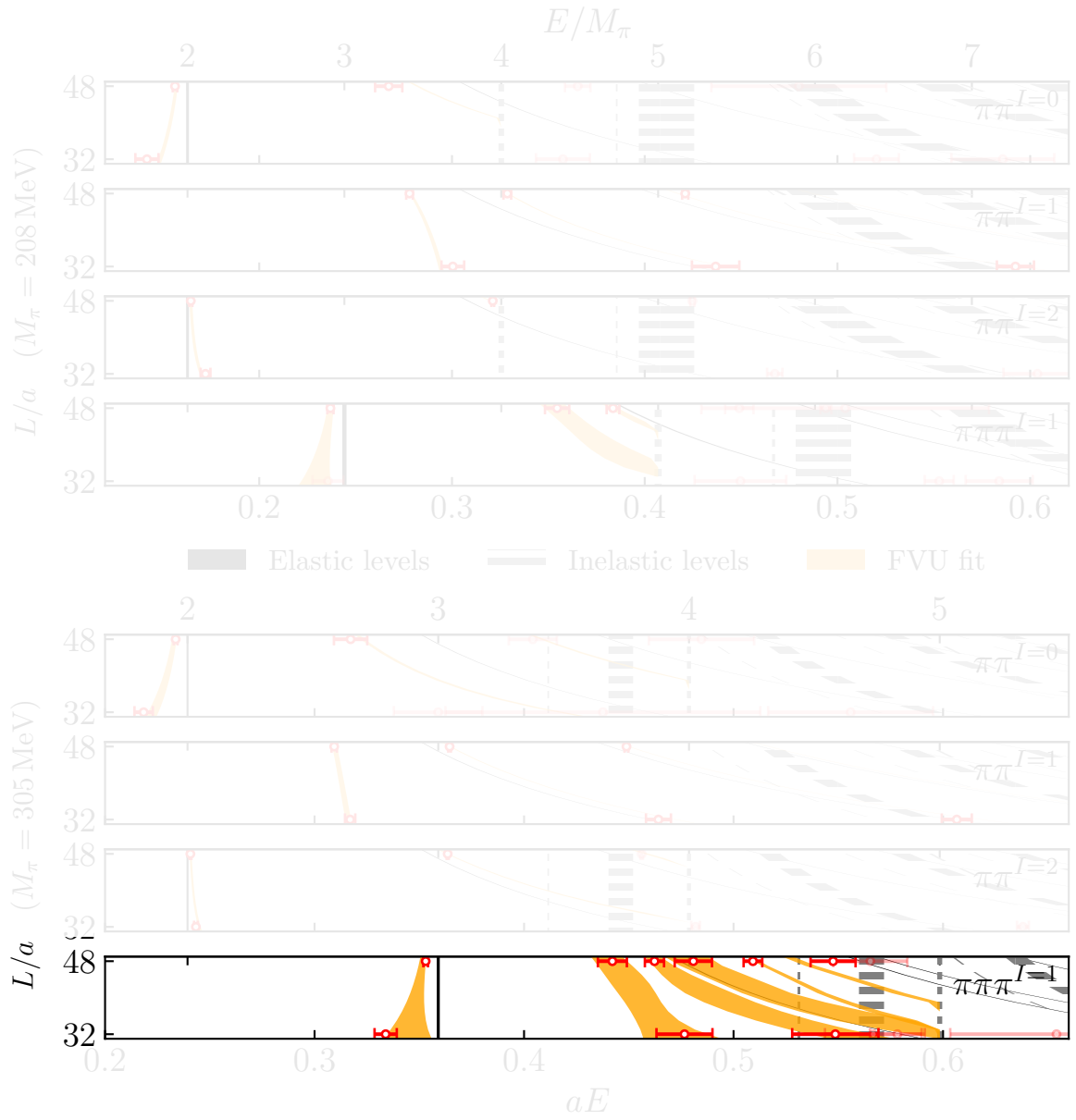

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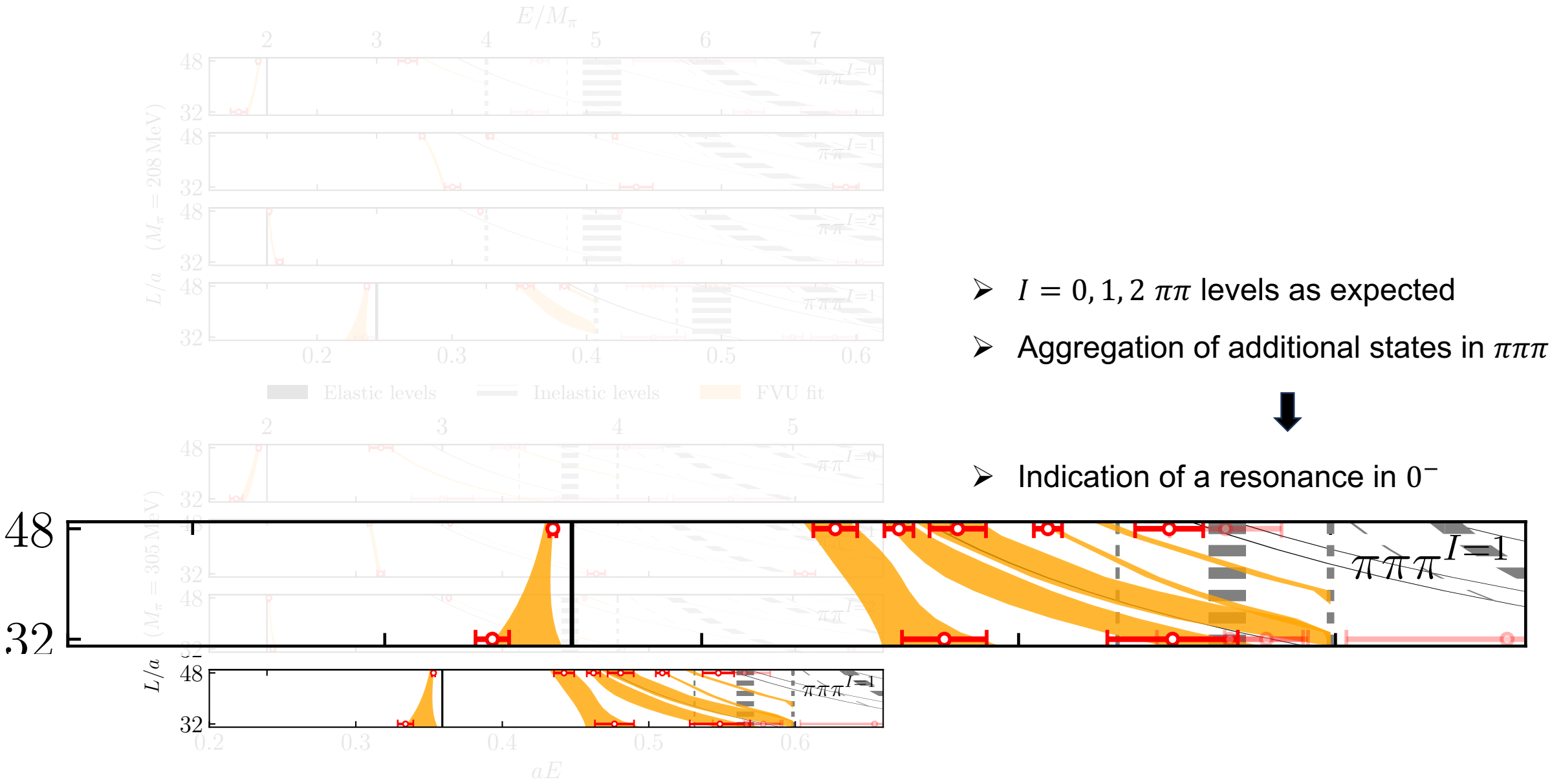

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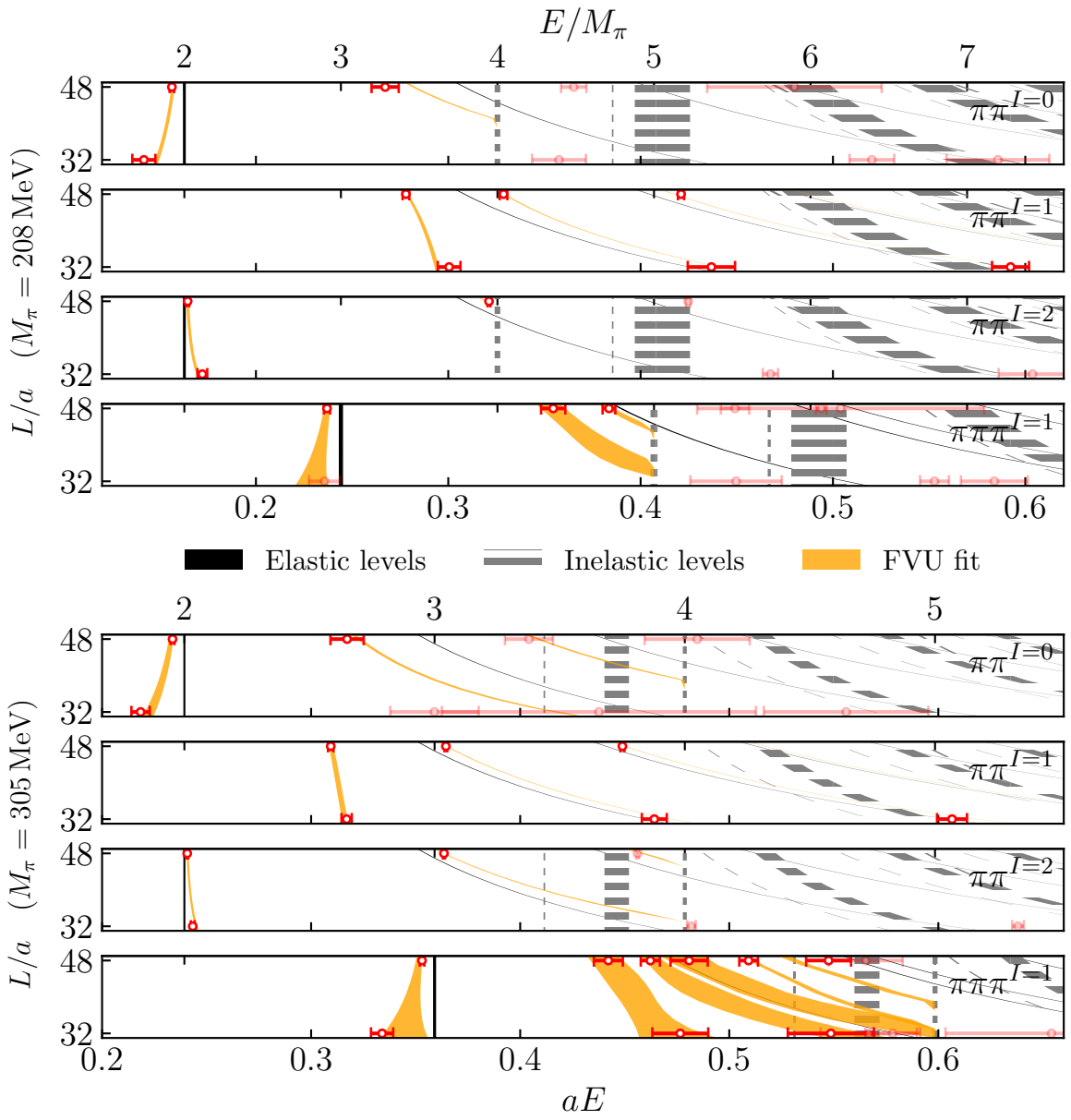

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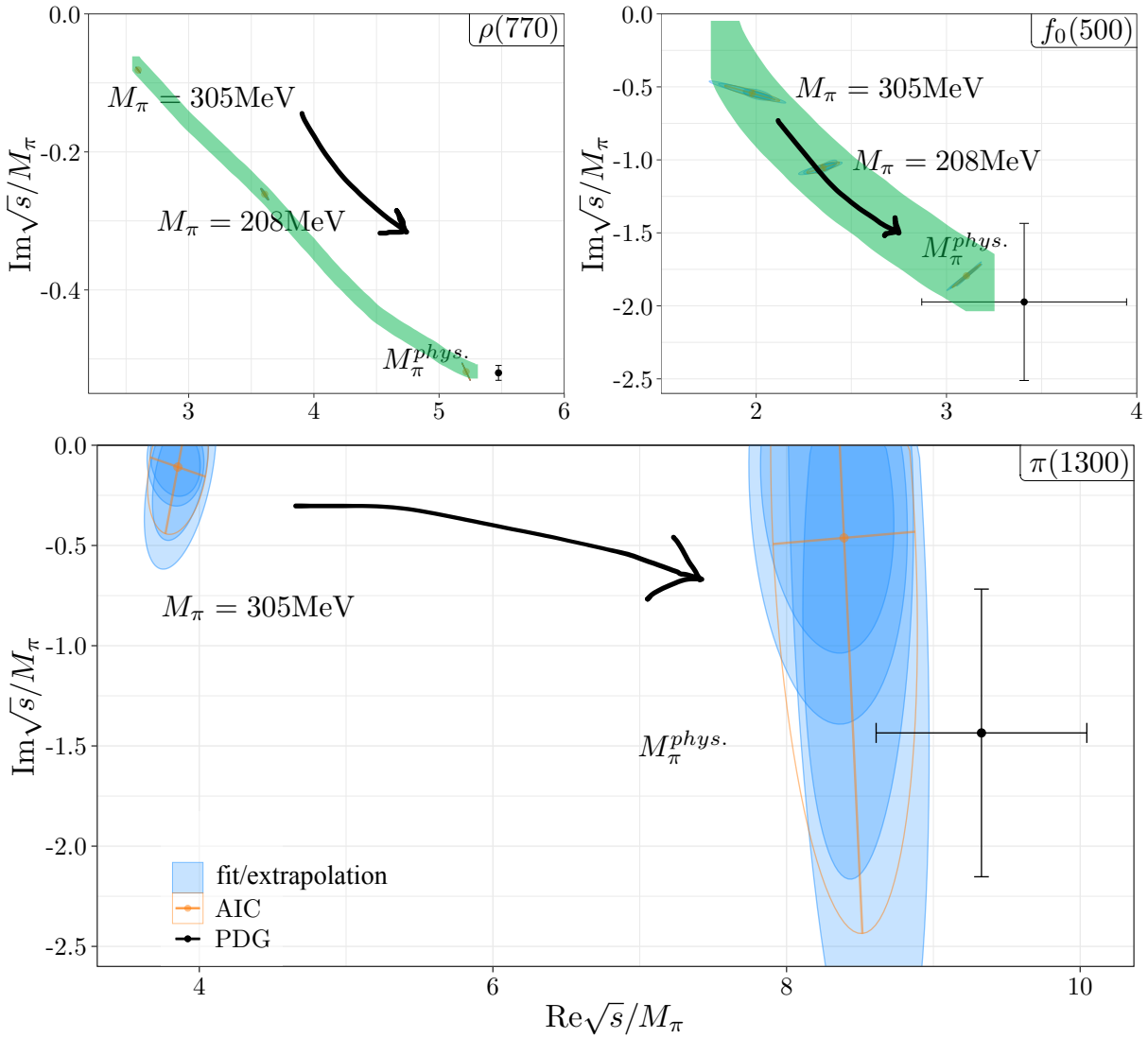
⬇

- Indication of a resonance in 0^-

Pole positions



- Extrapolation to the physical pion mass

$$M_{\pi(1300)} = (1169 \pm 46) - i(62 \pm 169)\ \mathrm{MeV}$$

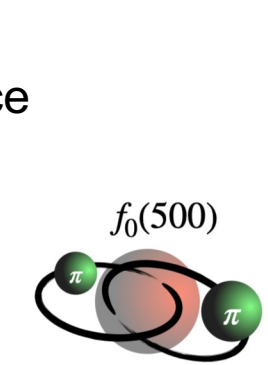
$$M_{\rho(770)} = (727 \pm 3) - i(72 \pm 1)\ \mathrm{MeV}$$

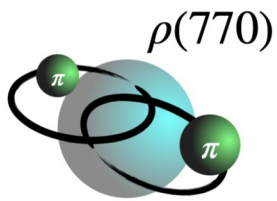
$$M_{f_0(500)} = (433 \pm 7) - i(250 \pm 7)\ \mathrm{MeV}$$

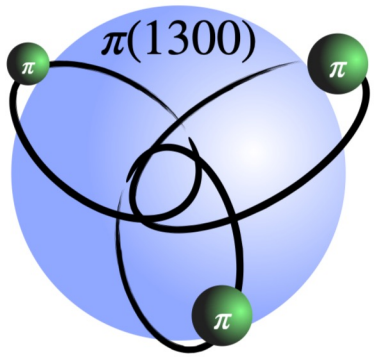
Haobo Yan (燕浩波), Maxim Mai, Marco Garofalo, Yuchuan Feng, Micheal Döring, Chuan Liu, Liuming Liu, Ulf-G. Meißner, Carsten Urbach, **Phys. Rev. Lett** accepted

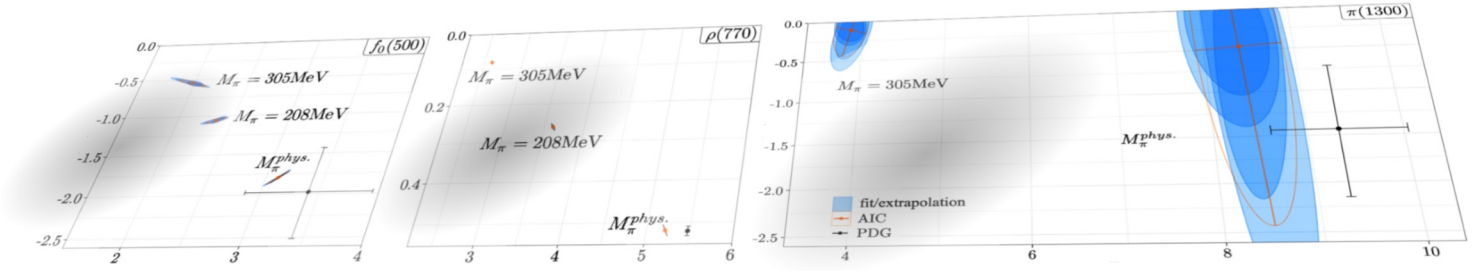
Summary

1. $D_0^*(2300)$: bound state → virtual state → resonance
2. $D_0^*(2300)$ two-pole structure is under test
3. Three-body problem: $\pi\pi\pi \to \omega(782)$
4. Three-body problem: $\pi\pi\pi \to \pi(1300)$
5. OpTion for operator construction
6. More coming up very soon! Roper? Tcc?









Thank you!